QCD Thermodynamic Geometry

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ABSTRACT

QCD phase transition is studied within the thermodynamic geometry. Through the definition of a metric in the thermodynamic space, one builds a scalar thermodynamic geometry curvature, R, in the usual way and investigates the nature of the interactions. R, indeed, reflects some important features of the system: e.g. the so-called interaction hypothesis, $|R| \sim \xi^d$, where ξ is the correlation length and d the effective spatial dimension of the underling thermodynamic system. Moreover, the sign of R seems to provide information on the system interactions (attractive or repulsive, fermionic or bosonic). We have studied R in different model: Nambu-Jona-Lasinio model with two and three flavors, Hadron Resonance Gas models and Lattice-QCD. In all of these models, R shows a characteristic behavior, different for each transition type (if present): I or II order or crossover.

THERMODYNAMIC GEOMETRY

Thermodynamic Geometry is considered a powerful tool to study statistical systems. The starting point is to equip the thermodynamic space with a metric. One can do this in different ways, but the most physically significant one is perhaps that introduced by Ruppeiner [1] throught the Hessian of the entropy density:

$$g_{\mu\nu} = -\frac{\partial s}{\partial X^\mu \partial X^\nu}\,, \tag{1}$$

with $X^\mu = (\varepsilon, n_1, \cdots)$, being ε the internal energy density and n_i the number densities of particles of different species.

The resulting distance is in inverse relation with the fluctuation probability between equilibrium states. For example in classical fluctuation theory one has

$$P \propto \exp\left\{-\frac{\Delta\ell^2}{2}\right\}\,, \tag{2}$$

where $\Delta\ell^2 = g_{\mu\nu}\Delta X^\mu \Delta X^\nu$ is the line element in the thermodynamic geometry. Moreover, it leads to the "interaction hypothesis", i.e the correspondence between the absolute value of the scalar curvature R (an intensive variable, with units of a volume, evaluated by the metric) and ξ^d, where ξ is the correlation length and d is the effective spatial dimension of the underling thermodynamic system. Indeed, a covariant and consistent thermodynamic fluctuation theory can be developed, which generalizes the classical fluctuations theory and offers a theoretical justification to the physical meaning of R.

Within the thermodynamic geometry approach, the physical meaning of the sign of R is still under debate but there are indications that it is directly related to the microscopic interactions, since R is positive for repulsive interactions and negative for attractive ones. A similar behavior has been found for quantum gases, but with a different meaning: R is positive for fermi statistical interactions and it is negative in the bosonic case. In this sense, therefore, a change in sign of R is an indication of the balance between effective interactions, even when no transition occurs.

In the analysis of the phase transitions [2–4] we considered a two dimensional manifold, where the intensive coordinates are $\beta = 1/T$ and $\gamma = -\mu/T$, with μ chemical potential. Moreover the metric (1) turns out to be related with the derivatives of the potential $\phi = p/T$, where p is the pressure:

$$g_{\mu\nu} = \begin{pmatrix} \phi_{,\beta\beta} & \phi_{,\beta\gamma} \\ \phi_{,\beta\gamma} & \phi_{,\gamma\gamma} \end{pmatrix}\,, \tag{3}$$

with the usual comma notation for derivatives. The scalar curvature R simply becomes

$$R = \frac{1}{2\,g^2}\begin{vmatrix} \phi_{,\beta\beta} & \phi_{,\beta\gamma} & \phi_{,\gamma\gamma} \\ \phi_{,\beta\beta\beta} & \phi_{,\beta\beta\gamma} & \phi_{,\beta\gamma\gamma} \\ \phi_{,\beta\beta\gamma} & \phi_{,\beta\gamma\gamma} & \phi_{,\gamma\gamma\gamma} \end{vmatrix}\,. \tag{4}$$

For low chemical potential, and thus for low γ, eq. (4) can be expresses as a Taylor expansion [2]:

$$R(\beta,\gamma) = R_{\mathcal{O}(0)}(\beta) + R_{\mathcal{O}(2)}(\beta)\gamma^2 + R_{\mathcal{O}(4)}(\beta)\gamma^4\,. \tag{5}$$

