Fluctuations of net Lambda distributions measured as a function of collision energy with the STAR detector at RHIC

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Main motivation(s)

- Mapping the freeze-out curve and probing the possible critical point through fluctuations of conserved quantum numbers:
  - net charge - $\Delta Q$, net baryons - $\Delta B$, net strangeness - $\Delta S$

- Non monotonic behavior of fluctuations of conserved quantum numbers as a function of collision energy could indicate the presence of a critical point.
- Fluctuations of conserved quantum numbers are related to susceptibilities in lattice QCD: can be used to extract chemical freeze-out parameters: $T$ & $\mu_B$ in a model-independent way.
Is there a flavor hierarchy?

The temperature at which the calculated HRG susceptibility ratio deviates from LQCD calculations indicates different temperatures depending on the quark flavor (strange or light).

WB Collaboration, PRL 111, 202302 (2013)

- The temperature at which the calculated HRG susceptibility ratio deviates from LQCD calculations indicates different temperatures depending on the quark flavor (strange or light).
Moments of net multiplicity distributions and susceptibilities

• Net charged particles, net protons and net kaons have been used as proxies for net charge, net baryon number and net strangeness, respectively.

• First 4 moments: Mean (\(M\)), St.dev. (\(\sigma\)), Skewness (\(S\)) & Kurtosis (\(k\)).

• Moment ratios/products of net multiplicity distributions relate to appropriate susceptibility ratios.

• Susceptibilities:

\[
\chi_{\text{lmmn}}^{QBS} = \frac{\partial^{l+m+n}p/T^4}{\partial (\mu_B/T)^l \partial (\mu_S/T)^m \partial (\mu_Q/T)^n}
\]

\[
\chi_{n}^{QBS} = \frac{1}{VT^3} \frac{\partial^n (\ln Z)}{\partial (\mu_{QBS}/T)^n}
\]

• Relationship of moment products/ratios with susceptibilities:

\[
\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}, \quad S_B \sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}, \quad \kappa_B \sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}
\]

• Moments can be experimentally measured and then compared to modeled (HRG, LQCD) susceptibilities in order to extract FO parameters.
Interpretation of previous STAR results

HRG calculations on the basis of STAR net-proton, net-charge and net-kaon measurements
R. Bellwied et al., arXiv.1805.00088

Measure net Lambda fluctuations in order to provide a more complete strangeness proxy together with net kaons and compare with HRG predictions for sequential hadronization.
The Solenoidal Tracker At RHIC (STAR)
## Data sets and statistics

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>Statistics (M)</th>
<th>Year</th>
<th>$\mu_B$ (MeV)</th>
</tr>
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<tbody>
<tr>
<td>19.6</td>
<td>$\sim 34$</td>
<td>2011</td>
<td>205</td>
</tr>
<tr>
<td>27</td>
<td>$\sim 71$</td>
<td>2011</td>
<td>155</td>
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<tr>
<td>39</td>
<td>$\sim 114$</td>
<td>2010</td>
<td>115</td>
</tr>
<tr>
<td>62.4</td>
<td>$\sim 40$</td>
<td>2010</td>
<td>70</td>
</tr>
<tr>
<td>200</td>
<td>$\sim 221$</td>
<td>2011</td>
<td>20</td>
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Lambda sample optimized for purity & efficiency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>cut set - 1</th>
<th>cut set - 2</th>
<th>cut set - 3</th>
<th>cut set - 4</th>
<th>cut set - 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCA (V⁰ to PV)</td>
<td>&lt; 0.35</td>
<td>&lt; 0.5</td>
<td>&lt; 0.65</td>
<td>&lt; 0.8</td>
<td>&lt; 0.95</td>
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<tr>
<td>DCA (P to PV)</td>
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<td>&gt; 0.5</td>
<td>&gt; 0.4</td>
<td>&gt; 0.3</td>
<td>&gt; 0.2</td>
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<tr>
<td>DCA (π to PV)</td>
<td>&gt; 1.75</td>
<td>&gt; 1.5</td>
<td>&gt; 1.25</td>
<td>&gt; 1.0</td>
<td>&gt; 0.75</td>
</tr>
<tr>
<td>DCA (P to π)</td>
<td>&lt; 0.5</td>
<td>&lt; 0.6</td>
<td>&lt; 0.7</td>
<td>&lt; 0.8</td>
<td>&lt; 0.9</td>
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<td>Background</td>
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<td>8608</td>
<td>22908</td>
<td>34184</td>
<td>82161</td>
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<tr>
<td>Signal</td>
<td>108654</td>
<td>160737</td>
<td>196537</td>
<td>213468</td>
<td>253431</td>
</tr>
<tr>
<td>S/B</td>
<td>33.00</td>
<td>18.67</td>
<td>8.58</td>
<td>6.24</td>
<td>3.08</td>
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<tr>
<td>Purity</td>
<td>97.14%</td>
<td>94.92%</td>
<td>89.56%</td>
<td>86.20%</td>
<td>75.52%</td>
</tr>
</tbody>
</table>
Corrections applied

- **p_T-dependent efficiency correction** (based on Nonaka et al., PRC95, 064912 (2017)): was applied and compared to p_T-independent correction and the difference was *found to be small in peripheral bins and negligible in central bins for both cumulant ratios* (c2/c1, c3/c2). p_T-dependence was applied for varying number of p_T-bins.

