From varying constants to the entangled cyclic multiverses

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Plan:

- 1. Introduction.
- 2. Dynamical constants vs singularities in cosmology.
- 3. Dynamical constants and cyclic universes.
- 4. Thermodynamics and a toy cyclic multiverse.
- 5. Inter-universal entanglement in the multiverse.
- 6. Conclusions.

References

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- A. Balcerzak, S. Barroso-Bellido, MPD, S. Robles-Perez - work in progress.

1. Introduction

- One of the main problems of cosmology is the **problem of singularities**. There are many approaches to construct non-singular models, but usually it is difficult to avoid singularities under some generic conditions.
- Besides, there are **various types of singularities** with quite different properties (like Big-Rip, Sudden Future, Finite Scale Factor, Little-Rip etc.) not necessarily geodesically incomplete ("weak singularities").
- Main concern of this talk:
- Is it possible to construct **a cyclic universe** with no singularities or ”weaker” singularities? **(multiverse in time)**
- Is it possible to construct any classical or quantum **”cyclic parallel universes” scenario**? **(multiverse in space & time)**
- Are there **any effects** which link these parallel universes? Are they **observable** in our universe?

2. Dynamical constants vs singularities in cosmology.

- According to Hawking and Penrose (1973) a spacetime is singular if there exists **at least one geodesic which is incomplete** i.e. which cannot be extended in at least one direction and has only a finite range of affine parameter (proper time or length for non-null geodesics).
- This is a kind of “minimalistic” approach which **does not tell us the full nature of these singularities**: e.g. how they influence the physical and geometrical quantities.
- An interesting set of **alternative/extended gravity cosmologies** are **dynamical constants cosmologies** which have been applied to solve some standard cosmology problems such as the horizon and flatness problem (e.g. Moffat 1993, Albrecht, Magueijo 1999; Barrow 1999, Uzan 2003).
- Our underlying idea was **to apply dynamical constants to remove or to change the strength** of singularities in cosmology (MPD, Marosek 2013; MPD, Balcerzak, Marosek 2014).

Variety of singularities: strength.

- Tipler's (Phys. Lett. A64, 8 (1977)) definition (of **a strong singularity**):
  $I^i_j(\tau) = \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. an extended object is **crushed to zero volume** (represented by three linearly independent, vorticity-free geodesic deviation vectors at p parallely transported along causal geodesic l) at the singularity by infinite tidal forces
- Królak's (CQG 3, 267 (1988)) definition (of a strong singularity):
  $I^i_j(\tau) = \int_0^\tau d\tau' |R^i_{ajb} u^a u^b|$
  diverges on the approach to a singularity at $\tau = \tau_s$
- i.e. the **expansion** of every future-directed congruence of null (timelike) geodesics emanating from point p and containing l **becomes negative** somewhere on l
- For null geodesics one replaces Riemann by the Ricci tensor components.

Variety of singularities: strength (MPD 2015).

Type	Name	t sing.	a(t_s)	$\varrho(t_s)$	p(t_s)	$\dot{p}(t_s)$ etc.	w(t_s)	T	I
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	str
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	str
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	str
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	we
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	we
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	we
III	Finite Scale Factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	str
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	we
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	we
...	...	...	...	...	...	...	...	...	.

Dynamical constants $G(t)$ and $c(t)$ vs singularities

We considered the simplest theory for the generalized Einstein-Friedmann equations in **varying speed of light (VSL)** theories (Barrow & Magueijo model - 1999) and **varying gravitational constant G** theories (ϱ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density)

$$\varrho(t) = \frac{3}{8\pi G(t)} \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right) , \tag{1}$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right) , \tag{2}$$

and the energy-momentum "conservation law" is (related to 2nd law of thermodynamics)

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a} \left(\varrho(t) + \frac{p(t)}{c^2(t)} \right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2} . \tag{3}$$

"All-in-one" scale factor.

We used a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which **allows for big-bang, big-rip, sudden future, finite scale factor and w-singularities** and reads as

$$a(t) = a_s \left(\frac{t}{t_s} \right)^m \exp \left(1 - \frac{t}{t_s} \right)^n , \tag{4}$$

with the constants t_s, a_s, m, n.

For $0 < m < 2/3$ we have **a big-bang singularity** - $a \to 0$, $\varrho \to \infty$, $p \to \infty$ at $t \to 0$;

For $m < 0$ we have **a big-rip singularity** - $a \to \infty$, $\varrho \to \infty$, $p \to \infty$ at $t = 0$;

For $1 < n < 2$ we have **a sudden future singularity** (SFS) which appears at $t = t_s$ ($a = a_s$, $\varrho =$ const., $p \to \infty$);

For $0 < n < 1$ we have **a (stronger) finite scale factor singularity** (FSF) at $t = t_s$ ($a = a_s$, $\varrho \to \infty$, $p \to \infty$).

More examples of regularizing other singularities

- To remove SFS: **light has to stop moving** at the singularity (same happens in loop quantum cosmology (LQC) where it is called the anti-newtonian limit $c = c_0\sqrt{1 - \varrho/\varrho_c} \to 0$ for $\varrho \to \varrho_c$ with ϱ_c being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \to c_0$).
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at a singularity (seems reasonable because of the requirement to **overcome an infinite (anti-)tidal forces** at the singularity).
- However, it makes another singularity - **a singularity of strong coupling** for a physical field such as $G \propto 1/\Phi$.
- Such problems were already dealt with for example in superstring and brane cosmology where both the **curvature singularity and a strong coupling singularity appeared**.

3. Dynamical constants and cyclic universes.

An idea is similar to some superstring/brane universe evolution scenarios like the **ekpyrotic/cyclic models** (Khoury et al. 2001, Steinhardt et al. 2002, Turok et al. 2005 etc.) where one has **some special coupling** of a scalar field $\beta(\phi)$ of gravity in the Lagrangian of a 4-dimensional theory (Einstein frame)

$$S = \int d^4x \sqrt{-g} \left[\frac{c^4}{16\pi G} R - \frac{1}{2} \partial_\mu \phi \partial^\nu \phi - V(\phi) + \beta^4(\phi)(\varrho_R + \varrho_m) \right] , \tag{5}$$

where the potential has an explicit form as

$$V(\phi) = V_0 \left(1 - e^{-s\phi}\right) F(\phi), \qquad F(\phi) \propto e^{-\frac{1}{g_s}} , \tag{6}$$

(ϱ_R is the energy density of radiation while ϱ_m is the energy density of matter, g_s - dilaton/string coupling constant) which **leads to cyclic models of the universe**.

Dynamical constants and cyclic universes.

- We construct cyclic models **using the idea of dynamical physical constants** instead of the special coupling (MPD & Marosek 2013) using eqs. (1), (2), and (3).
- First we assume that that the gravitational constant G varies (and $\dot{c} = 0$) as

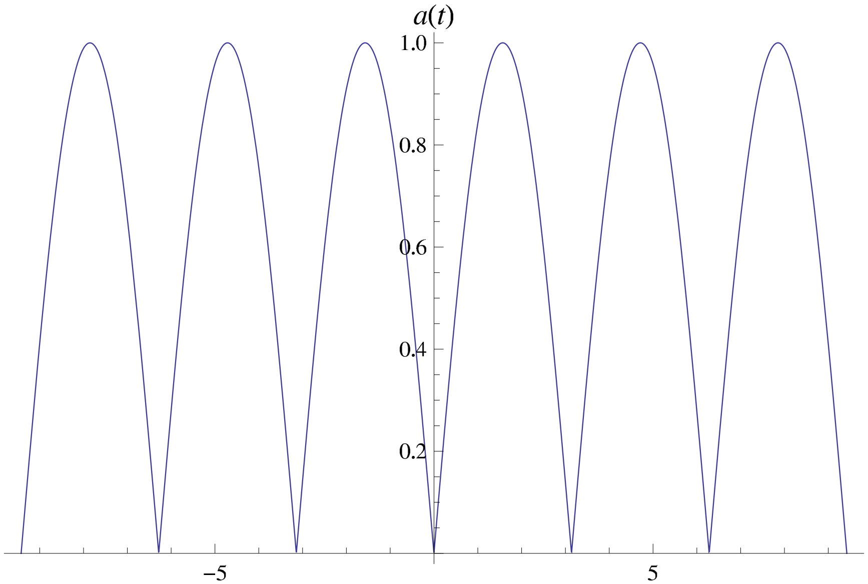
$$G\left(t\right) = \frac{G_0}{a^2(t)}, \tag{7}$$

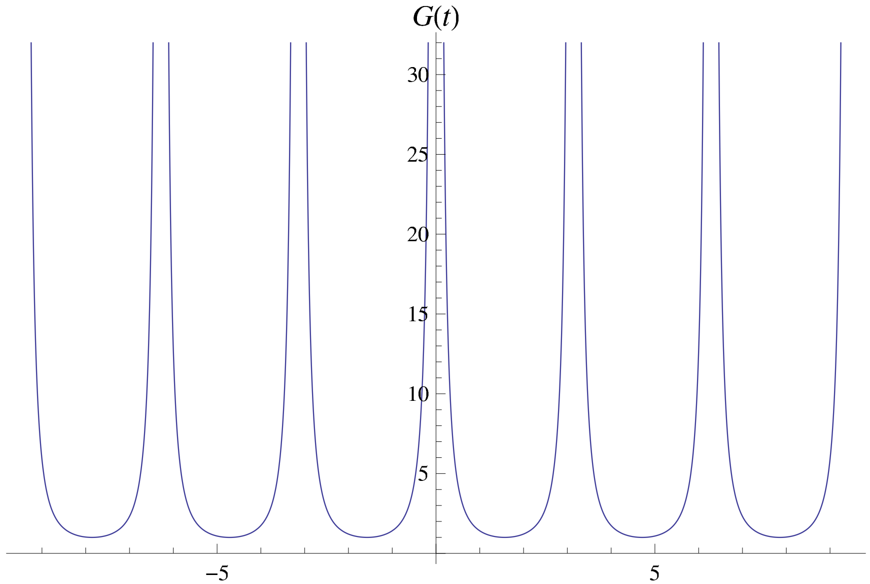
and for the positive curvature $(k = +1)$ we obtain **sinusoidal pulse Friedmann model** with the scale factor

$$a(t) = a_0 \left| \sin\left(\pi \frac{t}{t_c}\right) \right| \tag{8}$$

with a_0 =const., so **the scale factor is ”singular”** (reaching zero as in the big-bang scenario).