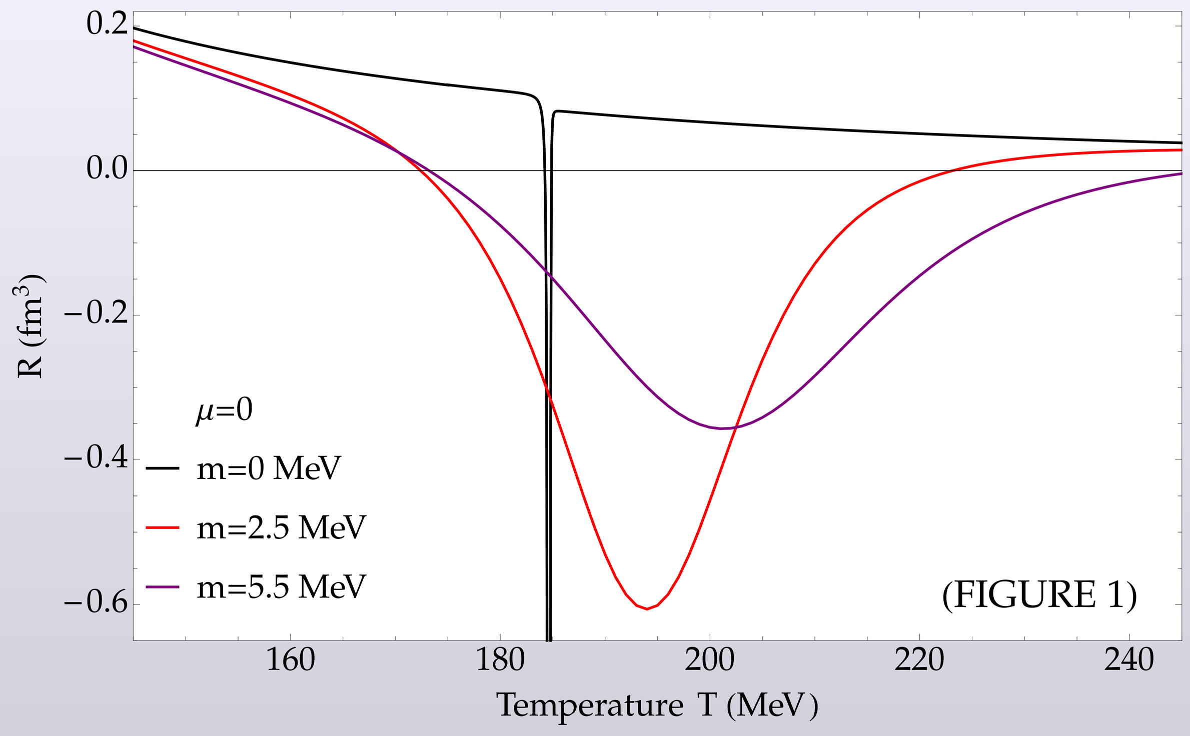
The coefficients $R_{\mathcal{O}(2n)}$ are functions of the cumulants χ_{2n} and of their derivatives with respect to β. For example, the zero-order term is

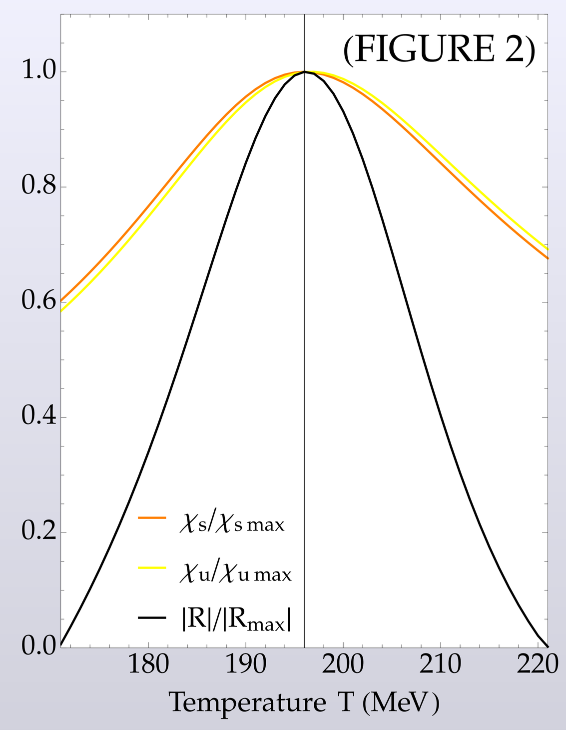
$$R_{\mathcal{O}(0)}(\beta) = \frac{1}{2\,\ddot{P}_0(T)}\left[3 + T\,\frac{\dot{\chi}_2(T)}{\chi_2(T)}\right] \times \left[\frac{\dddot{P}_0(T)}{\ddot{P}_0(T)} - \frac{\dot{\chi}_2(T)}{\chi_2(T)}\right]\,, \tag{6}$$

