From varying constants to the entangled cyclic multiverses

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ICNFP2019, Kolymbari 29 August 2019
Plan:

1. Introduction.
2. Dynamical constants vs singularities in cosmology.
3. Dynamical constants and cyclic universes.
4. Thermodynamics and a toy cyclic multiverse.
5. Inter-universal entanglement in the multiverse.
6. Conclusions.
References

1. Introduction

- One of the main problems of cosmology is the **problem of singularities**. There are many approaches to construct non-singular models, but usually it is difficult to avoid singularities under some generic conditions.

- Besides, there are **various types of singularities** with quite different properties (like Big-Rip, Sudden Future, Finite Scale Factor, Little-Rip etc.) not necessarily geodesically incomplete ("weak singularities").

- Main concern of this talk:
  - Is it possible to construct a **cyclic universe** with no singularities or ”weaker” singularities? (**multiverse in time**)
  - Is it possible to construct any classical or quantum ”**cyclic parallel universes**” scenario? (**multiverse in space & time**)
  - Are there **any effects** which link these parallel universes? Are they **observable** in our universe?
2. Dynamical constants vs singularities in cosmology.

- According to Hawking and Penrose (1973) a spacetime is singular if there exists **at least one geodesic which is incomplete** i.e. which cannot be extended in at least one direction and has only a finite range of affine parameter (proper time or length for non-null geodesics).

- This is a kind of “minimalistic” approach which **does not tell us the full nature of these singularities**: e.g. how they influence the physical and geometrical quantities.

- An interesting set of **alternative/extended gravity cosmologies** are **dynamical constants cosmologies** which have been applied to **solve some standard cosmology problems** such as the horizon and flatness problem (e.g. Moffat 1993, Albrecht, Magueijo 1999; Barrow 1999, Uzan 2003).

- Our underlying idea was **to apply dynamical constants to remove or to change the strength** of singularities in cosmology (MPD, Marosek 2013; MPD, Balcerzak, Marosek 2014).
Variety of singularities: strength.

- Tipler’s (Phys. Lett. A64, 8 (1977)) definition (of a strong singularity):
  \[ I^i_j(\tau) = \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' |R^i_{a,jb}u^a u^b| \]
  diverges on the approach to a singularity at \( \tau = \tau_s \)

- i.e. an extended object is **crushed to zero volume** (represented by three linearly independent, vorticity-free geodesic deviation vectors at \( p \) parallelly transported along causal geodesic \( l \)) at the singularity by infinite tidal forces

- Królik’s (CQG 3, 267 (1988)) definition (of a strong singularity):
  \[ I^i_j(\tau) = \int_0^\tau d\tau' |R^i_{a,jb}u^a u^b| \]
  diverges on the approach to a singularity at \( \tau = \tau_s \)

- i.e. the **expansion** of every future-directed congruence of null (timelike) geodesics emanating from point \( p \) and containing \( l \) **becomes negative** somewhere on \( l \)

- For null geodesics one replaces Riemann by the Ricci tensor components.
## Variety of singularities: strength (MPD 2015)

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From varying constants to the entangled cyclic multiverses – p. 7/54
Dynamical constants $G(t)$ and $c(t)$ vs singularities

We considered the simplest theory for the generalized Einstein-Friedmann equations in *varying speed of light (VSL)* theories (Barrow & Magueijo model - 1999) and *varying gravitational constant G* theories ($\varrho$ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density)

\begin{align*}
\varrho(t) &= \frac{3}{8\pi G(t)} \left( \frac{\ddot{a}^2}{a^2} + \frac{k c^2(t)}{a^2} \right), \quad (1) \\
p(t) &= -\frac{c^2(t)}{8\pi G(t)} \left( 2\frac{\ddot{a}}{a} + \frac{\ddot{\varrho} a^2}{a^2} + \frac{k c^2(t)}{a^2} \right), \quad (2)
\end{align*}

and the energy-momentum “conservation law” is (related to 2nd law of thermodynamics)

\begin{align*}
\dot{\varrho}(t) + 3\frac{\dot{a}}{a} \left( \varrho(t) + \frac{p(t)}{c^2(t)} \right) = -\varrho(t) \frac{\dot{G}(t)}{G(t)} + 3\frac{k c(t) \dot{c}(t)}{4\pi G a^2}. \quad (3)
\end{align*}
"All-in-one" scale factor.

We used a general form of the scale factor (MPD, K. Marosek, JCAP 02 (2013), 012), which allows for big-bang, big-rip, sudden future, finite scale factor and \( w \)-singularities and reads as

\[
a(t) = a_s \left( \frac{t}{t_s} \right)^m \exp \left( 1 - \frac{t}{t_s} \right)^n,
\]

with the constants \( t_s, a_s, m, n \).

