THE SPATIAL SUB-SEPARATION OF STRANGENESS FROM ANTI-STRANGENESS IN RHICs

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Outline

I. Motivation

II. Thermalization in relativistic heavy-ion collisions. Statistical model of ideal hadron gas

III. Strangeness in the central cell

IV. Freeze-out (dN/dt) of main hadron species

V. Spatial separation of S from anti-S:
   - in Thermal model predictions of Yields
   - directed flow for different particles
   - in lambda-antilambda polarization

VI. Conclusions
To do the analysis of the spatio-temporal evolution of all particles in the $T - \mu_B$, $T - \mu_S$ plane and the analysis of the finally emitted particles in $x - t$ plane.

See the spatial separation of strange particles from non strange (and of mesons from baryons).

Find average $T, \mu_B, \mu_S$ of different particles at freeze-out time.
Identified hadron yields

- Lots of particles, most newly created from the excited gluon fields ($E=mc^2$)
- Large variety of species:
  - $\pi^\pm(u\bar{d},d\bar{u})$, $m=140$ MeV
  - $K^\pm(u\bar{s},s\bar{u})$, $m=494$ MeV
  - $p(uud)$, $m=938$ MeV
  - $\Lambda(uds)$, $m=1116$ MeV
  - also: $\Xi(dss)$, $\Omega(sss)$, ...
- Abundancies follow mass hierarchy, except at low energies where remnants from the incoming nuclei are significant
- What do we learn?
Grand Canonical Ensemble

\[ \ln Z_i = \frac{Vg_i}{2\pi^2} \int_0^\infty dp \ln \left( 1 \pm \exp\left( \frac{-(E_i - \mu_i)}{T} \right) \right) \]

\[ n_i = \frac{N}{V} = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp((E_i - \mu_i)/T) \pm 1} \]

\[ \mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I^3_i \]

for every conserved quantum number there is a chemical potential \( \mu \) but can use conservation laws to constrain:

- Baryon number: \( V \sum_i n_i B_i = Z + N \rightarrow V \)
- Strangeness: \( V \sum_i n_i S_i = 0 \rightarrow \mu_S \)
- Charge: \( V \sum_i n_i I^3_i = \frac{Z - N}{2} \rightarrow \mu_{I_3} \)

This leaves only \( \mu_{b} \) and \( T \) as free parameter when \( 4\pi \) considered for rapidity slice fix volume e.g. by \( dN_{ch}/dy \)
Chemical freeze-out

- Thermal fits of hadron abundancies:

\[ n_i = N_i/V = \frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T]} \pm 1 \]

- Quantum numbers conservation

\[ \mu = \mu_B B + \mu_3 l_3 + \mu_S S + \mu_C C \]

- Hadron yields \( N_i \) can be obtained using only 3 parameters: \((T_{\text{chem}}, \mu_B, V)\)

- The hadron abundancies are in agreement with a thermally equilibrated system

\[ T_{\text{chem}} = 155-165 \text{ MeV} \]

\[ \mu_B \approx 0 \]
Central cell: Relaxation to (local) equilibrium
Equilibration in the Central Cell

Kinetic equilibrium:
Isotropy of velocity distributions
Isotropy of pressure

Thermal equilibrium:
Energy spectra of particles are described by Boltzmann distribution

Chemical equilibrium:
Particle yields are reproduced by SM with the same values of 

\[ N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp \left( \frac{\mu_i}{T} \right) \exp \left( -\frac{E_i}{T} \right) \]
Statistical model of ideal hadron gas

\[
\begin{align*}
\varepsilon^{\text{mic}} &= \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S), \\
\rho_B^{\text{mic}} &= \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S), \\
\rho_S^{\text{mic}} &= \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S).
\end{align*}
\]

Multiplicity

Energy

Pressure

Entropy density
**Kinetic Equilibrium**

Isotropy of pressure

Pressure becomes isotropic for all energies from 11.6 AGeV to 158 AGeV

L. Bravina et al., PRC 78 (2008) 014907
Net strangeness density in the central cell at 11 to 80 AGeV

Net strangeness in the cell is negative because of different interaction cross sections for Kaons and antiKaons with Baryons.
Thermal and chemical equilibrium seems to be reached

Particle yields

(a) UrQMD  $t = 13 \text{ fm/c}$

(b) QGSM  $t = 10 \text{ fm/c}$

Thermal and chemical equilibrium seems to be reached
How dense can be the medium?

