Equation of state of QCD matter within the Hagedorn bag-like model

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FIAS Frankfurt Institute for Advanced Studies

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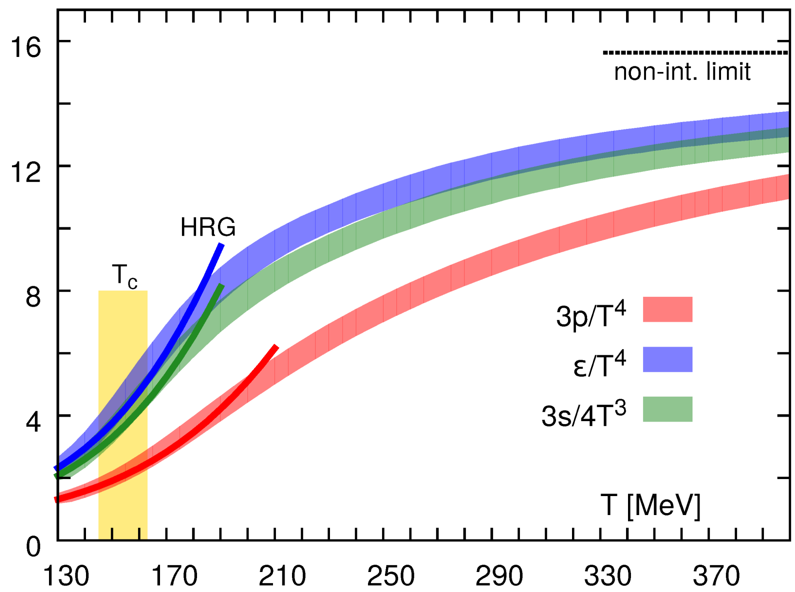
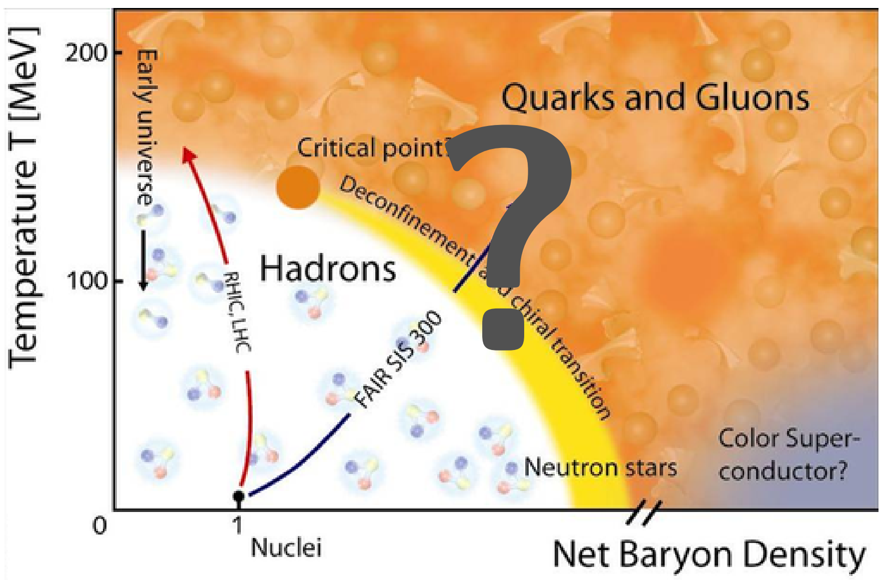
QCD equation of state

$\mu_B = 0$



$T - \mu_B$ plane



Approaches:

- Lattice parameterization, $\mu_B = 0$ directly, small μ_B through Taylor/fugacity expansion
- Merge models for hadronic and QGP phases, e.g. Maxwell construction (EOSQ) [Kolb, Sollfrank, Heinz, PRC '00], smooth switching function [Albright, Kapusta, Young, 1404.7540], etc.
- Effective models with hadronic and partonic degrees of freedom [Steinheimer et al., 1009.5239; Motornenko et al., 1905.00866; etc.]

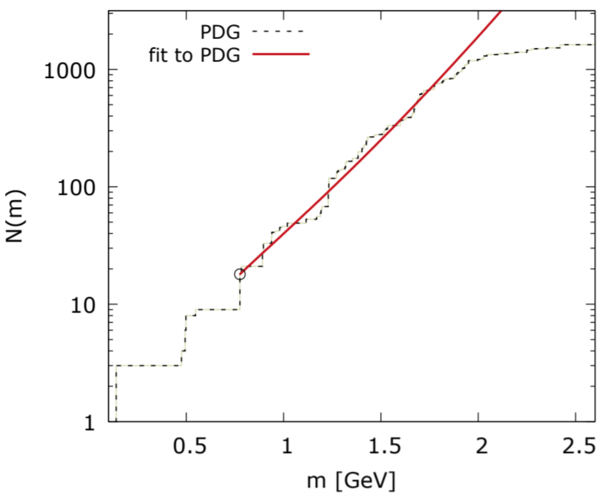
In most cases hadron-parton transition is put in "by hand"

Hagedorn mass spectrum

R. Hagedorn (1965): Statistical Bootstrap Model, fireballs consist of fireballs

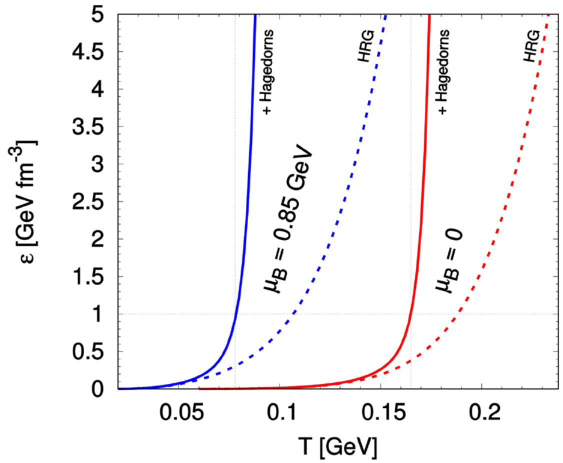
$$\rho(m) = A\, m^{-\alpha}\, exp\,(m/\,T_H)$$

fits the PDG spectrum



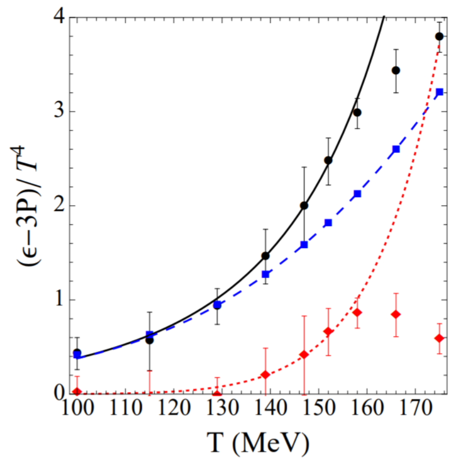
[P.M. Lo et al., 1507.06398]

limiting temperature T_H



[Beitel, Gallmeister, Greiner, 1402.1458]

thermodynamics near T_{pc}



[Majumder, Mueller, PRL '10]

- Fast equilibration of hadrons in HICs [Noronha-Hostler et al., PRL '08; PRC '10], an alternative to strings in transport codes [Beitel, Greiner, Stoecker, PRC '16], thermal distribution of hadrons as a consequence of decaying heavy Hagedorns [Beitel, Gallmeister, Greiner, PRC '14]
- Transport coefficients around T_{pc} [Noronha-Hostler, Noronha, Greiner, PRL '09]

Hagedorn bag-like model

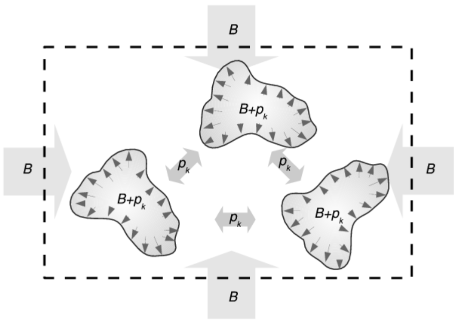
An analytic model of a (phase) transition between hadronic matter and QGP

[Gorenstein, Petrov, Zinovjev, PLB '81; Gorenstein, W. Greiner, Yang, JPG '98]

Ingredients:

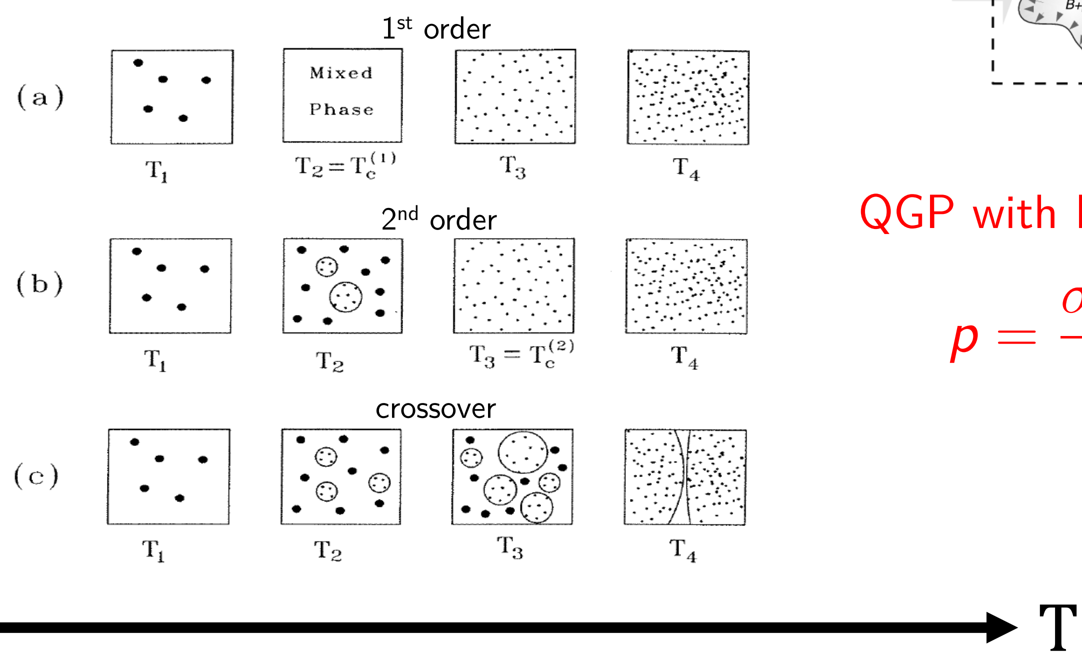
- Statistical mechanics of colorless quark-gluon bags (single partition function)
- Hagedorn spectrum from MIT bag model $\rho(m) = A\, m^{-\alpha}\, exp\,(m/T_H)$
- Compressible bags with finite eigenvolumes $V \to V - bN$

