# Primordial fluctuations and anisotropy in heavy-ion collisions

by

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Based on:

**- Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault 1902.07168**

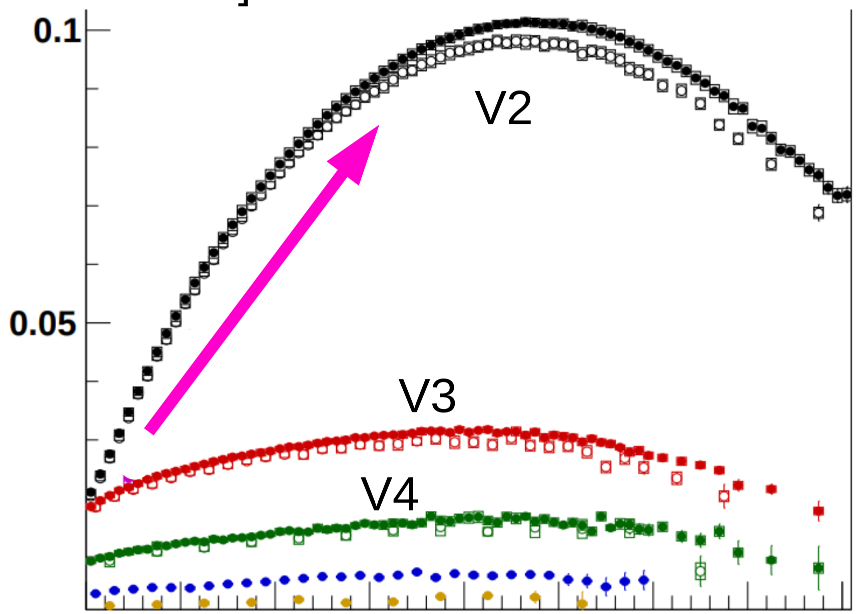
# The fluid paradigm. Anisotropy from anisotropy.

final state harmonic ← $$v_n = \kappa_n \varepsilon_n$$ → initial state harmonic

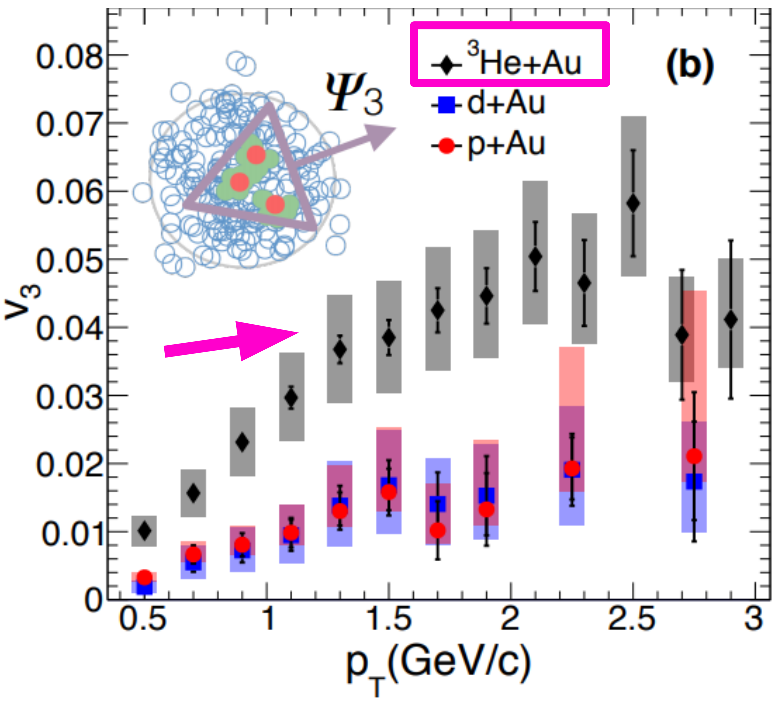
## Direct confirmations in the data.

[ALICE collaboration, 1804:02944]

**n=2**




**n=3**




[PHENIX collaboration, 1805:02973]

$\kappa_n$ → Just a number. Rather independent of centrality up to ~20-30%.

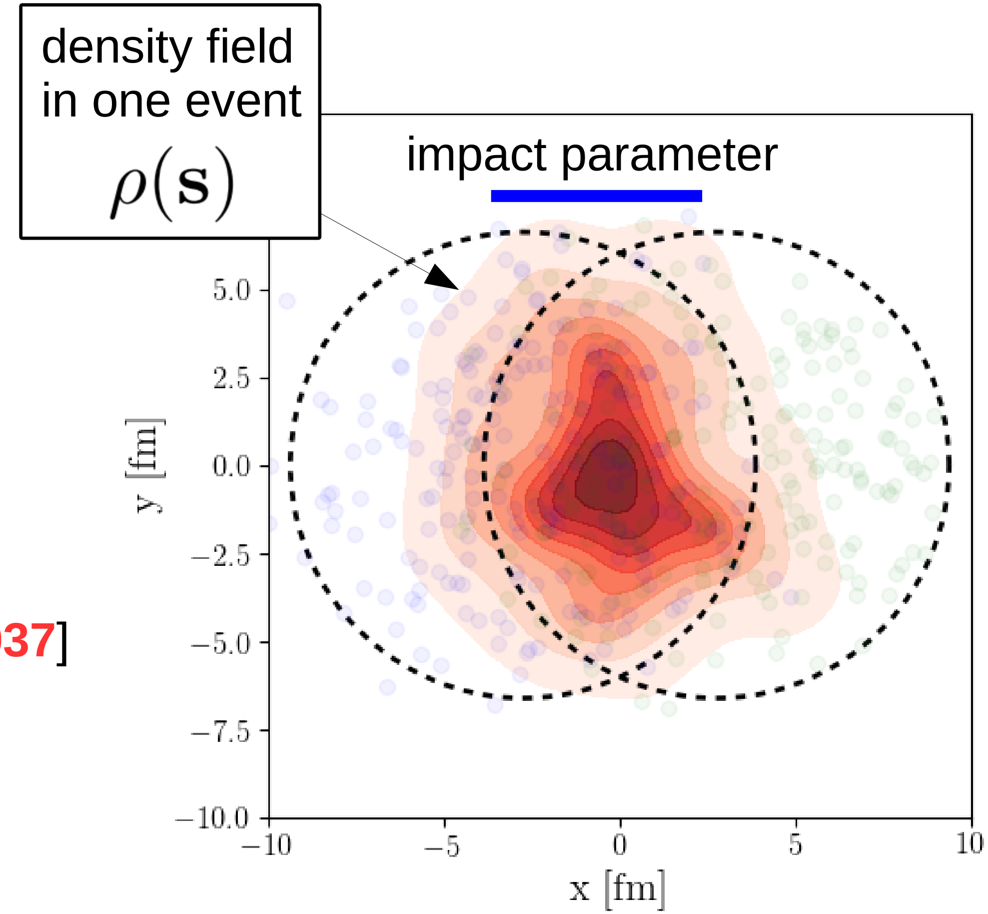
[Noronha-Hostler, Yan, Gardim, Ollitrault, arXiv 1511:03869]

# How do we calculate the initial anisotropy?

[Teaney, Yan **1010.1876**]

$$\varepsilon_n \equiv \frac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})}$$

$$\mathbf{s} = x + iy$$




**Origin of anisotropy:**

Elliptic flow → **geometry** + **fluctuations**
[PHOBOS Collaboration **nucl-ex/0610037**]

Triangular flow → **fluctuations only**
[Alver, Roland **1003.0194**]

