Higher order Symmetric Cumulants


Symmetric Cumulants (SC)

Symmetric Cumulants are flow observables introduced originally to quantify the relationship between event-by-event fluctuations of two different flow amplitudes \( v_m \) and \( v_n \).

SC\((m,n)\) are robust against nonflow, dependence on symmetry planes \( \psi \) is eliminated by definition, and they have better sensitivity to the details of n/s temperature dependence than individual flow harmonics.

SC have potential to discriminate contributions to anisotropic flow development coming from the initial conditions and from the transport properties of the Quark-Gluon Plasma.


Generalization

The generalization of SC observables to the correlations involving more than two flow harmonics is not trivial, especially from the experimental point of view, and it involves few subtle steps and even conceptual changes.

List of requirements for generalization:

1. No built-in trivial contribution
2. Genuine multi-harmonic correlations (cumulants)
3. Symmetry
4. Cleanliness
5. Isotropy
6. Uniqueness
7. Robustness against nonflow
8. Event weights

Example: Theory

\[
SC(k,l,m) = \left( \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle \right) - \left( \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle \right) - \left( \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle \right) + 2 \left( \langle v_k^2 \rangle \langle v_l^2 \rangle \langle v_m^2 \rangle \right)
\]

Example: Experiment

\[
SC(k,l) = \langle \cos(3\varphi_1 + 2\varphi_2 + \varphi_3) - \cos(\varphi_1 + \varphi_2 + \varphi_3) - \cos(3\varphi_1 - 2\varphi_2 - \varphi_3) \rangle
\]

Multi-harmonic correlations

Shift of paradigm: Cumulants of azimuthal angles \( \varphi_1, \varphi_2 \) ... do not lead necessarily to the cumulants of flow amplitudes \( v_m, v_n \).

Example: If one treats azimuthal angles as fundamental degrees of freedom and performs cumulant expansion for the 6-particle correlator

\[
\langle \cos(3\varphi_1 + 2\varphi_2 + \varphi_3 - 2\varphi_4 - \varphi_5) \rangle = v_1^2 v_2^2 v_3^2
\]

it follows:

\[
\langle v_1^2 v_2^2 v_3^2 \rangle = \langle v_1^2 v_2^2 \rangle \langle v_3^2 \rangle - \langle v_1^2 v_3^2 \rangle \langle v_2^2 \rangle - \langle v_2^2 v_3^2 \rangle \langle v_1^2 \rangle + 2 \langle v_1^2 \rangle \langle v_2^2 \rangle \langle v_3^2 \rangle
\]

This is not a valid cumulant of three flow amplitudes \( v_1, v_2, v_3 \), since the final expression:

1. has the contribution from other and independent degrees of freedom (symmetry planes \( \psi \))
2. is not identical to 0 for the case fluctuations in the subset or among all flow amplitudes are mutually independent from each other

In order to access the genuine multi-harmonic correlations, one needs to treat flow amplitudes as the fundamental degrees of freedom, and to perform cumulant expansion directly on them.

Multi-particle azimuthal correlators are only used to estimate different averages of flow amplitudes in the cumulant expansion.

Theoretical predictions

Predictions from IEBE-VISHNU for Pb-Pb collisions at 2.76 TeV:

Measurements of higher order SC are feasible at LHC energies, they exhibit a non-trivial centrality dependence, which is midcentral and peripheral collisions is strikingly different between initial and final state.

The higher order Symmetric Cumulants contain information which is inaccessible to individual flow harmonics and correlated fluctuations of only two flow harmonics.

Therefore, they provide further and independent constraints for the properties of Quark-Gluon Plasma in nuclear collisions.