eliminates the "limiting" temperature



dilute hadron gas

$p = Tn$

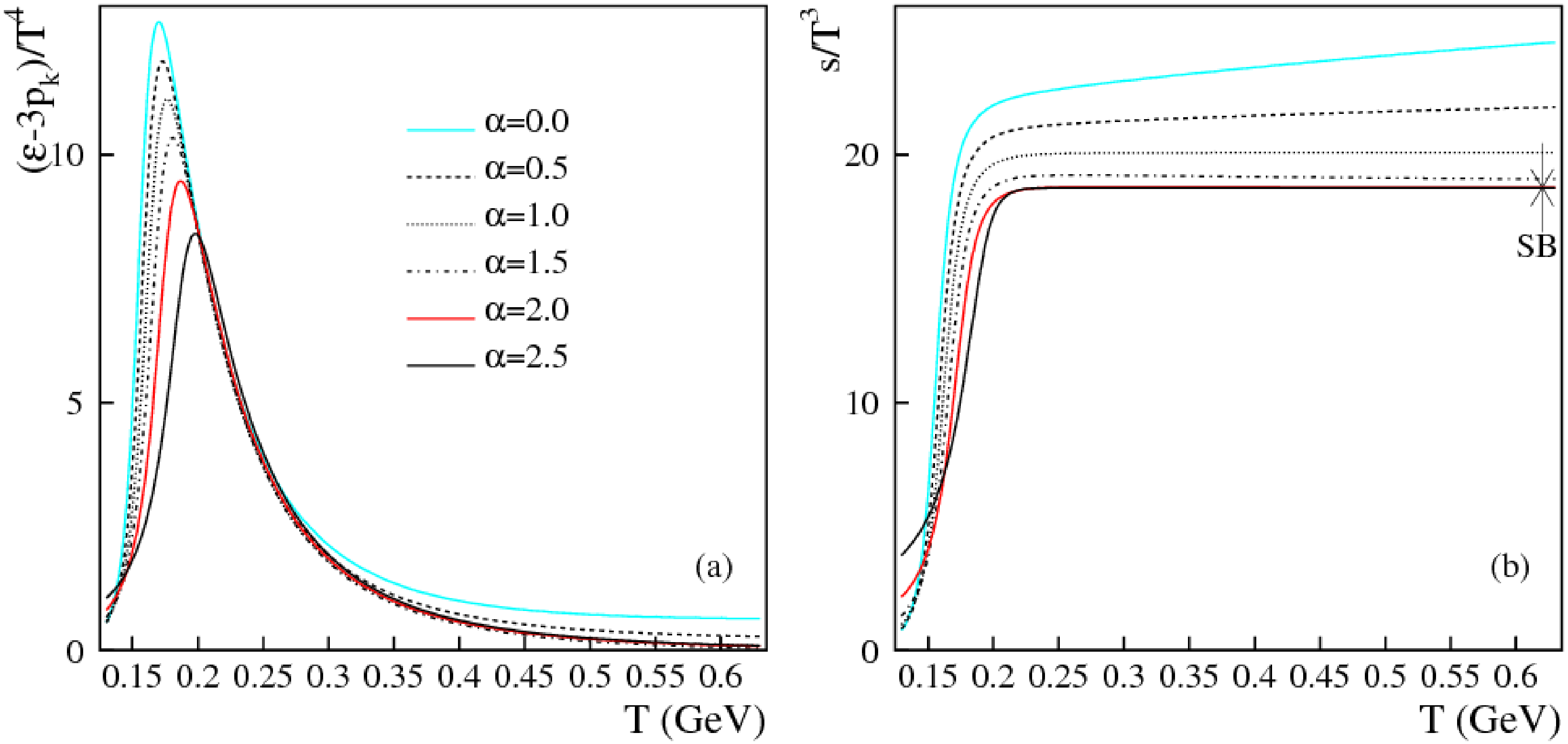


QGP with MIT bag model

$$p = \frac{\sigma_Q}{3} T^4 - B$$

Crossover transition: Prior studies

First quantitative analysis performed in [L. Ferroni, V. Koch, PRC 79, 034905 (2009)] for the crossover scenario

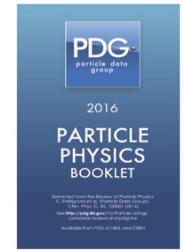


Crossover transition in bag-like model qualitatively compatible with LQCD quantitatively... not so much

Model implementation

Thermodynamic system of known hadrons and quark-gluon bags

Mass-volume density: $\rho(m, v; \lambda_B, \lambda_Q, \lambda_S) = \rho_H + \rho_Q$



$$\rho_H = \sum_{i \in \mathrm{HRG}} \lambda_B^{b_i} \, \lambda_Q^{q_i} \, \lambda_S^{s_i} \, d_i \, \delta(m - m_i) \, \delta(v - v_i)$$

PDG hadrons

$$\rho_Q = C \, v^{\gamma} \, (m - Bv)^{\delta} \, \exp\left\{ \frac{4}{3} [\sigma_Q(\lambda_B, \lambda_Q, \lambda_S)]^{1/4} \, v^{1/4} \, (m - Bv)^{3/4} \right\} \theta(v - V_0) \, \theta(m - Bv - M_0).$$

Quark-gluon bags [J. Kapusta, PRC '81; Gorenstein+, ZPC '84]

Excluded volume $\rightarrow$ **isobaric (pressure) ensemble**

[Gorenstein, Petrov, Zinovjev, PLB '81]

$$\hat{Z}(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int_0^{\infty} Z(T, V, \lambda_B, \lambda_Q, \lambda_S) \, e^{-sV} \, dV = [s - f(T, s, \lambda_B, \lambda_Q, \lambda_S)]^{-1}$$

$$f(T, s, \lambda_B, \lambda_Q, \lambda_S) = \int dv \int dm \, \rho(m, v; \lambda_B, \lambda_Q, \lambda_S) \, \phi(T, m) \, e^{-s\, v}$$

The system pressure is $p = T s^*$ with s^* being the *rightmost* singularity of $\hat{Z}$

Crossover transition

Type of transition is determined by exponents γ and δ of the bag spectrum

Crossover seen in lattice, realized in model for $\gamma + \delta \geq -3$ and $\delta \geq -7/4$, the rightmost singularity is a pole singularity, $s^* = f(T, s^*)$

[Begun, Gorenstein, W. Greiner, JPG '09]

Transcendental equation for pressure:

$$p(T, \lambda_B, \lambda_Q, \lambda_S) = T \sum_{i \in \mathrm{HRG}} d_i \, \phi(T, m) \, \lambda_B^{b_i} \, \lambda_Q^{q_i} \, \lambda_S^{s_i} \, \exp\left(-\frac{m_i p}{4BT}\right)$$

$$+ \frac{C}{\pi} \, T^{5+4\delta} \, [\sigma_Q]^{\delta+1/2} \, [B + \sigma_Q T^4]^{3/2} \left(\frac{T}{p - p_B}\right)^{\gamma+\delta+3} \Gamma\left[\gamma + \delta + 3, \frac{(p - p_B) V_0}{T}\right]$$

Solved numerically

Calculation setup:

$$\gamma = 0, \quad -3 \leq \delta \leq -\frac{1}{2}, \quad B^{1/4} = 250 \text{ MeV}, \quad C = 0.03 \text{ GeV}^{-\delta+2}, \quad V_0 = 4 \text{ fm}^3$$

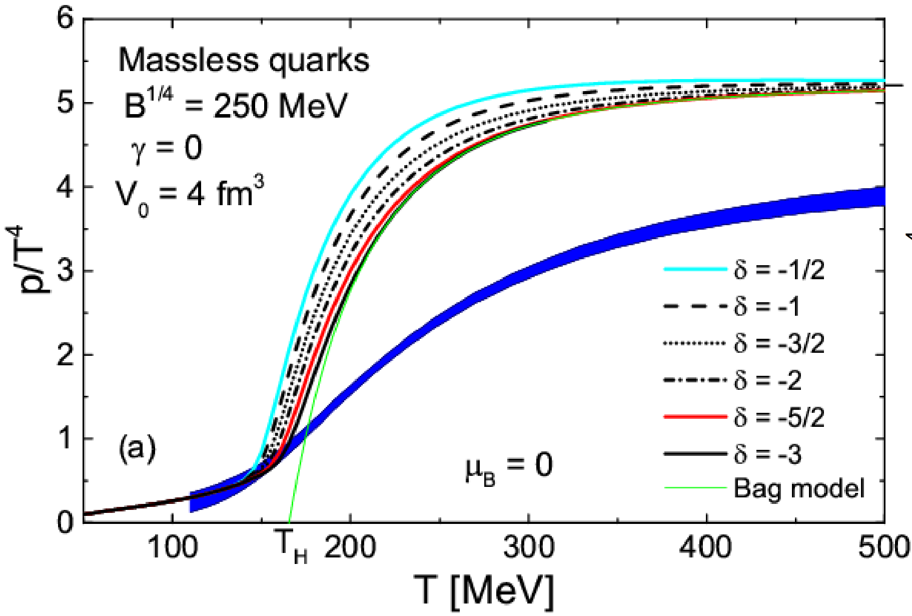
crossover

$$T_H = \left(\frac{3B}{\sigma_Q}\right)^{1/4} \simeq 165 \text{ MeV}$$

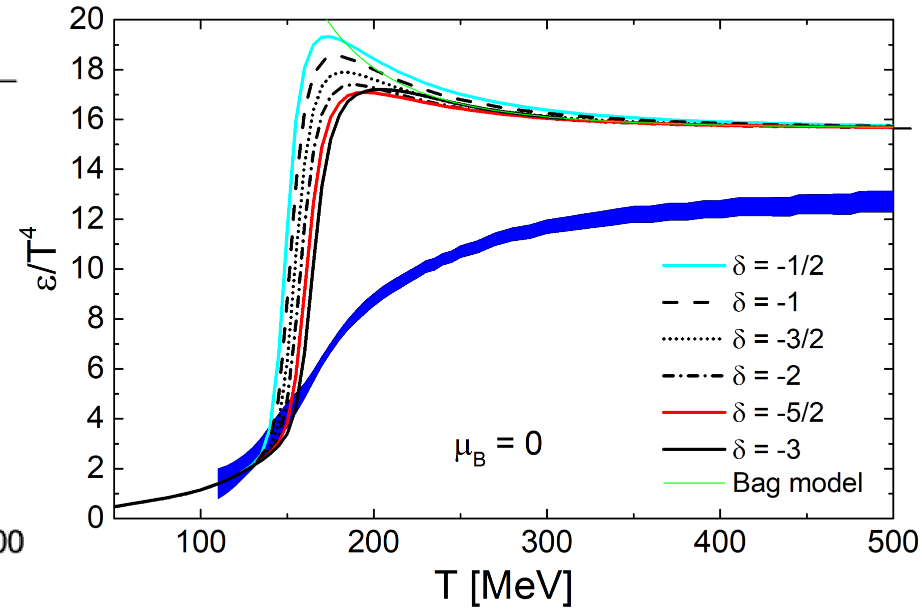
Hagedorn temperature

Thermodynamic functions

Pressure p/T^4



Energy density ε/T^4



Lattice data from 1309.5258 (Wuppertal-Budapest)

