

CALCULATING HARD PROBE RADIATIVE ENERGY LOSS BEYOND SOFT-GLUON APPROXIMATION: HOW VALID IS THE APPROXIMATION?

Bojana Ilic¹ (Blagojevic), Magdalena Djordjevic¹, Marko Djordjevic²

¹Institute of Physics Belgrade, University of Belgrade, Belgrade, Serbia

²Institute of Physiology and Biochemistry, Faculty of Biology, University of Belgrade, Belgrade, Serbia



Abstract

One of the most common assumptions when calculating **radiative energy loss** of **high p_\perp** particles in quark-gluon plasma is the **soft-gluon approximation**, which considers that initial parton losses only a small amount of its energy via gluon's bremsstrahlung. Despite its convenience, the approximation sustainability was questioned by the reported notable radiative energy loss within different theoretical models. To address this issue, **we relax the soft-gluon approximation within DGLV formalism**. The obtained analytic expression beyond soft-gluon approximation is significantly more involved than in soft-gluon case. Unexpectedly, however, the numerical results lead to similar predictions for the fractional radiative energy loss and the number of radiated gluons in these two cases. Furthermore, the effect on these two variables is of an opposite sign, and results in nearly overlapping suppression predictions with and without soft-gluon approximation. We also show that this surprising result can be understood by the interplay of initial parton's p_\perp distribution and its energy loss probability. Consequently, the results presented here provide confidence that, despite the concerns mentioned above, the soft-gluon approximation remains adequate within DGLV formalism. Finally, we also discuss generalizing this relaxation in the dynamical QCD medium, which suggests a more general applicability of the conclusions obtained here.

Introduction

Pros:

- **Its convenience:** The **soft-gluon (sg) approximation** (i.e. $x = \omega/E \ll 1$, $E \equiv$ initial parton energy and $\omega \equiv$ radiated gluon energy) is one of the most common analytic assumptions.
- It was used in radiative part of our **dynamical energy loss formalism**, whose angular averaged R_{AA} predictions were **successfully tested against comprehensive set of experimental data, implying reliability of the formalism and the approximation.**

Cons:

- Different theoretical models, assuming this approximation, obtained **significant radiative energy loss, questioning the validity of this approximation.**
- The approximation breaks down for intermediate momentum ranges ($5 < p_\perp < 10$ GeV), where experimental data are most abundant and with the smallest error-bars, and for **gluons** primarily, due to color factor $9/4$ compared to quarks.

Why is relaxing the soft-gluon approximation important?

- To establish its adequacy.
- To extend the model toward intermediate p_\perp region.
- To test the reliability of our predictions in the above case.

Upon obtaining analytical expressions beyond soft-gluon (**bsg**) approximation, we compare **bsg** and **sg** numerical predictions for fractional radiative energy loss $\frac{\Delta E^{(1)}}{E}$, number of radiated gluons $N_g^{(1)}$, fractional differential radiative energy loss $\frac{1}{E} \frac{dE^{(1)}}{dx}$, single gluon radiation spectrum $\frac{dN_g^{(1)}}{dx}$ and suppression R_{AA} , to assess the effect of relaxation.

Theoretical Framework

We address validity of the soft-gluon approximation within **DGLV** formalism, which assumes:

- Finite size, optically thin QGP.
- **Static scattering centers**, so the interactions with medium constituents are modeled by Debye color-screened Yukawa potential.
- Gluons, in finite temperature QGP, as massive transversely polarized plasmons with effective mass $m_g = \mu/\sqrt{2}$.

Generalization of the results on **dynamical medium** is discussed.

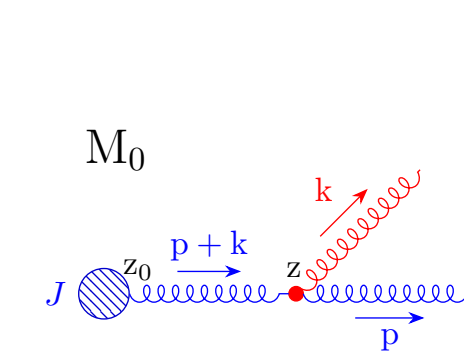


Fig. 1: 0th order

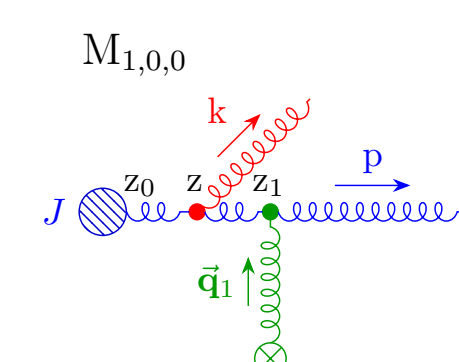


Fig. 2: Interaction with one scatterer.

