

Chemical equilibration of QGP in hadronic collisions

Aleksas Mazeliauskas

Institut für Theoretische Physik
Universität Heidelberg

June 11, 2019

A. Kurkela, AM (2018) [1, 2]

*Phys. Rev. D*99 (1811.03068)
Phys. Rev. Lett. 122 (1811.03040)



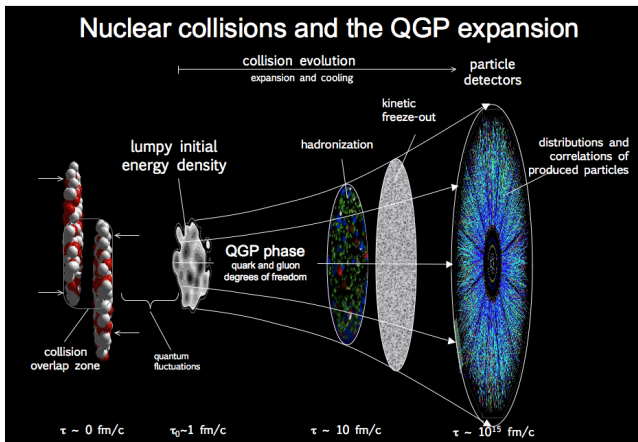
UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386



Isolated quantum systems and universality in extreme conditions

Heavy ion collisions

Nuclear collision programs at RHIC (since 2000) and LHC (since 2010).

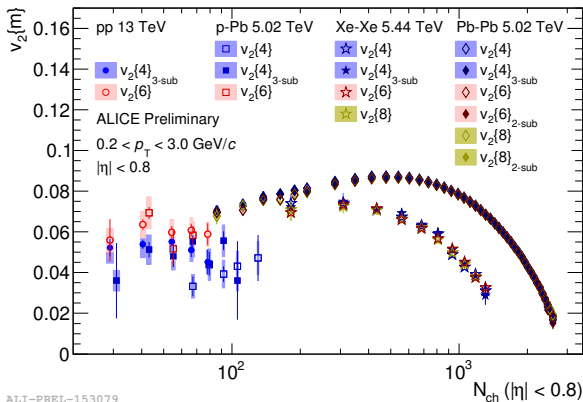


Sorensen, Quark-gluon plasma 4, 2010

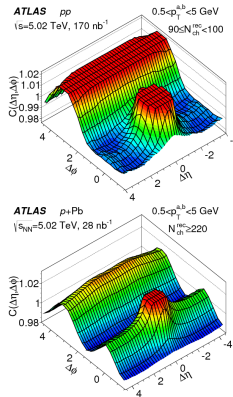
- Range of collision energies: 7.7-200 GeV, 2.76 TeV, 5.02 TeV.
- Different systems: PbPb, pPb, pp, AuAu, pAu, dAu, $^3\text{HeAu}$...

Evidence for equilibration of QGP

Collective particle flow seen even in the smallest collision systems.



ALI-PREL-153079

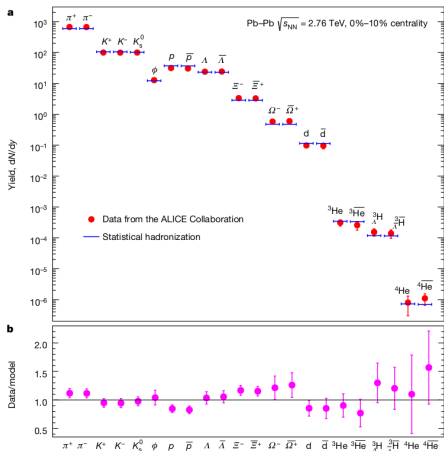
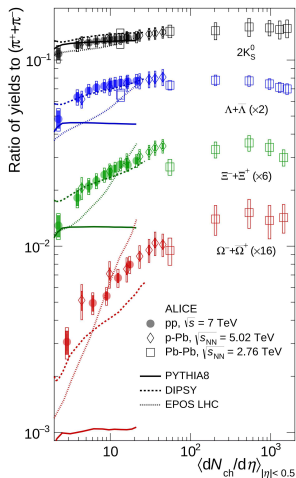


ATLAS Phys.Rev. C96 (2017) no.2, 024908

Is hydrodynamically flowing QGP formed in pp and pA collisions?

Strangeness enhancement in high multiplicity pp and pPb collisions

Qualitative change in hadron production.



ALICE Nature Physics (2017) Andronic, Braun-Munzinger, Redlich, Stachel, Nature (2018)

Is strangeness enhancement due to chemically equilibrating QGP?

