Constraint of Compact Star Observables for Walecka-type Nuclear Matter EoS

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Outline

Motivation
- Predict the uncertainty of the macroscopic Compact Star observables based on the theoretical uncertainties → Masquarade problem
- How strong constraints can be obtained from Compact star measurements

1) How much difference arise from different approximations?
- MINIMALISTIC 1-boson-1-fermion model with a Yukawa coupling at T=0
- Uncertainties at various levels: FRG, MF and 1-loop approximations

2) Uncertainties from the parameters of the realistic nuclear matter
- Parameter dependence in the extended Walecka model for symmetric matter
- Comparison between symmetric and asymmetric matter parameters
Motivation

EoS from exp & theory

Application in compact stars

Constraints by astrophysical observations

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1) How much difference arise from the different levels of approximations?

Motivation for FRG

- It is hard to get effective action for an interacting field theory: e.g.: EoS for superdense cold matter ($T \to 0$ and finite $\mu$)
- Taking into account quantum fluctuations using a scale, $k$
  - Classical action, $S = \Gamma_{k \to \Lambda}$ in the UV limit, $k \to \Lambda$
  - Quantum action, $\Gamma = \Gamma_{k \to 0}$ in the IR limit, $k \to 0$
- FRG Method
  - Smooth transition from macroscopic to microscopic
  - RG method for QFT
  - Non-perturbative description
  - Not depends on coupling
  - BUT: Technically it is NOT simple
Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- Scale dependent effective action (k scale parameter)

\[ \partial_k \Gamma_k = \frac{1}{2} \int d^D p \; S T r \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)}} + R_k \right] \]

- \( k = \Lambda \)
  - Classical action
- Integration
- \( k = 0 \)
  - Quantum fluctuations included

Wetterich equation
Ansatz: Interacting Fermi-gas model

Ansatz for the effective action:

\[
\Gamma_{\kappa} [\varphi, \psi] = \int d^4 x \left[ \bar{\psi} \left( i \partial - g \varphi \right) \psi + \frac{1}{2} \left( \partial_{\mu} \varphi \right)^2 - U_{\kappa}(\varphi) \right]
\]

**Fermions**: \(m=0,\) **Yukawa-coupling** generates mass

**Bosons**: the **potential** contains self interaction terms

**We study the scale dependence of the potential only!!**
Interacting Fermi-gas at finite temperature

Ansatz for the effective action in LPA:

\[
\Gamma_k [\varphi, \psi] = \int d^4 x \left[ \bar{\psi} \left( i \partial \! \! \! / - g \varphi \right) \psi + \frac{1}{2} \left( \partial_\mu \varphi \right)^2 - U_k (\varphi) \right]
\]

\[
\Gamma_k [\psi] = \int d^4 x \left[ \frac{1}{2} \psi_i K_{k,i,j} \psi_j + U_k (\psi) \right]
\]

\[
\partial_k U_k = \frac{k^4}{12 \pi^2} \left[ \frac{1 + 2 n_B (\omega_B)}{\omega_B} + 4 \frac{1 + n_F (\omega_F - \mu) + n_F (\omega_F + \mu)}{\omega_F} \right]
\]

\[
U_A (\varphi) = \frac{m_0^2}{2} \varphi^2 + \frac{\lambda_0}{24} \varphi^4 \quad \omega^2_F = k^2 + g^2 \varphi^2 \quad \omega^2_B = k^2 + \partial_\varphi U \quad n_B/F (\omega) = \frac{1}{1 + e^{-\beta \omega}}
\]

Wetterich-equation in LPA

Bosonic part

Fermionic part
Result: Phase structure of interacting Fermi gas model

Exact FRG solution counts all quantum fluctuations
1-Loop approximation has only tree diagrams
Mean Filed solution contains averaged effect of interactions

In the phase structure, FRG and 1L are very similar if the LO has the strongest contribution.
Result: Comparison of MF, 1L, & FRG-based EoS

Mean Filed is the stiffest
1-Loop approximation
Exact FRG solution softest
Result: Comparison of MF, 1L, & FRG-based EoS

- MF is 25% stiffer than the FRG
- 1L is 10% stiffer than the FRG
- Exact FRG solution softest
Result: Comparison to other EoS models

Compare FRG to SQM3, GNH3, WFF1
- Overlap with SQM3 at high $\varepsilon$
- Cutoff, $\varepsilon_{\text{Cut}}$ is also higher
- Approximations differ slightly
Result: Comparison of compressibility in the models

Compare FRG to 1L and MF

- Compressibility:

\[ \frac{1}{\chi} = n \frac{\partial P}{\partial n} = 2n^2 \frac{\partial}{\partial n}(E/A) + n^3 \frac{\partial^2}{\partial n^2}(E/A) \]

