

# Parton-Hadron-Quantum-Molecular Dynamics (PHQMD) - a Novel Microscopic N-Body Transport Approach for Heavy-Ion Dynamics and Hypernuclei Production

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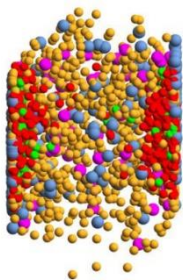
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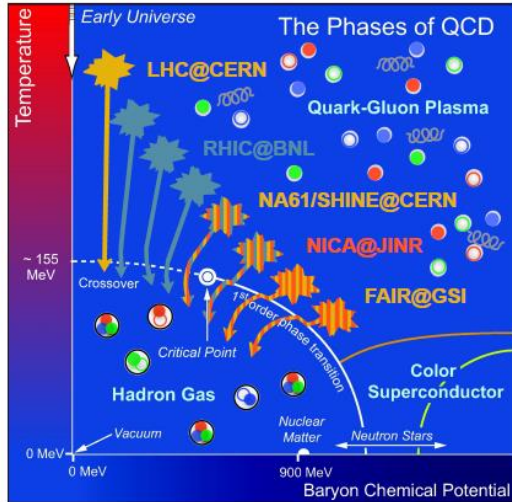


*The 18th International Conference  
on Strangeness in Quark Matter (SQM 2019)  
Bari, Italy, 10-15 June, 2019*



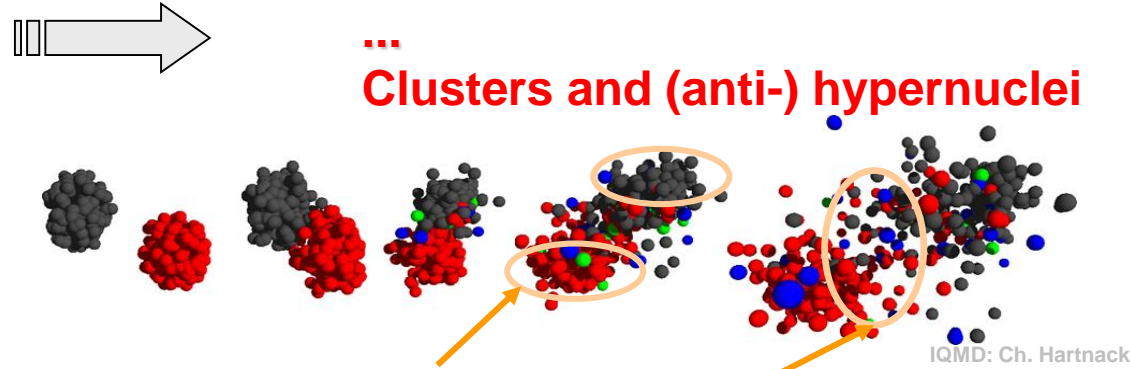
# The ,holy grail' of heavy-ion physics:

## The phase diagram of QCD



## Experimental observables:

...  
**Clusters and (anti-) hypernuclei**



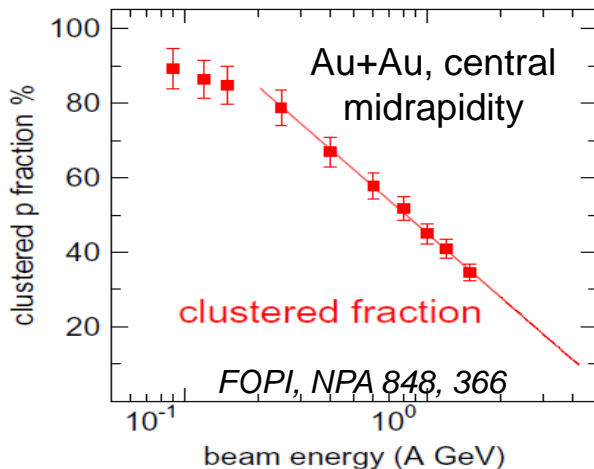
- projectile/target spectators → **heavy cluster formation**
- midrapidity → **light clusters**

! Hyperons are created in participant zone

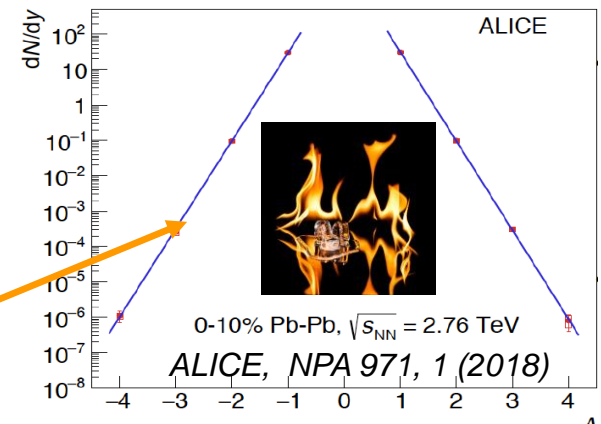
**(Anti-) hypernuclei production:**

- at mid-rapidity by  $\Lambda$  coalescence during expansion
- at projectile/target rapidity by rescattering/absorption of  $\Lambda$  by spectators

□ Clusters are very abundant at low energy



**High energy HIC:**  
**,Ice in a fire' puzzle:**  
how the weakly bound objects can be formed in a hot environment ?!



# Modeling of cluster and hypernuclei formation

## Existing models for clusters formation:

### □ statistical model:

- assumption of thermal equilibrium (difficult to justify at target and projectile rapidity)
- strong sensitivity of nuclei yields to choice of  $T_{ch}$
- binding energies are small compared to  $T_{ch}$

### □ coalescence model:

- determination of clusters at a given point in time by coalescence radii in coordinate and momentum spaces

→ don't provide information on the dynamics of clusters formation

In order to understand the **microscopic origin** of clusters formation one needs:

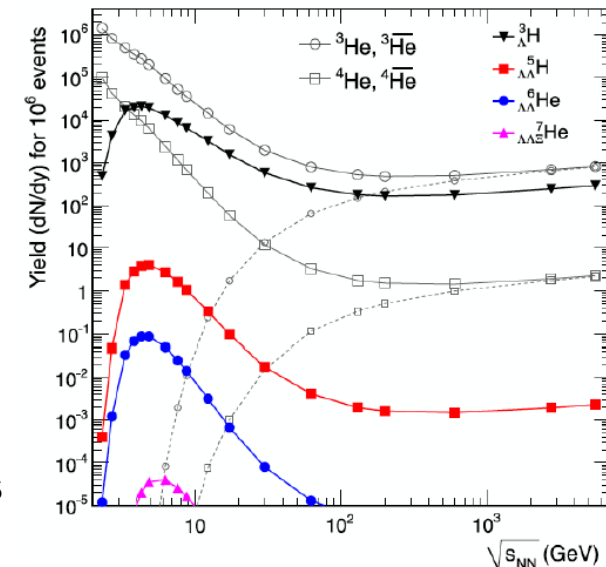
- a realistic model for the **dynamical time evolution** of the HIC → transport models
- **dynamical modeling of cluster formation** based on interactions