Sinusoidal pulse model: $a(t)$, $G(t)$.





Dynamical constants and cyclic universes.

- The mass density $\rho(t)$ and the pressure $p(t)$ are nonsingular and oscillatory

$$\rho\left(t\right) = \frac{3}{8\pi G_0}\left[\frac{\pi^2 a_0^2 \cos^2\left(\pi\frac{t}{t_c}\right)}{t_c^2} + c^2\right] \geq 0, \tag{9}$$

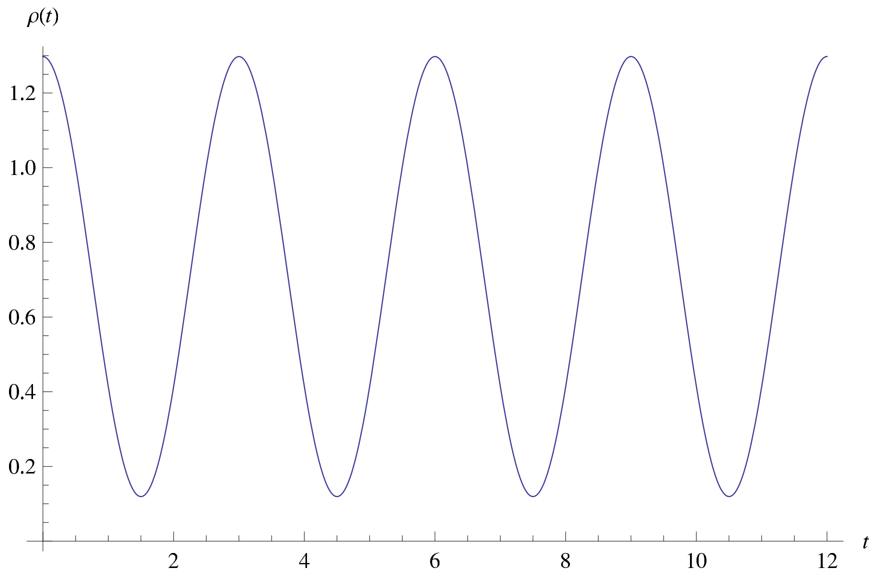
$$p\left(t\right) = -\frac{c^2}{8\pi G_0}\left[\frac{3\pi^2 a_0^2 \cos^2\left(\pi\frac{t}{t_c}\right)}{t_c^2} + c^2 - 2\frac{\pi^2 a_0^2}{t_c^2}\right] . \tag{10}$$

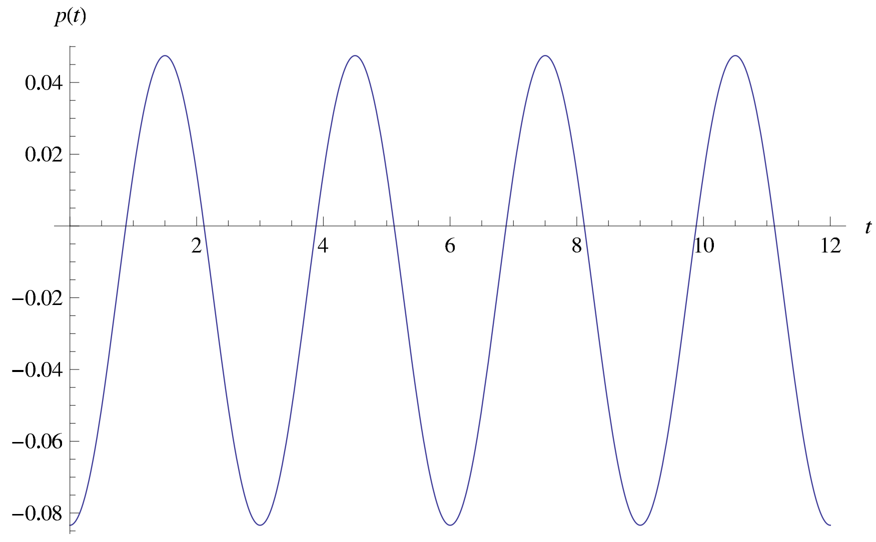
- For these models the null energy condition is always fulfilled **(a ”singular bounce”)**.

$$p(t) + \rho(t)c^2 = p(mt_c) \equiv p_a \geq 0 \quad m = 0, 1, 2, 3, ... \tag{11}$$

- The same is true for the weak energy condition which additionally requires (9) ($\rho \geq 0$).

Sinusoidal pulse model: density and pressure.





Strong energy condition is also fulfilled

$$\rho\left(t\right)c^{2}+3p\left(t\right) \quad = \quad \frac{3{a_0}^{2}\pi c^{2}}{4G_0{t_c}^{2}}\sin^{2}\left(\pi\frac{t}{t_c}\right)\geq 0, \qquad (12)$$

and the universe is decelerating $(\ddot{a}<0)$.

Dynamical constants and cyclic universes.

- In fact, in our model we have no proper bounce in the scale factor since

$$\dot{H} = -\frac{\pi^2}{t_c^2} - \frac{\dot{a}^2}{a^2} < 0. \tag{13}$$

  Defining

$$H_G = \frac{\dot{G}}{G} = -2\frac{\dot{a}}{a} = -2H \tag{14}$$

  we can see that $\dot{H}_G > 0$ always accompanies $\dot{H} < 0$ **(a contraction of space is always balanced by an expansion of gravity)** which makes an apparently singular and sharp bounce regular in matter density and pressure **due to the special form of running gravity**.

- In general, the strong energy condition means that gravity is attractive, but in our case **its atractivity is overbalanced by the strong coupling of** $G(t)$ at the Big-Bang type of singularity ($G \to \infty$).

Dynamical constants and cyclic universes.

- Another interesting case is **the tangential pulse model** with the scale factor

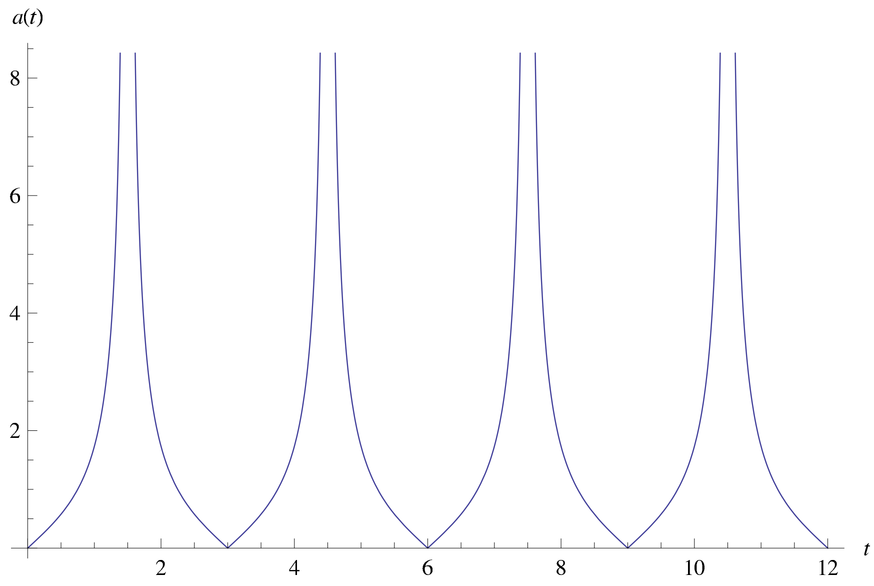
$$a(t) = a_0 \left| \tan\left(\pi \frac{t}{t_s}\right) \right| \quad , \tag{15}$$

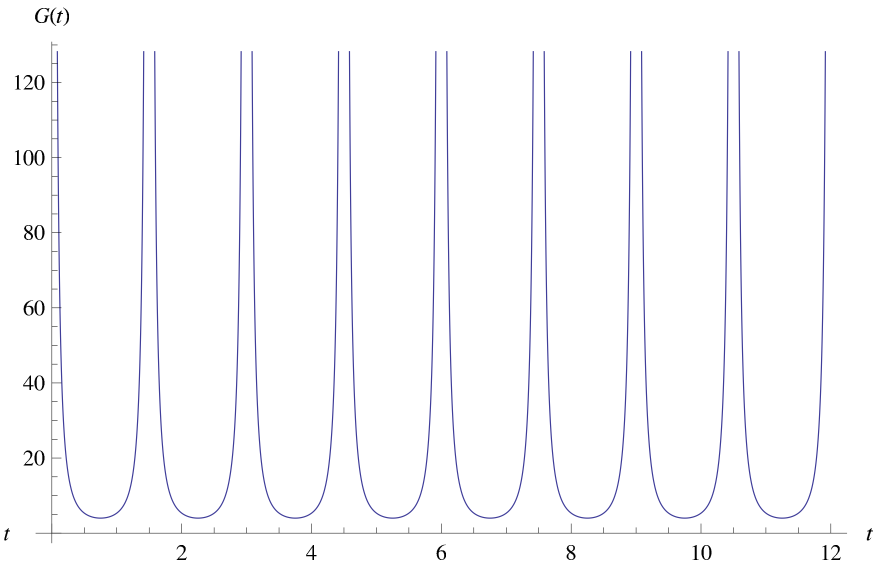
which can be accompanied by the gravitational constant varying as

$$G\left(t\right) = \frac{4G_s}{\sin^2\left(2\pi \frac{t}{t_s}\right)} \quad . \tag{16}$$

- This scale factor is infinite ($a \to \infty$) for $t = nt_s$ with $n = 1/2, 3/2, 5/2, \ldots$ (like at **big-rips**) and it is zero for $t = mt_s$ with $m = 0, 1, 2, 3, \ldots$ (like at **big-bangs**) so that we can say that we face "singular bounces".
- Each time the scale factor $a(t)$ attains a singular value (vanishes or reaches infinity), the **gravitational coupling becomes infinite** ($G \to \infty$).