Dramatic differences at the non-equilibrium stage; after beginning of kinetic equilibrium the energy densities and the baryon densities are the same for “small” and “big” cell.
COMPARISON BETWEEN MODELS

The phase trajectories at the center of a head-on Au+Au collisions

Green area : freeze-out region;
Yellow area : the phase coexistence region from schematic EOS that has a critical point at final density

Different models exhibit a large degree of mutual agreement

I. Arsene et al., PRC 75 (2007) 034902
Infinite hadron gas: a box with periodic boundary conditions
Initialization: (i) nucleons are uniformly distributed in a configuration space; (ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Model employed: UrQMD 55 different baryon species (N, Δ, hyperons and their resonances with $m \leq 2.25 \text{ GeV}/c^2$), 32 different meson species (including resonances with $m \leq 2 \text{ GeV}/c^2$) and their respective antistates. For higher mass excitations a string mechanism is invoked.

Test for equilibrium: particle yields and energy spectra
Saturation of yields after a certain time. Strange hadrons are saturated longer than others.
Nearly the same temperature and complete isotropy of $dN/dp_T$.
A rapid rise of $T$ at low $\varepsilon$ and saturation at high energy densities. Saturation temperature depends on number of resonances in the model. W/o strings and many-N decays – no limiting $T$ is observed.
Freeze-out of main hadron species
Figure 1: Particle densities in the central cell at times 1 – 20 fm/c.
Different particles are frozen at different space times with different values of $T-\mu_B-\mu_S$.

Figure 3: $dN/dt_{fr}$ for all space and for central cell.
Figure 4: $d^2N/dtdz$ for protons, lambdas, pions and kaons.
Figure 5: \(dN/dz\) for protons, lambdas, sigmas, pions and kaons for all rapidities (top figure) and for \(|y| < 1\) (bottom figure). One can see that there are much more particles with large \(z\) for all rapidities, than for \(|y| < 1\).
|        | All y | |        | All y |
|--------|-------| |        |       |
| $t$, fm/c | $|x|$, fm | $|y|$, fm | $|z|$, fm | $T$, MeV | $\mu_b$, MeV | $\mu_s$, MeV | $t$, fm/c | $|x|$, fm | $|y|$, fm | $|z|$, fm | $T$, MeV | $\mu_b$, MeV | $\mu_s$, MeV |
| All    | 18.0  | 4.7  | 6.4  | 112.4 | 473.1  | 72.1  | 18.1  | 4.9  | 5.3  | 110.6 | 492.3  | 70.8  |
| $p$    | 19.7  | 4.7  | 7.2  | 108.6 | 478.1  | 63.0  | 19.6  | 4.9  | 5.7  | 101.9 | 524.5  | 72.5  |
| $\bar{p}$ | 19.1  | 5.9  | 7.8  | 109.0 | 459.1  | 64.5  | 18.3  | 6.4  | 5.8  | 106.1 | 462.1  | 66.6  |
| $\Lambda$ | 24.6  | 5.5  | 8.1  | 90.4  | 539.8  | 50.4  | 24.4  | 5.7  | 7.1  | 92.2  | 532.3  | 49.4  |
| $\bar{\Lambda}$ | 23.3  | 6.6  | 8.6  | 98.2  | 487.0  | 58.0  | 22.7  | 6.8  | 7.2  | 96.4  | 497.4  | 54.1  |
| $\Sigma$ | 20.4  | 4.7  | 6.4  | 105.0 | 496.4  | 56.8  | 20.3  | 4.8  | 5.7  | 101.9 | 524.5  | 72.5  |
| $\bar{\Sigma}$ | 20.0  | 5.5  | 7.5  | 106.3 | 472.7  | 62.3  | 19.5  | 5.7  | 6.4  | 104.0 | 489.4  | 62.4  |
| $\pi$  | 16.9  | 4.7  | 6.1  | 116.8 | 448.5  | 69.0  | 17.0  | 4.9  | 5.1  | 114.6 | 471.2  | 73.4  |
| $K$    | 14.4  | 3.7  | 4.4  | 128.1 | 457.4  | 83.5  | 14.4  | 3.9  | 3.8  | 124.8 | 486.1  | 93.8  |
| $\bar{K}$ | 20.9  | 5.3  | 7.1  | 102.9 | 486.2  | 59.9  | 20.8  | 5.5  | 6.1  | 101.0 | 500.6  | 64.8  |