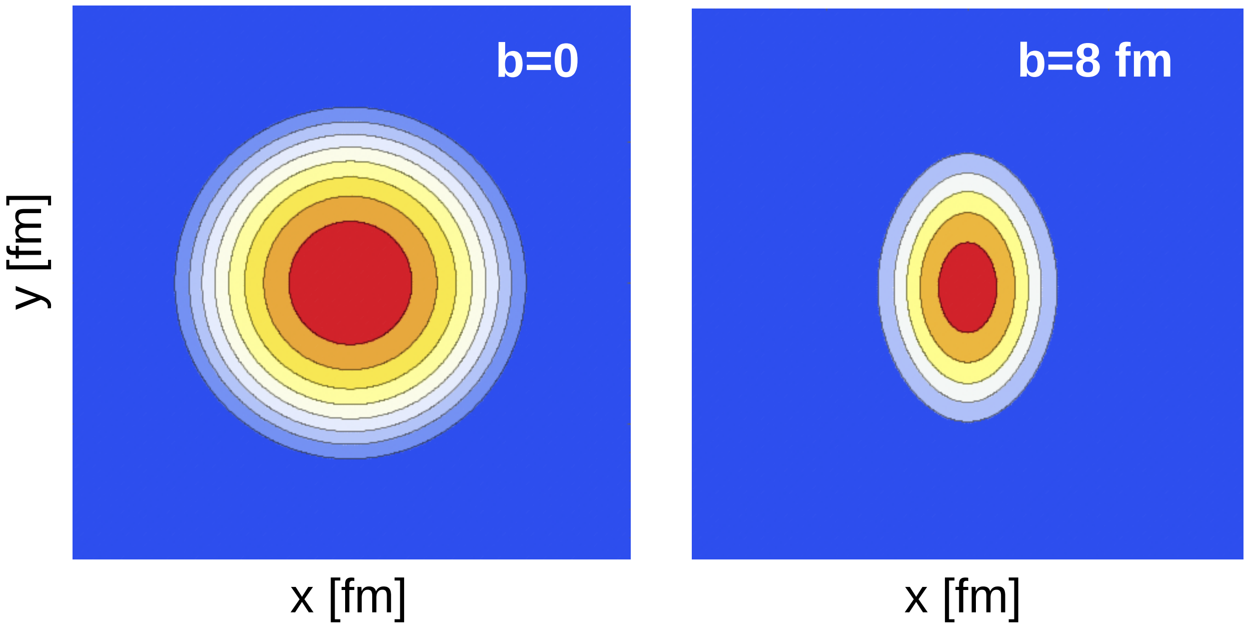
The theoretical input is a model for $\rho(\mathbf{s})$ and its fluctuations.

What do we need?

- $\langle \rho(\mathbf{s}) \rangle$

**The average density.**

Integrates to the average total energy.

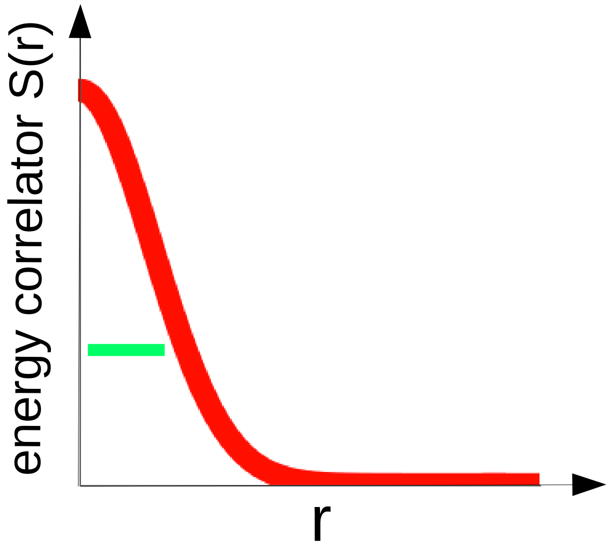



- **The fluctuations** around the average.

$$S(\mathbf{s}_1, \mathbf{s}_2) = \langle \rho(\mathbf{s}_1) \rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle$$

Density of variance.
Integrates to the variance of the total energy.
Quantifies the correlation of fluctuations.

$$\mathbf{r} = \mathbf{s}_1 - \mathbf{s}_2$$



Input from high-energy QCD (or CGC). Nuclei characterized by a scale:

$$Q_s^2(\mathbf{s}) \propto T(\mathbf{s})$$ ← Nuclear thickness

Proportional to the density of ‘color charges’, that source the energy density.
Average energy density after the collision known for a long time:

$$\langle \rho(\mathbf{s}) \rangle \propto Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s})$$

[Lappi **hep-ph/0606207**]
[Lappi, Venugopalan **nucl-th/0609021**]

Fluctuations calculated recently:

[Albacete, Guerrero-Rodriguez, Marquet **1808.00795**]

The leading order of the expansion, of order $N_c^0$, reads:

$$\begin{aligned}
&[\mathrm{Cov}[\epsilon_{\rm MV}](0^+;x_\perp,y_\perp)]_{N_c^0} \\
&= \Bigg[\frac{1}{g^4 r^8} e^{-\frac{r^2}{2}(Q_{s1}^2+Q_{s2}^2)} \Bigg(16+32e^{\frac{Q_{s1}^2 r^2}{2}} \\
&\quad -64e^{\frac{Q_{s1}^2 r^2}{4}} - 4e^{\frac{r^2}{4}(2Q_{s1}^2+Q_{s2}^2)} \left(Q_{s2}^4 r^4 - 2\left(4\pi\,\partial^2 L(0_\perp)\right)^2 \bar{Q}_{s1}^4 r^4 + 8\,Q_{s2}^2 r^2 + 48\right) \\
&\quad + \frac{1}{8} e^{\frac{r^2}{4}(Q_{s1}^2+Q_{s2}^2)} \left(Q_{s1}^4 Q_{s2}^4 r^8 + (4\,Q_{s1}^2 Q_{s2}^2 r^6 + 128\,r^2)(Q_{s1}^2+Q_{s2}^2) + 16\,r^4 \left(Q_{s1}^2+Q_{s2}^2\right)^2 + 1024\right) \\
&\quad + 2e^{\frac{r^2}{2}(Q_{s1}^2+Q_{s2}^2)} \left(\bar{Q}_{s1}^4 r^4 (Q_{s2}^2 r^2 - 4)\left(4\pi\,\partial^2 L(0_\perp)\right)^2 + 40\right)\Bigg)\Bigg] + [1 \leftrightarrow 2].
\end{aligned} \tag{4.48}$$

The next term, of order $N_c^{-2}$, reads:

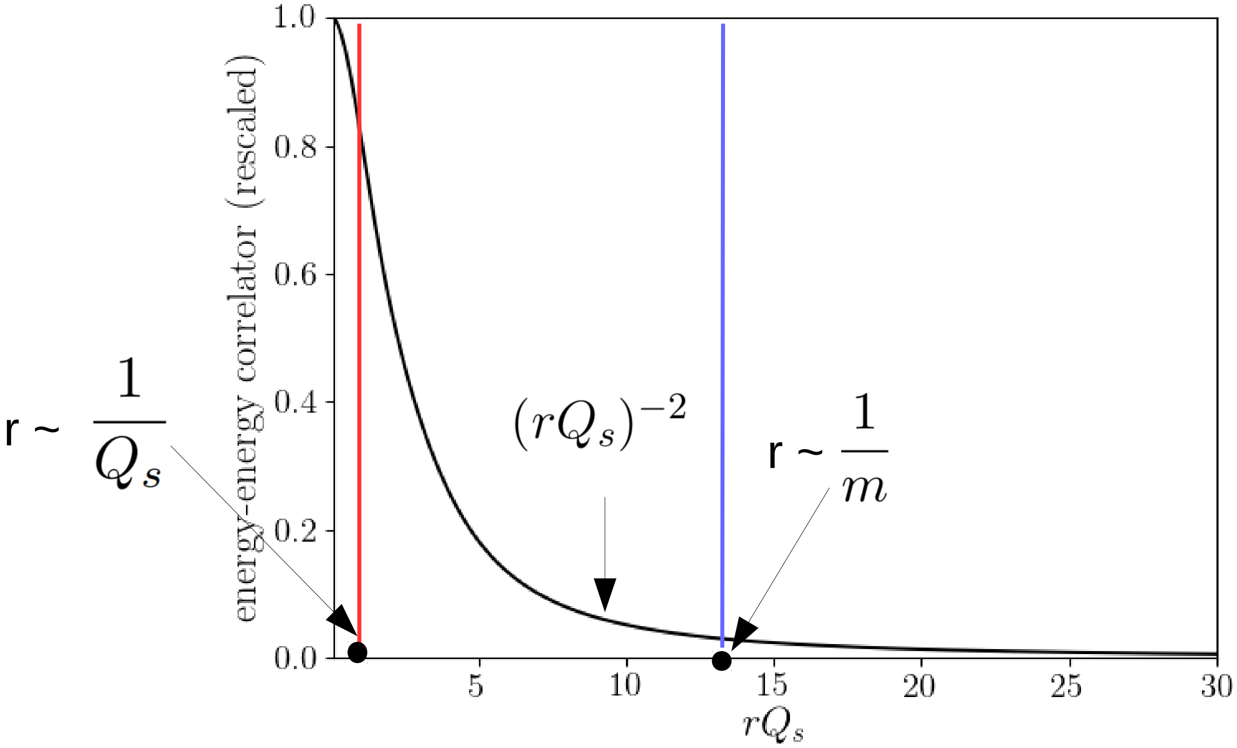
$$\begin{aligned}
&[\mathrm{Cov}[\epsilon_{\rm MV}](0^+;x_\perp,y_\perp)]_{N_c^{-2}} \\
&= \Bigg[\frac{1}{N_c^2 g^4 r^8} e^{-\frac{r^2}{2}(Q_{s1}^2+Q_{s2}^2)} \Bigg(2\left(Q_{s1}^2 r^2 + Q_{s2}^2 r^2 + 8\right)^2 \\
&\quad + 4\,Q_{s1}^2 r^2 (8+Q_{s1}^2 r^2) e^{\frac{Q_{s2}^2 r^2}{2}} - 8(8+Q_{s1}^2 r^2)(4+Q_{s1}^2 r^2) e^{\frac{Q_{s2}^2 r^2}{4}} \\
&\quad + 4\,e^{\frac{r^2}{4}(2Q_{s1}^2+Q_{s2}^2)} \left(Q_{s2}^4 r^4 - 2(4\pi\,\partial^2 L(0_\perp))^2 \bar{Q}_{s1}^4 r^4 + 8Q_{s2}^2 r^2 + 16 Q_{s1}^2 r^2\right) \\
&\quad - \frac{1}{8} e^{\frac{r^2}{4}(Q_{s1}^2+Q_{s2}^2)} \left(Q_{s1}^4 Q_{s2}^4 r^8 + (4Q_{s1}^2 Q_{s2}^2 r^6 + 128 r^2)(Q_{s1}^2+Q_{s2}^2) + 16\,r^4 (Q_{s1}^2+Q_{s2}^2)^2 - 1024\right) \\
&\quad - 2\,e^{\frac{r^2}{2}(Q_{s1}^2+Q_{s2}^2)} \left(\bar{Q}_{s1}^4 r^4 (Q_{s2}^2 r^2 - 4)(4\pi\,\partial^2 L(0_\perp))^2 + 32 Q_{s1}^2 r^2 - 4\,Q_{s1}^2 Q_{s2}^2 r^4\right)\Bigg)\Bigg] + [1 \leftrightarrow 2].
\end{aligned} \tag{4.49}$$

These expressions give:

$$S(\mathbf{s}_1, \mathbf{s}_2) = \langle \rho(\mathbf{s}_1) \rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle$$

in collisions of large nuclei.

Let us be practical...what are the relevant features of the correlator?




$$\frac{1}{Q_s} \ll \frac{1}{m}$$

- It is very sharp compared to the system size.
  Short-range correlations: $S(\mathbf{s}_1, \mathbf{s}_2) \approx \xi(\mathbf{s})\delta(\mathbf{r})$ , $\mathbf{s} = \frac{\mathbf{s}_1 + \mathbf{s}_2}{2}$

- Its integral is divergent and dominated by r^-2 tail.
An infrared cutoff naturally emerges, around the size of the nucleon.
Scales are separated: we can isolate the leading contribution.

$$\xi(\mathbf{s}) \equiv \int_{\mathbf{r}} S\left(\mathbf{s} + \frac{\mathbf{r}}{2}, \mathbf{s} - \frac{\mathbf{r}}{2}\right) \propto Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s}) \left[Q_A^2(\mathbf{s}) \ln\left(1 + \frac{Q_B^2(\mathbf{s})}{m^2}\right) + Q_B^2(\mathbf{s}) \ln\left(1 + \frac{Q_A^2(\mathbf{s})}{m^2}\right)\right]$$

**How do we use it in practice ? We follow** [Blaizot, Broniowski, Ollitrault, 1405.3572].

$$\rho(\mathbf{s}) = \langle\rho(\mathbf{s})\rangle + \delta\rho(\mathbf{s}), \quad \langle\rho(\mathbf{s})\rangle \gg \delta\rho(\mathbf{s})$$

(on long wavelengths)

**Perturbative expansion of the anisotropy:** $\varepsilon_n \equiv \frac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})}$

**Mean-squared anisotropies to first nontrivial order:**

$$\langle\varepsilon_2\varepsilon_2^*\rangle = \varepsilon_2\{2\}^2 = \sigma^2 + \bar{\varepsilon}_2^2$$

**and**

$$\varepsilon_3\{2\}^2 = \frac{\int_{\mathbf{s}} |\mathbf{s}|^6\, \xi(\mathbf{s})}{\left(\int_{\mathbf{s}} |\mathbf{s}|^3 \langle\rho(\mathbf{s})\rangle\right)^2}$$

$$\sigma^2 = \frac{\int_{\mathbf{s}} |\mathbf{s}|^4\, \xi(\mathbf{s})}{\left(\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\rho(\mathbf{s})\rangle\right)^2} \quad , \quad \bar{\varepsilon}_2 = \frac{\int_{\mathbf{s}} \mathbf{s}^2 \langle\rho(\mathbf{s})\rangle}{\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\rho(\mathbf{s})\rangle}$$

**From the CGC we have:**

$$\langle\rho(\mathbf{s})\rangle = \frac{4}{3g^2} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s})$$

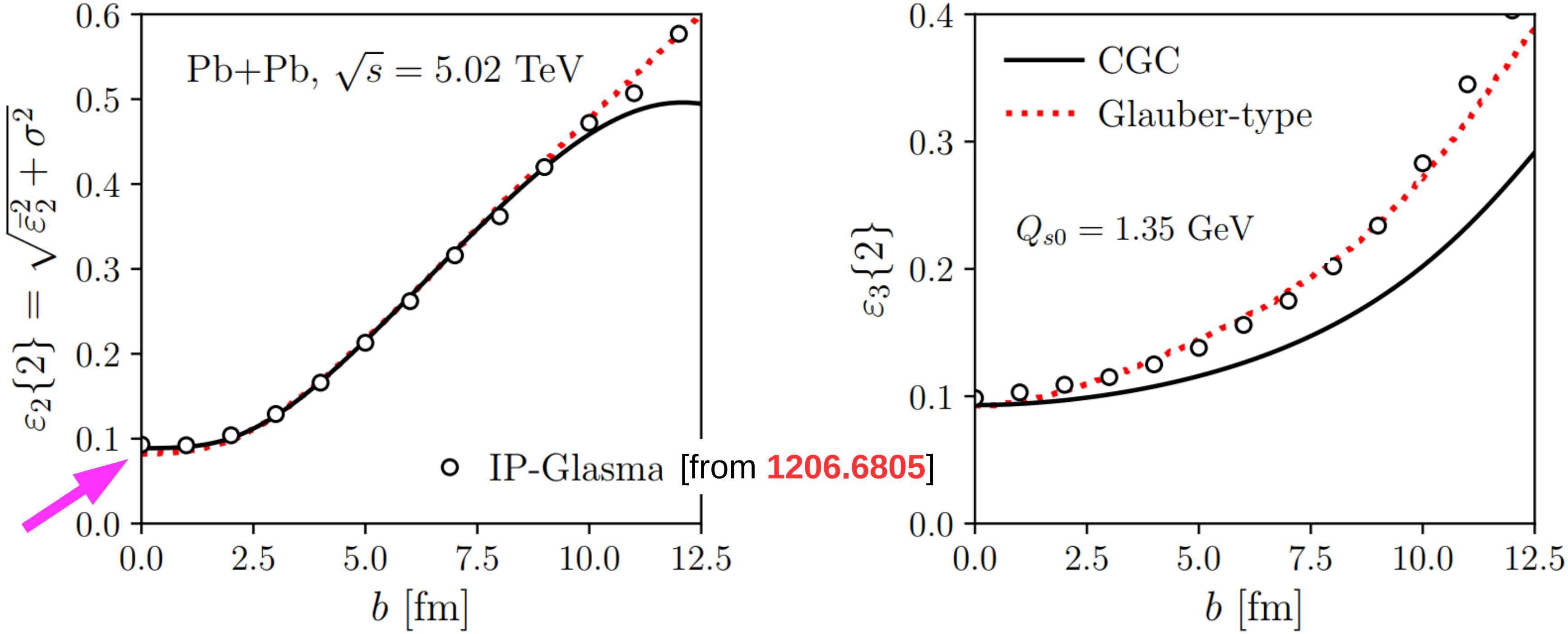
Prefactors from:
[Albacete, Guerrero-Rodriguez, Marquet 1808.00795]