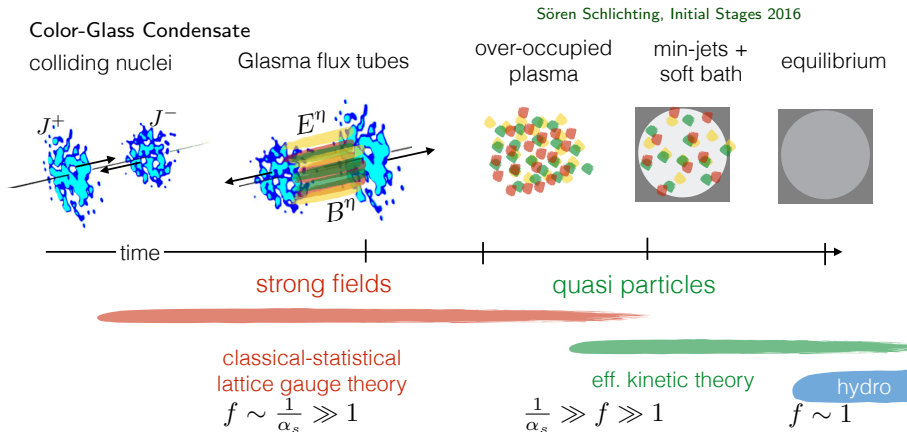
Weak coupling picture of QCD equilibration

At high energies the mid-rapidity is dominated by small Bjorken- x gluons

- strong gluon fields $A_\mu \sim \frac{1}{g} \implies$ classical field description
- particle scatterings isotropize and equilibrate plasma \implies
“Bottom-up” thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[3]

Berges, Boguslavski, Schlichting, Venugopalan (2014) [4] Kurkela and Zhu (2015) [5]



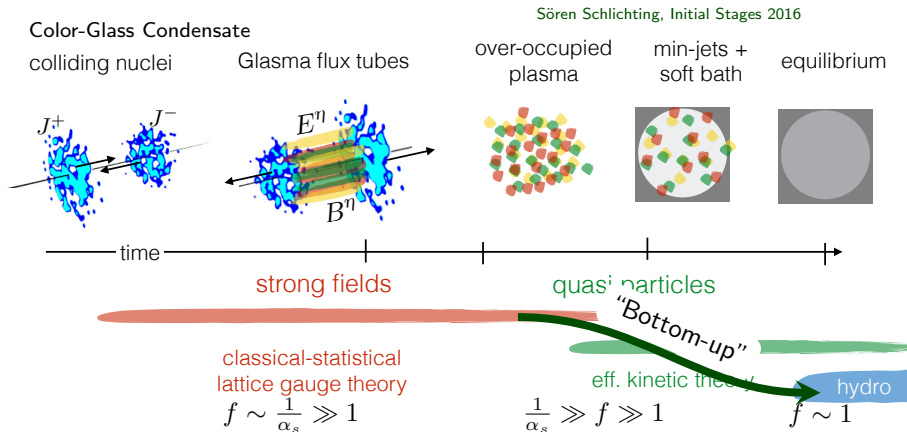
Weak coupling picture of QCD equilibration

At high energies the mid-rapidity is dominated by small Bjorken- x gluons

- strong gluon fields $A_\mu \sim \frac{1}{g} \implies$ classical field description
- particle scatterings isotropize and equilibrate plasma \implies
“Bottom-up” thermalization scenario

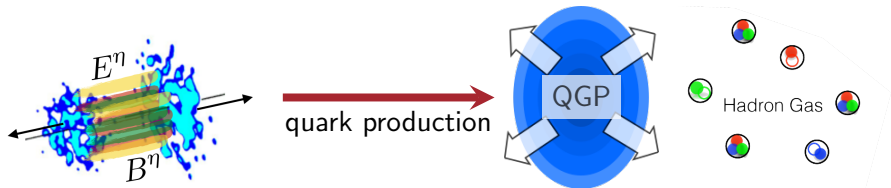
Baier, Mueller, Schiff, and Son (2001)[3]

Berges, Boguslavski, Schlichting, Venugopalan (2014) [4] Kurkela and Zhu (2015) [5]



Fermion production in weakly coupled QCD

Initial state is dominated by gluon fields



But final state is assumed to be in chemical equilibrium:

- QGP expansion described by 3 flavour equation of state (u, d, s).
- hadron production at freeze-out consistent with thermal ensemble.

How can we produce fermions?

- Quark production from strong color fields.
- Leading order kinetics: gluon fusion $gg \leftrightarrow q\bar{q}$ and splitting $g \leftrightarrow q\bar{q}$.

Tanji, Berges (2017) [6]
Martinez, Sievert, Wertepny (2018) [7]

Kurkela, AM (2018) [1, 2]

High temperature gauge kinetic theory $\Lambda_{QCD} \ll gT \ll T$


Kinetic theory of weakly coupled quark and gluon quasi-particles.

