Sequential coalescence with charm conservation

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Sequential coalescence:
2(3)-body Dirac equation + hydrodynamics + coalescence

Charm conservation

$D_s/D^0$ enhancement in A-A collisions

by Jiaxing Zhao & Shuzhe Shi and Nu Xu & PZ
Hadronization

- **Hadronization in vacuum**
  
a non-perturbative and unsolved problem.

- **Hadronization of quark matter**
  
  statistical distribution at freeze-out:

  \[ N_{meson} \sim \int d\sigma^\mu p_\mu W(x, p)f_q(x_1, p_1)f_{\bar{q}}(x_2, p_2) \]

  \[ P.Braun-Munzinger, J.Stachel, J.Wessels and N.Xu, PLB344, 43(1995) \]

  coalescence (recombination) models:

  \[ V.Greco, C.Ko and P.Levai, PRL90, 202302(2003) \]

  \[ R.Hwa and C.Yang, PRC70, 024905(2004), …… \]

- **Two assumptions**
  
  1) assumed coalescence probability (Wigner function)

  \[ W(x, p) \sim e^{-\frac{x^2}{\langle x \rangle^2}} e^{-\frac{p^2}{\langle p \rangle^2}} \text{ with parameters } \langle x \rangle \text{ and } \langle p \rangle \]

  2) assumption of simultaneous hadronization for all hadrons

- **Heavy quark hadronization**

  Sequential dissociation by (non-relaytivistic and relativistic) potential models

  \[ H.Satz, JPG32, R25(2006) \]

  \[ X.Guo, S.Shi and PZ, PLB718, 1439(2012) \]

  1) sequential dissociation temperature

  \[ T_{J/\psi} > T_{\psi'} \approx T_{\chi_c} \]

  \[ \rightarrow \text{ heavy flavor hadrons are sequentially produced}! \]

  2) self-consistent Wigner function \[ W(x, p) = \int d^4y e^{-iyp} \psi(x + \frac{y}{2})\psi^*(x - \frac{y}{2}) \]
Sequential heavy flavor production

■ Step 1: From 2(3)-body Dirac equations for heavy flavor mesons and baryons
  → sequential production temperature $T_h$
  and wave function $\psi(x)$ [Wigner function $W(x, p)$]

■ Step 2: From hydrodynamic equations for QGP evolution
  → $T(\vec{x}, t) = T_h$ → sequential production time $t_h(\vec{x}, T_h)$

■ Step 3: Sequential coalescence
  \[ N_m \sim \int d\sigma^\mu p_\mu W(x, p) f_q(x_1, p_1) f_{\bar{q}}(x_2, p_2) \]
  \[ N_b \sim \int d\sigma^\mu p_\mu W(x, p) f_q(x_1, p_1) f_q(x_2, p_2) f_q(x_3, p_3) \]
Step 1: Sequential production temperature

**For heavy flavor mesons**

\[ T=0: \text{see H.Crater, J.Yoon and C.Wong, PRD79, 034011(2009)} \]
\[ T>0: \text{see S.Shi, X.Guo and PZ, PRD88, 014021(2013)} \]

**For heavy flavor baryons**


N-body relativistic potential model


Schroedinger-like equation for baryon wave function

\[
\mathcal{V}_{ij} = 2m_{ij}S + S^2 + 2\epsilon_{ij}A - A^2 + \Phi_D + \sigma_i \cdot \sigma_j \Phi_{SS} + \mathbf{L}_{ij} \cdot (\sigma_i + \sigma_j)\Phi_{SO} + (\sigma_i \cdot \mathbf{r}_{ij})(\sigma_j \cdot \mathbf{r}_{ij})\mathbf{L}_{ij} \cdot (\sigma_i + \sigma_j)\Phi_{SOT} + \mathbf{L}_{ij} \cdot (\sigma_i - \sigma_j)\Phi_{SOD} + i\mathbf{L}_{ij} \cdot (\sigma_i \times \sigma_j)\Phi_{SOX} + (3(\sigma_i \cdot \mathbf{r}_{ij})(\sigma_j \cdot \mathbf{r}_{ij}) - \sigma_i \cdot \sigma_j)\Phi_T
\]

\[
\mathbf{V}_{qq}(r) = A_{qq}(r) + S_{qq}(r),
\]
\[
A_{qq}(r) = -\alpha_{qq}/r,
\]
\[
S_{qq}(r) = \sigma_{qq} r.
\]

\[ T_h/T_c \simeq \begin{cases} 
1.15 & \text{for } D_s \\
1.10 & \text{for } D^0 \\
1 & \text{for } \Lambda_c 
\end{cases} \]

O.Kaczmarek, EPJC 61, 811(2009)


S.Shi, J.Zhao and PZ, arXiv:1905.10627

Pengfei Zhuang, SQM2019, Bari, 20190610-15
Step 2: Sequential production time

Hydrodynamic equations:

\[ \partial \mu T_{\mu \nu} = 0 \]
\[ \partial \mu n_\mu = 0 \]

\[ \rightarrow \tau(\vec{x}|T_h) \]

\[ \tau_{J/\psi} < \tau_{D_S} < \tau_{D^0} < \tau_{\Lambda_c} < \tau_{\pi,K,N} \]
Step 3: Sequential coalescence

\[ N_b \sim \int \frac{P^\mu d\sigma^\mu(R)}{(2\pi)^3} f_q(r_1, p_1)f_q(r_2, p_2)f_q(r_3, p_3)W(r, p) \]

**Hydrodynamics**  
**Energy loss**  
**Dirac equations**

- **light quarks** \((u, d)\):
  - full energy loss, equilibrium distribution

  \[ f_q = \frac{N_q}{e^{(u\mu p_p - \mu q)/T} + 1} \]

- **strange quark** \(s\):
  - thermal equilibrium but not chemical equilibrium

  \[ f_s = \frac{N_s\lambda_s}{e^{u\mu p_p / T} + 1} \]

  \[ \lambda_s = \begin{cases} 
  0.85 & \text{at RHIC} \\
  1 & \text{at LHC} 
  \end{cases} \]

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Pengfei Zhuang, SQM2019, Bari, 20190610-15
Charm conservation

If all charmed hadrons are simultaneously produced, the charm conservation contributes only a normalization constant, → it does not change the particle ratios!

