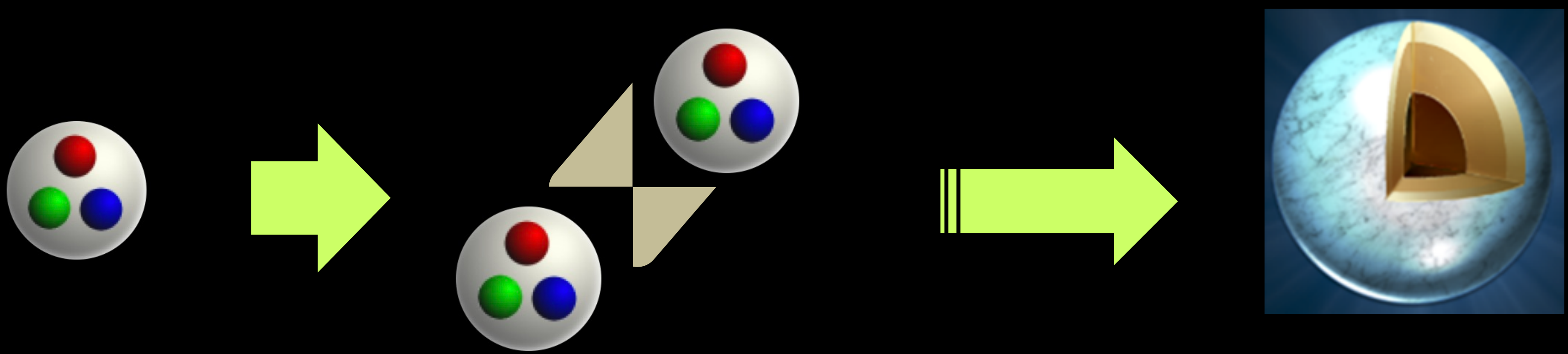
Recent lattice QCD results on hyperon-nucleon & hyperon-hyperon interactions from HAL QCD

Tetsuo Hatsuda

(RIKEN iTHEMS)

Strangeness in Quark Matter (Bari, June 11, 2019)



Single Baryon

Baryon Interaction

Compressed Baryonic Mater

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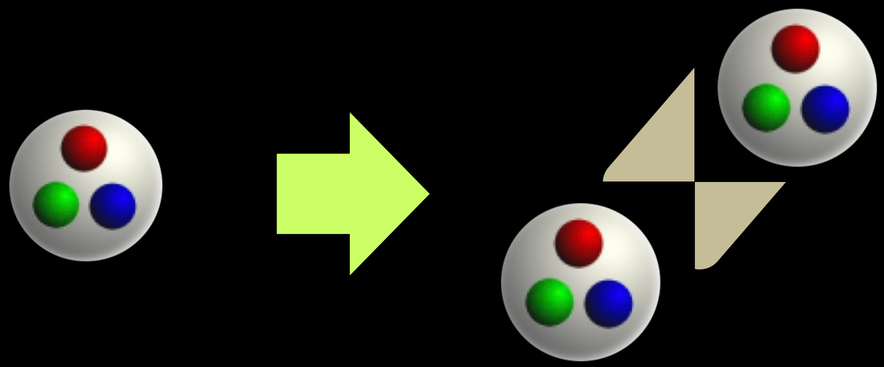