Tangential pulse model - $a(t)$, $G(t)$.





Dynamical constants and cyclic universes.

- The mass density and pressure are nonsingular

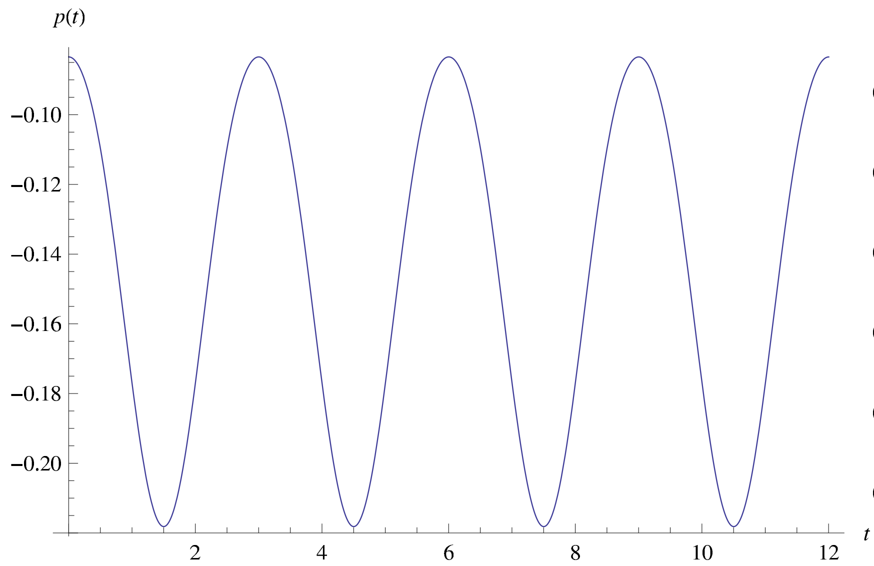
$$\rho\left(t\right) = \frac{3}{8\pi G_s}\left[\frac{\pi^2}{t_s^2} + \frac{3c^2\cos^4\left(\pi\frac{t}{t_s}\right)}{a_0^2}\right] \geq 0, \tag{17}$$

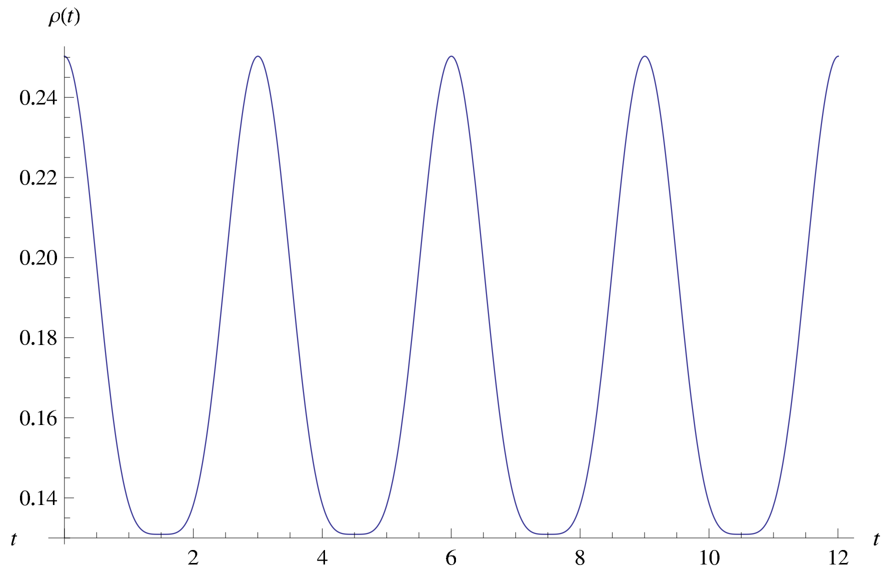
$$p\left(t\right) = -\frac{c^2}{8\pi G_s}\left[\frac{\pi^2}{t_s^2} + 4\frac{\pi^2\sin^2\left(\pi\frac{t}{t_s}\right)}{t_s^2} + \frac{c^2\cos^4\left(\pi\frac{t}{t_s}\right)}{a_0^2}\right], \tag{18}$$

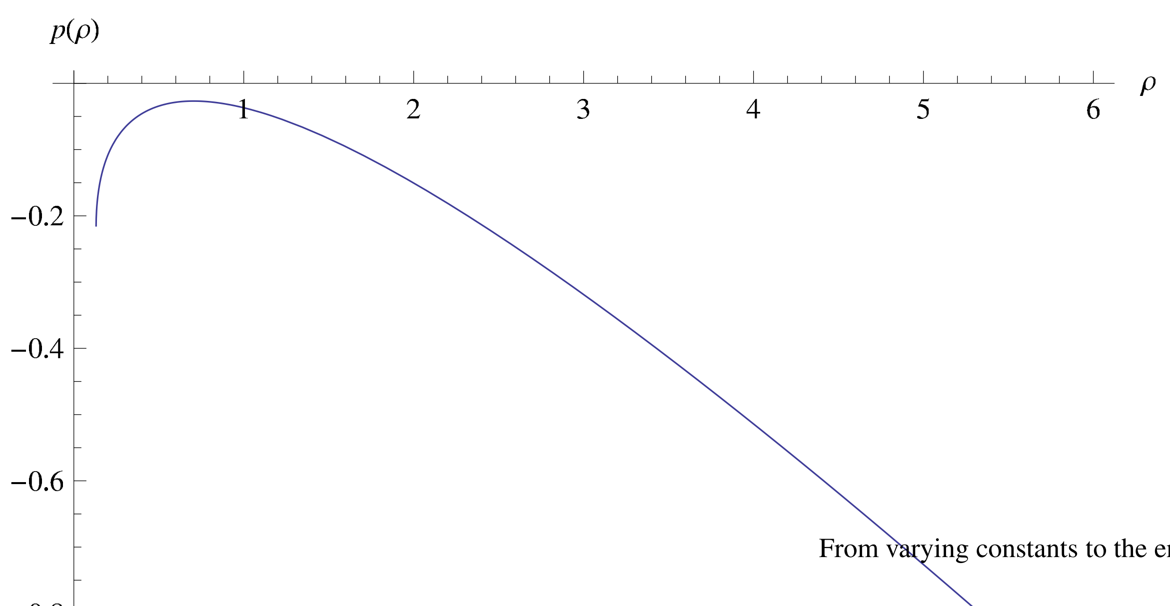
- The null energy condition is satisfied at $t = mt_s$ (**Big-Bang-like sings**) and violated at $t = nt_s$ (**Big-Rip-like sings**):

$$c^2\rho\left(t\right) + p\left(t\right) = \frac{c^2}{4\pi G_s}\left[\frac{\pi^2}{t_s^2} - 2\frac{\pi^2\sin^2\left(\pi\frac{t}{t_s}\right)}{t_s^2} + 4\frac{c^2\cos^4\left(\pi\frac{t}{t_s}\right)}{a_0^2}\right]. \tag{19}$$

Tangential pulse model: density and pressure.







Dynamical constants and cyclic universes.

- It proves that the former is a Big-Rip-like singularity and the latter is a Big-Bang-like singularity. The word "like" comes from the fact that the mass density and pressure are regular which is not the case at a Big-Bang and a Big-Rip.
- The pressure at both of these singularities $p(mt_s)$ and $p(nt_s)$ is negative.
- The strong energy condition reads as

$$c^2\rho\left(t\right)+3p\left(t\right)=\frac{3c^2}{4\pi G_s}\left[\frac{c^2\cos^4\left(\pi\frac{t}{t_s}\right)}{a_0^2}-2\frac{\pi^2\sin^2\left(\pi\frac{t}{t_s}\right)}{t_s^2}\right] \tag{20}$$

- so that

$$c^2\rho\left(mt_s\right)+3p\left(mt_s\right)=\frac{3c^4}{4\pi G_s {a_0}^2}\geq 0,\quad c^2\rho\left(nt_s\right)+3p\left(nt_s\right)=-\frac{3c^2}{2\pi G_s {t_s}^2}\leq 0.$$

4. Thermodynamics and a toy cyclic multiverse.

- Using previous cyclic models pattern we now **develop an idea of two independently evolving universes (doubleverse)** which allow the **same** evolution of the scale factor but **different** evolution of the physical constants in each universe.
- The idea **based on the thermodynamics** of varying c and varying G universes. We assume that **the multiverse obeys the 2nd law of thermodynamics**, though individual universes may not do so.
- From thermodynamical interpretation we conclude that the **entropy can be related to varying constants** as (e.g. Youm 2002)

$$S(t) = 2\frac{Nk_B}{\tilde{w}} \ln\left[A_0 c(t)\right], \qquad S(t) = Nk_B \ln\left[\frac{A_0}{G(t)}\right], \tag{21}$$

where A_0 is an integration constant (and can also be taken to be one), k_B - Boltzmann constant, $\tilde{w}$ - barotropic index, N-number of particles.

