Hyperons in thermal QCD

FASTSUM Collaboration

Gert Aarts, <u>Chris Allton</u>, Sergio Chaves, Davide de Boni, Jonas Glesaaen,
Simon Hands, Alan Kirby, Aleksandr Nikolaev, Sam Offler, Chrisanthi Praki
Swansea University

Benjamin Jäger
Southern Denmark University

Seyong Kim
Sejong University, Korea

Maria Paola Lombardo
INFN, Florence

Sinead Ryan
Trinity College, Dublin

Jon-Ivar Skullerud
National University of Ireland, Maynooth, Ireland

Liang-Kai Wu
Jiangsu University, China

Overview

- Baryons in medium: *rarely studied!*
- FASTSUM approach
  - Anisotropy
  - New, lighter ensemble + finer ensemble
- **Hyperons at non-zero T**
  - Hadron Resonance Gas (for “warm” baryons)
  - Parity doubling (for “hot” baryons)
- Other FASTSUM Results
  - **Bottomonium**
  - Conductivity
  - Dense matter mesons
- Conclusions

Baryons in a medium

Previous Work:

Lattice studies of baryons at finite temperature very limited, ***(all quenched)***

- screening masses De Tar and Kogut 1987
- with a small chemical potential
  QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

Effective models, mostly at $T \sim 0$ and nuclear density $\Rightarrow$ parity doubling models

De Tar & Kunihiro 89 Mukherjee, Schramm, Steinheimer & Dexheimer, Sasaki 2017

Our Work:

PRD 92 (2015) 014503 [arXiv:1502.03603]

JHEP 06 (2017) 034 [arXiv:1703.09246]

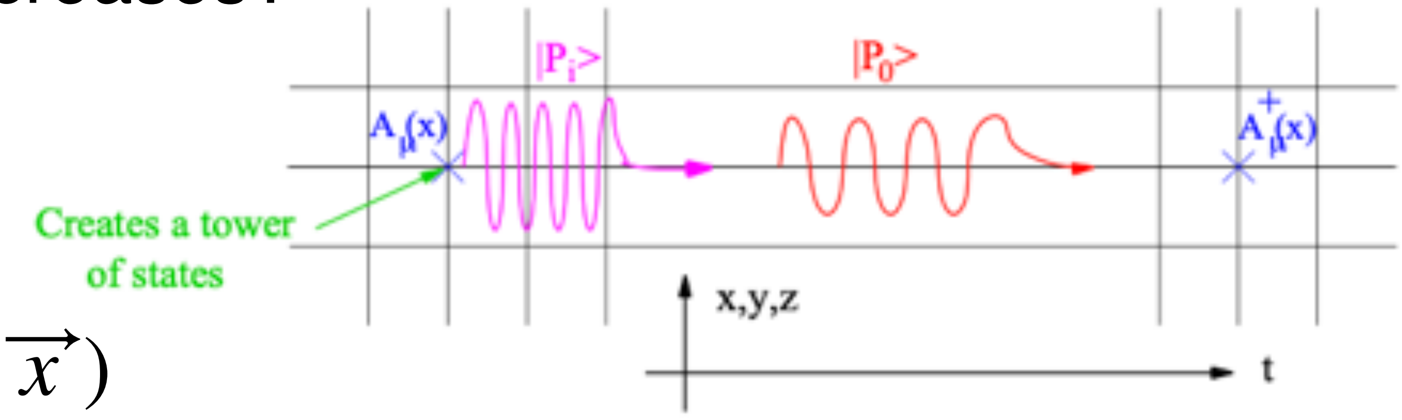
Phys.Rev. D99 (2019) no.7, 074503 [arXiv:1812.07393]

Parity

No parity doubling in (T=0) Nature:

+ve parity: $m_+ = m_N = 0.939$ GeV
-ve parity: $m_- = m_{N*} = 1.535$ GeV

Question: What happens as T increases?



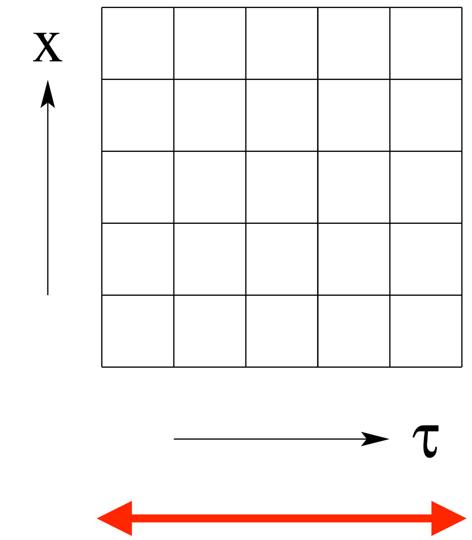
Lattice:
Parity operation:

$$P\mathcal{O}(\tau, \overrightarrow{x})P^{-1} = \gamma_4 \mathcal{O}(\tau, -\overrightarrow{x})$$

Construct correlation functions:

$$G_\pm(\tau) = \int d\mathbf{x} \, \langle \mathrm{tr} O(\mathbf{x}, \tau) P_\pm \overline{O}(\mathbf{0}, 0) \rangle \,, \qquad P_\pm = \frac{1}{2}(\mathbb{1} \pm \gamma_4)$$

Symmetries



Charge conjugation (at zero density):

$$G_\pm(\tau) = -\,G_\mp(1/T - \tau) \qquad (\ast)$$

i.e. **positive**/**negative** parity states propagate **forward**/**backward** in τ

Eg. for a single state: $G_+(\tau) = A_+ e^{-m_+\tau} + A_- e^{-m_-(1/T-\tau)}$

(Contrasts with meson sector)

Chiral symmetry:

Constrains spinor structure so that $G_+(\tau) = -\,G_-(\tau)$ ie. parity doubling: $m_+ = m_-$

Together with ($\ast$) $\longrightarrow G_+(\tau) = G_+(1/T - \tau)$ i.e. **forward/back** symmetry

Question: Does this happen in Nature in deconfined phase?

- assuming $m_q \sim 0$
- what about the strange-quark sector

Nucleon Correlators

Nucleon: $$O_N^\alpha(x) = \epsilon_{abc} u_a^\alpha(x) \left(d_b^T(x) C Y_n u_c(x) \right)$$

Different Operator Choices: $Y_4 = \gamma_5$, $Y_5 = \gamma_4\gamma_5$ and $Y_6 = \frac{1}{2}(Y_4 + Y_5)$

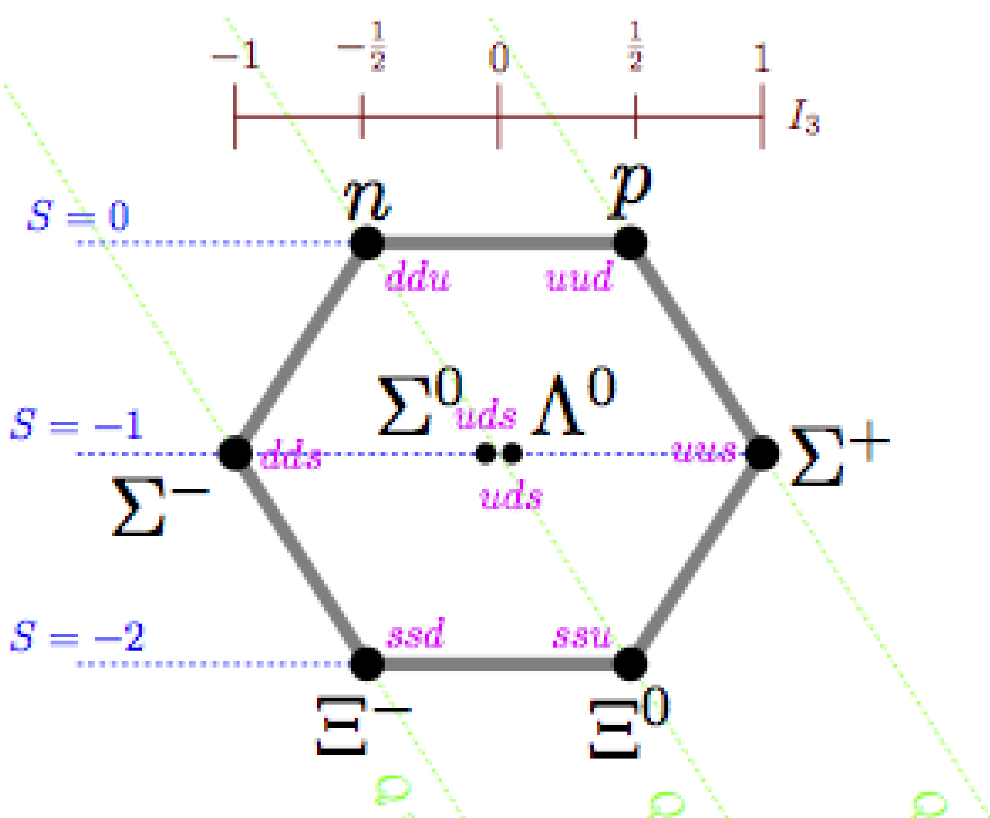
Delta: $$O_{\Delta,i}^\alpha(x) = \epsilon_{abc} \left[2u_a^\alpha(x) \left(d_b^T(x) C \gamma_i u_c(x) \right) + d_a^\alpha(x) \left(u_b^T(x) C \gamma_i u_c(x) \right) \right]$$

Omega: $$O_{\Omega,i}^\alpha(x) = \epsilon_{abc}\, s_a^\alpha(x) \left(s_b^T(x) C \gamma_i s_c(x) \right)$$

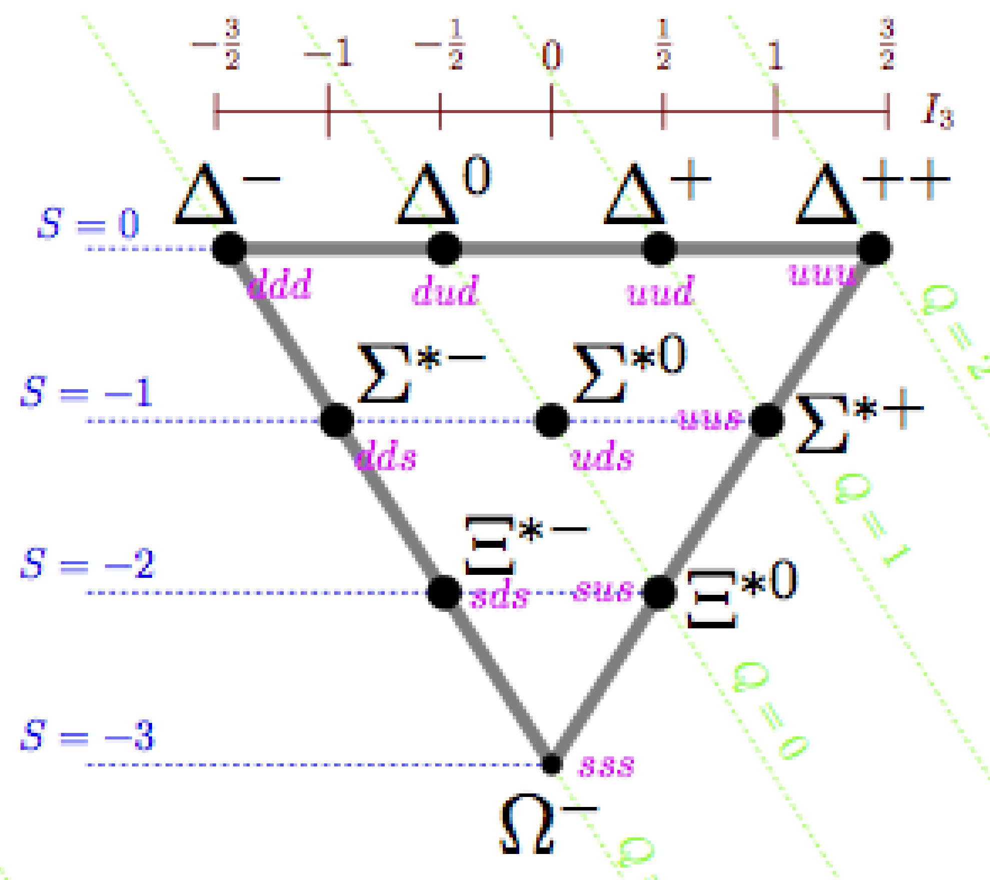
$$G_\pm(\tau) = \int d\mathbf{x} \left\langle \mathrm{tr} O(\mathbf{x},\tau) P_\pm \overline{O}(\mathbf{0},0) \right\rangle , \qquad P_\pm = \frac{1}{2}(\mathbb{1} \pm \gamma_4)$$

Baryons

Spin 1/2 octet

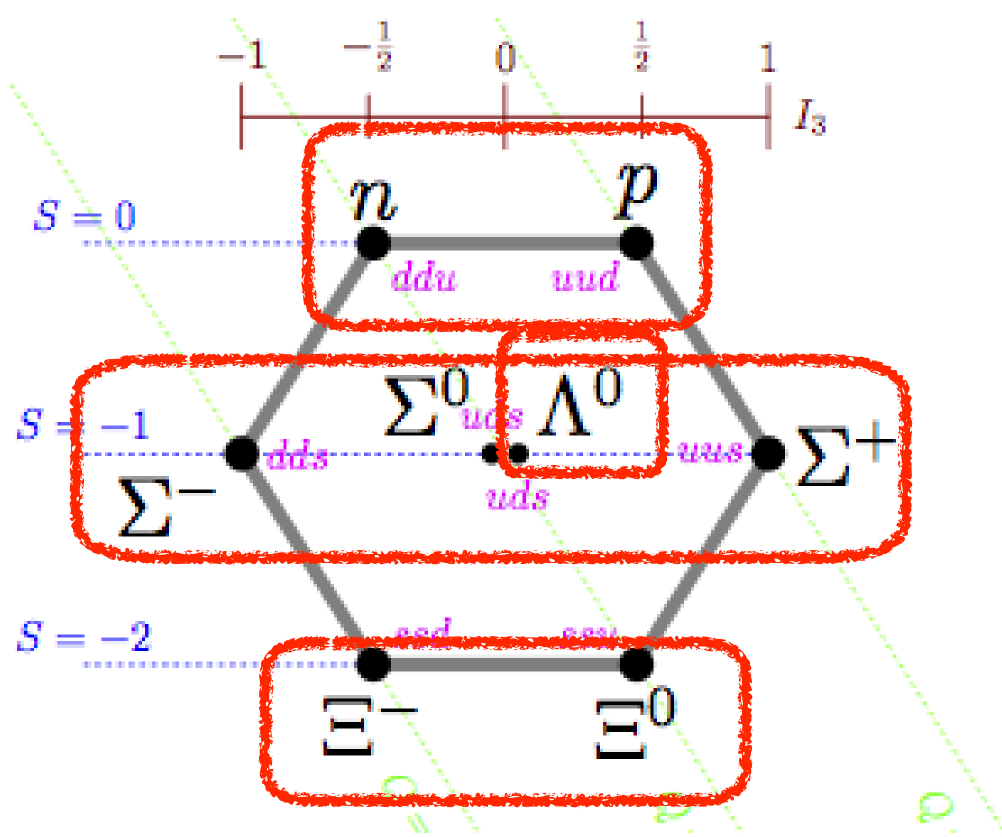


Spin 3/2 decuplet



Baryons: No isospin

Spin 1/2 octet



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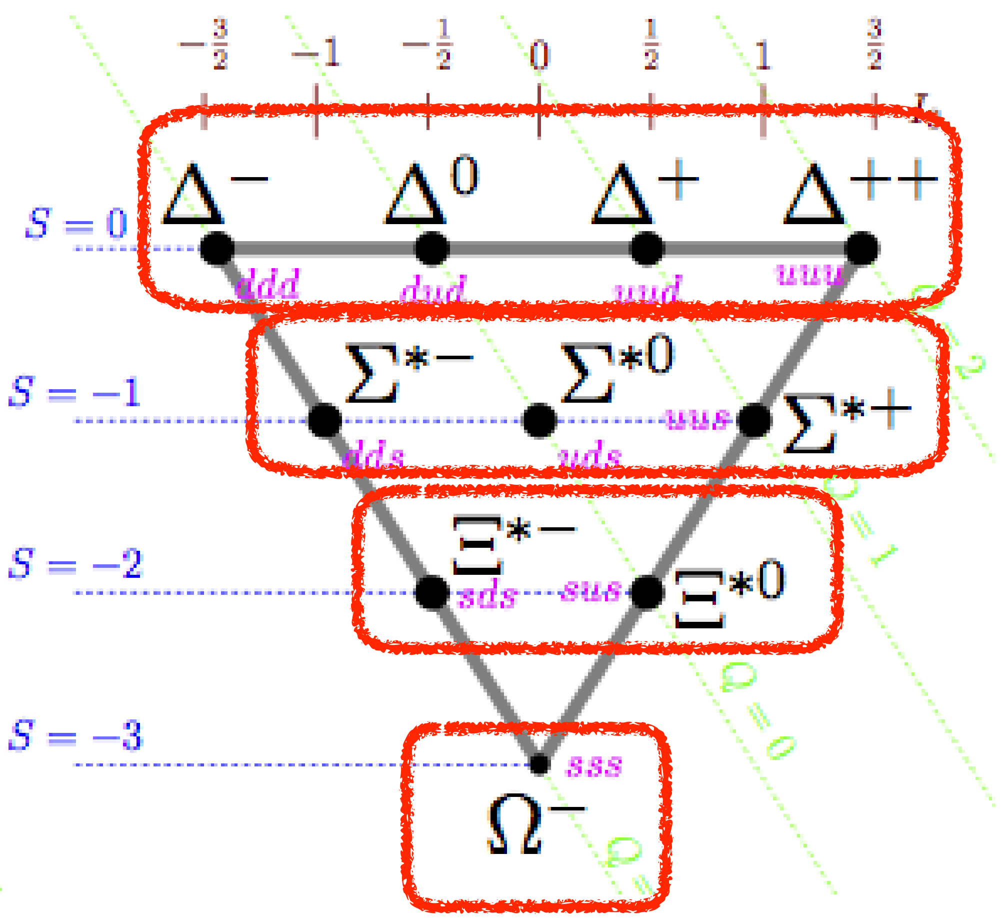
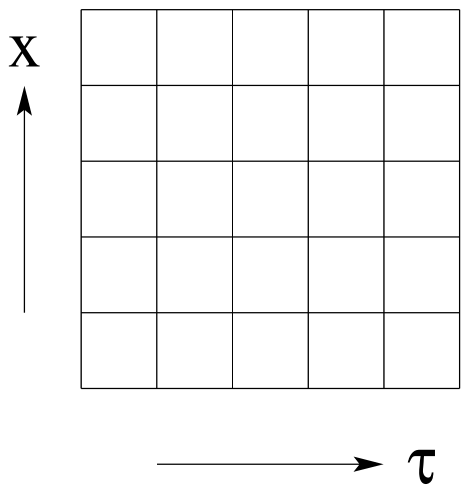
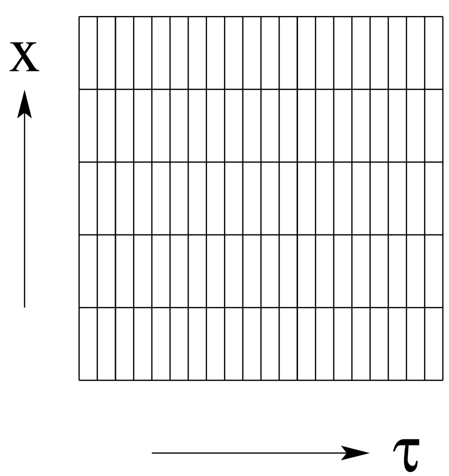


Lattice Parameters - Generation 2





$$T = 1/L_\tau$$
$$= 1/(a_\tau N_\tau)$$

(2+1) flavour

Quark Mass: M_π = 392(4) MeV

Lattice Spacing: a_s = 0.123 fm

Anisotropy: a_s/a_t = 5.6

Spatial Volume: $(3\text{fm})^3$ - $(4\text{fm})^3$

N_s	N_τ	T [MeV]	T/T_c	N_{src}	N_{cfg}
24	128	44	0.24	16	139
24	40	141	0.76	4	501
24	36	156	0.84	4	501
24	32	176	0.95	2	1000
24	28	201	1.09	2	1001
24	24	235	1.27	2	1001
24	20	281	1.52	2	1000
24	16	352	1.90	2	1001

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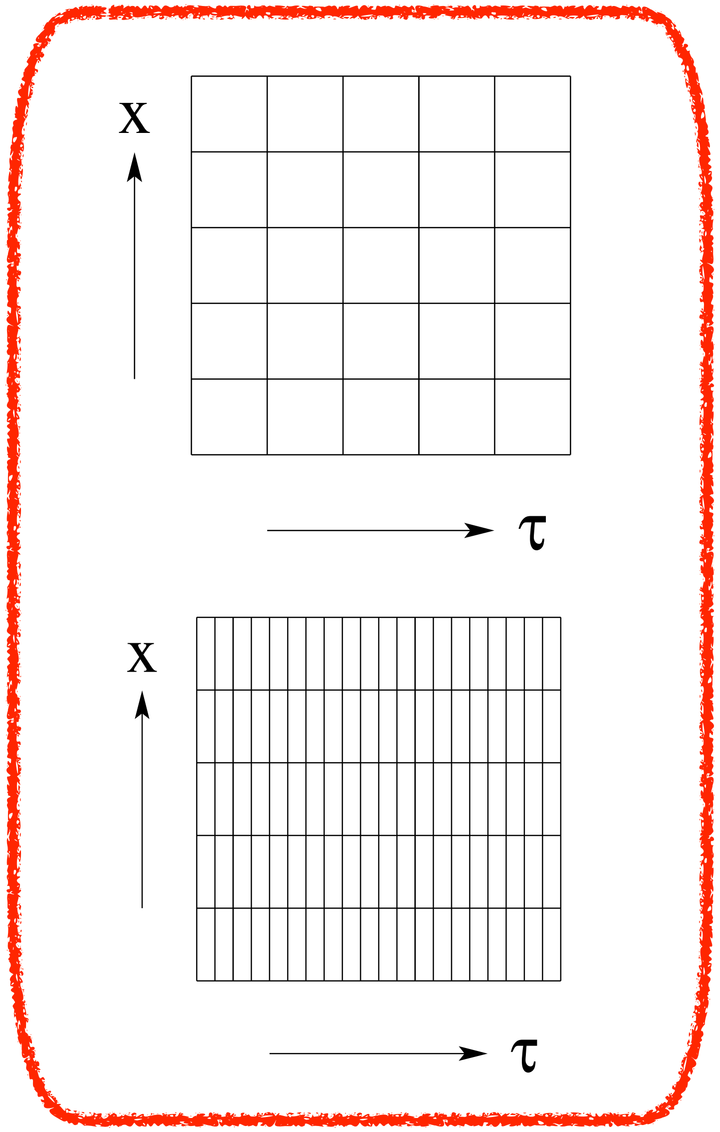

