First order dissipative hydrodynamics from an effective covariant kinetic theory

Samapan Bhadury[1]

Collaborators:

Manu Kurian[2], *Dr. Vinod Chandra*[2], *Dr. Amaresh Kumar Jaiswal*[1]

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[1]National Institute of Science Education and Research, Bhubaneswar

[2]IIT Gandhinagar

Outline:

Hydro Equations :

- In Landau frame $(u_\nu T^{\mu\nu} = \epsilon u^\mu)$:

$$T^{\mu\nu} = \underbrace{\epsilon \; u^\mu u^\nu - P\Delta^{\mu\nu}}_{Ideal \;\; part} + \Pi^{\mu\nu} \tag{1}$$

$$N^\mu = \underbrace{nu^\mu}_{Ideal \;\; part} + n^\mu \tag{2}$$

- ϵ, P, n, u^μ are energy density, pressure, number density and flow velocity respectively.
- $g^{\mu\nu} = diag(1, -1, -1, -1)$ and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$.
- Dissipative quantities $\rightarrow \quad \Pi^{\mu\nu} = -\Pi\Delta^{\mu\nu} + \pi^{\mu\nu}; \quad n^\mu$
- An observable is macroscopic but interactions are microscopic!

Hydro Equations (contd.):

- There are two conservation laws that must be obeyed even for a dissipative fluid system.
- $$\partial_\mu T^{\mu\nu} = 0; \qquad \partial_\mu N^\mu = 0$$
  Using these two laws, one finds the hydro equations[3] (evolution of thermodynamic quantities) for a relativistic fluid :

$$\dot{\epsilon} + (\epsilon + P + \Pi)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} = 0 \tag{3}$$

$$(\epsilon + P + \Pi)\dot{u}^\rho - \nabla^\rho(P + \Pi) + \Delta^\rho{}_\nu \partial_\mu \pi^{\mu\nu} = 0 \tag{4}$$

$$\dot{n} + n\theta + \partial_\mu n^\mu = 0 \tag{5}$$

Hydro-Equations(5) + EoS(1) = 6 equations.
$\epsilon(1) + P(1) + n(1) + u^\mu(3) + \Pi(1) + \pi^{\mu\nu}(5) + n^\mu(3)$
$= 15$ unknowns.

[3] Paul Romatschke, Int. J. Mod. Phys. E **19**, 1 (2010)

Outline:

Distribution Function :

- $f(x, p) \rightarrow$ momentum distribution at each space-time point.
- One can connect the macroscopic observables with $f(x, p)$, and examine the evolution of $f(x, p)$.
- The equilibrium distribution function can be expressed as:

$$f_{eq} = \frac{1}{e^{\beta(u \cdot p) - \alpha} + r} \tag{6}$$

- β is inverse absolute temperature, $\alpha = \mu / T$ where μ is the chemical potential.
- r = 0, +1, -1 for MB, FD, BE distributions respectively.

Implementing Kinetic Theory :

- Since the macroscopic observables can be found from $T^{\mu\nu}$ and N^{μ}, the first step is to relate these quantities with $f(x,p)$ which can be done as:

$$T^{\mu\nu} = \int \mathrm{dP}\ p^{\mu} p^{\nu} f \tag{7}$$

$$N^{\mu} = \int \mathrm{dP}\ p^{\mu} f \tag{8}$$

Where the measure is given by:

$$\int \mathrm{dP} \equiv \int \frac{\mathrm{d^4p}}{(2\pi)^4}\ 2\Theta(p^0)\ \delta(p^{\mu} p_{\mu} - m^2) \tag{9}$$

Boltzmann's Equation:

- The evolution of $f(x,p)$ is governed by the Boltzmann eqn:

$$p^{\mu}\partial_{\mu}f + F^{\mu}\partial_{\mu}^{(p)}f = -C[f] \tag{10}$$

- $F^{\mu} \rightarrow$ the force term.
- $C[f] \rightarrow$ the collision kernel[4].
- For non-equilibrium case we have, $f = f_{eq} + \delta f$. $(\delta f/f \ll 1)$

[4] We will use RTA for present case : $\frac{(u\cdot p)}{\tau_R}\delta f$

Dissipative Quantities:

- Using Boltzmann and hydro equations, we can determine the deviation part i.e. δf, which can in turn be be used to evaluate the dissipative quantities as follows:

$$\Pi = -\frac{1}{3}\Delta_{\alpha\beta}\int \mathrm{dp}\ p^{\alpha}p^{\beta}\left(\delta f + \delta\bar{f}\right) \tag{11}$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}{}_{\alpha\beta}\int \mathrm{dp}\ p^{\alpha}p^{\beta}\left(\delta f + \delta\bar{f}\right) \tag{12}$$

$$n^{\mu} = \Delta^{\mu}{}_{\alpha}\int \mathrm{dp}\ p^{\alpha}\left(\delta f - \delta\bar{f}\right) \tag{13}$$

Outline:

Quasiparticle Picture :

- The QCD Equation of State (EoS) varies with temperature and we have to deal with a dynamic EoS.
- In usual kinetic theory approach, EoS is however taken to be fixed. Thus the transport coefficents obtained in this framework should not reflect their true temperature dependence.
- In a quasiparticle picture, the temperature dependence is taken into account by introducing a temperature dependent factor in the equilibrium distribution function.
- Two of the available quasiparticle models under kinetic theory framework are:
  (i) Effective Fugacity Model(EQPM),
  (ii) Effective Mass Model.

Trace Anomaly :

- Lattice data is matched using a temperature dependent fugacity factor. Thus EoS shows realistic behaviour.[5]

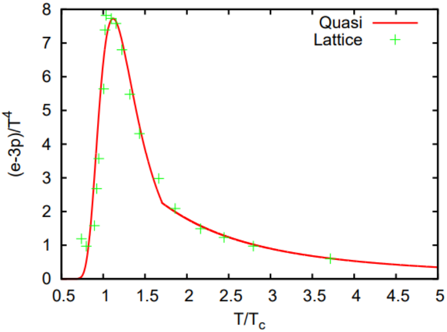


Figure: Trace Anomaly

[5] V. Chandra, V. Ravishankar, Phys.Rev. D84 (2011) 074013

Effective Fugacity Model (EQPM) :

Equilibrium distribution functions are :

$$f_q^0 = \frac{z_q \exp\left[-\beta(u^\mu p_\mu - \mu_q)\right]}{1 + z_q \exp\left[-\beta(u^\mu p_\mu - \mu_q)\right]}, \tag{14}$$

$$f_{\bar{q}}^0 = \frac{z_q \exp\left[-\beta(u^\mu p_\mu + \mu_q)\right]}{1 + z_q \exp\left[-\beta(u^\mu p_\mu + \mu_q)\right]}, \tag{15}$$

$$f_g^0 = \frac{z_g \exp\left[-\beta(u^\mu p_\mu)\right]}{1 - z_g \exp\left[-\beta(u^\mu p_\mu)\right]} \tag{16}$$

- z_q, z_g are the quasiquark and quasigluon fugacity factors respectively.
- These fugacities are functions of T/T_C where $T_c = .170$ GeV

Fugacity Factors :

- The fugacity factors are close to unity for high temperatures[6].

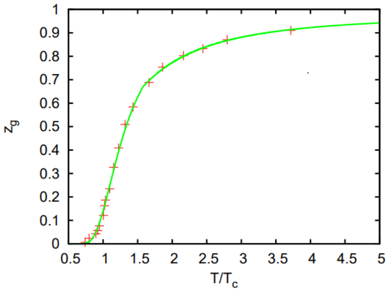


Figure: Variation of z_g

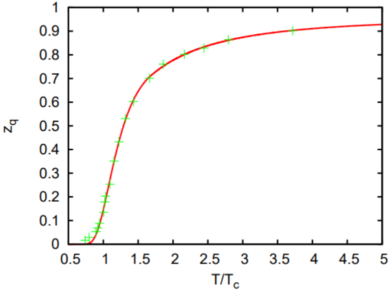


Figure: Variation of z_q

[6] V. Chandra, V. Ravishankar, Phys.Rev. D84 (2011) 074013

Conserved Quantities :

The energy-momentum tensor and particle number flow are still conserved but must be modified as:

Energy-Momentum Tensor :

$$T^{\mu\nu}(x) = \sum_{k=1}^{N} g_k \int d\tilde{P}\, \tilde{p}_k^{\mu} \tilde{p}_k^{\nu} f_k^0(x, \tilde{p}_k) + \sum_{k=1}^{N} \delta\omega_k g_k \int d\tilde{P} \frac{\langle \tilde{p}_k^{\mu} \tilde{p}_k^{\nu} \rangle}{E_k} f_k^0(x, \tilde{p}_k) \tag{17}$$

where we have[7]: $\langle \tilde{p}_k^{\mu} \tilde{p}_k^{\nu} \rangle \equiv \frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right)\tilde{p}_\alpha \tilde{p}_\beta$

$$\tilde{p}_k^{\mu} = p_k^{\mu} + \delta\omega_k u^{\mu}, \quad \delta\omega_k = T^2 \partial_T \ln(z_k)$$

[7] k represents the particle species.