- **Feed-down correction**: from multi-strange hyperons varies between 15% and 25% depending on collision energy and centrality. Feed-down changes the single cumulants, but *found to be negligible for both cumulant ratios*.

- **Centrality bin width correction** (based on Luo, Xu, arXiv:1701.02105): was applied and *found to be negligible for both cumulant ratios*.

- The data shown in the following are CBW corrected and have a p_T-dependent efficiency correction applied. They are not feed-down corrected.
### Statistical uncertainty estimation

- Sub-sampling method was used to estimate the statistical uncertainties.
- Number of samples to be greater than 10 in order to be consistent with bootstrapping.
- Both Delta method and sub-sampling gave similar results.

### Systematic uncertainty estimation

<table>
<thead>
<tr>
<th>Source</th>
<th>Variations</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCA of $P$ to $\pi$</td>
<td>$&lt; 0.6$ (default)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 0.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 0.55$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 0.65$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 0.70$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&gt; 0.5$ &amp; $&gt; 1.5$ (default)</td>
<td>11.2%</td>
</tr>
<tr>
<td></td>
<td>$&gt; 0.6$ &amp; $&gt; 1.7$</td>
<td></td>
</tr>
<tr>
<td>DCA of $\pi$ to $PV$ &amp; $P$ to $PV$</td>
<td>$&gt; 0.55$ &amp; $&gt; 1.6$</td>
<td>20.3%</td>
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<tr>
<td></td>
<td>$&gt; 0.45$ &amp; $&gt; 1.4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&gt; 0.4$ &amp; $&gt; 1.3$</td>
<td></td>
</tr>
<tr>
<td>$n\sigma$ ($\pi$) &amp; $n\sigma$ ($P$)</td>
<td>$&lt; 2.0$ &amp; $&lt; 2.0$ (default)</td>
<td>44.9%</td>
</tr>
<tr>
<td></td>
<td>$&lt; 2.5$ &amp; $&lt; 2.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 1.5$ &amp; $&lt; 1.5$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$ &amp; $\varepsilon$ (default)</td>
<td>$\varepsilon \times (1 + 2.25%)$ &amp; $\varepsilon \times (1 + 2.25%)$</td>
<td>23.6%</td>
</tr>
<tr>
<td>$\text{Eff} (\Lambda)$ &amp; $\text{Eff} (\bar{\Lambda})$</td>
<td>$\varepsilon \times (1 - 2.25%)$ &amp; $\varepsilon \times (1 - 2.25%)$</td>
<td></td>
</tr>
</tbody>
</table>
Baselines and model predictions

**Baselines**

- **Central limit theorem**: Cumulants as a function of number of participant nucleons.
- **Poisson**: Cumulants as a function of mean of individual particle distributions. (if $M = \sigma^2$)
- **Negative binomial distributions**: Cumulants as a function of mean and variance of individual particle distributions. (if $M < \sigma^2$)

**Models**

**UrQMD**: A hadronic transport model used to investigate hadron yields, transverse spectra, strangeness production. No QGP crossover. See https://urqmd.org/

**HRG**: Statistical model for the low temperature region of QCD phase diagram. Thermodynamic observables can be approximated (even at non-zero chemical potentials). See arXiv.1805.00088
Single cumulants

STAR Preliminary

Au + Au collisions at STAR

0.9 < p_t (GeV/c) < 2.0

|y| < 0.5
Cumulant ratio, $C_2/C_1$

(a). 19.6 GeV  
(b). 27 GeV  
(c). 39 GeV  

(d). 62.4 GeV  
(e). 200 GeV  

Au + Au collisions at STAR  
Net $\Lambda$, $C_2/C_1$  
$0.9 < p_T\ (GeV/c) < 2.0$  
$|y| < 0.5$  

- Net $\Lambda$  
- NBD  
- Poisson  
- UrQMD
Energy dependence of cumulant ratios

(a) \( \frac{C_2}{C_1} \)

- Net \( \Lambda \), 0-5%
- Net \( \Lambda \), 50-60%
- NBD
- Poisson
- UrQMD

Au + Au Collisions at STAR

(b) \( \frac{C_3}{C_2} \)

