Unimodular Pure dS Supergravity

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based on work done in collaboration with:
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Making contact between supergravity and nature

- **Problem:** (unbroken) supersymmetry doesn't allow de Sitter solutions (i.e. positive cosmological constant).

Easy to see from algebra

- Attempt to embed (A)dS algebra

$$[P_\mu, P_\nu] = s\frac{1}{4L^2}M_{\mu\nu}, \quad \text{with} \quad s = \begin{cases} -1 & \text{dS} \\ +1 & \text{AdS} \end{cases}$$

into SUSY algebra:

$$[P_\mu, Q_\alpha] = \frac{1}{4L}(\gamma_\mu Q)_\alpha$$
$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}(\gamma^\mu)_{\alpha\beta}P_\mu - \frac{1}{8L}(\gamma^{\mu\nu})_{\alpha\beta}M_{\mu\nu}$$

then Jacobi identity $[P_\mu, P_\nu, Q] = 0$ fixes

$$s = +1$$

Making contact between supergravity and nature

- **Problem:** (unbroken) supersymmetry doesn't allow de Sitter solutions (i.e. positive cosmological constant).
- **Solution:**SUSY is broken on dS solution.
- Requires the introduction of the *goldstino*, on which supersymmetry is realised non-linearly.
- Very difficult to achieve in pure supergravity theory.

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Constrained superfields

- Would like to incorporate the goldstino into a superfield (or supermultiplet), such that we can make full use of the machinery of SUSY/sugra, make contact with $\mathcal{N} > 1$, etc.
- Proposed solution is that the goldstino is part of a *nilpotent supefield* X [Rocek '78,Lindstrom, Rocek '79,Samuel, Wess '83,Casalbuoni,De Curtis,Dominici,Feruglio,Gatto '89,Komargodski, Seiberg '09,Kuzenko, Tyler '11]

$$X = \varphi + \sqrt{2}\theta G + \theta^2 F^X$$

$$X^2 = 0 \quad \Rightarrow \quad \varphi = \frac{G^2}{2F^X}$$

Constrained superfields

- Couple to supergravity via superpotential

$$\mathcal{W} = a(X^0)^3 + b(X^0)^2 X$$

with a and b arbitrary constants and X^0 is the compensator. This gives solutions with cosmological constant and gravitino mass

$$\Lambda \propto |b|^2 - |a|^2, \qquad m_\psi \propto a$$

- Much recent activity related to dS solutions in supergravity[Bandos,Bergshoeff,Dudas,Farakos,Ferrara,Freedman,Hasegawa,Kallosh,Kehagias,Porrati,Van Proeyen,Sagnotti,Scalisi,Yamada,...'14-18], inflationary models[Dall Agata,Antoniadis,Benakli,Dudas,Farakos,Ferrara,Kahn,Kallosh,Linde,Roberts,Sagnotti,Thaler,Zavala, Zwirner,...'14-18], brane models[Angelantonj,Antoniadis,Dudas,Mourad, Pradisi,Riccioni,Sagnotti,Uranga,Vercnocke,Zavala...'99-'18]

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Stueckelberg: gentle introduction

- Start with a Lagrangian with an explicitly broken symmetry:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu$$

where the gauge field A_μ lacks the usual gauge symmetry $\delta A_\mu = \partial_\mu \alpha$ due to the mass term. This can be restored by introducing an extra scalar field s, and writing

$$A_\mu = \tilde{A}_\mu + \partial_\mu s, \quad \begin{cases} \delta \tilde{A}_\mu = \partial_\mu \alpha \\ \delta s = -\alpha \end{cases}$$

and plugging back into the Lagrangian

$$\mathcal{L} = -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{m^2}{2} \partial_\mu s \partial^\mu s - \frac{m^2}{2} \left(\tilde{A}_\mu \tilde{A}^\mu + 2 A_\mu \partial^\mu s \right)$$

- The procedure can be interpreted as performing a gauge transformation on the field and promoting the parameter to a field (the Stueckelberg field).

Unimodular gravity

- Original formulation of unimodular gravity

$$S = \int d^4x \left[\sqrt{-g}R - 2\Lambda\left(\sqrt{-g} - \epsilon_0\right)\right]$$

where Λ is a Lagrange multiplier field which imposes the unimodularity condition

$$\sqrt{-g} = \epsilon_0$$

with ϵ_0 being a constant (which could be set to 1 if we want).

- The Einstein Equation also imposes:

$$\partial_\mu \Lambda = 0$$

and hence we interpret the v.e.v. of Λ as the cosmological constant.

