International Conference on New Frontiers in Physics,
Crete, Greece, August 2019

Implicit assumptions in the proof of the Bell's inequality

David Oaknin
Rafael Advanced Defense Systems, Haifa, Israel

International Conference on New Frontiers in Physics, Crete, Greece, August 2019

The Bell's theorem revisited: a subtle, though crucial, assumption has gone unnoticed ... and it might not be justified !

David Oaknin

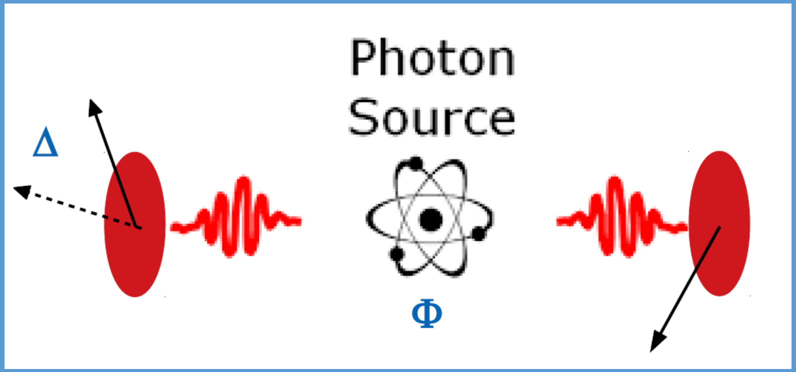
Rafael Advanced Defense Systems, Haifa, Israel

International Conference on New Frontiers in Physics,
Crete, Greece, August 2019

An explicitly local statistical model of hidden variables for the Bell's polarization states

David Oaknin
Rafael Advanced Defense Systems, Haifa, Israel

Bell’s experiment:



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle^{(A)}\,|\downarrow\rangle^{(B)} + e^{i\Phi}\,|\downarrow\rangle^{(A)}\,|\uparrow\rangle^{(B)}\right),$$

$\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$: eigenstates of Pauli operators $\sigma_Z^{(A,B)}$ along locally defined Z-axes.

The particles’ polarization is tested at two widely separated detectors along two arbitrary directions within the locally defined XY-planes.

Quantum Mechanics:

$$E(\Delta - \Phi) = -\cos(\Delta - \Phi)$$

$E(\Delta - \Phi)$: Statistical correlation between binary outcomes of the two detectors.

Δ : Relative angle between the orientations of the two detectors.

The Bell's inequality states in an experimentally testable way that this prediction cannot be reproduced by any model of hidden variables that shares certain intuitive features.

In particular, the CHSH version of the inequality states that for any such model of hidden variables:

$$|E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \delta) - E(\Delta_2 - \delta)| \leq 2$$

for every set of values $(\Delta_{1,}\Delta_{2,}\delta)$.

On the other hand, according to the predictions of Quantum Mechanics this magnitude reaches a maximum value of $2\sqrt{2}$ for $\Delta_1 = -\Delta_2 = \frac{\pi}{4} = \delta/2$

Very carefully designed experimental tests have confirmed the predictions of Quantum Mechanics and, thus, ruled out all such models of hidden variables.

The CHSH inequality's proof :

D : space of possible configurations

$\rho(\lambda)$: (density of) probability for each $\lambda \in D$ to happen in a single realization of the experiment.

$s_{\Omega_A}^{(A)}(\lambda)$
$s_{\Omega_B}^{(B)}(\lambda)$: binary values - either -1 or +1, which describe the outcomes in each one of the two detectors if the polarization of their corresponding particles would be tested along directions Ω_A and Ω_B, respectively.

Hence, for any $\lambda \in D$ and any two orientations Ω_A, Ω'_A for detector A and Ω_B, Ω'_B for detector B, we have:

$$H(\lambda) \equiv s^{(A)}_{\Omega_A}(\lambda) * \left(s^{(B)}_{\Omega_B}(\lambda) + s^{(B)}_{\Omega'_B}(\lambda) \right) + s^{(A)}_{\Omega'_A}(\lambda) * \left(s^{(B)}_{\Omega_B}(\lambda) - s^{(B)}_{\Omega'_B}(\lambda) \right) = \pm 2$$

The CHSH inequality is then obtained by integration over the space of all possible hidden configurations.

