Radial flow induced by inhomogeneous magnetic field in heavy ion collisions

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Overview

1. Magnetohydrodynamic in heavy ion collisions

2. Particle transverse momentum spectrum

Relativistic MHD

RMHD equations

The coupled RMHD equations are

$$
\begin{aligned}
d_\mu T^{\mu\nu} &= 0,\ T^{\mu\nu} = T^{\mu\nu}_{matt} + T^{\mu\nu}_{EM} && (1)\\
d_\mu F^{\mu\nu} &= -J^\nu,\ (d_\mu J^\mu = 0),\ J^\mu = \rho u^\mu + \sigma^{\mu\nu} e_\nu && (2)\\
d_\mu F^{*\mu\nu} &= 0,\ e^\mu = F^{\mu\nu} u_\nu,\ b^\mu = F^{*\mu\nu} u_\nu && (3)
\end{aligned}
$$

Where d_μ is covariant derivative.

Resistive RMHD

In the case of finite and homogeneous electrical conductivity σ of medium

$$
\begin{aligned}
D\epsilon + (\epsilon + P)\Theta &= e^\lambda J_\lambda, && (4)\\
(\epsilon + P) D u^\alpha + \nabla^\alpha P &= F^{\alpha\lambda} J_\lambda - u^\alpha e^\lambda J_\lambda, && (5)\\
d_\mu F^{\mu\nu} &= -J^\nu,\ (d_\mu J^\mu = 0),\ d_\mu F^{*\mu\nu} = 0 && (6)
\end{aligned}
$$

Relativistic MHD

Ideal RMHD

If we suppose the electrical conductivity of QGP to be infinite $\sigma \to \infty$ then we obtain electric field four vector $e_\mu = (0,0,0,0)$. Conservative equations and Maxwell equations are given by:

$$
\begin{aligned}
D(\epsilon + \frac{1}{2}b^2) + (\epsilon + P + b^2)\Theta + u_\mu b^\nu d_\nu b^\mu &= 0, && (7)\\
(\epsilon + P + b^2)Du^\mu + \nabla^\mu(P + \frac{1}{2}b^2) - b^\mu d_\nu b^\nu & \\
-b^\nu d_\nu b^\mu - u^\mu u_\nu b^\lambda d_\lambda b^\nu &= 0, && (8)\\
Db^\mu + \Theta b^\mu - u^\mu b^\nu Du_\nu - b^\nu d_\nu u^\mu &= 0 && (9)
\end{aligned}
$$

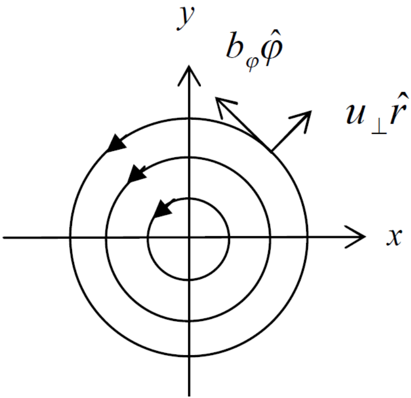
Where

$$D = u^\mu d_\mu, \quad \Theta = d_\mu u^\mu$$

Gabriele Inghirami et al, Eur. Phys. J. C (2016) 76:659.
M. Haddadi Moghaddam et al, Eur. Phys. J. C (2018) 78:255.

Induced radial flow in B-field

We consider the medium expands both radially and along the beam axis, the only nonzero components of $u_\mu = (u_\tau, u_\perp, 0, 0)$ are u_τ, which describes the boost-invariant longitudinal expansion, and $u_\perp$, which describes the transverse expansion.
And we suppose the external magnetic field to be located in transverse plane as $b_\mu = (0, 0, b_\phi, 0)$.



Transverse MHD $\mathbf{u} \cdot \mathbf{B} = 0$.