Unimodular gravity

- Original formulation of unimodular gravity

$$S = \int d^4x \left[\sqrt{-g}R - 2\Lambda\left(\sqrt{-g} - \epsilon_0\right)\right]$$

This version of the Lagrangian is not invariant under the full group of diffeomorphisms, but only under a subgroup of transformations calledd TDiffs, whose parameter satisfies $\nabla_\mu \xi^\mu = 0$. It is obvious that the problematic term is

$$2\int d^4x \Lambda \epsilon_0$$

where Λ transforms as a scalar.

Unimodular gravity with Stueckelberg

- One can restore the full general coordinate invariance through the Stueckelberg trick of introducing an extra field which transforms appropriately. We do this by performing a g.c.t. $x^\mu \to y^\mu(x)$ on the problematic term

$$2\int d^4x \Lambda(x)\epsilon_0 \quad \to \quad 2\int d^4y \Lambda'(y)\epsilon_0 = 2\int d^4x |J| \Lambda(x)\epsilon_0$$

where $|J| = Det\left(\frac{\partial y^\mu}{\partial x^\alpha}\right)$. We promote y^μ to be 4 functions of space-time $y^\mu \to \phi^\mu(x)$ and the new action is

$$S_{St} = \int d^4x \left[\sqrt{-g}R - 2\Lambda\left(\sqrt{-g} - Det\left(\frac{\partial \phi}{\partial x}\right)\epsilon_0\right)\right]$$

which is now invariant under the full group of g.c.t.'s, provided the ϕ's transforms as scalars:

$$\phi'^\mu(y(x)) = \phi^\mu(x)$$

Unimodular gravity with Stueckelberg

- We performed the Stueckelberg trick by acting with a *finite* transformation on the coordinates. However, this will not always be practical, and one can alternatively restore the diffeomorphism invariance by performing an *infinitessimal* transformation either in the passive form:

$$x^\mu \;\rightarrow\; x^\mu + \xi^\mu + \tfrac{1}{2}\xi^\rho \partial_\rho \xi^\mu + \dots$$

or in the active form, where we vary the scalar field Λ:

$$\delta\Lambda = -\xi^\rho \partial_\rho \Lambda - \tfrac{1}{2}\xi^\rho \partial_\rho \xi^\mu \partial_\mu \Lambda + \dots$$

and promoting the parameters to fields: $\xi^\mu \rightarrow s^\mu$. The Lagrangian can then be constructed order by order in the Stueckelberg field s^μ

$$\mathcal{L} = \mathcal{L}_0 + \Lambda \left[1 + \partial_\mu s^\mu + \tfrac{1}{2} s^\rho \partial_\rho \partial_\mu s^\mu + \tfrac{1}{2} \left(\partial_\mu s^\mu\right)\left(\partial_\rho s^\rho\right) + \dots \right]$$

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Chiral superspace

- We define a *chiral density* superfield Δ via its transformation property

$$\delta\Delta = -\partial_M \left[(-1)^M \eta^M \Delta\right]$$

where

$$\eta^\mu(\epsilon) = \Theta^\beta y^\mu_{1\beta}(\epsilon) + \Theta^2 y^\mu_2(\epsilon)$$
$$\eta^\alpha(\epsilon) = \epsilon^\alpha + \Theta^\beta \Gamma^\alpha_{1\beta}(\epsilon) + \Theta^2 \Gamma^\alpha_2(\epsilon)$$

with

$$\begin{aligned} y^\mu_{1\alpha}(\epsilon) =& 2i\left(\sigma^\mu\bar{\epsilon}\right)_\alpha \\ y^\mu_2(\epsilon) =& \bar{\psi}_\nu\bar{\sigma}^\mu\sigma^\nu\bar{\epsilon} \\ \Gamma^\alpha_{1\beta}(\epsilon) =& -i\left(\sigma^\mu\bar{\epsilon}\right)_\beta \psi^\alpha_\mu \\ \Gamma^\alpha_2(\epsilon) =& -i\omega^{\alpha\beta}_\mu\left(\sigma^\mu\bar{\epsilon}\right)_\beta + \frac{1}{3}M^*\epsilon^\alpha \\ & -\frac{1}{2}\psi^\alpha_\nu\left(\bar{\psi}_\mu\bar{\sigma}^\nu\sigma^\mu\bar{\epsilon}\right) + \frac{1}{6}b_\mu\left(\varepsilon\sigma^\mu\bar{\epsilon}\right)^\alpha \end{aligned}$$

- It naturally generalises scalar densities (such as $\sqrt{-g}$).