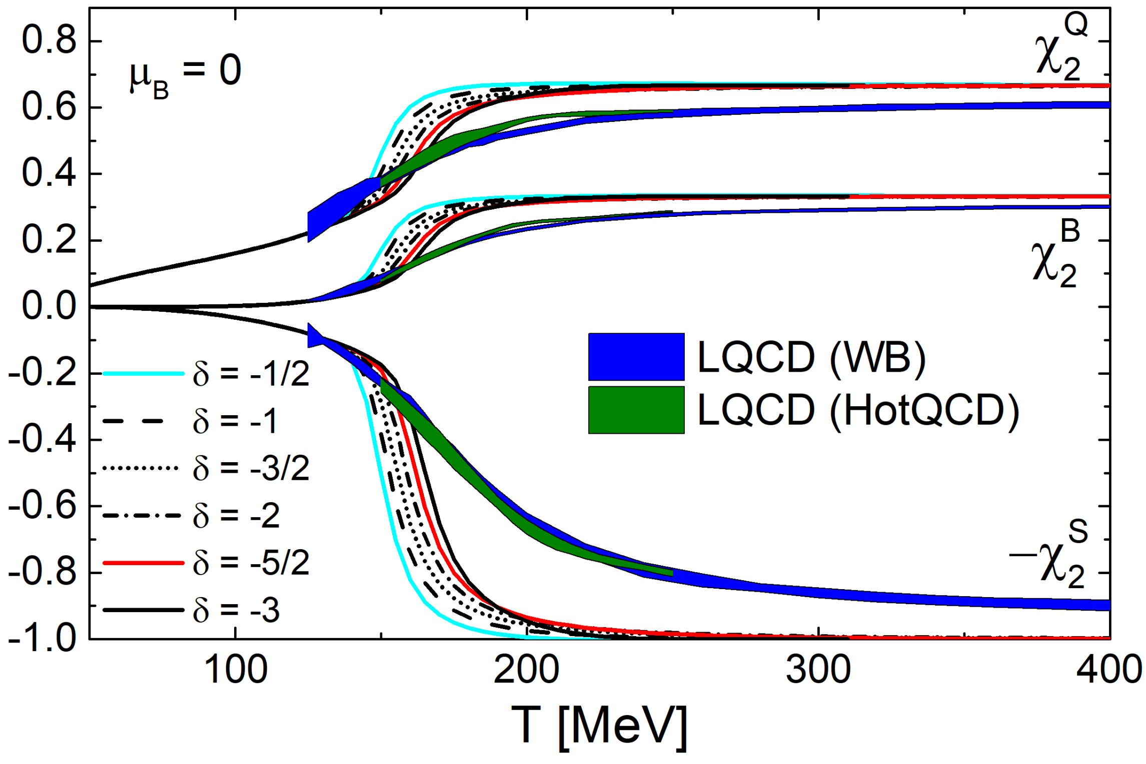
- Crossover transition towards bag model equation of state
- Dependence on δ is mild
- Approach to the Stefan-Boltzmann limit is too fast
- Peak in energy density, not seen on the lattice

Overall consistent with Ferroni-Koch results

Conserved charges susceptibilities

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l\,\partial(\mu_S/T)^m\,\partial(\mu_Q/T)^n}$$

Available from lattice QCD, not considered in this type of model before



Qualitatively compatible with lattice QCD

Bag model with massive quarks

The main source of quantitative disagreement is the inaccuracy of the standard MIT bag model with massless quarks for describing QGP

Quasiparticle models suggest sizable thermal masses of quarks and gluons in high-temperature QGP [Peshier et al., PLB '94; PRC '00; PRC '02]

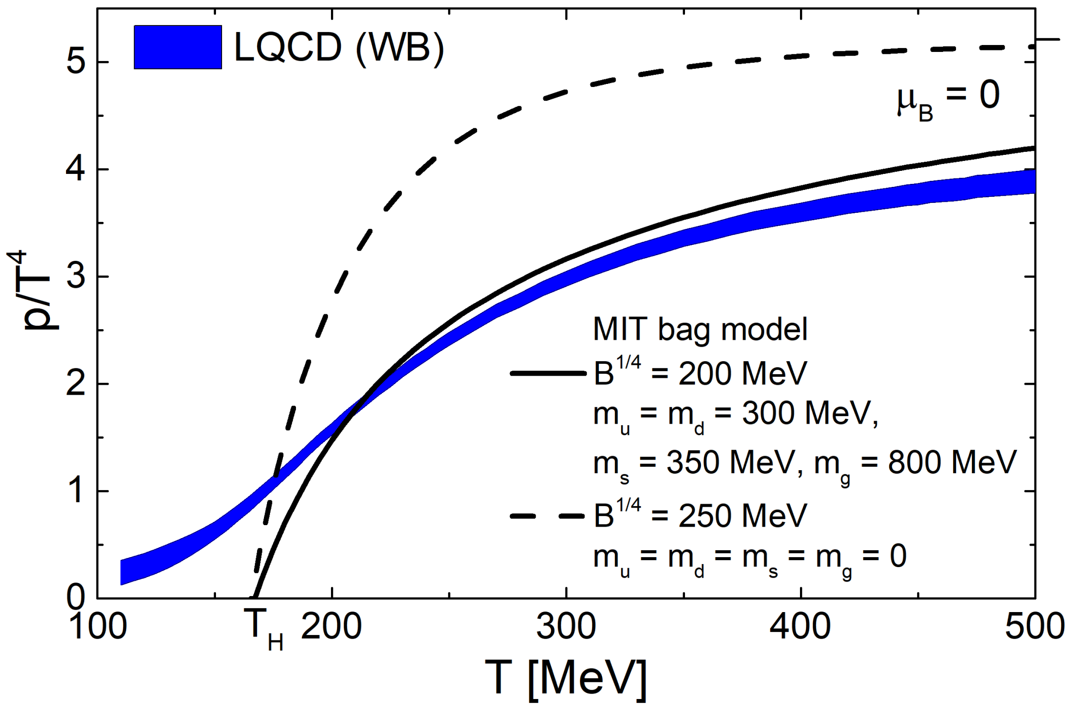
Heavy-bag model: bag model EoS with non-interacting *massive* quarks and gluons and the bag constant [Ivanov et al., PRC 72, 025804 (2005)]

Massive quarks and gluons instead of massless ones:

$$\sigma_Q(T,\lambda_B,\lambda_Q,\lambda_S) = \frac{8}{\pi^2 T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_g^2}}\left[\exp\left(\frac{\sqrt{k^2+m_g^2}}{T}\right)-1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2 T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f^{-1}\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$
$$+\sum_{f=u,d,s}\frac{3}{\pi^2 T^4}\int_0^\infty dk \frac{k^4}{\sqrt{k^2+m_f^2}}\left[\lambda_f\exp\left(\frac{\sqrt{k^2+m_f^2}}{T}\right)+1\right]^{-1}$$

Hagedorn model with massive quarks

Introduction of constituent masses leads to much better description of QGP



Parameters for the crossover model:

$$m_u = m_d = 300 \text{ MeV}, \quad m_s = 350 \text{ MeV}, \quad m_g = 800 \text{ MeV}, \quad B^{1/4} = 200 \text{ MeV}$$

$$\gamma = 0, \quad \delta = -2, \quad C = 0.03, \quad V_0 = 8 \text{ fm}^3$$

$T_H \simeq 167$ MeV

Hagedorn model with massive quarks

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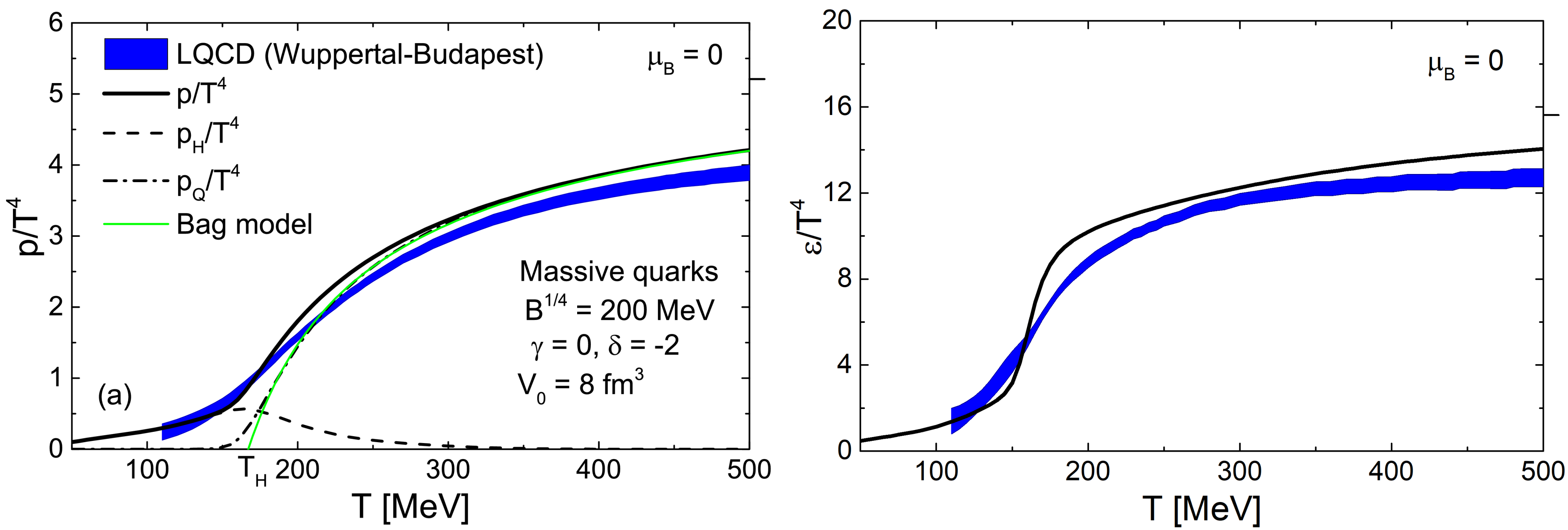