□ Cluster formation is sensitive to **nucleon dynamics**

→ One needs to **keep the nucleon correlations (initial and final)** by realistic **nucleon-nucleon interactions** in transport models:

- **QMD** (quantum-molecular dynamics) – allows to keep correlations
- **MF** (mean-field based models) – correlations are smeared out

A. Andronic et al., PLB 697, 203 (2011)



# PHQMD

**The goal:** to develop a **unified n-body microscopic transport approach** for the description of heavy-ion dynamics and dynamical cluster formation from low to ultra-relativistic energies

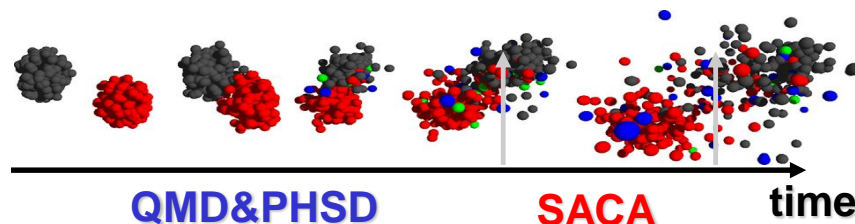
**Realization:** combined model **PHQMD = (PHSD & QMD) & SACA**

## Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:  
**QMD** (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons  
+ **collision integral** = interactions of hadrons and partons (QGP)  
from **PHSD** (Parton-Hadron-String Dynamics)

Clusters recognition:  
**SACA** (Simulated Annealing Clusterization Algorithm)  
vs. **MST** (Minimum Spanning Tree)





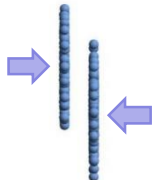
# Parton-Hadron-String-Dynamics (PHSD)

collision integral  
→ PHQMD

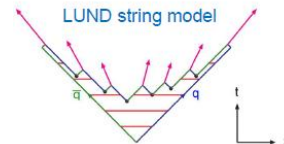
**PHSD** is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

**Dynamics:** based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A  
collision

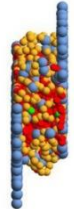


□ **Initial A+A collisions** :  
N+N → **string formation** → decay to pre-hadrons + leading hadrons

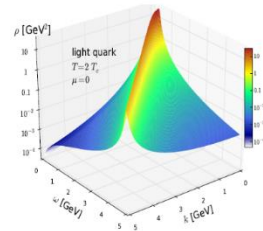


□ **Formation of QGP stage** if local  $\varepsilon > \varepsilon_{\text{critical}}$  :  
dissolution of **pre-hadrons** → partons

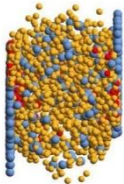
Partonic phase



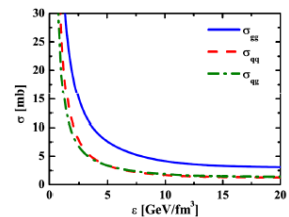
□ **Partonic phase - QGP:**  
QGP is described by the **Dynamical QuasiParticle Model (DQPM)**  
matched to reproduce **lattice QCD EoS** for finite T and  $\mu_B$  (crossover)



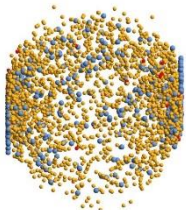
Hadronization



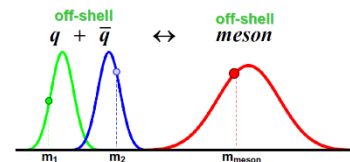
- **Degrees-of-freedom:** strongly interacting quasiparticles: **massive quarks and gluons ( $g, q, q_{\text{bar}}$ )** with sizeable collisional widths in a self-generated mean-field potential
- **Interactions:** (quasi-)elastic and inelastic collisions of partons



Hadronic phase



□ **Hadronization** to colorless **off-shell mesons and baryons:**  
Strict 4-momentum and quantum number conservation



□ **Hadronic phase:** hadron-hadron interactions – **off-shell HSD**

# QMD propagation

□ **Generalized Ritz variational principle:**  $\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0.$

Assume that  $\psi_N = \prod_{i=1}^N f(r_i, p_i, r_{i0}, p_{i0}, t)$  for N particles (neglecting antisymmetrization !)

↙  
single-particle Wigner density of the nucleon "i"

Trial wave function for one particle "i": **Gaussian** with width **L** centered at  $r_{i0}, p_{i0}$

$$L = 4.33 \text{ fm}^2$$

$$f(\mathbf{r}_i, \mathbf{p}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L} (\mathbf{r}_i - \mathbf{r}_{i0}(t))^2} e^{-\frac{L}{2\hbar^2} (\mathbf{p}_i - \mathbf{p}_{i0}(t))^2}$$

□ **Equations-of-motion (EoM)** for **Gaussian centers** in coordinate and momentum space:

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \quad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

**Hamiltonian:**  $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$

$$V_{i,j} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) = V_{\text{Skyrme}} + V_{\text{Coul}}$$

# QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H \rangle = \langle T \rangle + \langle V \rangle = \sum_i (\sqrt{p_{i0}^2 + m^2} - m) + \sum_i \langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle$$

□ **Skyrme potential (scalar)\* :**

$$\langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle = \alpha \left( \frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right) + \beta \left( \frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right)^\gamma$$

	$\alpha$ (MeV)	$\beta$ (MeV)	$\gamma$	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

