Universal gate design with beamsplitters and phase-shifters

based on 1906.06748

Saygin, Kondratyev, Dyakonov, Mironov, Straupe, Kulik

INR RAS, ITEP, MIPT, ITMP MSU

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Quantum computing

$$\boxed{\textit{Quantum algorithm}} = \boxed{\textit{Unitary matrix}}$$

Experimental implementations

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Linear optics

Universal programmable linear-optical interferometers

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Versatility and Universality

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(ability to set arbitrary unitary transformation of the input optical modes by tuning the phaseshifting elements inside the interferometer)

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unitary matrix factorization theorem

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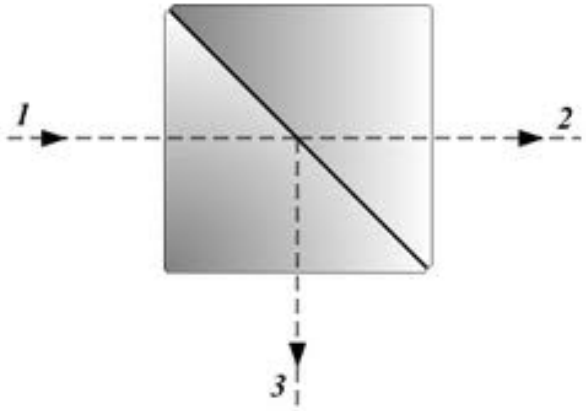

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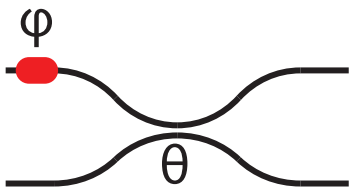
- The overall fidelity may be improved for some transformations by application of an optimization algorithm but the overall universality feature of the interferometer will be inevitably lost.
- We demonstrate a new methodology for the design of the high-dimensional mode transformations, which overcomes this problem.

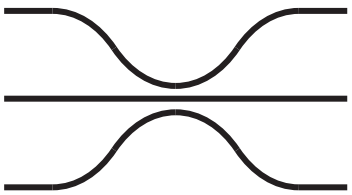
a

Beamsplitter

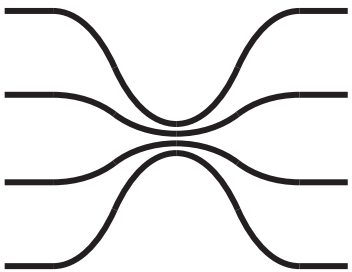


BS(θ,φ)