Table 1: Average coordinates of freezeout and $T$, $\mu_b$, $\mu_s$ at this coordinates.
|                  | All $y$          | $|y| < 1$             |
|-----------------|-----------------|----------------------|
| $t$, fm/c       | $|x|$, fm       | $|y|$, fm | $|z|$, fm | $T$, MeV | $\mu_b$, MeV | $\mu_s$, MeV | $t$, fm/c | $|x|$, fm | $|y|$, fm | $|z|$, fm | $T$, MeV | $\mu_b$, MeV | $\mu_s$, MeV |
| All             | 18.2            | 4.8                 | 8.4       | 120.8    | 396.0           | 57.9          | 17.7       | 5.2        | 6.3       | 112.0    | 419.9 | 55.2        |
| $p$             | 21.0            | 4.9                 | 10.0      | 113.1    | 406.5           | 51.0          | 19.9       | 5.2        | 6.9       | 105.2    | 447.6 | 47.2        |
| $\bar{p}$       | 20.0            | 6.4                 | 9.5       | 110.3    | 390.0           | 51.1          | 18.2       | 7.0        | 6.4       | 110.2    | 406.8 | 52.9        |
| $\Lambda$       | 26.0            | 5.9                 | 11.2      | 93.9     | 481.9           | 34.0          | 25.0       | 6.1        | 8.6       | 90.7     | 488.4 | 48.5        |
| $\bar{\Lambda}$ | 25.0            | 7.1                 | 11.5      | 98.7     | 435.9           | 50.8          | 23.7       | 7.5        | 8.7       | 95.5     | 463.2 | 41.4        |
| $\Sigma$        | 21.3            | 4.9                 | 9.0       | 106.4    | 444.0           | 51.8          | 20.7       | 5.1        | 7.0       | 103.8    | 444.9 | 49.0        |
| $\bar{\Sigma}$  | 21.1            | 6.2                 | 9.4       | 107.1    | 409.4           | 45.5          | 20.1       | 6.6        | 7.3       | 106.0    | 429.6 | 43.1        |
| $\pi$           | 16.9            | 4.8                 | 7.8       | 121.6    | 394.8           | 66.1          | 16.6       | 5.1        | 6.0       | 115.6    | 397.7 | 55.0        |
| $K$              | 15.1            | 4.0                 | 6.3       | 131.8    | 374.8           | 68.0          | 14.8       | 4.2        | 4.9       | 120.6    | 416.2 | 59.6        |
| $\bar{K}$       | 20.3            | 5.3                 | 8.8       | 110.5    | 419.1           | 44.7          | 19.7       | 5.6        | 6.8       | 105.2    | 447.6 | 47.2        |

Table 2: Average coordinates of freezeout and $T$, $\mu_b$, $\mu_s$ at this coordinates.
Table 3: Average coordinates of freezeout and $T$, $\mu_b$, $\mu_s$ at this coordinates.
Au+Au, $E_{lab}=40A$ GeV, $b = 0$ fm, all space