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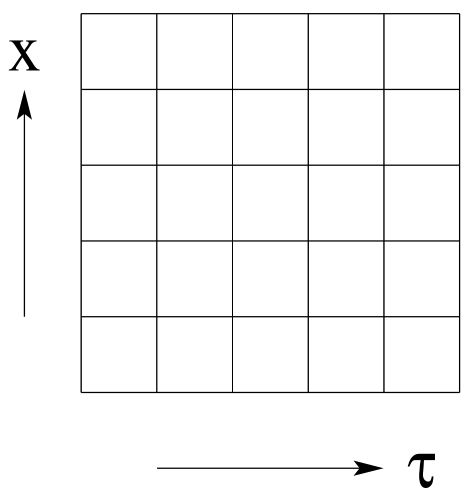

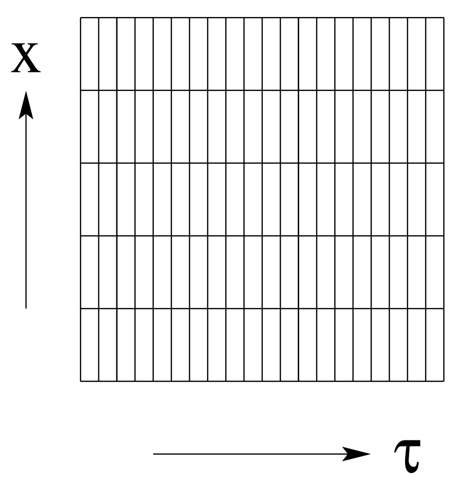

$$T = 1/L_\tau = 1/(a_\tau N_\tau)$$

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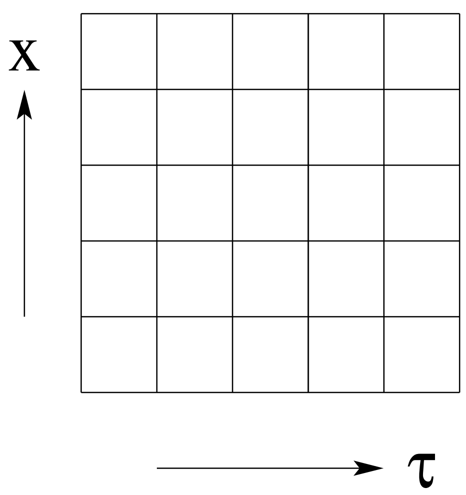

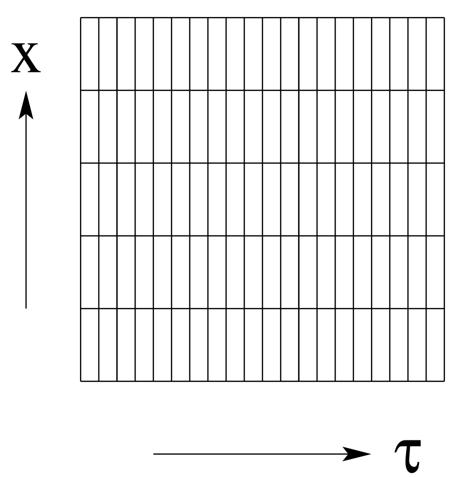

$T = 1/L_\tau$

$= 1/(a_\tau N_\tau)$

T_c = 185(4) MeV

N_τ=128 ensembles from

Hadron Spectrum Collaboration

N_s	N_τ	T [MeV]	T/T_c	N_{src}	N_{cfg}
24	128	44	0.24	16	139
24	40	141	0.76	4	501
24	36	156	0.84	4	501
24	32	176	0.95	2	1000
24	28	201	1.09	2	1001
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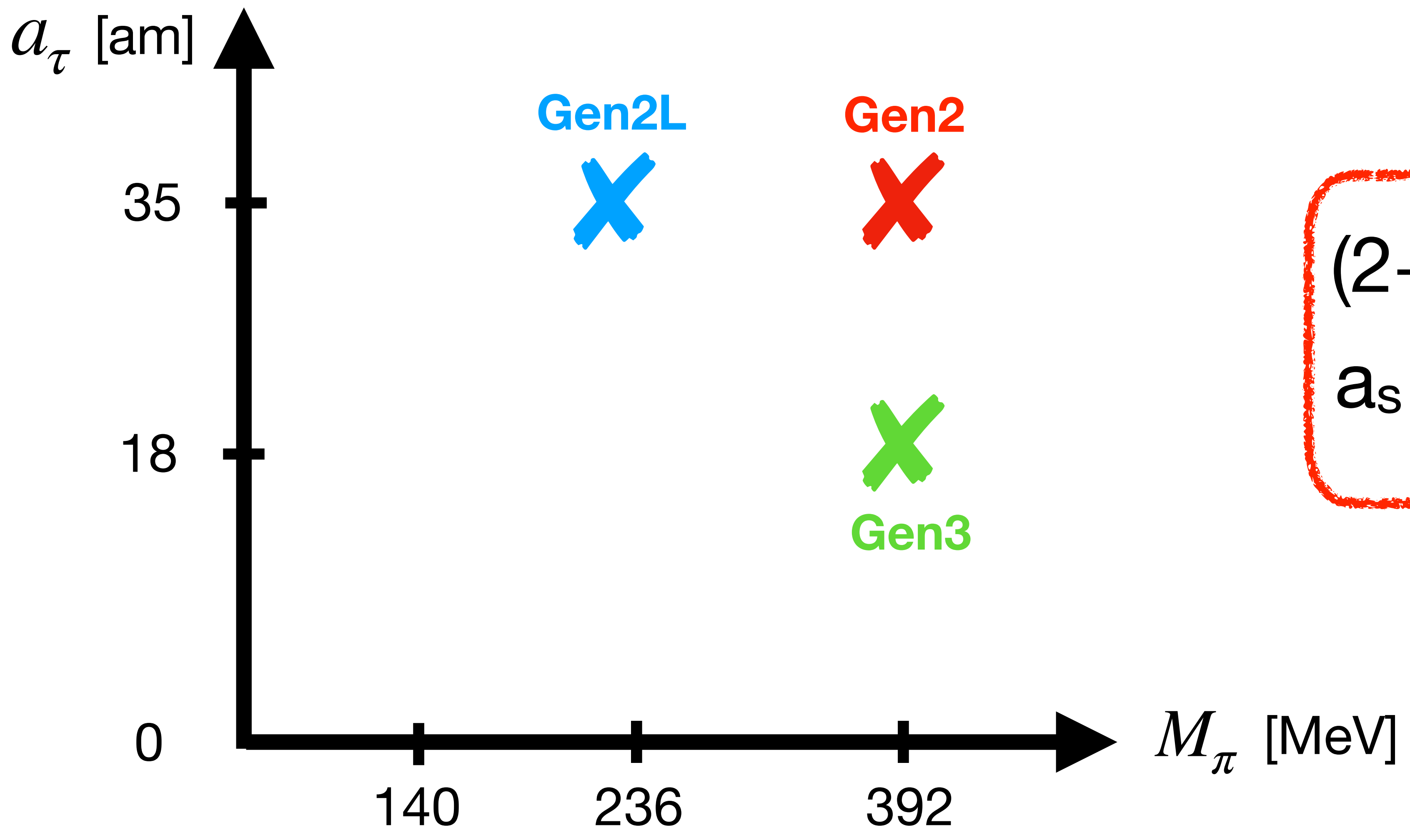
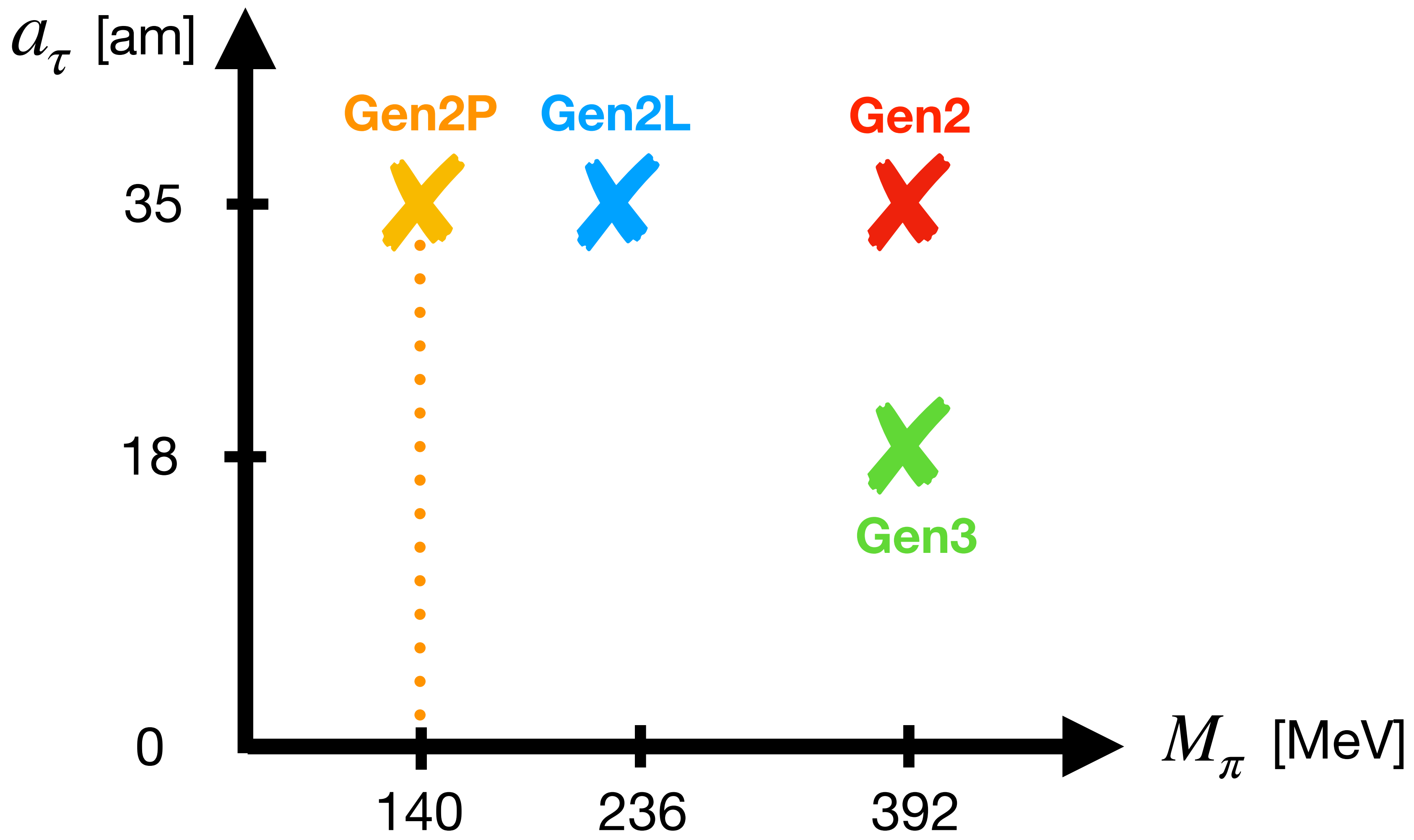
Lattice Parameters