□ **modified interaction density:**

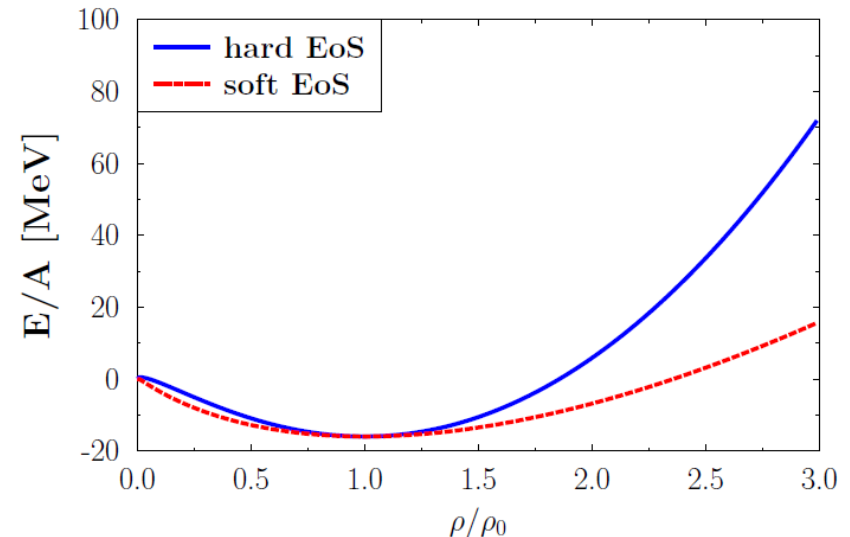
$$\rho_{int}(\mathbf{r}_{i0}, t) \rightarrow C \sum_j \left( \frac{4}{\pi L} \right)^{3/2} e^{-\frac{4}{L}(\mathbf{r}_{i0}^T(t) - \mathbf{r}_{j0}^T(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^L(t) - \mathbf{r}_{j0}^L(t))^2},$$

❖ **HIC ↔ EoS for infinite matter at rest**

○ **compression modulus K of nuclear matter:**

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2(E/A(\rho))}{(\partial\rho)^2} \Big|_{\rho=\rho_0}$$

**EoS for infinite matter at rest**



\* Work in progress: implementation of momentum dependent potentials

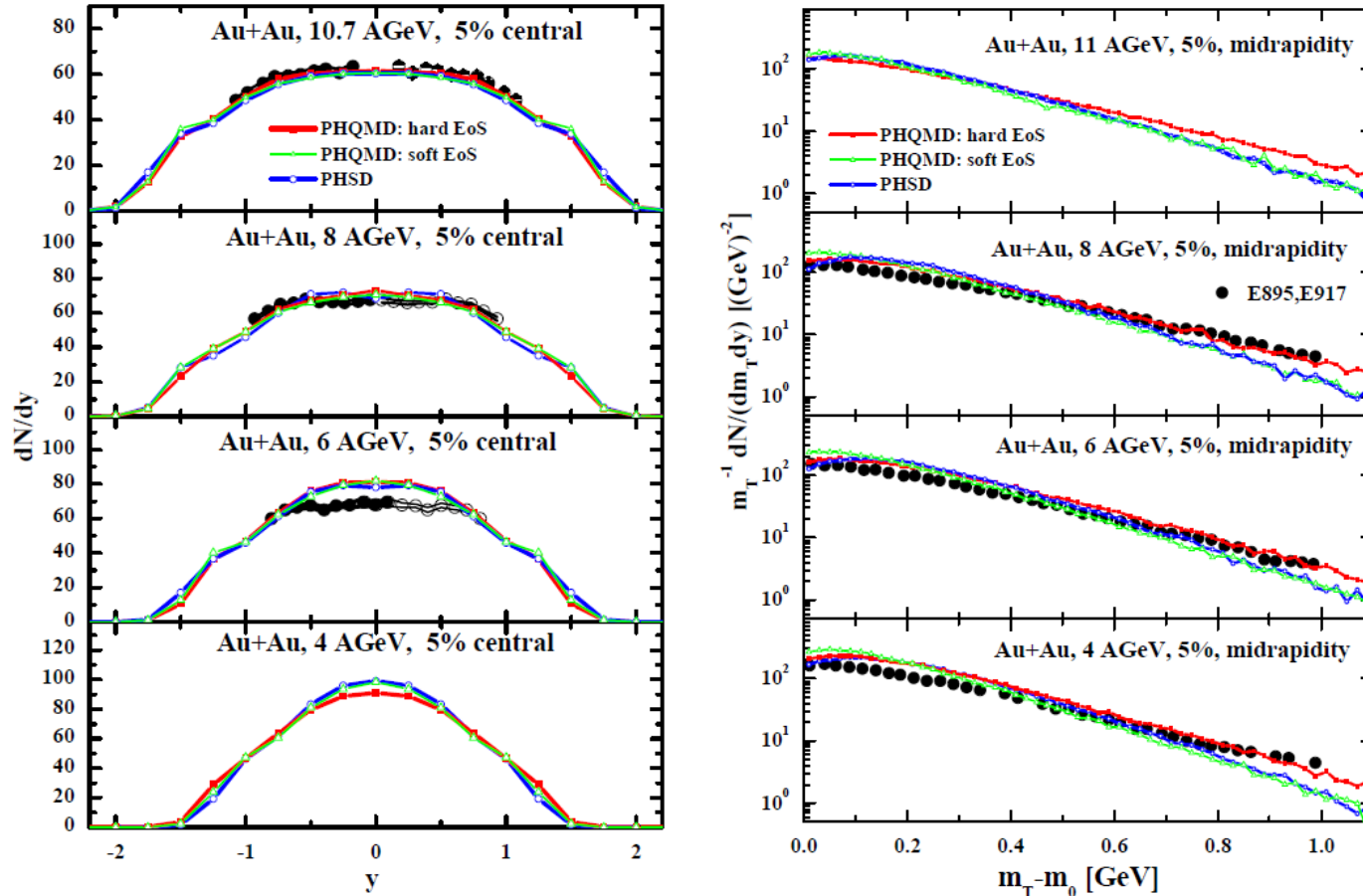
# **First PHQMD results on ‚bulk‘ observables**