Conserved Quantities (contd.) :

Particle 4-Flow :

$$N^{\mu}(x) = g_q \int d\tilde{P}\,\tilde{p}_q^{\mu} \left[f_q^0(x,\tilde{p}_k) - f_{\bar{q}}^0(x,\tilde{p}_k)\right] + \delta\omega_q g_q \int d\tilde{P}\,\frac{\langle\tilde{p}_q^{\mu}\rangle}{E_q} \left[f_q^0(x,\tilde{p}_k) - f_{\bar{q}}^0(x,\tilde{p}_k)\right] \tag{18}$$

Dissipative Quantities :

$$\pi^{\mu\nu} = \sum_k g_k \Delta^{\mu\nu}_{\alpha\beta} \int d\tilde{P} \tilde{p}^\alpha_k \tilde{p}^\beta_k \delta f_k + \sum_k \delta\omega_k g_k \Delta^{\mu\nu}_{\alpha\beta} \int d\tilde{P} \tilde{p}^\alpha_k \tilde{p}^\beta_k \frac{1}{E_k} \delta f_k \tag{19}$$

$$\Pi = -\frac{1}{3} \sum_k g_k \Delta_{\alpha\beta} \int d\tilde{P} \tilde{p}^\alpha_k \tilde{p}^\beta_k \delta f_k - \frac{1}{3} \sum_k \delta\omega_k g_k \Delta_{\alpha\beta} \int d\tilde{P} \tilde{p}^\alpha_k \tilde{p}^\beta_k \frac{1}{E_k} \delta f_k \tag{20}$$

$$n^\mu = g_q \Delta^\mu_\alpha \int d\tilde{P} \tilde{p}^\alpha_q (\delta f_q - \delta f_{\tilde{q}}) - \delta\omega_q g_q \Delta^\mu_\alpha \int d\tilde{P} \tilde{p}^\alpha_q \frac{1}{E_q} (\delta f_q - \delta f_{\tilde{q}}) \tag{21}$$

Transport Coefficients :

Using the dissipative quantities, the transport coefficients can be be found using the following relations:

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \quad \Pi = -\zeta\theta, \quad n^{\mu} = \kappa_n \nabla^{\mu}\alpha \tag{22}$$

$\eta \rightarrow$ shear viscosity; $\zeta \rightarrow$ bulk viscosity; $\kappa_n \rightarrow$ conductivity;

Effective Mass Model :

- In this model[8] [9], the mass of the quasiparticles are considered to be temperature dependent. [i.e. $m \equiv m(T)$]
- Considering a temperature dependent mass allows us to work with a dynamical Equation of State of the QCD matter.
- But there is a problem with validity of thermodynamics.
- This problem however can be resolved by modifying the definition of $T^{\mu\nu}$ as:

$$T^{\mu\nu} = \int \mathrm{dP}\ p^{\mu} p^{\nu} f - B(T)\ g^{\mu\nu} \tag{23}$$

- By working out the thermodynamics, $B(T)$ turns out to behave like a function similar to Bag pressure.

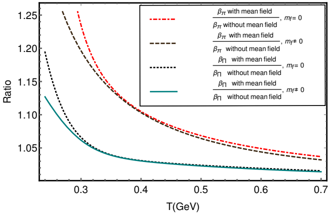
[8] Paul Romatschke, PRD 85, 065012 (2012)

[9] Tinti, Jaiswal, Ryblewski, PRD **95**, 054007 (2017)

Outline:

Effect of Mean Field on β_π, β_Π, β_n :

- The effects of mean field are more visible in the lower temp regime. Also the effect is suppressed for the massive case and for finite baryon chemical potentials too.
- In the high temperature region the ratios tend towards unity.



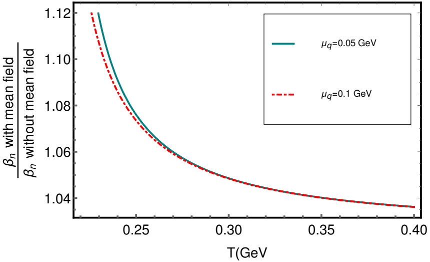


Figure: Effect on bulk viscous

Temperature Dependence of β_Π/β_π :

- Just like the previous plots, the effects of mean field and finite quark mass are more visible in the lower temp regime.
- The ratio β_Π/β_π tends to zero at higher temperature.