Arnold, Moore, Yaffe (2003)[8]

$$\partial_\tau f_{g,q} - \frac{p_z}{\tau} \partial_{p_z} f_{g,q} = -\mathcal{C}_{2 \leftrightarrow 2}[f] - \mathcal{C}_{1 \leftrightarrow 2}[f]$$

Leading order in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- Elastic scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $qg \leftrightarrow gq$, $gg \leftrightarrow q\bar{q}$


$$= |\mathcal{M}_{qq}^{gg}|^2 = \lambda^2 16 \frac{d_F C_F}{C_A^2} \left[C_F \left(\frac{u}{t} + \frac{t}{u} \right) - C_A \left(\frac{t^2 + u^2}{s^2} \right) \right]$$

Hard Thermal Loop resummed propagators, screening mass $m_D \sim gT$

Braaten, Pisarski (1990) [9]

High- pT QCD kinetic theory \Rightarrow parton energy loss
Low- pT QCD kinetic theory \Rightarrow thermalization

We will consider QCD with $N_c = 3$ and $N_f = 3$ massless quarks ($\mu_B = 0$).

High temperature gauge kinetic theory $\Lambda_{QCD} \ll gT \ll T$


Kinetic theory of weakly coupled quark and gluon quasi-particles.

Arnold, Moore, Yaffe (2003)[8]

$$\partial_\tau f_{g,q} - \frac{p_z}{\tau} \partial_{p_z} f_{g,q} = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$

Leading order in the coupling constant $\lambda = 4\pi\alpha_s N_c$:

- Medium induced colinear radiation: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$


$$= |\mathcal{M}_{qq}^g|^2 = \frac{k'^2 + p'^2}{k'^2 p'^2 p^3} \underbrace{\mathcal{F}_q(k'; -p', p)}_{\text{splitting rate}}$$

Resummed multiple scatterings with the medium
(Landau–Pomeranchuk–Migdal suppression).

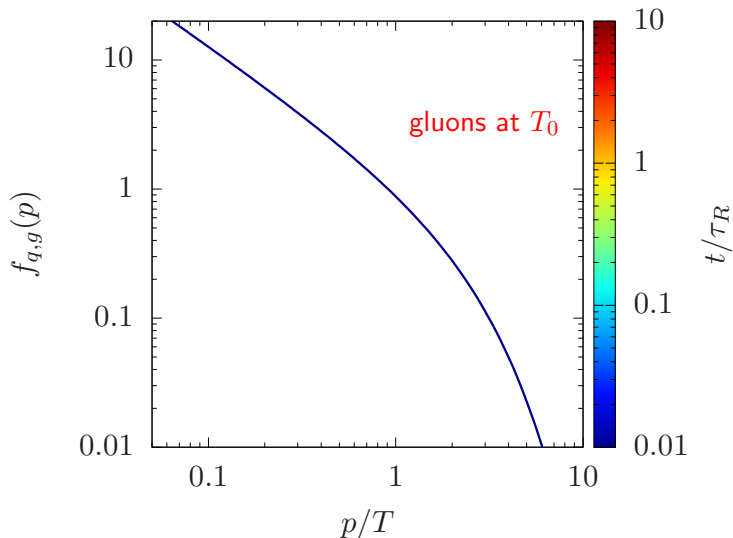
High- pT QCD kinetic theory \Rightarrow parton energy loss

Low- pT QCD kinetic theory \Rightarrow thermalization

We will consider QCD with $N_c = 3$ and $N_f = 3$ massless quarks ($\mu_B = 0$).

Well controlled example: gluon thermal bath with $\lambda = N_C g^2 = 0.1$

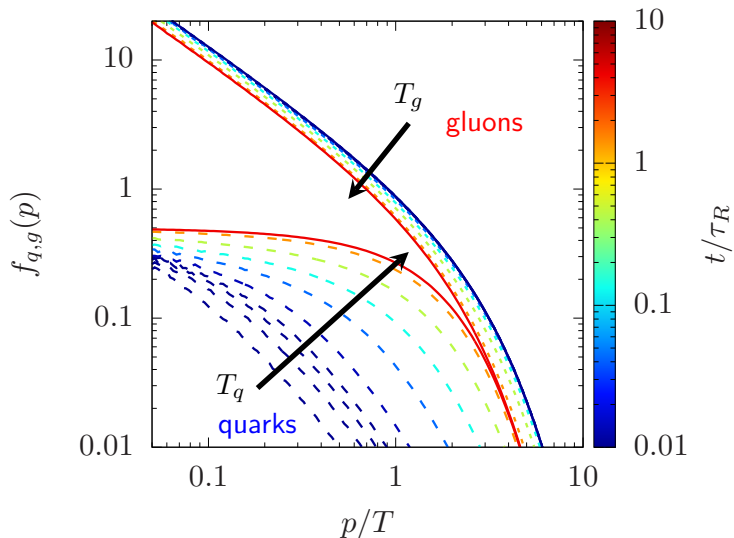
Quark production: gluon fusion $gg \rightarrow q\bar{q}$ and collinear radiation $g \rightarrow q\bar{q}$



Gluons maintain an approximate kinetic equilibrium, fermions do not.