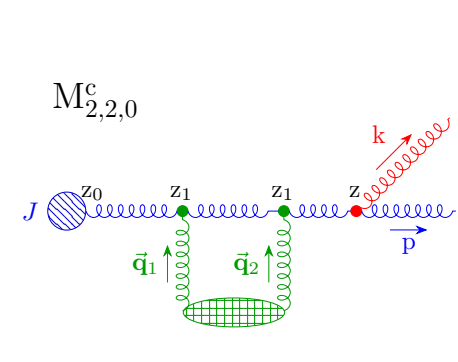


Fig. 3: Interaction with two scatterers in contact limit

We analytically relaxed the approximation for **high p_\perp gluon**, by calculating corresponding 11 Feynman diagrams within DGLV, under the following assumptions:

- **Initial gluon propagates along the longitudinal axis.**
- The soft-rescattering (eikonal) approximation.
- The first order in opacity approximation.

Analytical and Numerical Results

Beyond soft-gluon approximation

$$f_{bsg} = \frac{(1-x+x^2)^2}{x(1-x)} \left\{ \frac{(\mathbf{k}-\mathbf{q}_1)^2 + \chi}{\left(\frac{4x(1-x)E}{L}\right)^2 + ((\mathbf{k}-\mathbf{q}_1)^2 + \chi)^2} \left(2 \frac{(\mathbf{k}-\mathbf{q}_1)^2}{(\mathbf{k}-\mathbf{q}_1)^2 + \chi} - \frac{\mathbf{k} \cdot (\mathbf{k}-\mathbf{q}_1)}{\mathbf{k}^2 + \chi} - \frac{(\mathbf{k}-\mathbf{q}_1) \cdot (\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2 + \chi} \right) + \frac{\mathbf{k}^2 + \chi}{\left(\frac{4x(1-x)E}{L}\right)^2 + (\mathbf{k}^2 + \chi)^2} \left(\frac{\mathbf{k}^2}{\mathbf{k}^2 + \chi} - \frac{\mathbf{k} \cdot (\mathbf{k}-x\mathbf{q}_1)}{(\mathbf{k}-x\mathbf{q}_1)^2 + \chi} \right) + \left(\frac{(\mathbf{k}-x\mathbf{q}_1)^2}{((\mathbf{k}-x\mathbf{q}_1)^2 + \chi)^2} - \frac{\mathbf{k}^2}{(\mathbf{k}^2 + \chi)^2} \right) \right\}$$

Only this term remains in soft-gluon approximation and reduces to:

Soft-gluon approximation

$$f_{sg} = \frac{1}{x} \frac{(\mathbf{k}-\mathbf{q}_1)^2 + m_g^2}{\left(\frac{4x(1-x)E}{L}\right)^2 + ((\mathbf{k}-\mathbf{q}_1)^2 + m_g^2)^2} 2 \left\{ \frac{(\mathbf{k}-\mathbf{q}_1)^2}{(\mathbf{k}-\mathbf{q}_1)^2 + m_g^2} - \frac{\mathbf{k} \cdot (\mathbf{k}-\mathbf{q}_1)}{\mathbf{k}^2 + m_g^2} \right\}$$

Analytical Results

The obtained analytical expression for $\frac{dN_g^{(1)}}{dx}$ in **bsg** case:

- **Is more complicated than in sg case.**
- **Recovers sg result for $x \ll 1$.**
- **Is symmetric under the exchange of radiated (k) and final gluon (p).**

