Quarkonia and its fate in the anisotropic hot QGP medium

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Outline

- Quarkonia
- Medium Modified Potential
- Effective Fugacity Model
- Inclusion of Anisotropy
- Binding Energy and Thermal Width
- Results.
- Summary and Conclusion.
- Possible Future Directions.

- Quarkonia is a colorless and flavorless bound state of quarks and antiquarks, $Q\overline{Q}$, mainly created at the very early stages after the heavy-ion collisions.
- A typical lifetime of the medium produced in the $A - A$ collision is smaller than the lifetime of heavy quarkonia states.

Figure: lead-lead collision

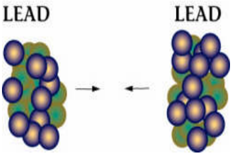
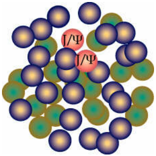


Figure: formation of QGP



- The survival/dissociation of quarkonia is a clean probe to study the produced medium.

- While passing through the QGP medium, their potential get modify and become complex.
- The imaginary part causes thermal width (TW) whereas the real part contribute to the binding energy (BE).
- At the dissociation temperature, the thermal width equals twice the binding energy.
  A. Mocsy and P. Petreczky, Phys. Rev. Lett. 99, 211602 (2007).
- Experimental observation of J/ψ (which is a bound state of $c\bar{c}$) suppression in the di-lepton mass spectrum is a direct indication of the QGP formation in relativistic heavy-ion collision experiments.

- We employ the vacuum Cornell potential, which describes the $Q\bar{Q}$ potential $V(r)$ as a combination of the Coulomb and linear potentials:

$$\mathrm{V}(r) = -\frac{\alpha}{r}\ (coulombic) + \sigma r\ (confining).$$

α is strong coupling constant,
σ is string tension and
r is effective radius of corresponding quarkonia state.

- The medium modification enters through the dielectric permittivity, $\epsilon(k)$ of the medium in the Fourier space,

$$\grave{V}(k) = \frac{\bar{\mathrm{V}}(k)}{\epsilon(k)}. \quad (1)$$

where $\bar{\mathrm{V}}(k)$, is the Fourier transform of $\mathrm{V}(r)$ and has a form,

$$\bar{\mathrm{V}}(k) = -\sqrt{\frac{2}{\pi}}\left(\frac{\alpha}{k^2} + 2\frac{\sigma}{k^4}\right). \quad (2)$$

- $\epsilon(k)$, can be calculated using the semi-classical transport theory.

Quasi-parton equilibrium distribution function

- EQPM maps the hot QCD medium effects in terms of the effective equilibrium distribution function of quasi-partons which describes the strong interaction effects in terms of effective fugacities ($z_{g,q}$).

$$f_g(p) = \frac{1}{z_g^{-1} e^{\beta E_p} - 1}, \quad f_q(p) = \frac{1}{z_q^{-1} e^{\beta E_p} + 1}$$

- These $z_{g/q}$ lead to non-trivial dispersion relation both of the gluonic and quark:

$$\omega_{g/q} = E_p + T^2 \partial_T \log(z_{g/q}).$$

Debye mass with EQPM

$$m_D^{2\,[EoS(i)]}(T) = \frac{8\ \alpha(T)}{\pi} T^2 \Big(N_c PolyLog[2, z_g^i] - N_f PolyLog[2, -z_q^i] \Big).$$

$\alpha(T)$ is the running coupling at finite temperature (T), $i-$ denotes the different EoSs. In the limit $z_{g,q} \to 1$, $m_D(T)$ reduces to the leading order (LO) or for ideal EoS:

$$m_D^{2\,[LO]}(T) = 4\pi\alpha(T)\ T^2 \Big(\frac{N_c}{3} + \frac{N_f}{6} \Big).$$

Caution: Model is valid only above T_c.