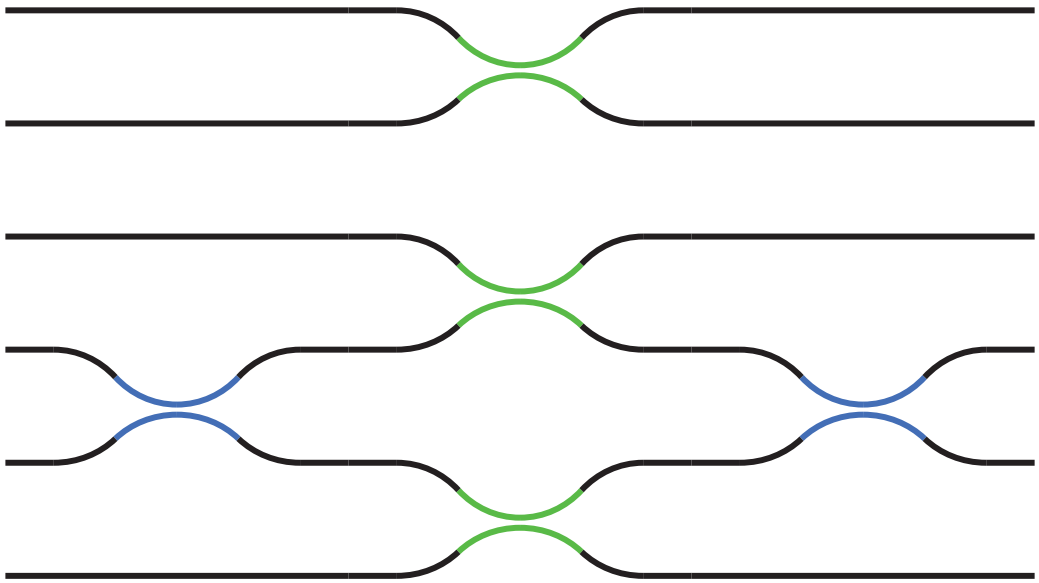
Tritter

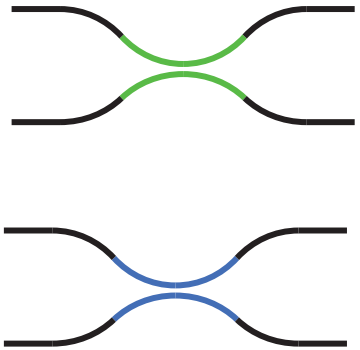


Quarter



Six-mode CNOT gate





Beamsplitter, R = 1/3

Beamsplitter, R = 1/2

b

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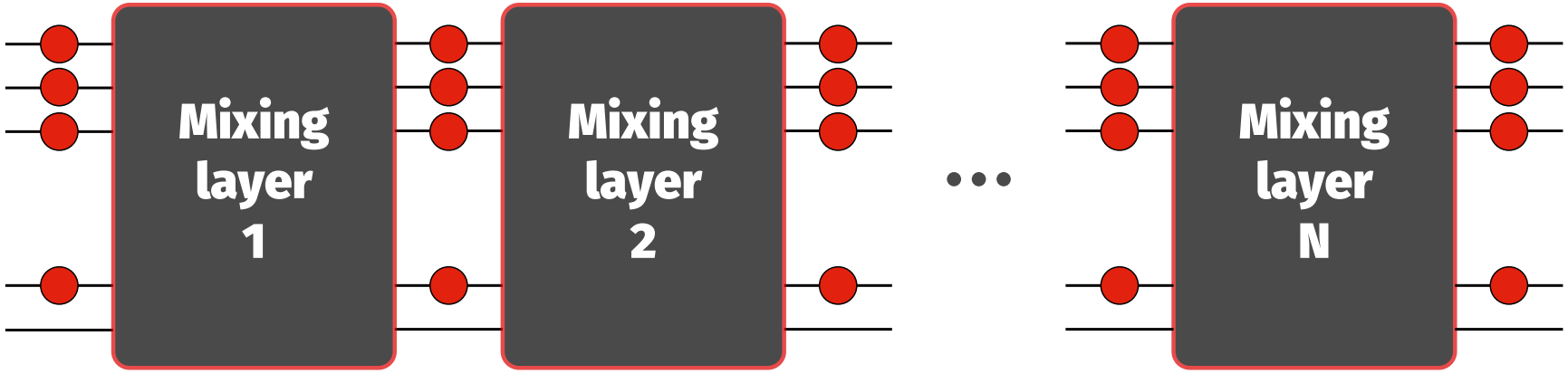

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  There are special points (submanifolds).

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Even simpler example is θ and ϕ angles on the S^2

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the investigation of the manifold structure shows that such points are regular, even flat.

metric is singular $\rightarrow$ Riemann tensor is regular

Same story in case of $SU(2)$: Euler angles

$$U = e^{\phi_1 \sigma_3} e^{\phi_2 \sigma_1} e^{\phi_3 \sigma_3} \xrightarrow[\phi_2 = 0]{} e^{\phi' \sigma_3}$$