$$-2 \le \int d\lambda\, \rho(\lambda) \cdot H(\lambda) \le +2$$

$$\int d\lambda\, \rho(\lambda) \cdot H(\lambda) = E(\Delta_1) + E(\Delta_2) + E(\Delta_1 - \delta) - E(\Delta_2 - \delta)$$

The proof requires three physically well-defined angles, say

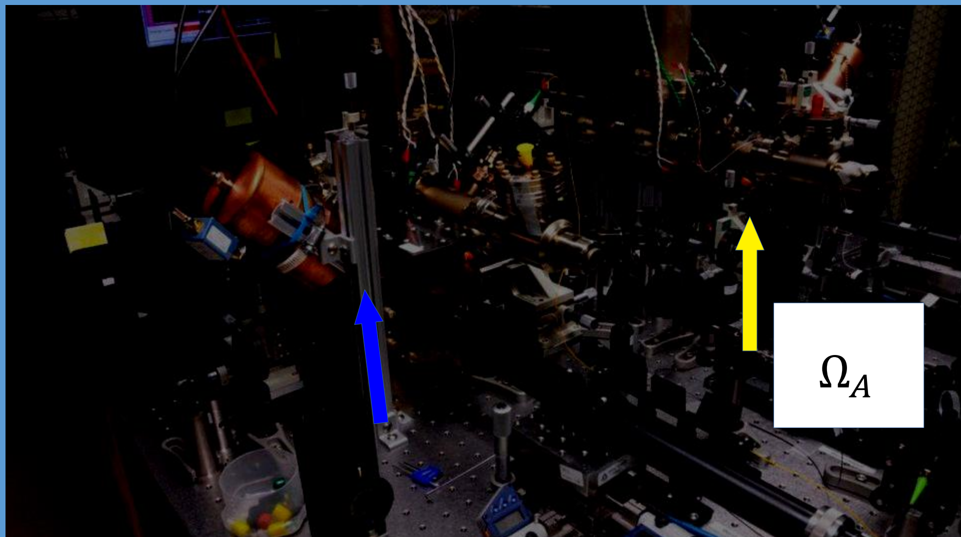
$$\Delta_1 = \vartheta(\Omega_B, \Omega_A), \qquad \Delta_2 = \vartheta(\Omega'_B, \Omega_A), \qquad \delta = \vartheta(\Omega'_A, \Omega_A),$$

The fourth direction, Ω_A , serves as a reference direction to define the other three.

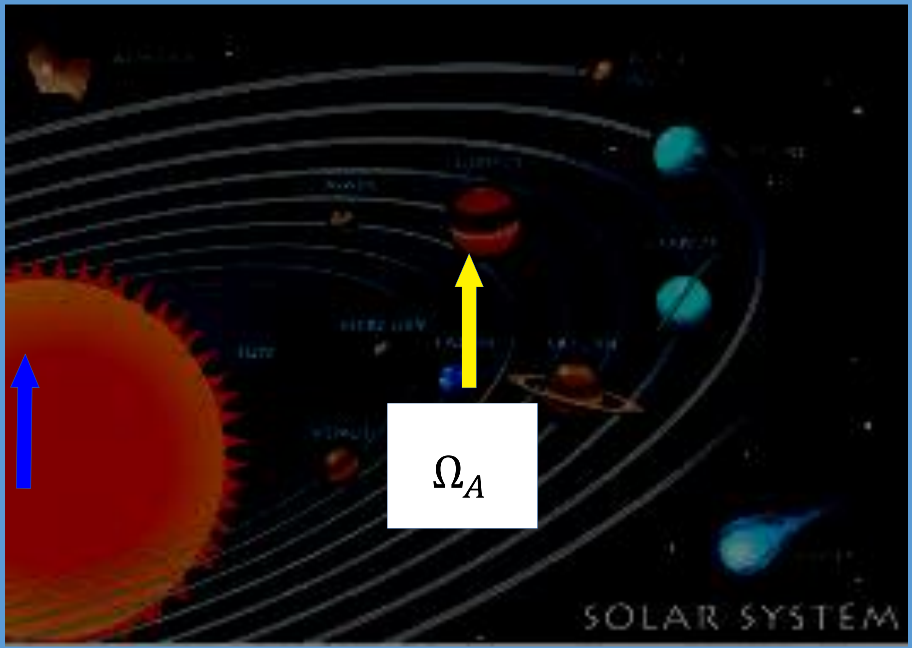
This fourth direction serves also as a reference to describe the possible hidden configurations $\lambda \in S$ of each pair of entangled particles: whatever λ is, it must be defined with respect to a reference frame (as any physical property).

