Universal gate design with beamsplitters and phase-shifters
based on 1906.06748

Saygin, Kondratyev, Dyakonov, Mironov, Straupe, Kulik

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Crete, OAC, August 28, 2019
Quantum computing

\[ \text{Quantum algorithm} = \text{Unitary matrix} \]

Experimental implementations
Quantum computing

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Quantum code is a matrix element of a Unitary matrix

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The Unitary matrix is constructed out of Quantum gates.

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trapped cold atoms or ions
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Linear optics
Universal programmable linear-optical interferometers
Universal programmable linear-optical interferometers

Versatility and Universality
Universal programmable linear-optical interferometers

Versatility and Universality

(ability to set arbitrary unitary transformation of the input optical modes by tuning the phaseshifting elements inside the interferometer)
Universal programmable linear-optical interferometers

Versatility and Universality

unitary matrix factorization theorem
Universal programmable linear-optical interferometers

Versatility and Universality

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Qubits realized by processing states of different modes of light through linear elements e.g. mirrors, beam splitters and phase shifters
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balanced beamsplitter
Universal programmable linear-optical interferometers

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Usual approach is vulnerable to manufacturing imperfections inevitable in any realistic experimental implementation, and the larger the circuit size grows, the more strict the tolerances become.
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Usually approach is vulnerable to manufacturing imperfections inevitable in any realistic experimental implementation, and the larger the circuit size grows, the more strict the tolerances become.

The overall fidelity may be improved for some transformations by application of an optimization algorithm but the overall universality feature of the interferometer will be inevitably lost.

We demonstrate a new methodology for the design of the high-dimensional mode transformations, which overcomes this problem.
Beamsplitter

BS(θ,φ)

Tritter

Quarter

Six-mode CNOT gate

Beamsplitter, R = 1/3

Beamsplitter, R = 1/2
2x2 Unitary Matrix: 2 optical modes $SU(2)$
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Phase shifter: $P_1 = \begin{pmatrix} e^{\phi_1} & 0 \\ 0 & e^{-\phi_1} \end{pmatrix} = e^{\phi_1 \sigma_3}$
2x2 Unitary Matrix: 2 optical modes $SU(2)$

Phase shifter: $P_1 = \begin{pmatrix} e^{\phi_1} & 0 \\ 0 & e^{-\phi_1} \end{pmatrix} = e^{\phi_1 \sigma_3}$

Beamsplitter: $H = \frac{1}{\sqrt{-2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = (Hadamard) = (Fourier)$
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Euler angles
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- Factorization Theorem
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Euler angles

Factorization Theorem

Factorization Theorem breaks if $H$ is slightly changed.
3x3 Unitary Matrix: 3 optical modes $SU(3)$
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\begin{pmatrix}
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\end{pmatrix}
= 
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\begin{pmatrix}
c_{11} & 0 & c_{13} \\
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Can be generalized to $SU(N)$.
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not fault tolerant to fabrication errors
- 3x3 Unitary Matrix: 3 optical modes
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extremely robust even to quite large fabrication errors
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- extremely robust even to quite large fabrication errors
- the mode mixing elements may be quite arbitrary mode-coupling elements
- the universality is not proven rigorously

Numerical experiments show strong evidence that this architecture is capable of realizing large-scale arbitrary unitary transformations with high fidelity can be generalized to $SU(N)$, number of matrices grows linearly.
3x3 Unitary Matrix: 3 optical modes

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- can be generalized to $SU(N)$
- number of matrices grows linearly
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\[ U = P_{12} TP_{34} TP_{56} TP_{78} \]
SU(3) the new architecture

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Where \( P_{ij} = \begin{pmatrix} \exp(l\phi_i) & 0 & 0 \\ 0 & \exp(l\phi_j) & 0 \\ 0 & 0 & \exp(-l(\phi_i + \phi_j)) \end{pmatrix}, \]
\[ SU(3) \text{ the new architecture} \]

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\[ T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{pmatrix}, \quad w = \exp\left(\frac{2\pi i}{3}\right). \]
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8-dimensional Lie group ↔ 8 parameters (angles) in the design
- 8 dimensional Lie group $\leftrightarrow$ 8 parameters (angles) in the design
- straightforward generalization to higher rank groups:

$$U_N = \underbrace{P \ T \ P \ ... \ P \ T \ P}_{N + 1 \text{ layers}}$$

Generically the parameters are independent but there are special points (submanifolds).
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Each $P$ has $N - 1$ parameters (angles)
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let us consider zero $\phi_3$ and $\phi_4$: $P_{34} = 1$
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\[ T^2 \approx 1 \Rightarrow U = P_{12} P_{56} TP_{78} = P_{1+5} P_{2+6} TP_{78} = P_{1'} P_{2'} TP_{78} \]
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The resulting matrix is 4-dimensional, not 6 \( \rightarrow \) singular point?
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the investigation of the manifold structure shows that such points are regular, even flat.
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Same story in case of \( SU(2) \): Euler angles
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Even simpler example is \( \theta \) and \( \phi \) angles on the \( S^2 \)
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Still, we could not prove rigorously, that any Unitary matrix can be realized in this way
But we have very strong evidence by numerical simulations
Conclusions

New architecture for Universal programmable linear-optical interferometers
Robust and beautiful, easy to implement technically
Experimental universality in simulations
Lacks mathematical proof
Corollary: two matrices are enough to generate any SU(N)
The minimal set of universal gates contains 2 elements!
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Corollary: two matrices are enough to generate any $SU(N)$

The minimal set of universal gates contains 2 elements!
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![Figure: The plot of the matrix elements of the unitary matrix randomly generated from two random unitary matrices: Re $U_{11}$ and Im $U_{11}$ (left); Re $U_{11}$ and Re $U_{23}$ (right).]
THANK YOU FOR YOUR ATTENTION!