Gubser Flow

Gubser explains a generalization of Bjorken flow where the medium has finite transverse size and expands both radially and along the beam axis. The local four-velocity in the flow is entirely determined by the assumption of symmetry under a subgroup of the conformal group. The profile suggested by Gubser:

$$u^{\mu} = (u^{\tau}, u^{\perp}, 0, 0) \tag{10}$$

Where,

$$u^{\tau} = \frac{1 + q^2\tau^2 + q^2 x_{\perp}^2}{2q\tau\sqrt{1+g^2}}, \; u^{\perp} = \frac{q x_{\perp}}{\sqrt{1+g^2}} \tag{11}$$

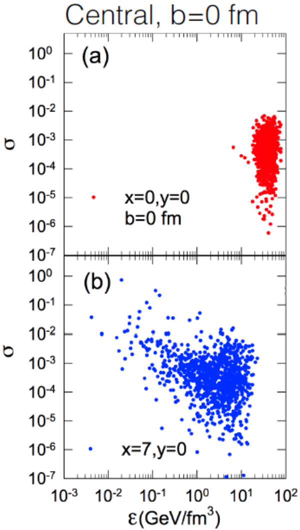
And

$$g = \frac{1 + q^2 x_{\perp}^2 - q^2\tau^2}{2q\tau} \tag{12}$$

q is a quantity with dimensions of inverse length.

S. Gubser, Phys. Rev D 82, 085027 (2010)

Why perturbation approach?



The typical magnetic field produced in Au-Au peripheral collisions at $\sqrt{s_{NN}} = 200 GeV$ reaches $|eB| \sim 10 m_\pi^2$. The estimate $\epsilon \sim 5.4 GeV/fm^3$ at about proper time $\tau = 1 fm$ is taken from Gubser flow. By taking $m_\pi \approx 150$ MeV and $e^2 = 4\pi/137$, one finds $B_c^2/\epsilon \sim 0.6$. This value in the central collisions is much smaller than in peripheral collisions, therefore, in our calculations we assumed $B_c^2/\epsilon = 0.015$ which correspond to $\rho = eB_c^2/2\epsilon \sim 0.002$.

Victor Roy et al, Phys. Rev C 92, 064902, (2015)

Analytical solution

We now seek the perturbative solution in the presence of a weak external magnetic field pointing along the ϕ direction in an inviscid fluid with infinite electrical conductivity:

$$u_\mu = (1, \lambda^2 u_\perp, 0, 0), b_\mu = (0, 0, \lambda b_\phi, 0), \quad b^2 \equiv b^\mu b_\mu$$
$$\epsilon = \epsilon_0(\tau) + \lambda^2 \epsilon_1(\tau, x_\perp), \;\; \epsilon_0(\tau) = \frac{\epsilon_c}{\tau^{4/3}} \tag{13}$$

Shi Pu et al, Phys. Rev. D 93,054042 (2016)

In such setup, the conservation equations (Energy and Euler eqs.) reduce to the following partial differential equations

$$u_\perp - \tau^2 \partial_\perp \left(\frac{u_\perp}{x_\perp}\right) - \tau^2 \partial_\perp^2 u_\perp - \tau \partial_\tau u_\perp + 3\tau^2 \partial_\tau^2 u_\perp$$
$$- \frac{3\tau^{7/3}}{x_\perp \epsilon_c} b_\phi^2 - \frac{3\tau^{7/3}}{4\epsilon_c} \partial_\perp b_\phi^2 - \frac{9\tau^{10/3}}{4 x_\perp \epsilon_c} \partial_\tau b_\phi^2 - \frac{3\tau^{10/3}}{4\epsilon_c} \partial_\perp \partial_\tau b_\phi^2 = 0. \tag{14}$$

Perturbative solution

When $b_\phi = 0$, our PDE is a homogeneous partial differential equation, which can be solved by separation of variables. The general solution are given by

$$u_\perp^{hom}(\tau, x_\perp) = \sum_k \left(c_1^k J_1(kx_\perp) + c_2^k Y_1(kx_\perp) \right) \times \left(c_1'^k \tau^{2/3} J_{1/3}(k\tau/\sqrt{3}) + c_2'^k \tau^{2/3} Y_{1/3}(k\tau/\sqrt{3}) \right) \quad (15)$$

For non-vanishing b_ϕ we assume a space-time profile of the magnetic field in central collisions in the form:

$$b_\phi^2(\tau, x_\perp) = B_c^2 \tau^n \sqrt{\alpha} x_\perp e^{-\alpha x_\perp^2}. \quad (16)$$

We see that the magnitude of b_ϕ is zero at $x_\perp = 0$. In order to find solutions for transverse velocity $u_\perp$ and energy density ϵ consistently with the assumed magnetic field, we found it convenient to first expand the magnetic field, Eq. (16) into a series of $x_\perp$-dependent functions:

$$b_\phi^2(\tau, x_\perp) = \sum_k \tau^n B_k^2 \; f(kx_\perp), \quad (17)$$

Perturbative solution

We consider the following ansatz for radial velocity:

$$u_\perp(\tau, x_\perp) = \sum_m \Big(a_m(\tau) J_1(m x_\perp) + b_m(\tau) Y_1(m x_\perp)\Big) \tag{18}$$