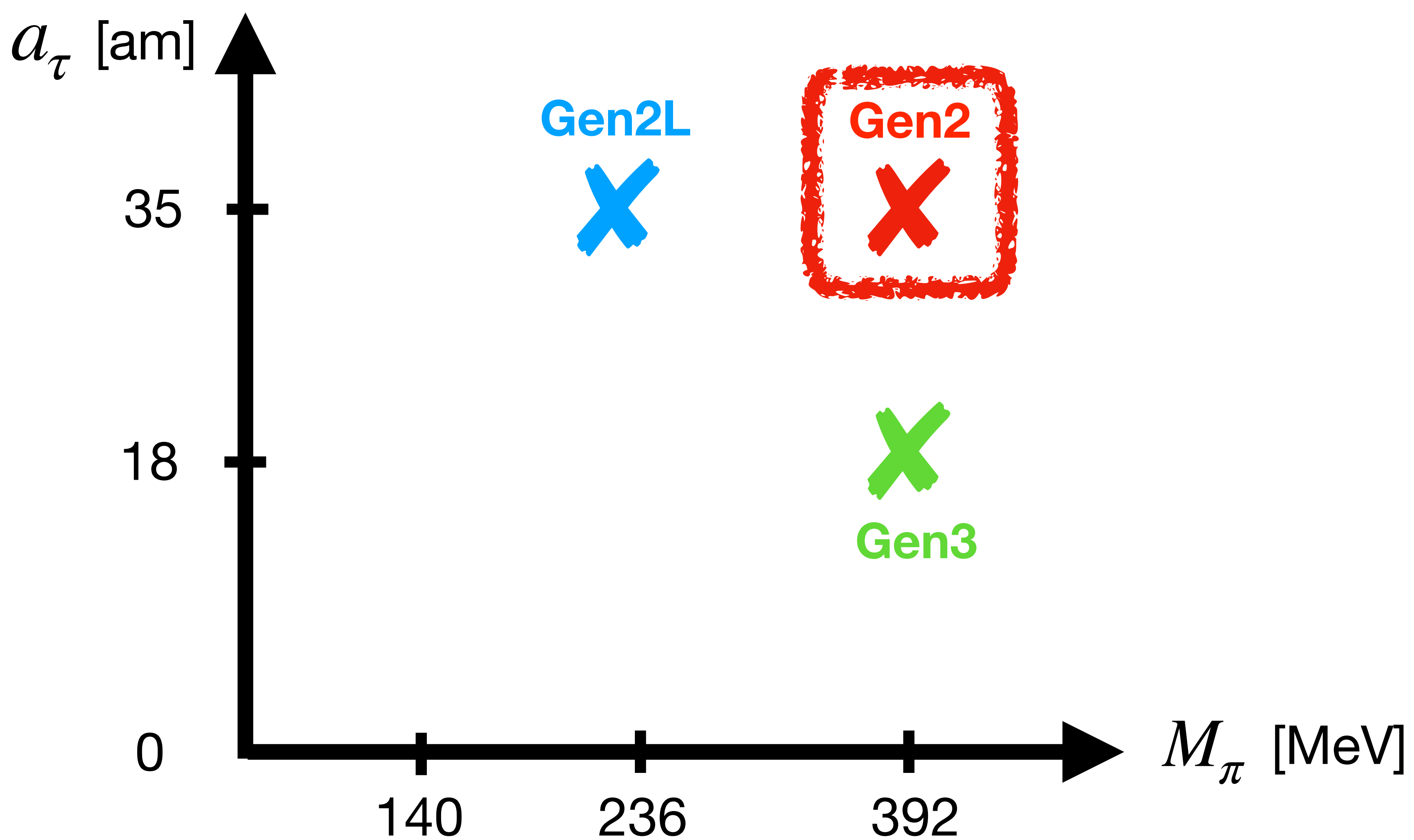
(2+1) flavour

a_s ~ 0.12 fm

Lattice Parameters

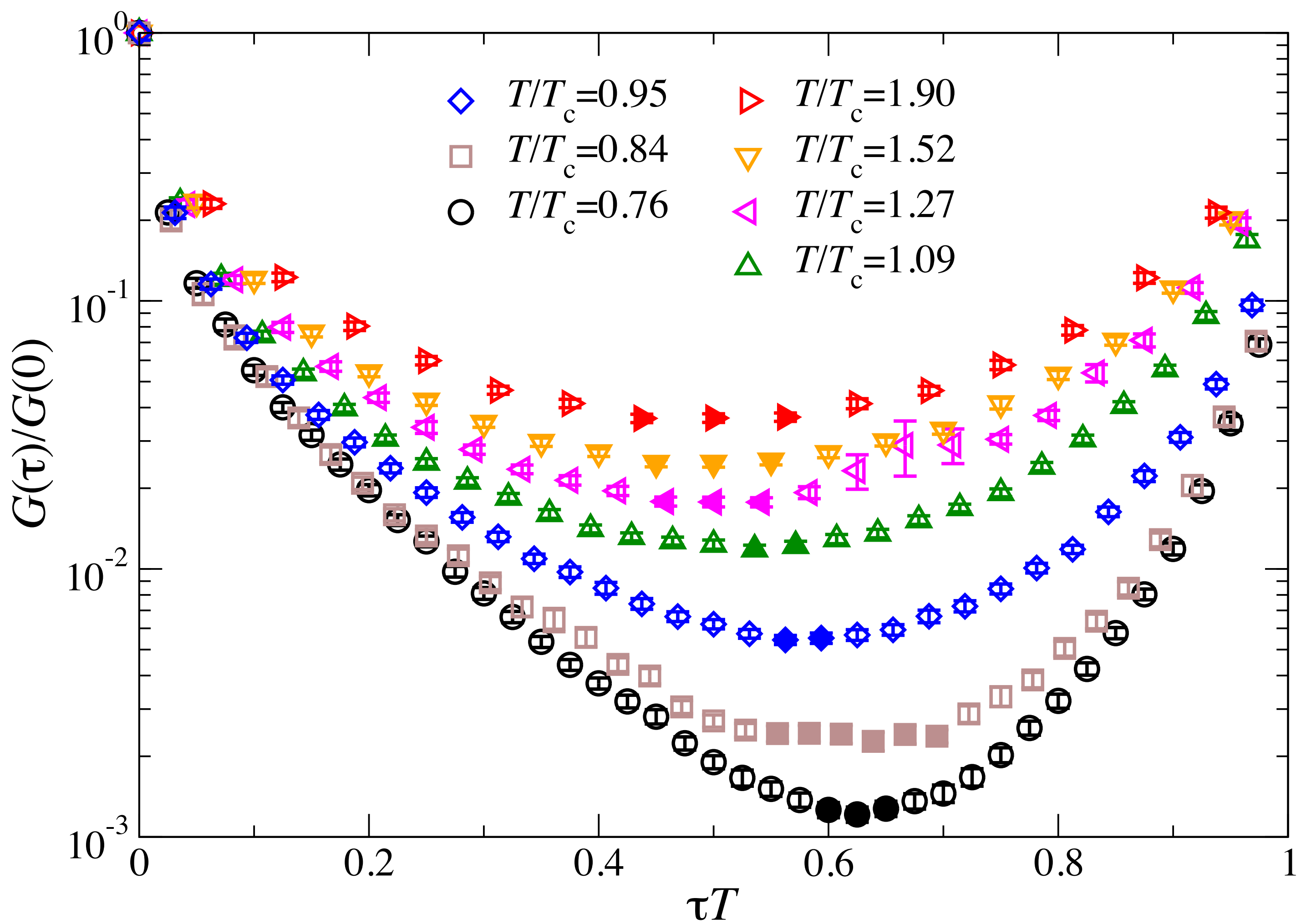


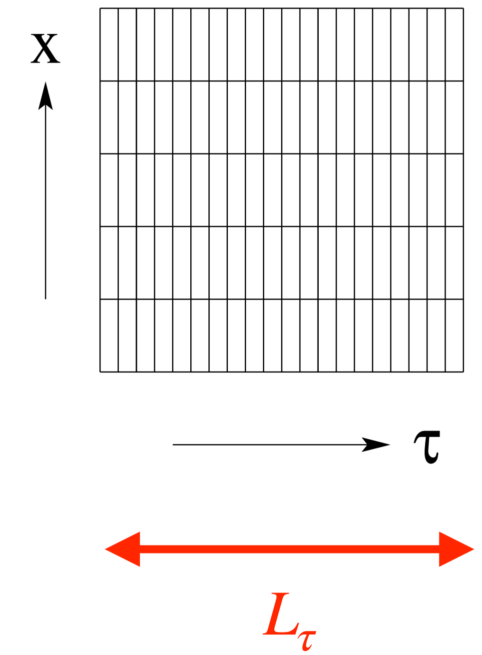
Generation 2 results



Lattice Nucleon Correlator: G_+

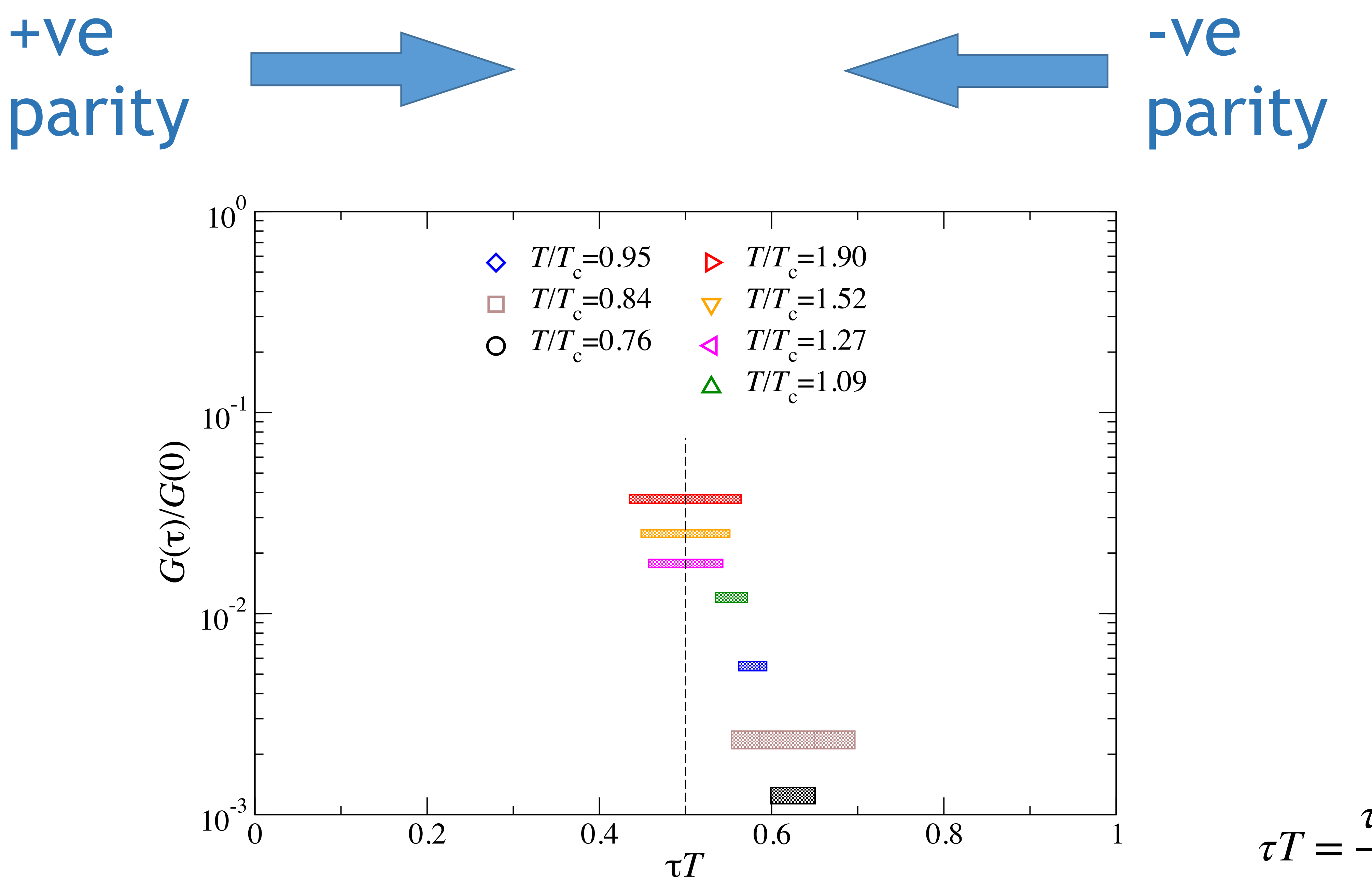
+ve parity → ← -ve parity





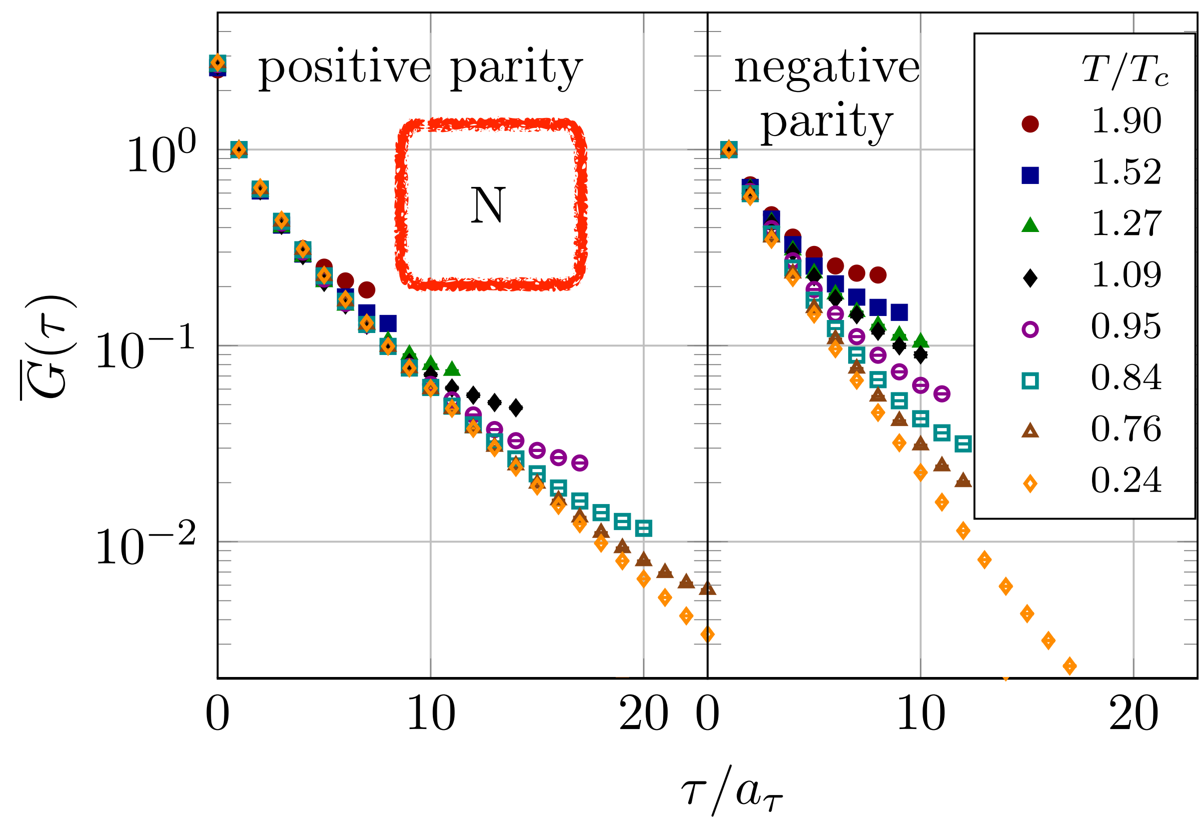
$$\tau T = \frac{\tau}{a_\tau N_\tau} \equiv \frac{\tau}{L_\tau}$$

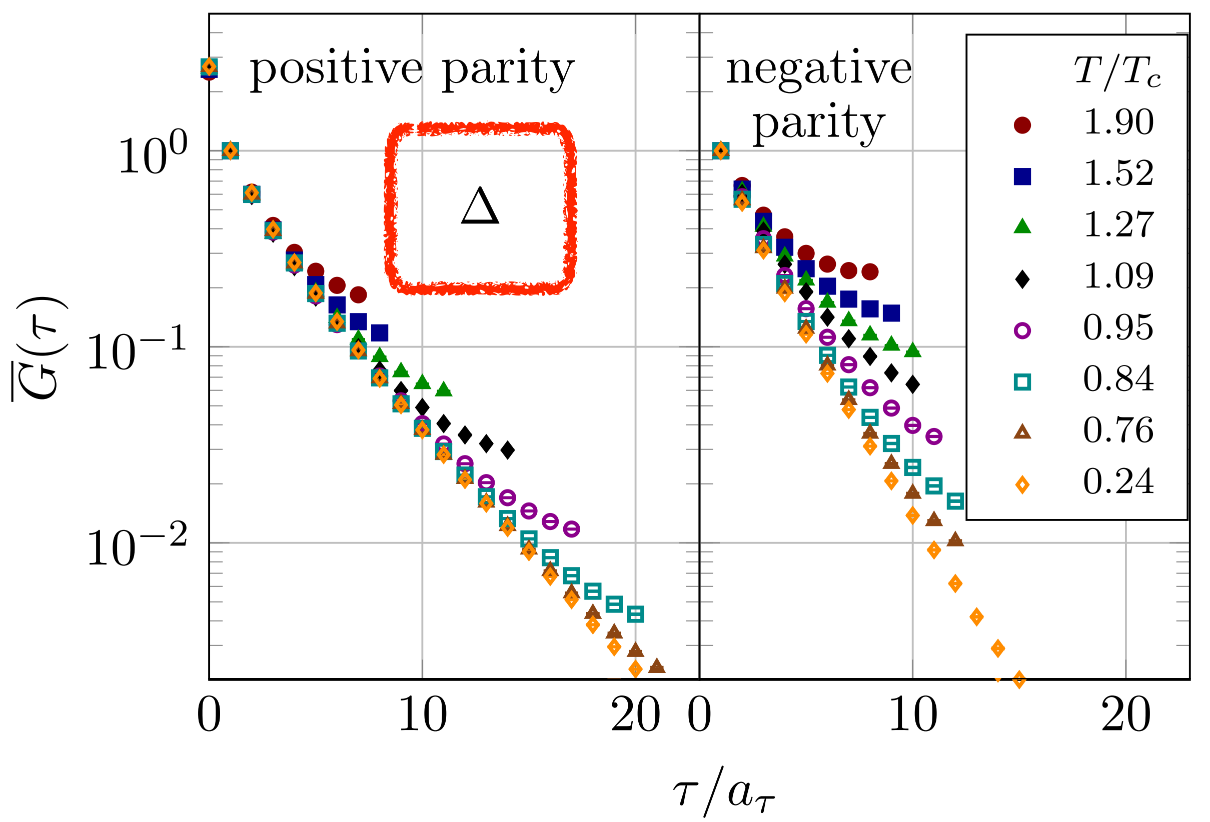
Lattice Nucleon Correlator: G_+

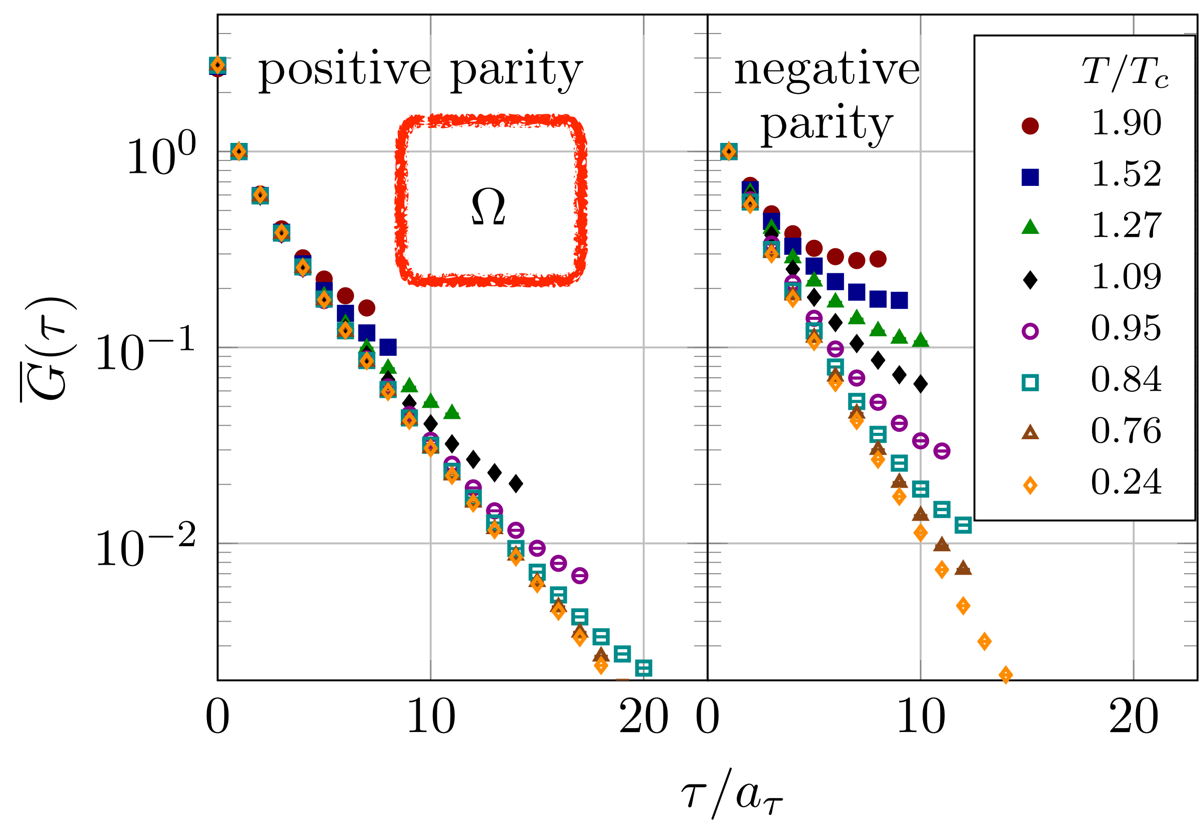


$$\tau T = \frac{\tau / a_\tau}{N_\tau}$$

Raw Correlators

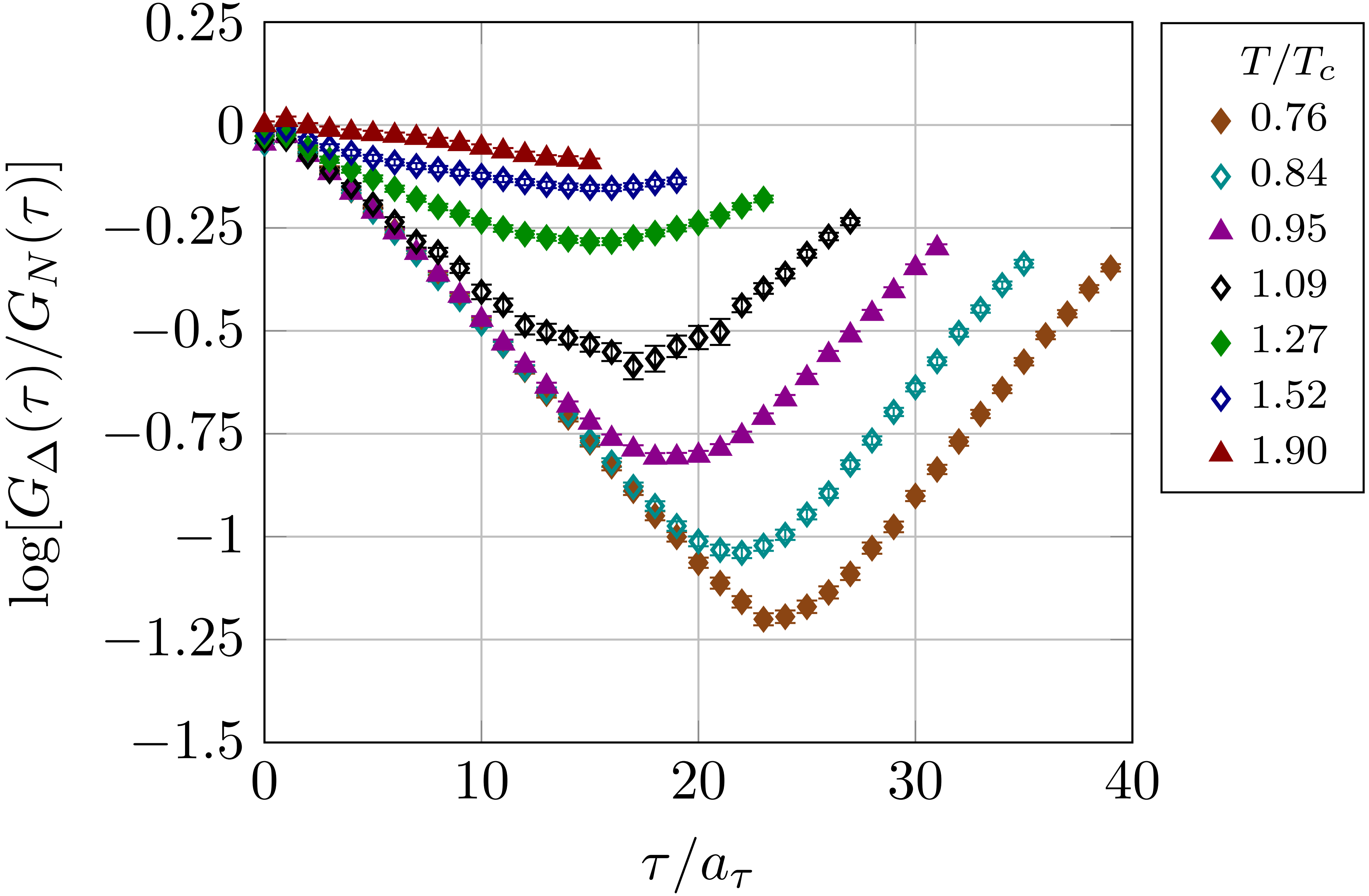






Delta cf Nucleon

$$G_+(\tau) \; =? \; A_+ e^{-m_+\tau} + A\_e^{-m\_(1/T-\tau)}$$

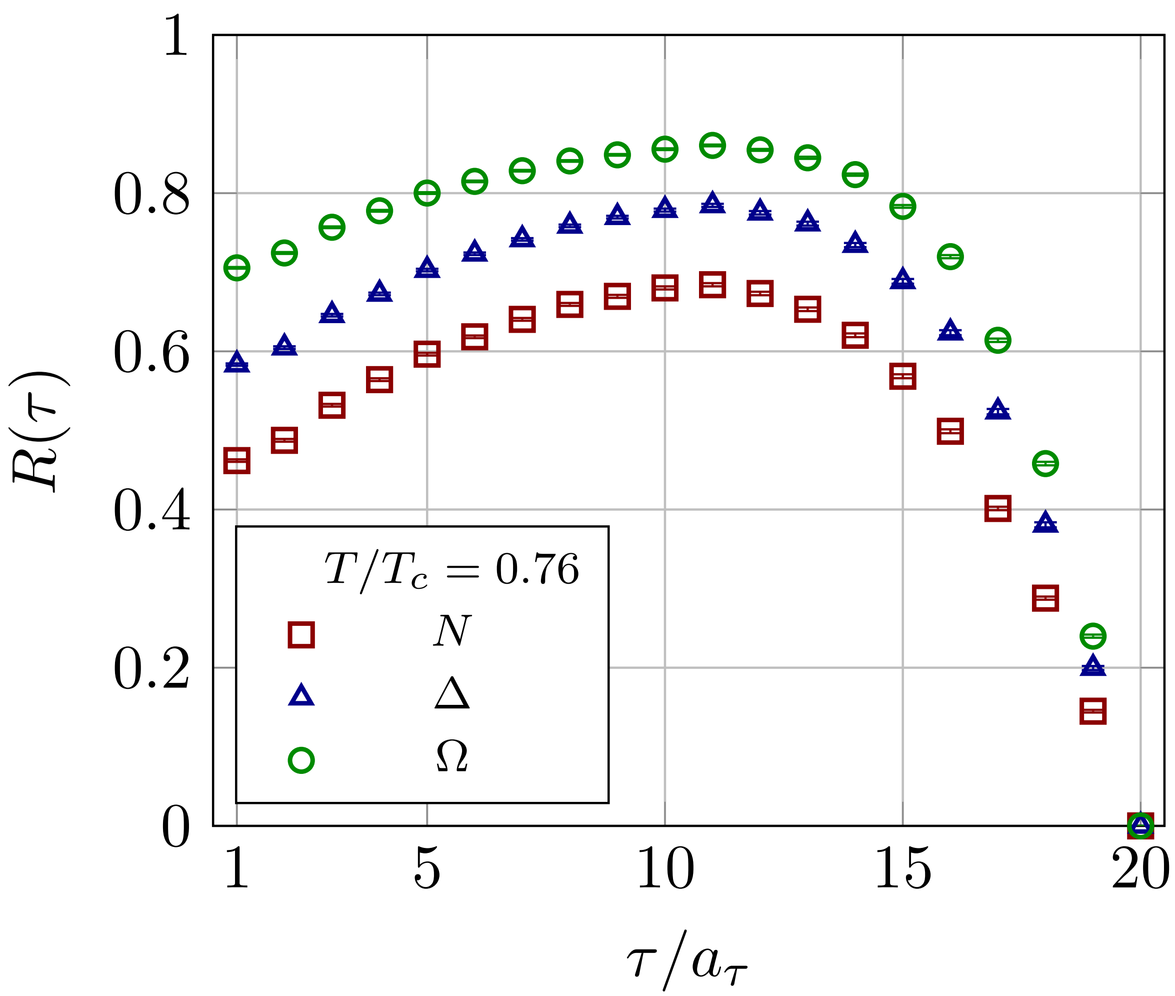


$$R(\tau) = \frac{G_+(\tau) - G_+(1/T - \tau)}{G_+(\tau) + G_+(1/T - \tau)}$$

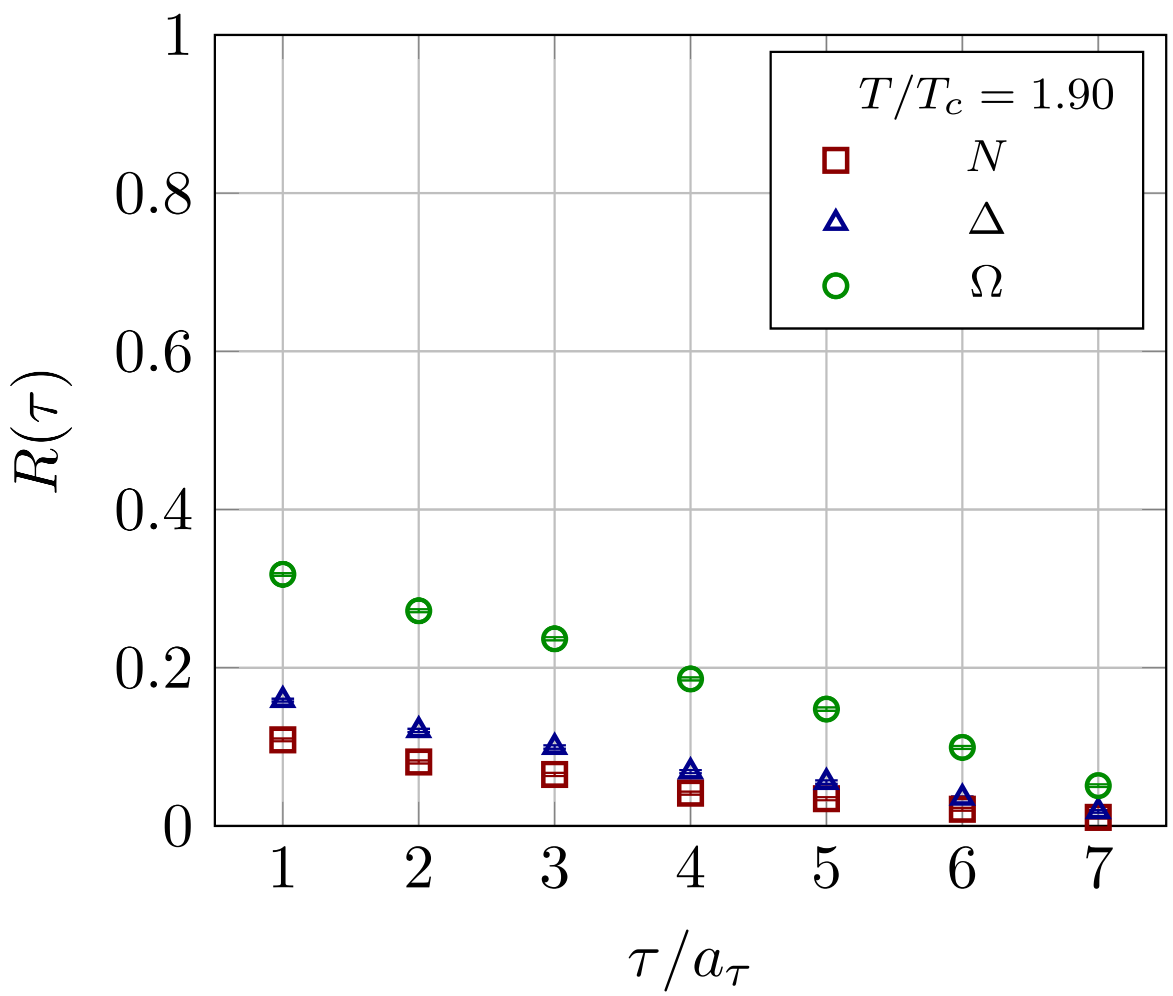
$R(\tau) \sim 0 \longrightarrow$ parity doubling

