Dualities of QCD phase diagram: dense quark matter with isospin and chiral imbalance

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The model and its thermodynamical potential

We study two-flavour chiral ($\mu_5 \neq 0$ and $\mu_{I5} \neq 0$) and isospin ($\mu_I \neq 0$) imbalanced dense ($\mu \neq 0$) quark matter in the framework of (3+1)-dimensional NJL model. Its Lagrangian has the form:

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \mu_5\gamma^0\gamma^5 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5\Big]q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big] \quad (1)$$

Here q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets (the summation in (1) over flavor, color, and spinor indices is implied); τ_k $(k = 1,2,3)$ are Pauli matrices; m_0 is the diagonal matrix in flavor space with bare quark masses $(m_u = m_d = m_0)$; μ_B is a baryon number chemical potential;

- μ_B is baryon chemical potential,
- μ_I is taken into account to introduce the non-zero imbalance between u and d quarks,
- μ_{I5} and μ_5 accounts for chiral isospin and chiral imbalances.

From here we use notations $\mu \equiv \mu_B/3$, $\nu = \mu_I/2$ and $\nu_5 = \mu_{I5}/2$.

One uses a semibosonized version of the Lagrangian (1), which contains composite bosonic fields $\sigma(x)$ and $\pi_a(x)$ $(a = 1,2,3)$

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \mu_5\gamma^0\gamma^5 + \nu_5\tau_3\gamma^0\gamma^5 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big]. \quad (2)$$

From the auxiliary Lagrangian (2) one gets the equations for the bosonic fields

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q). \quad (3)$$

The ground state expectation values $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ of the composite bosonic fields are determined by the saddle point equations or as a minimum of the TDP, meaning the coordinates (M_0, Δ_0), where

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = \langle\pi_3(x)\rangle = 0, \quad (4)$$

The TDP of the system is

$$\Omega(M,\Delta) = -\frac{1}{VN_c}\mathcal{S}_{\text{eff}}\{\sigma(x),\pi_a(x)\}\big|_{\sigma(x)=\langle\sigma(x)\rangle,\pi_a(x)=\langle\pi_a(x)\rangle}, \quad (5)$$

where $\mathcal{S}_{\text{eff}}$ is effective action.

Since the NJL model is a non-renormalizable theory we have to use fitting parameters for the quantitative investigation of the system. We use the following, widely used parameters:

$$m_0 = 5,5\,\text{MeV}; \qquad G = 15.03\,\text{GeV}^{-2}; \qquad \Lambda = 0.65\,\text{GeV}.$$

Dualities of the phase diagram

It is possible to demonstrate that the TDP is invariant with respect to the so-called duality transformation $\mathcal{D}_H$ of the order parameters and chemical potentials,

$$\mathcal{D}_H: \quad M \longleftrightarrow \Delta, \ \ \nu \longleftrightarrow \nu_5. \quad (6)$$

Moreover, there are two other dualities.

$$\mathcal{D}_{HM}: \quad \mu_5 \longleftrightarrow \nu_5 \qquad \text{if} \quad \Delta = 0$$

and

$$\mathcal{D}_{H\Delta}: \quad \mu_5 \longleftrightarrow \nu \qquad \text{if} \quad M = 0$$