Single Baryon

Baryon Interaction

I. Baryon on the Lattice

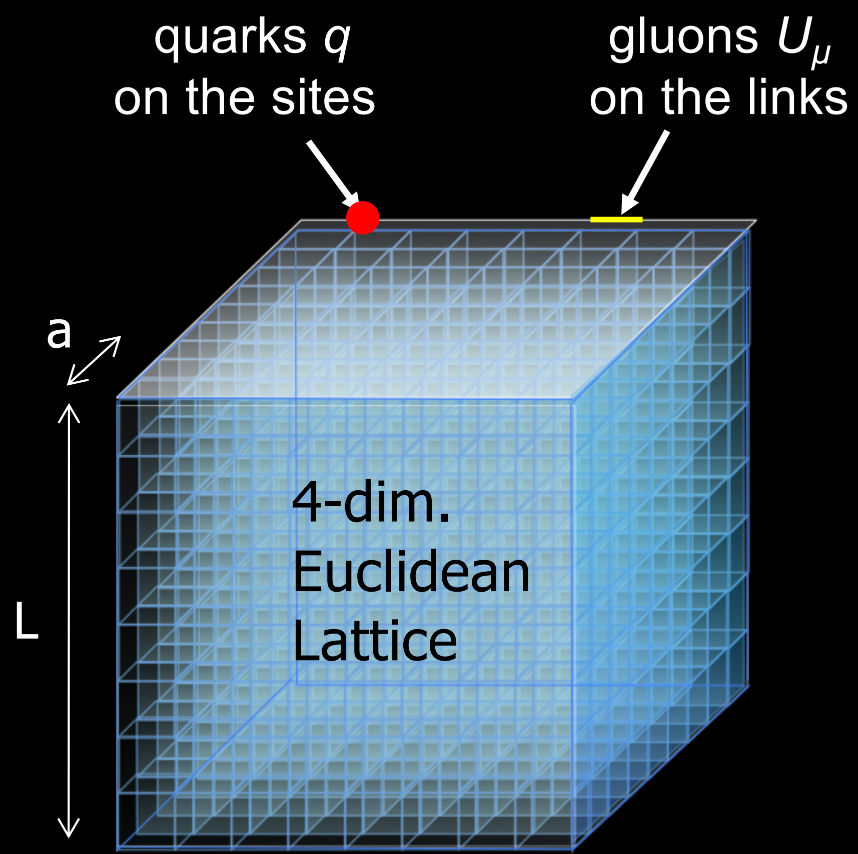
II. BB interactions on the Lattice

III. Recent YN and YY Results from HAL QCD

Lattice QCD Simulations

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$Z = \int [dU][dq d\bar{q}] \ \exp\left[- \int d\tau d^3x \mathcal{L}_{\rm E} \right]$$

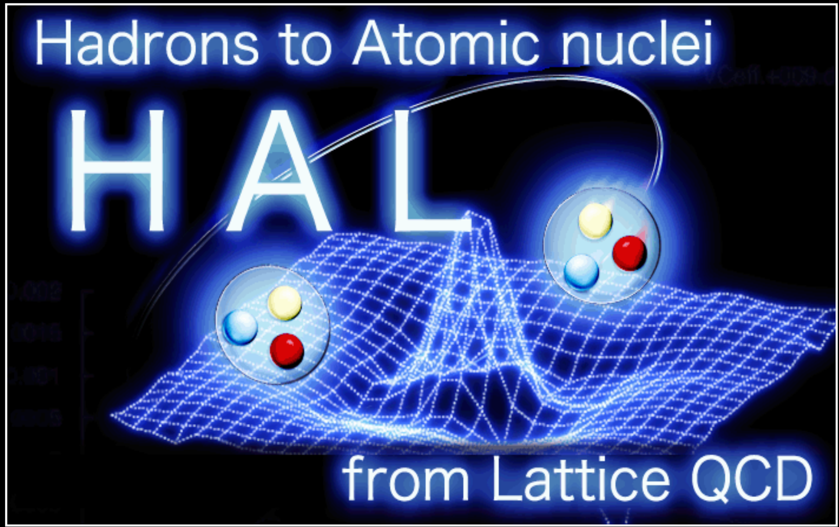


Huge integration variables
~ $10^{9\text{-}10}$ for 96^4 lattice

Importance Sampling
Hybrid MC = MD + Metropolis

Continuum & Thermodynamic Limits
($a \to 0$ & $L \to \infty$)

HAL QCD Collaboration



K Computer (10 Pflops) at RIKEN
2011–2019

(KEK) T. Aoyama
(RIKEN) T. Doi, T. M. Doi, S. Gongyo,
T. Hatsuda, T. Iritani, T. Sugiura
(Nihon) T. Inoue
(YITP) Y. Akahoshi, S. Aoki,
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(Birjand) F. Etminan