$$\xi(\mathbf{s}) = \frac{16\pi}{9g^4} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s}) \left( Q_A^2(\mathbf{s}) \ln\left(1 + \frac{Q_B^2(\mathbf{s})}{m^2}\right) + Q_B^2(\mathbf{s}) \ln\left(1 + \frac{Q_A^2(\mathbf{s})}{m^2}\right)\right)$$

**Saturation scale proportional to the integrated nuclear density:** $Q_s^2(\mathbf{s}) = Q_{s0}^2 T(\mathbf{s})/T(\mathbf{0})$

# RESULTS

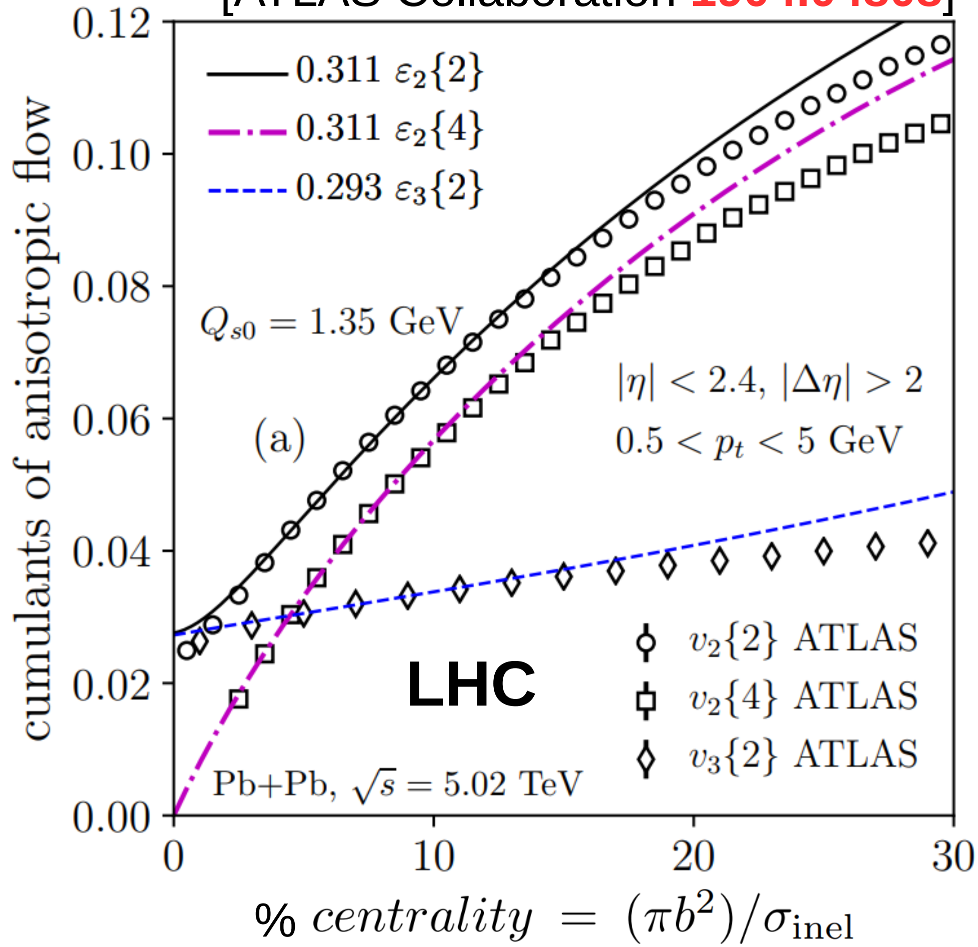
Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault
**[1902:07168]**



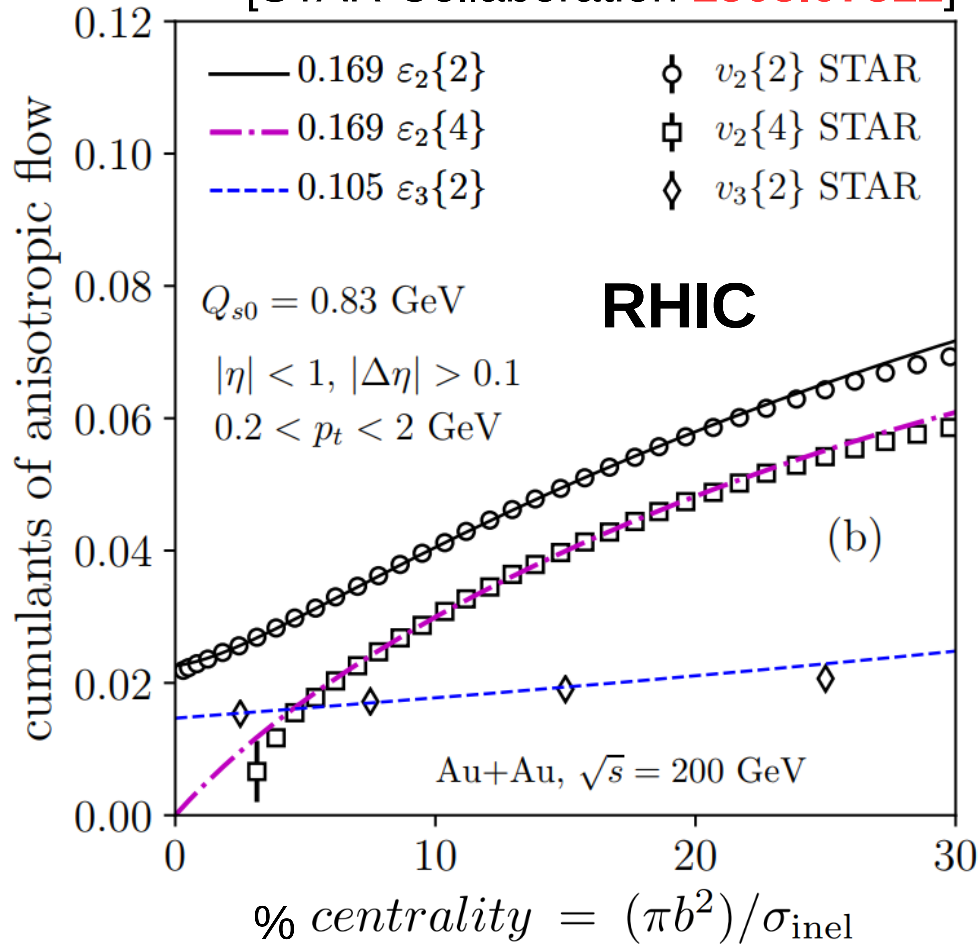

**We reproduce the Glauber results. Not a lucky coincidence!**

$$\varepsilon_2\{2\}^2 \approx \varepsilon_3\{2\}^2 \approx \frac{\log(Q^2/m^2)}{R^2Q^2} \longrightarrow \varepsilon_2\{2\} \approx 0.1 \text{ for } \begin{array}{l} m = 0.1 \text{ GeV} \\ Q = 1 \text{ GeV} \\ R = 5 \text{ fm} \end{array}$$