For \( 0 < m < 2/3 \) we have a big-bang singularity - \( a \to 0, \varrho \to \infty, p \to \infty \) at \( t \to 0 \);

For \( m < 0 \) we have a big-rip singularity - \( a \to \infty, \varrho \to \infty, p \to \infty \) at \( t = 0 \);

For \( 1 < n < 2 \) we have a sudden future singularity (SFS) which appears at \( t = t_s \) (\( a = a_s, \varrho = \text{const.}, p \to \infty \));

For \( 0 < n < 1 \) we have a (stronger) finite scale factor singularity (FSF) at \( t = t_s \) (\( a = a_s, \varrho \to \infty, p \to \infty \)).
More examples of regularizing other singularities

- To remove SFS: light has to stop moving at the singularity (same happens in loop quantum cosmology (LQC) where it is called the anti-newtonian limit \( c = c_0 \sqrt{1 - \frac{\rho}{\rho_c}} \to 0 \) for \( \rho \to \rho_c \) with \( \rho_c \) being the critical density (Cailettau et al. 2012). The low-energy limit \( \rho \ll \rho_0 \) gives the standard limit \( c \to c_0 \).

- To regularize an SFS, FSF by varying gravitational constant \( G(t) \) - the strength of gravity has to become infinite at a singularity (seems reasonable because of the requirement to overcome an infinite (anti-)tidal forces at the singularity).

- However, it makes another singularity - a singularity of strong coupling for a physical field such as \( G \propto 1/\Phi \).

- Such problems were already dealt with for example in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity appeared.
3. Dynamical constants and cyclic universes.

An idea is similar to some superstring/brane universe evolution scenarios like the **ekpyrotic/cyclic models** (Khoury et al. 2001, Steinhardt et al. 2002, Turok et al. 2005 etc.) where one has **some special coupling** of a scalar field $\beta(\phi)$ of gravity in the Lagrangian of a 4-dimensional theory (Einstein frame)

$$S = \int d^4 x \sqrt{-g} \left[ \frac{c^4}{16\pi G} R - \frac{1}{2} \partial\mu \phi \partial^\nu \phi - V(\phi) + \beta^4(\phi)(\varrho_R + \varrho_m) \right],$$

(5)

where the potential has an explicit form as

$$V(\phi) = V_0 \left(1 - e^{-s\phi}\right) F(\phi), \quad F(\phi) \propto e^{-\frac{1}{g_s}},$$

(6)

($\varrho_R$ is the energy density of radiation while $\varrho_m$ is the energy density of matter, $g_s$ - dilaton/string coupling constant) which **leads to cyclic models of the universe.**
We construct cyclic models using the idea of dynamical physical constants instead of the special coupling (MPD & Marosek 2013) using eqs. (1), (2), and (3).

First we assume that the gravitational constant $G$ varies (and $\dot{c} = 0$) as

$$G(t) = \frac{G_0}{a^2(t)}, \quad (7)$$

and for the positive curvature ($k = +1$) we obtain sinusoidal pulse Friedmann model with the scale factor

$$a(t) = a_0 |\sin \left( \pi \frac{t}{t_c} \right)| \quad (8)$$

with $a_0 =$const., so the scale factor is ”singular” (reaching zero as in the big-bang scenario).
Sinusoidal pulse model: $a(t), G(t)$. 
Dynamical constants and cyclic universes.

The mass density $\rho(t)$ and the pressure $p(t)$ are nonsingular and oscillatory

$$\rho(t) = \frac{3}{8\pi G_0} \left[ \frac{\pi^2 a_0^2 \cos^2 \left( \frac{\pi t}{t_c} \right)}{t_c^2} + c^2 \right] \geq 0,$$  

$$p(t) = -\frac{c^2}{8\pi G_0} \left[ \frac{3\pi^2 a_0^2 \cos^2 \left( \frac{\pi t}{t_c} \right)}{t_c^2} + c^2 - 2\frac{\pi^2 a_0^2}{t_c^2} \right].$$  

For these models the null energy condition is always fulfilled (a "singular bounce").

$$p(t) + \rho(t)c^2 = p(mt_c) \equiv p_a \geq 0 \quad m = 0, 1, 2, 3, \ldots$$  

The same is true for the weak energy condition which additionally requires \((\rho \geq 0).\)
Sinusoidal pulse model: density and pressure.

Strong energy condition is also fulfilled

\[ \rho(t) c^2 + 3p(t) = \frac{3a_0^2 \pi c^2}{4G_0 t_c^2} \sin^2 \left( \pi \frac{t}{t_c} \right) \geq 0, \quad (12) \]

and the universe is decelerating (\( \ddot{a} < 0 \)).
Dynamical constants and cyclic universes.