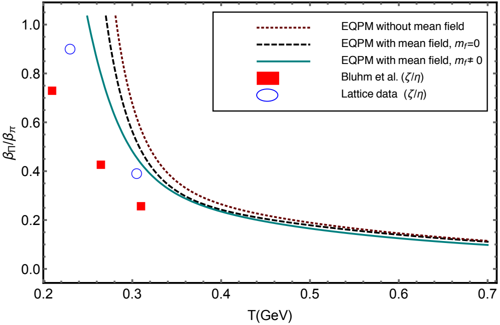


Figure: β_Π/β_π as a function of temperature for $\mu = 0.1$

Behaviour of conductivities to shear viscosity ratios :

- Conductivity is small compared to shear viscosity at low temp.
- Effect of mean field interaction is significant at low temp.

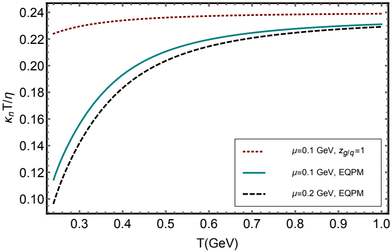


Figure: Charge Conductivity

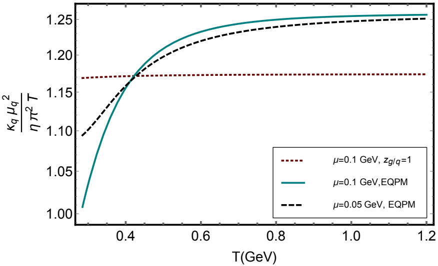


Figure: Thermal Conductivity

Longitudinal boost-invariant expansion :

- Using Milne coordinates (τ, x, y, η) to evaluate the evolution of energy density for a purely longitudinal expansion we have:

$$\tau = \sqrt{t^2 - z^2}; \quad \eta = \tanh^{-1} z/t;$$

- Then, the fluid four-velocity and metric tensor are modified to:

$$u^\mu = (1, 0, 0, 0); \quad g^{\mu\nu} = (1, -1, -1, -1/\tau^2)$$

- Then the energy density evolution equation for $\mu = 0$ is[10]:

$$\frac{d\varepsilon}{d\tau} = -\left(\frac{\varepsilon + P}{\tau}\right) + \left(\frac{\zeta + 4\eta/3}{\tau^2}\right) \tag{24}$$

- Pressure anisotropy is given by:

$$P_L/P_T \equiv (P + \Pi - \Phi)/(P + \Pi + \Phi/2) \tag{25}$$

Proper Time Evolution of Pressure Anisotropy and Temp :

- EQPM shows faster isotropization than non-interacting Boltzmann particles.
- Presence of viscous effects slows down temp. drop of medium.

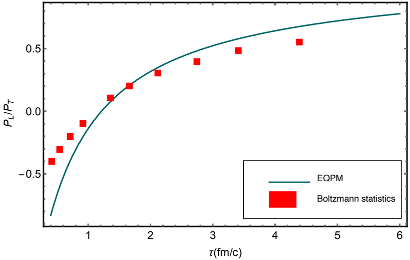


Figure: Pressure Anisotropy

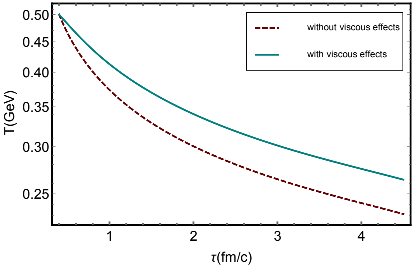


Figure: Temperature Evolution

Outline:

Summary:

- We derived the first order dissipative hydrodynamic evolution equations under EQPM, considering a grand canonical ensemble with finite baryon chemical potential μ_q and non-zero quark mass m_q.
- We observed that the mean field contributions produce significant modification to the first order coefficients of the $\pi^{\mu\nu}$, Π and n^{μ} of the hot QGP medium near T_c.
- We found that the charge conductivity is relatively smaller at the lower temperature.
- The effect of the baryon chemical potential is more visible in the temperature regime close to T_c .
- Proper time evolution of temperature and pressure anisotropy are seen to be sensitive to the viscous effects and the equation of state.

Thank you