[ATLAS Collaboration 1904.04808]




[STAR Collaboration 1505.07812]



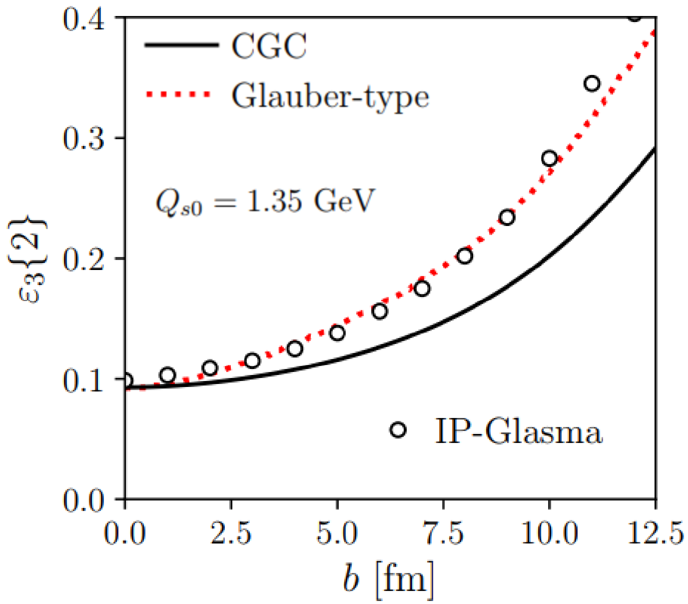

- We use $v_2\{4\} = \kappa_2 \varepsilon_2\{4\} \approx \kappa_2 \bar{\varepsilon}_2$ [Voloshin, Poskanzer, Tang, Wang, 0708.0800]
  **$v_2$\{4\} fixes the response coefficient $k_2$.**

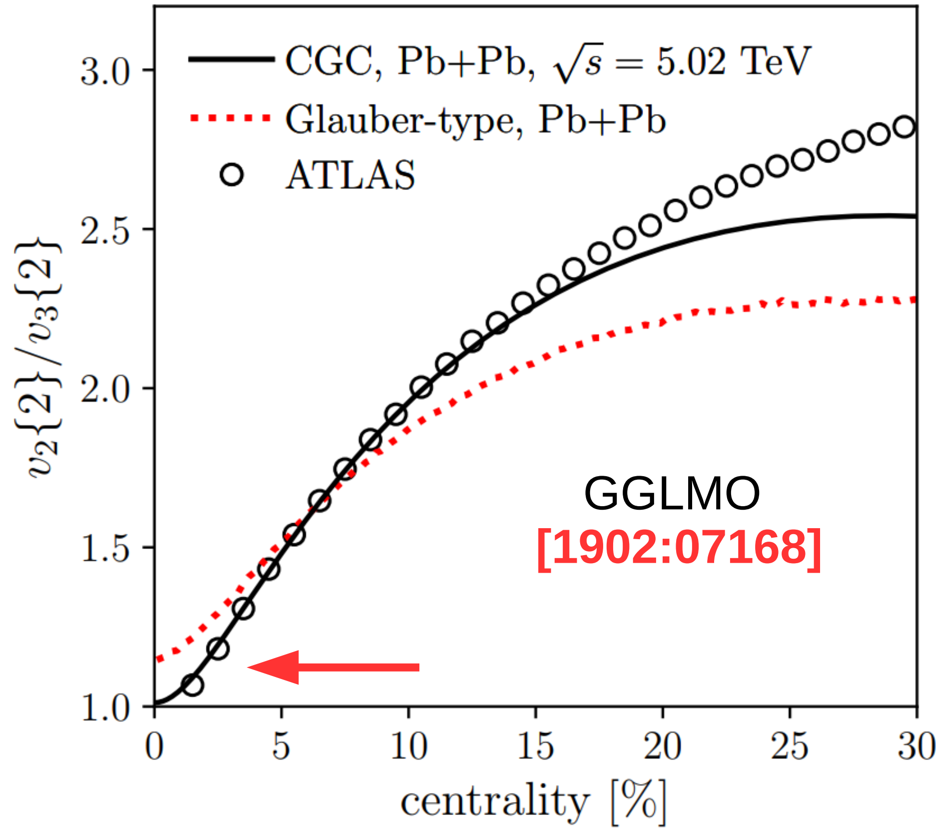
- The splitting between v2{2} and v2{4} is due to fluctuations:

$$Q_s[\text{LHC}] > Q_s[\text{RHIC}] \implies \text{ smaller splitting at LHC.}$$

# TRIANGULAR FLOW

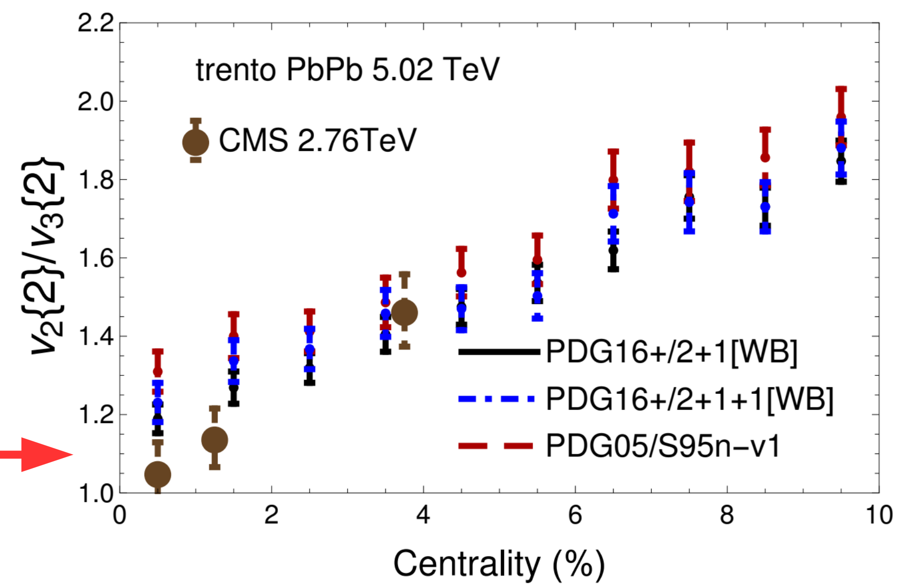
In the CGC, triangular flow grows more mildly than in a Glauber calculation.