Even simpler example is θ and ϕ angles on the S^2

$ds^2 = d\theta^2 + \sin^2 \theta d\phi^2$

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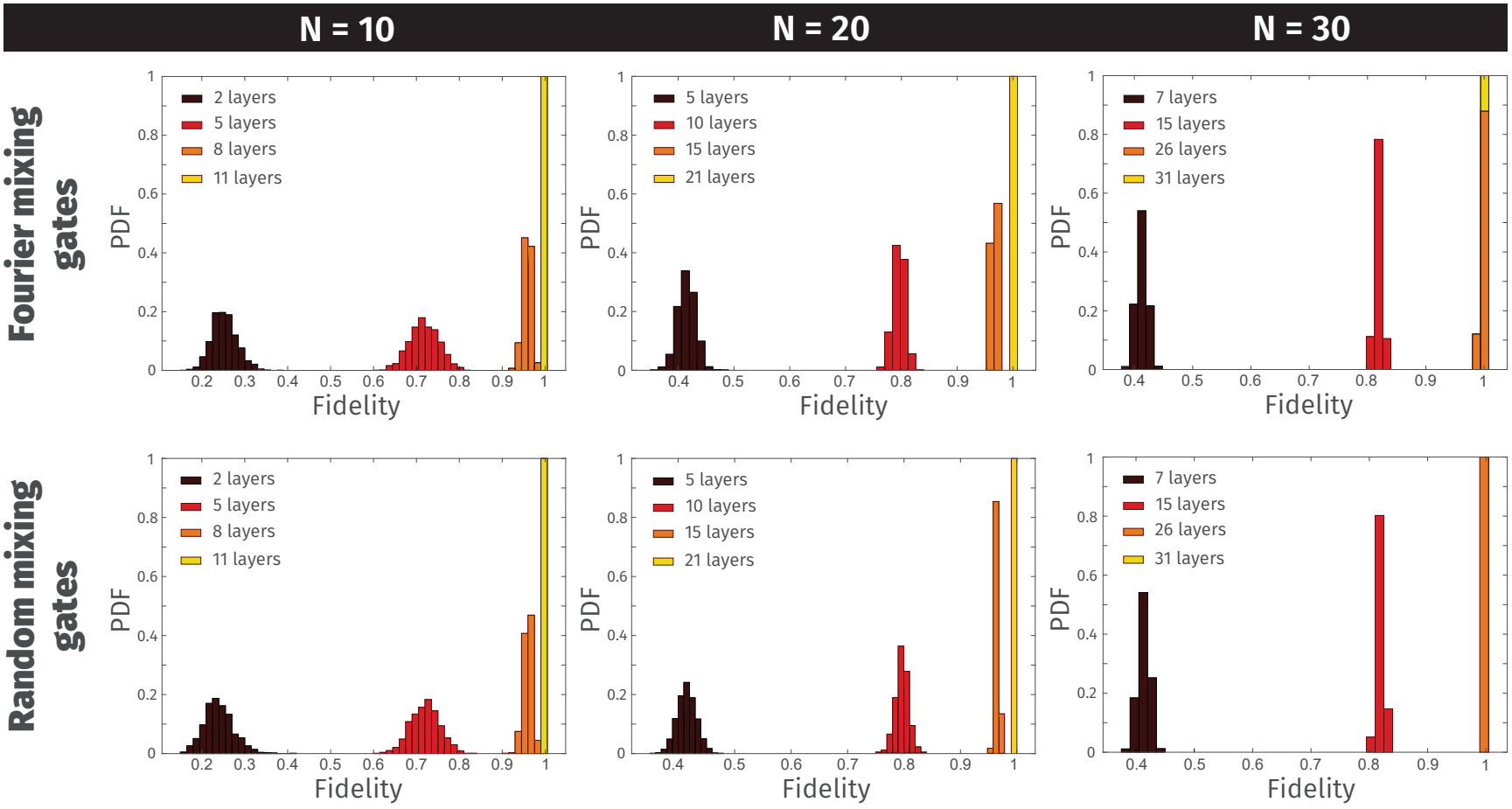
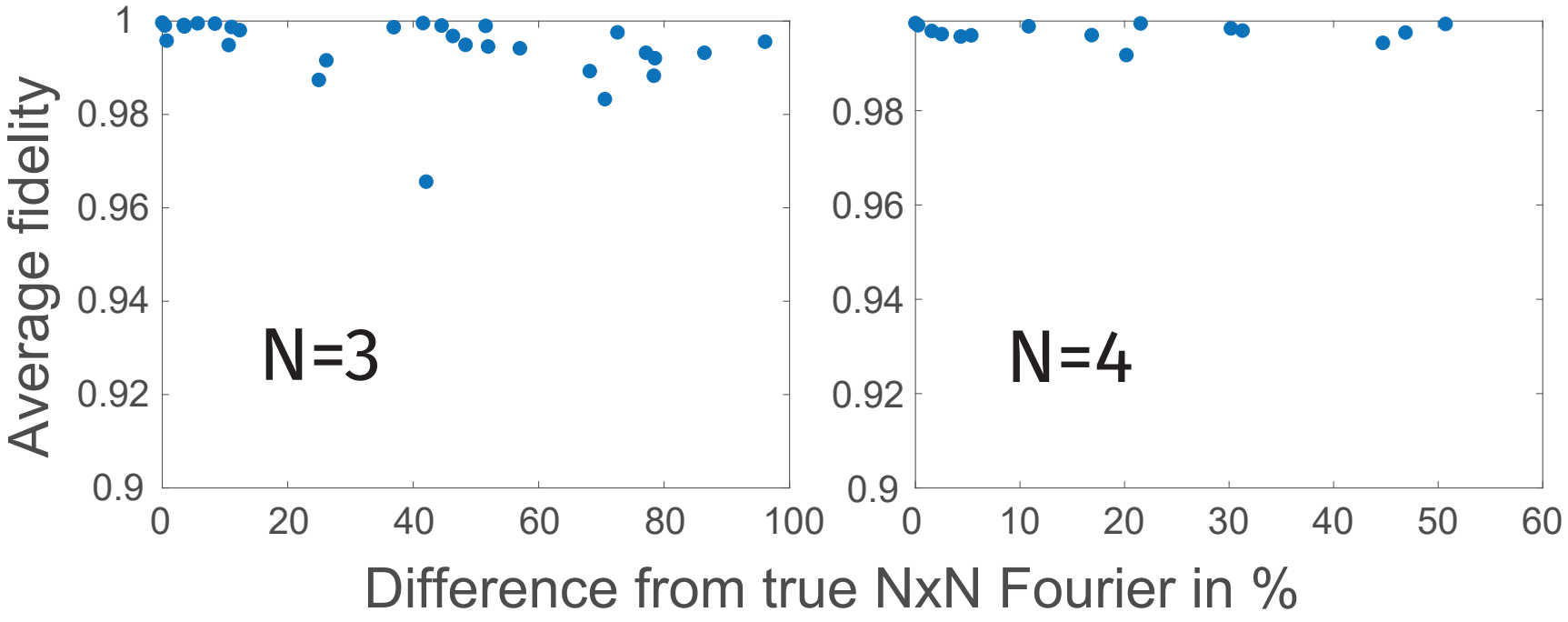
- But we have very strong evidence by numerical simulations