being " $\dot{}$ " the derivative with respect to T, $P_0(\beta)$ is the pressure and $\chi_2(T) = \partial^2(P/T^4)/\partial\gamma^2$, both at $\mu = 0$.

NAMBU - JONA LASINIO MODEL

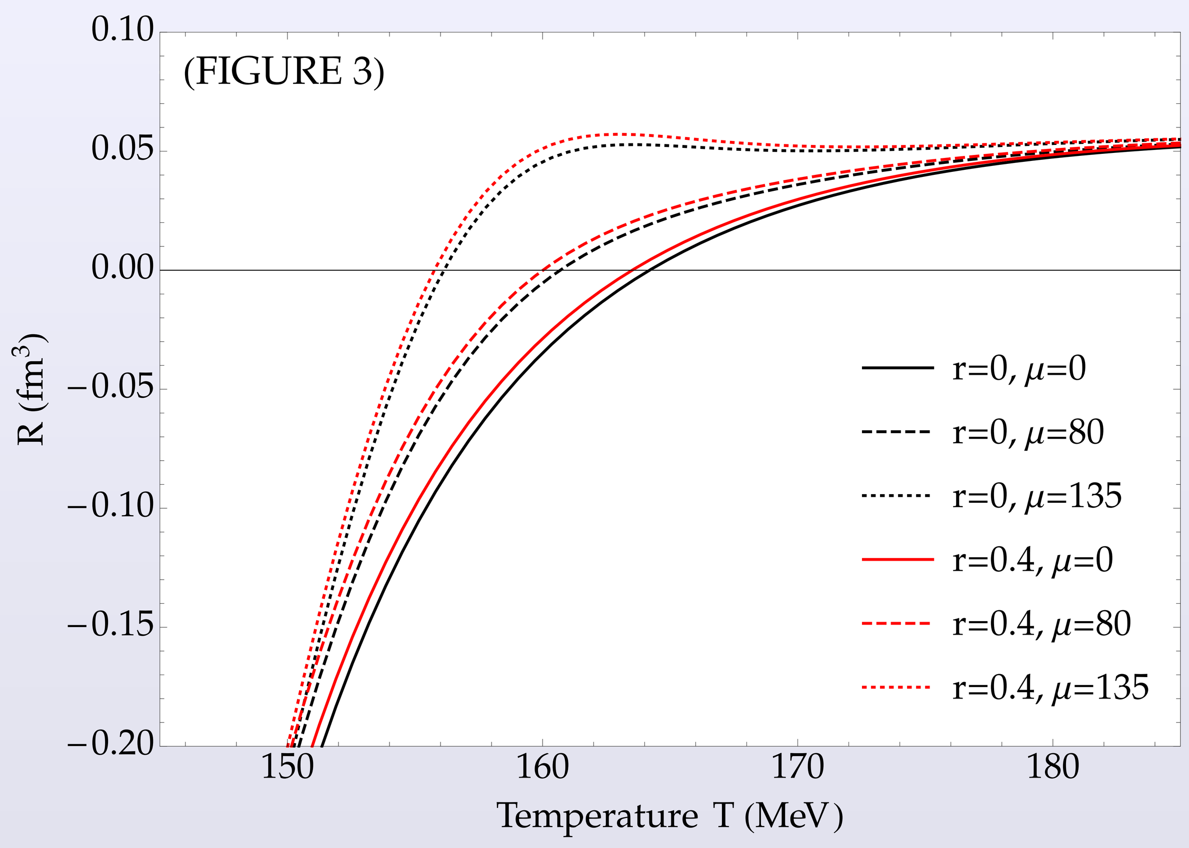
We study Nambu - Jona Lasinio model with two or three flavors and in the chiral limit or with physical masses. It's well know that, in the chiral limit, two flavor NJL model exhibits a II order phase transition. In terms of thermodynamic geometry this means that R diverges at the critical point (see black curve in FIG. 1). Moreover, for small μ and near the transition the curvature is negative, i.e. the interaction is mostly attractive, suggesting that the chiral symmetry restoration is due to thermal fluctuations. By adding a mass $m = m_u = m_d \neq 0$, the divergence of the II order phase transition turns into a minimum in the negative R region (see red and purple curves in FIG. 1). The transition temperature obtained by chiral susceptibility χ_f is in agreement with the one evaluated by the maximum of $|R|$ (see FIG. 2 for 3 flavors).