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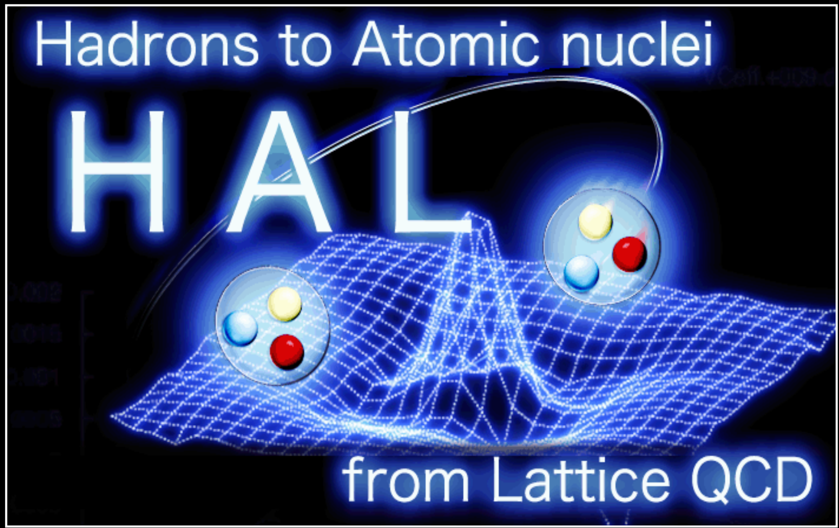

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$M_\pi = 146$ MeV
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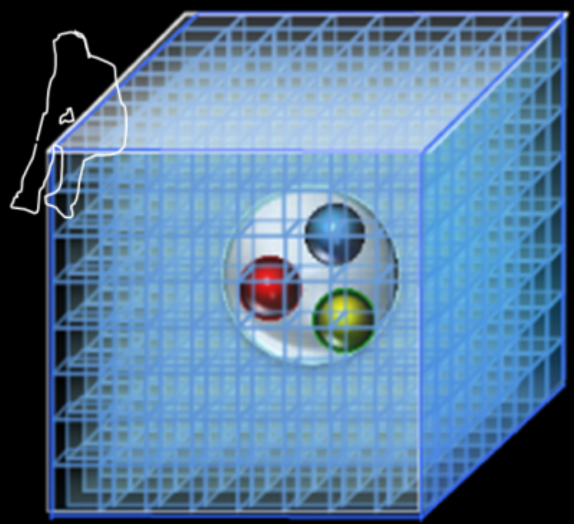
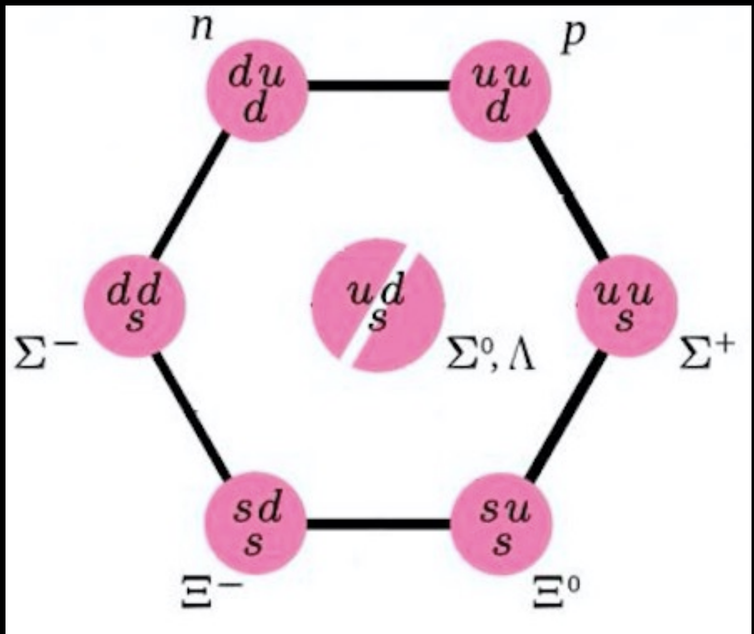
I. Baryon on the Lattice