- Compression modulus

\[ K = k_F^2 \frac{\partial^2}{\partial k_F^2}(E/A) = \frac{9}{n_0 \chi} \]

- The difference between the models is about \( \sim 10\% \)
Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3 → TOV result: density function

Compare FRG to 1L and MF
- Soft FRG make biggest star
- High-\(\epsilon\) part is similar for all
- Difference: \(\sim5\%\) (.1 M\(\odot\) and .5 km)

FRG to SQM3, GNH3
- FRG: small stars 1.4 M\(\odot\) and 8 km
- Other models: larger radii and less central density
Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram

Compare FRG to 1L and MF
- Soft FRG make biggest star
- High-\(\varepsilon\) part is similar for all
- Difference: \(~5\%\) (.1 \(M_\odot\) and .5 km)

FRG to SQM3, GNH3, WFF1
- Small stars 1.4 \(M_\odot\) and 8 km
- Overlap with SQM3 at high \(\varepsilon\)
- Interaction (\(\omega\)) will increase
Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram

\[ M_a \text{ vs. } M \]
Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram

![Graph showing mass versus radius for different EOS models](image_url)

- FRG result
- SQM3 Prakash, PRD 52, 661 (1995)
- WFF1 Wiringa et al, PRC 38, 1010 (1988)
Test: Can we test this by observations?

- Compare different EoS results on M(R) diagram: MF & FRG
- Maximal relative differences are also plotted
Test: Can we test this by observations?

- Compare different EoS results on M(R) diagram: MF & FRG
- Maximal relative differences are also plotted
The summary of the theoretical uncertainties

- The magnitude of the uncertainties of (astro)physical observables

- **Microscopical observables are maximum: 10-25%**

- **Macroscopical astrophysical ones are maximum: 5-10%**

- Measurement resolution limit is about: 10%

<table>
<thead>
<tr>
<th>Observable</th>
<th>Max theory uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential, ( U(\phi) )</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>Phase diagram (( g_c ))</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>EoS ( p(\mu),p(\epsilon) )</td>
<td>&lt; 25%</td>
</tr>
<tr>
<td>Compressibility</td>
<td>&lt; 10%</td>
</tr>
<tr>
<td>( \epsilon(R) )</td>
<td>~ 5%</td>
</tr>
<tr>
<td>M(R) diagram</td>
<td>&lt; 10% (M) &lt; 5% (R)</td>
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<td>Compactness</td>
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2) Uncertainties from the parameters of realistic nuclear matter

Modified $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \Psi_i \left( i\not\!D - m_N + g_\sigma \not\!\sigma - g_\omega \gamma^0 \not\!\omega_0 \right) \psi_i$$

Nucleon effective mass

$$-\frac{1}{2} m_\sigma \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4$$

Scalar meson self interaction terms

$$+ \frac{1}{2} m_\omega \omega_0^2$$

Extra terms

$$+ \frac{1}{2} m_\rho \rho^a_\mu \rho^\mu a$$

Vector meson

$$+ \Psi_e \left( i\not\!D - m_e \right) \Psi_e$$

Tensor meson

Electron in $\beta$-equilibrium

$$\mu_n = \mu_p + \mu_e$$
Modified $\sigma$-$\omega$ model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i \partial - m_N + g_\sigma \bar{\sigma} - g_\omega \gamma^0 \bar{\omega}_0 \right) \psi_i$$

**Proton and neutron**

Nucleon effective mass

**Scalar meson self interaction terms**

$$-\frac{1}{2} m_\sigma^2 \bar{\sigma}^2 - \lambda_3 \bar{\sigma}^3 - \lambda_4 \bar{\sigma}^4$$

**Extra terms**

$$+ \frac{1}{2} m_\omega^2 \bar{\omega}_0^2$$

**Vector meson**

**Tensor meson**

$$+ \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^{\mu a}$$

**Electron in $\beta$-equilibrium**

$$+ \bar{\Psi}_e \left( i \partial - m_e \right) \Psi_e$$

$$\mu_n = \mu_p + \mu_e$$

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\[ \mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i \gamma^0 \partial - m_N + g_\sigma \sigma - g_\omega \gamma^0 \omega_0 \right) \psi_i \]

Proton and neutron

Nucleon effective mass

- \( \frac{1}{2} m_\sigma^2 \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4 \)

Scalar meson self interaction terms

+ \( \frac{1}{2} m_\omega^2 \omega_0^2 \)

Vector meson

+ \( \frac{1}{2} m_\rho^2 \rho^\alpha_\mu \rho^\mu_\alpha \)