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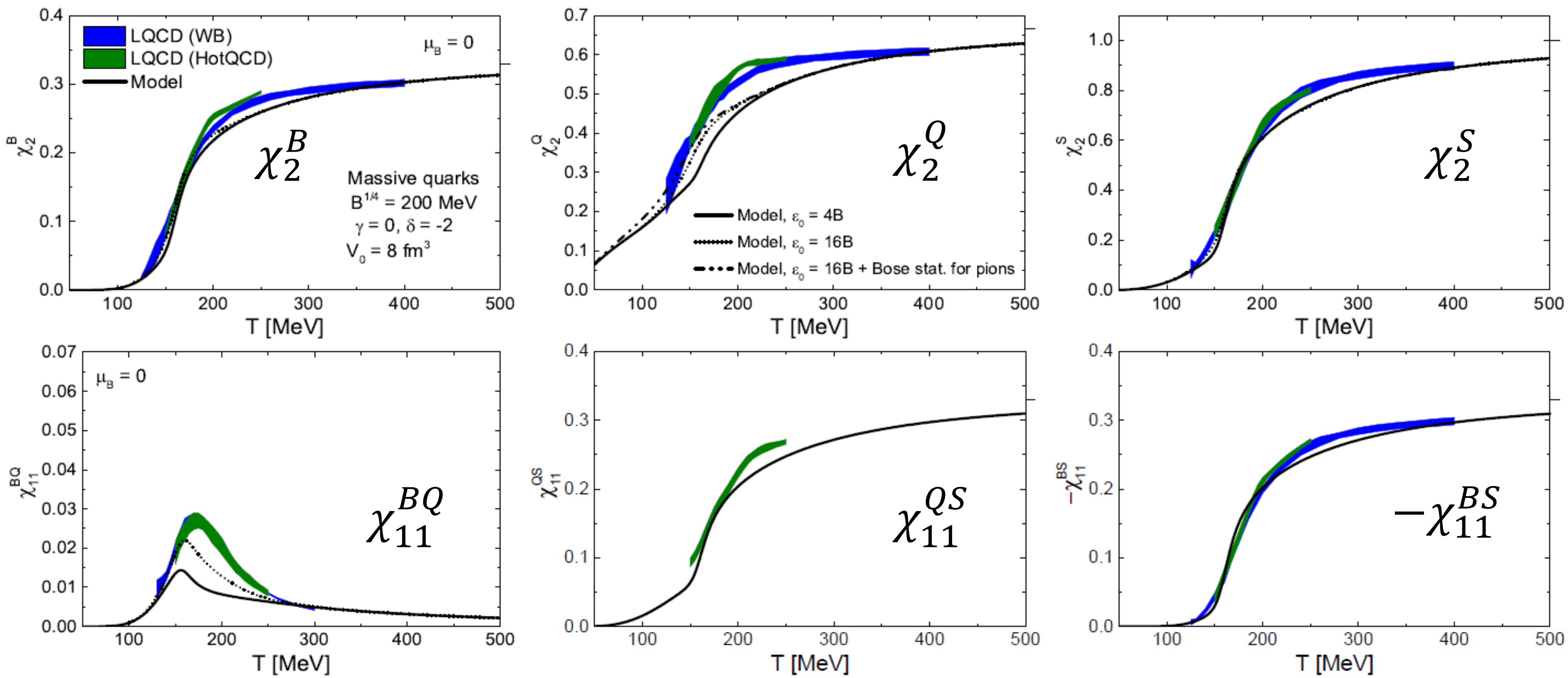
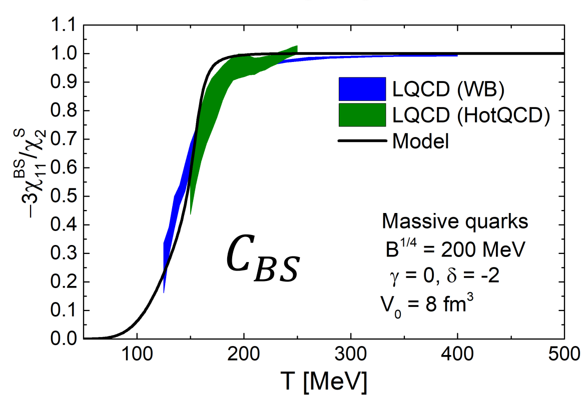
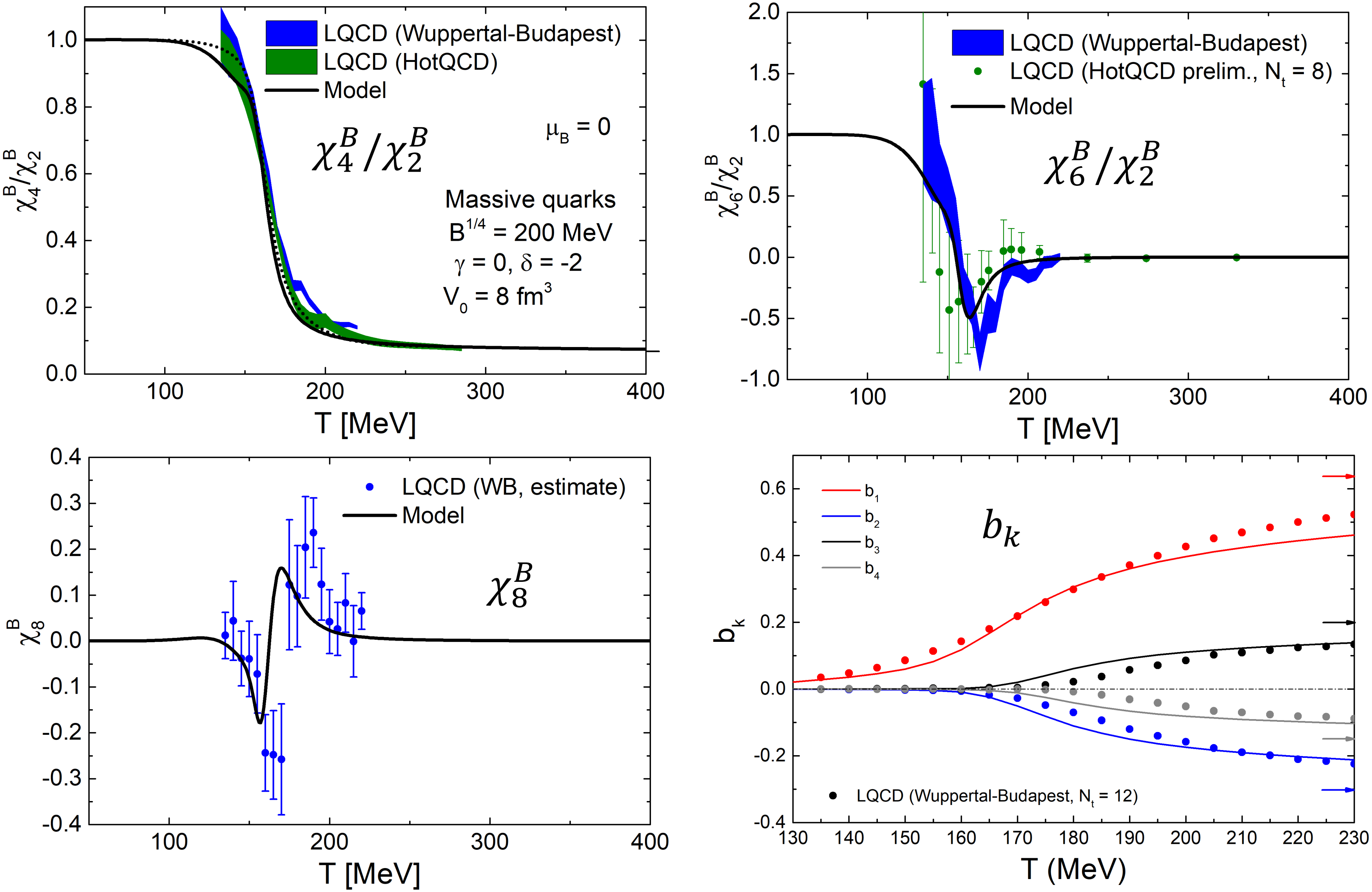
2^{nd} order susceptibilities





Lattice data from 1112.4416 (Wuppertal-Budapest), 1203.0784 (HotQCD)

Higher-order susceptibilities and Fourier coefs.

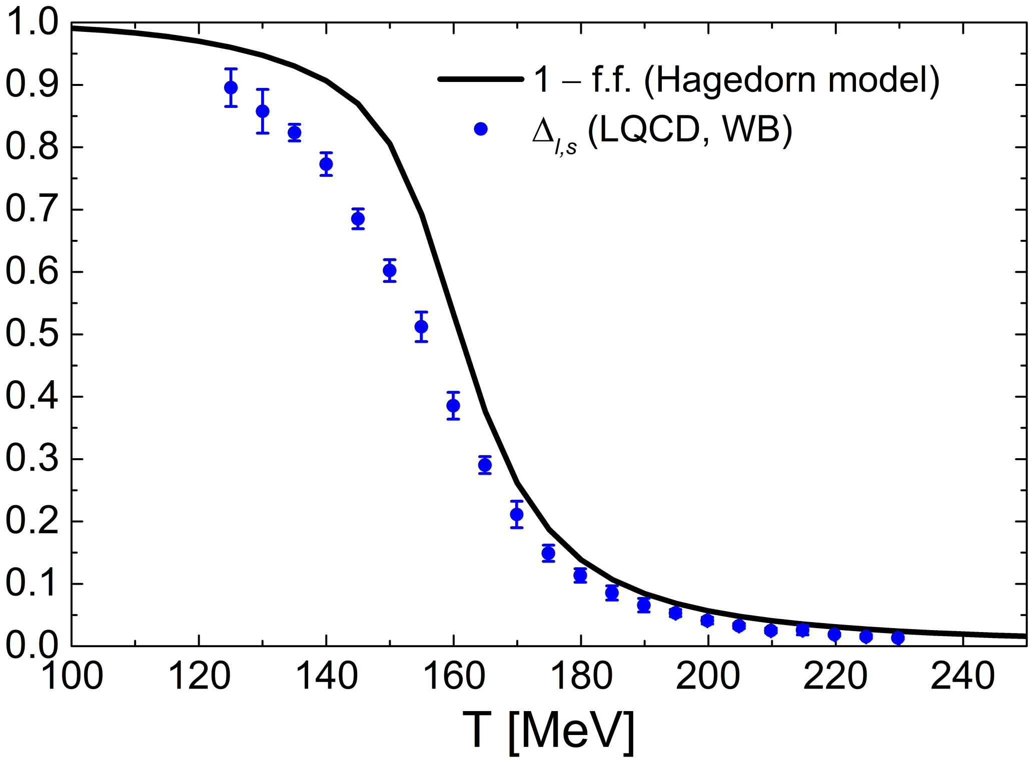


Lattice data from 1805.04445 & 1708.02852 (Wuppertal-Budapest), 1701.04325 (HotQCD)

Chiral transition

Picture: bag interior is chirally restored, vacuum is chirally broken

Proxy for the chiral condensate: $$\frac{<\psi\bar{\psi}>_{T\neq 0}}{<\psi\bar{\psi}>_{T=0}} \cong 1 - \frac{<V_{had}>}{V} = 1 - f.f.$$



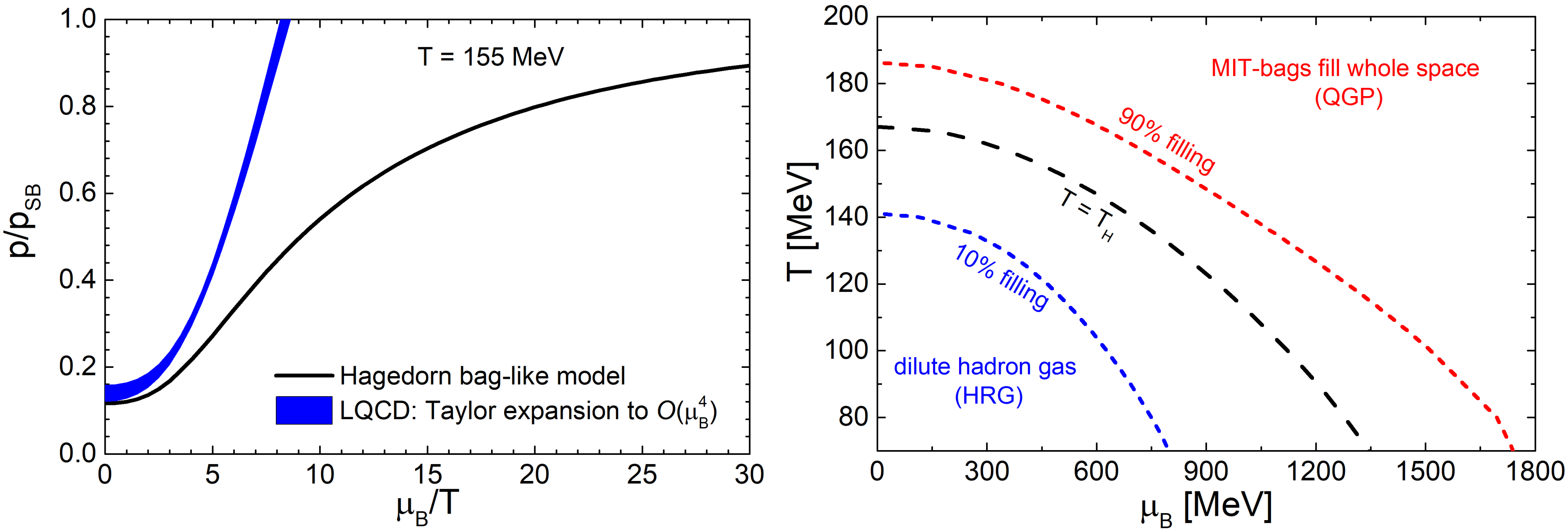
Lattice QCD:

$$\langle\bar{\psi}\psi\rangle_q = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

$$\Delta_{l,s} = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

Lattice data from 1005.3508 (Wuppertal-Budapest), see also 1111.1710 (HotQCD)