**Poster 145 by V. Kireyev**



# PHQMD: ,bulk' dynamics at AGS/FAIR/NICA energies

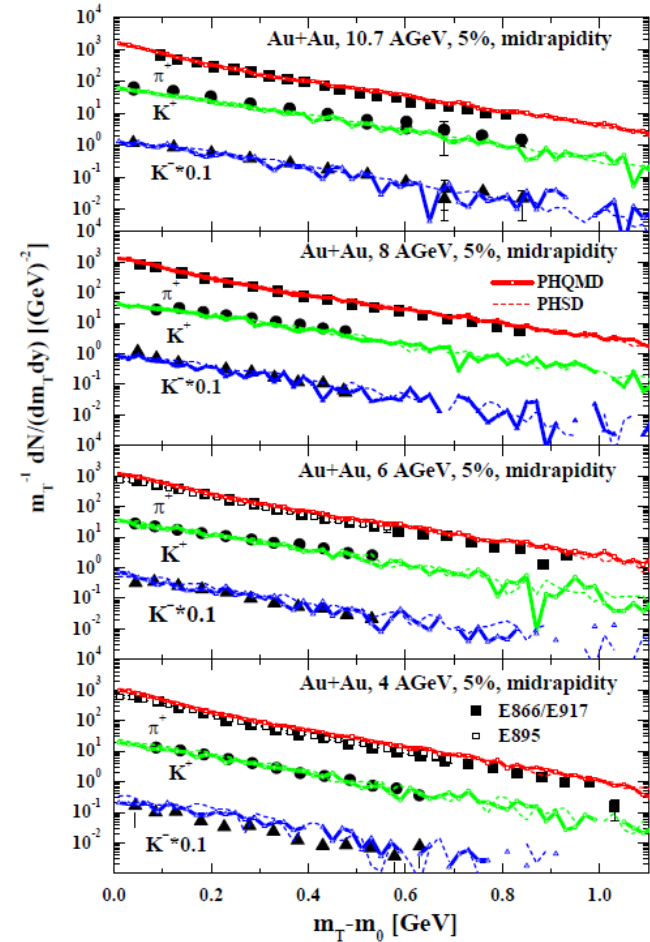
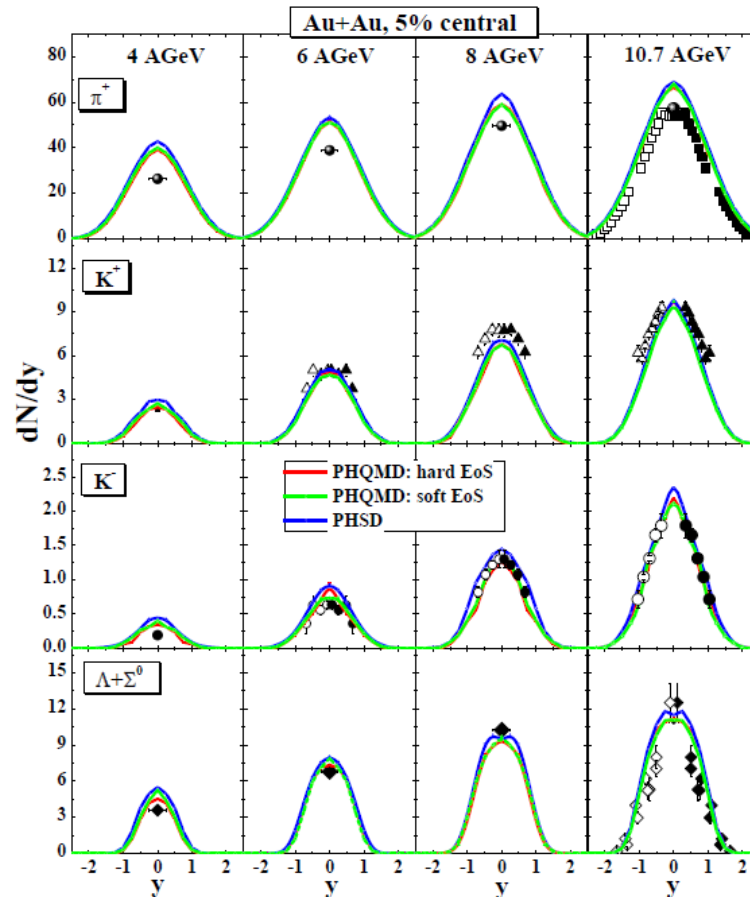
The rapidity and  $m_T$  distributions for protons from 5% central Au+Au collisions at 4, 6, 8, 10.7 A GeV



- the **influence of EoS** is slightly visible in rapidity spectra of protons
  - $m_T$  spectra of protons from PHQMD with a 'hard' EoS are harder than with 'soft' EoS
  - PHQMD results for the  $m_T$  spectra with 'soft' EoS are in a good agreement with the PHSD spectra (using 'soft' EoS in default PHSD4.0 version)
- QMD and MF dynamics gives similar results with similar EoS

# PHQMD: ,bulk' dynamics at AGS/FAIR/NICA energies

The rapidity and  $m_T$  distributions for  $\pi^+$ ,  $K^+$ ,  $K^-$ ,  $\Lambda+\Sigma^0$  from 5% central Au+Au collisions at 4, 6, 8, 10.7 A GeV



□ the influence of EoS is slightly visible in rapidity and  $m_T$  spectra of newly produced hadrons

# **Clusters in PHQMD: MST & SACA**

# Cluster recognition: Minimum Spanning Tree (MST)

The **Minimum Spanning Tree (MST)** is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

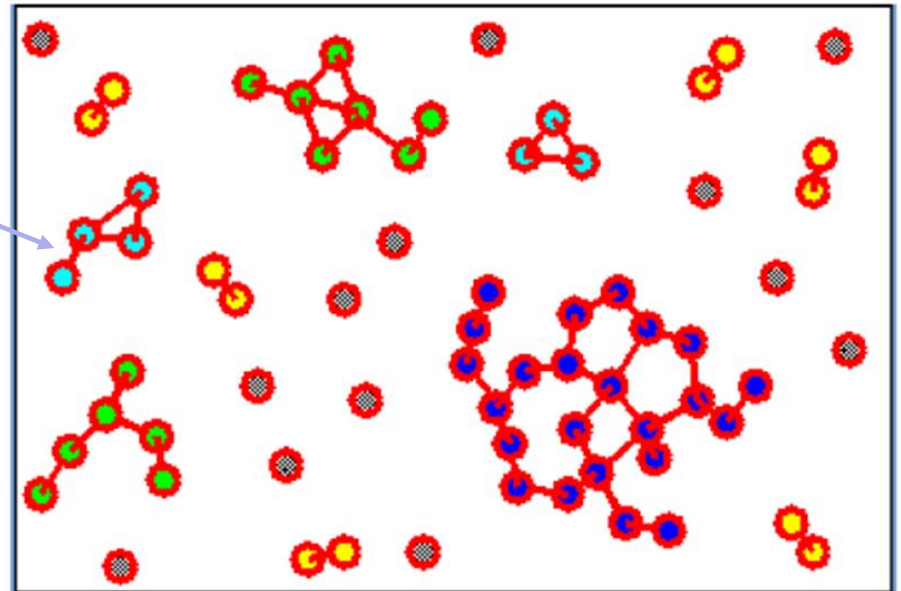
The MST algorithm searches for accumulations of particles in **coordinate space**:

1. Two particles are 'bound' if their **distance in coordinate space** fulfills

$$|\bar{r}_i - \bar{r}_j| \leq 2.5 \text{ fm}$$

2. Particle is **bound to a cluster** if it **bounds with at least one particle** of the cluster.

\* Remark:  
inclusion of an additional momentum cuts (coalescence) lead to a small changes: particles with large relative momentum are mostly not at the same position



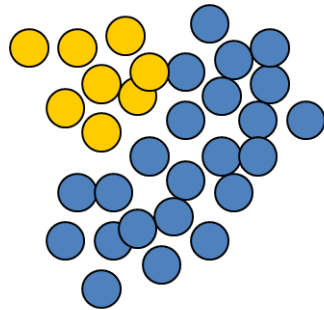
# Simulated Annealing Clusterization Algorithm (SACA)

## Basic ideas of clusters recognition by SACA:

Based on idea by Dorso and Randrup  
(Phys.Lett. B301 (1993) 328)

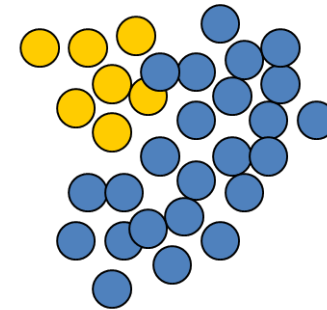
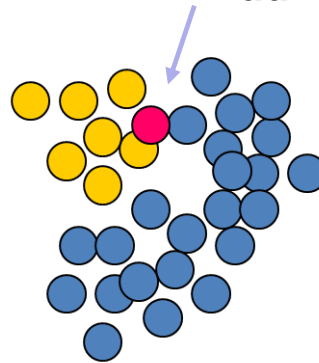
- Take the positions and momenta of all nucleons at time  $t$
- Combine them in all possible ways into all kinds of clusters or leave them as single nucleons
- Neglect the interaction among clusters
- Choose that configuration which has the **highest binding energy**:

Take **randomly 1 nucleon**  
out of a cluster



$$E = E_{kin}^1 + E_{kin}^2 + V^1 + V^2$$

Add it randomly to another cluster



$$E' = E'_{kin} + E'_{kin} + V^1 + V^2$$

If  $E' < E$  take a new configuration

If  $E' > E$  take the old configuration with a probability depending on  $E' - E$

Repeat this procedure many times

→ **Leads automatically to finding of the most bound configurations**

# Cluster recognition by SACA

- ❑ **SACA** searches for the most bound configurations → clusters  
Clusters are bound by **potential interactions between nucleons**  $V_i$   
 $V_i$  – Skyrme potential (as in PHQMD!)
- ❑ **Binding energy (SACA) vs. Weizsaecker formula**



There are two kinds of clusters:

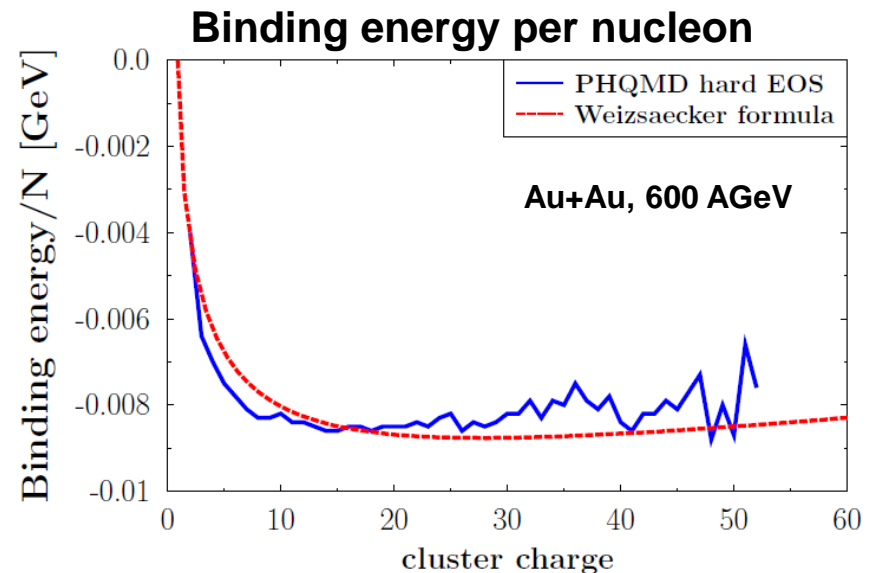
I) **Heavy clusters** formed from **spectator matter** close to beam and target rapidity:

- initial-final state correlations
- HIC makes spectator matter unstable

II) **Light clusters** formed from **participant matter** created during the expansion of the fireball →

“ice” ( $E_{\text{bind}} \approx -8$  MeV/N) in “fire” ( $T \geq 100$  MeV)

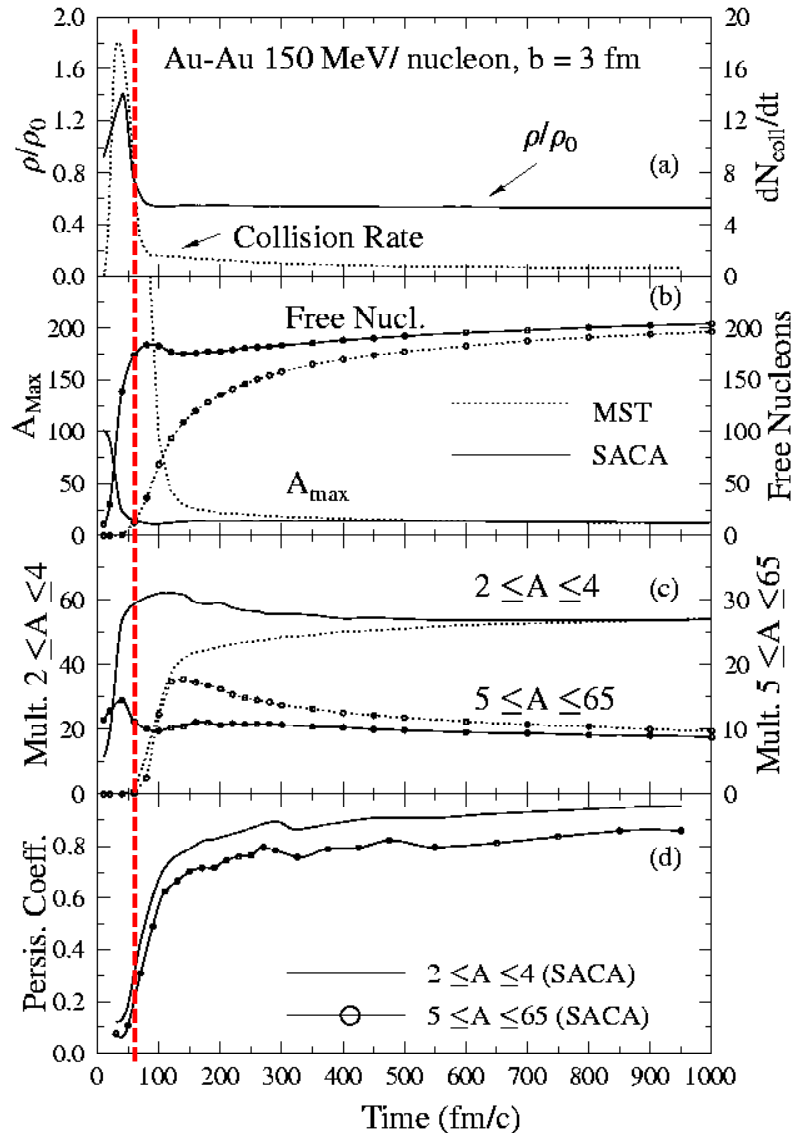
- origin is not well understood
- seen from SIS to RHIC
- quantum effects may be important