Cyclic multiverse - varying c.

- As first step assume that the entropy of the multiverse is constant:

$$\dot{S} = \sum_{i=1}^{n} \dot{S}_i = \dot{S}_1 + \dot{S}_2 + \dot{S}_3 + ... + \dot{S}_n = 0. \tag{22}$$

- For the "doubleverse" we have:

$$S_1 = \frac{2}{\tilde{w}} N_1 k_B \ln\left[c_1(t)\right], \tag{23}$$

$$S_2 = \frac{2}{\tilde{w}} N_2 k_B \ln\left[c_2(t)\right]. \tag{24}$$

- Make the following ansätze for $c(t)$:

$$c_1\left(t\right) = e^{\lambda_1 \phi_1 (t)}, \tag{25}$$

$$c_2\left(t\right) = e^{\lambda_2 \phi_2 (t)}, \tag{26}$$

where λ_1 and λ_2 are constants.

Cyclic multiverse.

- The **total entropy of the doubleverse** is

$$S = S_1 + S_2 = 2\frac{\widetilde{\rho_1} V_1 c_1^2}{T_1}\lambda_1\phi_1 + 2\frac{\widetilde{\rho_2} V_2 c_2^2}{T_2}\lambda_2\phi_2\left(t\right) \ . \tag{27}$$

- For simplicity we take (solves exactly):

$$\frac{\tilde{\varrho}_1 V_1 c_1^2}{T_1}\lambda_1 = \frac{\tilde{\varrho}_2 V_2 c_2^2}{T_2}\lambda_2 \ , \tag{28}$$

and using

$$N_1\lambda_1 = N_2\lambda_2 \ . \tag{29}$$

- This allows to pick up the following functional dependence:

$$\phi_1\left(t\right) = \sin^2\left(\pi\frac{t}{t_s}\right), \tag{30}$$

$$\phi_2\left(t\right) = \cos^2\left(\pi\frac{t}{t_s}\right), \tag{31}$$

Cyclic multiverse - varying c.

So that the entropies are

$$S_1 = \frac{2}{\tilde{w}} N_1 k_B \lambda_1 \sin^2\left(\pi \frac{t}{t_s}\right), \quad (32)$$

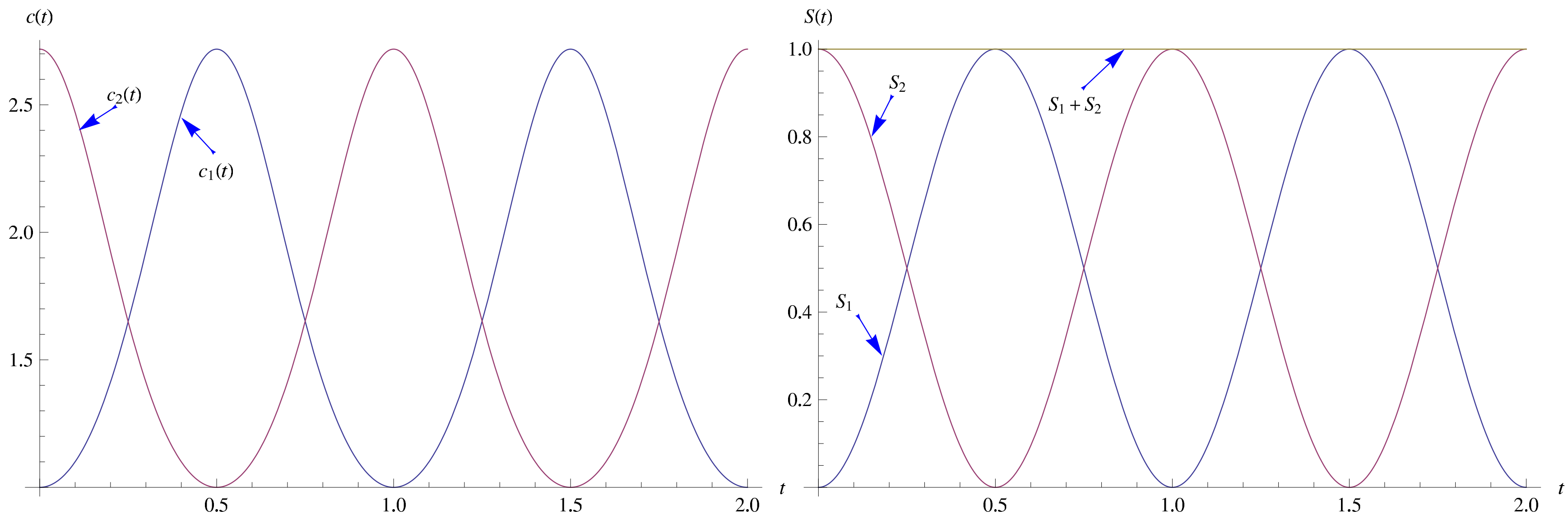
$$S_2 = \frac{2}{\tilde{w}} N_2 k_B \lambda_2 \cos^2\left(\pi \frac{t}{t_s}\right). \quad (33)$$

Two scale factors evolutions are equal and of the similar form as in cyclic universe models of the previous section:

$$a(t) = a_1(t) = a_2(t) = a_0 \left| \sin\left(\pi \frac{t}{t_s}\right) \right| \quad (34)$$

- Their evolution is evidently cyclic.
- The point is that although the **geometrical evolution of the ”parallel” universes is the same**, this is not the case with **the evolution of the physical constants $c_1(t)$ and $c_2(t)$ which is evidently different**.

Total entropy remains constant.



Similar considerations are valid for the models with varying G.

5. Inter-universal entanglement in the multiverse.

- Now we assume that **the universes 1 and 2 are quantum mechanically entangled** and there are **periods** of their evolution when the entanglement matters (e.g. at the maximum expansion point) and influences the behaviour of individual universes, though **most of the evolution of the individual universes is classical**.
- It means that we consider the multiverse with **two patches - the universes 1 and 2** which can be understood in the hierarchy given by Tegmark (2003) from level I (separate inflationary patches) to IV.
- These two patches are **classically disconnected**, but they can be quantum mechanically entangled (Robles-Perez et al. 2010, 2014, 2015) and the effect of entanglement can be **imprinted in individual universes** (for example in CMB spectrum, large-scale structure etc.).

Sinusoidal pulse multiverse entanglement.

For the previously considered sinusoidal pulse model we write the Friedmann equation in the form

$$H^2 = -\Lambda + \frac{1}{a^2}, \tag{35}$$

where

$$\Lambda \equiv \frac{\pi^2}{t_c^2} \quad \text{and} \quad a_0 = \frac{1}{\sqrt{\Lambda}}, \tag{36}$$

while for the tangential pulse

$$H^2 = \frac{1}{a^2}\left(1 + \Lambda a^2\right)^2 = \Lambda^2 a^2 + 2\Lambda + \frac{1}{a^2}, \tag{37}$$

where the first term on the right-hand side scales as phantom.

Wheeler-deWitt (second) quantization

Getting conjugate momentum as

$$p_a = -a\frac{da}{dt}, \tag{38}$$

the Hamiltonian constraint

$$p_a^2 - \omega^2(a) = 0, \tag{39}$$

can easily be derived from the Friedmann equations as

$$\omega_{\text{sin}}^2(a) \equiv a^2 - \Lambda a^4. \tag{40}$$

for the sinusoidal pulse and as

$$\omega_{\text{tan}}^2(a) \equiv \Lambda^2 a^6 + 2\Lambda a^4 + a^2. \tag{41}$$

for the tangential pulse.

Wheeler-deWitt (second) quantization

The Wheeler-DeWitt equation is **formally similar to the Klein-Gordon equation**

$$\ddot{\phi} + \omega^2(a)\phi = 0, \tag{42}$$

where $\phi \equiv \phi(a)$ is the wave function and $\dot{\phi} \equiv \frac{d\phi}{da}$. The WKB solutions of (42) (where two signs correspond to **two different branches of the universe**)

$$\phi_{\pm} \propto \frac{1}{\sqrt{2\omega}} e^{\pm iS}, \tag{43}$$

where, $\dot{S} = \omega$. For the sinusoidal pulse, we get

$$S = \int da\, \omega_{\sin}(a) = -\frac{\left(1 - \Lambda a^2\right)^{\frac{3}{2}}}{3\Lambda}. \tag{44}$$

For $a \in (0, a_0)$, the WKB wave function (43) represents a Lorentzian (classical) universe; for $a > a_0$, the wave function represents an exponential decay of the Euclidean regime or quantum barrier.