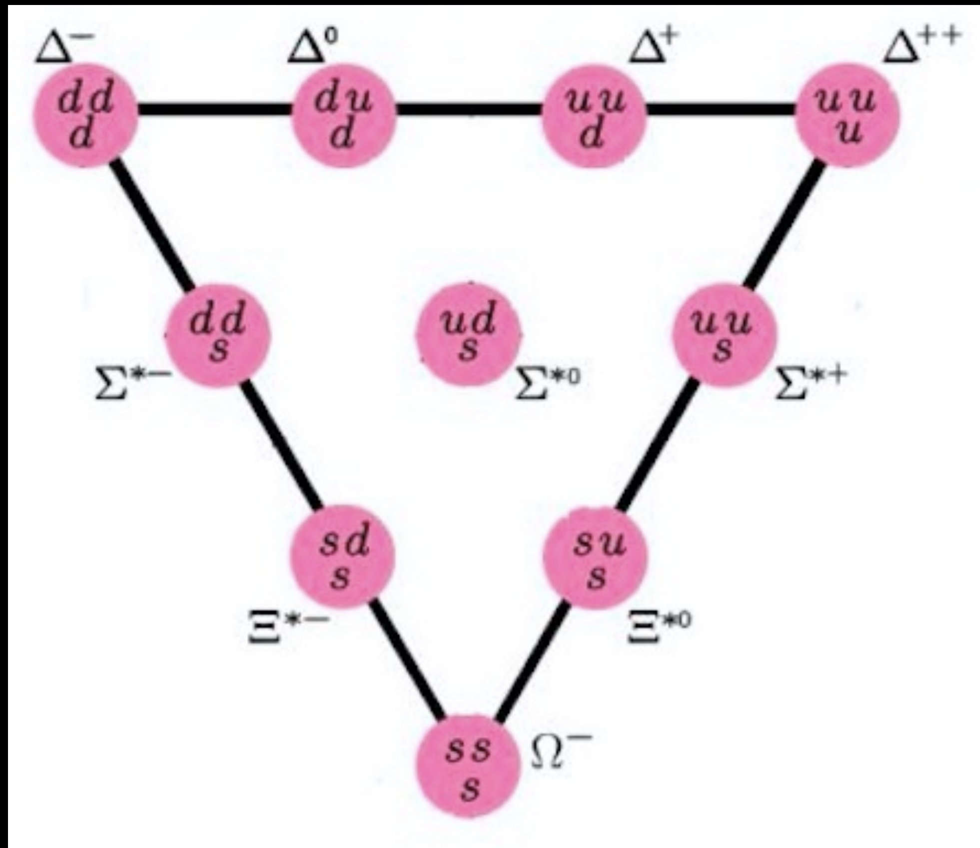


SU(3)$_F$ classification of single baryon

8 (Octet)



10 (Decuplet)



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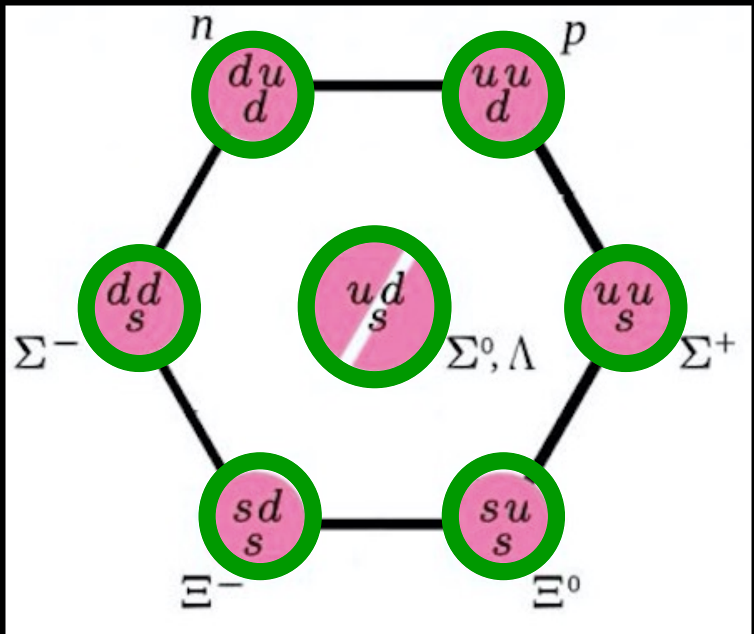