- If charmed hadrons are sequentially produced, however, more charm quarks are involved in the earlier production and less in the later production,

\[
r_h = \frac{\text{involved charm quarks}}{\text{total charm quarks } N_c} = \begin{cases} 
1 & \text{for } h = D_s \\
1 - \frac{N_{D_s}}{N_c} \ (\sim 90\%) & \text{for } h = D^0 \\
1 - \frac{N_{D_s} + N_{D^0}}{N_c} \ (\sim 60\%) & \text{for } h = \Lambda_c
\end{cases}
\]

K.Zhou, Z.Chen, C.Greiner and PZ., PLB758, 434(2016)

M.Gorenstein, A.Kostyuk, H.Stoecker and W.Greiner, PLB509, 277(2001)
S.Plumari, V.Minissale, S.Das and V.Greco, EPJC78, 348(2018)
Charm quark distribution

Charm quarks are not fully thermalized:

\[ f_c = r_h \rho_c(x) \left[ \alpha f_{th}(p) + \beta f_{pp}(p) \right] \]

\( r_h \) is the charm conservation factor,
thermalization fraction \( \alpha \) depends on the coalescence time:

\[
(\alpha, \beta) = \begin{cases} 
(0.4, 0.6) & D_s \\
(0.5, 0.5) & D^0 \\
(0.6, 0.4) & \Lambda_c 
\end{cases}
\]

charm quark density

\[ \rho_c(x) = T_A(x_T)T_B(x_T - b) \frac{\cosh \eta}{\tau} \frac{d\sigma_{pp}^{c\bar{c}}}{d\eta} \]

Normalized rapidity and transverse momentum distributions with PYTHIA8
$D_s/D^0$ enhancement

**Strong $D_s/D^0$ enhancement at RHIC**

![Graph showing $D_s/D^0$ ratio vs. transverse momentum $p_T$](image)

$L. Zhou [STAR], NPA967, 620(2017)$

$D_s$ enhancement due to strangeness enhancement in quark matter


$M. He, R. Fries and R. Rapp, PRL 110, 112301(2013)$

However, $D_s$ enhancement cannot fully explain the $D_s/D^0$ enhancement!

$D_s$ enhancement + charm conservation induce $D^0$ suppression

$\rightarrow$ a further $D_s/D^0$ enhancement!

![Graphs showing $D_s/D^0$ ratio vs. $p_T$](image)

$L. Zhou [STAR], NPA967, 620(2017)$

$J. Zhao, S. Shi, N. Xu and PZ, arXiv: 1805.10858$

$J. Adam et. al. [ALICE], JHEP 1603, 082(2016)$

solid lines: with charm conservation, dashed lines: without charm conservation

Pengfei Zhuang, SQM2019, Bari, 20190610-15
\[ \Lambda_c/D^0 \text{ and } \Xi_c/D^0 \]

solid lines: with charm conservation, dashed lines: without charm conservation

\( D_s \) is produced first, then \( D^0 \), and finally \( \Lambda_c \) and \( \Xi_c \).
**Baryon density effect**

- **quark distributions at finite baryon density**

\[ f_{u,d} = \frac{N_{u,d}}{e^{(u_p \mu_B - \mu_B)/T} + 1} \]
\[ f_s = \frac{N_s \lambda_s}{e^{u_p \mu / T} + 1} \]

at high baryon density, more \( u \) and \( d \) quarks, less \( \bar{u} \) and \( \bar{d} \) quarks, and probably \( n_{\bar{u}} < n_{\bar{s}} \)!

**Calculated quark number density at \( \vec{r} = 0 \):**

- **\( D_s(c\bar{s}) \) enhancement and \( D^0(c\bar{u}) \) suppression at high \( \mu_B \)!

J.Zhao, S.Shi, N.Xu and PZ, in progress

\[ \text{pp limit} \quad \sqrt{s} \]

\[ D_s/D^0 \quad \text{Au+Au Central, } |y|<0.1 \]

\[ u(d) \quad s(\bar{s}) \quad \bar{u}(\bar{d}) \quad c(\bar{c}) \]

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$D_s/D^0(\sqrt{s})$

A significantly strong $D_s/D^0$ enhancement at about $\sqrt{s} = 10$ GeV where the baryon density is the largest.
Comparison with $K/\pi$

The behavior of $D_s^+ / D^0$ ($D_s^- / \bar{D}^0$) is similar to $K^+ / \pi^+$ ($K^- / \pi^-$).

The two peaks locate at the largest baryon density.
Summary

- We developed a sequential coalescence model for heavy flavor hadron production: 2(3)-body Dirac equations + hydrodynamic equations + coalescence.

- Charm conservation enhances significantly the ratio $D_s/D^0$.

- $D_s/D^0$ is further enhanced at high baryon density.