Sinusoidal branches

We thus obtain **two classical branches**

$$a(t) = \frac{1}{h} \sin[h(t - t_0)], \tag{45}$$

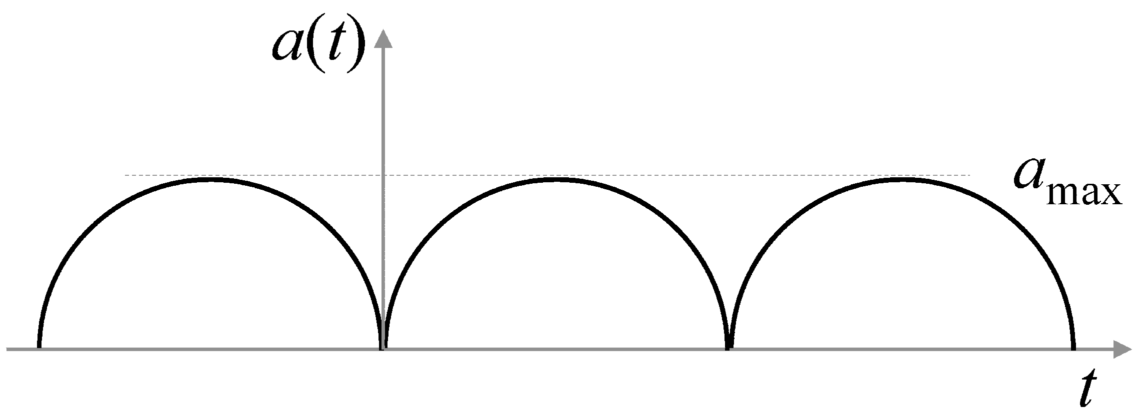
and the other with scale factor given by

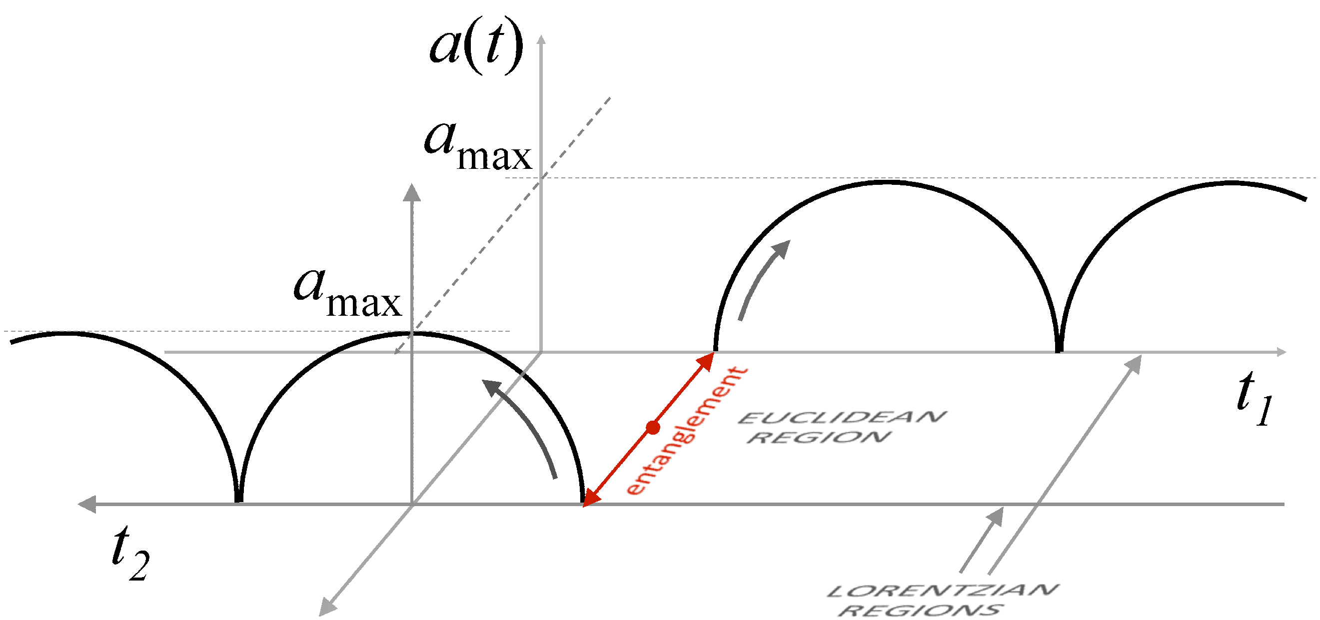
$$a(t) = \frac{1}{h} \sin[h(t_0 - t)], \tag{46}$$

which are related by time symmetry, $t \to -t$ ($t_0 \to -t_0$).

- The same universe for any internal observer provided that: 1. the universes are created in entangled pairs; 2. observers' time variables follow an antipodal-like symmetry (e.g. Linde1988, Robles-Perez 2014).
- Before reaching the big crunch singularities, **one branch of the universe can undergo a quantum transition to the the other branch universe**, appearing there as a newborn universe, forming thus a continuous cyclic multiverse.

Quantum transitions between universes in the multiverse





Tangential branches

For the **tangential pulse**, we arrive at

$$S = \int da\, \omega_{\tan}(a) = \frac{1}{4} a^2 \left(2 + \Lambda a^2\right) . \quad (47)$$

Analogously as for the sinusoidal pulse, the **evolution that corresponds to the plus and minus signs** of $\phi_\pm$ in (43) is given now by

$$\frac{da}{dt} = \pm \left(h^2 a^2 + 1\right) . \quad (48)$$

We thus obtain

$$a(t) = \frac{1}{h} \tan[h(t - t_0)], \quad (49)$$

and

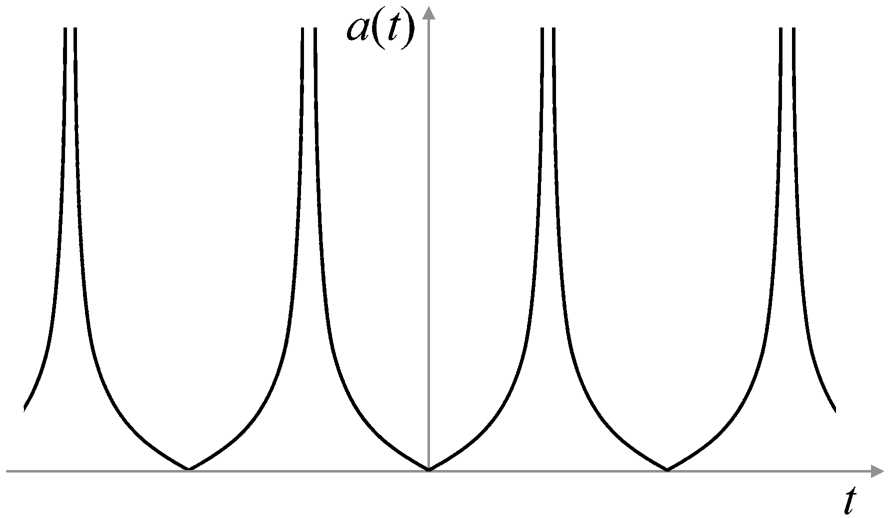
$$a(t) = \frac{1}{h} \tan[h(t_0 - t)], \quad (50)$$

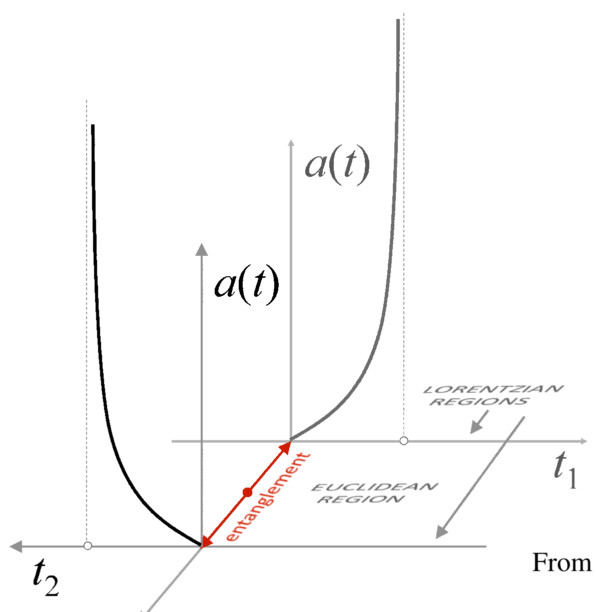
for the two branches of the tangential pulse.

Creation of cyclic universes in entangled pairs.

- **Quantum effects dominant** close to $a = 0$ (though p and ϱ regular) - say at $a = a_{\rm min}$.
- For $a < a_{\rm min}$ - no real solution found. Only double Euclidean instantons appear.
- They give rise (in the Lorentzian regime) to **an entangled pair of universes whose quantum states are quantum-mechanically correlated**.
- Observer living in the universe with time t_1 considers his branch expanding; the one with t_2 - contracting.
- Both observers see their universes expanding (antipodal symmetry) - two branches can be combined to form a universe that is **classically indistinguishable** from the cyclic single time picture.

Quantum creation of entangled pairs of the universes





Third quantization procedure of WdW equation - 3Q

- QFT formalism in which WdW wave function ϕ becomes an operator.
- This is due to fact that WdW equation (42) can be obtained from the Hamilton eqs of the **(third-quantized) Hamiltonian**

$$\mathrm{H} = \frac{1}{2} P_\phi^2 + \frac{\omega^2(a)}{2} \phi^2, \tag{51}$$

where, $P_\phi \equiv \dot{\phi}$.

- **3Q makes ϕ and P_ϕ operators.**
- We have

$$\hat{\phi}(a) = \frac{1}{\sqrt{2\omega}} e^{iS(a)} \hat{b}_+ + \frac{1}{\sqrt{2\omega}} e^{-iS(a)} \hat{b}_-^\dagger, \tag{52}$$

where, $\hat{b}_+ \equiv \hat{b}_+(a_{\text{min}})$ and $\hat{b}_-^\dagger \equiv \hat{b}_-^\dagger(a_{\text{min}})$, are constant operators given at some initial value, $a = a_{\text{min}}$.

