

Introduction

The non-extensive statistical description of the identified final state particles measured in high energy collisions is well-known by its wide range of applicability. However, there are many open questions that need to be answered, including the question of the observed mass scaling of massive hadrons or the size and multiplicity dependence of the model parameters. This latter is especially relevant, since the amount of the available experimental data with high multiplicity at small systems is very limited so far.

In this contribution the role of the size of the colliding system and multiplicity dependence of the parameters in the non-extensive hadronization model is investigated with HIJING++ calculations. We present cross-check comparisons of HIJING++ with existing experimental data to verify its validity in our range-of-interest, as well as calculations at high-multiplicity regions where we have insufficient experimental data.

HIJING++ tuning process

HIJING++ is based on the convolution of sequential collisions of nucleon-nucleon pairs in each nucleus-nucleus interaction → important to have a solid proton-proton collision baseline.

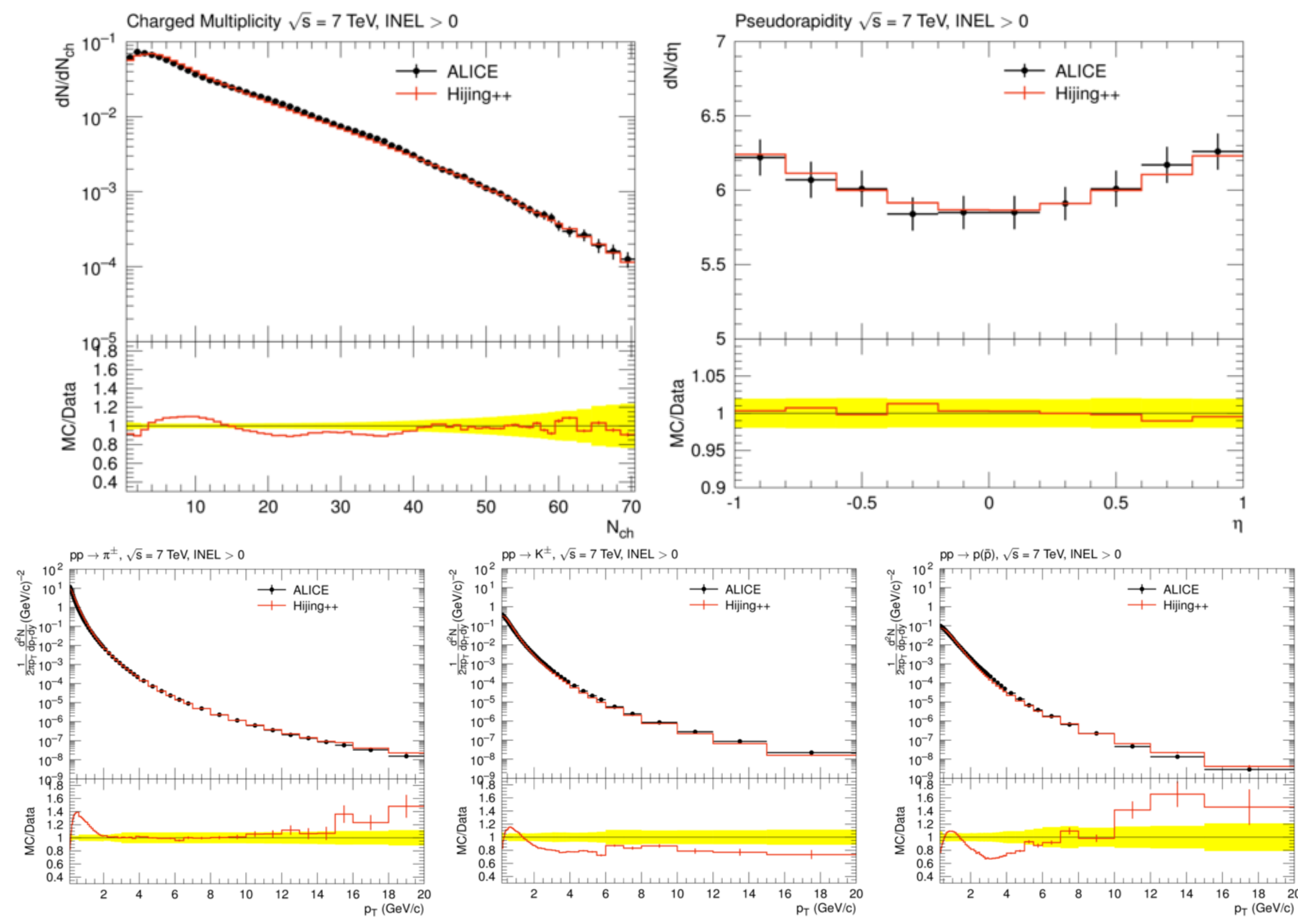
The following $\sqrt{s} = 7$ TeV proton-proton ALICE experimental data were used:

- p_T spectra of identified charged pions, kaons and (anti)protons with $INEL > 0$ normalization (at least one charged particle in the $|\eta| < 1.0$ region is required) up to $p_T = 20$ GeV/c
- charged hadron multiplicity distribution in the range of $\langle dN_{ch}/d\eta \rangle = 0-70$, where N_{ch} is the number of charged particles
- charged hadron pseudorapidity distribution at mid-pseudorapidity $|\eta| < 1.0$

HIJING++ has about ~20 phenomenological parameters to be tuned, such as:

Parameter	Description
p_0	soft-hard separation scale: minimum p_T transfer of hard or semihard scatterings
σ_{soft}	the inclusive cross section for soft interactions
σ_0	the cross section that characterizes the geometrical size of a nucleus
H_0	the parameter in the scaled eikonal function of nucleus used to calculate total cross-section
K	K-factor for the differential jet cross sections in the lowest order pQCD calculation

- HIJING++ reproduces the event multiplicity well, within ~15% for the multiplicity and ~1% for the pseudorapidity distribution
- The charged pion and kaon spectra also show a good agreement above $p_T = 2$ GeV/c
- The production is slightly overestimated at lower p_T values
- Best agreement for the π^\pm results is ~1% between 2 and 15 GeV/c
- For kaons, the yield is slightly underestimated above 2 GeV/c: the agreement is ~15-20%
- The proton yield is overestimated in the large p_T region, the agreement is ~20-30%



Non-Extensive Hadronization Model

Our goal is to study the transverse momentum spectra of identified hadrons with Tsallis - Pareto type distributions, revealing non-trivial dependence on the center-of-mass energy, the hadron mass and the event multiplicity (i.e. on the size of the system).

- Blast-wave assumptions: the fireball is azimuthally symmetric and is expanding with a v radial flow velocity (in units of $c = 1$)
- The freeze-out occurs instantly on a hypersurface according to the Cooper - Frye formulation at a given freeze-out temperature
- The following simple form of the invariant yield is used:

$$\frac{d^2N}{p_T dp_T dy} \Big|_{INEL > 0} = A \cdot m_T \cdot \left(1 + \frac{E}{nT}\right)^{-n}$$