Anisotropic distribution

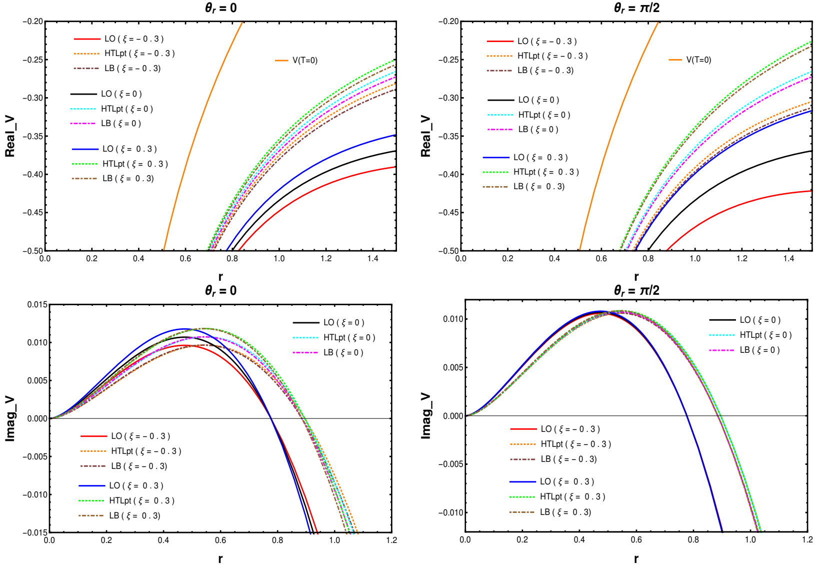
The anisotropic distribution function is obtained by stretching/ squeezing the isotropic one along the direction specified by an anisotropy vector ($\mathbf{n}$), with anisotropic strength (ξ).

$$f_\xi(\mathbf{p}) = C_\xi f\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}.\mathbf{n})^2}\right),$$

$\mathbf{n}^2 = 1$.
θ_n, is the angle between p and n,
$\xi > 0$, correspond to contraction, oblate case
$\xi < 0$, correspond to stretching, prolate case
$\xi = 0$, took us back to the case of isotropy.

- If one normalizes the Debye mass, in the small-ξ limit, C_ξ comes out as,

$$C_\xi = \begin{cases} 1 - \frac{\xi}{3} + O\left(\xi^{\frac{3}{2}}\right) & \text{if} \quad -1 \leq \xi < 0 \\ 1 + \frac{\xi}{3} + O\left(\xi^{\frac{3}{2}}\right) & \text{if} \quad \xi \geq 0. \end{cases}$$



θ_r is the angle between $\mathbf{r}$ and $\hat{n}$.

- To obtain the binding energy with heavy quark potential, one needs to solve the Schrödinger equation using the obtained potential.

Doing so for the isotropic case we obatined:

$$E_b(T) = \frac{m_Q\ \sigma^2}{m_D^4(T)\ n^2} + \alpha\ m_D(T)$$

In the anisotropic case:

$$E_b(T) = \frac{m_Q\ \sigma^2}{m_D^4(T)\ n^2} + \alpha\ m_D(T) + \frac{\xi}{3}\left(\frac{m_Q\ \sigma^2}{m_D^4(T)\ n^2} + \alpha\ m_D(T) + \frac{2\ m_Q\ \sigma^2}{m_D^4(T)\ n^2}\right)$$

Here, n is the radial quantum numbers and m_Q is the quark mass.

- The imaginary part of in-medium potential provides an estimate for thermal width for a particular resonance state given as,

$$\Gamma(T) = -\int d^3\mathbf{r}\ |\Psi(r)|^2\, \mathrm{Im}\ V(\mathbf{r}). \tag{3}$$

- The modified potential, at high temperature, has long-range Coulombic tail that dominates over all the other terms, one can opt $\Psi(r)$ as Coulombic wave function.

Thermal width for $1s$ case:

$$\Gamma_{1s}(T) \quad = \quad T\left(\frac{4}{\alpha\ m_Q^2} + \frac{12\sigma}{\alpha^4\ m_Q^4}\right)\left(1-\frac{\xi}{6}\right)m_D^2 \log\left(\frac{m_D}{\alpha\ m_Q}\right).$$

Thermal width for $2s$ case:

$$\Gamma_{2s}(T) = \frac{8\ m_D^2 T}{\alpha^4\ m_Q^4}\left(1-\frac{\xi}{6}\right)\left(7\alpha^3 m_Q^2 + 192\ \sigma\right) \log\left(\frac{2\ m_D}{\alpha\ m_Q}\right).$$

- Now, we have $E_b(T)$ (BE) and $\Gamma(T)$ for different quarkonia states.