|            | All $y$ |            | $|y| < 1$ |
|------------|---------|------------|------------|
|            | $t$,   | $|x|, |y|, |z|, |T|, $\mu_b$, $\mu_s$, | $t$,   | $|x|, |y|, |z|, |T|, $\mu_b$, $\mu_s$, |
|            | fm/c   | fm, fm, fm, MeV, MeV | fm/c   | fm, fm, fm, MeV, MeV |
| All        | 19.1   | 5.0, 10.6, 122.7, 321.6, 41.3 | 17.7   | 5.4, 7.2, 116.5, 358.8, 41.6 |
| $p$        | 23.4   | 5.2, 13.6, 115.8, 343.2, 37.8 | 20.7   | 5.5, 7.9, 105.2, 403.5, 37.8 |
| $\bar{p}$ | 21.5   | 6.7, 11.5, 114.9, 340.4, 32.7 | 18.8   | 7.2, 7.0, 109.0, 351.8, 33.6 |
| $\Lambda$ | 28.3   | 6.2, 14.9, 96.2, 423.3, 20.2 | 25.9   | 6.5, 10.0, 91.0, 459.3, 33.3 |
| $\bar{\Lambda}$ | 27.0   | 7.4, 14.0, 96.8, 413.7, 30.7 | 24.5   | 7.9, 9.4, 95.4, 423.0, 31.8 |
| $\Sigma$  | 23.1   | 5.2, 12.0, 107.8, 391.3, 25.4 | 21.6   | 5.5, 8.2, 102.9, 416.3, 37.2 |
| $\bar{\Sigma}$ | 22.7   | 6.4, 11.8, 107.8, 380.0, 28.7 | 20.7   | 6.9, 7.9, 103.6, 402.3, 42.6 |
| $\pi$     | 17.8   | 5.0, 9.8, 125.6, 323.4, 39.1 | 16.7   | 5.4, 6.9, 117.5, 359.8, 40.5 |
| $K$       | 16.6   | 4.3, 8.6, 126.3, 332.3, 42.1 | 15.5   | 4.5, 5.9, 120.2, 371.5, 52.8 |
| $\bar{K}$ | 20.7   | 5.4, 10.6, 113.6, 359.6, 30.6 | 19.3   | 5.7, 7.5, 111.9, 379.5, 34.9 |

Table 4: Average coordinates of freezeout and $T$, $\mu_b$, $\mu_s$ at this coordinates.
Consequences of the different space-time freeze-out:
- Differences in yields in SM
The difference between average freeze-out and freeze-out for particular species is very large.

**Figure 4:** Particle densities at average freeze-out coordinates of all particles (left column) and at freeze-out coordinates of each particle type (right column) from statmodel; at average freeze-out coordinates of all particles (star) and at freeze-out coordinates of each particle type (pentagon) from UrQMD. $E = 40A$ GeV.
Figure 2: $T(\mu_B)$ in the central cell. Average freezeout times of different particles in the central cell are marked by colored markers.
Figure 5: $T$ and $\varepsilon$ spatial distributions.
Figure 6: $\mu_b$ and $\rho_b$ spatial distributions.
Figure 7: $\mu_s$ and $\rho_s$ spatial distributions.
Consequences of the different space-time freeze-out:

- Directed flow
At $\sqrt{s} = 19.6\text{GeV}$ $\Lambda$ are mostly located near hot and dense regions and $\bar{\Lambda}$ are distributed more uniformly near system center.
Space distribution of Lambdas

At low energies $\Lambda$ and $\bar{\Lambda}$ are produced and emitted from the same regions as protons and antiprotons respectively. $\Lambda$'s are concentrated also near hot and dense spectators, whereas $\bar{\Lambda}$'s are mostly produced in central region.

Mean flow is calculated as:

$$< v_1 > = \int \text{sign}(y)v_1(y) \frac{dN_{\text{par}}}{dy} dy / \int \frac{dN_{\text{par}}}{dy} dy$$

Collective velocities are shown on the picture to demonstrate that particles which have positive product of velocities $v_x v_z$ produce normal component of flow and particles with $v_x v_z < 0$ produce anti-flow component of directed flow. [Bravina et al, EPJ Web of Conferences 191, 05004 (2018)]
Directed flow for Lambdas and kaons

\( V_1 \) for \( \Lambda \) changes sign at midrapidity with decreasing collision energy, whereas \( V_1 \) for kaons has negative slope (antiflow)
Different slopes of different particles: URQMD and Data
Consequences of the different space-time freeze-out:
- Difference in Polarization for lambdas and antilambdas
Polarization energy dependency

Polarization of $\Lambda$ and $\bar{\Lambda}$ decreases with energy as in the experiment. $\Lambda$’s global polarization agrees well with experimental data. $\bar{\Lambda}$ polarization has right energy dependence.