The orientation of this reference direction is an spurious (unphysical/irrelevant) gauge degree of freedom.

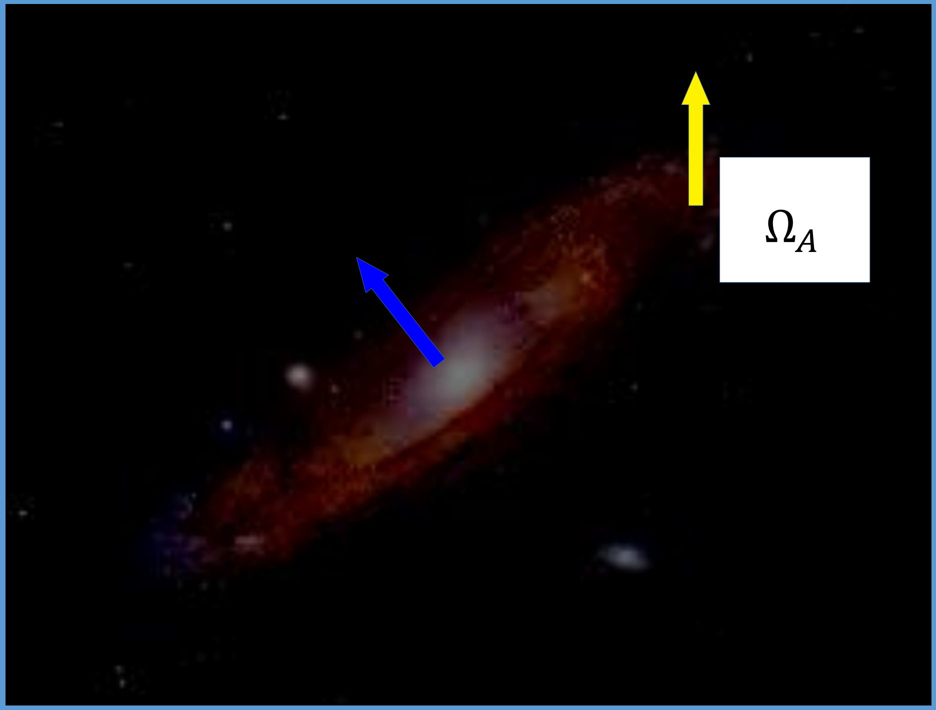
Lab frame: laboratory's table



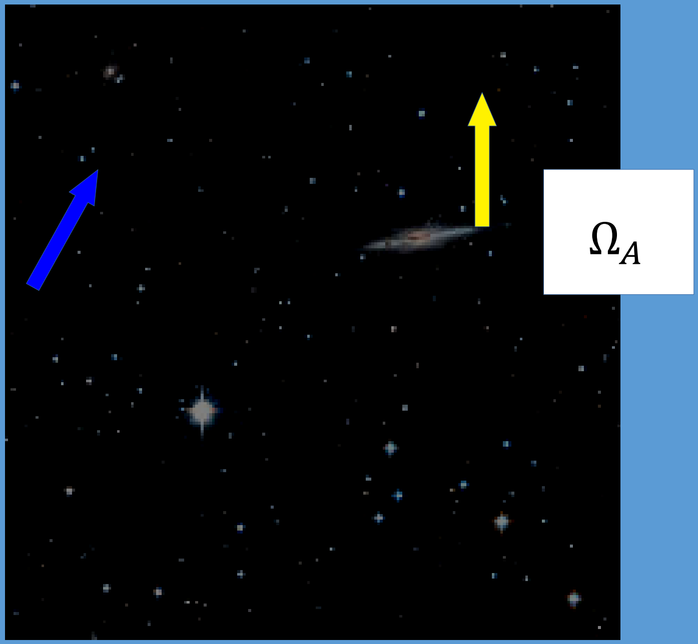
Lab frame: Sun's axis



Lab frame: the Galaxy's center



Lab frame: the center of the local Supercluster (the Great Attractor)



THE PROOF OF THE CHSH INEQUALITY SEEMS STRAIGHTFORWARD AND INDISPUTABLE.