Universe annihilation and creation operators

- "-" branch $a(t) = \frac{1}{h}\sin[h(t-t_0)]$ - $\hat{b}_-$ and $\hat{b}_-^\dagger$
- "+" branch - $\hat{b}_+$ and $\hat{b}_+^\dagger$
- similarly for the tangential pulse
- **analogue of creation an entangled pairs of particles with opposite momentum $\pm k$**
- **symmetry of WDW equation wrt $\pm\omega$ is translated into symmetry $\pm\phi$ in 3Q picture**

Representations of the multiverse vacua: b and c representations

- Vacuum state for $(b_{\pm}, b_{\pm}^{\dagger})$ representation – given by $|0_+, 0_-\rangle$.
- Not unique because of scale factor dependence of $\omega = \omega(a)$ (along minisuperspace geodesic).
- An invariant representation for the harmonic oscillator like (42) is (Lewis and Riesenfeld JMP, 1458 (1969), Robles-Perez and Gonzalez-Diaz 2010, 2014)

$$c_+ = \sqrt{\frac{1}{2}} \left(\frac{1}{R}\phi + i(RP_\phi - \dot{R}\phi) \right), \tag{53}$$

$$c_-^{\dagger} = \sqrt{\frac{1}{2}} \left(\frac{1}{R}\phi - i(RP_\phi - \dot{R}\phi) \right), \tag{54}$$

where $R = \sqrt{\phi_1^2 + \phi_2^2}$, with ϕ_1 and ϕ_2 being two real solutions of (42) satisfying

$$\phi_1 \dot{\phi}_2 - \dot{\phi}_1 \phi_2 = 1. \tag{55}$$

Representations of the multiverse vacua: $\hat{b}$ and $\hat{c}$ representations

- $\hat{c}-$representation defines an invariant vacuum state **independently of the classical evolution of the universe**
- in $\hat{b}-$representation there is **creation or annihilation of pairs of the universes** with well-determined value of the momentum in minisuperspace
- representations related by **Bogoliubov transformations**

$$\hat{c}_- = \alpha\hat{b}_- - \beta\hat{b}_+^\dagger, \quad (56)$$

$$\hat{c}_-^\dagger = \alpha^*\hat{b}_- - \beta^*\hat{b}_+, \quad (57)$$

where α, β - Bogoliubov coefficients fulfill $\mid \alpha \mid^2 - \mid \beta \mid^2 = 1$.

Invariant representation of vacuum

In terms of the invariant representation (53)–(54), the Hamiltonian (51) reads

$$H = H_0^- + H_0^+ + H_I, \tag{58}$$

where

$$H_0^{\pm} = \Omega(a)\left(c_{\pm}^{\dagger}c_{\pm} + \frac{1}{2}\right), \tag{59}$$

and,

$$H_I = \gamma(a)c_+^{\dagger}c_-^{\dagger} + \gamma^* c_+ c_-, \tag{60}$$

is the **Hamiltonian of interaction** (describing a non-local interaction by an entangled pair) of the universes while

$$\Omega(a) = \frac{1}{4}\left(\frac{1}{R^2} + R^2\omega^2 + \dot{R}^2\right), \tag{61}$$

$$\gamma(a) = -\frac{1}{4}\left\{\left(\dot{R} + \frac{i}{R}\right)^2 + \omega^2 R^2\right\}. \tag{62}$$

Entanglement thermodynamics - general framework

- The **multiverse is in the vacuum state** described by the ground state of the invariant representation in the minisuperspace, $|0_+ 0_-\rangle_c$ - **a pure state with zero entropy** - evolves unitary so that the entropy is constantly zero.
- However, there is **non-zero entropy of entanglement in each single universe** (for an internal observer) which eventually can give rise to an arrow of time.
- The state of the universe **for an internal observer** in terms of b-representation reads (Robles-Perez and Gonzalaez-Diaz 2014)

$$|0_+ 0_-\rangle_c = \frac{1}{|\alpha|} \sum_{n=0}^{\infty} \left(\frac{|\beta|}{|\alpha|} \right)^n |n_-, n_+\rangle_b, \qquad (63)$$

where $|n_-, n_+\rangle_b$ are the entangled mode states of the b-representation, and α and β are the Bogoliubov coefficients that relate both representations ($|\alpha|^2 - |\beta|^2 = 1$)

$$\hat{c}_- = \alpha \hat{b}_- - \beta \hat{b}_+^\dagger, \quad \hat{c}_-^\dagger = \alpha^* \hat{b}_-^\dagger - \beta^* \hat{b}_+. \qquad (64)$$

Entanglement thermodynamics

The **quantum state of a single universe of the entangled pair** can be obtained by **tracing out** the degrees of freedom of the partner universe

$$\rho_- = \mathrm{Tr}_+ \rho \equiv \sum_{n=0}^{\infty} {}_b\langle n_+ | \rho | n_+ \rangle_b, \tag{65}$$

where the density matrix is

$$\rho = |0_+ 0_-\rangle_c \langle 0_+ 0_-| \quad = \frac{1}{|\alpha|^2} \sum_{n,m} \left(\frac{|\beta|}{|\alpha|} \right)^{n+m} |n_-, n_+\rangle_b \langle m_-, m_+|. \tag{66}$$

As a result **we get a thermal state** (Robles-Perez and Gonzalez-Diaz 2014)

$$\rho_- = \frac{1}{|\alpha|^2} \sum_{n,m,l} \left(\frac{|\beta|}{|\alpha|} \right)^{n+m} \langle l_+ | m_+ \rangle |n_-\rangle_b \langle n_- | \langle m_+ | l_+ \rangle = \frac{1}{Z} \sum_n e^{-\frac{\omega}{T}(n+\frac{1}{2})} |n_-\rangle_b \langle n_-|, \tag{67}$$

where the partition function $Z^{-1} = 2 \sinh \frac{\omega}{2T}$.

Entanglement thermodynamics

Now we have the temperature of entanglement

$$T \equiv T(a) = \frac{\omega(a)}{2 \ln \coth r}, \quad \tanh r \equiv \frac{|\beta|}{|\alpha|}, \tag{68}$$

where r – the entanglement parameter (Nakagawa 2016, Baskal 2016).
The **entropy of entanglement** is given by the von Neumann formula (Horodecki et al. 2009, Nakagawa 2016, Baskal 2016)

$$S(\rho) = -\mathrm{Tr}\left(\rho \ln \rho\right), \tag{69}$$

applied to the thermal state ρ_-, and yields (Robles-Perez and Gonzlez-Diaz 2014)

$$S_{\text{ent}}(a) = \cosh^2 r \, \ln \cosh^2 r - \sinh^2 r \, \ln \sinh^2 r. \tag{70}$$

Entanglement thermodynamics - key points

- **We obtained a thermal state of a single universe of an entangled pair** for an internal observer from the zero entropy vacuum state of the superspace **of an external observer**.
- The entropy of entanglement for a single universe observer depends on the scale factor - it is not unitary due to non-local interaction producing entanglement.
- For an external observer the evolution of an entangled pair is unitary - no information paradox.
- The mean value of the Hamiltonian $\hat{H}_- = \omega(\hat{b}_-^\dagger \hat{b}_- + 1/2)$ is

$$E_-(a) \equiv \langle \hat{H}_- \rangle = \mathrm{Tr} \hat{\rho}_- \hat{H}_- = \omega \left(\langle \hat{N}(a) \rangle + \frac{1}{2} \right), \quad \langle \hat{N}(a) \rangle = \sinh^2 r. \tag{71}$$

Entanglement quantities - sinusoidal pulse

For the sinusoidal entangled pair of the universes we have

$$\alpha = \frac{1}{2}\left(\frac{1}{R\sqrt{\omega}} + R\sqrt{\omega} - \frac{i\dot{R}}{\sqrt{\omega}}\right), \tag{72}$$

$$\beta = -\frac{1}{2}\left(\frac{1}{R\sqrt{\omega}} - R\sqrt{\omega} - \frac{i\dot{R}}{\sqrt{\omega}}\right), \tag{73}$$

with the WKB solutions

$$\phi_1 = \frac{1}{\sqrt{\omega}}\cos S, \quad \phi_2 = \frac{1}{\sqrt{\omega}}\sin S, \tag{74}$$

which yields $R\sqrt{\omega} = 1$, and

$$\alpha = 1 + \frac{i\dot{\omega}}{4\omega^2}, \quad \beta = -\frac{i\dot{\omega}}{4\omega^2}, \tag{75}$$

with $|\alpha|^2 - |\beta|^2 = 1$, and $\dot{R} = -\frac{1}{2}\dot{\omega}\omega^{-\frac{3}{2}}$.