$R(\tau) \sim 1 \longrightarrow$ parity max broken

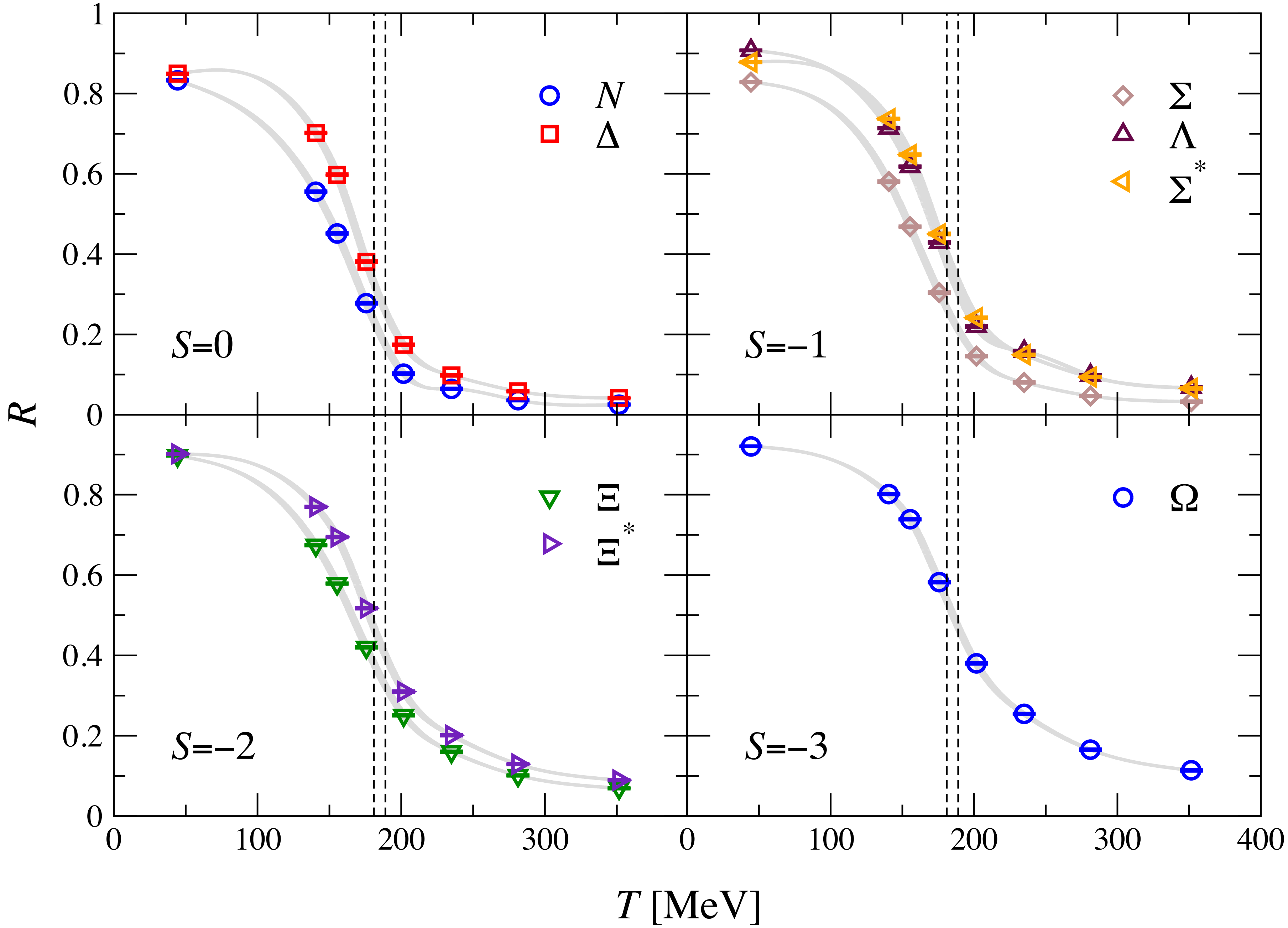
T/Tc = 0.76



T/Tc = 1.90

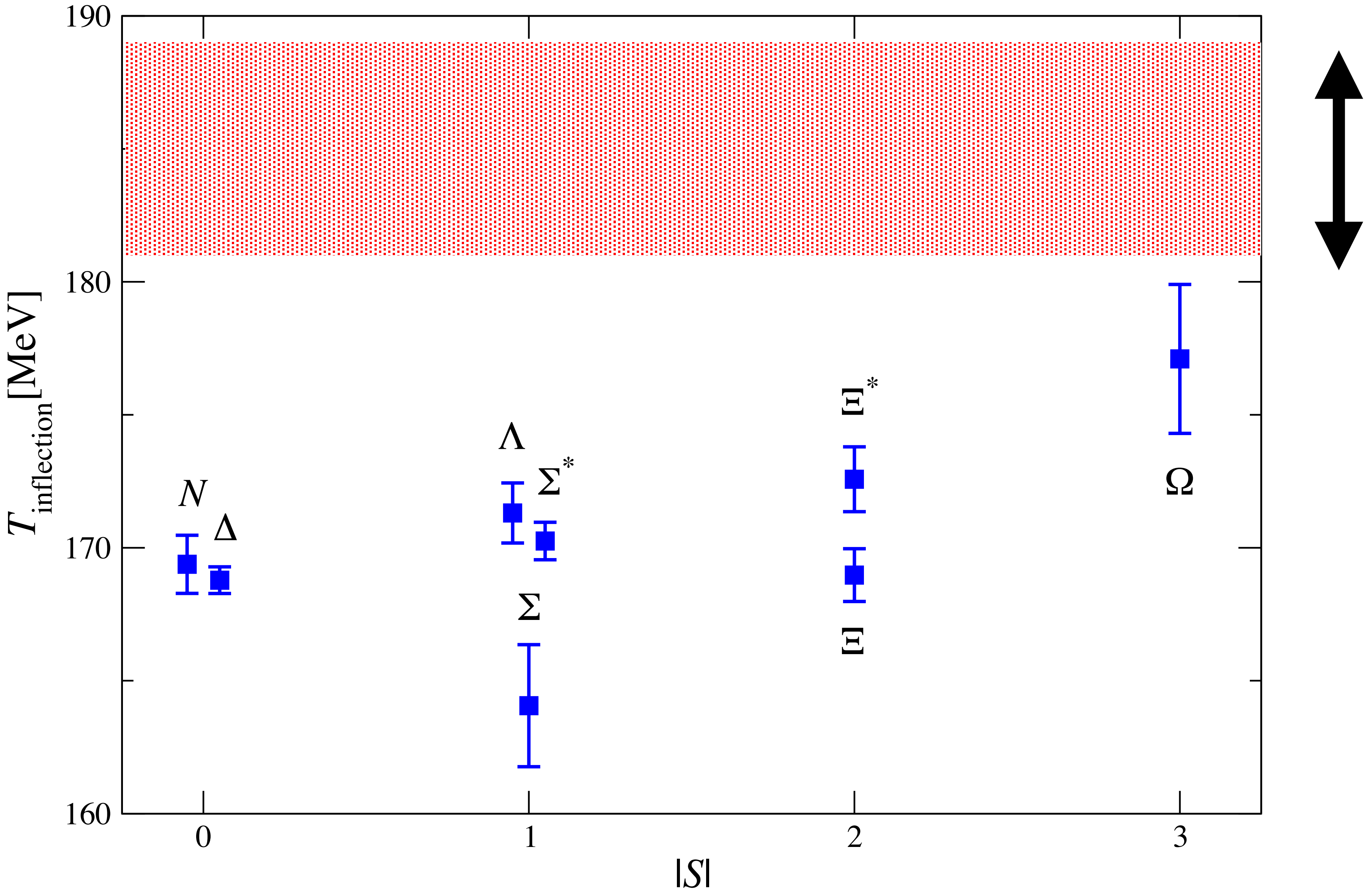


$$R \equiv \frac{\sum_{n=1}^{N_\tau/2-1} R(\tau_n)/\sigma^2(\tau_n)}{\sum_{n=1}^{N_\tau/2-1} 1/\sigma^2(\tau_n)}$$



- Cross-over occurs ~T_c
- effect of heavier s-quark visible

Point of Inflection versus T_C



T_C from
Polyakov Loop

Masses from exponential fits

(confined phase)

$$G(\tau) = A_+ e^{-M_+\tau} + A_- e^{-M_-\tau}$$

S		$I(J^P)$	$T/T_c = 0.24$	0.76	0.84	0.95	PDG
0	N	$\frac{1}{2}(\frac{1}{2}^+)$	1159(13)	1192(39)	1169(53)	1104(40)	939
		$\frac{1}{2}(\frac{1}{2}^-)$	1778(52)	1628(104)	1425(94)	1348(83)	1535
	Δ	$\frac{3}{2}(\frac{3}{2}^+)$	1459(58)	1521(43)	1449(42)	1377(37)	1232
		$\frac{3}{2}(\frac{3}{2}^-)$	2138(117)	1898(106)	1734(97)	1526(74)	1710
-1	Σ	$1(\frac{1}{2}^+)$	1277(13)	1330(38)	1290(44)	1230(33)	1193
		$1(\frac{1}{2}^-)$	1823(35)	1772(91)	1552(65)	1431(51)	1750
	Λ	$0(\frac{1}{2}^+)$	1248(12)	1293(39)	1256(54)	1208(26)	1116
		$0(\frac{1}{2}^-)$	1899(66)	1676(136)	1411(90)	1286(75)	1405–1670
	Σ^*	$1(\frac{3}{2}^+)$	1526(32)	1588(40)	1536(43)	1455(35)	1385
		$1(\frac{3}{2}^-)$	2131(62)	1974(122)	1772(103)	1542(60)	1670–1940
-2	Ξ	$\frac{1}{2}(\frac{1}{2}^+)$	1355(9)	1401(36)	1359(41)	1310(32)	1318
		$\frac{1}{2}(\frac{1}{2}^-)$	1917(27)	1808(92)	1558(76)	1415(50)	1690–1950
	Ξ^*	$\frac{1}{2}(\frac{3}{2}^+)$	1594(24)	1656(35)	1606(40)	1526(29)	1530
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-3	Ω	$0(\frac{3}{2}^+)$	1661(21)	1723(32)	1685(37)	1606(43)	1672
		$0(\frac{3}{2}^-)$	2193(30)	2092(91)	1863(76)	1576(66)	2250

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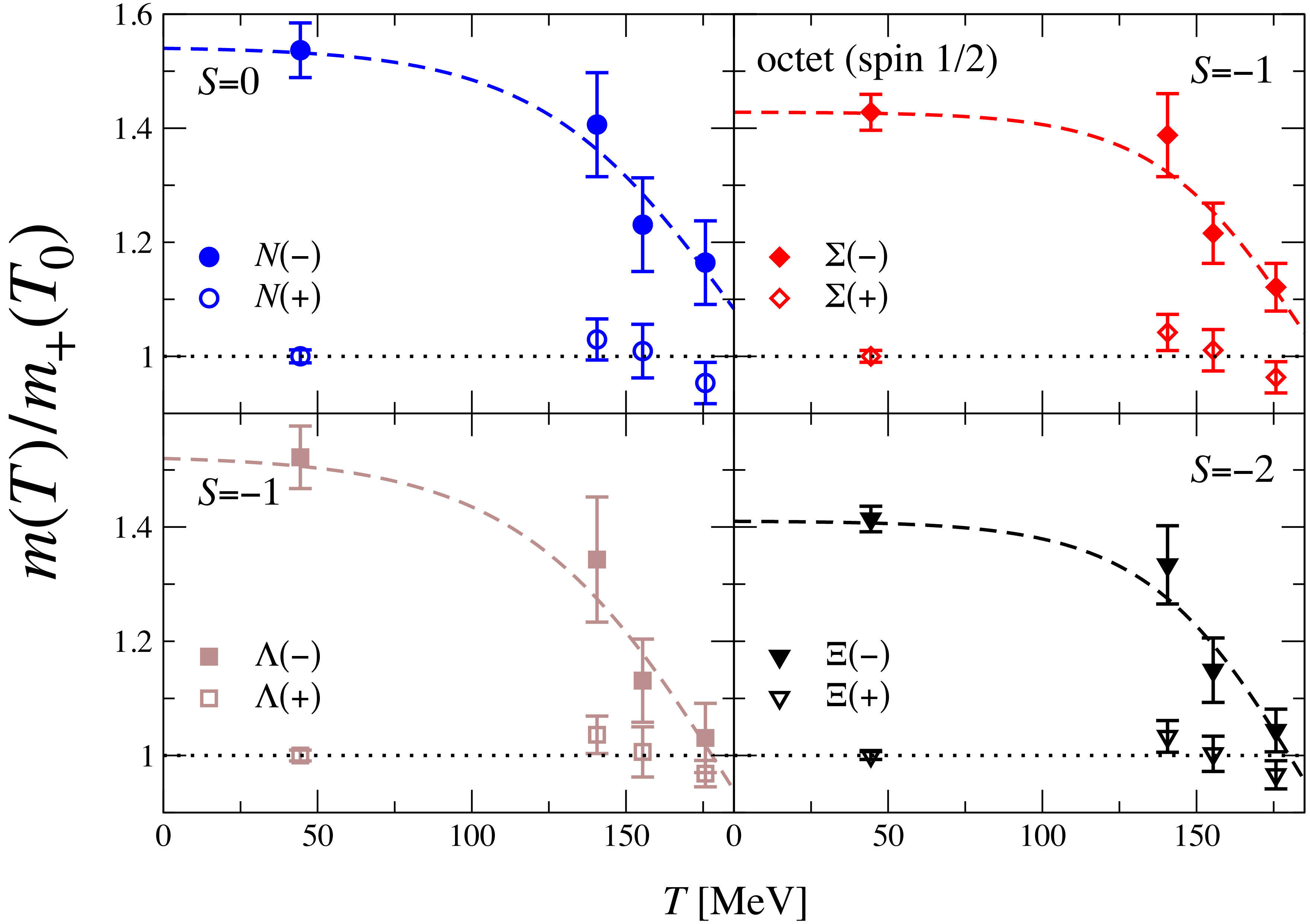
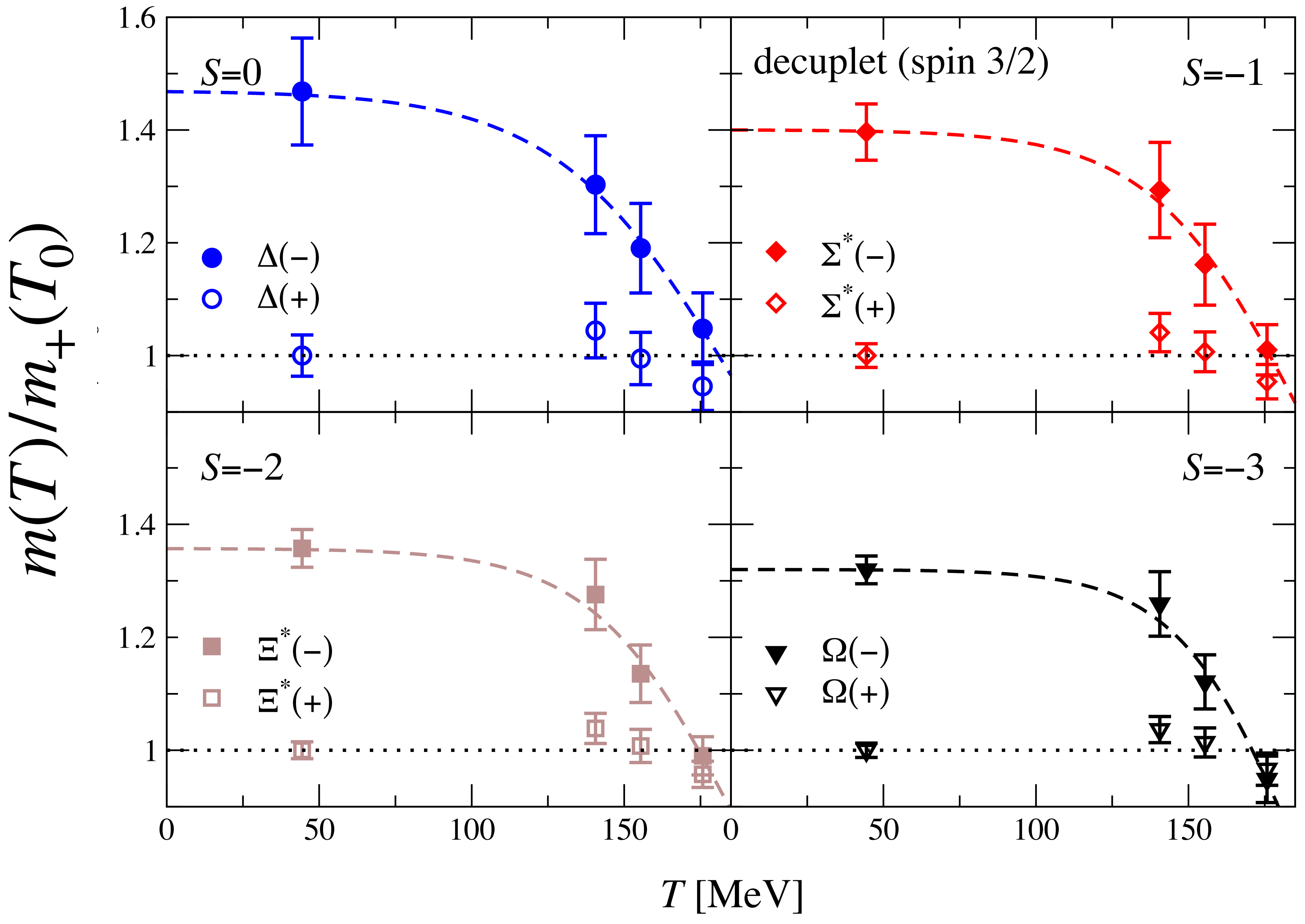
~300 MeV

Octet Masses versus T



Decuplet Masses versus T

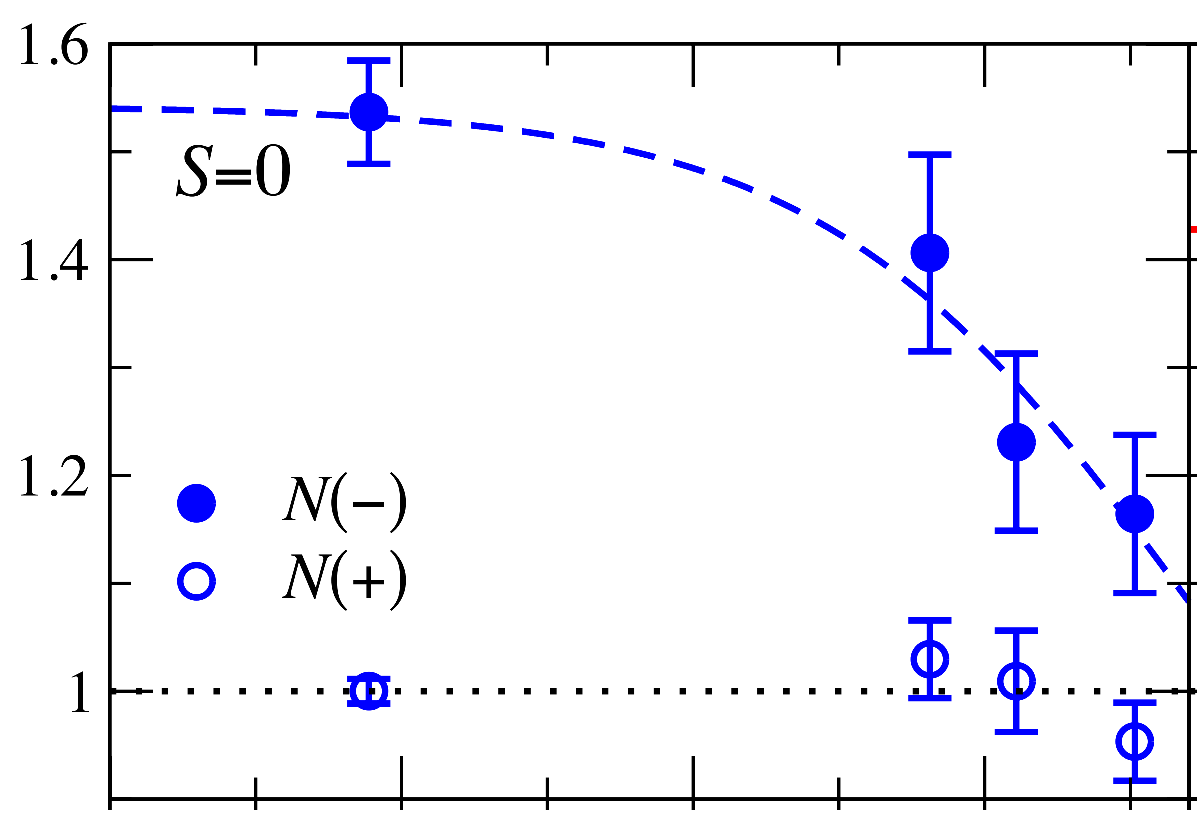


Hadron Resonance Gas

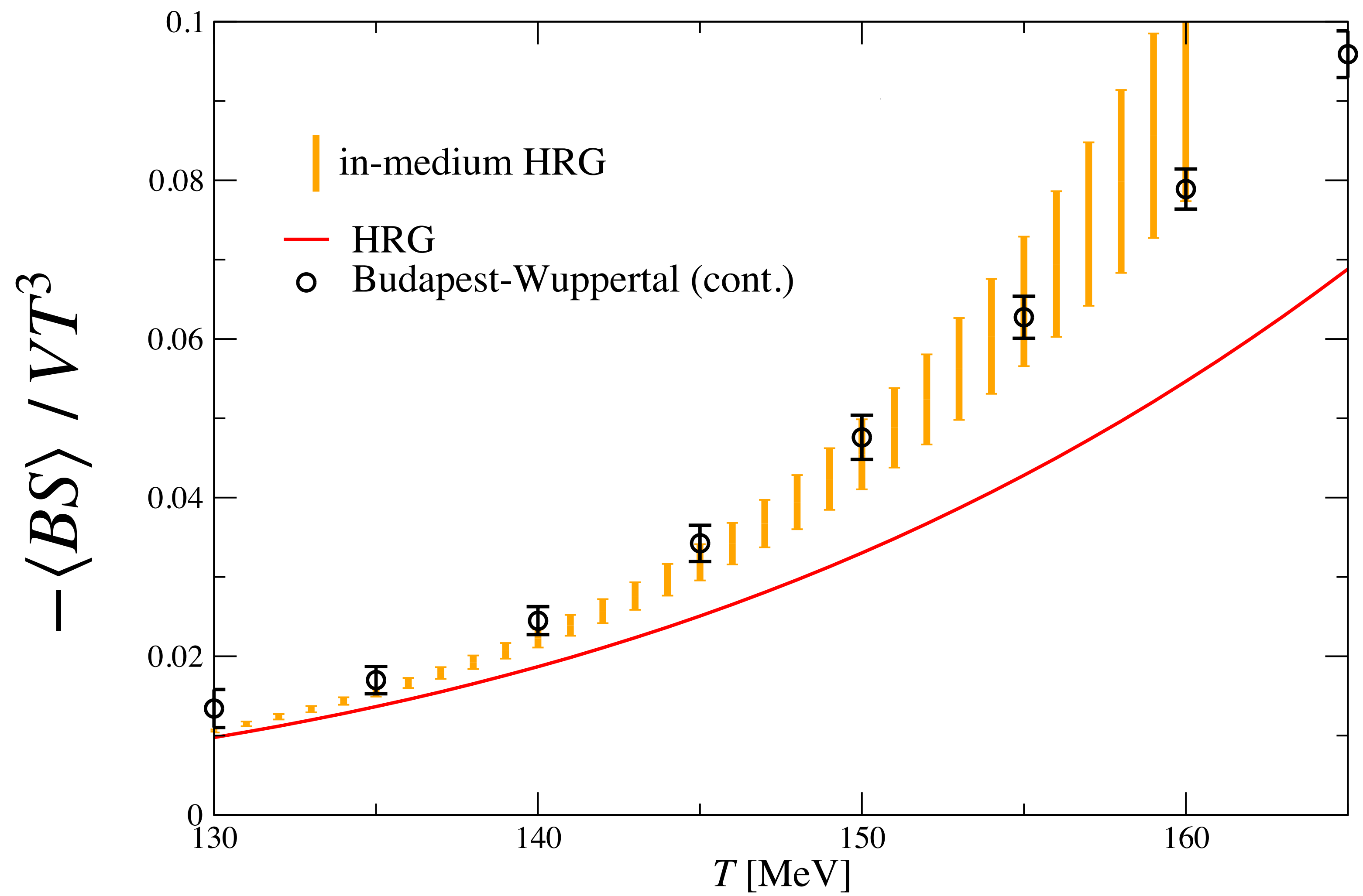
- applicable in confined phase
- non-interacting gas of (bound) hadrons
- thermodynamic partition function, multiplicity given by Boltzmann weight