Because when $m = 0$ then $u_\perp(\tau, x_\perp) = 0$ so from initial condition we obtain $b_m(\tau) = 0$. Finally, is solved from the following ordinary differential equation

$$\begin{aligned} &J_1(k x_\perp)\Big(1 + \tau^2 k^2 - \tau\partial_\tau + 3\tau^2\partial_\tau^2\Big) a_k(\tau) \\ &-\frac{3\tau^{7/3+n}}{4\epsilon_c} B_k^2 \Big(\frac{f(x_\perp)}{x_\perp}(4+3n) + \partial_\perp(f(x_\perp))k(1+n)\Big) = 0. \end{aligned} \tag{19}$$

One easily finds that for $a_k(\tau)$

$$\Big(1 + \tau^2 k^2 - \tau\partial_\tau + 3\tau^2\partial_\tau^2\Big) a_k(\tau) - \frac{3k\tau^{7/3+n}}{4\epsilon_c} B_k^2 = 0 \tag{20}$$

Mathematical setup for magnetic field

From the previous calculations we found that spatial function $f(x_\perp)$ obey in the following ODE

$$(1+n)kx_\perp\partial_\perp f(x_\perp)+(4+3n)f(x_\perp)=kx_\perp J_1(kx_\perp). \quad (21)$$

The general solution is given by

$$f(kx_\perp) = \frac{k^2x_\perp^2\Gamma\left(\frac{2nk+2k+3n+4}{2nk+2k}\right)\,{}_1F_2\left(\frac{2nk+2k+3n+4}{2nk+2k};2,\frac{4nk+4k+3n+4}{2nk+2k};-\frac{1}{4}k^2\right.}{4(n+1)\Gamma\left(\frac{4kn+4k+3n+4}{2kn+2k}\right)} + d_1(k^2(n+1)x_\perp)^{-\frac{3n+4}{kn+k}},$$

where ${}_1F_2$ is the hypergeometric function. The first term is a well-defined function, but the second one diverges in $x_\perp=0$ for any n except $n=-4/3$; hence, d_1 must be zero. For $n=-1$, wich will be considered in details; the solution of Eq. (21) takes a simple form:

$$f(kx_\perp)=kx_\perp J_1(kx_\perp) \quad (\text{for} \quad n=-1) \quad (23)$$

Magnetic field profile for the case $n = -1$

The square of magnetic field:

$$b_\phi^2(\tau, x_\perp) = \sum_k \tau^{-1} B_k^2 \beta_{1k} \frac{x_\perp}{a} J_1(\beta_{1k} \frac{x_\perp}{a}) \tag{24}$$

where the coefficients B_k^2 are given by

$$B_k^2 = \frac{2a}{a^2 \beta_{1k} [J_2(\beta_{1k})]^2} \int_0^a J_1(\beta_{1k} \frac{x_\perp}{a}) b_\phi^2 dx_\perp \tag{25}$$

where β_{1k} is the kth zero of J_1. In above integral, one can substitute the appropriate profile for b_ϕ^2 as we supposed.

Radial velocity for the case $n = -1$

The transverse velocity takes the form

$$u_\perp(\tau, x_\perp) = \sum_k a_k(\tau) J_1(k x_\perp). \tag{26}$$

$$\begin{aligned} a_k(\tau) &= c_1^k \tau^{2/3} J_{\frac{1}{3}}\left(\frac{k\tau}{\sqrt{3}}\right) + c_2^k \tau^{2/3} Y_{\frac{1}{3}}\left(\frac{k\tau}{\sqrt{3}}\right) + \frac{\pi k B_k^2}{48\Gamma(\frac{2}{3})\Gamma(\frac{7}{6})\Gamma(\frac{4}{3})\epsilon_c \sqrt[3]{k\tau}} \\ &\Big(-2^{2/3}\sqrt[3]{3}\tau^{4/3}\Gamma(\frac{2}{3})\Gamma(\frac{7}{6})(k\tau)^{2/3} J_{\frac{1}{3}}\left(\frac{k\tau}{\sqrt{3}}\right) {}_1F_2\left(\frac{1}{2}; \frac{4}{3}, \frac{3}{2}; -\frac{1}{12}k^2\tau^2\right) \\ &+ 2\sqrt[3]{2}3^{2/3}\tau^{4/3}\Gamma(\frac{4}{3})\Gamma(\frac{1}{6}) J_{\frac{1}{3}}\left(\frac{k\tau}{\sqrt{3}}\right) {}_1F_2\left(\frac{1}{6}; \frac{2}{3}, \frac{7}{6}; -\frac{1}{12}k^2\tau^2\right) \\ &+ 2^{2/3}3^{5/6}\tau^{4/3}\Gamma(\frac{2}{3})\Gamma(\frac{7}{6})(k\tau)^{2/3} Y_{\frac{1}{3}}\left(\frac{k\tau}{\sqrt{3}}\right) {}_1F_2\left(\frac{1}{2}; \frac{4}{3}, \frac{3}{2}; -\frac{1}{12}k^2\tau^2\right)\Big) \end{aligned}$$