NONETHELESS, IT INVOLVES A SUBTLE, THOUGH CRUCIAL, ASSUMPTION THAT:

a) IT IS NOT FULFILLED BY THE ACTUAL EXPERIMENTAL SET-UP

b) IT IS NOT REQUIRED BY FUNDAMENTAL PHYSICAL PRINCIPLES

How can we properly define the relative orientation δ between Ω_A and Ω'_A, if we are defining the orientation of detector A as our reference ?

It would be possible if the four binary values

$$s^{(A)}_{\Omega_A}(\lambda), \quad s^{(A)}_{\Omega'_A}(\lambda), \quad s^{(B)}_{\Omega_B}(\lambda), \quad s^{(B)}_{\Omega'_B}(\lambda)$$

could be obtained for single pairs of entangled particles.

In the actual experimental set-up the polarization of each particle in a single pair can be tested only along one direction !

How can we properly define the relative orientation δ between Ω_A and Ω'_A, if we are defining the orientation of detector A as our reference ?

It could still be possible if there would exist an absolute preferred frame of reference to which we could refer.

Principle of relativity: we are fully entitled to choose the orientation of detector A as a reference direction to describe the hidden configuration of each single pair of entangled particles !

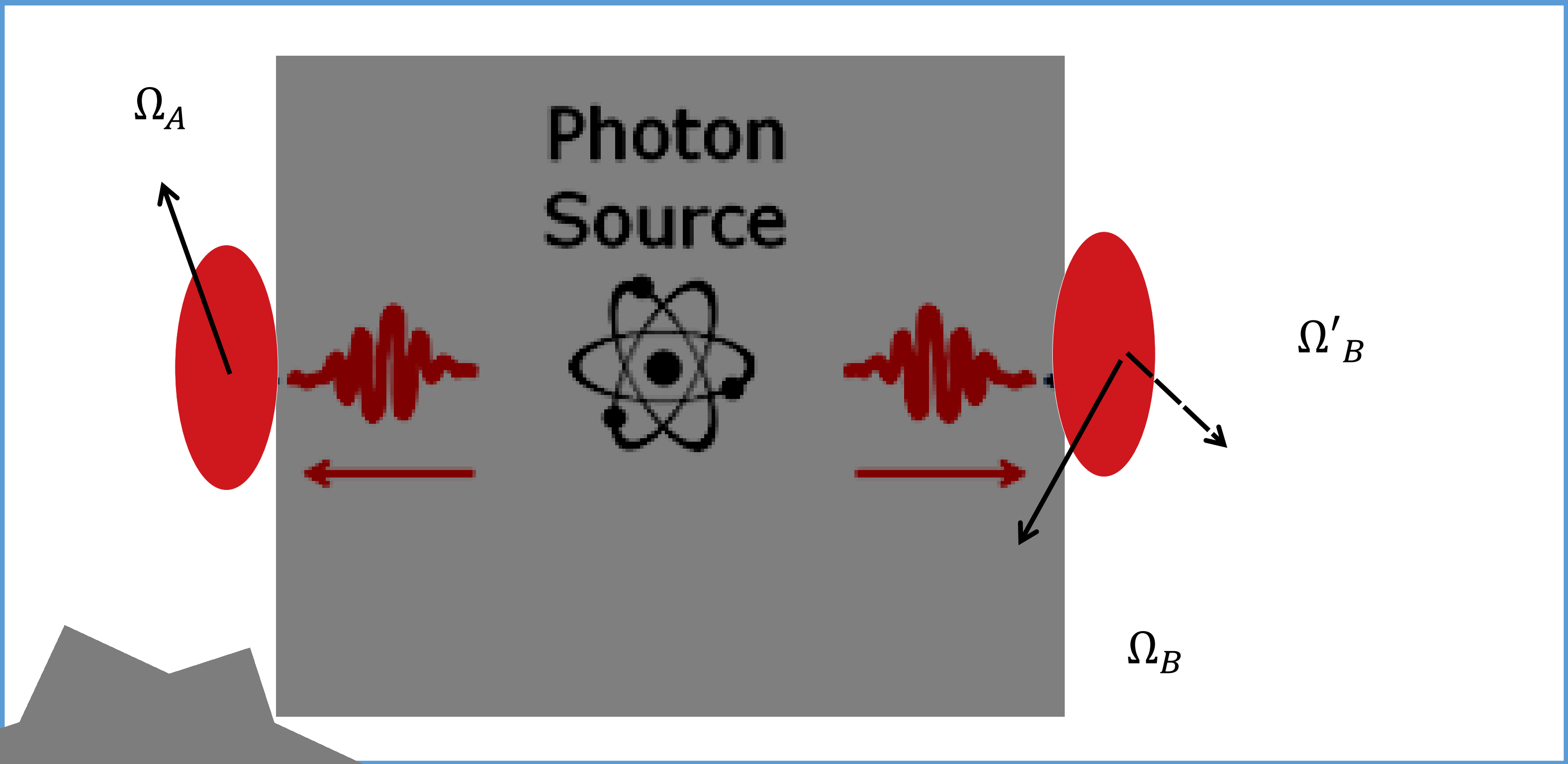
In the actual experimental set-up only the relative angle between the orientation of the two detectors is a physically well-defined observable, while their absolute orientation is an unphysical/irrelevant gauge degree of freedom.

