Dualities of QCD phase diagram: dense quark matter with isospin and chiral imbalance

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The model and its thermodynamical potential
We study two-flavor chiral (ψν ≠ 0 and μν ≠ 0) and isospin (μν ≠ 0) imbalanced dense (μ ≠ 0) quark matter in the framework of (3+1)-dimensional NJL model. Its Lagrangian has the form:

\[ \mathcal{L} = \bar{\psi}(v) D \psi + \frac{1}{2} \partial^\mu \psi \partial^\nu \psi - m \bar{\psi} \psi - \frac{1}{2} G \bar{\psi} \gamma^\mu \gamma^\nu \psi \]  

(1)

Here \( \gamma \) is the flavor doublet, \( \gamma = (\gamma_5, \gamma_8) \), where \( \gamma_5 \) and \( \gamma_8 \) are four-component Dirac spinors as well as color \( N = \text{tr} \) (the summation in (1) over flavor, color, and spinor indices is implied); \( \gamma_5 \) is the diagonal matrix in flavor space with bare quark masses (\( m_0 = m_d = m_s \)); \( \mu \) is a baryon number chemical potential; 

- \( \mu \) is taken into account to introduce the non-zero imbalance between \( v \) and \( d \) quarks, 
- \( \mu N \) accounts for chiral isospin and chiral imbalances.

From here we use notations \( \mu \equiv \mu_N/3, \nu = \mu / 2 \) and \( \rho = \mu / 2 \).

One uses a semibosonized version of the Lagrangian (1), which contains composite bosonic fields \( \sigma(v) \) and \( \pi_2(v) \) (\( n = 1, 2, 3 \)):

\[ \mathcal{L} = \bar{\psi}(v) D \psi + \frac{1}{2} \partial^\mu \psi \partial^\nu \psi - m \bar{\psi} \psi - \frac{1}{2} G \bar{\psi} \gamma^\mu \gamma^\nu \psi \]

(2)

From the auxiliary Lagrangian (2) one gets the equations for the bosonic fields

\[ \sigma(v) = - \frac{1}{2} G \gamma^\mu \gamma^\nu \psi \]

(3)

The ground state expectation values \( \langle \sigma(v) \rangle \) and \( \langle \pi_2(v) \rangle \) of the composite bosonic fields are determined by the saddle point equations or as a minimum of the TDP, meaning the coordinates \( (\nu, \mu) \), where

\[ \sigma(v) = M \langle \sigma(v) \rangle , \quad \pi_2(v) = \Delta \sin(2\nu x) \]

(4)

The TDP of the system is

\[ \Omega(\nu, \mu, \Delta) = \frac{1}{2} \int \delta[v(v), v(x)] \delta[\pi_2(v), \pi_2(x)] \delta[\sigma(v), \sigma(x)] \]  

(5)

where \( \delta \) is effective action.

Since the NJL model is a non-renormalizable theory we have to use fitting parameters for the quantitative investigation of the system. The use of the following, widely used parameters:

\[ m_0 = 5.5 \text{ MeV}, \quad G = 15.03 \text{ GeV}^{-2}, \quad \Lambda = 0.65 \text{ GeV}. \]

Dualities of the phase diagram
It is possible to demonstrate that the TDP is invariant with respect to the so-called duality transformation \( D_\mu \) of the order parameters and chemical potentials,

\[ D_\mu: \quad \mu \rightarrow \mu, \quad \nu \rightarrow \nu \]  

(6)

Moreover, there are two other dualities,

\[ D_{\mu(\nu)}: \quad \mu \rightarrow \nu, \quad \nu \rightarrow \mu \]

and

\[ D_{\nu(\mu)}: \quad \mu \rightarrow \mu, \quad \nu \rightarrow \mu \]

Phase portrait of the model

Figure 1. (Left) \( (\nu, \mu) \) phase diagram at \( \rho = 0 \) and \( \nu = -400 \text{ MeV} \). (Middle) \( (\nu, \mu) \) phase diagram at \( \rho = 0 \). (Right) \( (\nu, \mu) \) phase diagram at \( \rho = 0 \) and \( \nu = 100 \text{ MeV} \).

• The chiral imbalance leads to the generation of charged pion condensation in dense quark matter, so called \( PC_2 \) phase.

Duality in lattice QCD

\[ \nu_0(\mu) = \frac{1}{\nu} \int \nu D_\mu \nu \]  

(7)

and

\[ D_\phi(\nu_0): \quad \nu_0 \rightarrow \nu_0 \]

(8)

Inhomogeneous condensates

In vacuum, at \( \nu = \mu = 0 \) and \( \nu_0 = 0 \), the \( (\nu, \mu) \) and \( (\nu_0, \nu) \) do not depend on space coordinate \( x \). However, in a dense medium \( \mu \neq 0 \), they might depend on spatial coordinates. Though the chiral limit is an excellent approximation to QCD, one knows that in reality the current quark masses are nonzero. If current quark mass \( m_0 \neq 0 \) (physical point), we use the following the ansatz

\[ \sigma(x) = M \cos(2\nu x), \quad \pi_2(x) = M \sin(2\nu x) \]

(9)

We showed that the TDP is invariant with respect to the duality in inhomogeneous case has the form

\[ D_\mu: \quad \nu \rightarrow \nu, \quad \nu_0 \rightarrow \nu_0 \]  

(10)

\[ D_\phi(\nu_0): \quad \nu_0 \rightarrow \nu_0 \]

(11)

Conclusions

- There exist several dualities of the phase diagram.
- Chiral imbalance generate charged pion condensation in dense quark matter.
- Obtained phase portraits are in qualitative accordance with the recent lattice simulations.
- Duality holds with very good accuracy in lattice QCD results.
- Inhomogeneous phase structure of dense quark matter with chiral imbalance is obtained using the duality notion.

References