0.9 \( < p_T \) (GeV/c) \( < 2.0 \)

|yl| < 0.5

STAR Preliminary
Comparison to STAR net-p and net-k measurements

Net proton, STAR Coll., PRL112, 032302 (2014)
Net kaon, STAR Coll., PLB 785, 551 (2018)

(a) 0-5% central collisions
- Net + data
- Net kaon, data
- Net proton, data

STAR Preliminary

(b) Net : 0.9 < p_T (GeV/c) < 2.0, |y| < 0.5
Net proton : 0.4 < p_T (GeV/c) < 0.8, |y| < 0.5
Net kaon : 0.2 < p_T (GeV/c) < 1.6, |y| < 0.5

STAR Preliminary
Higher order ratios are prone to dynamical effects, thus not reliable in the context of FO interpretation (P. Alba, et. al., PLB 738 (305) 2014)

see talk by Jamie Stafford in this session
Rapidity dependence

Baryon Number Conservation?:
Rustamov, PBM, Stachel, NPA 960, 114 (2017)

\[ \alpha_{acc} = \frac{\langle N_{acc}^B \rangle}{\langle N_{4\pi}^B \rangle} \]

\[ \frac{C_2 (n_B - n_{\bar{B}})}{C_2 (\text{Skellam})} = 1 - \alpha_{acc} \]

\[ \alpha_1 = \frac{\langle N_{\Lambda}^{acc} \rangle}{\langle N_{\Lambda}^{4\pi} \rangle} \]

\[ 19.6 \text{ GeV, data} \]
\[ 200 \text{ GeV, data} \]
\[ 19 \text{ GeV, } 1 - (\alpha_S + \alpha_B) \]
\[ 200 \text{ GeV, } 1 - (\alpha_S + \alpha_B) \]

 works if B and S conservation are treated additive
Baryon-Strangeness Correlations

Determined by the ratio of off-diagonal to diagonal cumulants: \( \kappa_{B,S}^{1,1}/\kappa_S^2 \)

The related susceptibility ratio: \( C_{B,S} = -3\chi_{B,S}^{1,1}/\chi_S^2 \)

(-3 is just a normalization factor so that the asymptotic value = +1)

How fast does the BS correlator approach the asymptotic value?
Is there room for bound states above \( T_c \)?

Koch, Majumder, Randrup (2005), Mueller, Majumder (2006)

Ratti, Bellwied, Cristoforetti, Barbaro (2011)
Contributions to the 2\textsuperscript{nd}-order off-diagonal BS cumulant

STAR measurements: pK contribution, arXiv:1903.05370

HRG predictions: see talk by P. Parotto, this afternoon

\[ C_{p,K} = \frac{\sigma_{p,K}}{\sigma_{K}} \]

\[ 0.4 < p_T < 1.6 \text{ GeV/c} \]

\[ |\eta| < 0.5 \]

\[ \sqrt{s_{NN}} \text{ (GeV)} \]

\[ 0.4 \text{ GeV/c} \leq p_T \leq 4.0 \text{ GeV/c} \]

\[ |y| \leq 0.5 \]

\[ \Lambda \text{ contribution} \]

\[ C_2(\text{net-K}) / C_2(\text{net-K}) \]

\[ 0 \leq 0.5 \]

\[ |p_T| \leq 2.0, |y| < 0.5 \]

\[ |p_T| \leq 1.6, |y| < 0.5 \]

\[ \sqrt{s} \text{ [GeV]} \]

\[ \text{STAR Preliminary} \]

\[ \text{STAR Au+Au, 0-5\%} \]

\[ \text{HRG PDG16+ (} T_0 = 166\text{MeV)} \]

\[ \text{HRG PDG16+ (} T_0 = 145\text{MeV)} \]

\[ \text{STAR Au+Au, 0-5\%} \]
• The first three single cumulants and cumulant ratios of net Lambda distributions were analyzed as a function of collision centrality, energy and rapidity. Statistical and systematic uncertainties were estimated.

• Results were corrected for reconstruction efficiency in a $p_T$-dependent way. Feed-down and centrality bin width corrections show negligible effects on the cumulant ratios.

• Results can be described by Poisson and NBD expectations.

• There is no non-monotonic behavior of cumulants or cumulant ratios as a function of collision centrality or energy between 19.6 and 200 GeV.

• The variance show significant deviation from UrQMD, which increases as a function of collision centrality and energy.

• The net Lambda fluctuations can be described by the latest HRG model.
Physics conclusions

- The chemical freeze-out parameters calculated from the net-Lambda $c_2/c_1$ in a HRG model are consistent with kaon freeze-out conditions and significantly above proton freeze-out conditions.

- The rapidity dependence of the net-Lambda variance shows small deviations from NBD expectations for larger rapidity coverage, which could be attributed to the effect of baryon number and strangeness conservation.