N = 10
N = 20
N = 30
Fourier mixing gates
Random mixing gates
2 layers
5 layers
8 layers
11 layers
5 layers
10 layers
15 layers
21 layers
7 layers
15 layers
26 layers
31 layers
PDF
Fidelity


Average fidelity
1
0.98
0.96
0.94
0.92
0.9
N=3
0
20
40
60
80
100
N=4
0
10
20
30
40
50
60
Difference from true NxN Fourier in %

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The minimal set of universal gates contains 2 elements!

The fact is that just <u>two</u> matrices are sufficient to approximate <u>any</u> unitary.

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Let us illustrate this phenomenon for $SU(4)$: let us take two randomly chosen unitary 4×4 matrices U_1 and U_2. Now one can check if they generate all the unitary matrices just via generation of the random sequences $\{n_i\}$ and then looking at the matrix elements of $U = U_1^{n_1} U_2^{n_2} U_1^{n_3} \ldots$. A typical plot obtained this way looks like:

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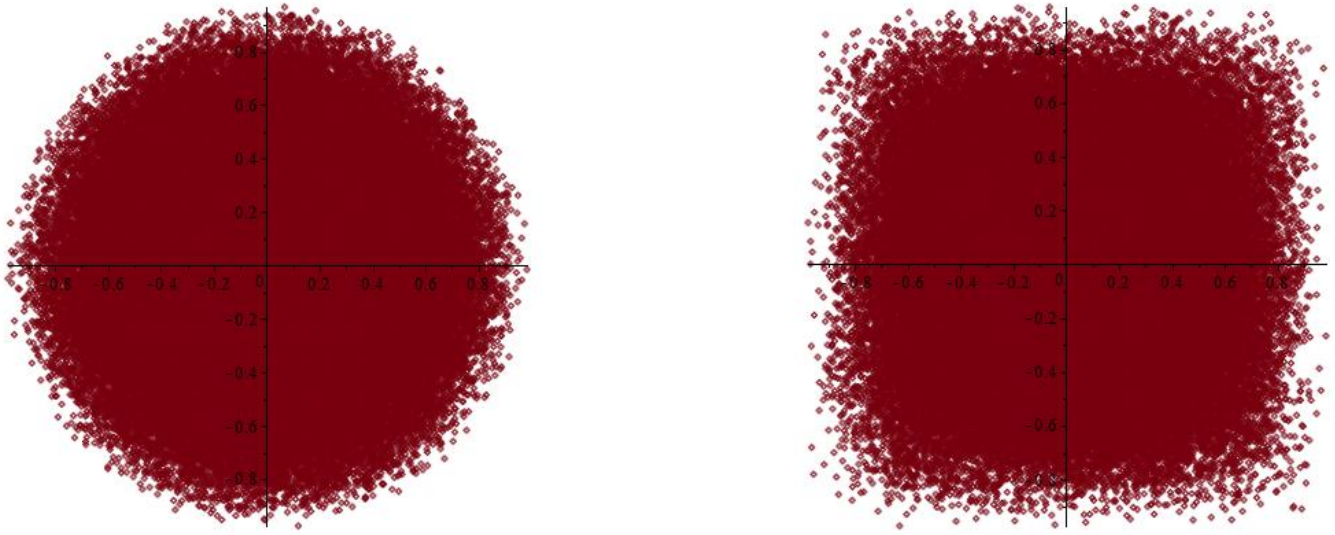

Figure: The plot of the matrix elements of the unitary matrix randomly generated from two random unitary matrices: Re U_{11} and Im U_{11} (left); Re U_{11} and Re U_{23} (right).

THANK YOU FOR YOUR ATTENTION!