Gauge Symmetry: In order to build a model of hidden variables we may only need to define the binary values (gauge-fixing condition):

$$s_{\Omega_A}^{(A)}(\lambda), \quad s_{\Omega_B}^{(B)}(\lambda), \quad s_{\Omega'_B}^{(B)}(\lambda), \quad s_{\Omega_B-\Delta}^{(B)}(\lambda), \quad s_{\Omega'_B-\Delta}^{(B)}(\lambda)$$

However, it is straightforward to check that the proof of the CHSH inequality does not necessarily hold for such models.

$$s_{\Omega_A}^{(A)}(\lambda) * \left(s_{\Omega_B}^{(B)}(\lambda) + s_{\Omega'_B}^{(B)}(\lambda) + s_{\Omega_B-\Delta}^{(B)}(\lambda) - s_{\Omega'_B-\Delta}^{(B)}(\lambda) \right)$$



Lab frame !

Pencil

It makes no sense to attempt to distinguish a situation in which the pencil/lab is kept fixed while the reference detector is rotated (crucial for the proof of CHSH inequality !), from a situation in which the reference direction is kept fixed while the pencil/lab is rotated (which is irrelevant !)

How do these arguments apply to the standard formalism of Quantum Mechanics ?

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle^{(A)}\,|\downarrow\rangle^{(B)} + \mathrm{e}^{i\Phi}\,|\downarrow\rangle^{(A)}\,|\uparrow\rangle^{(B)}\right),$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}^{(A,B)}$ are bases of eigenstates of the Pauli operators $\sigma_Z^{(A,B)}$ along Z-axes locally defined at the sites of each one of the particles.

These eigenstates are defined up to a phase and, therefore, the phase Φ in the above expression is not properly defined.

How can we properly define it ?

Choose an arbitrary experimental setting and use it as a definition of parallel directions $\Delta = 0$ between the orientations of the two detectors.

Use the experimental correlations between their outcomes to properly define the phase Φ of the entangled state with respect to the arbitrary chosen reference setting.

With respect to the reference setting we can now properly define a relative rotation Δ in the orientation of the two detectors.

Thus, the experimental setting is fully described by the pair (Φ, Δ) defined with respect to an arbitrary reference setting, but only the difference $\Delta - \Phi$ is independent of the chosen setting. Hence, the correlation between the outcomes must be a function of this single parameter.

However, since we need an arbitrary reference setting of the detectors to define the pair (Φ, Δ) there is no way that we can properly define separately the orientations of each one of the detectors.

The proof would still hold, if protected by symmetry considerations …

But symmetries may be (spontaneously) broken !

This is the quid to build an explicitly local model of hidden variables that reproduces the predictions of Quantum Mechanics.

Back to the proof of the CHSH inequality: what's missing there ?

Let say that detector A in position Ω_A defines a set of coordinates $\lambda_A \in \mathrm{D}$ over the space of all possible hidden configurations D, with a well-defined response function $s(\lambda_A)$.

Then detector A in position Ω'_A could define, in principle, a different set of coordinates $\lambda'_A \in \mathrm{D}$ over the space D, with its response given by $s(\lambda'_A)$.