Well controlled example: gluon thermal bath with $\lambda = N_C g^2 = 0.1$

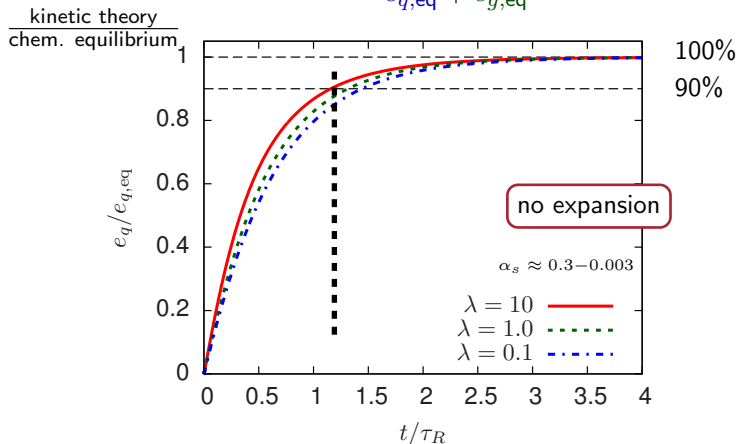
Quark production: gluon fusion $gg \rightarrow q\bar{q}$ and collinear radiation $g \rightarrow q\bar{q}$



Gluons maintain an approximate kinetic equilibrium, fermions do not.

Chemical equilibration in non-expanding systems

Equilibrium quark energy fraction $\frac{e_{q,\text{eq}}}{e_{q,\text{eq}} + e_{g,\text{eq}}} \approx 0.66$

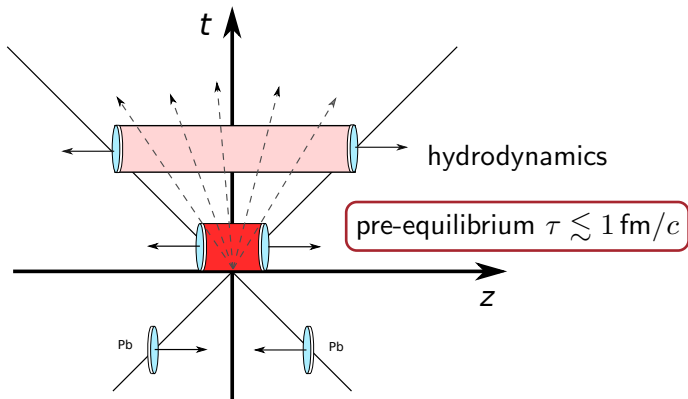


- Scaling with kinetic relaxation time $\tau_R = \frac{4\pi\eta/s}{T_{\text{final}}} \sim \frac{1}{\lambda^2 T} \sim l_{\text{mfp}}$.
 $\eta/s \approx 1, 35, 1900$
- 90% of fermion equilibrium energy produced at $t_{\text{chem}} \sim 1.2\tau_R$.

Boost invariant expansion

In thermal equilibrium—conservation of entropy per unity rapidity Bjorken (1983)

$$\frac{dN}{d\eta} \sim \frac{dS}{d\eta} = \langle s\tau \rangle = \text{const} \implies T_{\text{id.}}(\tau) \propto \frac{\langle s\tau \rangle^{1/3}}{\tau^{1/3}}$$

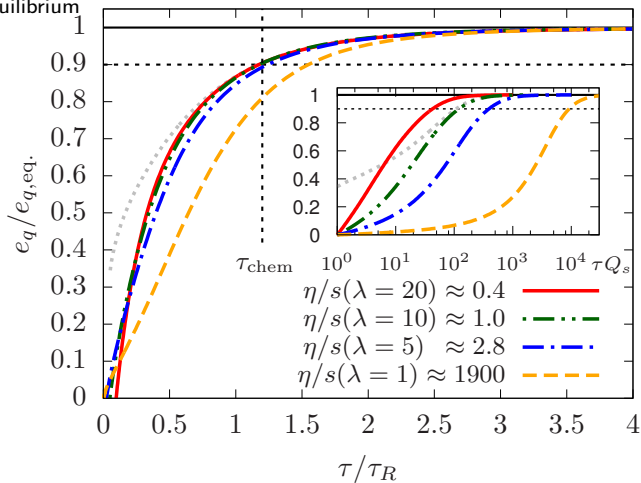


"Measure" energy with respect to ideal expansion $e_{\text{eq}} \propto T_{\text{id.}}^4$.