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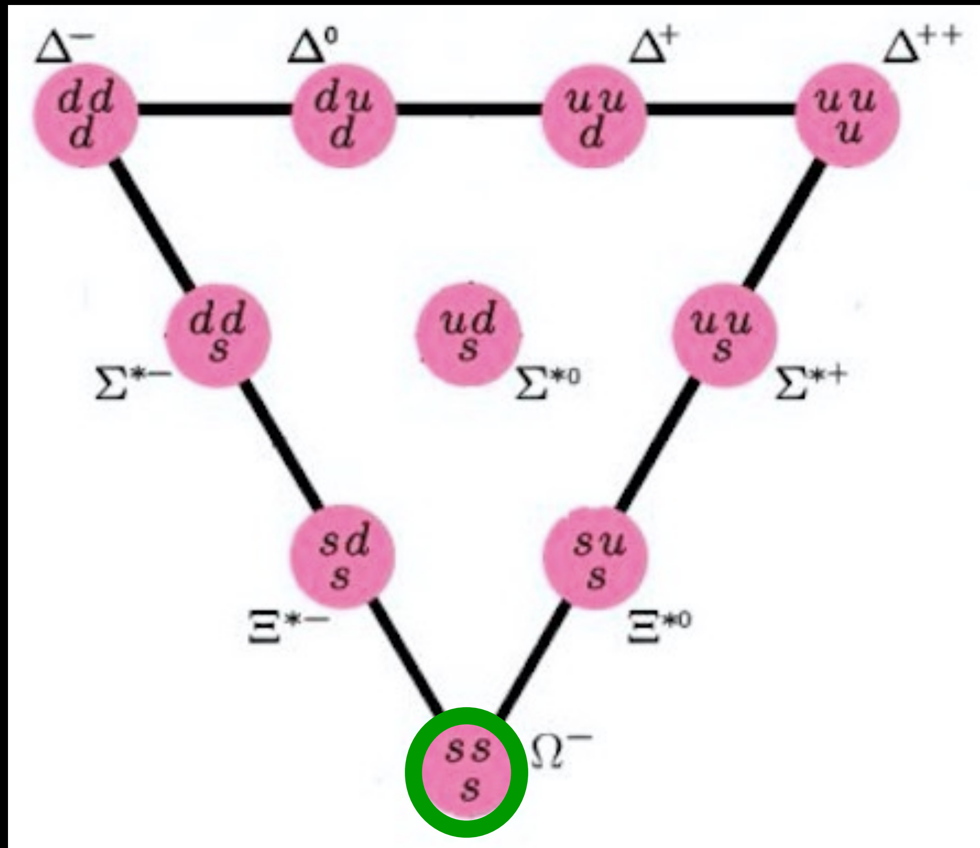