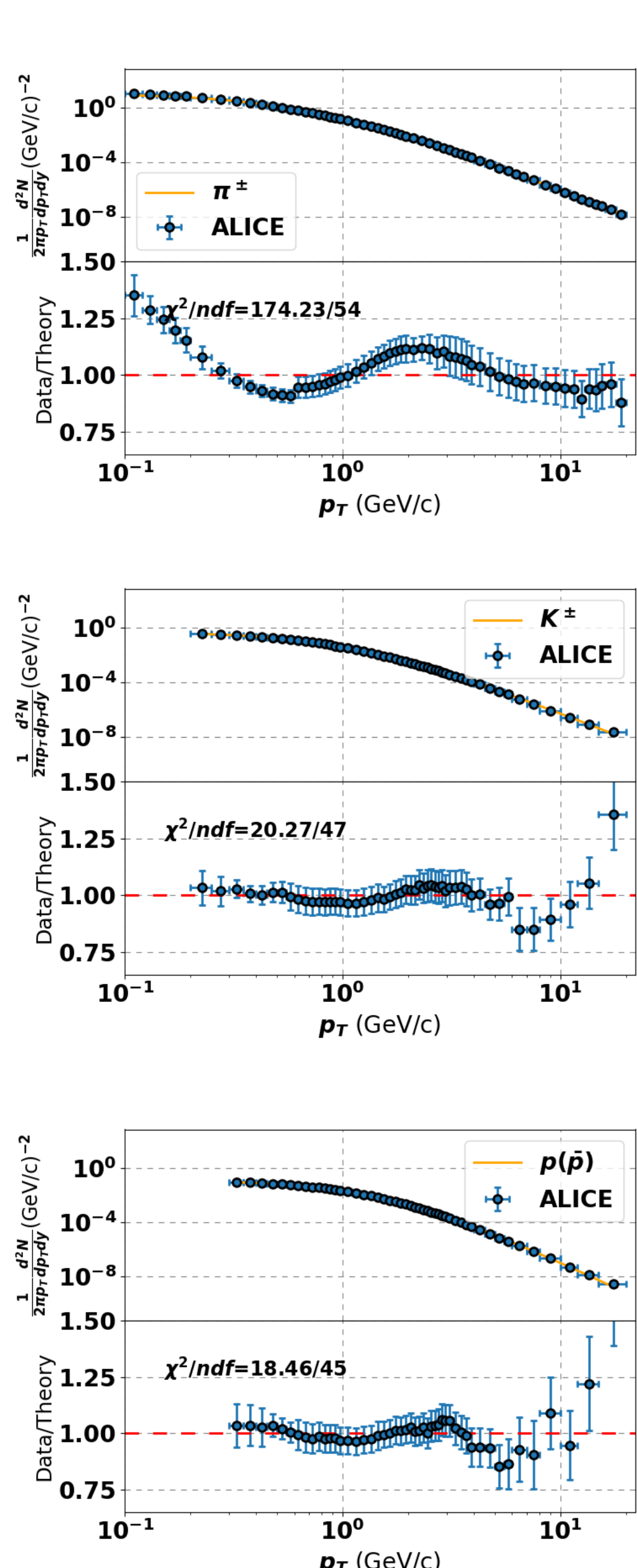
where A is the amplitude incorporating the irrelevant spin degeneracy and constant factors as well as the invariant volume, $m_T = \sqrt{p_T^2 + m^2}$ is the transverse mass, $E = \gamma(m_T - vp_T) - m$ is the one-particle energy in the co-moving coordinate system, v is the radial flow velocity, $\gamma = 1/\sqrt{1-v^2}$ is the Lorentz-factor, T is a parameter with temperature unit and finally $n = 1/(q-1)$ is the non-extensivity parameter, characterizing the temperature fluctuations.

- The parameters and curves fitted on the experimental "minimum bias" (in the sense, that there is no event multiplicity classification) data

- The fits are good on the investigated p_T range for all hadrons

Hadron	n	q	T (GeV)	A	v	χ^2/ndf
π^\pm	7.415 ± 0.033	1.135 ± 0.005	0.089 ± 0.010	73.188 ± 9.700	0.000 ± 0.119	$174.225 / 54$
K^\pm	7.539 ± 0.086	1.133 ± 0.013	0.155 ± 0.010	0.915 ± 0.095	0.000 ± 0.066	$20.274 / 47$
$p(\bar{p})$	8.805 ± 0.184	1.114 ± 0.023	0.191 ± 0.012	0.124 ± 0.013	0.000 ± 0.054	$18.462 / 45$

The results are consistent with our earlier observations: among the investigated hadron species, the proton (which is the heaviest) has the largest temperature and the smallest q and amplitude. The fitted radial flow velocity is zero for each hadrons.