Chemical equilibration with expansion

Time dependent mean free path $\Rightarrow \tau_R \equiv \frac{4\pi\eta/s}{T_{id}(\tau)}$

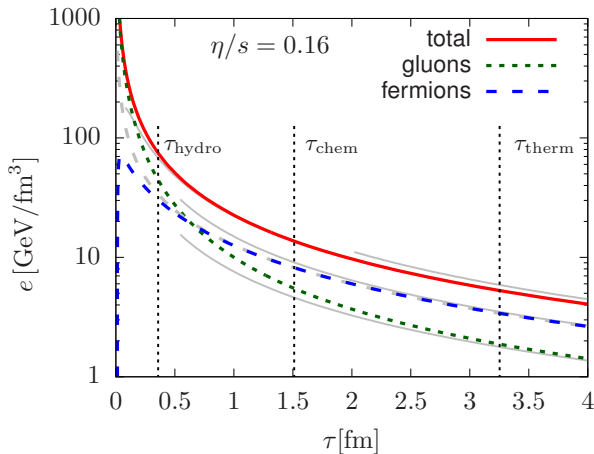
$\frac{\text{kinetic theory}}{\text{chem. equilibrium}}$



Chemical equilibration at $\tau_{chem} \sim 1.2\tau_R(\tau)$ for $\alpha_s \sim 0.3$

Physical equilibration time-scales in hadronic collisions

$$\tau = \underbrace{\left(\frac{\tau}{\tau_R}\right)^{3/2}}_{\text{scaled time variable}} \times \underbrace{\left(4\pi\eta/s\right)^{3/2} \times \langle s\tau \rangle^{-1/2} \times \left(4\pi^2\nu_{\text{eff}}/90\right)^{1/2}}_{\text{phenomenological input}}$$



Input:

$$\eta/s \approx 0.16$$

$$\langle s\tau \rangle \approx 4.1 \text{ GeV}^2$$

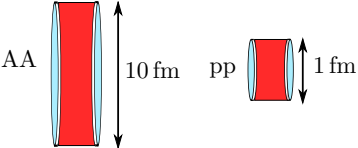
$$\nu_{\text{eff}} \approx 40$$

$$\underbrace{\tau_{\text{hydro}}}_{\pm 10\% \text{ viscous } e(\tau)} < \underbrace{\tau_{\text{chem}}}_{\pm 10\% \text{ fermion eq. } e(\tau)} < \underbrace{\tau_{\text{therm}}}_{\pm 10\% \text{ ideal } e(\tau)}$$

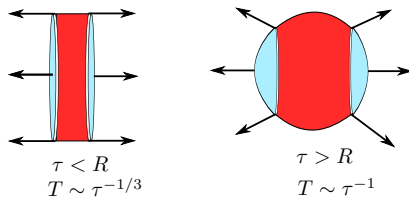
System size and chemical equilibration time

Will chemical equilibrium be reached for a given system size $dN_{\text{ch}}/d\eta$?

- Hotter system \Rightarrow faster equilibration.

$$\langle \tau_s \rangle \propto \frac{dN_{\text{ch}}/d\eta}{\pi R^2}$$


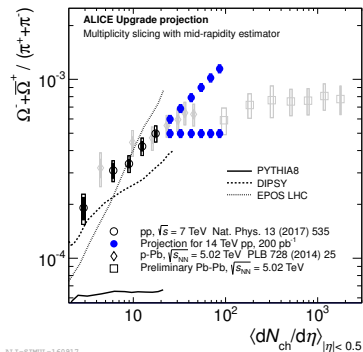
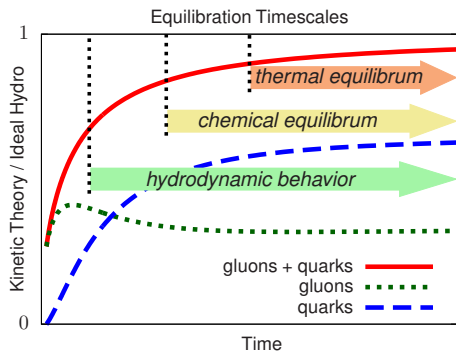
- Larger system \Rightarrow more time to equilibrate.



$$\frac{dN_{\text{ch}}}{d\eta} \gtrsim 110 \left(\frac{\eta/s}{0.16} \right)^3 \left(\frac{\tau_{\text{chem}}}{1.2\tau_R} \right)^3 \left(\frac{\tau_{\text{chem}}}{R} \right)^{-2}$$

Chemical equilibrium happens at the same $dN_{\text{ch}}/d\eta$ for all systems.