- In fact, in our model we have no proper bounce in the scale factor since

\[ \dot{H} = -\frac{\pi^2}{t_c^2} - \frac{\dot{a}^2}{a^2} < 0. \]  

(13)

Defining

\[ H_G = \frac{\dot{G}}{G} = -2\frac{\dot{a}}{a} = -2\dot{H} \]  

(14)

we can see that \( \dot{H}_G > 0 \) always accompanies \( \dot{H} < 0 \) (a contraction of space is always balanced by an expansion of gravity) which makes an apparently singular and sharp bounce regular in matter density and pressure due to the special form of running gravity.

- In general, the strong energy condition means that gravity is attractive, but in our case its attractiveness is overbalanced by the strong coupling of \( G(t) \) at the Big-Bang type of singularity (\( G \to \infty \)).
Another interesting case is the **tangential pulse model** with the scale factor

\[ a(t) = a_0 \left| \tan \left( \pi \frac{t}{t_s} \right) \right| , \]  

(15)

which can be accompanied by the gravitational constant varying as

\[ G(t) = \frac{4G_s}{\sin^2 \left( 2\pi \frac{t}{t_s} \right)} . \]  

(16)

This scale factor is infinite \((a \to \infty)\) for \(t = nt_s\) with \(n = 1/2, 3/2, 5/2, \ldots\) (like at **big-rips**) and it is zero for \(t = mt_s\) with \(m = 0, 1, 2, 3, \ldots\) (like at **big-bangs**) so that we can say that we face ”singular bounces”.

Each time the scale factor \(a(t)\) attains a singular value (vanishes or reaches infinity), the **gravitational coupling becomes infinite** \((G \to \infty)\).
Tangential pulse model - $a(t), G(t)$. 

![Graph showing $a(t)$ and $G(t)$ over time $t$.](attachment:graph.png)
Dynamical constants and cyclic universes.

- The mass density and pressure are nonsingular

\[
\rho(t) = \frac{3}{8\pi G_s} \left[ \frac{\pi^2}{t_s^2} + \frac{3c^2 \cos^4\left(\pi \frac{t}{t_s}\right)}{a_0^2} \right] \geq 0, \tag{17}
\]

\[
p(t) = -\frac{c^2}{8\pi G_s} \left[ \frac{\pi^2}{t_s^2} + 4 \frac{\pi^2 \sin^2\left(\pi \frac{t}{t_s}\right)}{t_s^2} + \frac{c^2 \cos^4\left(\pi \frac{t}{t_s}\right)}{a_0^2} \right], \tag{18}
\]

- The null energy condition is satisfied at \( t = mt_s \) (Big-Bang-like sings) and violated at \( t = nt_s \) (Big-Rip-like sings):

\[
c^2 \rho(t) + p(t) = \frac{c^2}{4\pi G_s} \left[ \frac{\pi^2}{t_s^2} - 2 \frac{\pi^2 \sin^2\left(\pi \frac{t}{t_s}\right)}{t_s^2} + 4 \frac{c^2 \cos^4\left(\pi \frac{t}{t_s}\right)}{a_0^2} \right]. \tag{19}
\]
Tangential pulse model: density and pressure.
Dynamical constants and cyclic universes.

- It proves that the former is a Big-Rip-like singularity and the latter is a Big-Bang-like singularity. The word "like" comes from the fact that the mass density and pressure are regular which is not the case at a Big-Bang and a Big-Rip.

- The pressure at both of these singularities \( p(mt_s) \) and \( p(nt_s) \) is negative.

- The strong energy condition reads as

\[
c^2 \rho (t) + 3p (t) = \frac{3c^2}{4\pi G_s} \left[ \frac{c^2 \cos^4 \left( \frac{\pi t}{t_s} \right)}{a_0^2} - 2 \frac{\pi^2 \sin^2 \left( \frac{\pi t}{t_s} \right)}{t_s^2} \right] \tag{20}
\]

- so that

\[
c^2 \rho (mt_s) + 3p (mt_s) = \frac{3c^4}{4\pi G_s a_0^2} \geq 0, \quad c^2 \rho (nt_s) + 3p (nt_s) = -\frac{3c^2}{2\pi G_s t_s^2} \leq 0.
\]
4. Thermodynamics and a toy cyclic multiverse.

- Using previous cyclic models pattern we now develop an idea of two independently evolving universes (doubleverse) which allow the same evolution of the scale factor but different evolution of the physical constants in each universe.

- The idea based on the thermodynamics of varying $c$ and varying $G$ universes. We assume that the multiverse obeys the 2nd law of thermodynamics, though individual universes may not do so.