Finite baryon density and phase structure



- Crossover transition to a QGP-like phase in both the T and μ_B directions
- Essentially a built-in "switching" function between HRG and QGP, thermodynamically consistent by construction (single partition function)
- Three conserved charges: μ_B, μ_Q, μ_S

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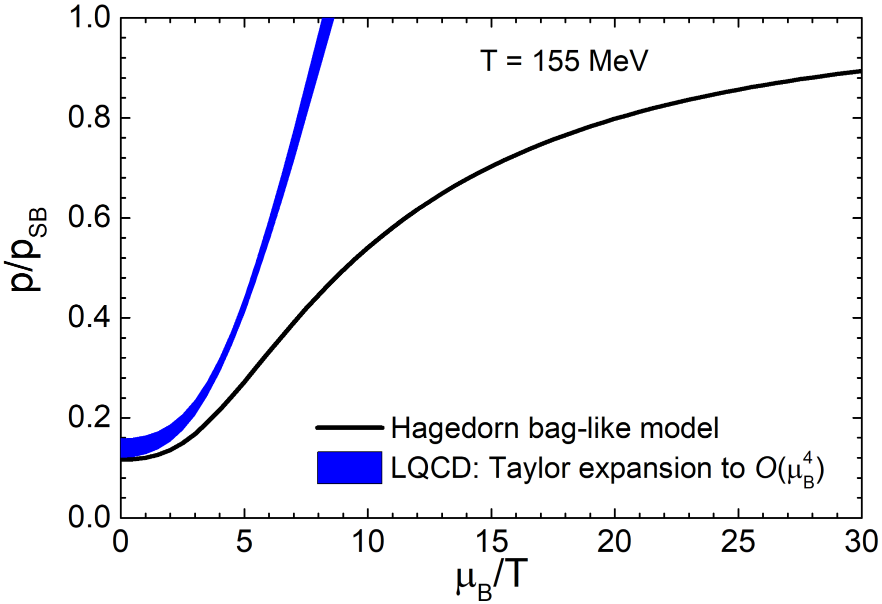

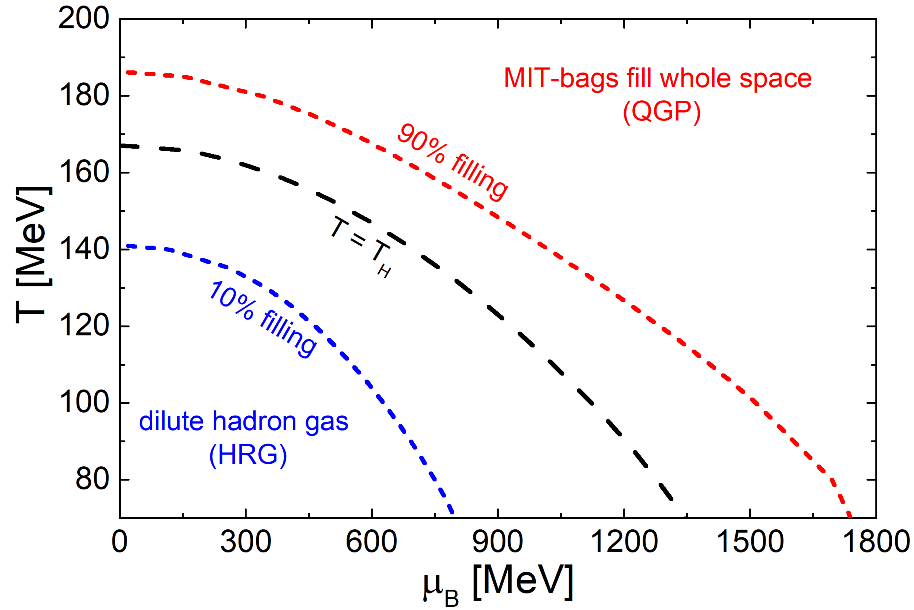

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- Essentially a built-in "switching" function between HRG and QGP, thermodynamically consistent by construction (single partition function)
- Three conserved charges: μ_B, μ_Q, μ_S

Outlook: Critical point/phase transition at finite μ_B can be incorporated through μ_B-dependence of γ and δ exponents in the bag spectrum, then predict signatures

as in Gorenstein, Gazdzicki, Greiner, Phys. Rev. C (2005)

Summary

- Hagedorn bag-like model with quasiparticle-type parton masses provides a reasonable description of hadron-QGP crossover within a single partition function. Includes three conserved charges and thus suitable for heavy-ion collisions.

- Inclusion of exponentially increasing Hagedorn states as well as excluded volume interactions are in line with various high order susceptibilities of lattice QCD

- Pure crossover scenario consistent with the present lattice data. Adjusting parameters for a hypothetical critical point at finite baryon density to predict its signatures.

Summary

- Hagedorn bag-like model with quasiparticle-type parton masses provides a reasonable description of hadron-QGP crossover within a single partition function. Includes three conserved charges and thus suitable for heavy-ion collisions.

- Inclusion of exponentially increasing Hagedorn states as well as excluded volume interactions are in line with various high order susceptibilities of lattice QCD

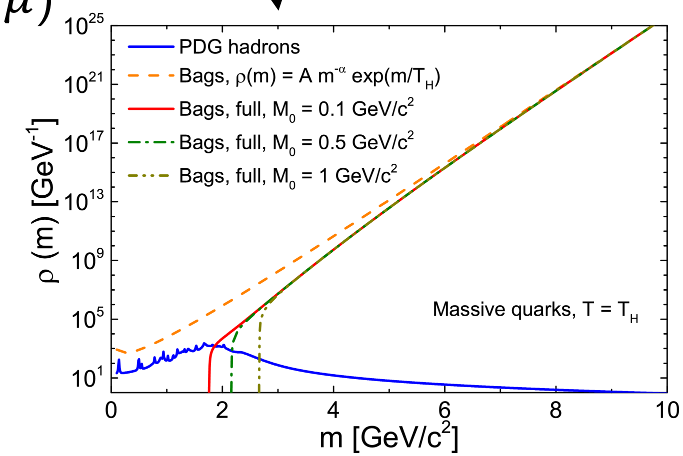
- Pure crossover scenario consistent with the present lattice data. Adjusting parameters for a hypothetical critical point at finite baryon density to predict its signatures.

Thanks for your attention!

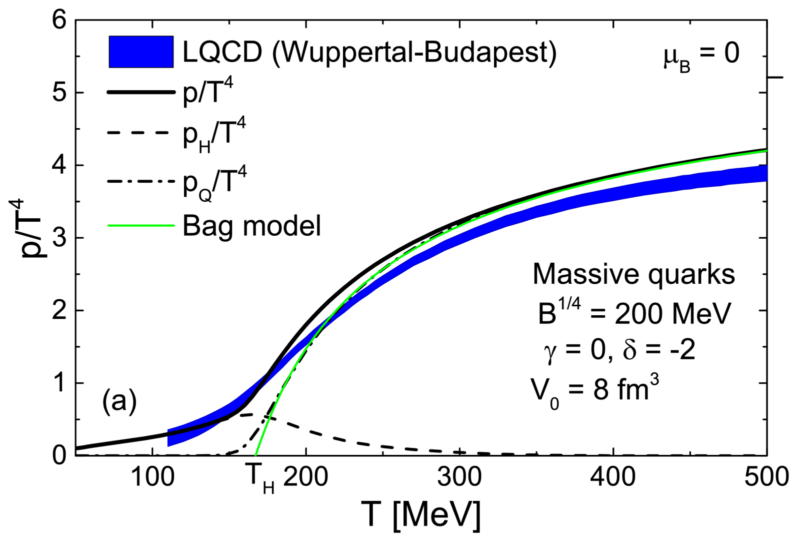
Backup slides

Hagedorn bag-like model: formulation

- HRG + quark-gluon bags $\rho_Q(m,v) = C\, v^{\gamma}\,(m - Bv)^{\delta}\, \exp\left\{\frac{4}{3}[\sigma_Q]^{1/4}\, v^{1/4}\,(m-Bv)^{3/4}\right\}$
- Non-overlapping particles (excluded volume correction) $V \to V - bN$
- Isobaric (pressure) ensemble $(T, V, \mu) \to (T, s, \mu)$
- *Massive* (thermal) partons (**new element**)



Resulting picture: transition (crossover, 1st order, 2nd order, etc.) between HRG and MIT bag model EoS, within single partition function



"Crossover" parameter set

$\gamma = 0, \quad \delta = -2, \quad C = 0.03, \quad V_0 = 8\ \text{fm}^3$
$m_u = m_d = 300\ \text{MeV}, \quad m_s = 350\ \text{MeV}$
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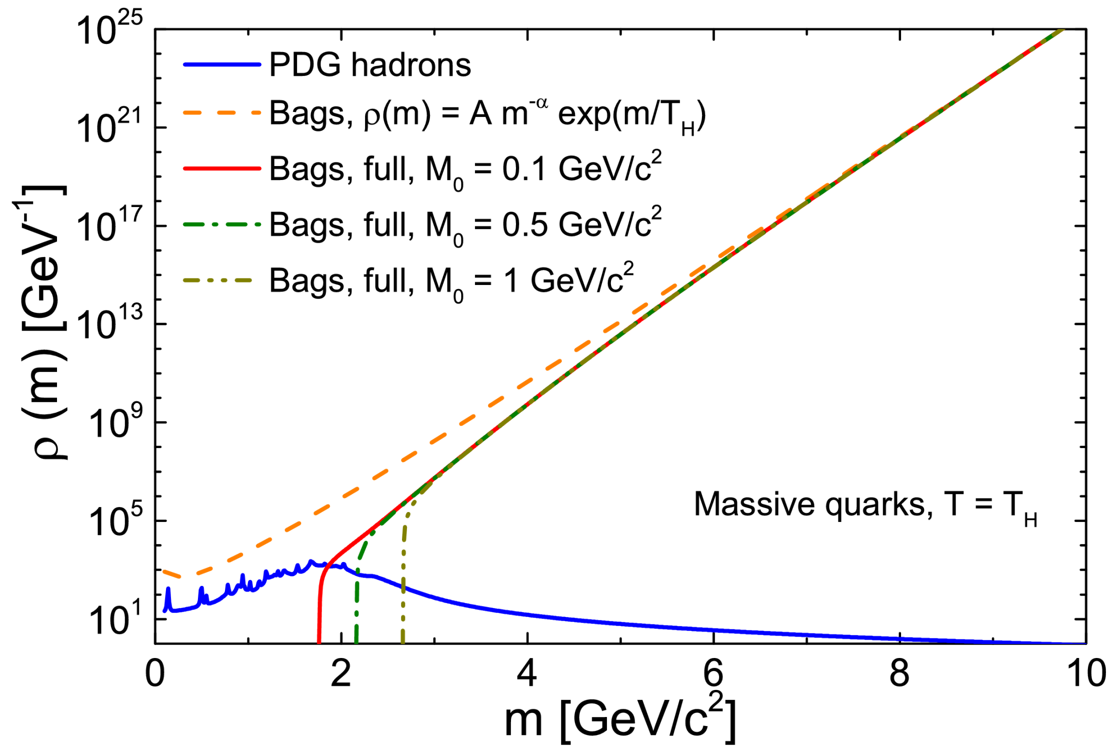
$T_H \simeq 167\ \text{MeV}$