Summary



High Luminosity LHC Run 3 Yellow Report (2018)

Ordering of equilibration timescales in QCD kinetic theory

$$\tau_{\text{hydro}} < \tau_{\text{chem}} < \tau_{\text{therm}}$$

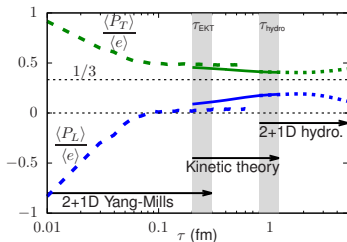
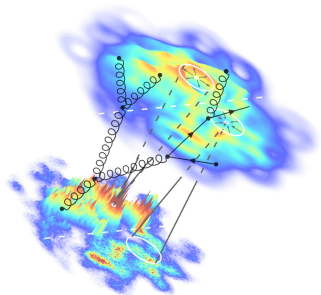
Direct link between transport properties, system size and chemistry.

$$\tau_{\text{chem}} = 1.5 \text{ fm} \times \left(\frac{\eta/s}{0.16} \right)^{\frac{3}{2}}, \quad \frac{dN_{\text{ch}}^{\text{chem}}}{d\eta} = 110 \times \left(\frac{\eta/s}{0.16} \right)^3.$$

Outlook: transverse pre-equilibrium evolution

KoMPoST— event-by-event kinetic pre-equilibrium for heavy ion collisions.

$$\underbrace{\delta T_{\mathbf{x}}^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}')}_{\text{goes into hydro}} = \int d^2 \mathbf{x}' \underbrace{G_{\alpha\beta}^{\mu\nu}(\mathbf{x} - \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}})}_{\text{linear response function}} \underbrace{\delta T_{\mathbf{x}'}^{\alpha\beta}(\tau_{\text{EKT}}, \mathbf{x}')}_{\text{initial}}.$$



<https://github.com/KMPoST/KoMPoST> [12] Kurkela, AM, Paquet, Schlichting and Teaney (2018)[10, 11]

To do:

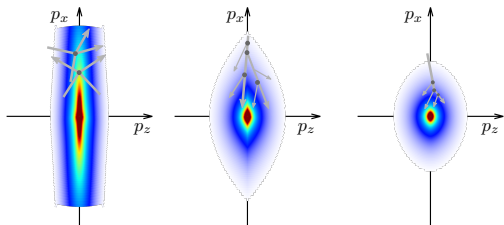
- radial expansion \implies important for small systems.
- quark response functions \implies photon production.
- finite fermion mass \implies breaking of conformal and flavour symmetries.

“Bottom-up” thermalization scenario

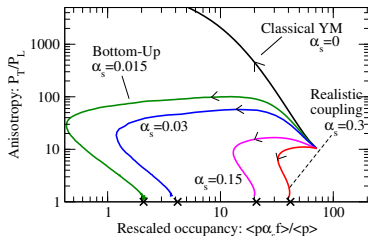
Baier, Mueller, Schiff, and Son (2001)[3]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

- | | | |
|-------------------------|---|---|
| I) over-occupied | $p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$ | $1 \ll Q_s \tau \ll \alpha_s^{-3/2}$ |
| II) under-occupied | $p_z \sim \sqrt{\alpha_s} Q_s$ | $\alpha_s^{-3/2} \ll Q_s \tau \ll \alpha_s^{-5/2}$ |
| III) mini-jet quenching | $p_z \sim \alpha_s^3 Q_s (Q_s \tau)$ | $\alpha_s^{-5/2} \ll Q_s \tau \ll \alpha_s^{-13/5}$ |



2 ↔ 2 broadening collinear cascade mini-jet quench



Berges, Boguslavski, Schlichting, Venugopalan (2014) [4]

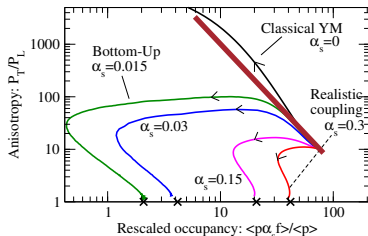
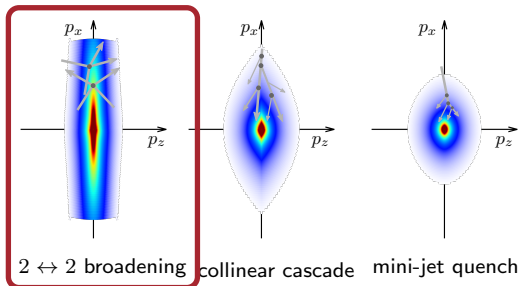
Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)
[5, 13, 11, 10]

“Bottom-up” thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[3]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