Chiral superspace

- We define a *chiral density* superfield Δ via its transformation property

$$\delta\Delta = -\partial_M \left[(-1)^M \eta^M \Delta\right]$$

- It naturally generalises scalar densities (such as $\sqrt{-g}$).
- The product of a chiral density with a chiral superfield is a chiral density

$$\begin{aligned}\delta\Delta\Phi &= -\partial_M \left[(-1)^M \eta^M \Delta\right] \Phi - \Delta\eta^M \partial_M \Phi \\ &= -\partial_M \left[(-1)^M \eta^M \Delta\Phi\right]\end{aligned}$$

- It can be used to construct invariant actions.

Chiral superspace

- The pure supergravity action in chiral superspace $(X^M = (x^\mu, \Theta^\alpha))$ is

$$S = \int d^4x d^2\Theta \mathcal{E} R + h.c.$$

with R a chiral superfield with components

$$\begin{aligned} R| &= -\frac{1}{6} M \\ \mathcal{D}_a R| &= -\frac{1}{6}\left(\sigma^\alpha \bar{\sigma}^\beta \psi_{\alpha\beta} + i b^\alpha \psi_\alpha - i \sigma^\alpha \bar{\psi}_\alpha M\right)_a \\ \mathcal{D}^2 R| &= -\frac{1}{3} e_a^\mu e_b^\nu \mathcal{R}_{\mu\nu}{}^{ab} + \frac{2}{3} i \bar{\psi}^\mu \bar{\sigma}^\nu \psi_{\mu\nu} + \ldots \end{aligned}$$

Chiral superspace

- The pure supergravity action in chiral superspace $(X^M = (x^\mu, \Theta^\alpha))$ is

$$S = \int d^4x d^2\Theta \mathcal{E} R + h.c.$$

with $\mathcal{E}$ a chiral density with components

$$\mathcal{E} = a \; + \; \sqrt{2}\Theta\rho \; + \; \Theta\Theta f, \qquad \text{with}$$
$$a = \frac{1}{2}e$$
$$\rho = \frac{i\sqrt{2}}{4}e\sigma^\mu\bar{\psi}_\mu$$
$$f = -\frac{1}{2}eM^* - \frac{1}{8}e\bar{\psi}_\mu\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right)\bar{\psi}_\nu$$

Unimodular Supergravity

- We define the unimodular supergravity action to be:

$$S = \int d^4x d^2\Theta \left[\mathcal{E} R - 2\Lambda \left(\mathcal{E} - \mathcal{E}_0\right)\right] + h.c.$$

where Λ is now a lagrange multiplier chiral superfield, and we define

$$\mathcal{E}_0 = \epsilon_0 - \frac{i}{2} m \Theta^2$$

with ϵ_0, m some real constants. Varying over Λ, we get

$$\mathcal{E} = \mathcal{E}_0,$$

which is the SUSY analogue of the unimodularity condition. The action is not invariant under full SUSY+diffeo, but subset preserving condition above.

The Super-Stueckelberg trick

- We perform a transformation $X^M \to Y^M(X)$ on the problematic term

$$2\int d^4x d^2\Theta \Lambda(X)\mathcal{E}_0 \quad \to$$

$$2\int d^4y d^2\Gamma \Lambda'(Y)\mathcal{E}_0 = 2\int d^4x d^2\Theta |sJ| \Lambda(X)\mathcal{E}_0$$

where $|sJ| = Ber\left(\frac{\partial Y^M}{\partial X^N}\right)$. Crucially, unlike in the gravity case, Y^M will *not* be a general superfunction of X^M, but will depend on the diffeomorphism and SUSY parameters in a very particular way.

The Super-Stueckelberg trick -perturbatively

- At linear order in the parameters, we have the usual transformation

$$\delta^{(1)} X^M = \Xi^M$$

with

$$\Xi^M = (\xi^\mu + \eta^\mu(\epsilon), \eta^\alpha(\epsilon))$$

- At second order, we have

$$\delta^{(2)} X^M = \frac{1}{2}\Xi^R \partial_R \Xi^M + \frac{1}{2}\delta^{(s)}\left(\Xi^M\right)$$

where the first term is the usual one at second order, while the second one takes into account the fact that the objects η^μ, η^α appearing in the SUSY transformations of the coordinates are not arbitrary functions, but depend on the supergravity felds $e_\mu, \psi_\mu, b_\mu, M$.