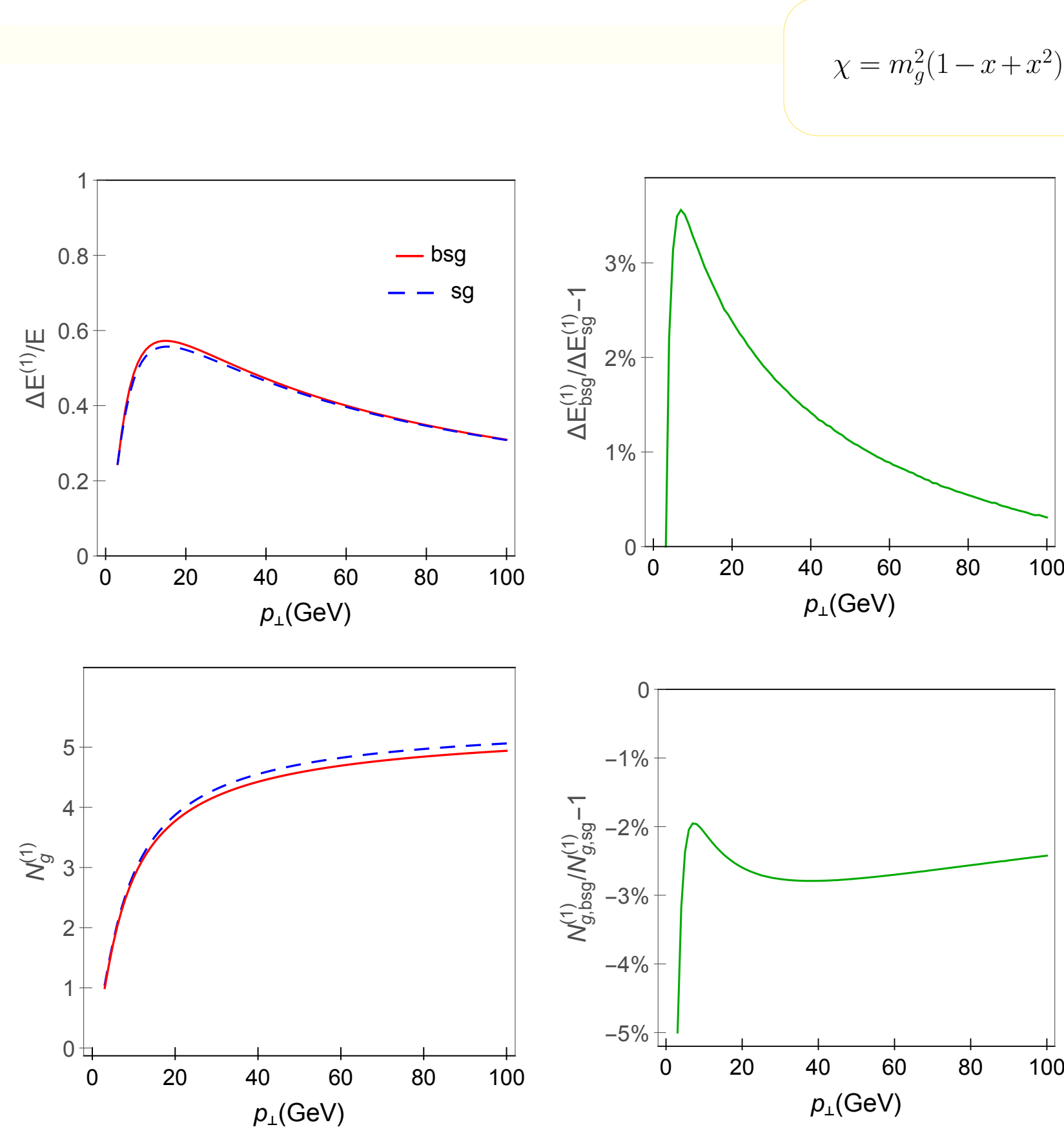


Fig. 4: The effect of relaxing the soft-gluon approximation on $\frac{\Delta E^{(1)}}{E}$ and $N_g^{(1)}$.