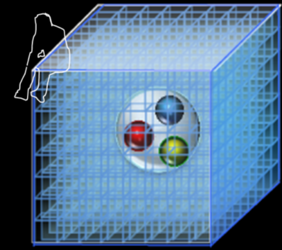
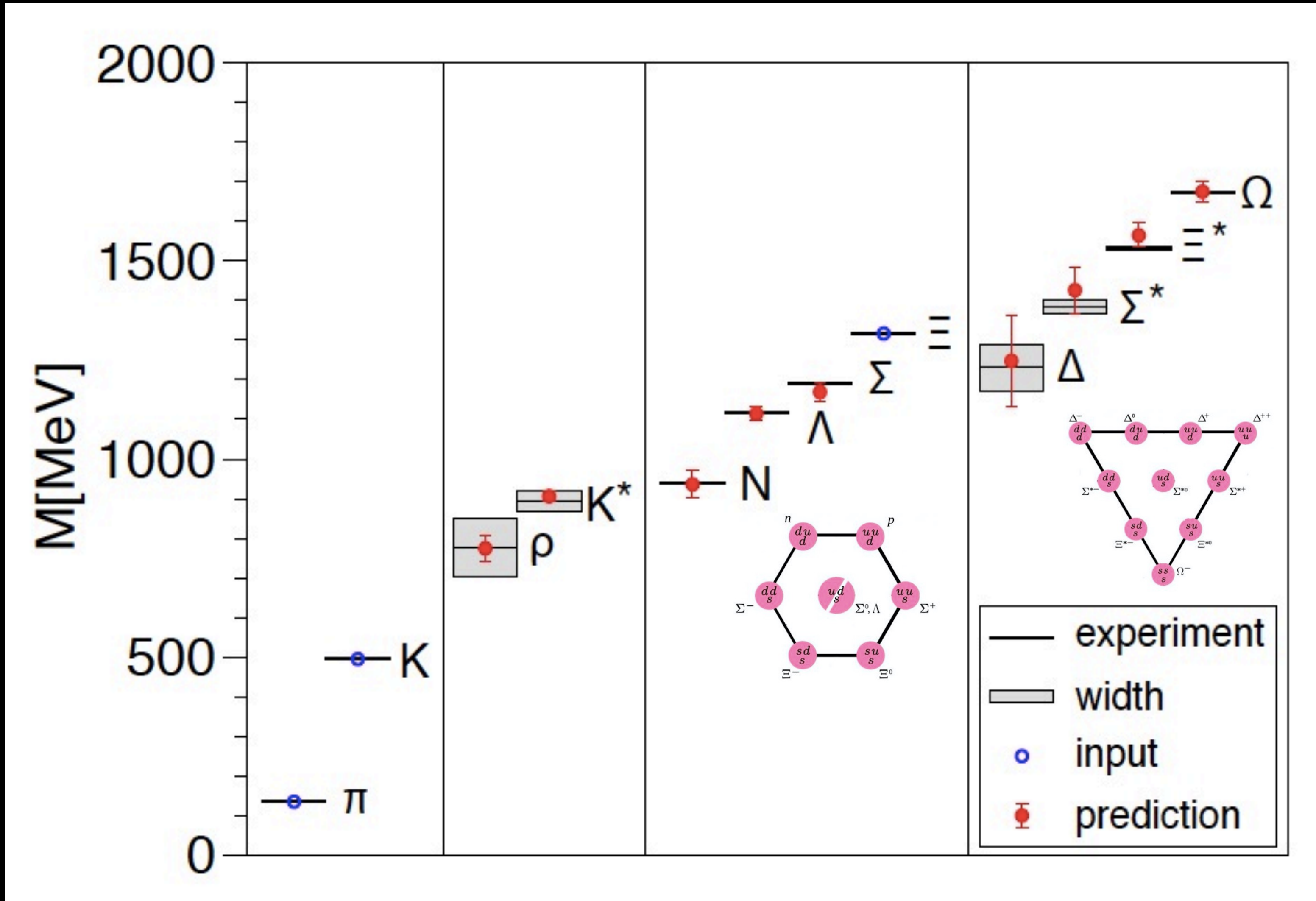