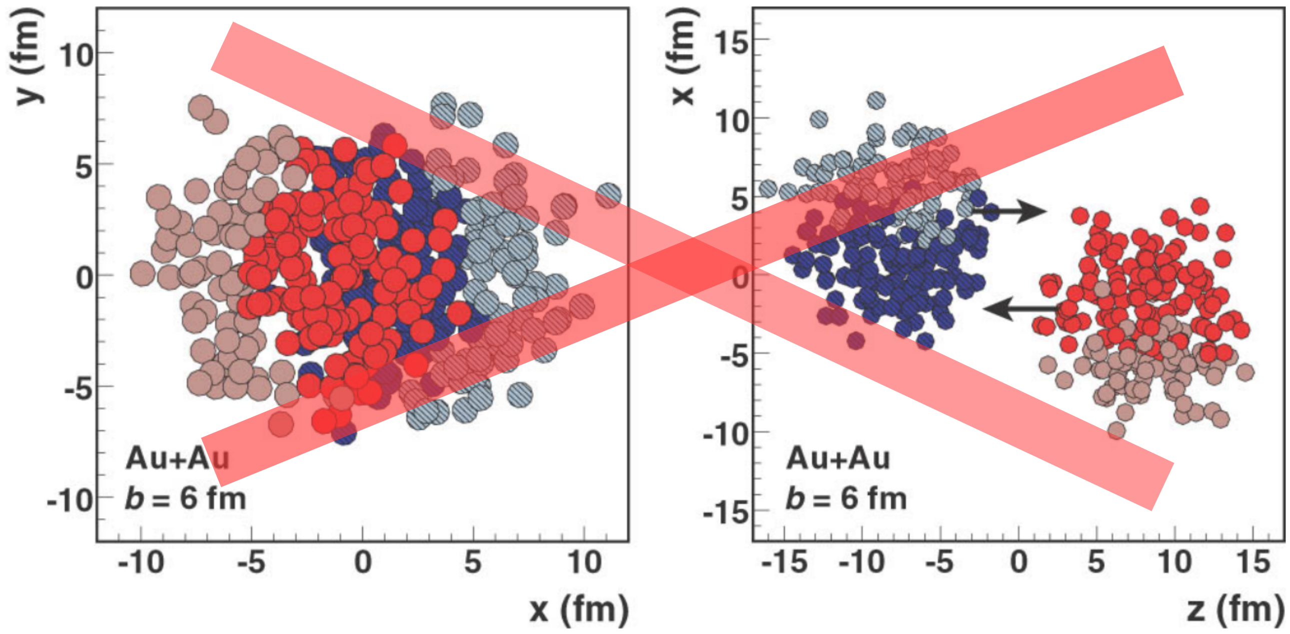

**This fixes a longstanding problem of hydro-to-data comparisons:**
**The ratio $v_2\{2\}/v_3\{2\}$ grows quickly with centrality.**

e.g. [Shen, Qiu, Heinz, 1502.04636]




[Alba, Mantovani Sarti, Noronha, Noronha-Hostler, Parotto, Portillo Vasquez, Ratti 1711.05207]

# A NEW PARADIGM FOR FLUCTUATIONS.




[Miller, Reygers, Sanders, Steinberg **nucl-ex/0701025**]

We free the description from the Glauber Monte Carlo Ansatz:

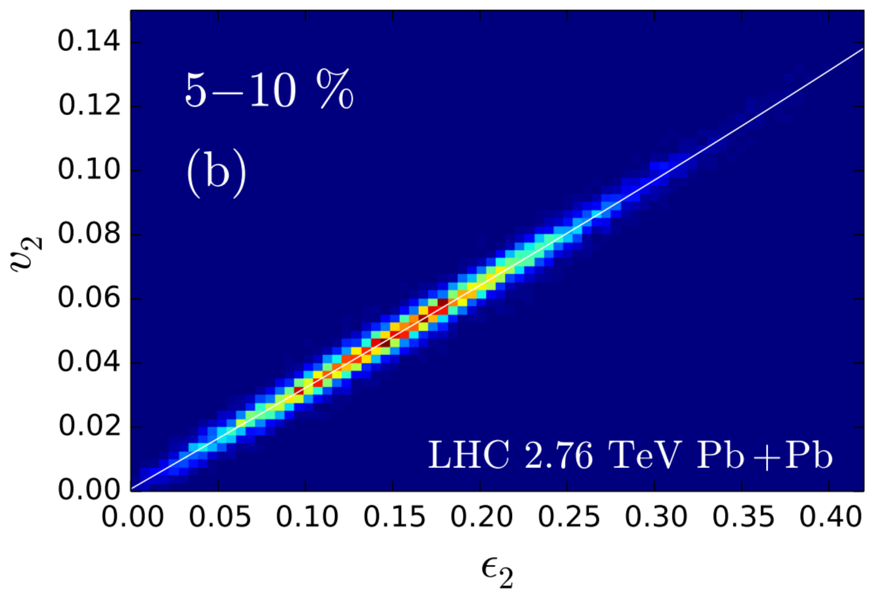
- No random sampling of nucleons.

- No ad hoc prescriptions about the deposition of energy.

- Nonperturbative physics only through the mass parameter.

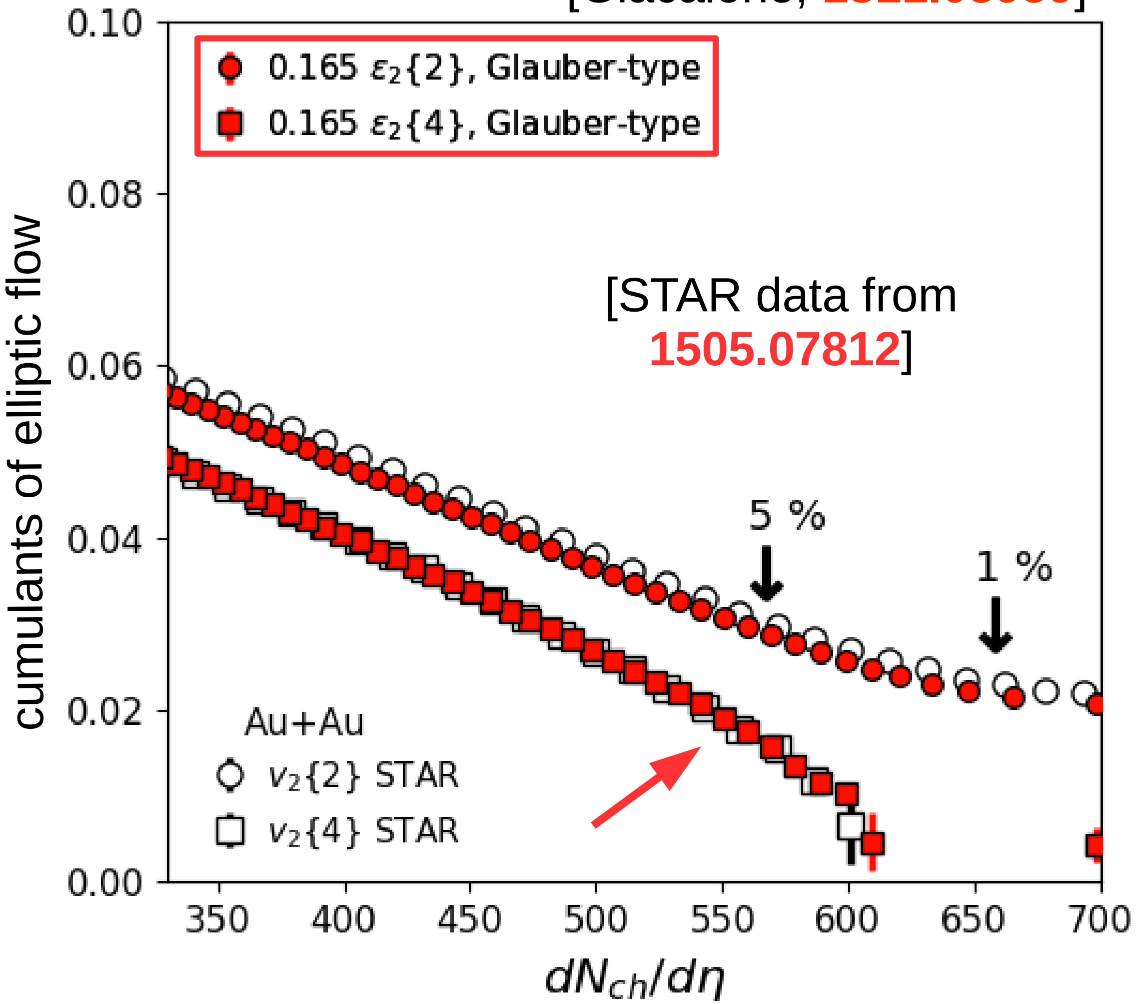
# BACKUP

[Niemi, Eskola, Patelaainen, arXiv 1505:02677]