→ average binding energy of the clusters identified by SACA at late times (150 fm/c) is **in agreement with Weizsaecker formula**

# SACA vs MST

## IQMD with SACA and MST



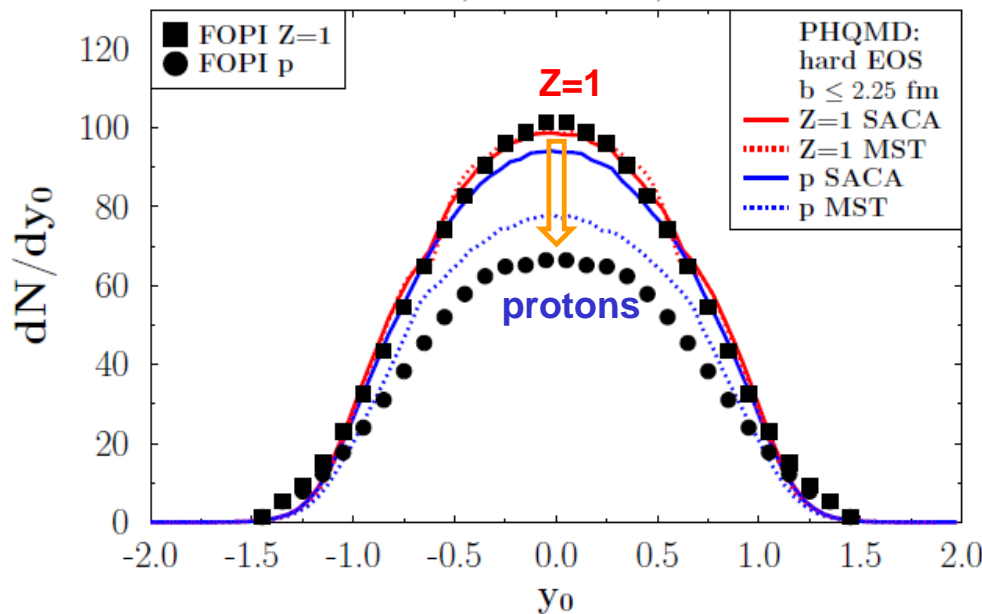
- ❑ **SACA** can identify the fragment pattern very **early** in time as compared to the **Minimum Spanning Tree (MST)** which requires a maximal distance in coordinate space between two nucleons to form a cluster
- ❑ → MST applicable at the latest stage of reactions
- ❑ **Advantage of SACA** : the origin of fragment formation can be studied (role of Coulomb energies, density, flow details, strangeness...)

P.B. Gossiaux, R. Puri, Ch. Hartnack,  
J. Aichelin, Nuclear Physics A 619 (1997)

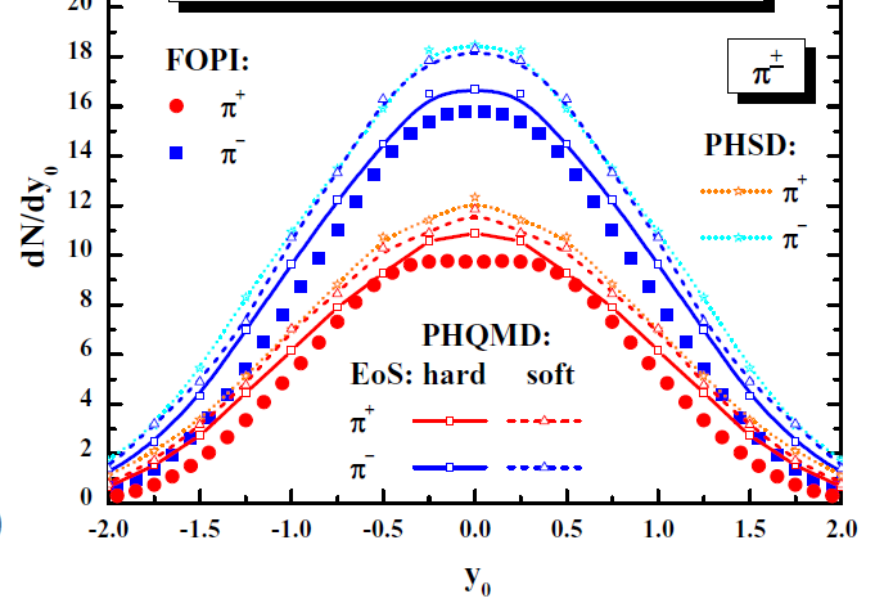
# PHQMD: light clusters and 'bulk' dynamics at SIS

Scaled rapidity distribution  $y_0 = y/y_{proj}$  in central Au+Au reactions at 1.5 AGeV

Au + Au, 1.5 AGeV, central



Au+Au, 1.5 A GeV,  $b < 2.25$  fm



- **30% of protons are bound in clusters at 1.5 A GeV**
- Presently MST is better identifying light clusters than SACA
  - ➔ To improve in SACA: more realistic potentials for small clusters, quantum effects
- Pion spectra are sensitive to EoS: better reproduced by PHQMD with a 'hard' EoS
- PHQMD with soft EoS is consistent with PHSD (default – soft EoS)
- \* To improve in PHQMD: momentum dependent potentials