$$c_1^k = \frac{\sqrt[3]{k}\left(3\pi^{3/2}\Gamma(\frac{7}{6}) - \sqrt{\pi}\Gamma^2(\frac{1}{6})\Gamma(\frac{5}{6})\right)B_k^2}{24\sqrt[3]{2}\sqrt[6]{3}\Gamma(\frac{5}{6})\Gamma(\frac{7}{6})\epsilon_c}, \quad c_2^k = -\frac{\sqrt[3]{\frac{3}{2}}\pi^{3/2}\sqrt[3]{k}B_k^2}{8\Gamma(\frac{5}{6})\epsilon_c}. \tag{27}$$

Modified energy density for the case $n = -1$

The correction of energy density ϵ_1 is obtained from the following equations:

$$\partial_\tau \epsilon_1 - \frac{4\epsilon_c}{3\tau^{4/3}}\left(\frac{u_\perp}{x_\perp} + \frac{\partial u_\perp}{\partial x_\perp}\right) + \frac{4\epsilon_1}{3\tau} + \frac{1}{2}\partial_\tau b_\phi^2 + \frac{b_\phi^2}{\tau} = 0 \tag{28}$$

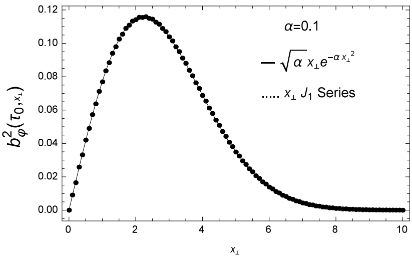
$$\partial_\perp \epsilon_1 - \frac{4\epsilon_c}{\tau^{4/3}}\partial_\tau u_\perp + \frac{4\epsilon_c}{3\tau^{7/3}} u_\perp + \frac{3}{2}\partial_\perp b_\phi^2 + \frac{3b_\phi^2}{x_\perp} = 0. \tag{29}$$

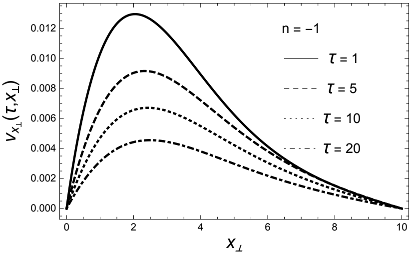
$$\epsilon_1(\tau, x_\perp) = \sum_k h(\tau) + \sum_k \frac{1}{24k\tau^{7/3}}\Big(32\epsilon_c(J_0(kx_\perp) - 1)(a_k(\tau) - 3ta_k'(\tau)) -9B_k^2 k\tau^{4/3}(k^2x_\perp^2\, {}_0F_1(2; -\frac{1}{4}k^2x_\perp^2) + 2kx_\perp J_1(kx_\perp) - 8J_0(kx_\perp) + 8)\Big), \tag{30}$$

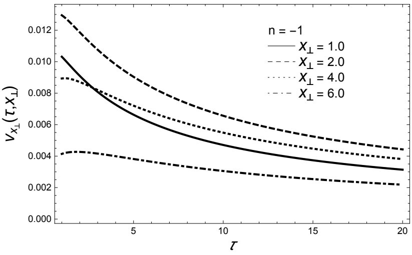
where $h(\tau)$ is the constant of integration and can be obtained form

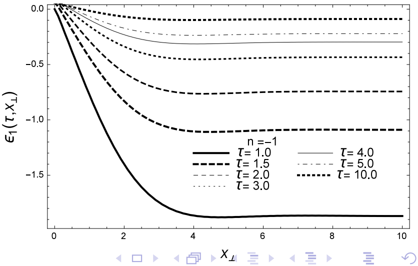
$$h(\tau) = \frac{\int_1^\tau \frac{4}{3}k\epsilon_c a_k(s)\, ds}{\tau^{4/3}}. \tag{31}$$