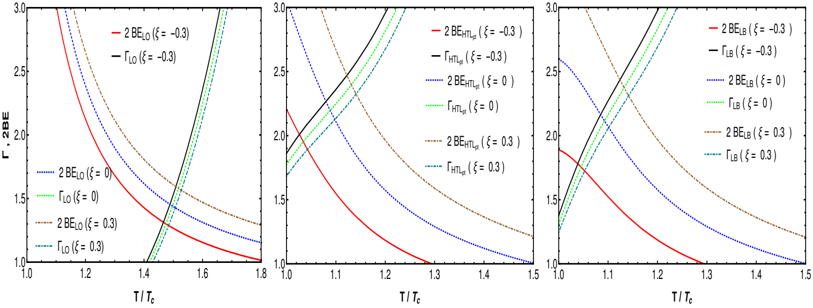


Figure: Γ, 2BE vs T/T_c for J/ψ at $T_c = 0.17GeV$.

- Exploiting the criteria mentioned earlier, we plotted $2BE$ along with $\Gamma(T)$ and obtain the dissociation temperature, T_D as their intersection point.
- Similarly, we obtained for the other states and found a similar pattern, but numbers were different.

- To give a detailed overview, the results at different anisotropies are shown.

The LO results for both the anisotropic as well as isotropic cases

Temperatures are in the unit of T_c			
Anisotropy →	$\xi = -0.3$	$\xi = 0.0$ Our, Lattice*	$\xi = 0.3$
1s states ↓			
Υ	2.861	2.964, 2.31	3.062
J/ψ	1.487	1.520, 1.10	1.551
2s states ↓			
Υ'	1.447	1.478, 1.10	1.508
ψ'	1.054	1.066, 0.20	1.078

- T_D is found to be smaller for prolate, $\xi < 0$ whereas it is higher for oblate, $\xi > 0$ as compared to isotropic case, $\xi = 0$ for all states studied here.
- The excited states dissociate at a lower temperature than their corresponding ground state.

* S. Digal, P. Petreczky and H. Satz, hep-ph/0110406 (Lattice).

Table for 3-loop HTL perturbative calculation results

Temperatures are in the unit of T_c			
Anisotropy $\rightarrow$	$\xi = -0.3$	$\xi = 0.0$	$\xi = 0.3$
1s states $\downarrow$			
Υ	2.427	2.540	2.639
J/ψ	1.054	1.119	1.172
2s states $\downarrow$			
Υ'	1.008	1.067	1.118
ψ'	< 1	< 1	< 1

Table for $(2+1)$- lattice results

Temperatures are in the unit of T_c			
Anisotropy $\rightarrow$	$\xi = -0.3$	$\xi = 0.0$	$\xi = 0.3$
1s states $\downarrow$			
Υ	2.451	2.564	2.665
J/ψ	1.063	1.121	1.172
2s states $\downarrow$			
Υ'	1.023	1.074	1.120
ψ'	< 1	< 1	< 1

- The quarkonia suppression of bottonium and charmonium ($1s$ and $2s$ states) have been discussed within the potential model approach.
- The vacuum inter-quark potential becomes complex in the presence of isotropic/ anisotropic hot QCD medium.
- The real part leads to the binding energies, whereas the imaginary part gives rise to the thermal width.
- $\Gamma(T)$ equals $2BE$, provides the dissociation temperature for a particular quarkonia state.
- Employing EQPM, the numbers were found to be lower as compared to the LO.
- T_D is found to be smaller for prolate, $\xi < 0$ whereas it is higher for oblate, $\xi > 0$ as compared to isotropic case, $\xi = 0$ for all states studied here.
- Further, the excited states dissociate at a lower temperature than their corresponding ground state.
- Based on the above discussion, one can say that the anisotropy, as well as the hot QCD medium interaction effects, play an essential role in deciding the fate of heavy-quarkonia states.