The Super-Stueckelberg trick

We can thus proceed, order by order in the transformation parameters, and write:

$$X^M \quad \rightarrow \quad X^M + \delta^{(1)} X^M + \delta^{(2)} X^M + \ldots \equiv Y^M(X, \Xi),$$

Again, we promote the parameters to fields to get:

$$\left.\begin{array}{c} \xi^\mu \rightarrow \phi^\mu \\ \epsilon \rightarrow \zeta \end{array}\right\} \Rightarrow$$

$$\begin{aligned} \Xi^M = (\xi^\mu + \eta^\mu(\epsilon), \eta^\alpha(\epsilon)) \quad &\rightarrow \quad \varphi^M = (\phi^\mu + \eta^\mu(\zeta), \eta^\alpha(\zeta)) \\ Y^M(X, \Xi) \quad &\rightarrow \quad \Phi^M(X, \varphi) \end{aligned}$$

At this point, we can construct the Lagrangian:

$$S = \int d^4x d^2\Theta \left[\mathcal{E} R - 2\Lambda \left(\mathcal{E} - Ber \left(\frac{\partial \Phi^M}{\partial X^N} \right) \mathcal{E}_0 \right) \right] + h.c.$$

The Stueckelberg superfield

- The superfield

$$S = Ber\left(\frac{\partial\Phi^M}{\partial X^N}\right) \equiv S_0 + \sqrt{2}\Theta S_1 + S_2\Theta^2$$

is a chiral density superfield, thus allowing us to restore SUSY to the action.

- Its components S_0, S_1, S_2 transform *linearly*, but they are functions of the Stueckelberg fields f^μ, ζ, which transform *non-linearly* and of the sugra fields $e_\mu, \psi_\mu, b_\mu, M$.

Stueckelberg action in components

In components, the action will be (setting $\epsilon_0 = \frac{1}{2}$)

$$S = \frac{1}{2\kappa^2}\int \Big[\sqrt{-g}\left[R - \tfrac{2}{3}M^*M + \tfrac{2}{3}b^\mu b_\mu + \varepsilon^{\mu\nu\rho\sigma}\left(\bar\psi_\mu\bar\sigma_\nu\tilde{\mathcal{D}}_\rho\psi_\sigma - \psi_\mu\sigma_\nu\tilde{\mathcal{D}}_\rho\bar\psi_\sigma\right)\right]$$

$$+\tfrac{1}{2}\sqrt{-g}\left[-2\Lambda_2 + \sqrt{2}i\Lambda_1\sigma^\mu\bar\psi_\mu + 2\Lambda_0\left(\bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu + M^*\right)\right] + h.c.$$

$$+\Big[\Lambda_0\left(\partial_\mu y_2^\mu - m - m\partial_\mu f^\mu\right) - \Lambda_1^a\left(\tfrac{\sqrt{2}}{2}\partial_\mu y_{a1}^\mu + \sqrt{2}\Gamma_{a2} - \sqrt{2}m\zeta_a\right)$$

$$+\Lambda_2\left(1 + \partial_\mu f^\mu - \Gamma^m_{m1}\right)\Big] + h.c. + \mathcal{O}^2$$

with :

$$y^\mu_{a1} = 2i\left(\sigma^\mu\bar\zeta\right)_a, \quad y_2^\mu = \bar\psi_\nu\bar\sigma^\mu\sigma^\nu\bar\zeta, \quad \Gamma^m_{a1} = -i\left(\sigma^\mu\bar\zeta\right)_a\psi^m_\mu, \quad \Gamma^m_2 = -i\omega^{mn}_\mu\left(\sigma^\mu\bar\zeta\right)_n + \tfrac{1}{3}M^*\zeta^m$$

Transformations

- The Stueckelberg fields will transform as:

$$\delta f^\mu = -\xi^\mu + \tfrac{1}{2}\left[\zeta^r \tilde{y}^\mu_{r1} - \epsilon^r y^\mu_{r1}\right] + \dots$$
$$\delta\zeta^m = -\epsilon^m + \tfrac{1}{2} f^\rho \partial_\rho \epsilon^m + \dots$$

  Then S transforms as required for a chiral density superfield

$$\delta S = -\partial_M \left[\eta^M S (-1)^M\right]$$

- We have restored full SUSY invariance to the action !

E.o.M and deSitter solution

- Write e.o.m. for all our fields (supergravity multiplet components, Λ components and Stueckelberg superfields f^μ and ζ) we see that our theory admits solutions with c.c. of arbitrary sign:

$$\Lambda \propto \langle \Lambda_2 \rangle - m^2$$

$$m_\psi \propto m$$

In progress

- Extend to non-perturbative constructions, via conformal construction.
- Consequences for proposed solutions to c.c. problem.
- Relation to constrained superfields.
- Relation to brane constructions for goldstino.