Fit: $$m\_(T) = \omega(T,\gamma)\, m\_(0) + [1 - \omega(T,\gamma)]\, m\_(T_c)$$

where $$\omega(T,\gamma) = \frac{tanh[(1 - T/T_c)/\gamma]}{tanh[1/\gamma]}$$ γ ~ width



Hadron Resonance Gas



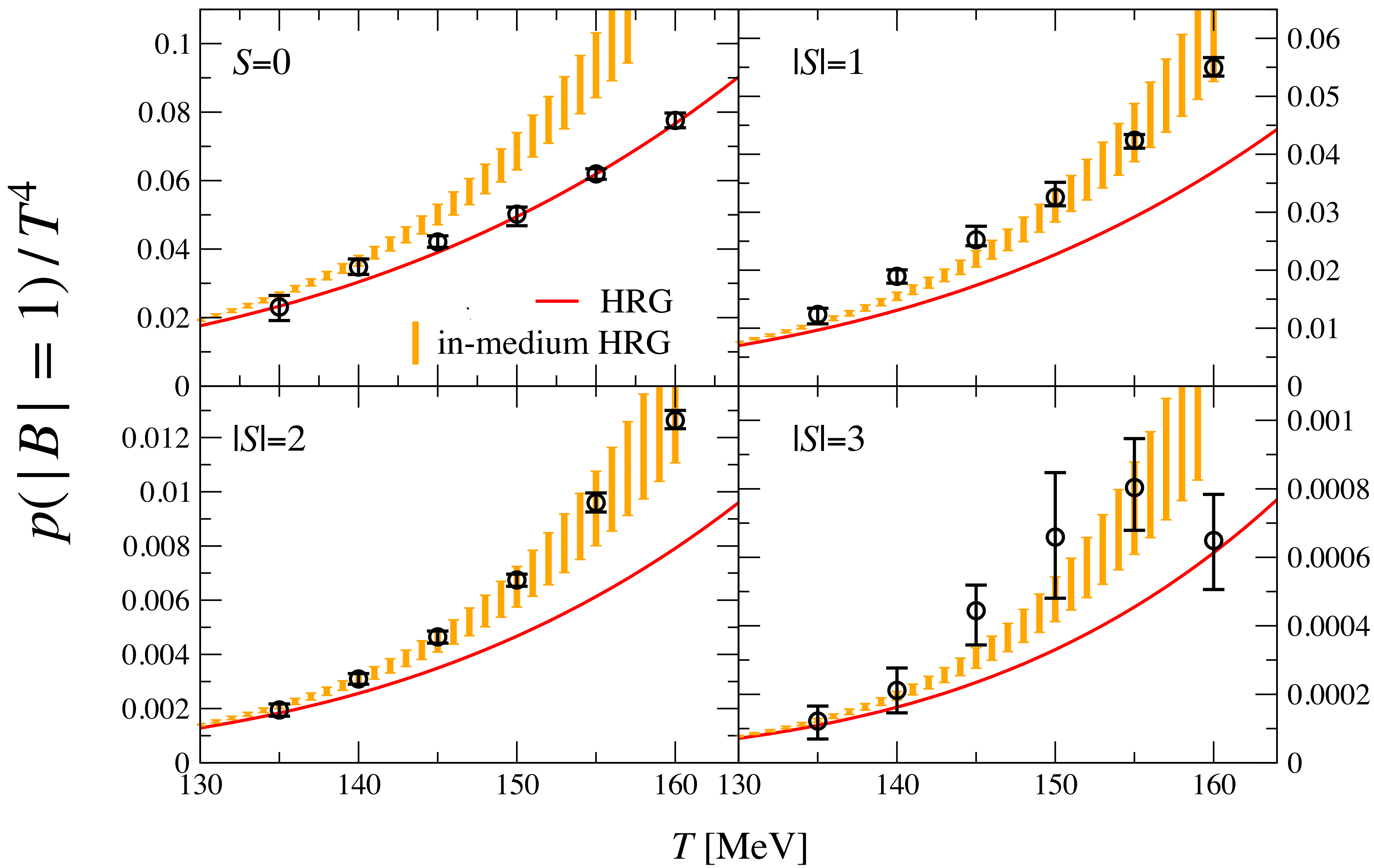
Lattice data from:

Budapest-Wuppertal: JHEP 1201 (2012) 138

Phys. Rev. D 92 (2015) no.11, 114505

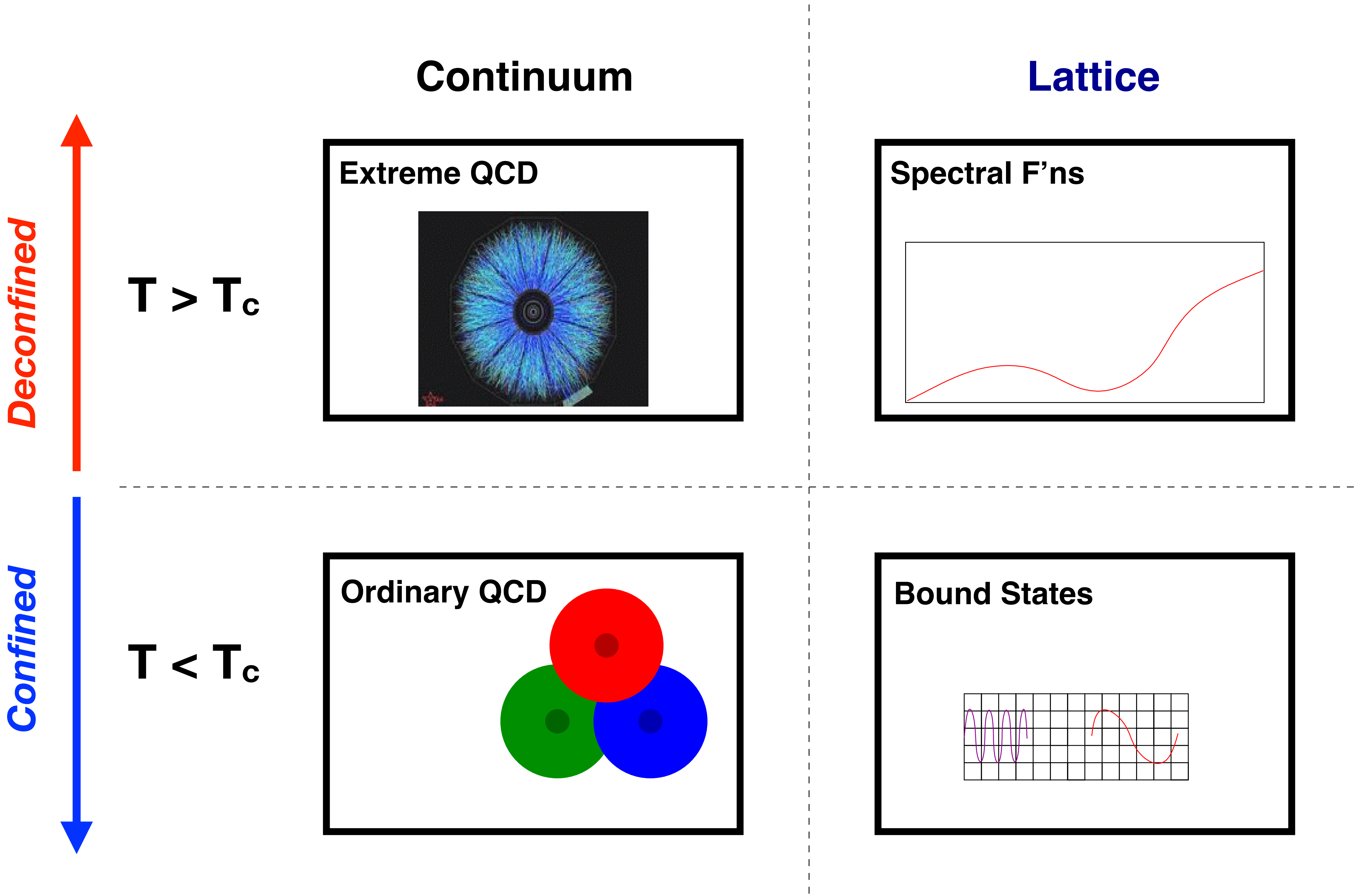
Pressure from HRG

Contributions from strange baryons



Lattice data from:

P. Alba et al., Phys. Rev. D 96 (2017) no.3, 034517



Continuum
Lattice
Deconfined
T > Tc
Extreme QCD
Spectral F’ns
Confined
T < Tc
Ordinary QCD
Bound States

Spectral Functions

$$G_{\pm}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\pm}(\omega)$$

$\uparrow$ Euclidean Correlator

$\nearrow$ (Lattice) Kernel

$\downarrow$ Spectral Function

$$K(\tau, \omega) = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}}$$

Input Data: $G_{\pm}(\tau), \;\; \tau = 1, \ldots, \mathcal{O}(10)$

Output Data: $\rho_{\pm}(\omega), \;\; \omega \sim 1, \ldots, \mathcal{O}(1000)$

ill-posed !

Maximum Entropy Method

Need to maximise $P(F|D)$

Bayes Theorem:

$$P(F|D)P(D) = P(D|F)P(F)$$

i.e. $$P(F|D) = \frac{P(D|F)P(F)}{P(D)}$$

But $P(D|F) \sim e^{-\chi^2}$ $\longrightarrow$ minimising χ^2 $\neq$ maximising $P(F|D)$

$\longrightarrow$ *Maximum Likelihood Method* wrong??

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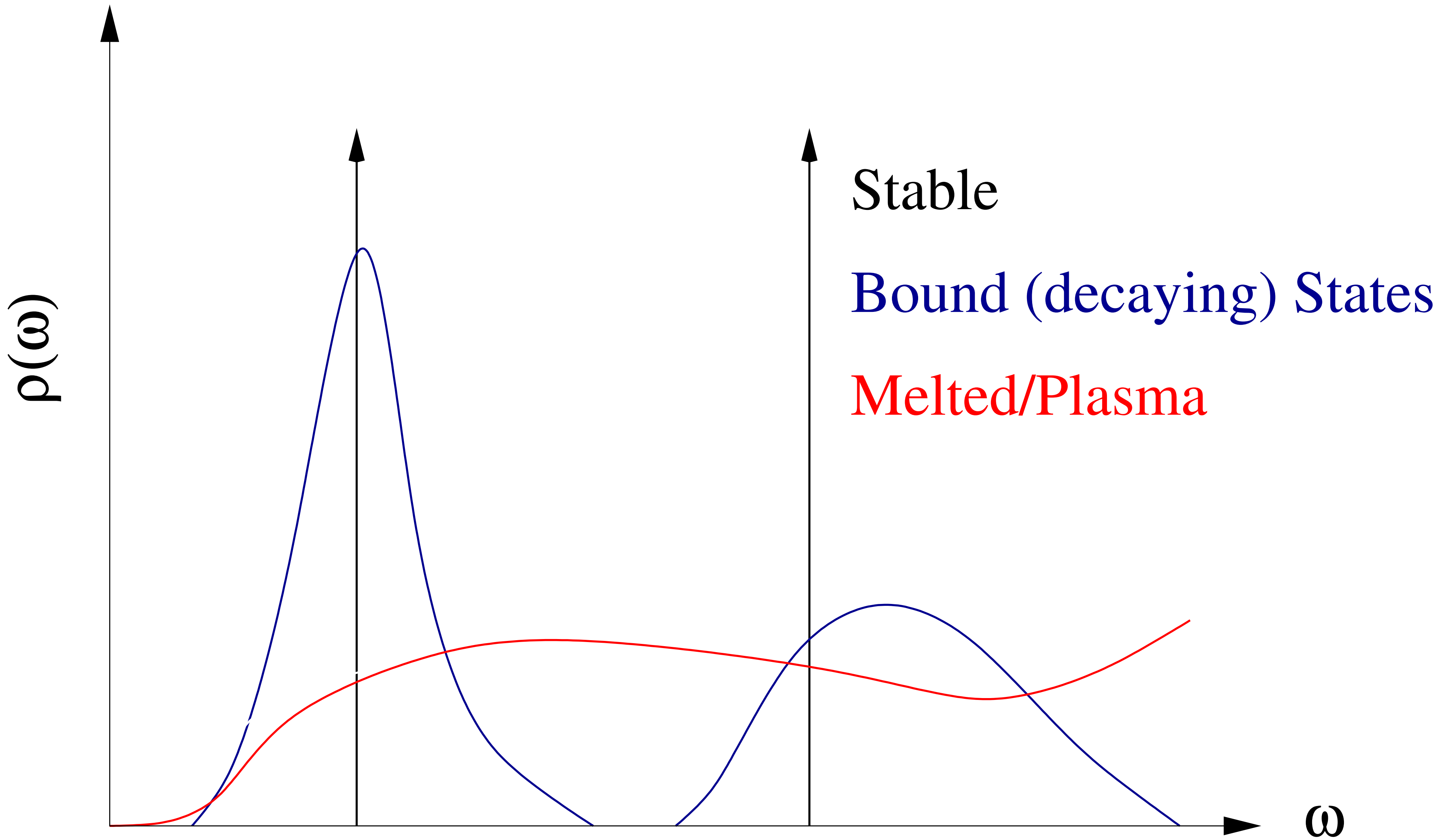
$\longrightarrow$ *Maximum Likelihood Method* wrong??

P(F) ~ e^S^ Shannon-Jaynes entropy: $S = \int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$

Competition between minimising χ^2 and maximising S

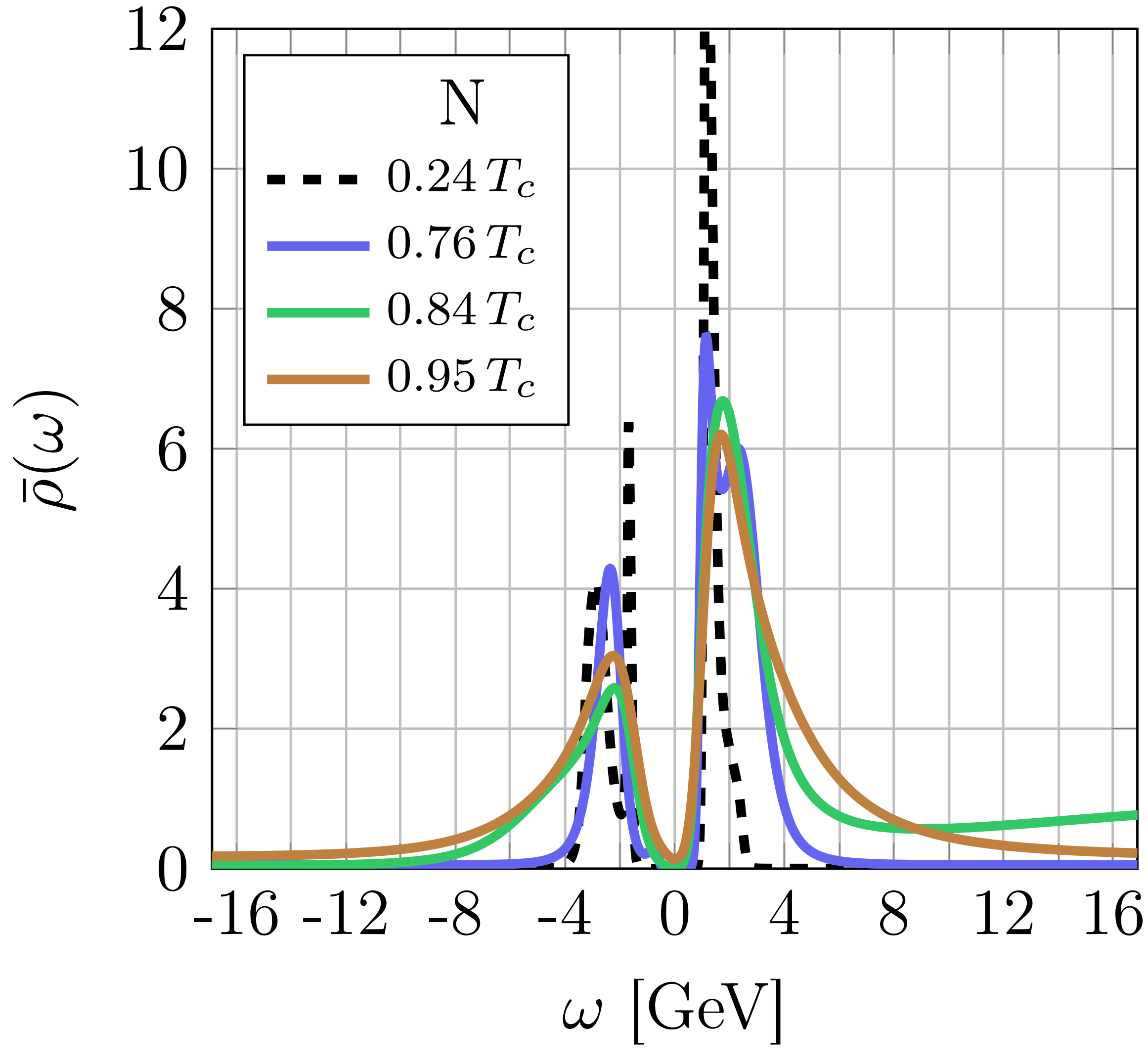
Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46(2001)259