- From thermodynamical interpretation we conclude that the entropy can be related to varying constants as (e.g. Youm 2002)

$$S(t) = 2 \frac{Nk_B}{\bar{w}} \ln [A_0 c(t)], \quad S(t) = Nk_B \ln \left[ \frac{A_0}{G(t)} \right], \quad (21)$$

where $A_0$ is an integration constant (and can also be taken to be one), $k_B$ - Boltzmann constant, $\bar{w}$ - barotropic index, $N$-number of particles.
Cyclic multiverse - varying $c$.

As first step assume that the entropy of the multiverse is constant:

$$\dot{S} = \sum_{i=1}^{n} \dot{S}_i = \dot{S}_1 + \dot{S}_2 + \dot{S}_3 + \ldots + \dot{S}_n = 0. \quad (22)$$

For the “doubleverse” we have:

$$S_1 = \frac{2}{\widetilde{w}} N_1 k_B \ln [c_1(t)], \quad (23)$$

$$S_2 = \frac{2}{\widetilde{w}} N_2 k_B \ln [c_2(t)]. \quad (24)$$

Make the following ansätze for $c(t)$:

$$c_1(t) = e^{\lambda_1 \phi_1(t)}, \quad (25)$$

$$c_2(t) = e^{\lambda_2 \phi_2(t)}, \quad (26)$$

where $\lambda_1$ and $\lambda_2$ are constants.
Cyclic multiverse.

- The **total entropy of the doubleverse** is

\[
S = S_1 + S_2 = 2 \frac{\tilde{\rho}_1 V_1 c_1^2}{T_1} \lambda_1 \phi_1 + 2 \frac{\tilde{\rho}_2 V_2 c_2^2}{T_2} \lambda_2 \phi_2 (t) .
\]  
(27)

- For simplicity we take (solves exactly):

\[
\frac{\tilde{\rho}_1 V_1 c_1^2}{T_1} \lambda_1 = \frac{\tilde{\rho}_2 V_2 c_2^2}{T_2} \lambda_2 ,
\]  
(28)

and using

\[
N_1 \lambda_1 = N_2 \lambda_2 .
\]  
(29)

- This allows to pick up the following functional dependence:

\[
\phi_1 (t) = \sin^2 \left( \pi \frac{t}{t_s} \right) ,
\]  
(30)

\[
\phi_2 (t) = \cos^2 \left( \pi \frac{t}{t_s} \right) ,
\]  
(31)
Cyclic multiverse - varying $c$.

So that the entropies are

$$S_1 = \frac{2}{\tilde{w}} N_1 k_B \lambda_1 \sin^2 \left( \frac{\pi t}{t_s} \right), \tag{32}$$

$$S_2 = \frac{2}{\tilde{w}} N_2 k_B \lambda_2 \cos^2 \left( \frac{\pi t}{t_s} \right). \tag{33}$$

Two scale factors evolutions are equal and of the similar form as in cyclic universe models of the previous section:

$$a(t) = a_1(t) = a_2(t) = a_0 \left| \sin \left( \frac{\pi t}{t_s} \right) \right| \tag{34}$$

- Their evolution is evidently cyclic.
- The point is that although the geometrical evolution of the "parallel" universes is the same, this is not the case with the evolution of the physical constants $c_1(t)$ and $c_2(t)$ which is evidently different.
Total entropy remains constant.

Similar considerations are valid for the models with varying $G$. 
5. Inter-universal entanglement in the multiverse.

- Now we assume that the universes 1 and 2 are quantum mechanically entangled and there are periods of their evolution when the entanglement matters (e.g. at the maximum expansion point) and influences the behaviour of individual universes, though most of the evolution of the individual universes is classical.

- It means that we consider the multiverse with two patches - the universes 1 and 2 which can be understood in the hierarchy given by Tegmark (2003) from level I (separate inflationary patches) to IV.

- These two patches are classically disconnected, but they can be quantum mechanically entangled (Robles-Perez et al. 2010, 2014, 2015) and the effect of entanglement can be imprinted in individual universes (for example in CMB spectrum, large-scale structure etc.).
For the previously considered **sinusoidal pulse** model we write the Friedmann equation in the form

\[ H^2 = -\Lambda + \frac{1}{a^2}, \]  

(35)

where

\[ \Lambda \equiv \frac{\pi^2}{t_c^2} \text{ and } a_0 = \frac{1}{\sqrt{\Lambda}}, \]  

(36)

while for the **tangential pulse**

\[ H^2 = \frac{1}{a^2} \left(1 + \Lambda a^2\right)^2 = \Lambda^2 a^2 + 2\Lambda + \frac{1}{a^2}, \]  