Hagedorn bag-like model: mass spectrum



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$T_H \simeq 167$ MeV

Cluster expansion in fugacities

Expand in fugacity $\lambda_B = e^{\mu_B/T}$ instead of μ_B/T – a relativistic analogue of Mayer's cluster expansion:

$$\frac{p(T,\mu_B)}{T^4} = \frac{1}{2}\sum_{k=-\infty}^{\infty} p_{|k|}(T)\, e^{k\mu_B/T} = \frac{p_0(T)}{2} + \sum_{k=1}^{\infty} p_k(T)\cosh(k\mu_B/T)$$

Net baryon density: $$\frac{\rho_B(T,\mu_B)}{T^3} = \sum_{k=1}^{\infty} b_k(T)\sinh(k\mu_B/T), \quad b_k \equiv k p_k$$

Analytic continuation to imaginary μ_B yields trigonometric Fourier series

$$\frac{\rho_B(T,\mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i}\sum_{k=1}^{\infty} b_k(T)\,\sin\left(\frac{k\,\tilde{\mu}_B}{T}\right)$$

with Fourier coefficients $$b_k(T) = \frac{2}{\pi\, T^4}\int_0^{\pi T} d\tilde{\mu}_B\,[\mathrm{Im}\,\rho_B(T,i\tilde{\mu}_B)]\,\sin(k\,\tilde{\mu}_B/T)$$

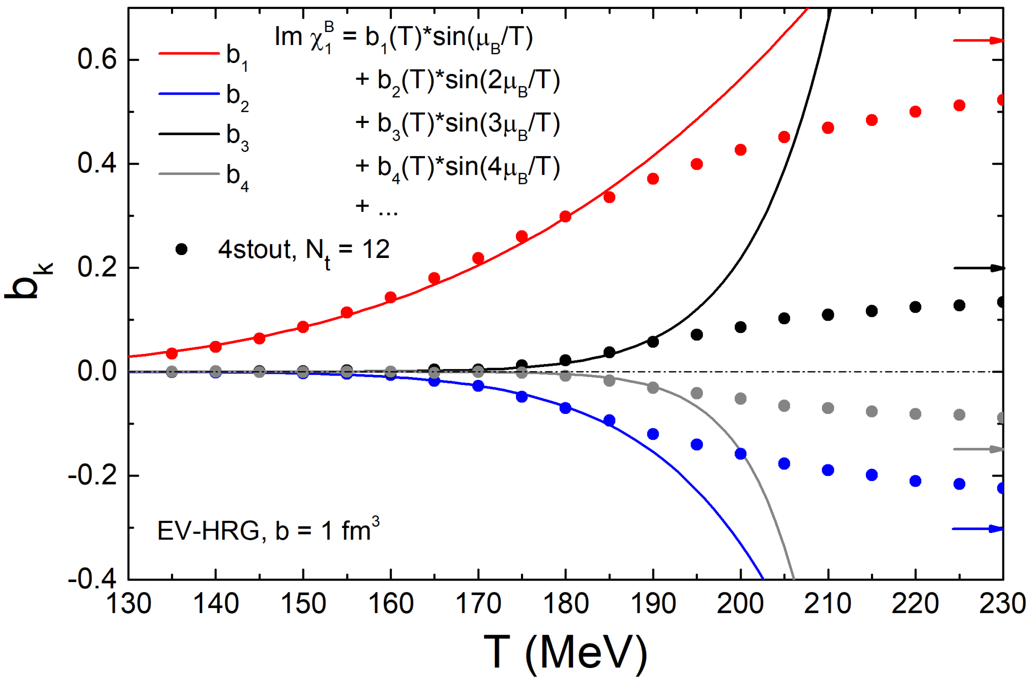
Four leading coefficients b_k computed in LQCD at the physical point

[**V.V.**, A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852]

HRG with repulsive baryonic interactions

Repulsive interactions with excluded volume (EV) $V \to V - bN$

[Hagedorn, Rafelski, '80; Dixit, Karsch, Satz, '81; Cleymans et al., '86; Rischke et al., Z. Phys. C '91]



HRG with baryonic EV:

$$p_B(T, \mu_B) = p_B^{\rm id}(T, \mu_B - b p_B)$$

$$b_k^{ev}(T) = (-1)^{k-1} \frac{2\,k^k}{k!} (b\,T^3)^{k-1} \left[\frac{\phi_B(T)}{T^3}\right]^k$$

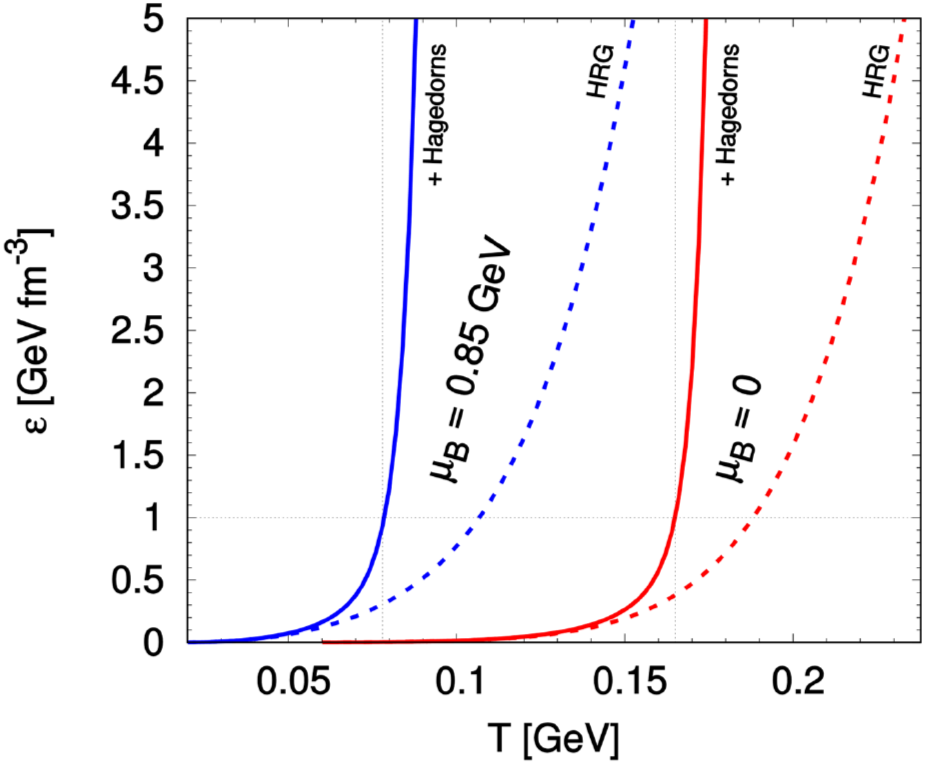
V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

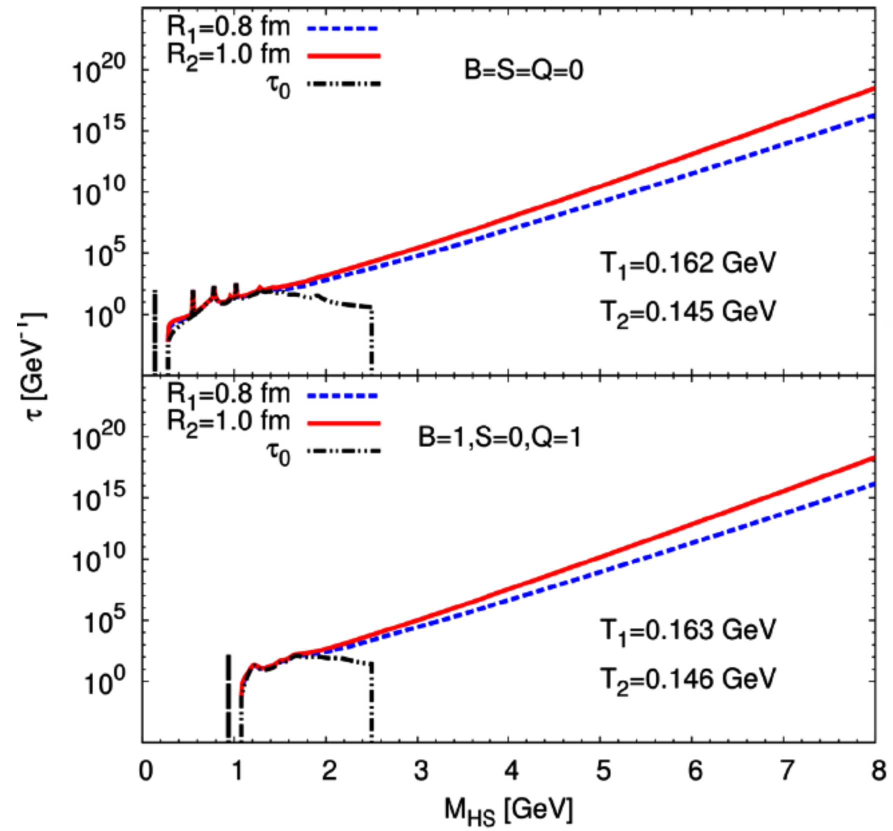
- Non-zero $b_k(T)$ for $k \geq 2$ signal deviation from ideal HRG
- EV interactions between baryons ($b \approx 1$ fm^3) reproduce lattice trend

Hagedorn resonance gas

HRG + exponential Hagedorn mass spectrum, e.g. as obtained from the bootstrap equation [Hagedorn ‘65; Frautschi, ‘71]

$$\rho(m) = A\, m^{-\alpha} \exp(m/T_H)$$

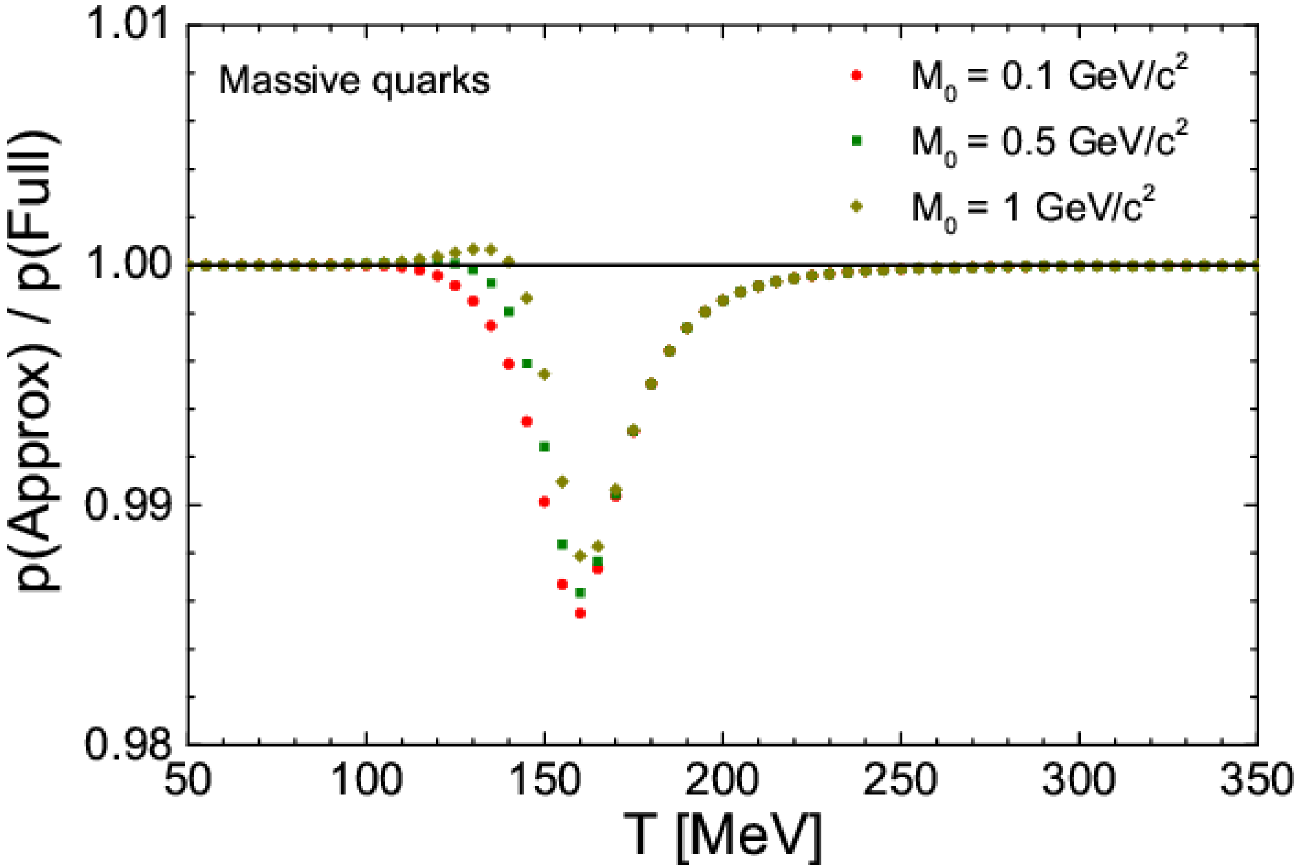




[Beitel, Gallmeister, Greiner, 1402.1458]