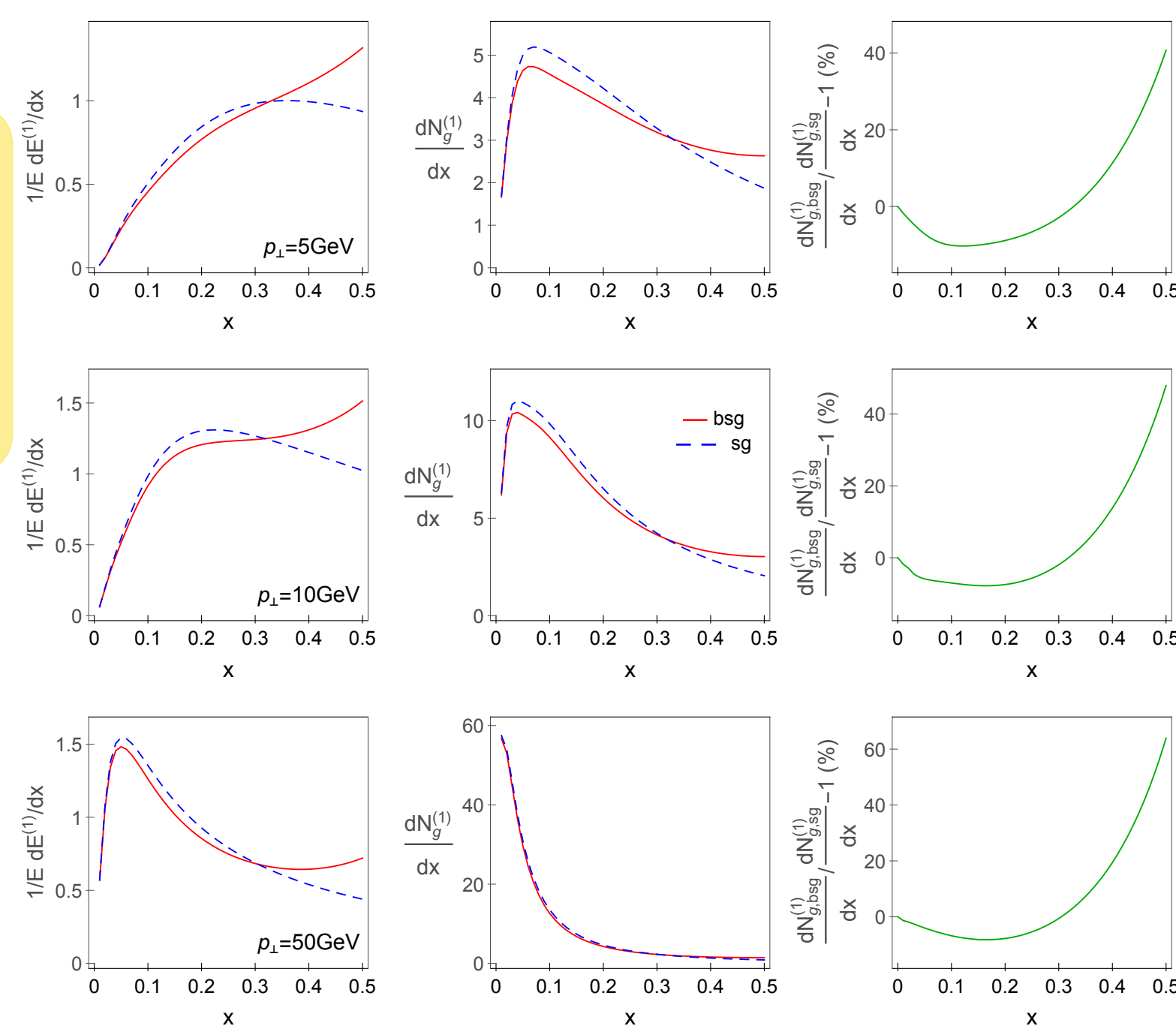


Fig. 5: The effect of relaxing the soft-gluon approximation on $\frac{1}{E} \frac{dE^{(1)}}{dx}$ and $\frac{dN_g^{(1)}}{dx}$.

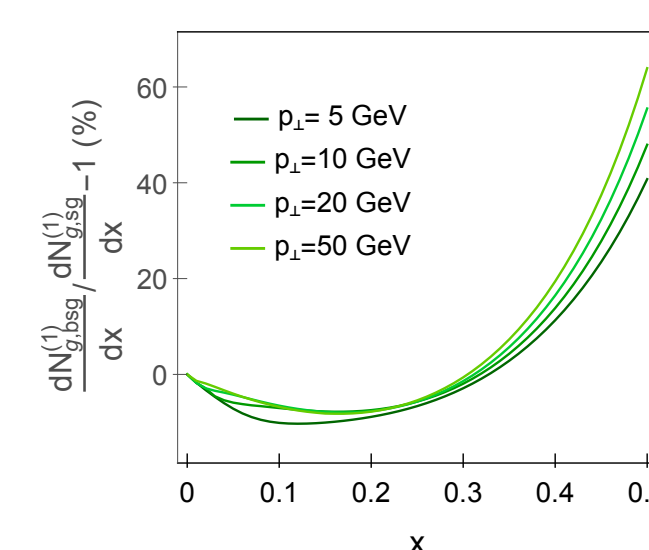


Fig. 6: The effect on $\frac{dN_g^{(1)}}{dx}$ for different p_\perp .