Future aspects

- To obtain an expression for the inter-quark potential for the moving medium and investigate its phenomenological aspect.
- To obtain the survival probability of various quarkonia states to map the theoretical results with the experimental observations.

Collaborators:

- Prof. Bedangadas Mohanty (NISER Bhubaneswar, India)
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- Dr. Avdhensh Kumar (Institute of Nuclear Physics, Poland).
- Dr. Vineet Agotiya (CU, Ranchi, India)
- Dr. Indrani Nilima (CU, Ranchi, India).

Thank you

Picture: Gate way of India

The real and the imaginary part of the modified potential:

Isotropic case

$$
\begin{aligned}
Re[V(\mathbf{r},T)] &= \frac{s\ \sigma}{m_D(T)} - \frac{\alpha\ m_D(T)}{s}\left(1+\frac{s^2}{2}\right).\\
\mathrm{Im}[V(r,T)] &= -\frac{s^2\ T}{3}\left(\alpha + \frac{s^2\ \sigma}{10\ m_D^2(T)}\right)\log\left(\frac{1}{s}\right).
\end{aligned}
$$

$s = r\ m_D(T)$

Anisotropic case

$$
\begin{aligned}
Re[V(r,\xi,T)] &= \frac{s\ \sigma}{m_D(T)}\left(1+\frac{\xi}{3}\right) - \frac{\alpha\ m_D(T)}{s}\left[1+\frac{s^2}{2}+\xi\left\{\frac{1}{3}+\frac{s^2}{16}\left(\frac{1}{3}+\cos\left(2\theta_r\right)\right)\right\}\right].\\
Im[V(r,\xi,T)] &= \frac{\alpha\ s^2\ T}{3}\left\{\frac{\xi}{60}(7-9\cos 2\theta_r)-1\right\}\log\left(\frac{1}{s}\right) + \frac{s^4\ \sigma\ T}{m_D^2(T)}\left\{\frac{\xi}{35}\left(\frac{1}{9}-\frac{1}{4}\cos 2\theta_r\right)\right.\\
&- \left.\frac{1}{30}\right\}\log\left(\frac{1}{s}\right).
\end{aligned}
$$

θ_r is the angle between $\mathbf{r}$ and $\mathbf{n}$.

- While considering the small anisotropy, one can solve the Schrödinger equation and obtained the binding energy by just considering the isotropic part with the first order perturbation in anisotropy parameter, ξ.

$$\hat{H}\psi_\nu(x) = E_\nu\psi_\nu(x), \qquad \hat{H} = -\frac{\nabla^2}{2M_R} + V(x) + M_1 + M_2.$$

Here, $M_R = \frac{M_1 M_2}{M_1+M_2}$. ν represent a list of relevant quantum number n, l and m.

- Once the ground state wave-function is found, we can compute its energy eigenvalue via

$$E_\nu = \frac{<\psi_\nu \hat{H} \psi_\nu>}{<\psi_\nu|\psi_\nu>} = \frac{\int d^3x \psi_\nu^* \hat{H} \psi_\nu}{\int d^3x \psi_\nu^* \psi_\nu}$$

- To obtain the binding energy of a state, $E_{\nu,bind}$, we subtract the quark masses and the potential at infinity,

$$E_{\nu,bind} = E_\nu - M_1 - M_2 - \frac{<\psi_\nu|V(|\mathbf{r}| \to \infty)|\psi_\nu>}{<\psi_\nu|\psi_\nu>}$$

- The idea of dividing potential by dielectric tensor to modify it came from the analogy of QED.
- If one perturbatively calculates the non-relativistic potential $V(r)$ between two unlike static charges, say, in QED, the usual Coulomb-like behaviour is modified by the photon self-energy $\Pi(\omega = 0, k)$ such that,

$$V(r) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{-e^2}{k^2 + \Pi_L} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{-e^2}{k^2(1 + \frac{\Pi_L}{k^2})} \tag{4}$$

- The dielectric permittivity can be defined as

$$\epsilon(\omega = 0, \mathbf{k}) \quad = \quad 1 + \frac{\Pi_L}{k^2} \tag{5}$$