LATTICE QCD

Scalar curvature from eq. (5) is plotted in FIG. 3, for different values of μ and of the ratio $r = n_Q/n_B = 0$ or 0.4 (where n_Q and n_B are the charge and baryon number densities respectively) [2]. Here the transition is driven by the condition $R = 0$, as shown in FIG. 4, that shows the chiral susceptibility, χ, at $\mu = 0$ MeV and as a function of the scalar curvature R for physical value of the strange quark mass, m_s, and $m_s/m_\ell = 20$ or $m_s/m_\ell = 27$ (both at $r = 0$) [4].



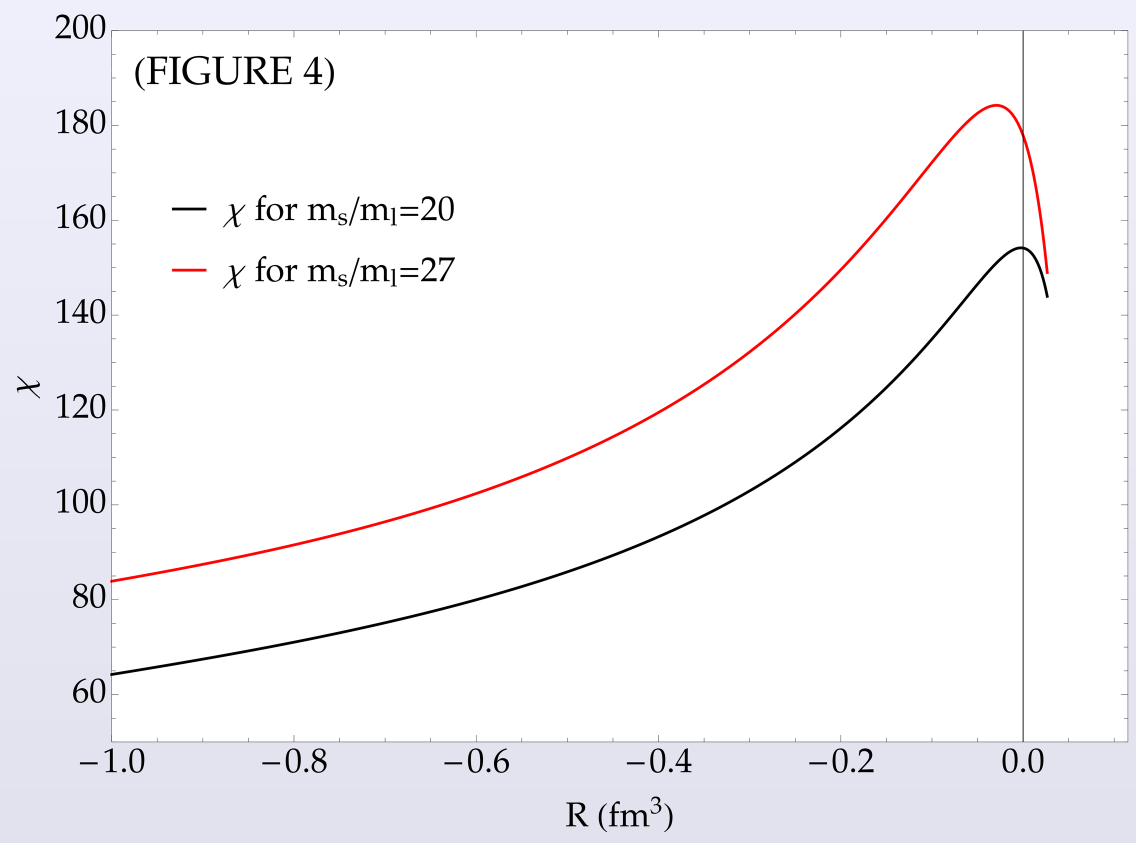
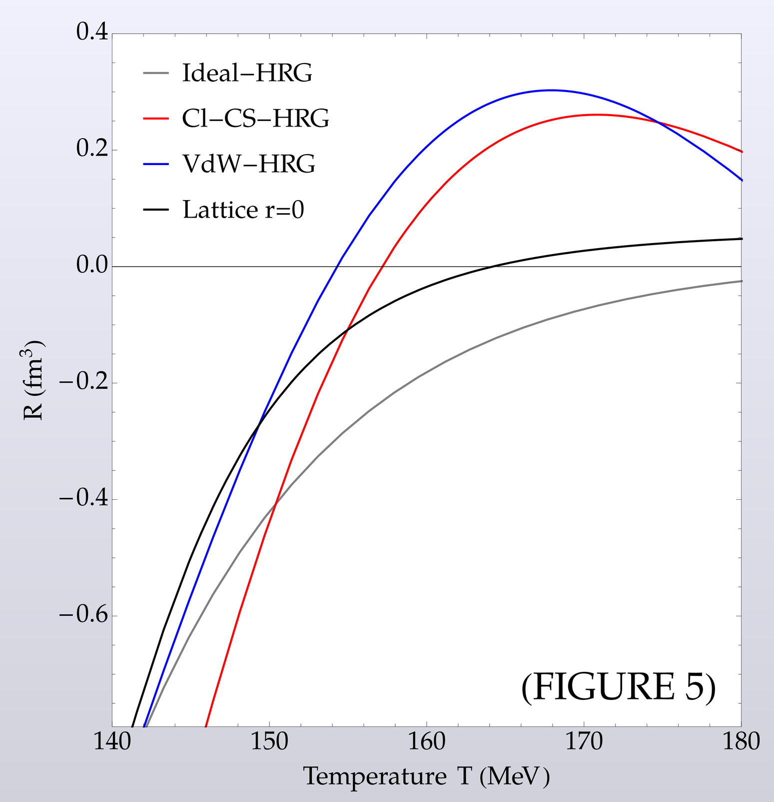