Spectral Function Dictionary

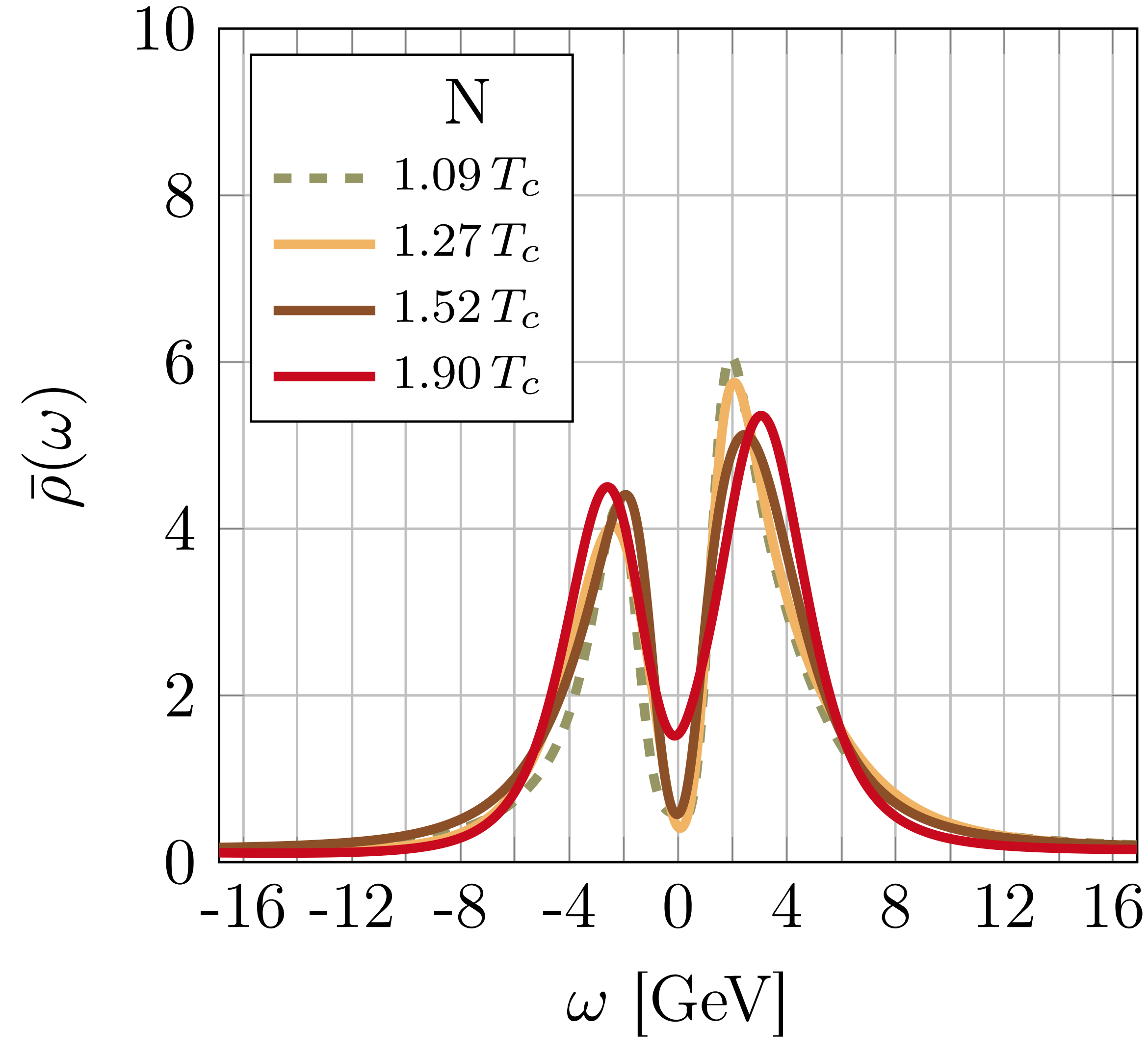


Nucleon spectral function via MEM

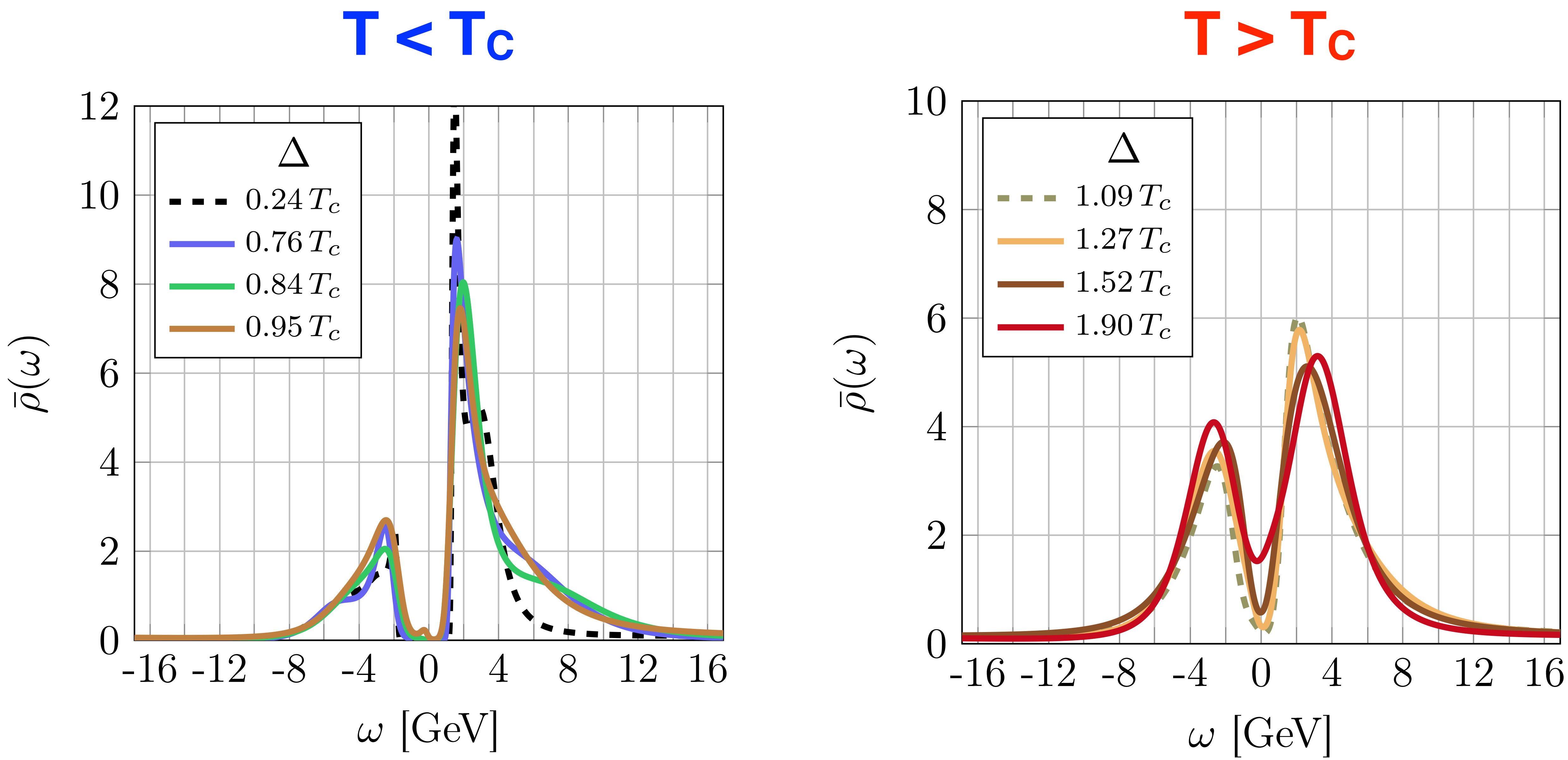
T < Tc



T > Tc

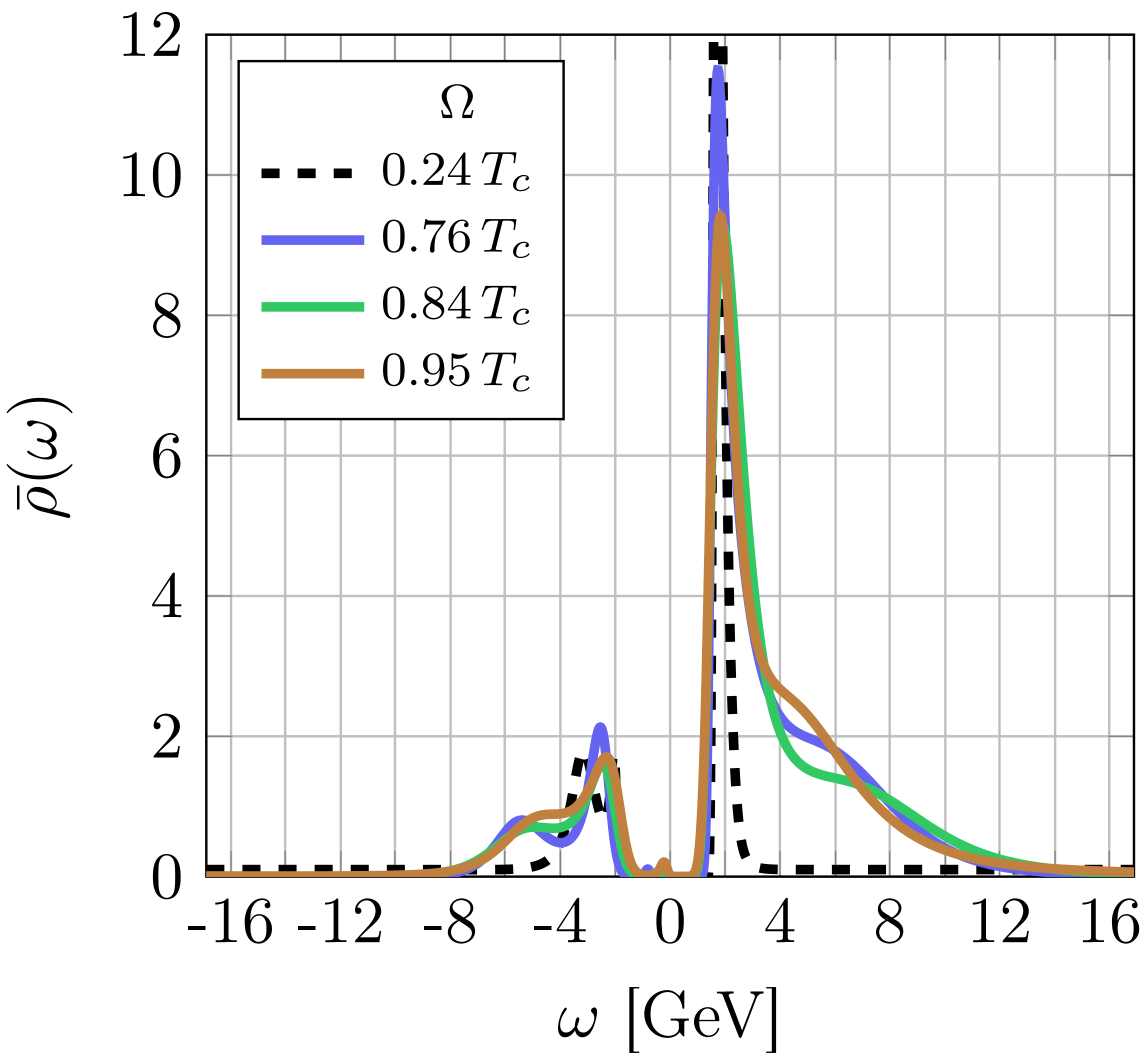


Δ spectral function via MEM

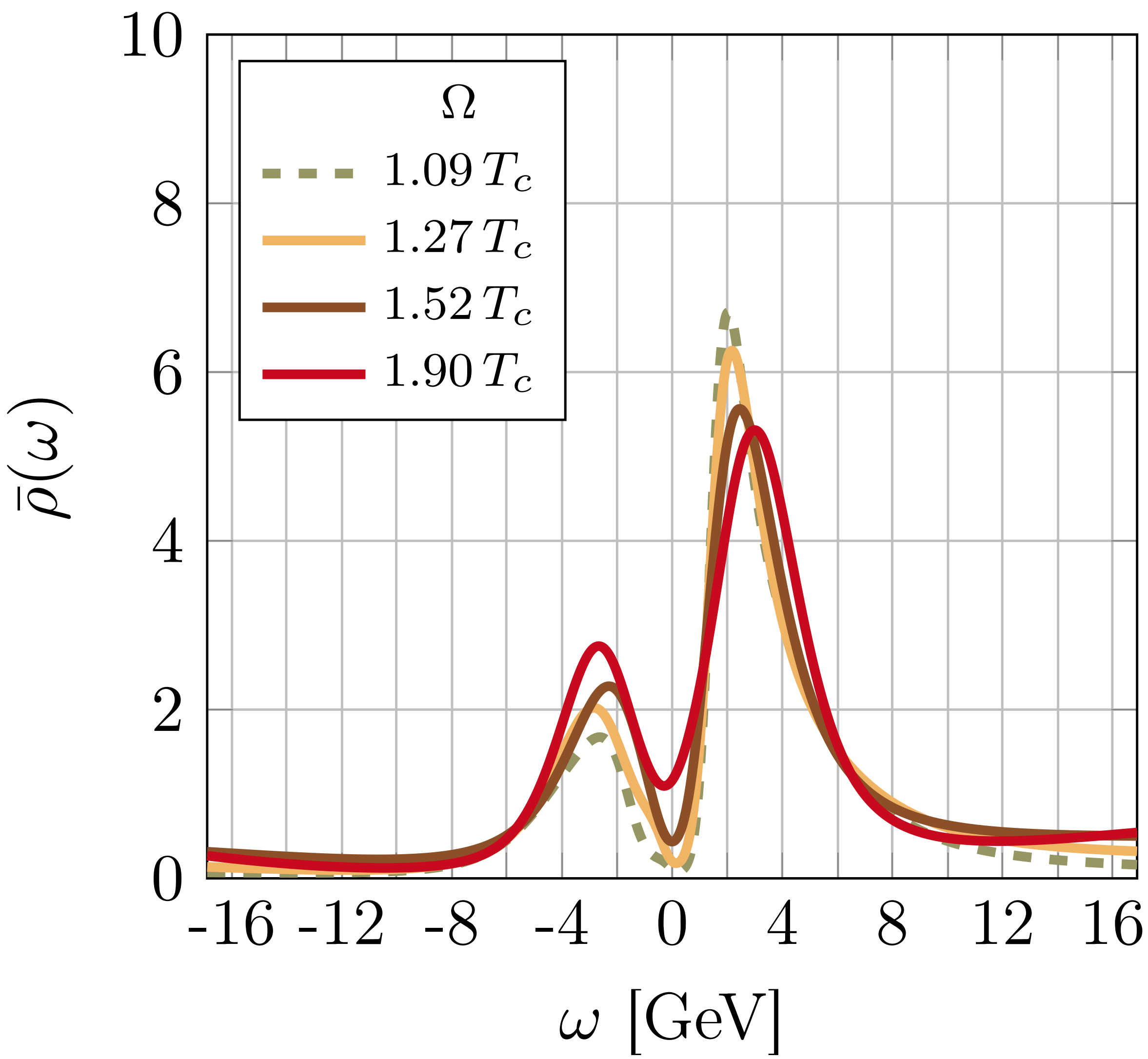


Ω spectral function via MEM

T < Tc

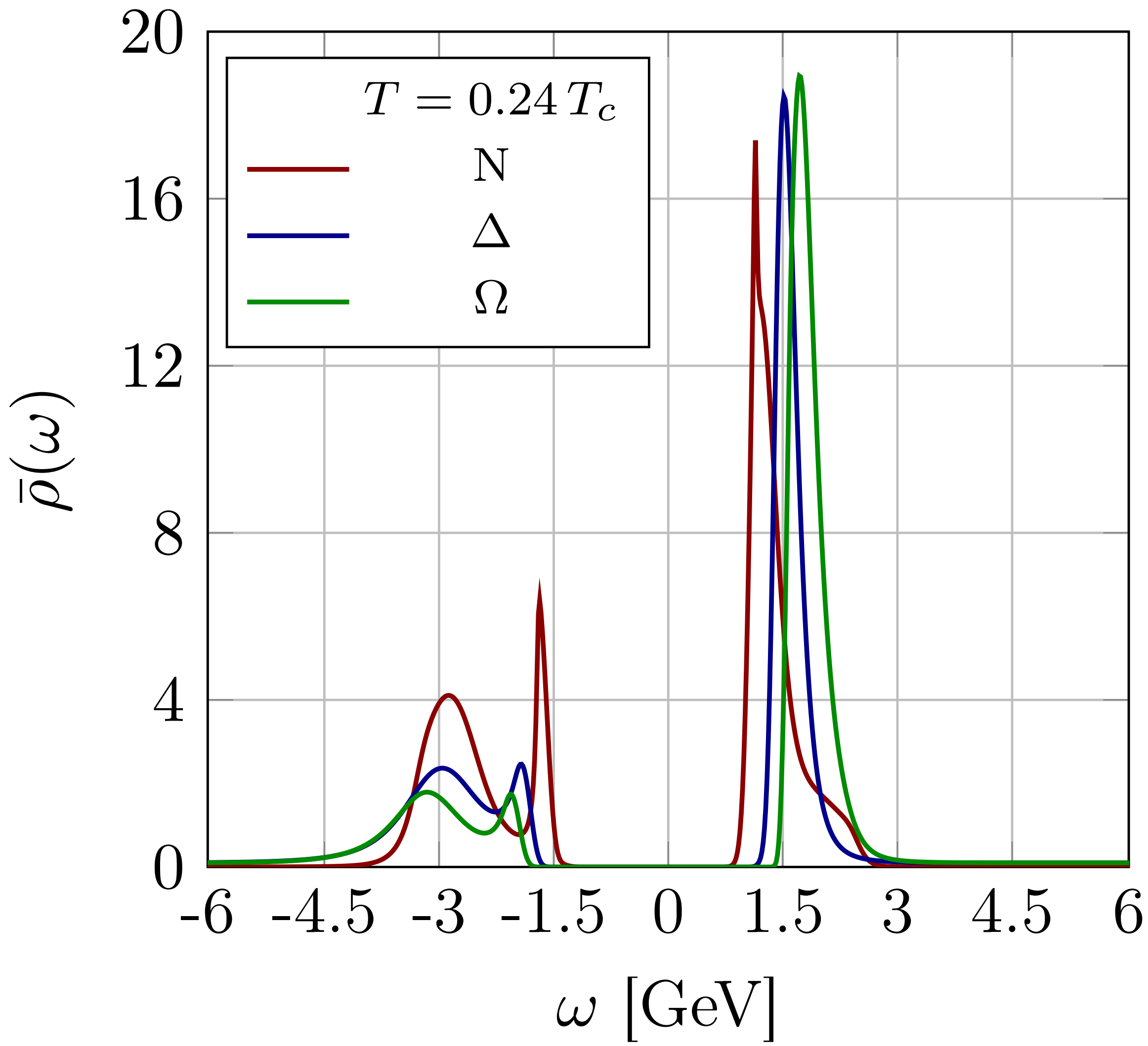


T > Tc

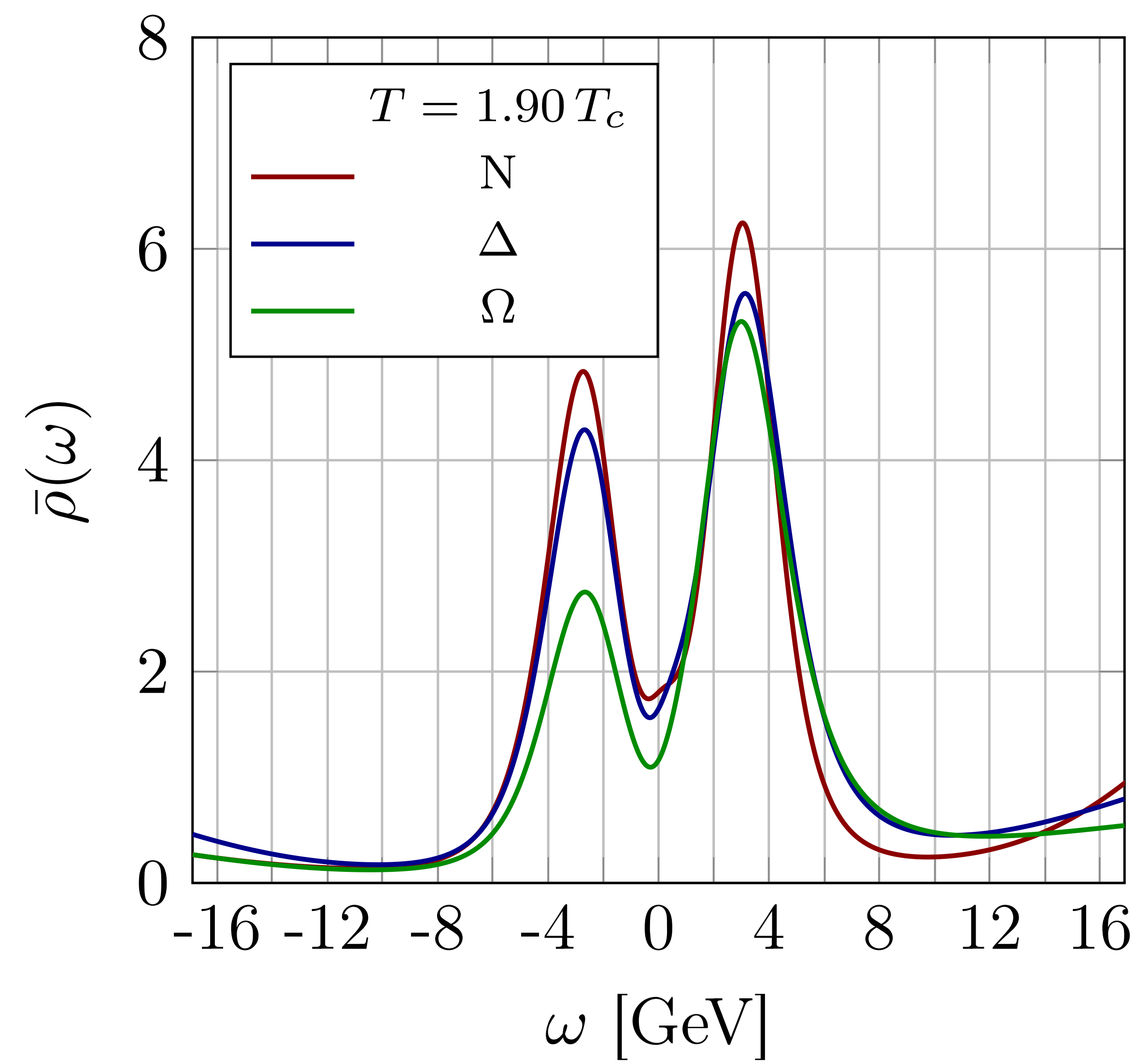


Comparison between baryons

T = 0.24 Tc

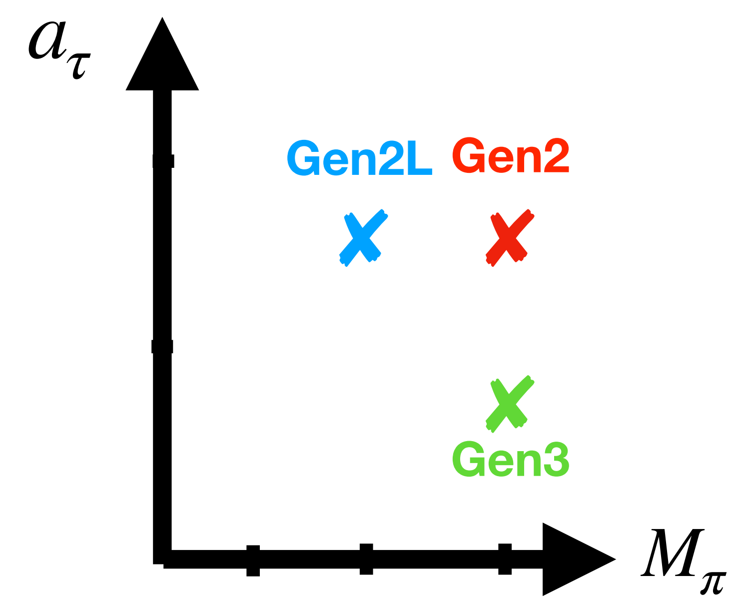


T = 1.90 Tc



Summary - Hyperon Spectrum

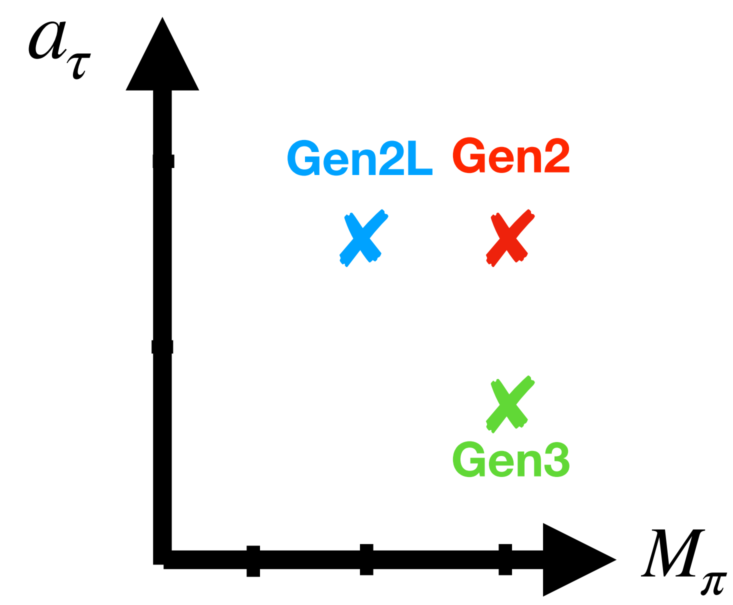
- Used
  - raw correlators
  - conventional exp fits
  - spectral f'ns (MEM)