Phase portrait of the model

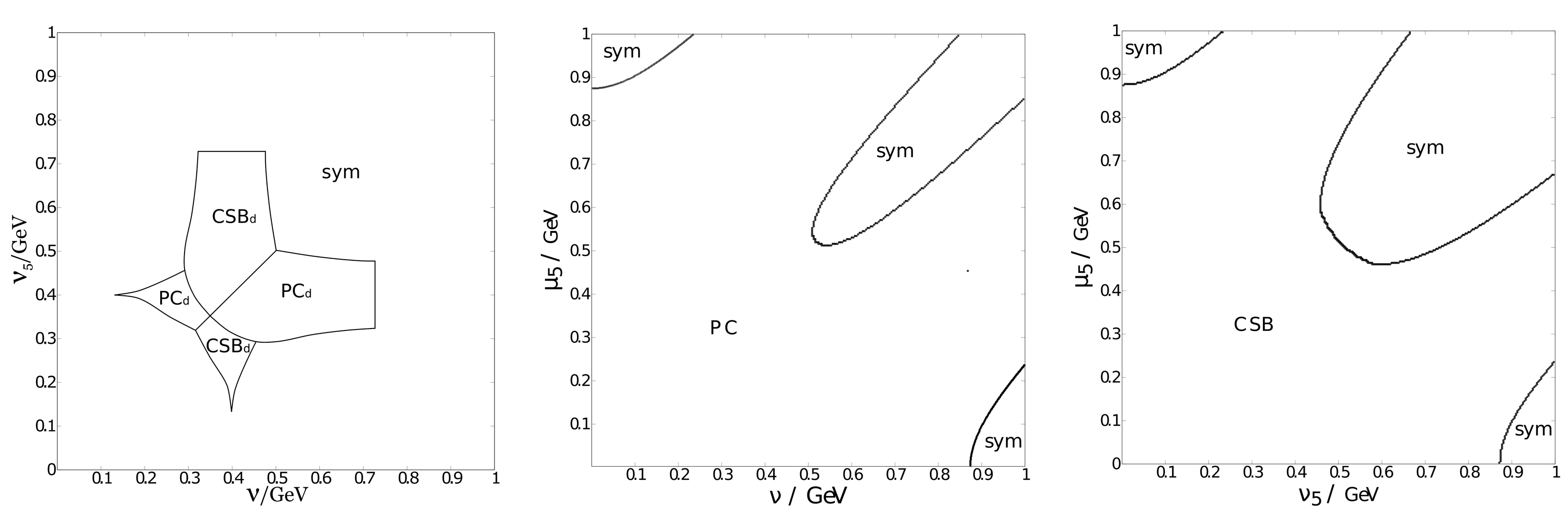

Figure 1. (Left) (ν,ν_5) phase diagram at $\mu_5 = 0$ and $\mu = 400$ MeV. (Middle) (ν,μ_5) phase diagram at $\nu_5 = \mu = 0$. (Right) (ν_5,μ_5) phase diagram at $\nu = 0$ and $\mu = 100$ MeV.

- The chiral imbalance leads to the generation of charged pion condensation in dense quark matter, so called PC$_d$ phase.

Duality in lattice QCD

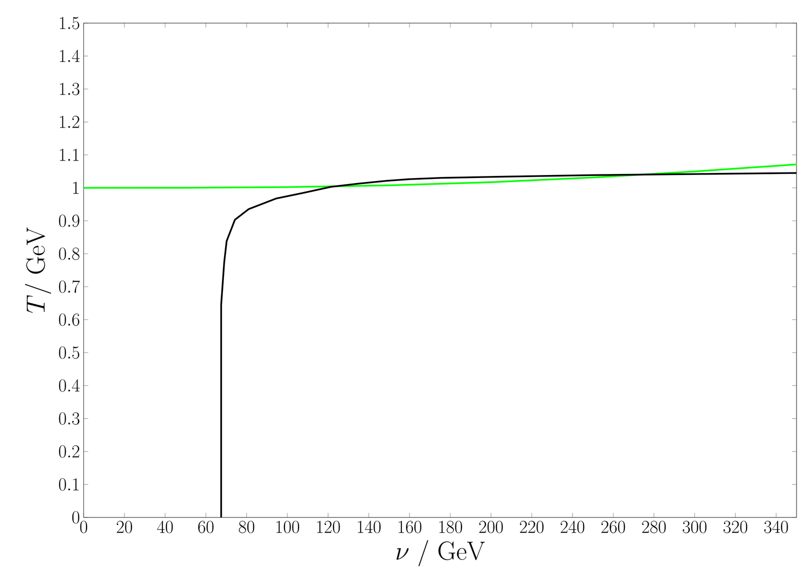

Figure 1: Critical temeperature T_c as a function of ν at $\mu_5 = 0$ and as a function of μ_5 at $\nu = 0$.

NJL model works very well in describing QCD phase diagram, but it is effective model for QCD and not QCD itselt. The duality was shown only in the large N_c approximation (or mean field approximation). It is also exact only in the chiral limit. So it is interesting to investigate whether duality can be seen in lattice QCD simulations results or not. Lattice QCD simulations are almost impossible at $\mu \neq 0$, but one can search for the fingerprints of the duality in the particular cases, for example, for $\mu = 0$.

Lattice studies:

- non-zero μ_I: B. Brandt, G. Endrodi et al, Phys. Rev. D 97 (2018) no.5, 054514; PoS LATTICE 2016 (2016) 039
- non-zero μ_5: V. V. Braguta et al

One can see from lattice QCD results that at ν and μ_5 greater that the half of the pion mass m_π the duality is a very good approximation.