(37)

where the first term on the right-hand side scales as phantom.
Wheeler-deWitt (second) quantization

Getting conjugate momentum as

\[ p_a = -a \frac{da}{dt}, \quad (38) \]

the Hamiltonian constraint

\[ p_a^2 - \omega^2(a) = 0, \quad (39) \]

can easily be derived from the Friedmann equations as

\[ \omega_{\text{sin}}^2(a) \equiv a^2 - \Lambda a^4. \quad (40) \]

for the sinusoidal pulse and as

\[ \omega_{\text{tan}}^2(a) \equiv \Lambda^2 a^6 + 2\Lambda a^4 + a^2. \quad (41) \]

for the tangential pulse.
The Wheeler-DeWitt equation is formally similar to the Klein-Gordon equation

\[ \ddot{\phi} + \omega^2(a) \phi = 0, \quad (42) \]

where \( \phi \equiv \phi(a) \) is the wave function and \( \dot{\phi} \equiv \frac{d\phi}{da} \). The WKB solutions of (42) (where two signs correspond to two different branches of the universe)

\[ \phi_\pm \propto \frac{1}{\sqrt{2\omega}} e^{\pm iS}, \quad (43) \]

where, \( \dot{S} = \omega \). For the sinusoidal pulse, we get

\[ S = \int da \omega_{\sin}(a) = -\frac{(1 - \Lambda a^2)^{3/2}}{3\Lambda}. \quad (44) \]

For \( a \in (0, a_0) \), the WKB wave function (43) represents a Lorentzian (classical) universe; for \( a > a_0 \), the wave function represents an exponential decay of the Euclidean regime or quantum barrier.
Sinusoidal branches

We thus obtain two classical branches

\[ a(t) = \frac{1}{h} \sin[h(t - t_0)], \]  

(45)

and the other with scale factor given by

\[ a(t) = \frac{1}{h} \sin[h(t_0 - t)], \]  

(46)

which are related by time symmetry, \( t \rightarrow -t \) \( (t_0 \rightarrow -t_0) \).

- The same universe for any internal observer provided that: 1. the universes are created in entangled pairs; 2. observers’ time variables follow an antipodal-like symmetry (e.g. Linde1988, Robles-Perez 2014).

- Before reaching the big crunch singularities, one branch of the universe can undergo a quantum transition to the the other branch universe, appearing there as a newborn universe, forming thus a continuous cyclic multiverse.
Quantum transitions between universes in the multiverse

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Tangential branches

For the **tangential pulse**, we arrive at

\[ S = \int da \omega_{\text{tan}}(a) = \frac{1}{4} a^2 (2 + \Lambda a^2) . \]  
\[ (47) \]

Analogously as for the sinusoidal pulse, the **evolution that corresponds to the plus and minus signs** of \( \phi_{\pm} \) in (43) is given now by

\[ \frac{da}{dt} = \pm (h^2 a^2 + 1) . \]  
\[ (48) \]

We thus obtain

\[ a(t) = \frac{1}{h} \tan[h(t - t_0)] , \]  
\[ (49) \]

and

\[ a(t) = \frac{1}{h} \tan[h(t_0 - t)] , \]  
\[ (50) \]

for the two branches of the tangential pulse.
Creation of cyclic universes in entangled pairs.

- **Quantum effects dominant** close to $a = 0$ (though $p$ and $\rho$ regular) - say at $a = a_{\text{min}}$.

- For $a < a_{\text{min}}$ - no real solution found. Only double Euclidean instantons appear.

- They give rise (in the Lorentzian regime) to an **entangled pair of universes whose quantum states are quantum-mechanically correlated**.

- Observer living in the universe with time $t_1$ considers his branch **expanding**; the one with $t_2$ - **contracting**.

- Both observers see their universes expanding (antipodal symmetry) - two branches can be combined to form a universe that is **classically indistinguishable** from the cyclic single time picture.
Quantum creation of entangled pairs of the universes

From varying constants to the entangled cyclic multiverses – p. 35/54
Third quantization procedure of WdW equation - 3Q

- QFT formalism in which WdW wave function $\phi$ becomes an operator.
- This is due to fact that WdW equation (42) can be obtained from the Hamilton eqs of the (third-quantized) Hamiltonian

$$H = \frac{1}{2} P_\phi^2 + \frac{\omega^2(a)}{2} \phi^2,$$

(51)

where, $P_\phi \equiv \dot{\phi}$.

- 3Q makes $\phi$ and $P_\phi$ operators.