Results for $n = -1$

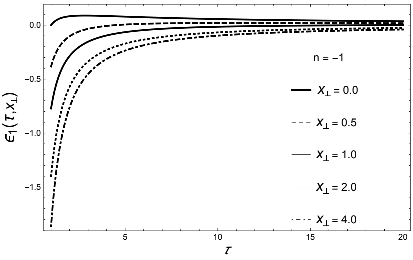


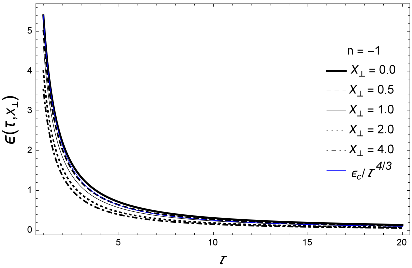


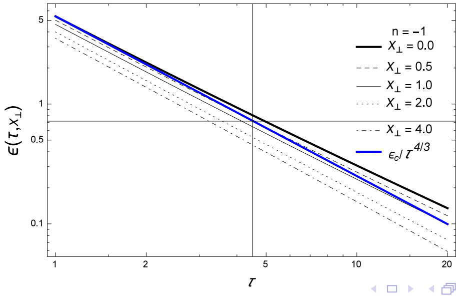




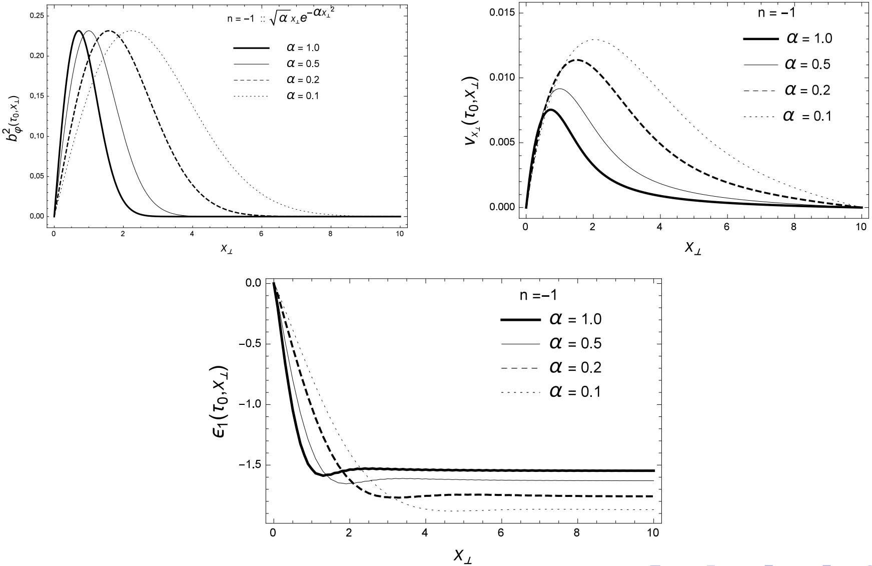
Results for $n = -1$



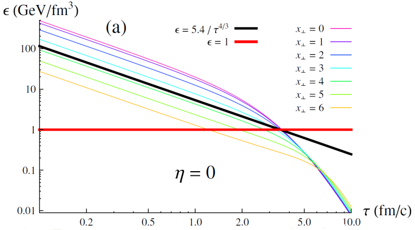


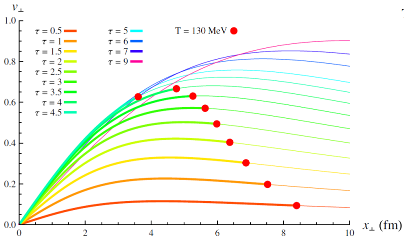


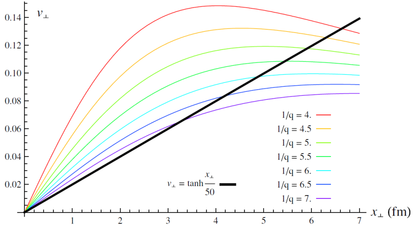
Scaling parameter α for $n = -1$



Gubser Flow







In our work the $v_\perp$ gets smaller when α is made larger as well as Gubser, $v_\perp$ get smaller when $1/q$ is made larger. It seems that, parameter $\sqrt{\alpha}$ play the role of parameter $1/q$.