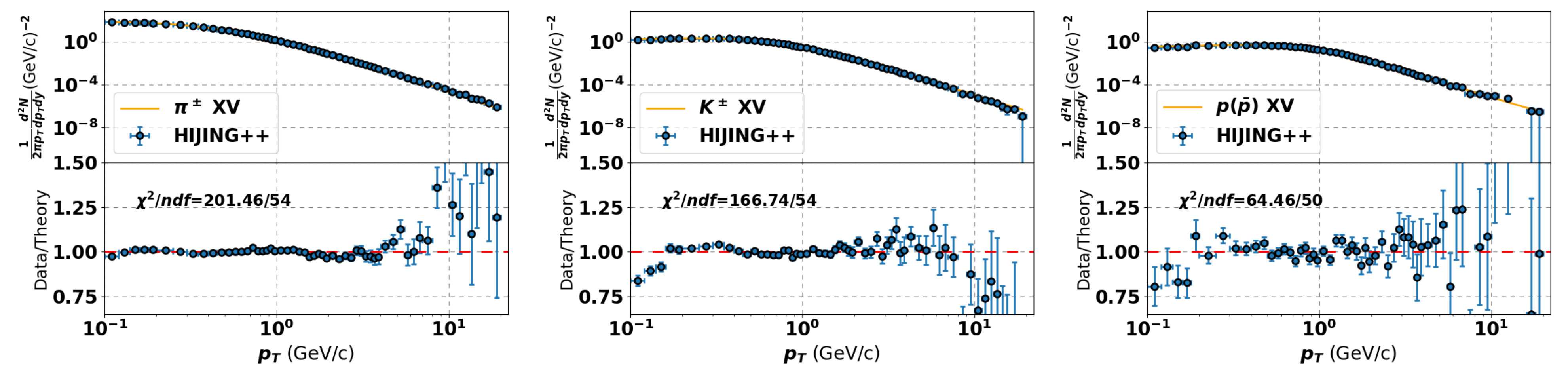
Results

Using the non-extensive hadronization framework we aim to investigate the multiplicity dependence of the Tsallis-Pareto parameters. We take advantage of the power of HIJING++ and extract the parameters in event multiplicity classes at the tuned $\sqrt{s} = 7$ TeV center-of-mass energy.

- The event classes of the HIJING++ run are classified as the multiplicity ranges:

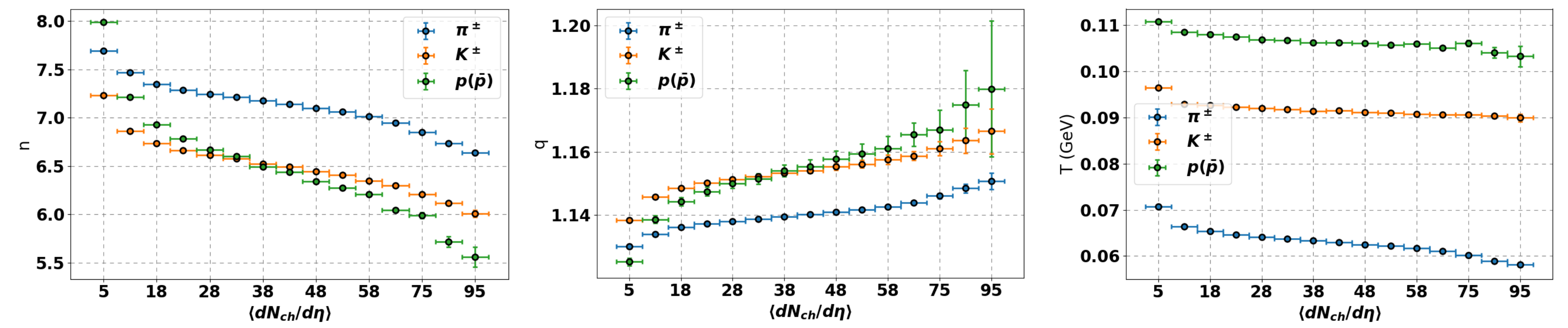
Class	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
$\langle dN_{ch}/d\eta \rangle_{min}$	0	10	15	20	25	30	35	40	45	50	55	60	70	80	90
$\langle dN_{ch}/d\eta \rangle_{max}$	10	15	20	25	30	35	40	45	50	55	60	70	80	90	100

- Using this event classification, we calculated the mid-rapidity transverse momentum spectra of charge averaged pions, kaons, and protons in $INEL > 0$ events in the $0.1 \text{ GeV}/c < p_T < 20 \text{ GeV}/c$ region, generating 200M events
- The spectra at the highest multiplicity class along with the fitted Tsallis-Pareto curves are presented:

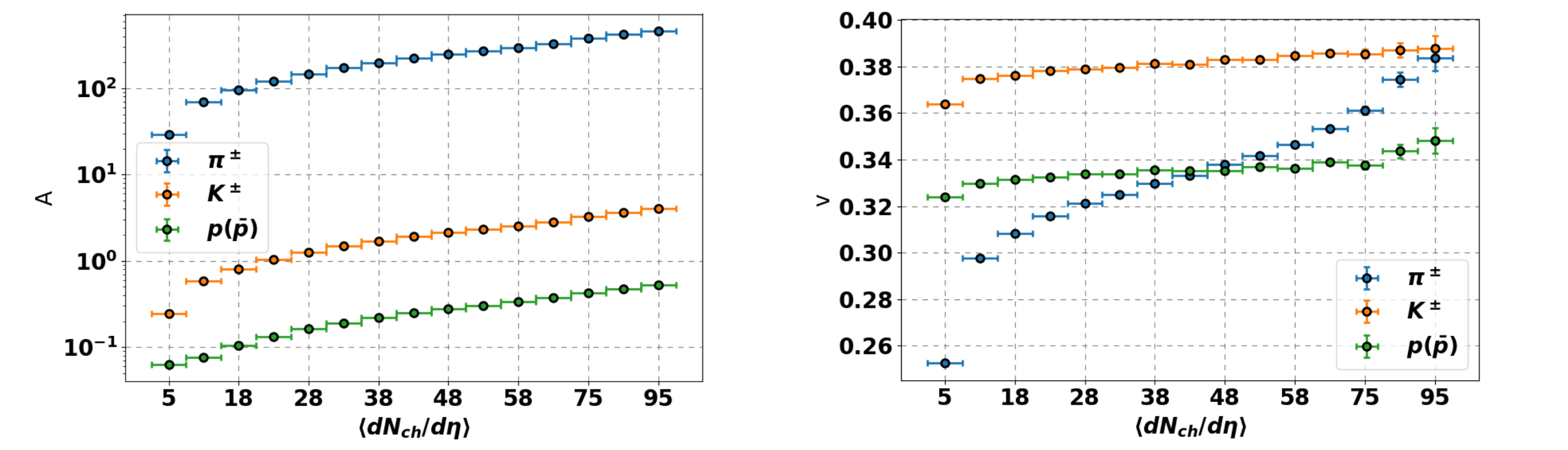


Studying the parameters in the function of the event multiplicity we can draw the following conclusions:

- With increasing multiplicity the q parameter increases for each hadron but with different slopes: the increase of q (or the decrease of n) is the largest for the heaviest hadron
- The temperature decreases slowly with the increasing pseudorapidity density
- The previously observed $T_{\pi^\pm} < T_{K^\pm} < T_{p(\bar{p})}$ mass hierarchy appears



- The radial flow velocity is not zero and also increases with the multiplicity, but also with different rates: at low multiplicity the pions (with the smallest mass) have the smallest v , but it increases rapidly with the increasing multiplicity
- The rate of increase in the case of protons and kaons are approximately the same
- The amplitudes are increasing for each hadron species with the multiplicity
- The value of pions is much higher than those of the heavier hadrons, which indicates that with increasing multiplicity the number of the produced pions grows faster than the number of kaons and protons



Summary

- Tuned HIJING++ agrees well with ALICE data such as multiplicity and p_T distributions at $\sqrt{s} = 7$ TeV
- Extracted non-extensive hadronization model parameters from $\sqrt{s} = 7$ TeV proton-proton collisions using ALICE data and tuned HIJING++ with various event multiplicity classifications
- Mass hierarchy emerges in the Tsallis parameters from both experimental and Monte Carlo data
- The parameters are sensitive to the event size and may serve as a **thermometer** of the collision
 - With increasing multiplicity: increasing q non-extensivity parameter and slight decrease of T temperature
 - In case of HIJING++, all hadrons result in a non-zero, mass dependent, increasing radial flow velocity

Acknowledgement

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