Tensor meson

\[ + \bar{\Psi}_e \left( i \gamma^0 \partial - m_e \right) \Psi_e \]

Isospin asymmetry

Electron in β-equilibrium

\[ \mu_n = \mu_p + \mu_e \]
\[ \mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i \partial - m_N + g_\sigma \bar{\sigma} - g_\omega \gamma^0 \bar{\omega}_0 \right) \psi_i \]

Nucleon effective mass

Proton and neutron

- \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 - \lambda_3 \bar{\sigma}^3 - \lambda_4 \bar{\sigma}^4

Scalar meson self interaction terms

+ \frac{1}{2} m_\omega^2 \bar{\omega}_0^2

Vector meson

+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_\mu^a

Tensor meson

+ \bar{\Psi}_e \left( i \partial - m_e \right) \Psi_e

Electron in \( \beta \)-equilibrium

\[ \mu_n = \mu_p + \mu_e \]
\[ \mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left( i\hat{\phi} - m_N + g_\sigma \sigma - g_\omega \gamma^0 \omega_0 \right) \psi_i \]

- Proton and neutron

\[ -\frac{1}{2} m_\sigma^2 \sigma^2 - \lambda_3 \sigma^3 - \lambda_4 \sigma^4 \]

- Scalar meson self interaction terms

\[ + \frac{1}{2} m_\omega^2 \omega_0^2 \]

- Extra terms

\[ + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_\mu^a \]

- Vector meson

\[ + \bar{\Psi}_e \left( i\hat{\phi} - m_e \right) \Psi_e \]

- Tensor meson

Electron in β-equilibrium:

\[ \mu_n = \mu_p + \mu_e \]
_modified σ-ω model in mean field

- **Theoretical mean field model:**
  
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  \[- \lambda_3 \bar{\sigma}^3 - \lambda_4 \bar{\sigma}^4\]

  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for β-equilibrium: \( \mu_n = \mu_p + \mu_e \)

- **Parameters of the theoretical model**

  - Fit couplings/masses/etc. according to the Rhoades–Ruffini theorem in agreement with experimental data.

  - Parameters are usually non-independent: optimization of the parameters need to perform \( \rightarrow \) similar EoS

- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors \( \rightarrow \) Theoretical uncertainties**
Parameters to fit normal nuclear matter
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Incompressibility

\[
K = k_F^2 \frac{\partial^2 (e/n)}{\partial k_F^2} = 9 \frac{\partial p}{\partial n}
\]

Landau mass

\[
m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,\text{eff}}^2}
\]
Parameters to fit normal nuclear matter

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The effective mass and Landau mass are **NOT** independent! The can not be fitted simultaneously.

**Incompressibility**

\[ K = k_F^2 \left( \frac{\partial^2(\epsilon/n)}{\partial k_F^2} \right) = 9 \left( \frac{\partial p}{\partial n} \right) \]

**Landau mass**

\[ m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N_{\text{eff}}}^2} \]
The Equation of State of different model fits

Adding higher-order terms
- helps, at lower pressure
- more parameter more constraints:

Effective mass fit $m_{\text{Eff}} = 0.6 \, m_N$

Different models give similar EoS
The Equation of State of different model fits

Adding higher-order terms
- helps, at lower pressure
- more parameter more constraints:
  - Landau mass fit $m_{\text{Eff}} = 0.83 \, m_N$
  - Effective mass fit $m_{\text{Eff}} = 0.6 \, m_N$
  - Original Walecka model

Different models give similar EoS
Depending on the ‘fit-type’
→ bands appear
The Equation of State of different model fits

Adding higher-order terms
- helps, at lower pressure
- more parameter more constraints:

Landau mass fit \( m_{\text{Eff}} = 0.83 \, m_N \)

Effective mass fit \( m_{\text{Eff}} = 0.6 \, m_N \)

Original Walecka model

Realistic nuclear matter EoSs, like WFF1, AP4 (SQM) support the Landau mass fits well.
The M-R diagrams: EoS & effective mass fit

SYMMETRIC nuclear matter EoS
- Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures in the M-R diagram
The M-R diagrams: EoS & effective mass fit

SYMMETRIC nuclear matter EoS
- Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures in the M-R diagram

Landau mass fit $m_{\text{Eff}} = 0.83 \, m_N$

Effective mass fit $m_{\text{Eff}} = 0.6 \, m_N$
The M-R diagrams: EoS & effective mass fit

SYMMETRIC nuclear matter EoS
- Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures in the M-R diagram