Numerical Results

The effect of relaxing the **sg** approximation is:

- **Small for:**
 - $\Delta E^{(1)}/E$ up to $\sim 3\%$.
 - $N_g^{(1)}$ up to $\sim -5\%$, and of an **opposite sign** for the two variables.
- **Relatively small for:**
 - $1/E \times dE^{(1)}/dx$ and
 - $dN_g^{(1)}/dx$ for $x \lesssim 0.4$ (up to $\sim 10\%$), whereas notable for higher x (up to $\sim 60\%$).
- Practically the same for both $1/E \times dE^{(1)}/dx$ and $dN_g^{(1)}/dx$, across the whole x region, regardless of initial gluon p_\perp .

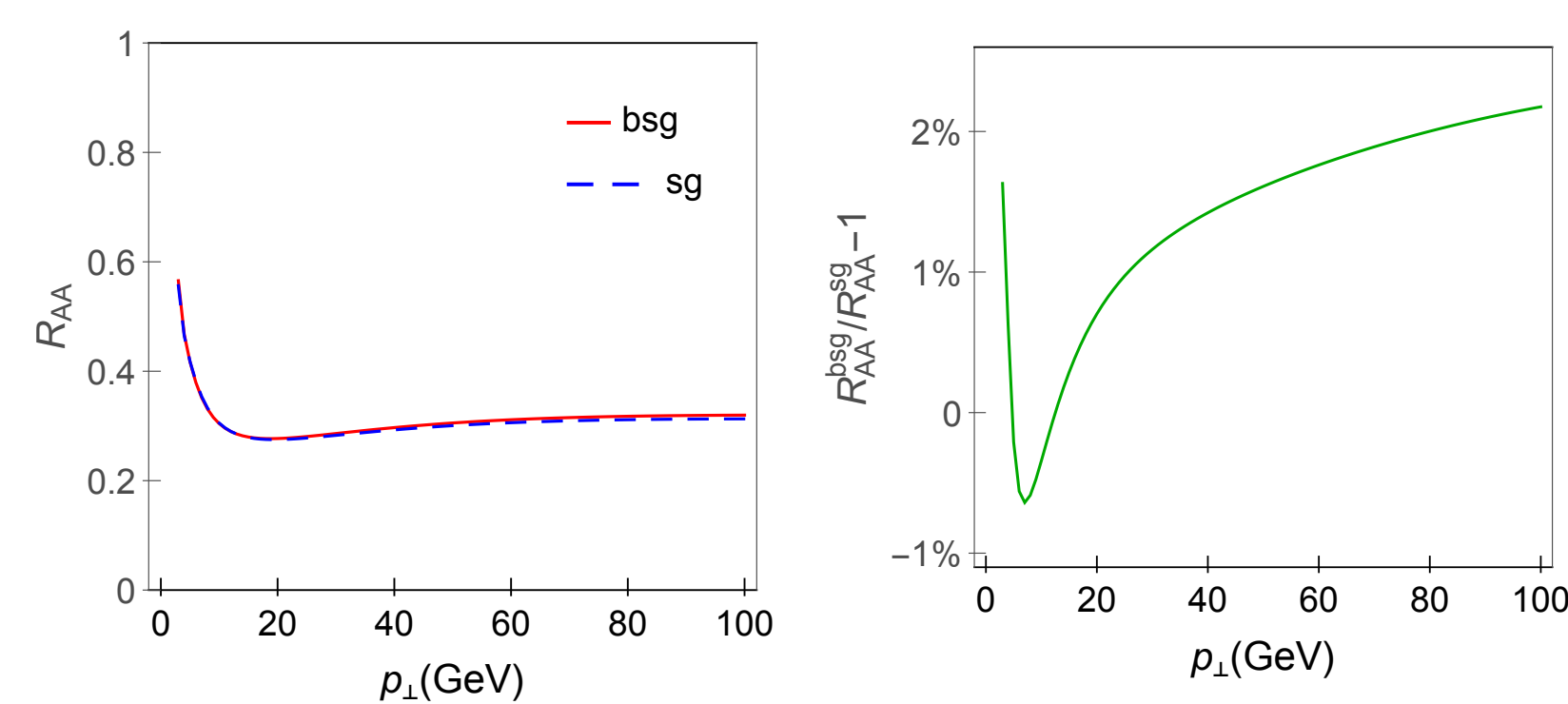
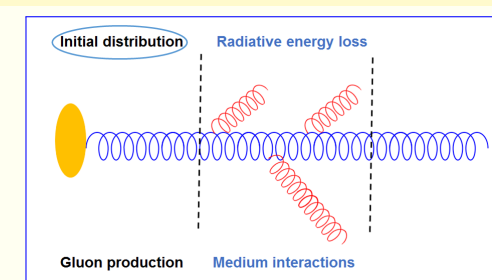