Ω^- (1672)
Weak decay (→ΛK, Ξπ)
Mean Life ~ 0.8 x 10^{-10} sec

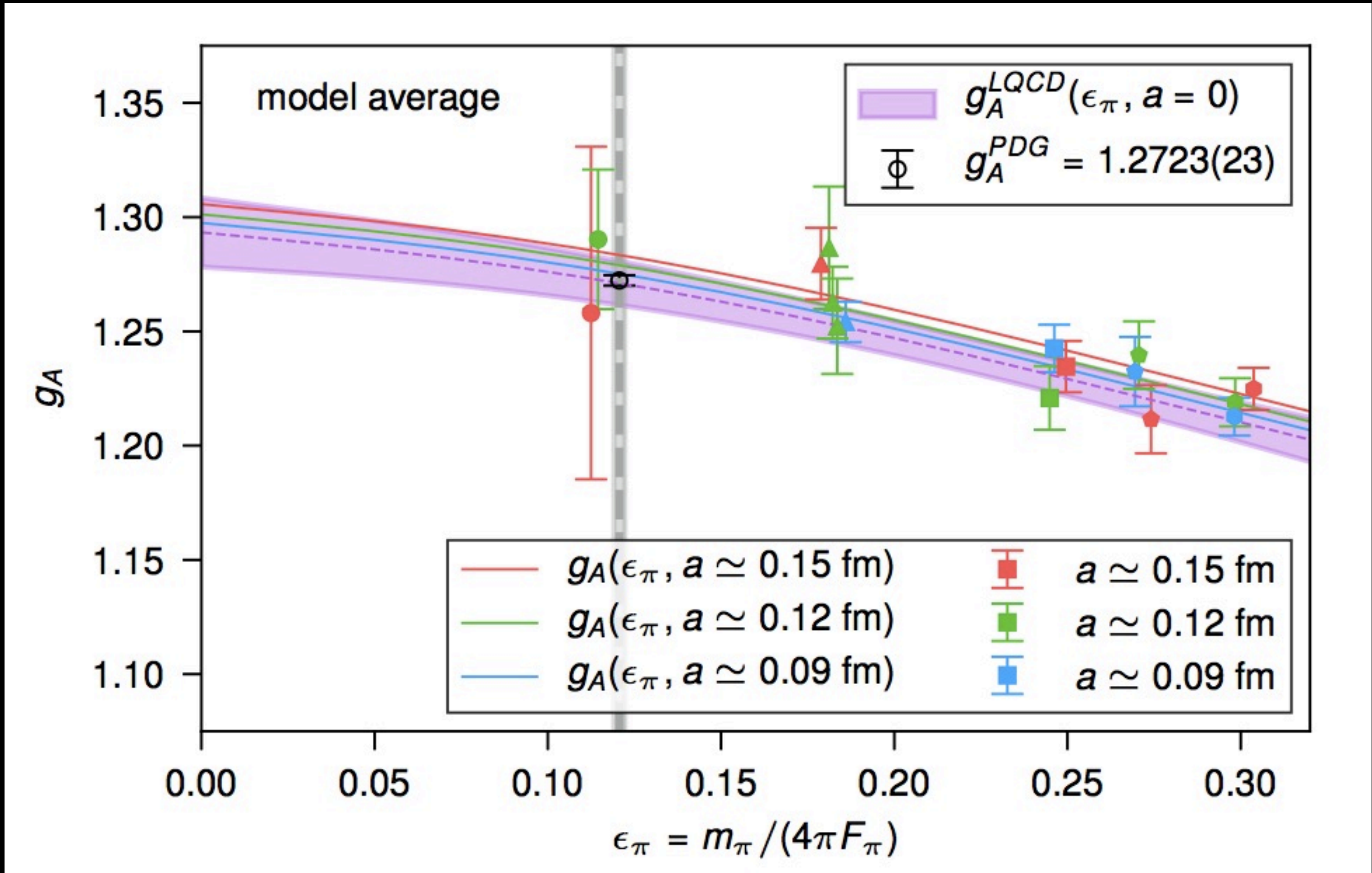
Hadron masses from LQCD





Review: Fodor and Hoelbling, Rev. Mod. Phys. 84 (2012) 449

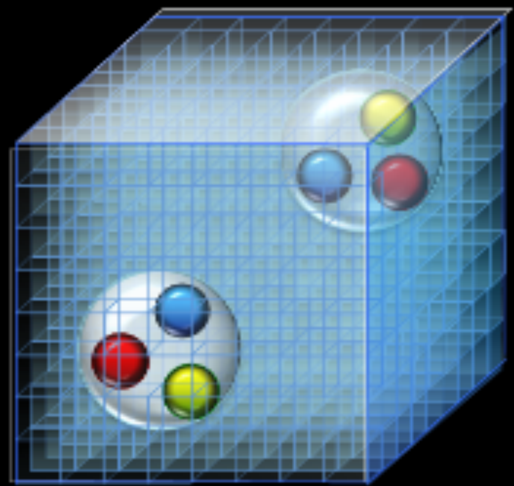
Nucleon axial charge (g_A) from LQCD



$(g_A)_{LQCD} = 1.2711(13)$ vs. $(g_A)_{expt} = 1.2723(23)$

C.C. Chang (RIKEN/LBNL)+, Nature 558 (2018) 91

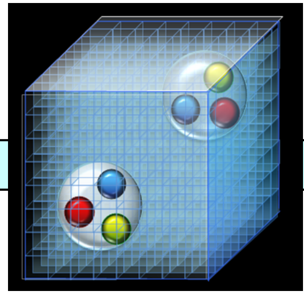
II. BB Interactions on the Lattice



Lattice QCD

Direct Method

$R(\tau)$



HAL QCD Method

$\mathscr{R}(\mathbf{r}, \tau)$

Observables

(phase shift, binding energy)

Lattice QCD

Direct Method

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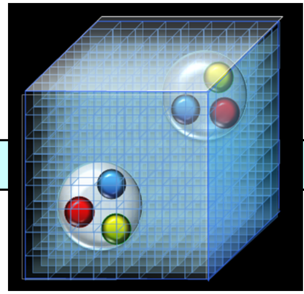

Iritani+ [HAL QCD Coll.]
JHEP1610 (2016) 101
PRD96 (2017) 034521
PRD99 (2019) 014514
JHEP1903 (2019) 007