FIGURE 6 shows the transition temperature obtained via the $R = 0$ condition in LATTICE QCD and HRG models, compared with the freeze-out hadronization curve and the pseudo-critical temperature by lattice data.

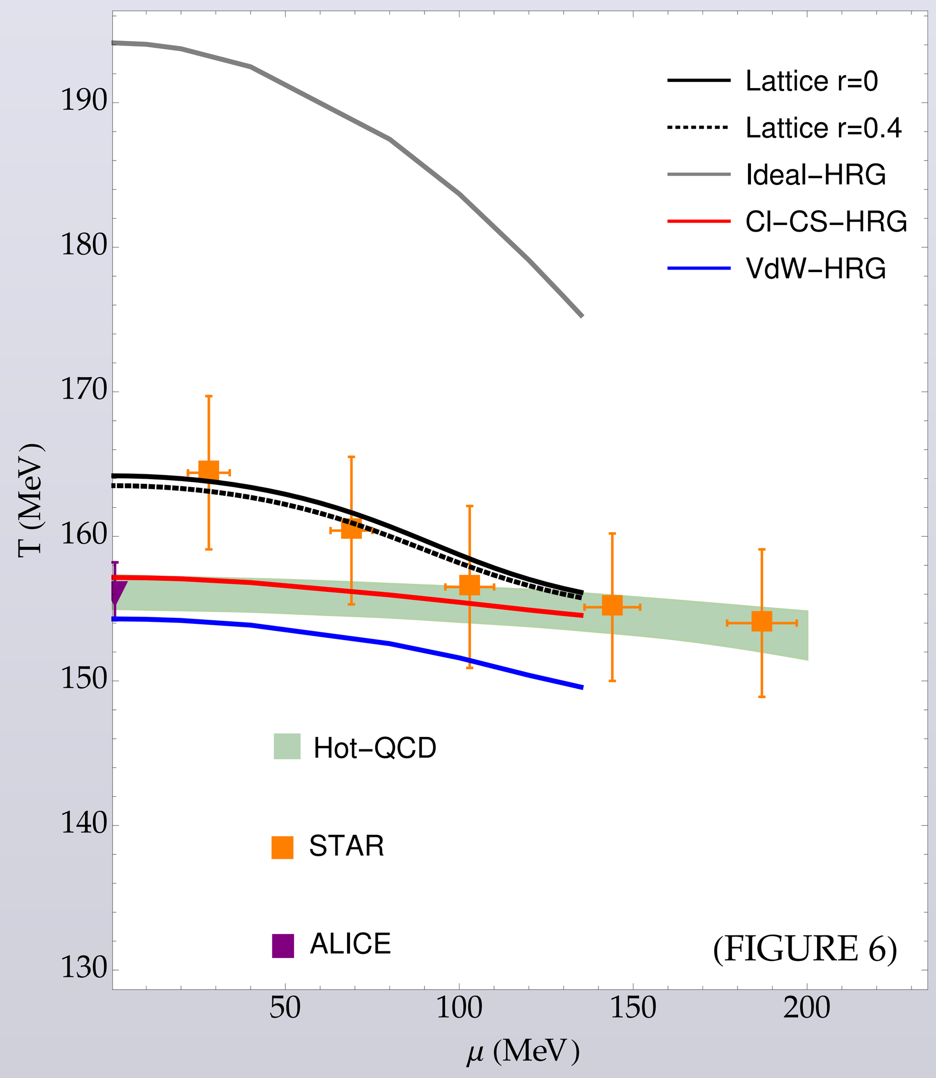
HADRON RESONANCE GAS MODEL



We study [3] Hadron Resonance Gas (HRG) models with different attractive and repulsive corrections and by an expansion at low chemical potential to $\mathcal{O}(\gamma^4)$ (eq. (5)):

- ideal HRG model of point-like constituents;
- Van der Walls HRG model;
- Clausius - Carnahan-Starling - HRG model, where the repulsive excluded volume interaction is given by the Carnahan-Starling term $f_{CS} = \exp\left\{-\frac{(4-3\eta)\eta}{(1-\eta)^2}\right\}$ (being η the packing fraction) and the attractive one by the Clausius form $u_{Cl} = -\frac{an}{1+bn}$ (n is the number density).

The scalar curvature R's are plotted in FIG. 5 together with that obtained from lattice QCD with $r = n_B/n_Q = 0$



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COMMENTS AND CONCLUSIONS

We find that, in NJL model, thermodynamic geometry reliably describes the phase diagram, both in the chiral limit and for finite quark masses. Different R scenarios are found as the transition is a I or II order or a crossover. Particularly, a non zero quark mass m reduces the value of $|R|$ at the transition from infinity to a finite value and the transition temperature is well evaluated by the maximum of $|R|$. However, NJL model misses color confinement and therefore there is no a priori reason to apply the same geometric criterium for non perturbative QCD dynamics.

In Lattice QCD the transition temperature is identified by a different criterion, i.e. $R = 0$. It indicates the transition from a mostly fermionic system (as the quark-gluon plasma) to an essentially bosonic one (as the hadron resonance gas) but, as shown in FIG. 4, it exactly corresponds to the maximum of chiral susceptibility, confirming the well known interplay between confinement and chiral symmetry breaking. The transition temperature evaluated by $R = 0$ is in agreement with the freeze-out hadronization curve and with the pseudo-critical temperature by lattice data within 10%.

The comparison between the geometrical study of NJL model and of (2+1) Quantum Chromodynamics at high temperature and small baryon density shows a clear connection between chiral symmetry restoration/breaking and deconfinement/confinement regimes. The behavior of the scalar curvature in HRG models also show a clear dependence of the thermodynamic geometry from volume excluded effect or attractive corrections.