- We have

$$\hat{\phi}(a) = \frac{1}{\sqrt{2\omega}} e^{iS(a)} \hat{b}_+ + \frac{1}{\sqrt{2\omega}} e^{-iS(a)} \hat{b}_+^\dagger,$$

(52)

where, $\hat{b}_+ \equiv \hat{b}_+(a_{\text{min}})$ and $\hat{b}_+^\dagger \equiv \hat{b}_+^\dagger(a_{\text{min}})$, are constant operators given at some initial value, $a = a_{\text{min}}$. 
Universe annihilation and creation operators

- "-" branch \( a(t) = \frac{1}{\hbar} \sin[h(t - t_0)] \) - \( \hat{b}_- \) and \( \hat{b}_-^{\dagger} \)
- "+" branch - \( \hat{b}_+ \) and \( \hat{b}_+^{\dagger} \)
- similarly for the tangential pulse
- analogue of creation an entangled pairs of particles with opposite momentum \( \pm k \)
- symmetry of WDW equation wrt \( \pm \omega \) is translated into symmetry \( \pm \phi \) in 3Q picture
Representations of the multiverse vacua: $b$ and $c$ representations

- Vacuum state for $(b_{\pm}, b_{\pm}^\dagger)$ representation – given by $|0_+, 0_-\rangle$.

- Not unique because of scale factor dependence of $\omega = \omega(a)$ (along minisuperspace geodesic).

- An invariant representation for the harmonic oscillator like (42) is (Lewis and Riesenfeld JMP, 1458 (1969), Robles-Perez and Gonzalez-Diaz 2010, 2014)

\[
c_+ = \sqrt{\frac{1}{2}} \left( \frac{1}{R} \phi + i(RP_\phi - \dot{R}\phi) \right), \tag{53}
\]

\[
c_\dagger_- = \sqrt{\frac{1}{2}} \left( \frac{1}{R} \phi - i(RP_\phi - \dot{R}\phi) \right), \tag{54}
\]

where $R = \sqrt{\phi_1^2 + \phi_2^2}$, with $\phi_1$ and $\phi_2$ being two real solutions of (42) satisfying

\[
\phi_1 \dot{\phi}_2 - \dot{\phi}_1 \phi_2 = 1. \tag{55}
\]
Representations of the multiverse vacua: $\hat{b}$ and $\hat{c}$ representations

- $\hat{c}$—representation defines an invariant vacuum state **independently of the classical evolution of the universe**
- in $\hat{b}$—representation there is **creation or annihilation of pairs of the universes** with well-determined value of the momentum in minisuperspace
- representations related by **Bogoliubov transformations**

\[
\hat{c}_- = \alpha \hat{b}_- - \beta \hat{b}_+^\dagger, \tag{56}
\]
\[
\hat{c}_+^\dagger = \alpha^* \hat{b}_- - \beta^* \hat{b}_+^\dagger, \tag{57}
\]

where $\alpha, \beta$ - Bogoliubov coefficients fulfill $| \alpha |^2 - | \beta |^2 = 1$. 

From varying constants to the entangled cyclic multiverses – p. 39/54
In terms of the invariant representation (53)–(54), the Hamiltonian (51) reads

\[ H = H^-_0 + H^+_0 + H_I, \] (58)

where

\[ H^{\pm}_0 = \Omega(a) \left( c^\dagger_\pm c_\pm + \frac{1}{2} \right), \] (59)

and,

\[ H_I = \gamma(a) c^\dagger_+ c^\dagger_- + \gamma^* c_+ c_-, \] (60)

is the Hamiltonian of interaction (describing a non-local interaction by an entangled pair) of the universes while

\[ \Omega(a) = \frac{1}{4} \left( \frac{1}{R^2} + R^2 \omega^2 + \dot{R}^2 \right), \] (61)

\[ \gamma(a) = -\frac{1}{4} \left\{ \left( \dot{R} + \frac{i}{R} \right)^2 + \omega^2 R^2 \right\}. \] (62)
Entanglement thermodynamics - general framework

- The **multiverse is in the vacuum state** described by the ground state of the invariant representation in the minisuperspace, $|0_+0_-\rangle_c$ - a pure state with zero entropy - evolves unitary so that the entropy is constantly zero.

- However, there is **non-zero entropy of entanglement in each single universe** (for an internal observer) which eventually can give rise to an arrow of time.