Fig. 7: The effect of relaxing the soft-gluon approximation on R_{AA} .

1. Why is R_{AA} barely affected by this relaxation?
2. Why the differential variables discrepancies at $x > 0.4$ do not influence R_{AA} ?



Even smaller effect on R_{AA} !

Both $\frac{\Delta E^{(1)}}{E}$ and $N_g^{(1)}$ non-trivially enter R_{AA} .

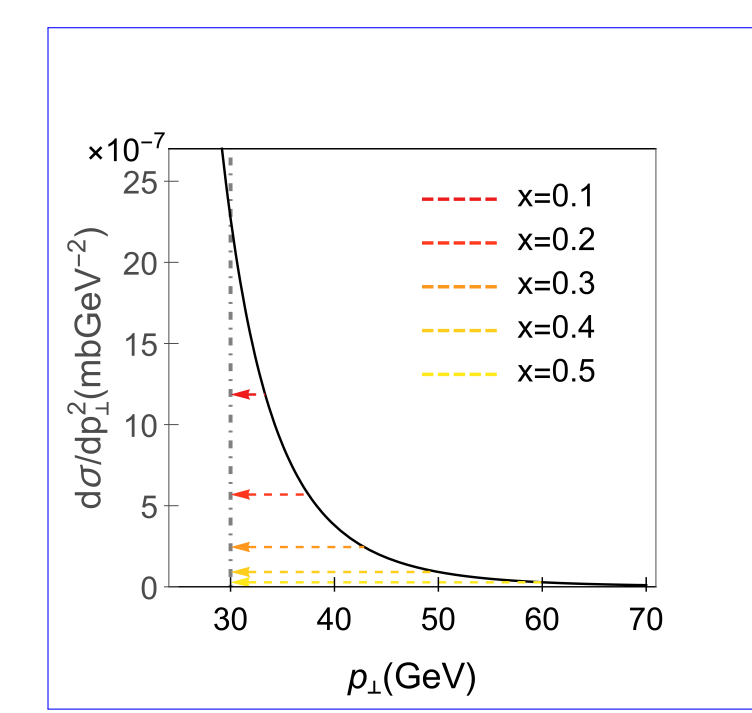


Fig. 9: The initial gluon distribution constrains the relevant x region.

- **Negligible** for R_{AA} (max 2%), and practically insensitive to gluon's momentum.
- The effect on R_{AA} is qualitatively the superposition of the effects on $\Delta E^{(1)}/E$ and $N_g^{(1)}$.
- The relevant x region for generating the predictions, due to exponentially decreasing initial distribution, is $x \lesssim 0.4$.

$x \lesssim 0.4$ is the **most relevant region** for distinguishing **bsg** from **sg** predictions, due to exponentially decreasing initial gluon p_\perp distribution.

Conclusions and Outlook

- Few theoretical models reported considerable radiative energy loss, imposing a question: is the soft-gluon approximation well-founded?
- To that end, we relaxed the approximation for high p_\perp gluons, which are most affected by it, within DGLV formalism, and although **analytical results differ greatly in bsg and sg cases, numerical predictions are nearly indistinguishable.**

- Consequently, high p_\perp quark is even less likely to be affected by the relaxation.

- This implies that **soft-gluon approximation works well within DGLV formalism.**

- **To our knowledge, this presents the introduction of effective gluon mass bsg radiative energy loss for the first time.**

- We expect that the soft-gluon approximation remains well-founded when dynamical medium is considered as well - this remains to be rigorously tested.

References

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Acknowledgments

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