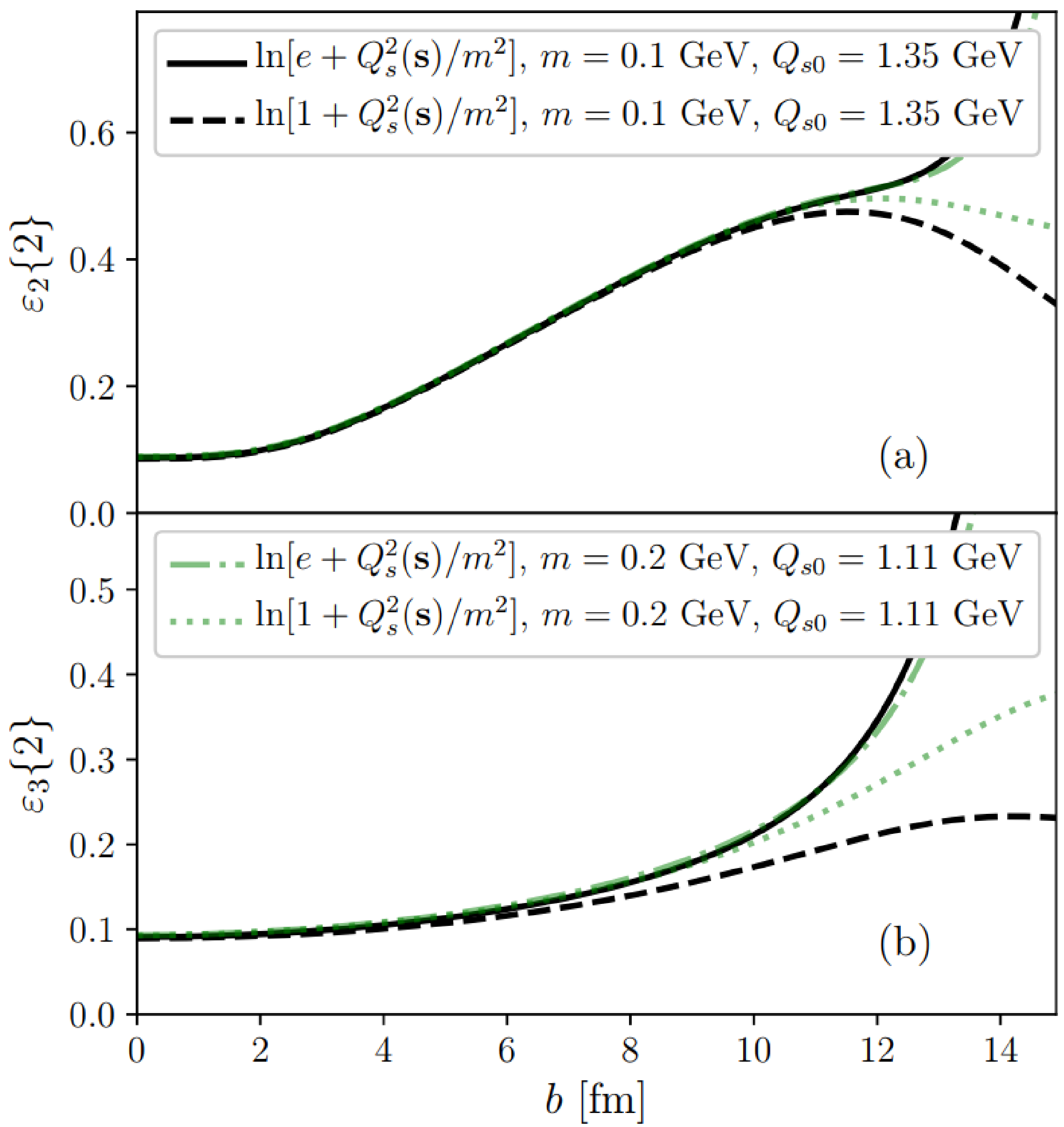



# The relation is linear.

[Giacalone, 1811.03959]

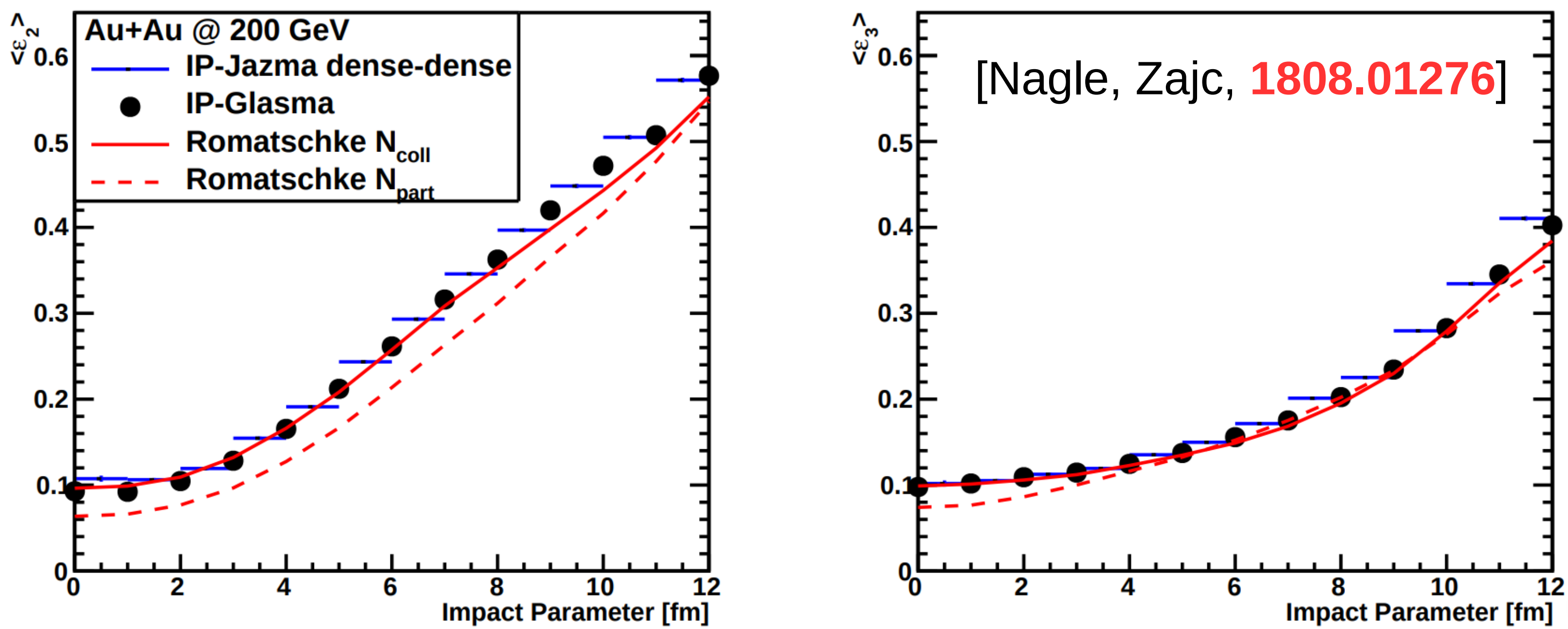


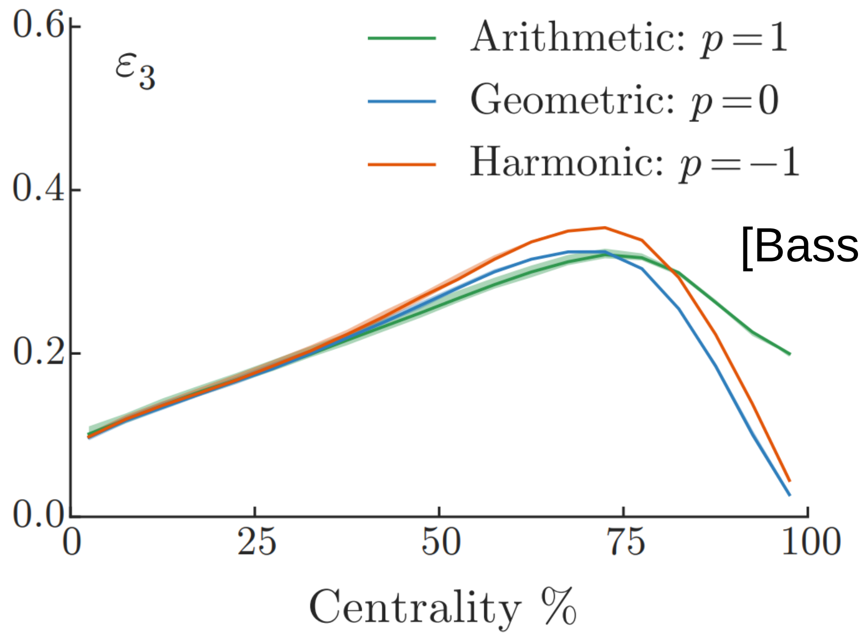
# How robust is the formalism ?