Landau mass fit $m_{\text{Eff}} = 0.83 \ m_N$

Effective mass fit $m_{\text{Eff}} = 0.6 \ m_N$

→ Landau mass fits provide compact star with lower $M_{\text{max}}$ but closer to the observations

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ASYMMETRIC nuclear matter EoS

- Cases with extra $x^3$ and/or $x^4$ terms provide similar band structures in the M-R diagram

Landau mass fit $m_{\text{Eff}} = 0.83 m_N$

Effective mass fit $m_{\text{Eff}} = 0.6 m_N$

→ Nuclear ASYMMETRY has weak decreasing effect on the $M_{\text{max}}$
Evolution/scaling in $M_{\text{max}}$ appears
- The $M_{\text{max}}$ is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters
Scaling: maximum star mass vs. nuclear parameters

Evolution/scaling of the maximum mass/radius of the compact star
- The $M_{\text{max}}$ is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters
- Fit errors are small < 1%
- $M_{\text{max}}$ depends linearly by parameters $m_L, m_{\text{Eff}} > 10x, K > 10x a_{\text{sym}}$
- Good approximation using effective mass, independently of the scalar interaction term
- Similar scaling for $R_{\text{max}}$

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Scaling: maximum star mass vs. nuclear parameters

**SYMMETRIC nuclear matter**
Maximal mass (in $M_\odot$)
- $M_{\text{maxM}} = 5.51 - 0.005$ $m_L$
- $M_{\text{maxM}} = 1.79 + 0.001$ $K$

**ASYMMETRIC nuclear matter**
Maximal mass (in $M_\odot$)
- $M_{\text{maxM}} = 5.50 - 3.64$ $m_L$
- $M_{\text{maxM}} = 1.61 + 0.24$ $K$
- $M_{\text{maxM}} = 1.85 + 0.01$ $a_{\text{sym}}$
To take away...

- **Theoretical (maximal) uncertainties were tested in FRG**
  - Microscopical level (EoS, phases, compressibility): 10-25%
  - Macroscopical astrophysical level (M,R,compactness): 5-10%

- **Uncertainties by the realistic nuclear matter parameters**
  - Linear dependence on the $m_L$, $m_{\text{Eff}} > 10 \times K > 10 \times a_{\text{sym}}$
  - Varying $m_L$, $m_{\text{Eff}}$ cause $\sim 10\%$ uncertainty on M and R
  - Differences on symmetric/asymmetric matter is $\sim 1-3\%$
BACKUP
Motivation for FRG

- **Observation:** Considering a point charge, which polarizes the medium seems like point charge with a modified charge.

- **Basic idea:** Due to the interaction, the measurable (effective) properties differs from the bare quantities.

- **Quantum corrections:**
  - Heisenberg uncertainty
    high-energy reaction for a short time is allowed
  - Pair production & annihilation
    bosonic propagator is modified due to the pair production
  - Self-interaction
    Interaction is a sum of many tiny- and self interaction
Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.

- **Scale dependent effective action (k scale parameter)**

\[
\partial_k \Gamma_k = \frac{1}{2} \int dp^D S T r \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]
\]

- **Ansatz** for the integration,
  - not need to be perturbative
  - scale-dependent coupling

Wetterich equation

\[
\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l
\]

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Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- **Scale dependent effective action (k scale parameter)**

\[
\partial_k \Gamma_k = \frac{1}{2} \int dp^D \ ST \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)}} + R_k \right]
\]

- **Regulator**
  - Determines the modes present on scale, k
  - Physics is regulator independent
Local Potential Approximation (LPA)

What does the ansatz exactly mean?

LPA is based on the assumption that the contribution of these two diagrams are close. (momentum dependence of the vertices is suppressed)

This implies the following ansatz for the effective action:

$$
\Gamma_k [\psi] = \int d^4x \left[ \frac{1}{2} \bar{\psi}_i K_{k,ij} \psi_j + U_k (\psi) \right]
$$
Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

\[
\Gamma_k [\varphi, \psi] = \int d^4 x \left[ \bar{\psi} \left( i \partial - g \varphi \right) \psi + \frac{1}{2} \left( \partial_{\mu} \varphi \right)^2 - U_k (\varphi) \right]
\]

\[
\partial_k U_k = \frac{k^4}{12 \pi^2} \left[ \frac{1 + 2n_B (\omega_B)}{\omega_B} + 4 \frac{-1 + n_F (\omega_F - \mu) + n_F (\omega_F + \mu)}{\omega_F} \right]
\]

\[
U_\Lambda (\varphi) = \frac{m_0^2}{2} \varphi^2 + \frac{\lambda_0}{24} \varphi^4 \quad \omega_F^2 = k^2 + g^2 \varphi^2 \quad \omega_B^2 = k^2 + \partial^2 \varphi U \quad n_{B/F} (\omega) = \frac{1}{1 + e^{-\beta \omega}}
\]
Interacting Fermi-gas at zero temperature