- The state of the universe for an internal observer in terms of $b$-representation reads (Robles-Perez and Gonzalaez-Diaz 2014)

$$
|0_+0_-\rangle_c = \frac{1}{|\alpha|} \sum_{n=0}^{\infty} \left( \frac{|\beta|}{|\alpha|} \right)^n |n_-, n_+\rangle_b,
$$  \hspace{1cm} (63)

where $|n_-, n_+\rangle_b$ are the entangled mode states of the $b$-representation, and $\alpha$ and $\beta$ are the Bogoliubov coefficients that relate both representations ($|\alpha|^2 - |\beta|^2 = 1$)

$$
\hat{c}_- = \alpha \hat{b}_- - \beta \hat{b}_+^\dagger, \quad \hat{c}_+^\dagger = \alpha^* \hat{b}_-^\dagger - \beta^* \hat{b}_+.
$$  \hspace{1cm} (64)
Entanglement thermodynamics

The quantum state of a single universe of the entangled pair can be obtained by tracing out the degrees of freedom of the partner universe

$$\rho_- = \text{Tr}_+ \rho = \sum_{n=0}^{\infty} b \langle n_+ | \rho | n_+ \rangle_b,$$

(65)

where the density matrix is

$$\rho = |0_+0_-\rangle_c \langle 0_+0_-| = \frac{1}{|\alpha|^2} \sum_{n,m} \left( \frac{\beta}{|\alpha|} \right)^{n+m} |n_-, n_+\rangle_b \langle m_-, m_+|.$$

(66)

As a result we get a thermal state (Robles-Perez and Gonzalez-Diaz 2014)

$$\rho_- = \frac{1}{|\alpha|^2} \sum_{n,m,l} \left( \frac{\beta}{|\alpha|} \right)^{n+m} \langle l_+ | m_+ \rangle_b \langle n_- | m_+ | l_+ \rangle = \frac{1}{Z} \sum_n e^{-\frac{\omega}{2T} (n+\frac{1}{2})} |n_-\rangle_b \langle n_-|,$$

(67)

where the partition function $Z^{-1} = 2 \sinh \frac{\omega}{2T}$. From varying constants to the entangled cyclic multiverses – p. 42/54
Now we have the temperature of entanglement

\[ T \equiv T(a) = \frac{\omega(a)}{2 \ln \coth r}, \quad \tanh r = \frac{|\beta|}{|\alpha|}, \quad (68) \]

where \( r \) – the entanglement parameter (Nakagawa 2016, Baskal 2016).

The entropy of entanglement is given by the von Neumann formula (Horodecki et al. 2009, Nakagawa 2016, Baskal 2016)

\[ S(\rho) = -\text{Tr} (\rho \ln \rho), \quad (69) \]

applied to the thermal state \( \rho_\theta \), and yields (Robles-Perez and Gonzalez-Diaz 2014)

\[ S_{\text{ent}}(a) = \cosh^2 r \ln \cosh^2 r - \sinh^2 r \ln \sinh^2 r. \quad (70) \]
Entanglement thermodynamics - key points

- **We obtained a thermal state of a single universe of an entangled pair** for an internal observer from the zero entropy vacuum state of the superspace of an external observer.

- The entropy of entanglement for a single universe observer depends on the scale factor - it is **not unitary** due to non-local interaction producing entanglement.

- For an external observer the evolution of an entangled pair is **unitary** - no information paradox.

- The mean value of the Hamiltonian $\hat{H}_- = \omega (\hat{b}_-^\dagger \hat{b}_- + 1/2)$ is

  \[
  E_- (a) \equiv \langle \hat{H}_- \rangle = \text{Tr} \hat{\rho}_- \hat{H}_- = \omega \left( \langle \hat{N}(a) \rangle + \frac{1}{2} \right), \quad \langle \hat{N}(a) \rangle = \sinh^2 r.
  \]  
  \[(71)\]
Entanglement quantities - sinusoidal pulse

For the sinusoidal entangled pair of the universes we have

\[ \alpha = \frac{1}{2} \left( \frac{1}{R\sqrt{\omega}} + R\sqrt{\omega} - \frac{i\dot{R}}{\sqrt{\omega}} \right), \]  
(72)

\[ \beta = -\frac{1}{2} \left( \frac{1}{R\sqrt{\omega}} - R\sqrt{\omega} - \frac{i\dot{R}}{\sqrt{\omega}} \right), \]  
(73)

with the WKB solutions

\[ \phi_1 = \frac{1}{\sqrt{\omega}} \cos S, \quad \phi_2 = \frac{1}{\sqrt{\omega}} \sin S, \]  
(74)

which yields \( R\sqrt{\omega} = 1 \), and

\[ \alpha = 1 + \frac{i\omega}{4\omega^2}, \quad \beta = -\frac{i\omega}{4\omega^2}, \]  
(75)

with \( |\alpha|^2 - |\beta|^2 = 1 \), and \( \dot{R} = -\frac{1}{2} \omega \omega^{-\frac{3}{2}} \).
We then obtain

\[
\tanh r = \frac{|\beta|}{|\alpha|} = \frac{\dot{\omega}}{\sqrt{16\omega^4 + \dot{\omega}^2}} = \frac{1}{\sqrt{1 + \left(\frac{4\omega^2}{\dot{\omega}}\right)^2}},
\]