- Confined phase:
  - +ve parity masses ~constant ≠ f(T)
  - -ve parity masses ↘ as T↗
- Deconfined phase:
  - degeneracy of parity ground states
  - some signs of degeneracy amongst baryon channels
- In progress: Gen2L (and Gen3)

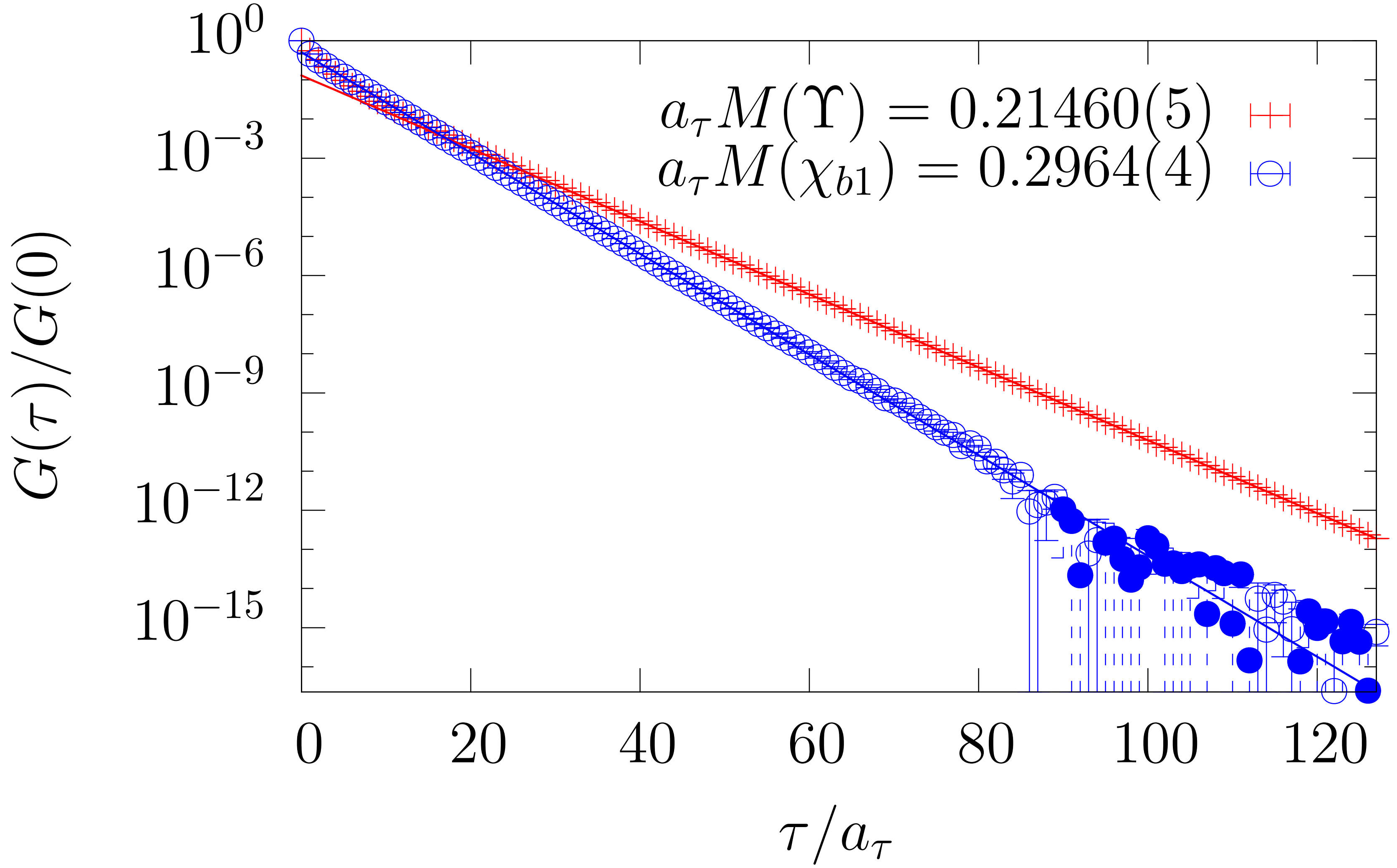
BOTTOMONIUM

- NRQCD approach for b-quark
- Main results from Gen 2
- Checks against
    - Gen 2L (light)
    - Gen 3 (finer temporal lattice)



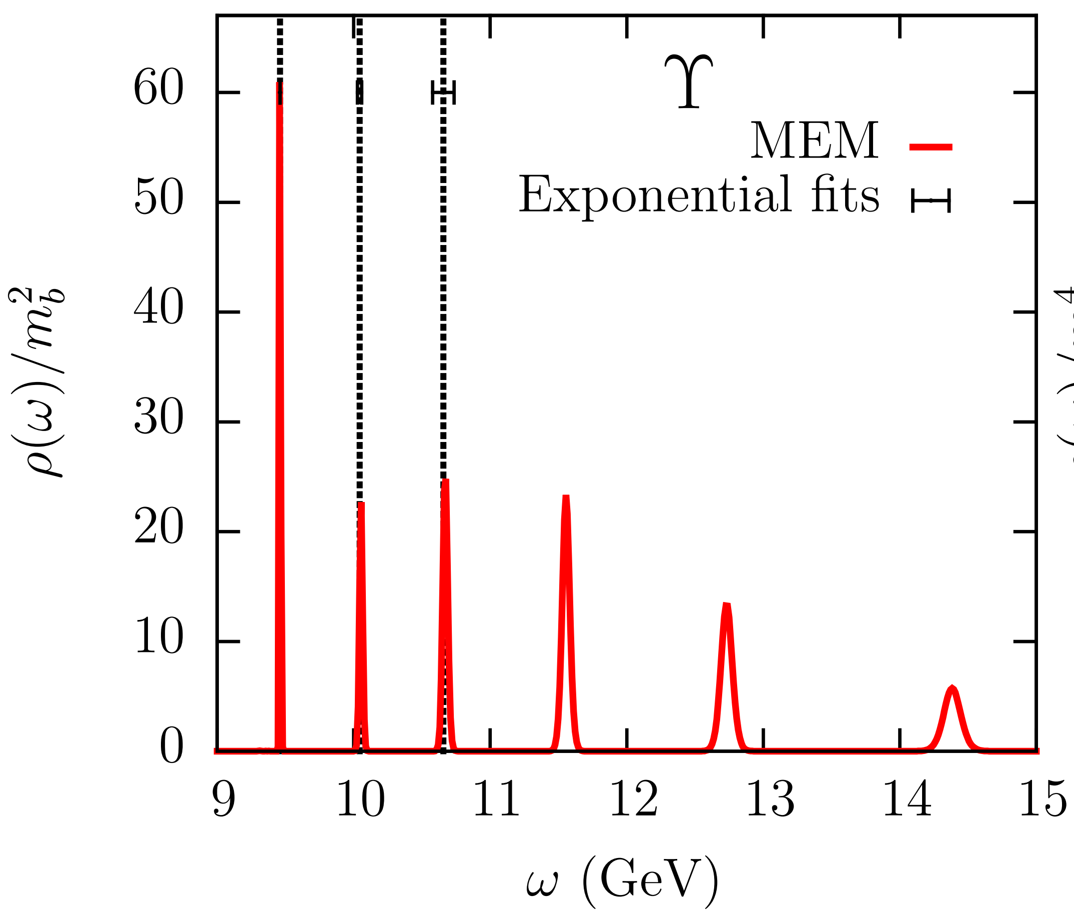
T=0 Correlators

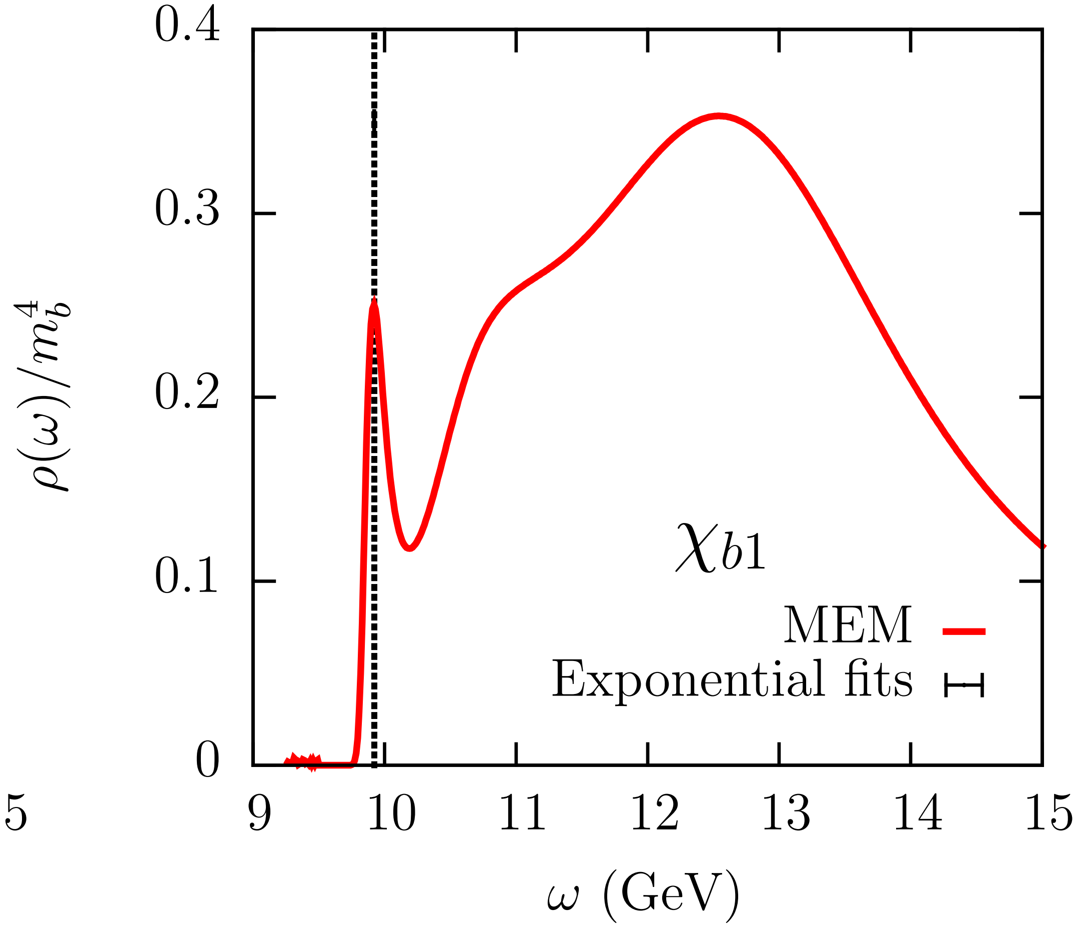
$$G(\tau) \equiv \sum_x \langle 0|J(x,\tau)J^\dagger(0,0)|0\rangle \quad \stackrel{\tau\to\infty}{\longrightarrow} \quad \frac{|\langle 0|J|\mathrm{gnd}\rangle|^2}{2M}\, e^{-M\tau}$$



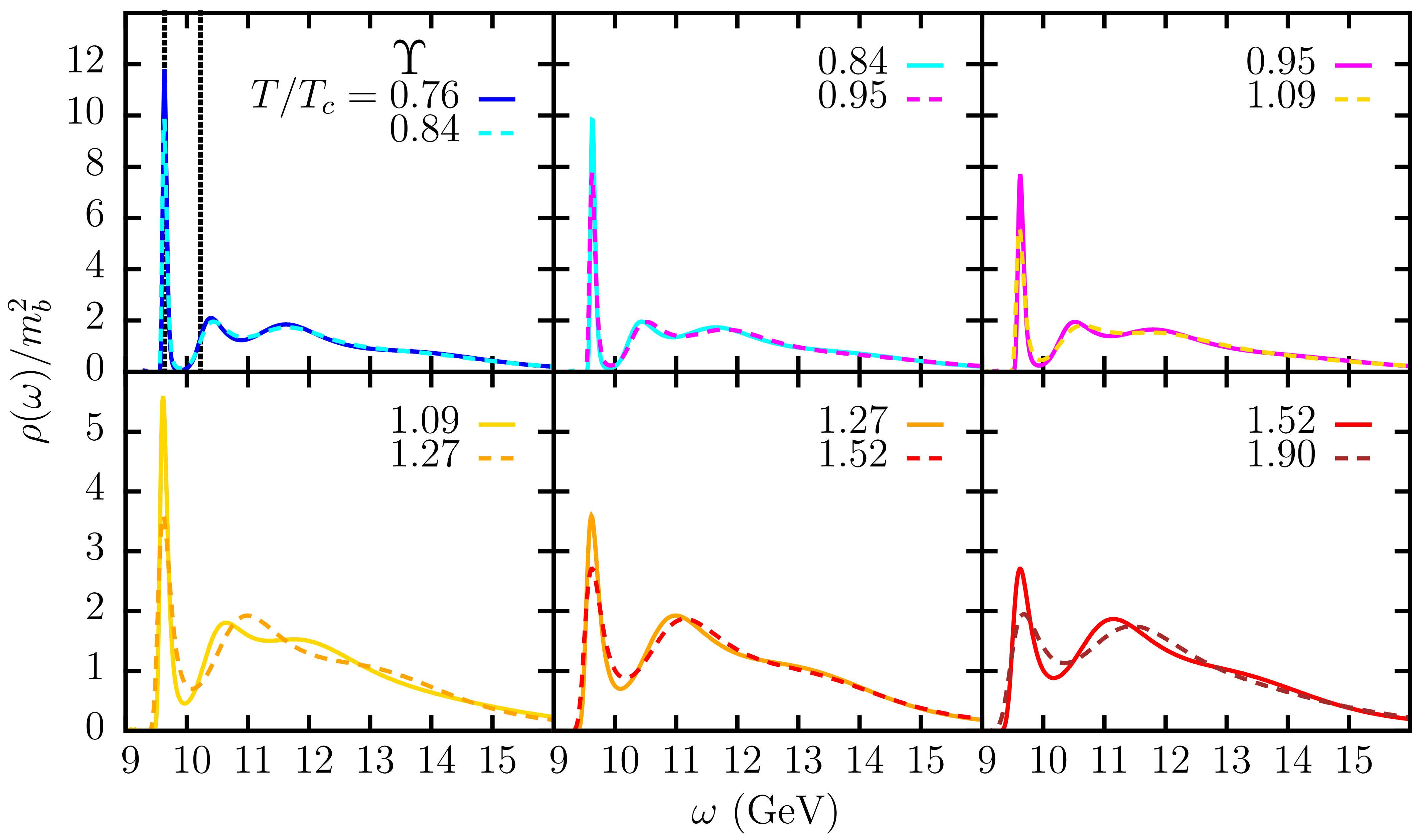
T=0 spectral functions

$$G(\tau) = \int_{\omega_{\min}}^{\omega_{\max}} \frac{\mathrm{d}\omega}{2\pi} K(\tau,\omega)\,\rho(\omega), \qquad K(\tau,\omega) = e^{-\omega\tau}.$$

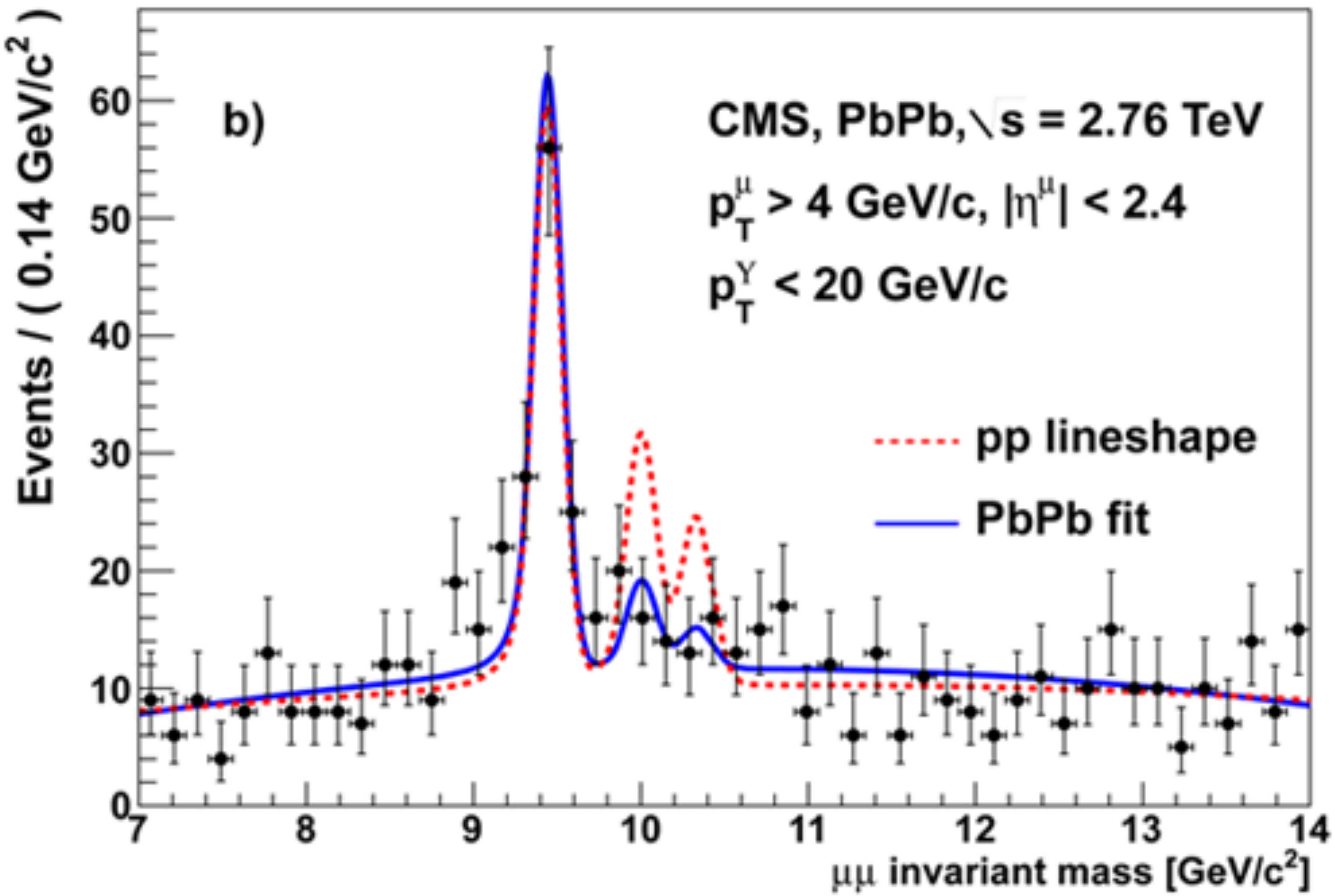




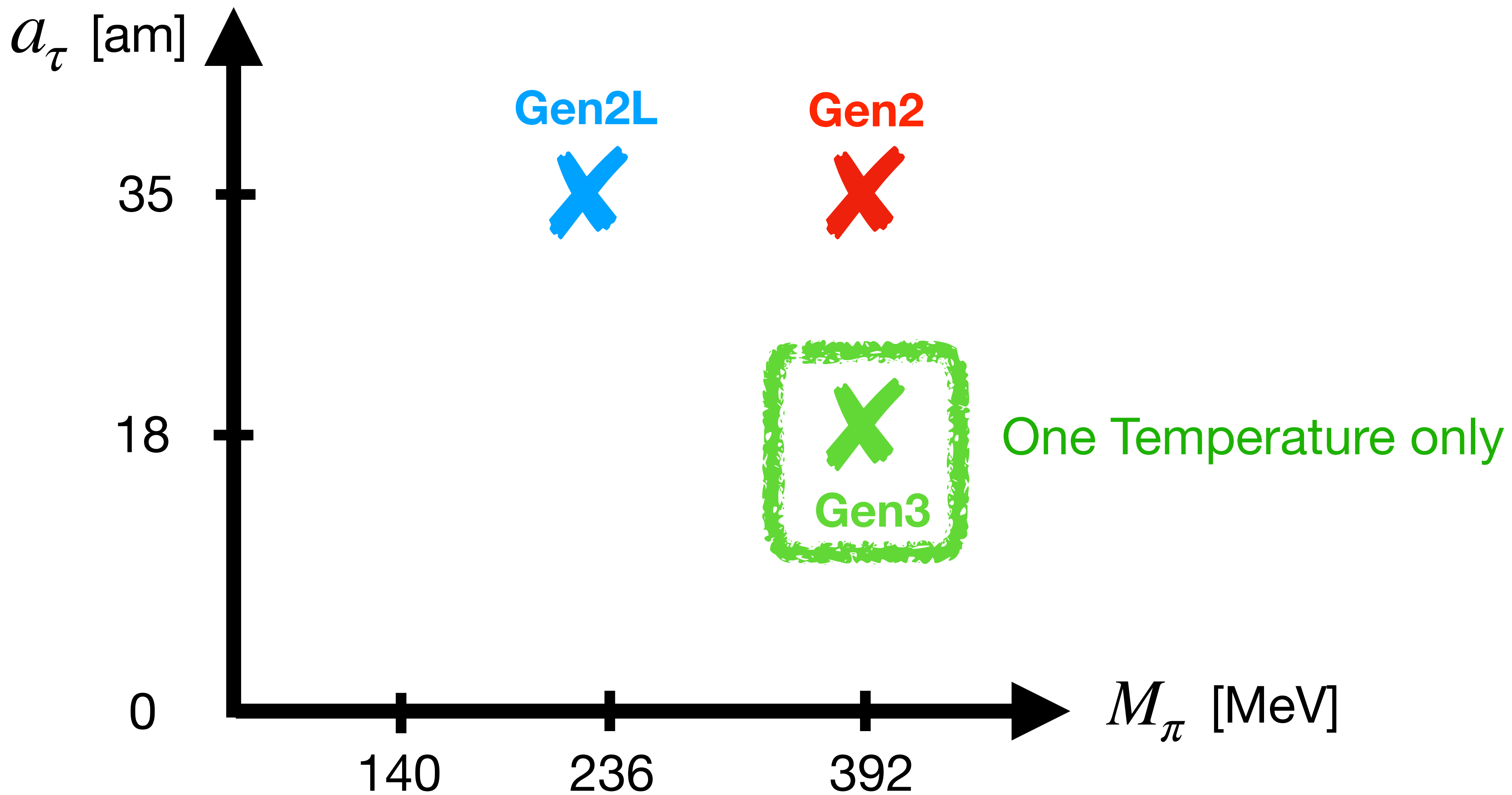
Themal modification of $\boldsymbol{\Upsilon}$ spectral function