If Hagedorns are point-like, *T_H is the limiting temperature*

Accuracy of Laplace's method

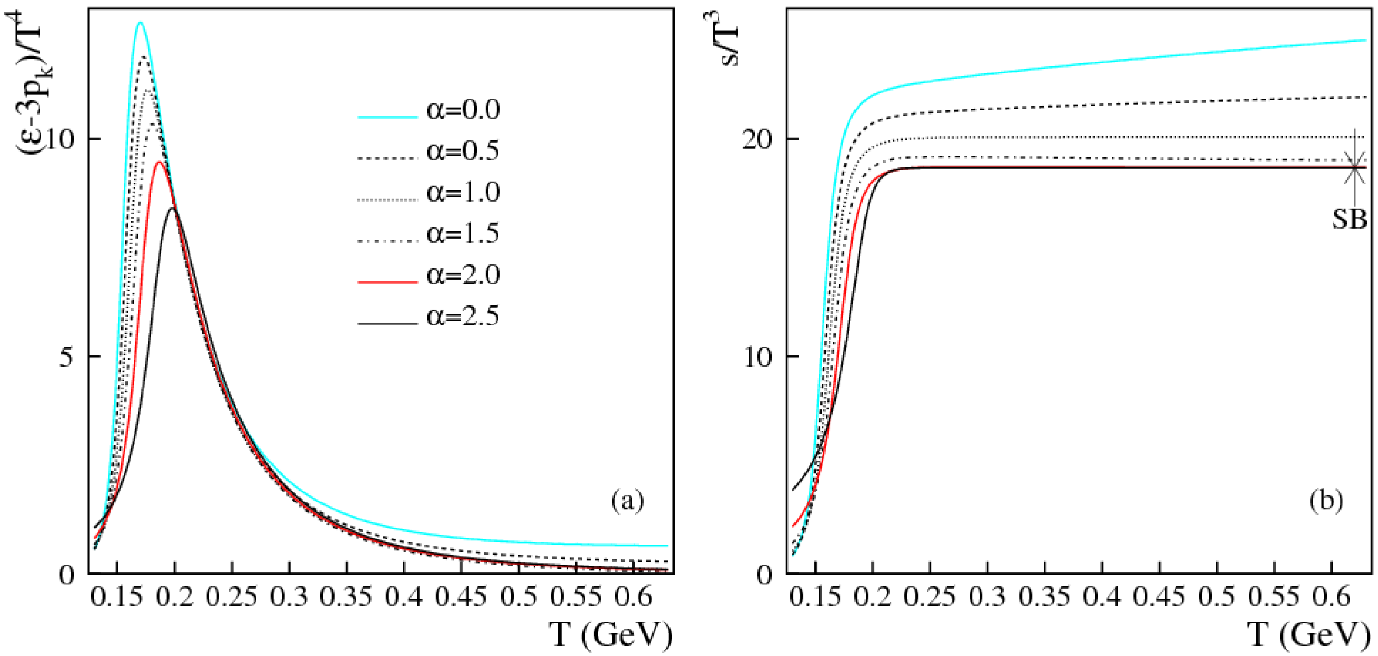


Laplace's method accurate within 1-2%, the value of M_0 is all but irrelevant

From limiting temperature to crossover

- A gas of extended objects → excluded volume
- Exponential spectrum of *compressible* QGP bags
- Both phases described by single partition function

[Gorenstein, Petrov, Zinovjev, PLB '81; Gorenstein, W. Greiner, Yang, JPG '98; I. Zakout et al., NPA '07]



[Ferroni, Koch, PRC 79, 034905 (2009)]

Crossover transition in bag-like model qualitatively compatible with LQCD

Mechanism for transition to QGP

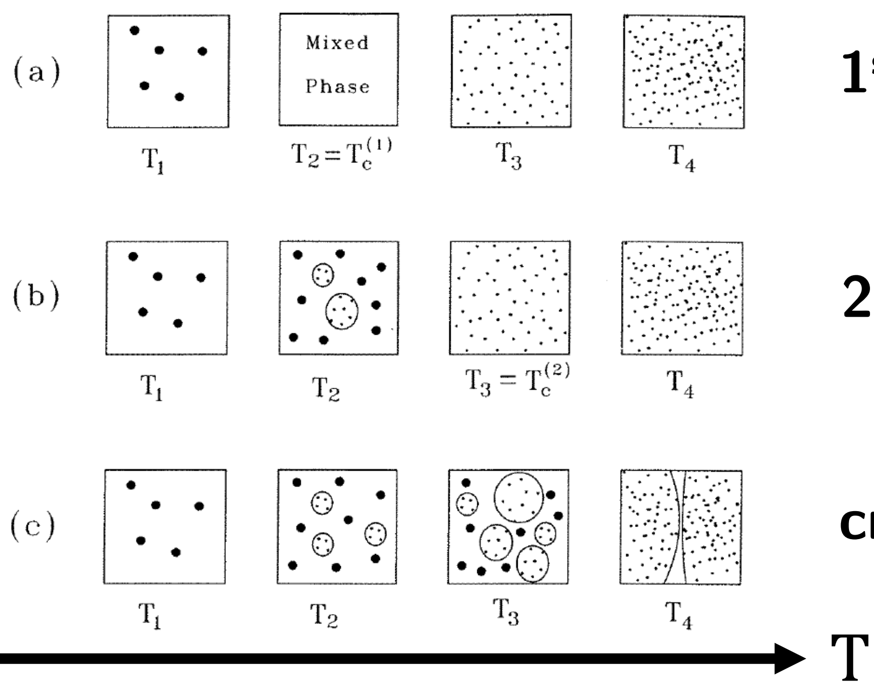
The isobaric partition function, $\hat{Z}(T,s,\lambda) = [s - f(T,s,\lambda)]^{-1}$, has

- pole singularity $s_H = f(T, s_H, \lambda)$ **"hadronic" phase**
- singularity s_B in the function $f(T,s,\lambda)$ due to the exponential spectrum

$$p_B = T\,s_B = \frac{\sigma_Q}{3} T^4 - B$$

MIT bag model EoS for QGP

[Chodos+, PRD '74; Baacke, APPB '77]