(76)

with, \(\dot{\omega} \equiv \frac{d\omega}{da}\), and \(\omega(a)\) given by (40), so that

\[
\tanh r = \frac{1}{\sqrt{1 + 16a^4 \frac{(1-\Lambda a^2)^3}{(1-2\Lambda a^2)^2}}} \equiv q.
\]

(77)

Finally we have for the temperature and entropy of entanglement

\[
T = -\frac{a\sqrt{1-\Lambda a^2}}{2 \ln q},
\]

(78)

\[
S = \frac{1}{1-q^2} \ln \left[ \frac{1}{1-q^2} \right] - \frac{q^2}{1-q^2} \ln \left[ \frac{q^2}{1-q^2} \right].
\]

(79)
Entanglement of entropy - results (sinusoidal pulse)

Result: Entropy is large (infinite) for big-bang \((a = 0)\), big-crunch \((a = 2 - \text{not plotted})\) and also for the maximum expansion (!!!) \((a = 1)\) regions
Creation of entangled pairs at maximum expansion
Result: Entropy is **large** (infinite) for big-bang ($a = 0$) region, but **vanishes** for big-rip ($a = \infty$) region!

Problem: Is the entropy of entanglements the proper measure of **quantumness**? (We know big-rip achieves Planck density)
Scale factor (blue, dotted), entanglement parameter $q$ (green, dashed), entropy of entanglement (yellow, solid line), and temperature of entanglement (red, dot-dashed) for the sinusoidal pulse. Unlike the entropy of entanglement, the parameter $q$ turns out to be a non-divergent (finite) measure of the entanglement.
Results: temperature of entanglement - tangential pulse.

Scale factor (blue, dotted), entanglement parameter $q$ (green, dashed), entropy of entanglement (yellow, solid line), and temperature of entanglement (red, dot-dashed) for the tangential pulse. The temperature of entanglement can be an **indicator of the quantumness** of the universes.
Observational signatures of the multiverse.

- Bearing in mind a harmonic oscillator analogue of the WdW equation one may calculate *the entanglement energy*

\[ E = \frac{\omega_{\text{eff}}}{2} = \omega \left( \sinh^2 r + \frac{1}{2} \right), \]  

(80)

which is analogous to a quantized oscillator in a vacuum state related to the frequency \( \omega = \omega(a) \) in eq. (42) when there is no entanglement i.e. if \( r = 0 \).

- An analogue of the Friedmann equation (39) for an entangled universe would then be

\[ \frac{da}{dt} = \frac{\omega_{\text{eff}}}{a} = \frac{\omega}{a} \left( 1 + 2 \sinh^2 r \right) \]  

(81)

which *gives a quantum entanglement correction* to the classical evolution of a universe contained in the multiverse.
Observational signatures of the multiverse.

- Quantum entanglement effect of the multiverse can be observed due to an appropriate term of quantum interaction in any universe of the multiverse i.e. also in Our Universe.

- Practical realization by an extra term in the Friedmann equation

\[ H^2 = \left(\frac{8\pi G}{3c^4}\right)\rho + \text{quantum entanglement} \quad (82) \]

- Entanglement signal can be imprinted in the spectrum of the cosmic microwave background (CMB) in the form of an extra dipole which is a cause of dark matter flow (Mersini-Houghton, Holman 2008; Kinney 2016)

- Entanglement also influences the potential of a scalar field which drives cosmological inflation and so a change of the CMB temperature (Di Valentin, Mersini-Houghton 2017, 2018; Bouhmadi-Lopez 2018)
Using varying constants \((c, G)\) theories we created cyclic universes and extended them into the **cyclic multiverse scenarios with same geometry and different evolution of the coupling constants** still obeying the overall 2nd law of thermodynamics (”dobubleverses”).

Quantum methods (from 2nd (WDW) to the 3rd quantization scheme) allowed us to consider possible **cyclic universes’ pair creation** and their quantum entanglement.

**The entropy of entanglement** is large at small values of the scale factor \(a \approx 0\) (big-bang, big-crunch) as well as at the **maximum expansion** point \(a \approx a_{\text{max}}\) suggesting strong ”quantumness” of these minisuperspace points.

The entropy of entanglements vanishes at large values of the scale factor \(a \rightarrow \infty\) (big-rip) despite presumably strong ”quantumness” of this point. However, **the temperature of entanglement** is large for this point and perhaps is more appropriate to measure its ”quantumness”.