Entanglement quantities - sinusoidal pulse

We then obtain

$$\tanh r = \frac{|\beta|}{|\alpha|} = \frac{\dot{\omega}}{\sqrt{16\omega^4 + \dot{\omega}^2}} = \frac{1}{\sqrt{1 + \left(\frac{4\omega^2}{\dot{\omega}}\right)^2}}, \tag{76}$$

with, $\dot{\omega} \equiv \frac{d\omega}{da}$, and $\omega(a)$ given by (40), so that

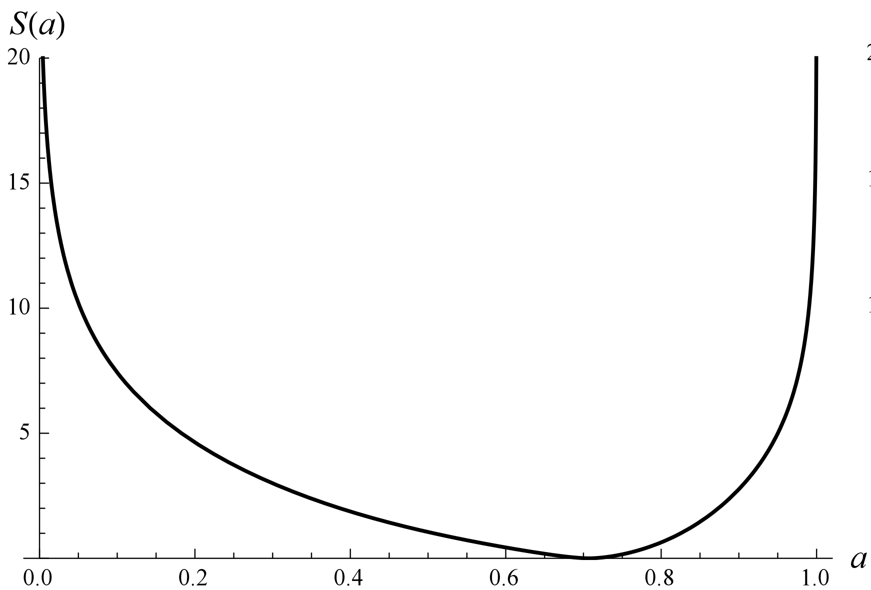
$$\tanh r = \frac{1}{\sqrt{1 + 16a^4 \frac{(1-\Lambda a^2)^3}{(1-2\Lambda a^2)^2}}} \equiv q. \tag{77}$$

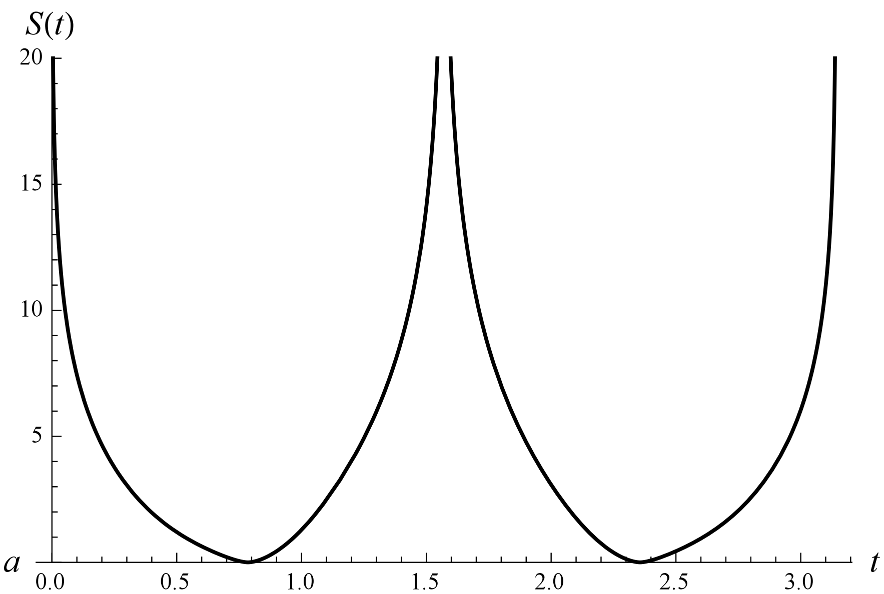
Finally we have for the **temperature and entropy of entanglement**

$$T = -\frac{a\sqrt{1-\Lambda a^2}}{2\ln q}, \tag{78}$$

$$S = \frac{1}{1-q^2}\ln\left[\frac{1}{1-q^2}\right] - \frac{q^2}{1-q^2}\ln\left[\frac{q^2}{1-q^2}\right]. \tag{79}$$

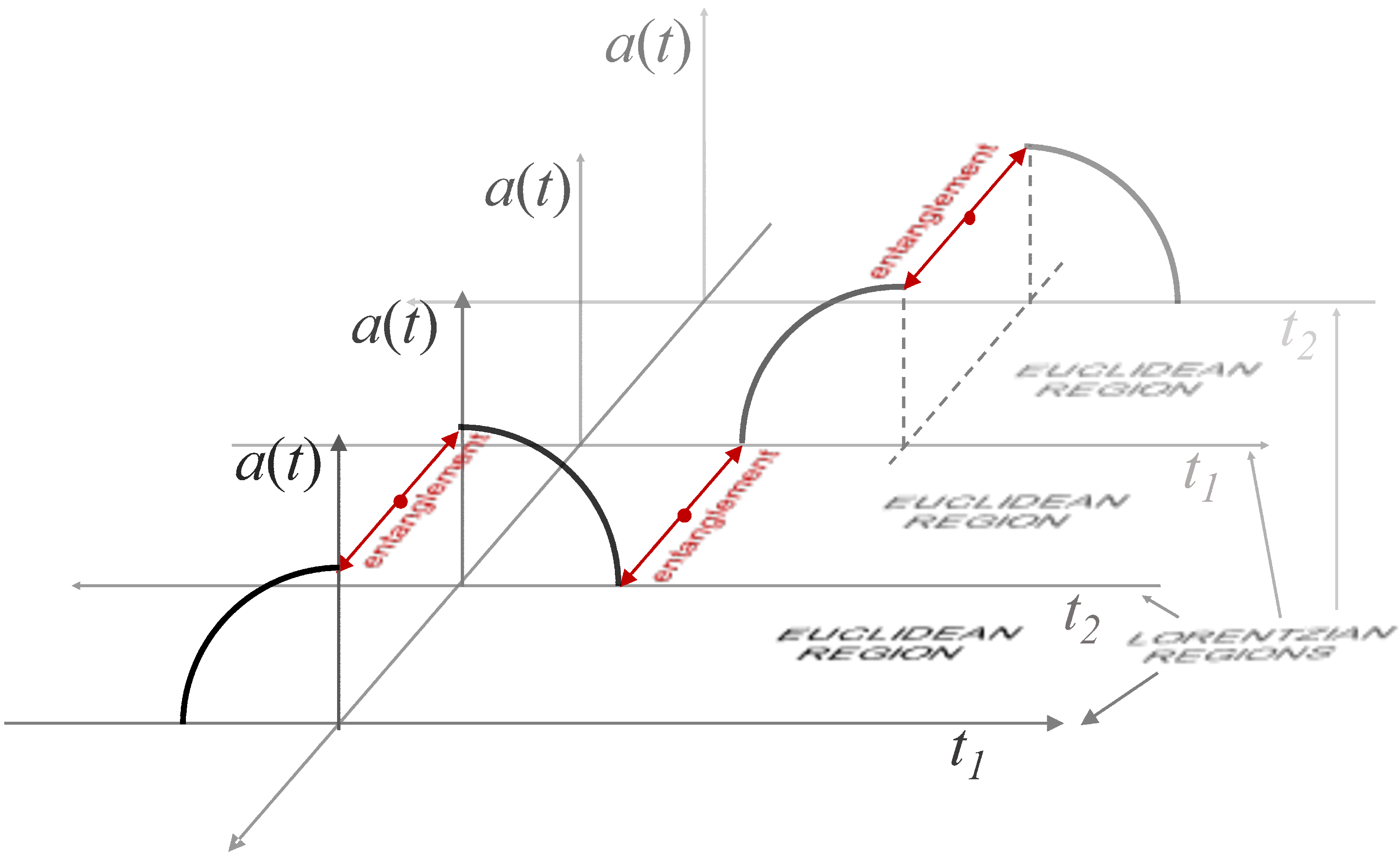
Entropy of entanglement - results (sinusoidal pulse)



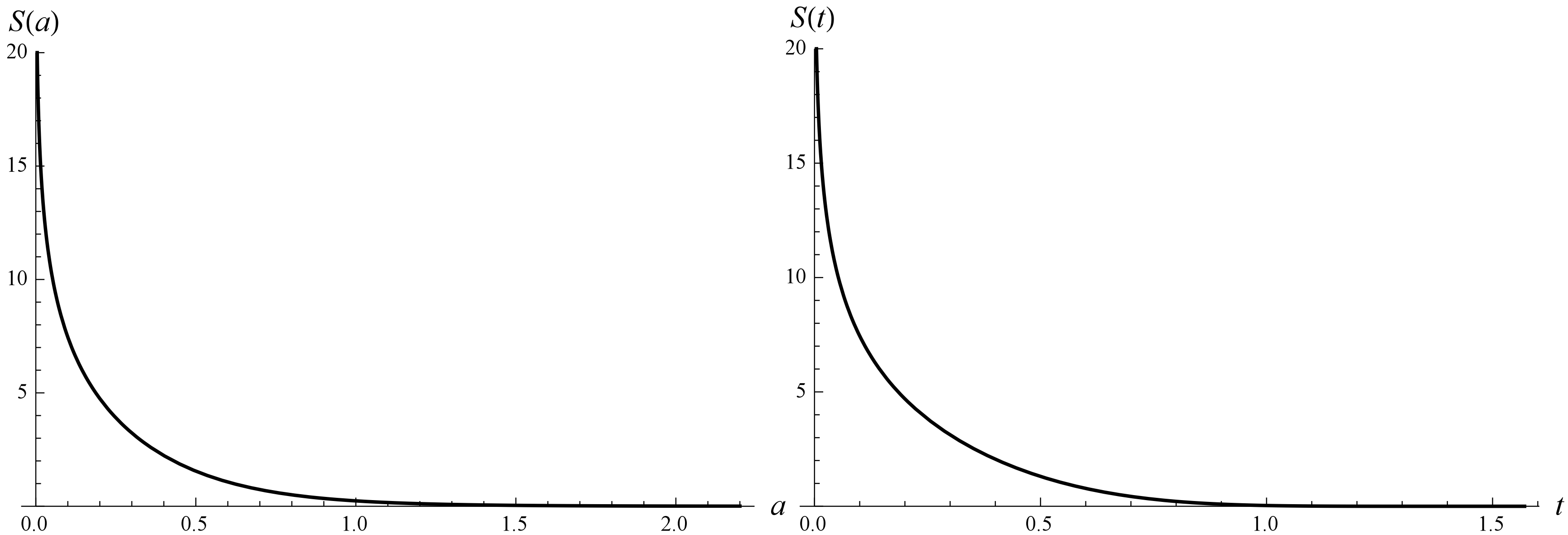


Result: Entropy is large (infinite) for big-bang ($a = 0$), big-crunch ($a = 2$ - not plotted) and also for the maximum expansion (!!!) ($a = 1$) regions