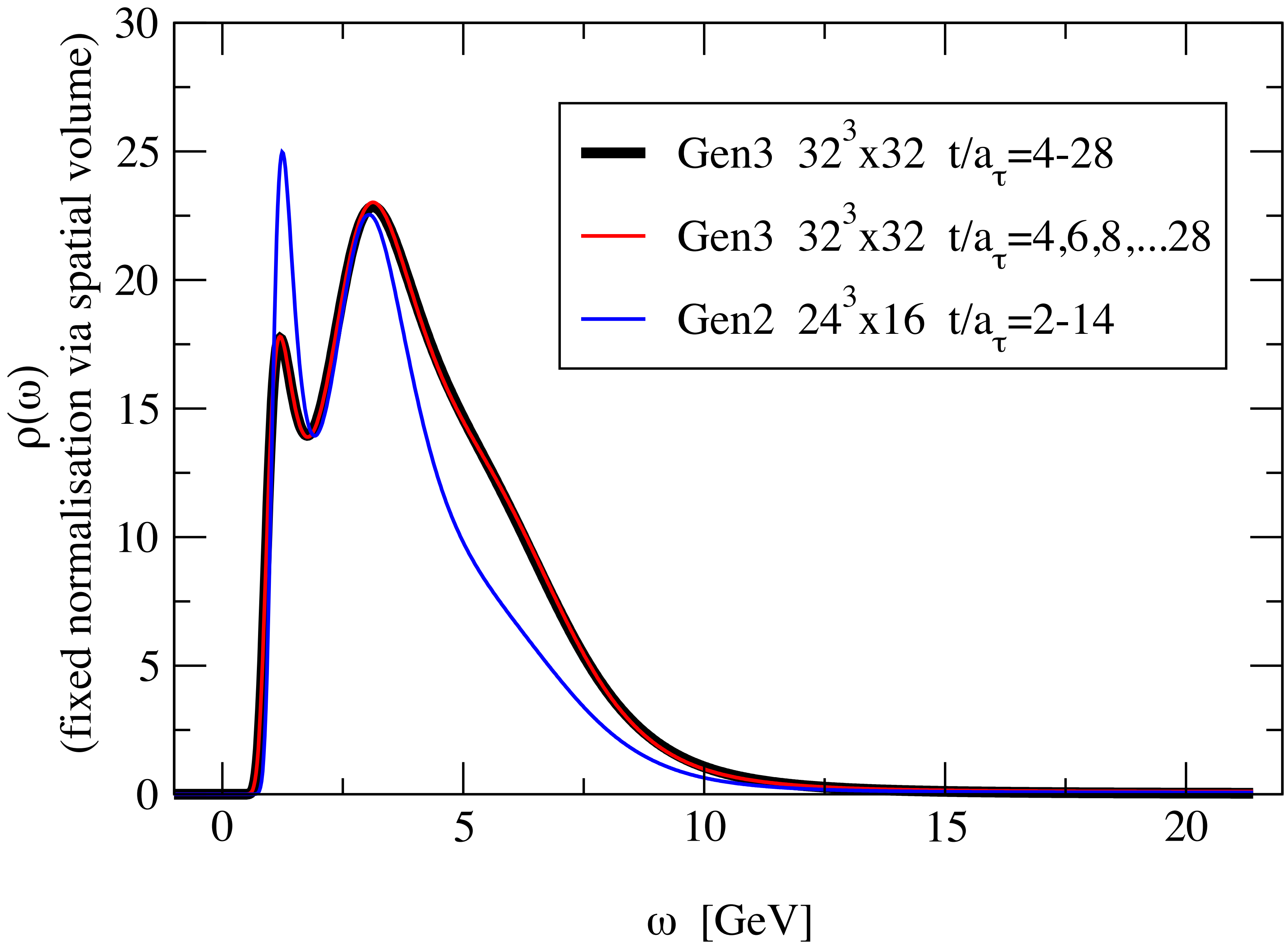
CMS pp versus PbPb



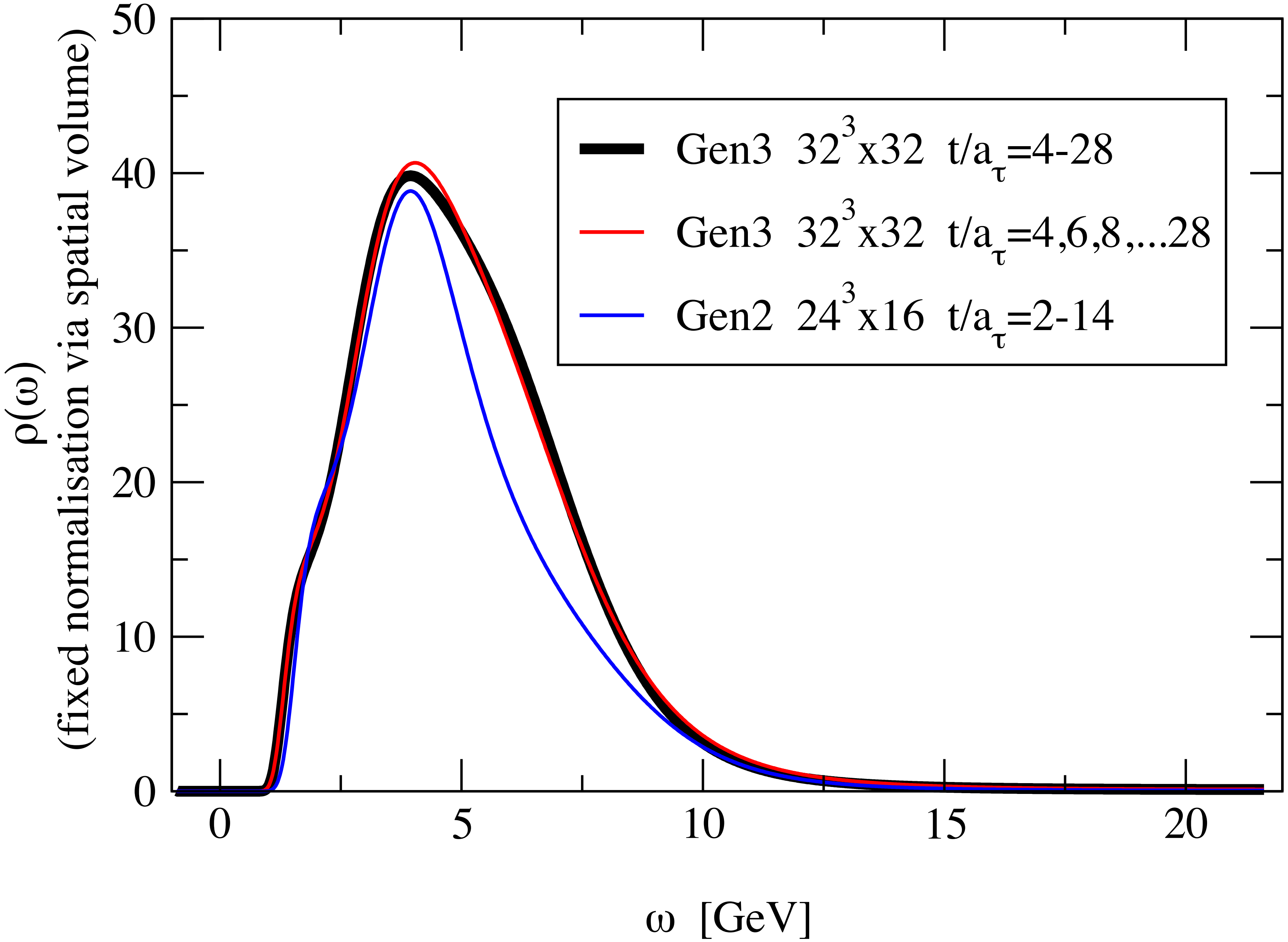
Generation 3 Results



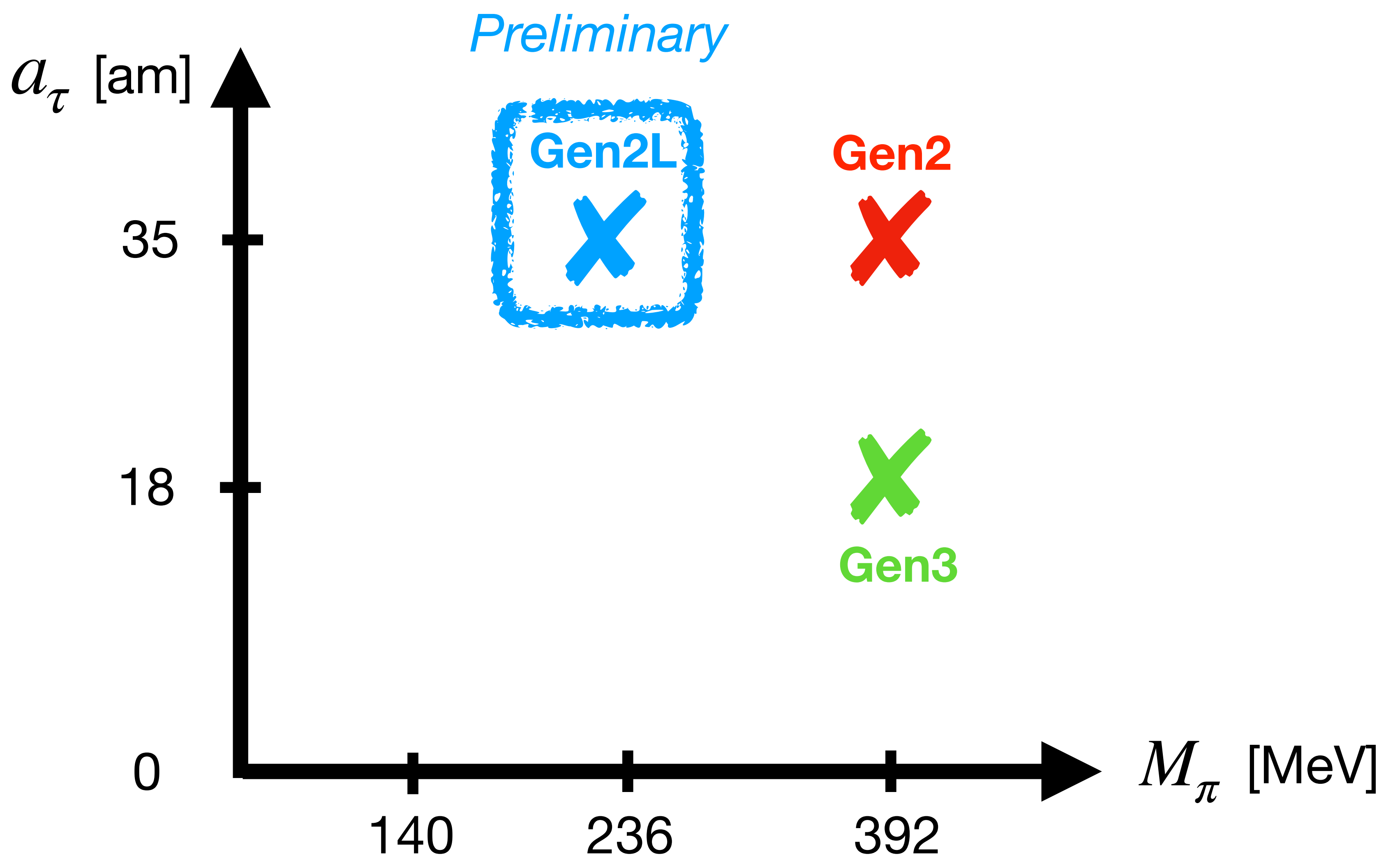
Upsilon: Gen2 vs Gen3
Going towards (temporal) continuum



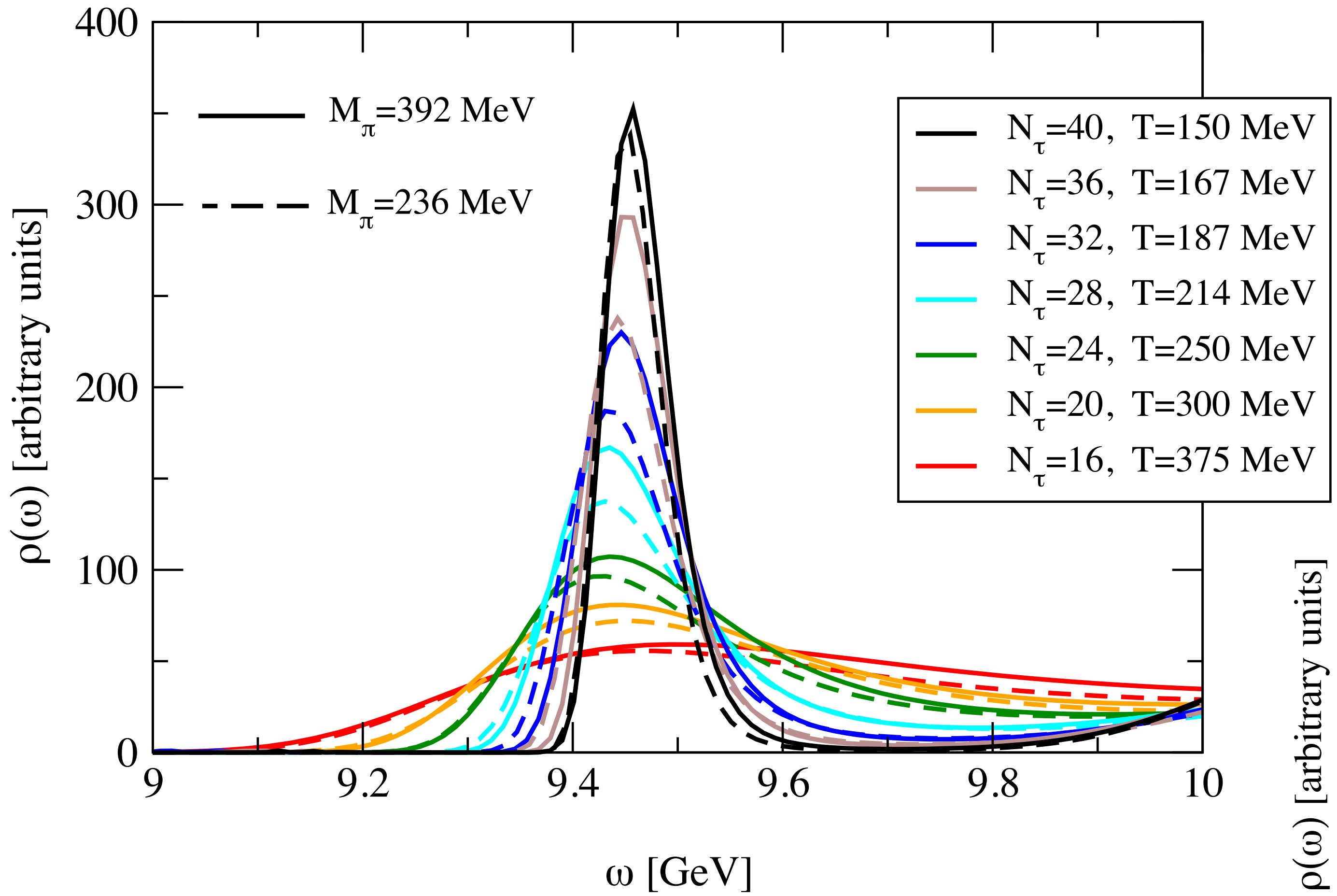
χ_{b1} : Gen2 vs Gen3
Going towards (temporal) continuum

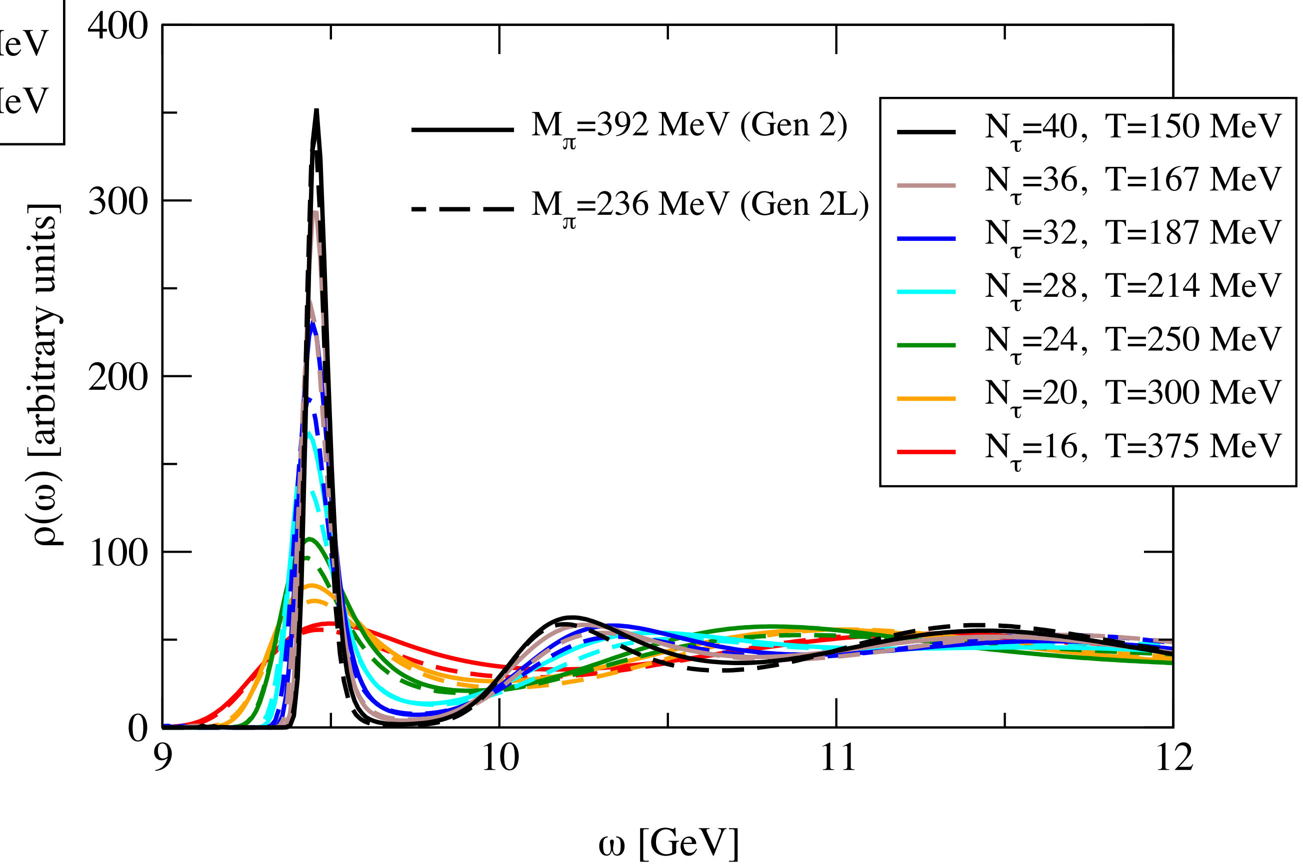


Generation 2L Results



Upsilon: Gen2 vs Gen2L
Going lighter

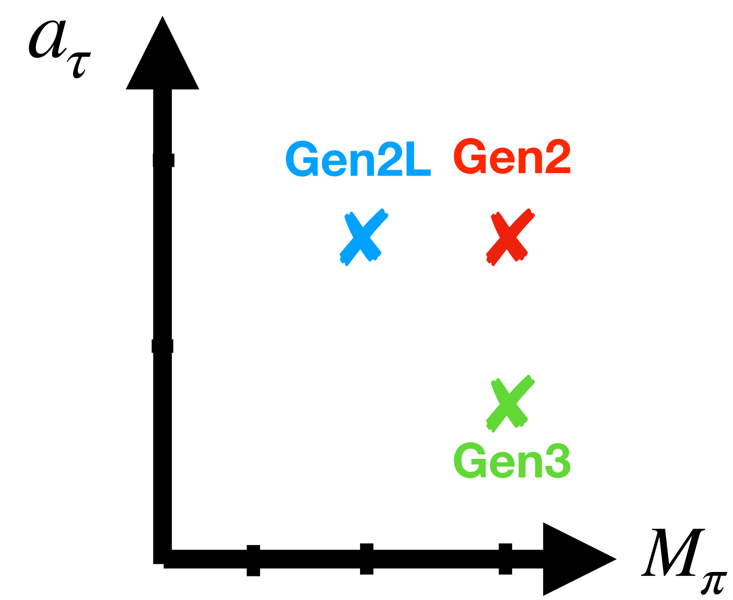




Preliminary

Summary - Bottomonium

- FASTSUM has analysed three different ensembles
    - "Gen2"
    - "Gen2L" (Preliminary)
    - "Gen3" (one T only)



- Produced results for for bottomonium using NRQCD
- Main results:
    - S-wave Υ & η_b stable well above T_c
    - P-wave χ_{b1} melts not far above T_c

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