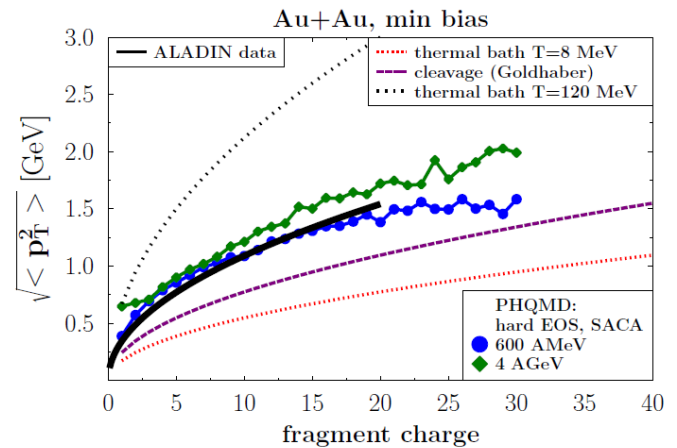
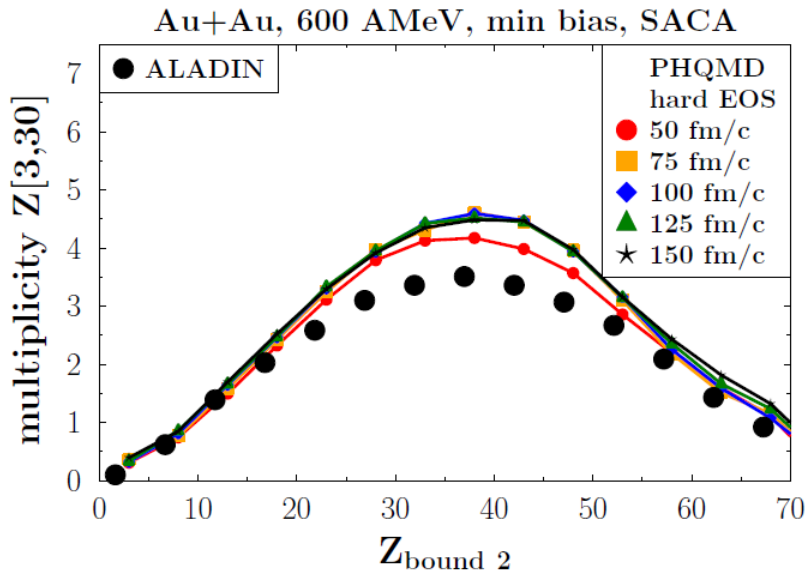
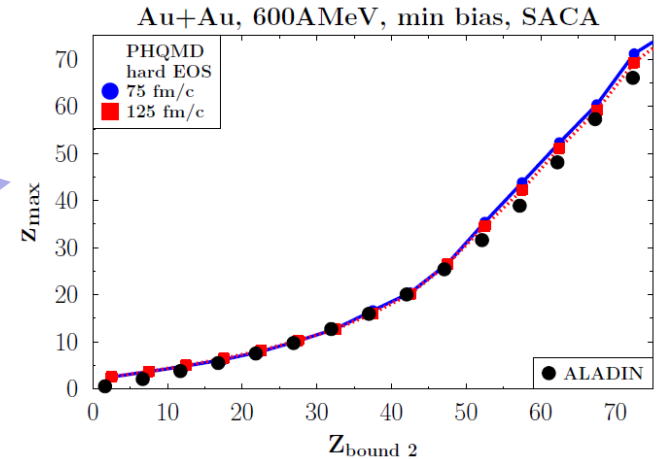


# PHQMD: heavy clusters

**Heavy clusters (spectator fragments):**  
 experim. measured up to  $E_{\text{beam}} = 1$  AGeV (ALADIN Collab.)

**PHQMD with SACA** shows an agreement with ALADIN data for very complex cluster observables as

- ❑ Largest fragments ( $Z_{\text{bound}}$ )
- ❑ Energy independent 'rise and fall'
- ❑ Rms  $p_T^2$

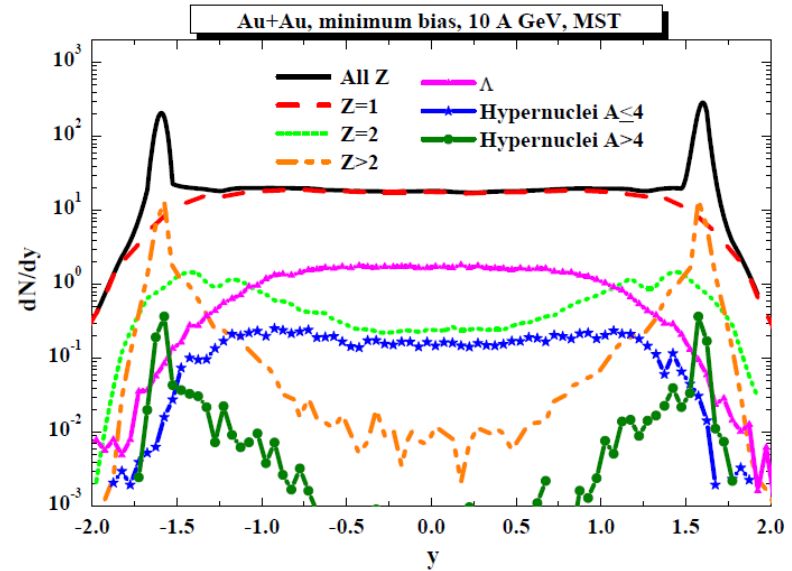
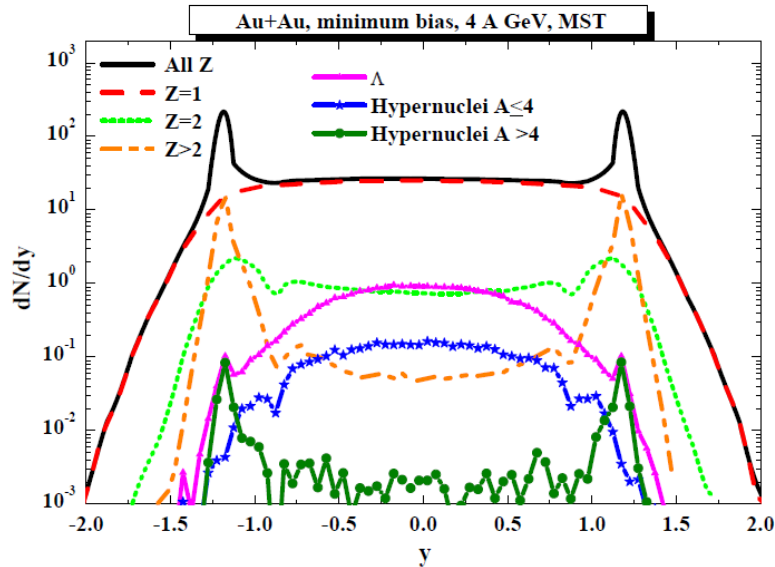


$$Z_{\text{bound } 2} = \sum_i Z_i \Theta(Z_i - (1 + \epsilon)) \quad (\epsilon < 1)$$