Freeze-out

UrQMD-3.4, Au+Au, b = 6 fm

$\Lambda$'s and $\bar{\Lambda}$'s with $|y| < 1$ and $0.2 < p_t < 3$ GeV/c were analyzed.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>7.7</th>
<th>11.5</th>
<th>14.5</th>
<th>19.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean freeze-out time $\Lambda$ [fm/c]</td>
<td>19.7806</td>
<td>21.0302</td>
<td>21.959</td>
<td>23.1288</td>
</tr>
</tbody>
</table>
Temperature extracted with statistical model is not uniform. There are two main regions. More hot regions with $T \approx 100\text{MeV}$ are connected to dense spectators. The other part is related to fireball with temperature $\sim 60\text{MeV}$.
Thermal vorticity in reaction plane

\[ \text{Au+Au, } \sqrt{s} = 7.7 \text{ GeV, } b = 6 \text{ fm, } t = 15 \text{ fm/c} \]

\[ \mathbf{\omega}_{zx} \]

Thermal vorticity component \( \omega_{zx} \) has quadruple-like structure in reaction plane which is stable in time but magnitude decreases due to system expansion. First and third quadrant are connected with central region which has small negative vorticity. This connection part becomes smaller when energy increases.
At $\sqrt{s} = 7.7\text{ GeV}$ $\Lambda$ and $\bar{\Lambda}$ are mainly emitted from regions with small negative vorticity, thus they should have non-zero positive polarization. $\bar{\Lambda}$ has mean value of $\omega_{zx}$ with larger magnitude than $\Lambda$ ($\sim -0.04$ and $\sim -0.017$ respectively).
Emission of $\Lambda$ and $\bar{\Lambda}$

At $\sqrt{s} = 19.6$ GeV $\Lambda$ and $\bar{\Lambda}$ are also mainly emitted from regions with small negative vorticity, but distributions are more symmetric and wide. Thus mean values of $\overline{\omega}_{z\chi}$ for $\Lambda$ and $\bar{\Lambda}$ drop ($\sim -0.009$ and $\sim -0.011$ respectively).
Polarization time evolution

Polarization of $\Lambda$ hyperon decreases with time. At the beginning lambda's are preferably formed in hot and dense regions with high polarization. But later lambda's are formed uniformly in fireball and average polarization is almost zero.
Conclusions

• MC models favor chemical equilibration of hot and dense nuclear matter at $t \approx 7 \text{ fm/c}$

• The EOS has a simple form: $P/\varepsilon = \text{const}$ (hydro!) even at far-from-equilibrium stage. The speed of sound $C_s^2$ varies from 0.12 (AGS) to 0.14 (40 AGeV), and to 0.15 (SPS & RHIC) => saturation

• In MC models different particles are frozen at different times: $K$-$\pi$-anti-$\Sigma$-$\Sigma$, antip-$p$-anti-$\Lambda$-$\Lambda$
  and in different space regions with different $T$-$\mu_B$-$\mu_S$
  It naturally explains such effects as directed flow for $p$, $\Sigma$, $\Lambda$
  and antiflow for $K$-anti-$\Sigma$-, antip-anti-$\Lambda$,
  higher polarization for anti-$\Lambda$ than for $\Lambda$
Back-up Slides
Single-particle method for extraction $T-\mu B-\mu S$