And detector B, as well, would define, at each one of its two possible orientations its own sets of coordinates over the space D, $\lambda_\mathrm{B} \in \mathrm{D}$ and $\lambda'_B \in \mathrm{D}$, with its response given by $s(\lambda_\mathrm{B})$ and $s(\lambda'_\mathrm{B})$, respectively.

Since these four sets of coordinates parameterize the same space D there must exist some transformation law that relates them:

$$\lambda'_A = \Upsilon(\lambda_A;\ \delta)$$

$$\lambda_B = \Upsilon(\lambda_A;\ \Delta_1)$$

$$\lambda'_B = \Upsilon(\lambda_A;\ \Delta_2)$$

such that the probability of each configuration to happen in a single realization remains invariant (free-will).

$$d\lambda_A\, \rho(\lambda_A) = d\lambda'_A\, \rho(\lambda'_A) = d\lambda_B\, \rho(\lambda_B) = d\lambda'_B\, \rho(\lambda'_B)$$

Hence, for any $\lambda_A \in D$ we should write:

$$H(\lambda_A) \equiv s(\lambda_A) * \left(s(\lambda_B) + s(\lambda'_B)\right)$$

$$+s(\lambda'_A) * \left(s(\lambda_B) - s(\lambda'_B)\right) = \pm 2$$

Thus, at the integration over the whole space D we get

$$-2 \le \int d\lambda_A \;\rho(\lambda_A) \cdot \left[s(\lambda_A) * \left(s(\lambda_B(\lambda_A)) + s(\lambda'_B(\lambda_A))\right)\right] +$$

$$+\int d\lambda_A \;\rho(\lambda_A) \cdot \left[s(\lambda'_A(\lambda_A)) * \left(s(\lambda_B(\lambda_A)) + s(\lambda'_B(\lambda_A))\right)\right] \le 2$$

$$\int d\lambda_A \;\rho(\lambda_A) \cdot \left[s(\lambda_A) * \left(s(\lambda_B(\lambda_A)) + s(\lambda'_B(\lambda_A)) \right) \right] +$$

$$+ \int d\lambda_A \;\rho(\lambda_A) \cdot \left[s(\lambda'_A(\lambda_A)) * \left(s(\lambda_B(\lambda_A)) + s(\lambda'_B(\lambda_A)) \right) \right]$$

The first integral does produce the wanted correlations

$$\int d\lambda_A \,\rho(\lambda_A) \cdot \left[s(\lambda_A) * \left(s(\lambda_B(\lambda_A)) + s(\lambda'_B(\lambda_A)) \right) \right] = E(\Delta_1) + E(\Delta_2)$$

But the second integral may not !

$$\int d\lambda_A \;\; \rho(\lambda_A) \cdot \left[s\big(\lambda'_A(\lambda_A)\big) * \Big(s\big(\lambda_B(\lambda_A)\big) + s\big(\lambda'_B(\lambda_A)\big) \Big) \right] =$$

$$= \int d\lambda'_A \;\; \rho(\lambda'_A) \cdot \left[s(\lambda'_A) * \big(s(\lambda_B(\lambda_A(\lambda'_A))) + s(\lambda'_B(\lambda_A(\lambda'_A))) \big) \right]$$

If the sets of coordinates accumulates a geometric phase along a cyclic transformation

$$\Upsilon(\lambda'_A;\, \Delta_1 - \delta) \;\; \neq \Upsilon(\Upsilon(\lambda'_A;\, -\delta);\, \Delta_1)$$

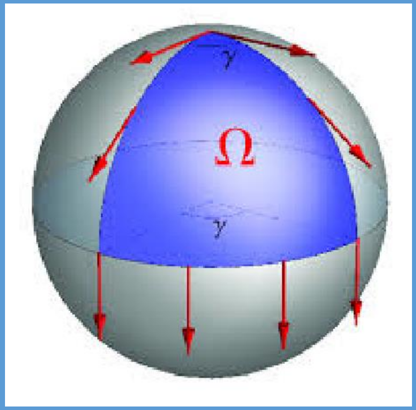
the second integral does not produce the wanted correlations

$$\int d\lambda'_A \, \rho(\lambda'_A) \cdot \left[s(\lambda'_A) * \Big(s\big(\tilde{\lambda}_B\big) + s\big(\tilde{\lambda}'_B\big) \Big) \right] = E(\Delta_1 - \delta) + E(\Delta_2 - \delta)$$

$$\tilde{\lambda}_B = \Upsilon(\lambda'_A;\, \Delta_1 - \delta), \qquad \tilde{\lambda}_B = \Upsilon(\lambda'_A;\, \Delta_2 - \delta)$$