Dynamical transverse effect on particles

From the local equilibrium hadron distribution the transverse spectrum is calculated via the Cooper-Frye formula in the freeze out surface

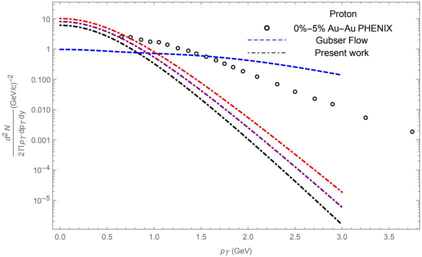
$$S = E\frac{d^3N}{dp^3} = \frac{g_i}{2\pi^2}\int_0^{x_f} x_\perp\ \tau_f(x_\perp)\ dx_\perp\ \Big[m_T K_1(\frac{m_T u_\tau}{T_f}) I_0(\frac{m_T u_\perp}{T_f}) + p_T R_f K_0(\frac{m_T u_\tau}{T_f}) I_1(\frac{m_T u_\perp}{T_f})\Big] \quad (32)$$

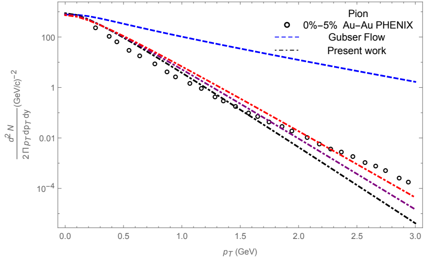
Where $\tau_f(x_\perp)$ is the solution of the $T(\tau_f, x_\perp) = T_f$ and the degeneracy is $g_i = 2$ for both the pions and the protons. The above integral over $x_\perp$ on the freeze-out surface is evaluated numerically.

U. Gursoy et al, Phys. Rev C 89, 054905 (2014)

Transverse momentum spectrum

The spectrum Eq. (32) is illustrated in the following figures for three different values of the freeze out temperature (140, 150 and 160 MeV) and compared with experimental results obtained at PHENIX. in central collisions.





M. Haddadi Moghaddam et al, arXiv: 1710.01037 [nucl-th]

Conclusions and future perspectives

- Magnetic fields may produce some relevant effects on several observable quantities like velocity, energy density, etc.
- As preliminary results, hadrons with different masses have different sensitivities to the electromagnetic fields.
- The difference between the charge-dependent flow of light pions and heavy protons might arise because the former are more affected by the weak magnetic field than the heavy protons.
- In future: effects of electric and magnetic field can be considered. (In preparation)

Thank you

Backup Slides

Relativistic MHD

Energy-momentum tensor and four vector fields

$$T^{\mu\nu}_{pl} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} \quad (33)$$

$$T^{\mu\nu}_{em} = F^{\mu\eta} F_\eta{}^\nu - \frac{1}{4} F^{\eta\rho} F_{\eta\rho} g^{\mu\nu} \quad (34)$$

$$F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \varepsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa, \quad (35)$$

$$F^{\star\alpha\beta} = u^\mu b^\nu - u^\nu b^\mu - \varepsilon^{\mu\nu\lambda\kappa} e_\lambda u_\kappa \quad (36)$$

$$\text{Levi - Civita} : \varepsilon^{\mu\nu\lambda\kappa} = 1/\sqrt{-\det g}[\mu\nu\lambda\kappa], \quad (37)$$

$$\text{Electric four vector} : e^\alpha = \gamma[\mathbf{v} \cdot \mathbf{E}, (\mathbf{E} + \mathbf{v} \times \mathbf{B})]^T, (\text{Cartezian}) \quad (38)$$

$$\text{Magnetic four vector} : b^\alpha = \gamma[\mathbf{v} \cdot \mathbf{B}, (\mathbf{B} - \mathbf{v} \times \mathbf{E})]^T, (\text{Cartezian}) \quad (39)$$

Where $\vec{v}, \vec{B}, \vec{E}$ are measured in lab frame and γ is Lorentz factor.

Why to study magnetic field in HIC?

Strong magnetic field may produce many effects:

1. The Chiral Magnetic Effect (CME)
2. The Chiral Magnetic Wave (CMW)
3. The Chiral separation Hall effect (CSHE)
4. Influence on the elliptic flow (v_2)
5. Influence on the directed flow (v_1)
6. ...