- | | | |
|-------------------------|---|---|
| I) over-occupied | $p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$ | $1 \ll Q_s \tau \ll \alpha_s^{-3/2}$ |
| II) under-occupied | $p_z \sim \sqrt{\alpha_s} Q_s$ | $\alpha_s^{-3/2} \ll Q_s \tau \ll \alpha_s^{-5/2}$ |
| III) mini-jet quenching | $p_z \sim \alpha_s^3 Q_s (Q_s \tau)$ | $\alpha_s^{-5/2} \ll Q_s \tau \ll \alpha_s^{-13/5}$ |



Berges, Boguslavski, Schlichting, Venugopalan (2014) [4]

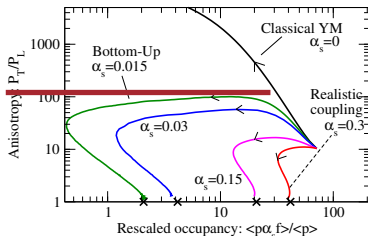
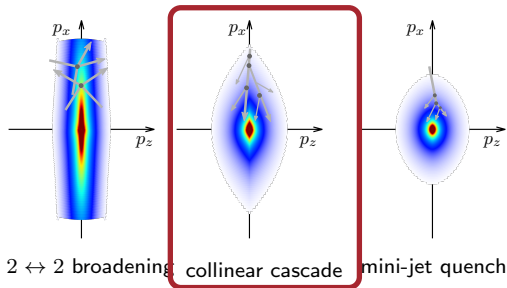
Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)
[5, 13, 11, 10]

“Bottom-up” thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[3]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

- | | | |
|-------------------------|---|---|
| I) over-occupied | $p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$ | $1 \ll Q_s \tau \ll \alpha_s^{-3/2}$ |
| II) under-occupied | $p_z \sim \sqrt{\alpha_s} Q_s$ | $\alpha_s^{-3/2} \ll Q_s \tau \ll \alpha_s^{-5/2}$ |
| III) mini-jet quenching | $p_z \sim \alpha_s^3 Q_s (Q_s \tau)$ | $\alpha_s^{-5/2} \ll Q_s \tau \ll \alpha_s^{-13/5}$ |



Berges, Boguslavski, Schlichting, Venugopalan (2014) [4]

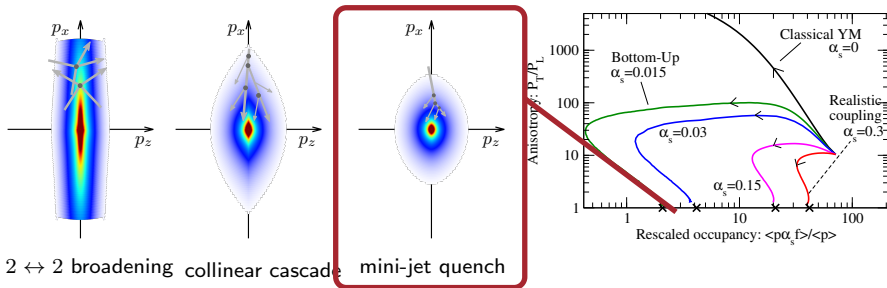
Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)
[5, 13, 11, 10]

"Bottom-up" thermalization scenario

Baier, Mueller, Schiff, and Son (2001)[3]

Evolution of initially over-occupied hard gluons $p \sim Q_s \gg \Lambda_{\text{QCD}}$

- | | | |
|-------------------------|---|---|
| I) over-occupied | $p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$ | $1 \ll Q_s \tau \ll \alpha_s^{-3/2}$ |
| II) under-occupied | $p_z \sim \sqrt{\alpha_s} Q_s$ | $\alpha_s^{-3/2} \ll Q_s \tau \ll \alpha_s^{-5/2}$ |
| III) mini-jet quenching | $p_z \sim \alpha_s^3 Q_s (Q_s \tau)$ | $\alpha_s^{-5/2} \ll Q_s \tau \ll \alpha_s^{-13/5}$ |

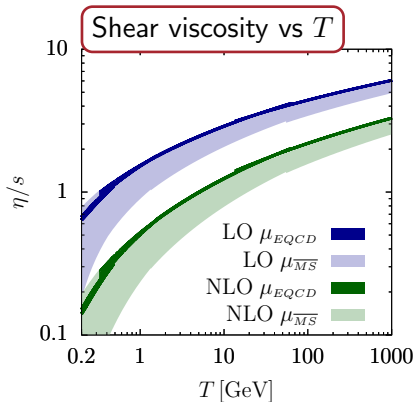


Berges, Boguslavski, Schlichting, Venugopalan (2014) [4]