The appearance of a geometric phase through a cyclic symmetry transformation

$$\Upsilon(\lambda'_A;\, \Delta_1 - \delta) \neq \Upsilon(\Upsilon(\lambda'_A;\, -\delta);\, \Delta_1)$$



is a known phenomena in theories involving gauge symmetries and, therefore, we should allow it when pursuing an underlying model of hidden variables for the Bell's states.

However, the Bell's theorem cannot account for such models.

We have taken advantage of these considerations to build an explicitly local statistical model that reproduces the predictions of Quantum Mechanics for the Bell's states.

[1] "Solving the EPR paradox: an explicitly local statistical model for the singlet quantum states", D. Oaknin, arXiv:1411:5704

[2] "The Bell's theorem revisited: a subtle, though crucial, assumption has gone unnoticed … and it might not be justified!", D. Oaknin, hal-01862953

THE MODEL:

We consider an infinite set of possible hidden configurations distributed over the unit circle.

The orientation of detector A sets a reference direction along this circle and its associated set of coordinates $\lambda_A \in [-\pi, \pi]$. The probability density of each hidden configuration to happen is given by:

$$g(\lambda_A) = -\left(\frac{1}{4}\right)|\sin(\lambda_A)|$$

Similarly, the orientation of detector B sets its own reference direction along this circle with its own associated set of coordinates $\lambda_B \in [-\pi, \pi]$.

Both sets of coordinates are related by a transformation law

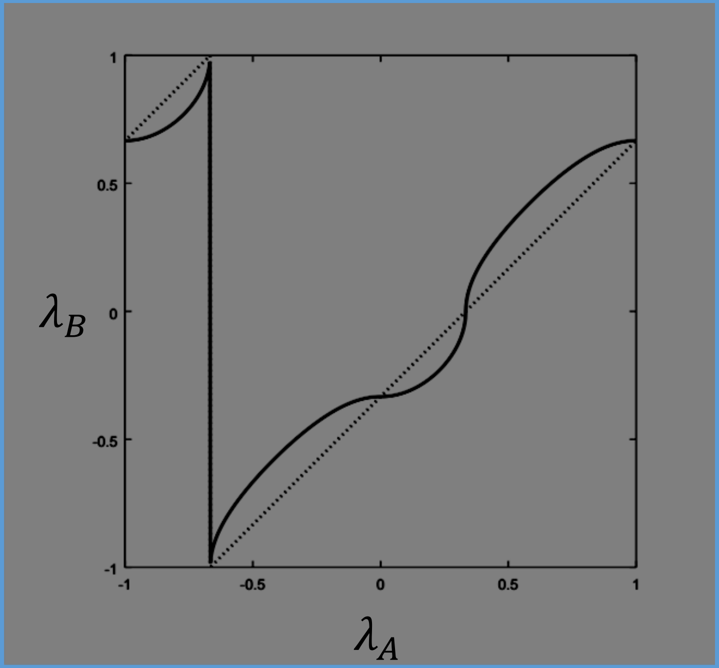
$$\lambda_B \ = \ -L(\lambda_A; \Delta - \Phi)$$

Moreover, symmetry considerations demand that the probability density of each hidden configuration to happen must given in the new set of coordinates by:

$$g(\lambda_B) = -\left(\frac{1}{4}\right)|\sin(\lambda_B)|$$

Since the probability of each possible hidden configuration must be independent of the set of coordinates (“free-will”) we must have:

$$g(\lambda_B)\, d\lambda_B = g(\lambda_A)\, d\lambda_A$$



That is,

$$|d[\cos(\lambda_B)]| = |d[\cos(\lambda_A)]|$$

This demand fixes the transformation law

$$\lambda_B \ = \ -L(\lambda_A; \Delta - \Phi)$$

as a function of the parameter $\Delta - \Phi$

This transformation law is additive in the following sense:

Let L_0 be a setting for which $\lambda_B = -L(\lambda_A; 0)$

If we use it as a definition of parallel directions between the two detectors, the entangled state corresponds to $\Delta = 0$, $\Phi = 0$.