1^{st} order PT

"collision" of singularities

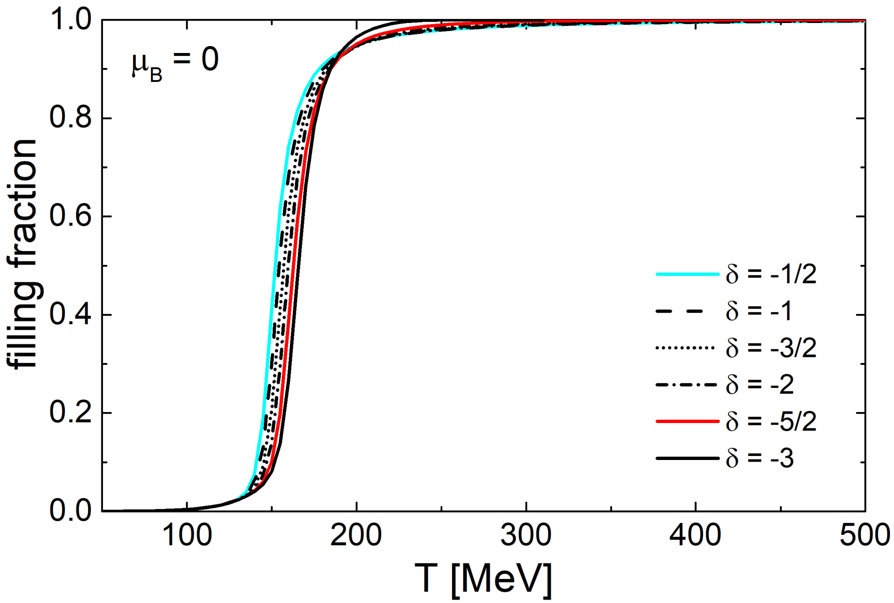
$$s_H(T_C) = s_B(T_C)$$

2^{nd} order PT

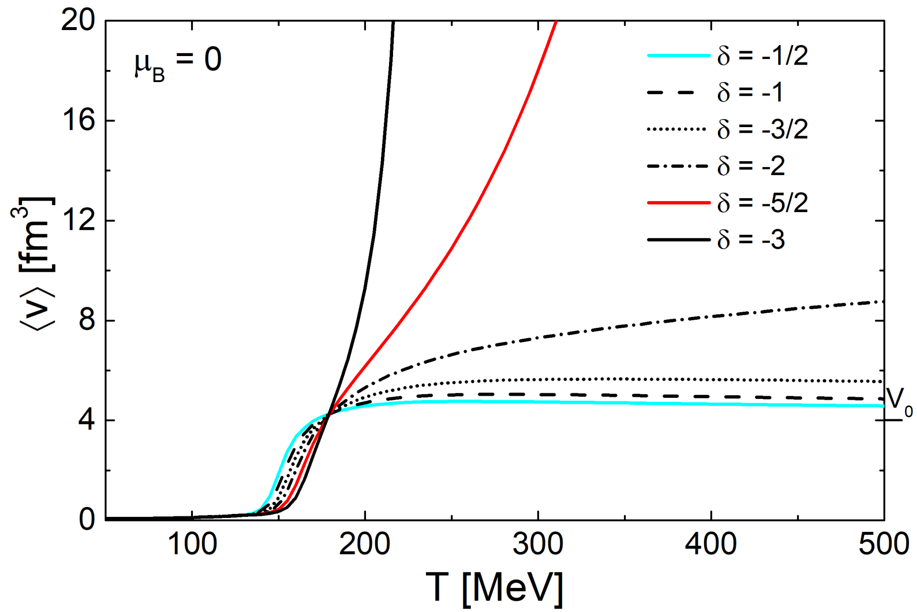
crossover $s_H(T) > s_B(T)$ at all T

Nature of the transition

Filling fraction $= \frac{<V_{had}>}{V}$



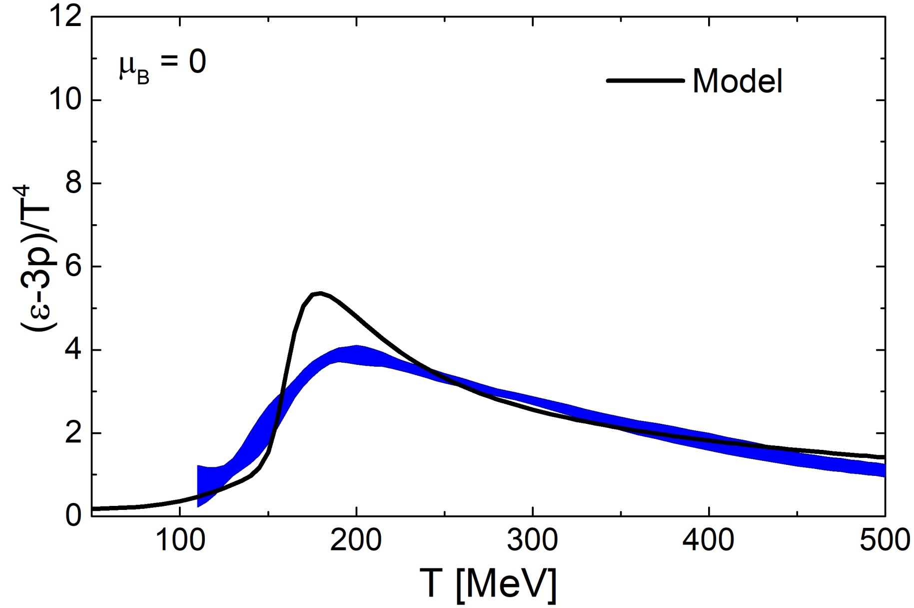
Mean hadron volume $< v >$



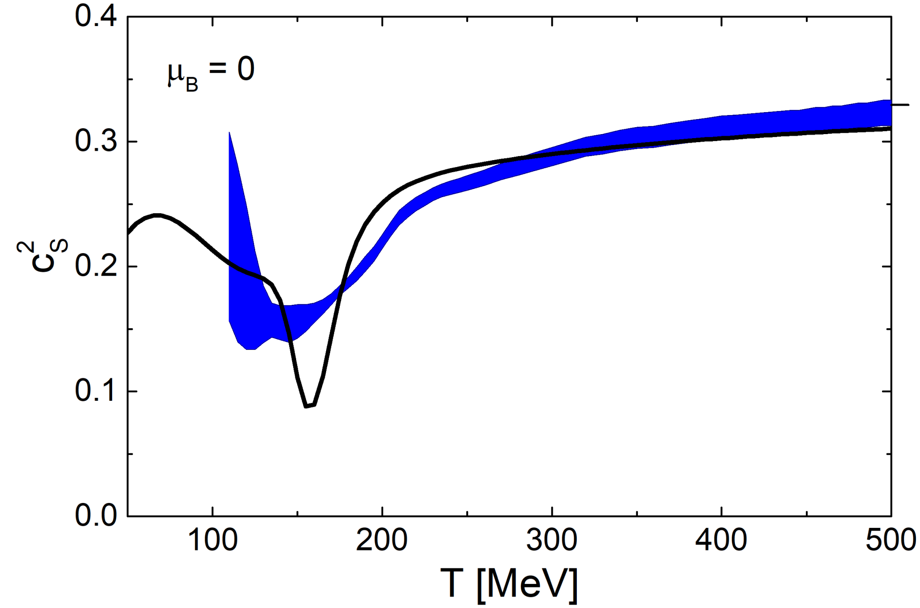
- Bags occupy almost whole space at large temperatures
- Strongest changes take place in the vicinity of T_H
- At $\delta < -7/4$ and $T \to \infty$ whole space — large bags with QGP

Hagedorn model: Thermodynamic functions

Trace anomaly $(\varepsilon - 3p)/T^4$



Speed of sound $c_s^2 = dp/d\varepsilon$

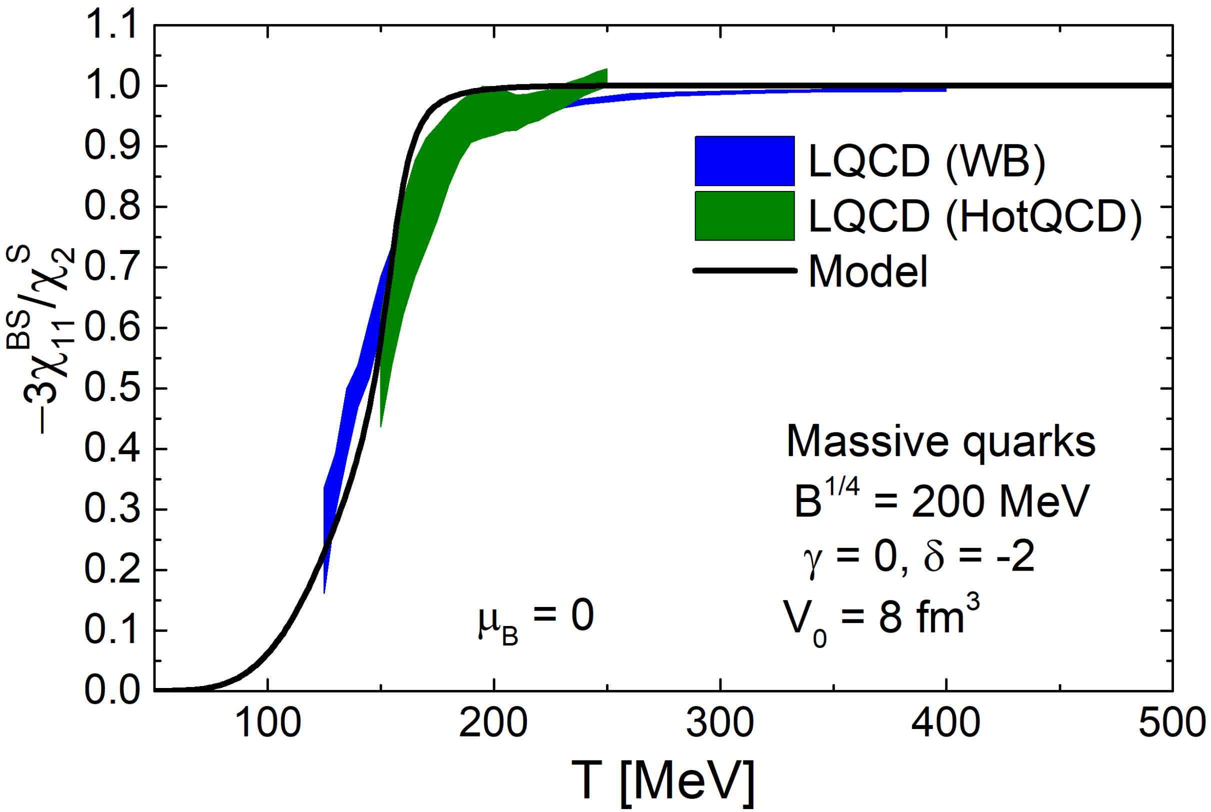


Hagedorn model: Baryon-strangeness ratio

$$C_{BS} = -\frac{3\chi_{11}^{BS}}{\chi_2^S}$$

Useful diagnostic of QCD matter

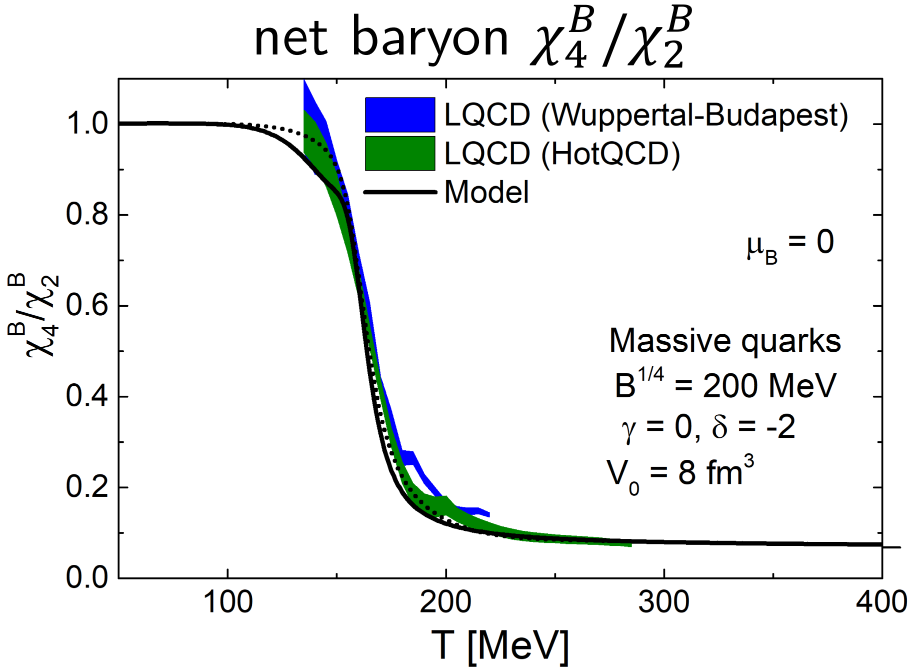
[V. Koch, Majumder, Randrup, PRL 95, 182301 (2005)]

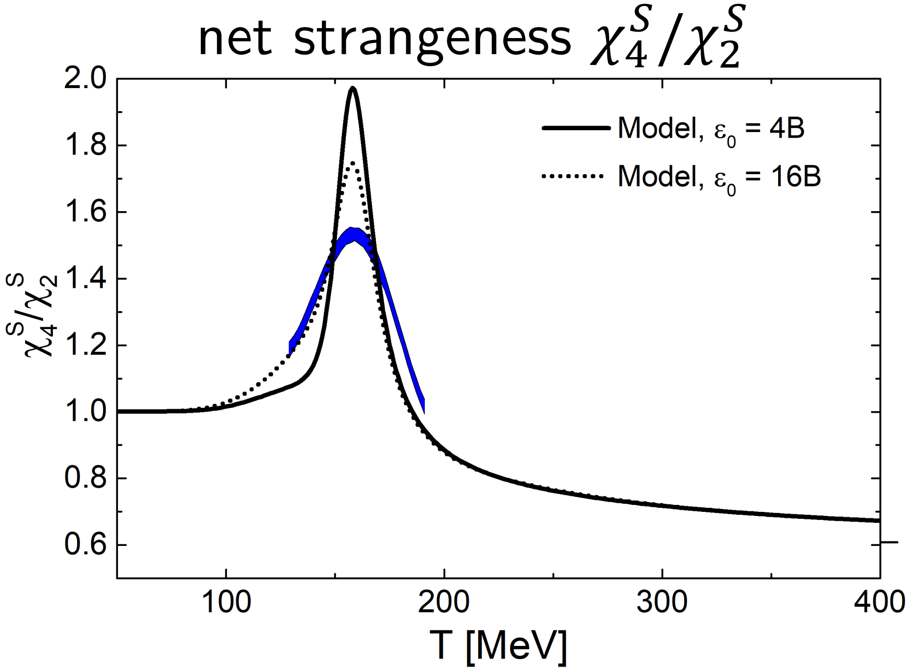


Consistent with lattice QCD

Hagedorn model: Higher-order susceptibilities

Higher-order susceptibilities are particularly sensitive probes of the parton-hadron transition and possible remnants of criticality at $\mu_B = 0$





Lattice data from 1305.6297 & 1805.04445 (Wuppertal-Budapest), 1708.04897 (HotQCD)

- Drop of χ_4^B/χ_2^B caused by repulsive interactions which ensure crossover transition to QGP
- Peak in χ_4^S/χ_2^S is an interplay of the presence of multi-strange hyperons and repulsive interactions