We have two equations for the two values of the step function each valid on different domain

\[ T=0, \mu \neq 0 \]

\[ n_F(\omega) \rightarrow \Theta(-\omega) \]
Interacting Fermi-gas at zero temperature

We have two equations for the two values of the step function each valid on different domain.

\[ T=0, \mu \neq 0 \quad \Rightarrow \quad n_F(\omega) \rightarrow \Theta(-\omega) \]

Fermi-surface

\[ k_F = \sqrt{\mu^2 - g^2\varphi^2}, \]
Interacting Fermi-gas at zero temperature

We have two equations for the two values of the step function each valid on different domain

\[ T=0, \mu \neq 0 \quad n_F(\omega) \rightarrow \Theta(-\omega) \]

\[ \partial_k U_k = \frac{k^4}{12 \pi^2} \left[ \frac{1}{\omega_B} - \frac{4}{\omega_F} \right] \]

\[ k_F = \sqrt{\mu^2 - g^2 \varphi^2} \]

Fermi-surface
Interacting Fermi-gas at zero temperature

We have two equations for the two values of the step function, each valid on different domain:

\[ n_F(\omega) \rightarrow \Theta(-\omega) \]

\[ T=0, \mu \neq 0 \]

\[ \partial_k U_k = \frac{k^4}{12\pi^2} \left[ \frac{1}{\omega_B} - \frac{4}{\omega_F} \right] \]

\[ k_F = \sqrt{\mu^2 - g^2 \varphi^2} \]

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel.
Integration of the Wetterich-equaiton

1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the fermi surface

3.) Calculate the initial conditions for the other equation inside the fermi surface

4.) Integrate the equation which is valid below the Fermi-surface
BUT...

To use the original method, we need an initial condition which does not have this mixing.

The boundary condition mixes the $k$ and $g\phi$. 
Solution: Need to transform the variables

We can transform the variables to make the quarter circle into a rectangle.

BUT now we have a well defined boundary condition too!
Coordinate transformation is required with:
- mapping the Fermi-surface to rectangle
- Keep the symmetries of the diff. eq.
- Circle-rectangle transformation:

\[ (k, \varphi) \mapsto (x, y) \]

\[ x = \varphi_F(k), \quad y = \frac{\varphi}{x} \]

Transformation of the potential:
with boundary condition at the Fermi-surface, \( V_0 \)

\[ \tilde{U}(x, y) = V_0(x) + \tilde{u}(x, y) \]

Transformed Wetterich-eq:

\[ x \partial_x \tilde{u} = -x V_0' + y \partial_y \tilde{u} - \frac{g^2 (k x)^3}{12 \pi^2} \frac{1}{\sqrt{(k x)^2 + \partial_y^2 \tilde{u}}} \]

and the new boundary conditions:

\[ \tilde{u}(x = 0, y) = \tilde{u}(x, y = \pm 1) = 0. \]
Solution of transformed Wetterich by an orthogonal system

Solution is expanded in an orthogonal basis to accommodate the strict boundary condition in the transformed area

\[ \tilde{u}(x, y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_{0}^{1} dy \, h_n(y) h_m(y) = \delta_{nm}. \]

The square root in the Wetterich-equation is also expanded:

\[ x e'_n(x) = \int_{0}^{1} dy \, h_n(y) \left[ -x V'_0 + y \partial_y \tilde{u} - \frac{g^2 (k x)^3}{12 \pi^2} \sum_{p=0}^{\infty} \left( -\frac{1}{2} \right)^p \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right] \]

Where:

\[ \omega^2 = (k x)^2 + M^2 \]

We use harmonic base:

\[ h_n(y) = \sqrt{2} \cos q_n y, \quad q_n = (2n + 1) \frac{\pi}{2}. \]
Result: The Effective Potential & Comparison

\[ \mu_c = 1.053\mu_{MF} \]

Potential in one-loop approximation

Fermi-surface in the field variable

G.G. Barnafoldi: SQM2019, Bari, Italy
Result: The Effective Potential & Comparison

Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is convex → coarse grained action

Solution changes only below Fermi-surface, since switch to another equation
Result: The Effective Potential & Comparison

Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is convex → coarse grained action

In the concave part of the potential solution is slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamical reasons → Maxwell construction
Test: Can we test this by observations?

- Compare Compactness by FRG, MF, 1L, SQM3, and WFF1 EoS
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