Consider now a new setting L_1 that is obtained from the former by a relative rotation of the detectors by an angle Δ. The two new sets of coordinates are related by

$$\lambda'_B = -L(\lambda_A; \Delta)$$

If we use this new setting as a reference $\Delta_1 = 0$, the entangled state corresponds to $\Phi_1 = -\Delta$

Hence, if we now consider a third experimental setting L_2 that is obtained from L_1 by a relative rotation of the two detectors by an angle Δ_2 the two final sets of coordinates are related by

$$\lambda''_B = -L(\lambda_A; \Delta_2 - \Phi_1) = -L(\lambda_A; \Delta_2 + \Delta_1)$$

That is, the last setting L_2 is related to the original one L_0 by a relative rotation of the detectors by an angle $\Delta_1 + \Delta_2$

Finally, we define the response function of detector A as:

$$s^{(A)}(\lambda_A) = \begin{cases} +1, if\ \lambda_A \in [0, +\pi) \\ -1, if\ \lambda_A \in [-\pi, 0) \end{cases}$$

Similarly, we define the response function of detector B as:

$$s^{(B)}(\lambda_B) = \begin{cases} +1, if\ \lambda_B \in [0, +\pi) \\ -1, if\ \lambda_B \in [-\pi, 0) \end{cases}$$

These definitions are explicitly local since they depend only on the orientation of the hidden configuration with respect to the corresponding detector !

Therefore,

$$s^{(A)}(\lambda_A) = +1 \;\wedge\; s^{(B)}\big(\lambda_B(\lambda_A)\big) = +1, if\ \lambda_A \in [0, \Delta - \Phi)$$
$$s^{(A)}(\lambda_A) = +1 \;\wedge\; s^{(B)}\big(\lambda_B(\lambda_A)\big) = -1, if\ \lambda_A \in [\Delta - \Phi, \pi)$$
$$s^{(A)}(\lambda_A) = -1 \;\wedge\; s^{(B)}\big(\lambda_B(\lambda_A)\big) = +1, if\ \lambda_A \in [\Delta - \Phi - \pi, 0)$$
$$s^{(A)}(\lambda_A) = -1 \;\wedge\; s^{(B)}\big(\lambda_B(\lambda_A)\big) = -1, if\ \lambda_A \in [-\pi, \Delta - \Phi - \pi)$$

Hence, the correlation between the outcomes of the two detectors is given by:

$$E(\Delta - \Phi) = \int_0^{\Delta-\Phi} g\,(\lambda_A)\,d\lambda_A + \int_{-\pi}^{\Delta-\Phi-\pi} g\,(\lambda_A)\,d\lambda_A - \int_{\Delta-\Phi}^{\pi} g\,(\lambda_A)\,d\lambda_A - \int_{\Delta-\Phi-\pi}^{0} g\,(\lambda_A)\,d\lambda_A =$$
$$= -\cos(\Delta - \Phi)$$

SUMMARY & CONCLUSIONS:

The proof of the Bell's inequality relies on an unjustified implicit assumption, which:

a) is not fulfilled by the actual set-up of the experiments that test it;

b) is not required by fundamental physical principles and, indeed, it is at odds with the principle of relativity.

Namely, the proof of the inequality implicitly assumes the existence of an absolute preferred frame of reference with respect to which the orientation of the devices that test the particles' polarization can be defined.

Hence, the inequality cannot actually distinguish between the predictions of quantum mechanics for Bell's states and those of models of hidden variables that do not comply with this unjustified assumption.

SUMMARY & CONCLUSIONS:

It is possible to build a local model of hidden variables that reproduces the predictions of Quantum Mechanics for the Bell's polarization states of two entangled particles and fulfills the constraints of 'free-will', once the gauge degrees of freedom involved are properly identified.

D.Oaknin, "Solving the EPR paradox: an explicit local statistical model for the singlet", arxiv:1411.5704

Similar analysis has been done for the GHZ polarization state of three entangled particles and for a single spin-1 particle (qutrit).

D.Oaknin, "Solving the Greenberger-Horne-Zeilinger paradox: an explicit local statistical model for the GHZ state", arxiv:1709.00167

D.Oaknin, "Bypassing the Kochen-Specker theorem: an explicit local statistical model for the qutrit", arxiv:1805.04935