Breaks down at about b=12 fm.

All MC-Glauber-based models have the same eccentricities.




[Bass, Bernhard, Moreland
1412.4708]

# ELLIPTIC FLOW FLUCTUATIONS

Fluctuations of elliptic flow produce the **splitting between $v_2\{2\}$ and $v_2\{4\}$.** Experimental data indicate that fluctuations are larger at RHIC energy.

Energy dependence of the saturation scale from fits of DIS data:

$$\frac{Q_s^2(x_1)}{Q_s^2(x_2)} = \left(\frac{\sqrt{s_1}}{\sqrt{s_2}}\right)^{0.28}$$

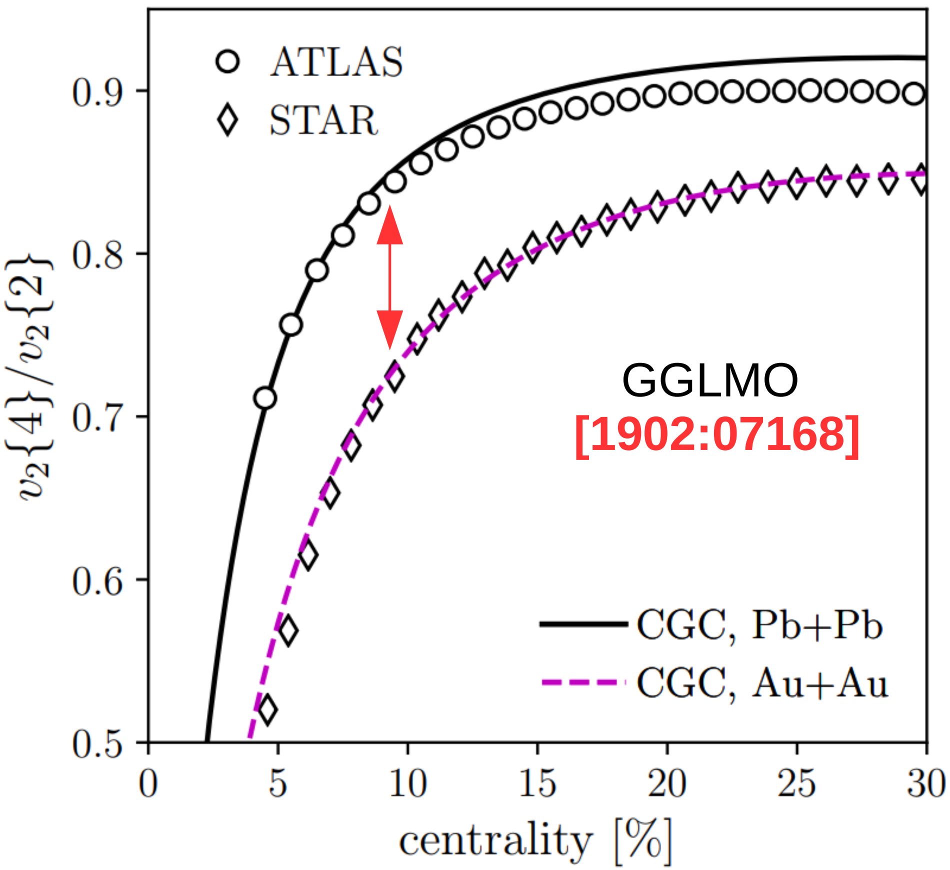
See e.g. [Albacete, Marquet, **1401.4866**]

Increase of **~1.6** from RHIC to LHC energy.

**Compatible with the evolution of $Q_{s0}$ found in our fit of anisotropic flow data:**

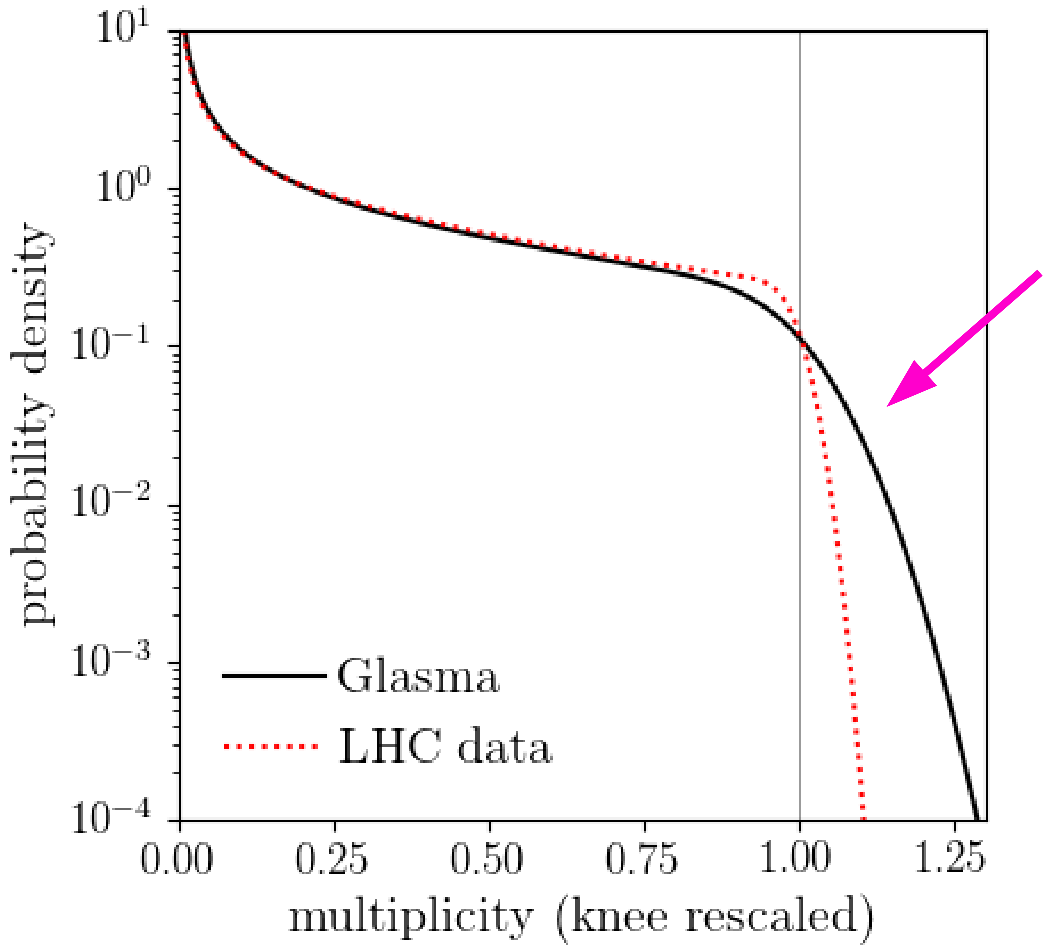
**$Q_{s0}$ (LHC) ~ 1.3 GeV**
**$Q_{s0}$ (RHIC) ~ 0.8 GeV**

**Very transparent physical explanation!**




**NB:** the Glauber-type calculation does not make any specific predictions for this ratio.

The fluctuations of the primordial energy density are too large compared to the fluctuations of the final-state multiplicity observed at LHC.




Need full pre-equilibrium dynamics. Nontrivial task.