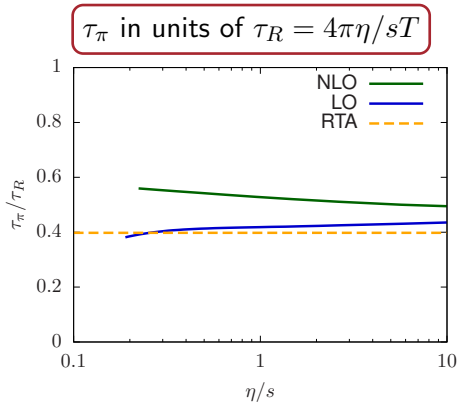
Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)
[5, 13, 11, 10]

Next-to-leading order transport coefficients

Can leading order results be informative for small η/s ?



Ghiglieri, Moore, Teaney (2018) [14, 15]



- Large NLO corrections to $\eta/s(T)$.
- Small *relative* correction in 2nd/1st order transport coefficient.
- *Extrapolation scheme: adjust $\tau_R = 4\pi\eta/sT$ to match phenomenology.*

Bibliography I

- [1] [Aleksi Kurkela and Aleksas Mazeliauskas.](#)
Chemical equilibration in hadronic collisions.
Phys. Rev. Lett., 122:142301, 2019, 1811.03040.
- [2] [Aleksi Kurkela and Aleksas Mazeliauskas.](#)
Chemical equilibration in weakly coupled QCD.
Phys. Rev., D99(5):054018, 2019, 1811.03068.
- [3] [R. Baier, Alfred H. Mueller, D. Schiff, and D. T. Son.](#)
'Bottom up' thermalization in heavy ion collisions.
Phys. Lett., B502:51–58, 2001, hep-ph/0009237.
- [4] [J. Berges, K. Boguslavski, S. Schlichting, and R. Venugopalan.](#)
Turbulent thermalization process in heavy-ion collisions at ultrarelativistic energies.
Phys. Rev., D89(7):074011, 2014, 1303.5650.
- [5] [Aleksi Kurkela and Yan Zhu.](#)
Isotropization and hydrodynamization in weakly coupled heavy-ion collisions.
Phys. Rev. Lett., 115(18):182301, 2015, 1506.06647.
- [6] [Naoto Tanji and Juergen Berges.](#)
Nonequilibrium quark production in the expanding QCD plasma.
Phys. Rev., D97(3):034013, 2018, 1711.03445.

Bibliography II

- [7] Mauricio Martinez, Matthew D. Sievert, and Douglas E. Weretepny. Multiparticle Production at Mid-Rapidity in the Color-Glass Condensate. *JHEP*, 02:024, 2019, 1808.04896.
- [8] Peter Brockway Arnold, Guy D. Moore, and Laurence G. Yaffe. Effective kinetic theory for high temperature gauge theories. *JHEP*, 01:030, 2003, hep-ph/0209353.
- [9] Eric Braaten and Robert D. Pisarski. Soft Amplitudes in Hot Gauge Theories: A General Analysis. *Nucl. Phys.*, B337:569–634, 1990.
- [10] Aleksi Kurkela, Aleksas Mazeliauskas, Jean-François Paquet, Sören Schlichting, and Derek Teaney. Effective kinetic description of event-by-event pre-equilibrium dynamics in high-energy heavy-ion collisions. *Phys. Rev.*, C99(3):034910, 2019, 1805.00961.
- [11] Aleksi Kurkela, Aleksas Mazeliauskas, Jean-François Paquet, Sören Schlichting, and Derek Teaney. Matching the Nonequilibrium Initial Stage of Heavy Ion Collisions to Hydrodynamics with QCD Kinetic Theory. *Phys. Rev. Lett.*, 122(12):122302, 2019, 1805.01604.

Bibliography III

- [12] [Aleksi Kurkela, Aleksas Mazeliauskas, Jean-François Paquet, Sören Schlichting, and Derek Teaney.](#)
KøMPøST: linearized kinetic theory propagator of initial conditions for heavy ion collisions.
[GitHub](#), April 2018.
- [13] [Liam Keegan, Aleksi Kurkela, Aleksas Mazeliauskas, and Derek Teaney.](#)
Initial conditions for hydrodynamics from weakly coupled pre-equilibrium evolution.
JHEP, 08:171, 2016, 1605.04287.
- [14] [Jacopo Ghiglieri, Guy D. Moore, and Derek Teaney.](#)
QCD Shear Viscosity at (almost) NLO.
JHEP, 03:179, 2018, 1802.09535.
- [15] [Jacopo Ghiglieri, Guy D. Moore, and Derek Teaney.](#)
Second-order Hydrodynamics in Next-to-Leading-Order QCD.
Phys. Rev. Lett., 121(5):052302, 2018, 1805.02663.