- Lambda fluctuations are the dominant contribution to the non-diagonal BS correlator, and their strength compared to the measured pK correlations in STAR is in agreement with expectations from the HRG model.
Backup
Contributions to the 2\textsuperscript{nd} order off-diagonal BS cumulant

HRG predictions: see talk by P. Parotto, this afternoon

\[ \mu_B = 0 \]

\[ 0.4 \text{ GeV/c} \leq p_T \leq 4.0 \text{ GeV/c} \]

\[ |y| \leq 0.5 \]
\[ p_T\text{-dependent efficiency correction} \]


\[ P(n) = \sum_{N} P(N) B_{p,N}(n) \]
\[ B_{p,N}(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \]
\[ P(N) = P (N_1, N_2, \ldots, N_M) \]
\[ q(r,s) = q(a^r/\varepsilon^s) = \sum_{i=1}^{M} (a^r_i/\varepsilon^s_i) n_i \]

First 3 cumulants:
\[ C_1 = M_1 \]
\[ C_2 = M_2 - M_1^2 \]
\[ C_3 = 2M_2^3 - 3M_1M_2 + M_3 \]
Net Lambda distributions

Event by event Net-Lambda ($\Lambda - \bar{\Lambda}$) distributions at STAR:
0.9 < $p_T$ (GeV/c) < 2.0
$|y| < 0.5$

- a). 19.6 GeV
- b). 27 GeV
- c). 39 GeV
- d). 62.4 GeV
- e). 200 GeV

Counts

$\Lambda - \bar{\Lambda}$
Net Lambda moments - uncorrected

(a) Au + Au collisions
   Net $\Lambda$
   $0.9 < p_T \text{ (GeV/c)} < 2.0$
   $|y| < 0.5$

(b) STAR Preliminary

(c) STAR Preliminary

(d) 19.6 GeV
    27 GeV
    39 GeV
    62.4 GeV
    200 GeV
    CLT

27
Effect of centrality bin-width correction (CBWC)
Effect of feed-down correction
# Track and V0 cuts

<table>
<thead>
<tr>
<th>Particle</th>
<th>Cut parameter</th>
<th>Cut boundary</th>
</tr>
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<tbody>
<tr>
<td>Proton</td>
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</tr>
<tr>
<td></td>
<td>Rapidity</td>
<td>$</td>
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<tr>
<td></td>
<td>nFitPoints</td>
<td>$&gt; 15$</td>
</tr>
<tr>
<td></td>
<td>nHitsFit/NFitPoss</td>
<td>$&gt; 0.52$</td>
</tr>
<tr>
<td></td>
<td>PID</td>
<td>$n\sigma_p &lt; 2.0$</td>
</tr>
<tr>
<td></td>
<td>DCA to PV</td>
<td>$&gt; 0.5cm$</td>
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<tr>
<td></td>
<td>DCA to $\pi$</td>
<td>$&lt; 0.6cm$</td>
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<td>Pion</td>
<td>Transverse momentum</td>
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<td>$V^0$</td>
<td>Transverse momentum</td>
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<td>Decay length</td>
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<td>Pointing away from PV</td>
<td>$(r_{v0} - r_{pv}) \cdot p_{v0} &gt; 0$</td>
</tr>
</tbody>
</table>
Factorial moments

Number of positive/negatively charged particles

Indices of the factorial moment

Upper bound

\[
f_{i,k}(n_p, n_{\bar{p}}) = \left< \frac{n_p!}{(n_p - i)!} \frac{n_{\bar{p}}!}{(n_{\bar{p}} - k)!} \right> = \sum_{n_p=i}^{\infty} \sum_{n_{\bar{p}}=k}^{\infty} p(n_p, n_{\bar{p}}) \frac{n_p!}{(n_p - i)!} \frac{n_{\bar{p}}!}{(n_{\bar{p}} - k)!}
\]

\[
F_{i,k}(N_p, N_{\bar{p}}) = \frac{f_{i,k}(n_p, n_{\bar{p}})}{(e_p)^i (e_{\bar{p}})^k}
\]

Cumulants:

\[
N = \langle N_p \rangle + \langle N_{\bar{p}} \rangle = \frac{\langle n_p \rangle}{e_p} + \frac{\langle n_{\bar{p}} \rangle}{e_{\bar{p}}}
\]

\[
C_1 \equiv K_1 = \langle N_p \rangle - \langle N_{\bar{p}} \rangle
\]

\[
C_2 \equiv K_2 = N - K_1^2 + F_{02} - 2F_{11} + F_{20}
\]

\[
C_3 \equiv K_3 = K_1 + 2K_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30} - 3K_1 (N + F_{02} - 2F_{11} + F_{20})
\]

[No. of events in a given cent.]