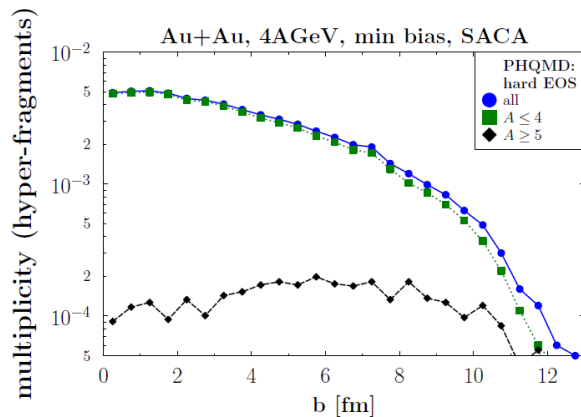
**PHQMD shows  $\sqrt{\langle p_T^2 \rangle(Z)} \propto \sqrt{Z}$ . dependence as exp. data**

# PHQMD: hypernuclei

PHQMD results (with a **hard EoS** and **MST algorithm**) for the rapidity distributions of all charges,  $Z = 1$  particles,  $Z=2$ ,  $Z>2$ , as well as  $\Lambda$ 's, hypernuclei  $A \leq 4$  and  $A > 4$  for Au+Au at 4 and 10 AGeV



The multiplicity of light hypercluster vs. impact parameter  $b$  for Au+Au, 4 AGeV

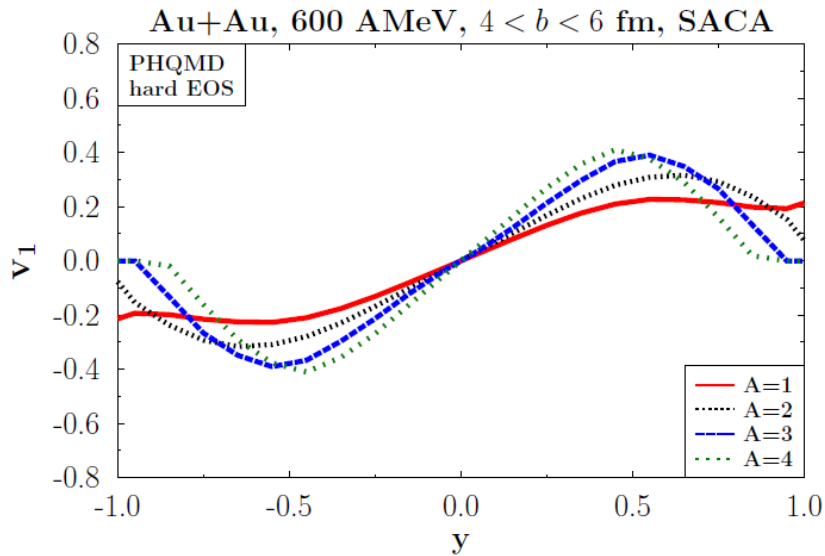


- Central collisions → light hypernuclei
- Peripheral collisions → heavy hypernuclei

Penetration of  $\Lambda$ 's, produced at midrapidity, to target/projectile region due to rescattering

➔ Possibility to study  $\Lambda N$  interaction

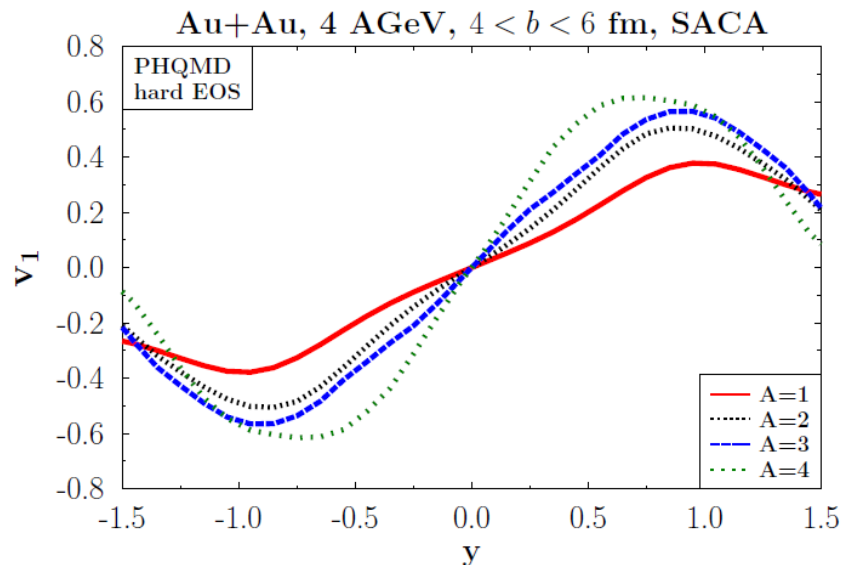
# PHQMD: collectivity of clusters



PHQMD with hard EoS, with SACA:  
 $v_1$  of light clusters ( $A=1,2,3,4$ ) vs rapidity  
for mid-central Au+Au at 600 AMeV, 4 AGeV



- $v_1$  : quite different for nucleons and fragments (as seen in experiments)
- Nucleons come from participant regions ( $\rightarrow$  small density gradient) while fragments from interface spectator-participant (strong density gradient)
- $v_1$  increases with  $E_{\text{beam}}$   $\rightarrow$  larger density gradient



# Summary

The **PHQMD** is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster formation

combined model **PHQMD = (PHSD & QMD) & SACA**

## PHQMD

- provides the good description of **hadronic 'bulk' observables**
  - from SIS to RHIC energies
- shows **sensitivity to EoS**:  $m_T$  spectra of baryons
- predicts the **dynamical formation of clusters** from low to ultra-relativistic energies
- allows to understand the proton spectra and the **properties of clusters** ( $dn/dp_T dy$ ,  $v_1, v_2$ , fluctuations)
- allows to understand clusters formation in the **participant and spectator region** (consistent with available fragment data by ALADIN collaboration)
- allows to understand the **formation of hypernuclei**

❖ **PHQMD : under development**



**Thank you for your attention !**

**Thanks to the Organizers !**