Inhomogeneous condensates

In vacuum, at $\mu = \nu = \mu_5 = 0$ and ν_5, $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x. However, in a dense medium $\mu \neq 0$, they might depend on spatial coordinates. Though the chiral limit is an excellent approximation to QCD, one knows that in reality the current quark masses are nonzero. If current quark mass $m_0 \neq 0$ (physical point), we use the following the ansatz

$$\langle\sigma(x)\rangle = M\cos(2kx^1) - m_0, \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \Delta\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \Delta\sin(2k'x^1), \quad (7)$$

We showed that the TDP is invariant with respect to the duality that in inhomogeneous case has the form

$$\mathcal{D}_I: \quad M \longleftrightarrow \Delta, \ \ \nu \longleftrightarrow \nu_5, \ \ k \longleftrightarrow k'. \quad (8)$$

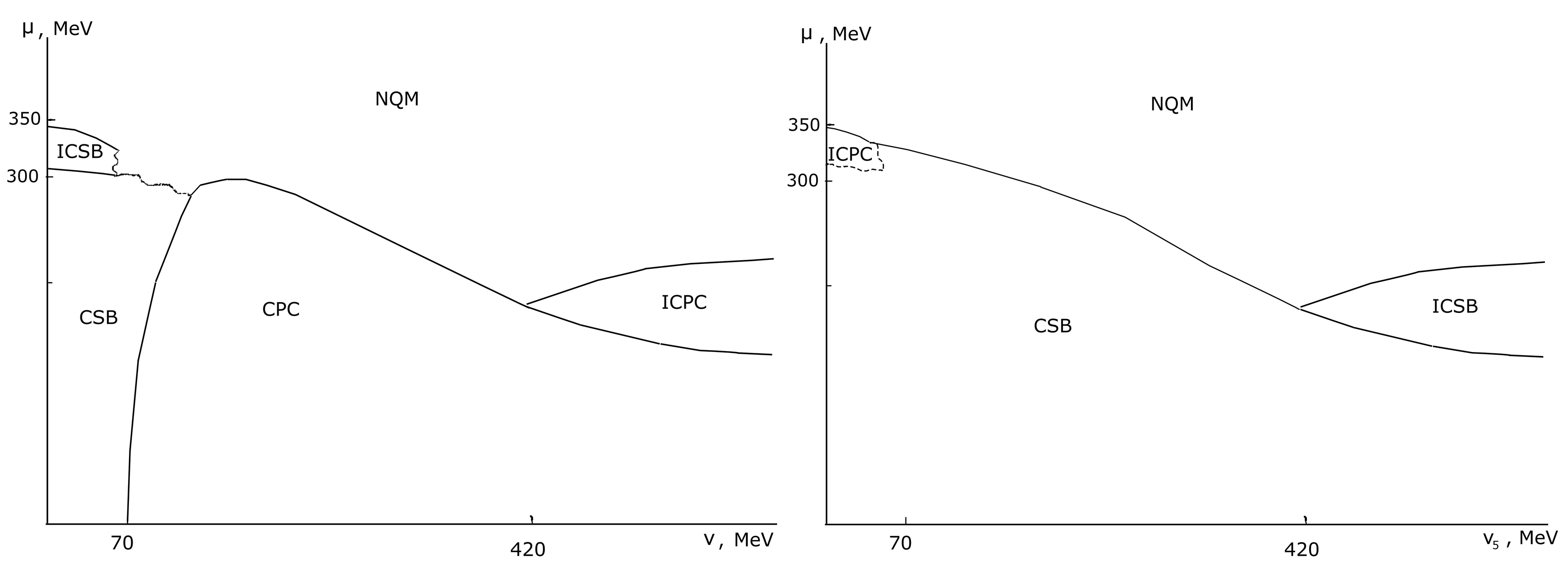

Figure 1. (Left). Combined schematic (ν,μ)-phase diagram at $\mu_5 = \nu_5 = 0$.
(Rigth). (ν_5,μ)-phase diagram at $\nu = \mu_5 = 0$. This plot is duality conjugated to the left one.

Conclusions

- There exist several dualities of the phase diagram
- Chiral imbalance generate charged pion condensation in dense quark matter
- Obtained phase portraits are in qualitative accordance with the recent lattice simulations
- Duality holds with very good accuracy in lattice QCD results
- Inhomogeneous phase structure of dense quark matter with chiral imbalance is obtained using the duality notion

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