gives more precise estimation of average $T-\mu_B-\mu_S$

$\text{Au+Au, } \sqrt{s} = 7.7 \text{ GeV, } b = 6 \text{ fm, UrQMD-3.4}$

$\Lambda$'s with $|y|<1$ and $0.2<p_{t}<3 \text{ GeV/c}$

$\bar{\Lambda}$'s with $|y|<1$ and $0.2<p_{t}<3 \text{ GeV/c}$
Thermal Approach

In local thermal equilibrium, the ensemble average of the spin vector for spin-1/2 fermions with four-momentum $p$ at space-time point $x$ is obtained from the statistical-hydrodynamical model as well as the Wigner function approach and reads

$$S^\mu(x, p) = -\frac{1}{8m} (1 - n_F) \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x),$$

where the thermal vorticity tensor is given by

$$\varpi_{\mu\nu} = \frac{1}{2} \left( \partial_\nu \beta_\mu - \partial_\mu \beta_\nu \right),$$

with $\beta^\mu = u^\mu / T$ being the inverse-temperature four-velocity. The number density of $\Lambda$'s is very small so that we can make the approximation $1 - n_F \simeq 1$ Therefore:

$$S^\mu(x, p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x).$$
By decomposing the thermal vorticity into the following components,

\[ \omega_T = (\omega_{0x}, \omega_{0y}, \omega_{0z}) = \frac{1}{2} \left[ \nabla \left( \frac{\gamma}{T} \right) + \partial_t \left( \frac{\gamma v}{T} \right) \right], \]

\[ \omega_S = (\omega_{yz}, \omega_{zx}, \omega_{xy}) = \frac{1}{2} \nabla \times \left( \frac{\gamma v}{T} \right), \]

Equation can be rewritten as

\[ S^0(x, p) = \frac{1}{4m} p \cdot \omega_S, \quad S(x, p) = \frac{1}{4m} (E_p \omega_S + p \times \omega_T), \]

where \( E_p, p, m \) are the \( \Lambda \)'s energy, momentum, and mass, respectively. The spin vector of \( \Lambda \) in its rest frame is denoted as \( S^{\mu*} = (0, S^*) \) and is related to the same quantity in the c.m. frame by a Lorentz boost. Finally:

\[ P = \frac{\langle S^* \rangle \cdot J}{||\langle S^* \rangle||J||}, \]

[F. Becattini et al, Phys. Rev. C 95, 054902 (2017)]
UrqMD

- Represents a Monte Carlo method for the time evolution of the various phase space densities of particle species.
- Based on the covariant propagation of all hadrons on classical trajectories, stochastic binary scatterings, resonance and string formation with their subsequent decay.
- Provides the solution of the relativistic Boltzmann equation.
- The collision criterion (black disk approximation):
  \[ d < d_0 = \sqrt{\sigma_{\text{tot}}(\sqrt{s}, \text{type})/\pi} \]
- 55 baryons and 32 mesons are included. All antiparticles and isospin-projected states are implemented.
- Cross sections are taken from PDG.
- Resonances are implemented in Breit–Wigner form.

Input from UrQMD:
\[
\varepsilon_{\text{UrQMD}} = \frac{1}{V} \sum_i E_i \\
\rho B_{\text{UrQMD}} = \frac{1}{V} \sum_i B_i \\
\rho S_{\text{UrQMD}} = \frac{1}{V} \sum_i S_i
\]

Stat. Physics:
\[
\varepsilon_{\text{stat}} = \sum_i \varepsilon_i(T, \mu_B, \mu_S) \\
\rho B_{\text{stat}} = \sum_i B_i n_i(T, \mu_B, \mu_S) \\
\rho S_{\text{stat}} = \sum_i S_i n_i(T, \mu_B, \mu_S)
\]

\[
\chi^2 = \frac{(\varepsilon_{\text{UrQMD}} - \varepsilon_{\text{stat}})^2}{\sigma_{\varepsilon}^2} + \\
+ \frac{(\rho B_{\text{UrQMD}} - \rho B_{\text{stat}})^2}{\sigma_{\rho B}^2} + \\
+ \frac{(\rho S_{\text{UrQMD}} - \rho S_{\text{stat}})^2}{\sigma_{\rho S}^2}
\]

Minuit2 numerical minimizer

Output:
\[T, \mu_B, \mu_S\]