Creation of entangled pairs at maximum expansion

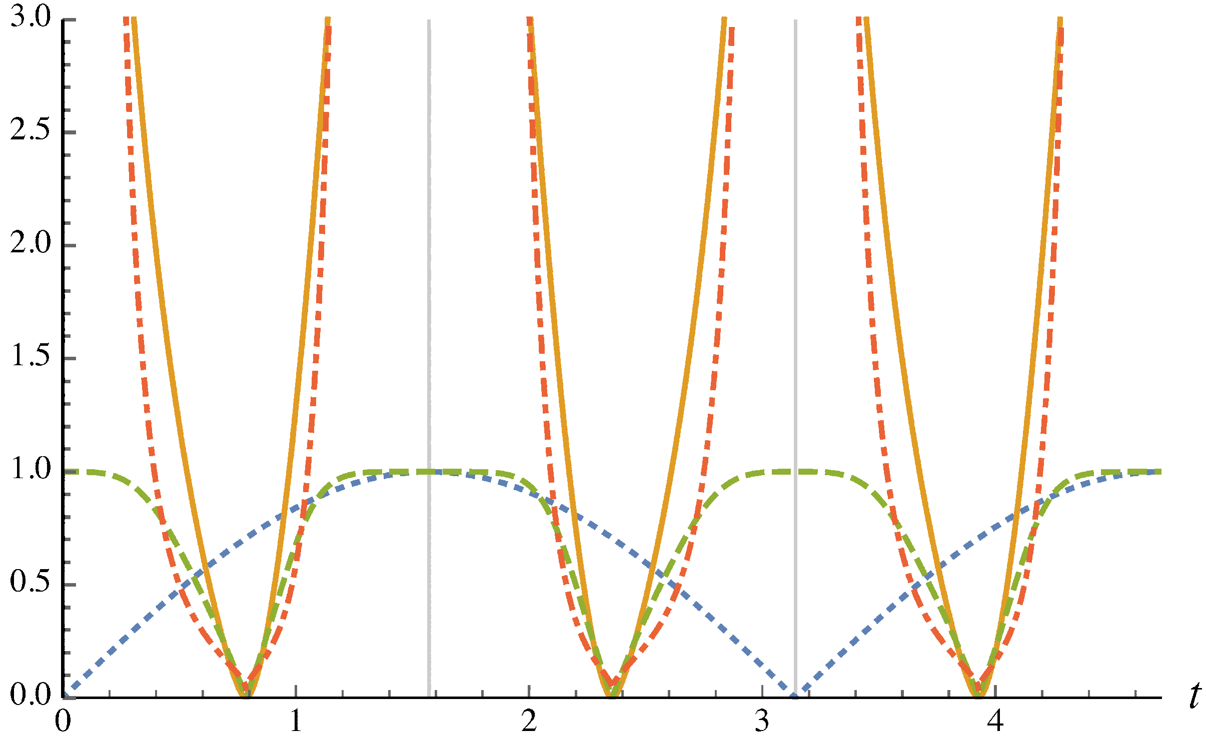


Entropy of entanglement - results (tangential pulse).



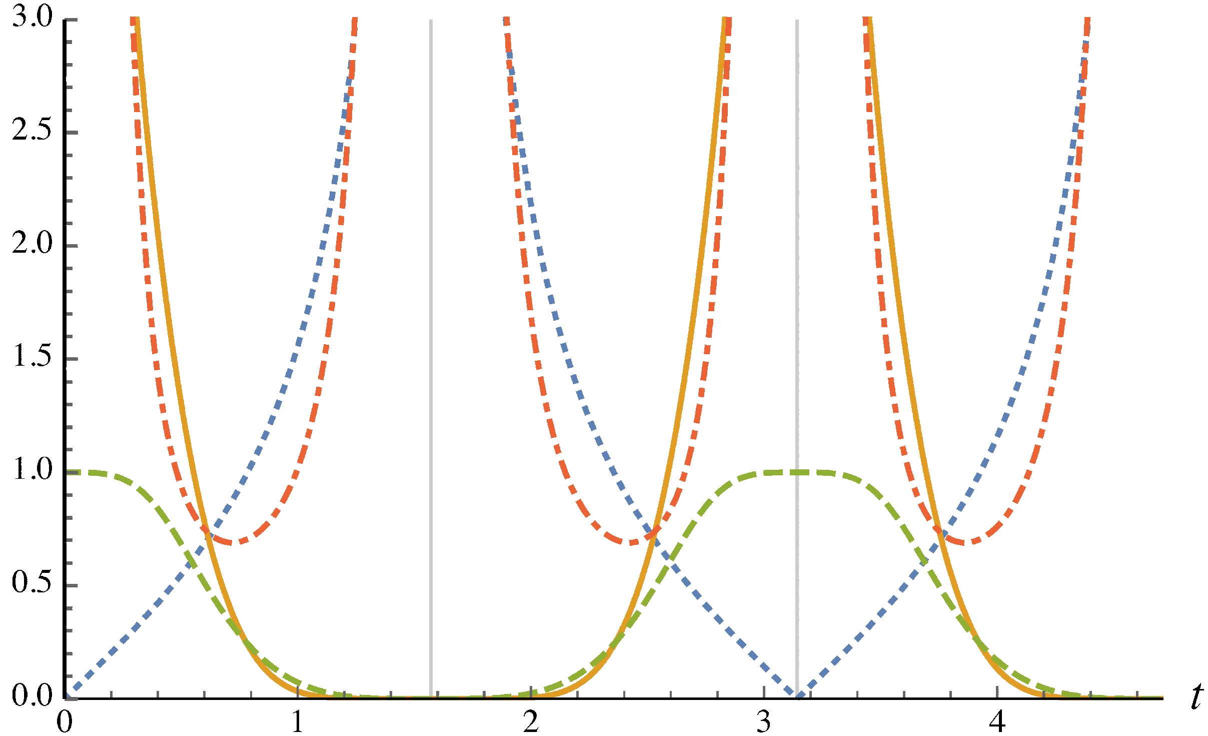
- Result: Entropy is **large** (infinite) for big-bang ($a = 0$) region, but **vanishes** for big-rip ($a = \infty$) region!
- Problem: Is the entropy of entanglements the proper measure of **quantumness**? (We know big-rip achieves Planck density)

Results: temperature of entanglement - sinusoidal pulse



Scale factor (blue, dotted), entanglement parameter q (green, dashed), entropy of entanglement (yellow, solid line), and temperature of entanglement (red, dot-dashed) for the sinusoidal pulse. Unlike the entropy of entanglement, the **parameter q turns out to be a non-divergent (finite) measure of the entanglement.**

Results: temperature of entanglement - tangential pulse.



Scale factor (blue, dotted), entanglement parameter q (green, dashed), entropy of entanglement (yellow, solid line), and temperature of entanglement (red, dot-dashed) for the tangential pulse. The temperature of entanglement can be **an indicator of the quantumness** of the universes.

Observational signatures of the multiverse.

- Bearing in mind a harmonic oscillator analogue of the WdW equation one may calculate **the entanglement energy**

$$E = \frac{\omega_{eff}}{2} = \omega \left(\sinh^2 r + \frac{1}{2} \right) , \tag{80}$$

which is analogous to a quantized oscillator in a vacuum state related to the frequency $\omega = \omega(a)$ in eq. (42) when there is no entanglement i.e. if $r = 0$.

- An analogue of the Friedmann equation (39) for an entangled universe would then be

$$\frac{da}{dt} = \frac{\omega_{eff}}{a} = \frac{\omega}{a} \left(1 + 2 \sinh^2 r \right) \tag{81}$$

which **gives a quantum entanglement correction** to the classical evolution of a universe contained in the multiverse.

Observational signatures of the multiverse.

- Quantum entanglement effect of the multiverse can be observed due to an appropriate term of **quantum interaction** in any universe of the multiverse i.e. also in Our Universe.
- Practical realization by **an extra term in the Friedmann equation**

$$H^2 = (8\pi G/3c^4)\rho + \quad \text{quantum} \quad \text{entanglement} \tag{82}$$

- entanglement signal can be **imprinted in the spectrum** of the cosmic microwave background (CMB) in the form of an extra dipole which is a cause of dark matter flow (Mersini-Houghton, Holman 2008; Kinney 2016)
- entanglement also influences the potential of a scalar field which drives cosmological inflation and so a **change of the CMB temperature** (Di Valentino, Mersini-Houghton 2017, 2018; Bouhmadi-Lopez 2018)

6. Conclusions

- Using varying constants (c, G) theories we created cyclic universes and extended them into the **cyclic multiverse scenarios with same geometry and different evolution of the coupling constants** still obeying the overall 2nd law of thermodynamics (”dobubleverses”).
- Quantum methods (from 2nd (WDW) to the 3rd quantization scheme) allowed us to consider possible **cyclic universes’ pair creation** and their quantum entanglement.
- **The entropy of entanglement** is large at small values of the scale factor $a \approx 0$ (big-bang, big-crunch) as well as at the **maximum expansion** point $a \approx a_{max}$ suggesting strong ”quantumness” of these minisuperspace points.
- The entropy of entanglements vanishes at large values of the scale factor $a \to \infty$ (big-rip) despite presumably strong ”quantumness” of this point. However, **the temperature of entanglement** is large for this point and perhaps is more appropriate to measure its ”quantumness”.