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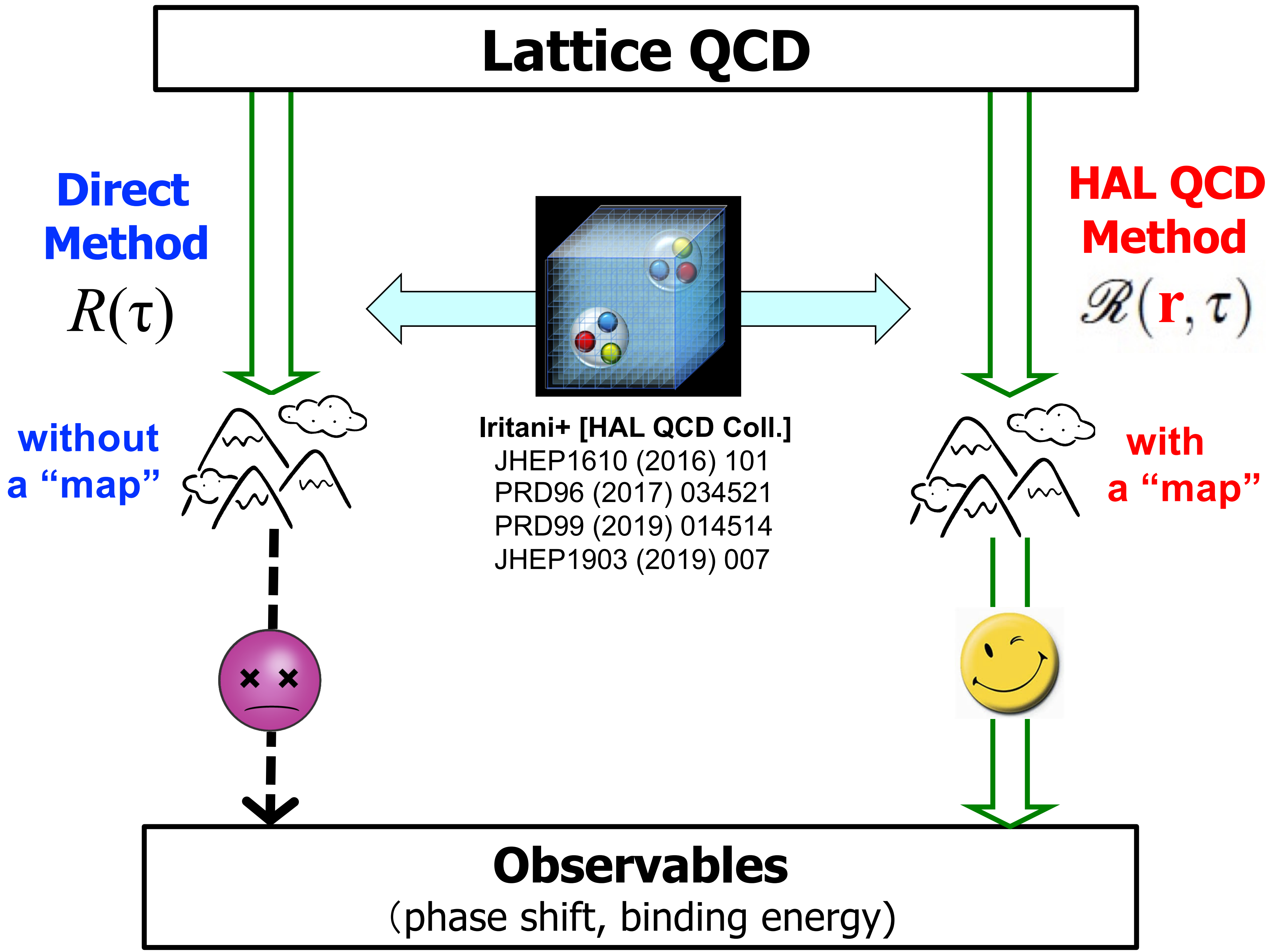

Lattice QCD
Direct Method
$R(\tau)$
HAL QCD Method
$\mathscr{R}(\mathbf{r},\tau)$
without a "map"
with a "map"
Iritani+ [HAL QCD Coll.]
JHEP1610 (2016) 101
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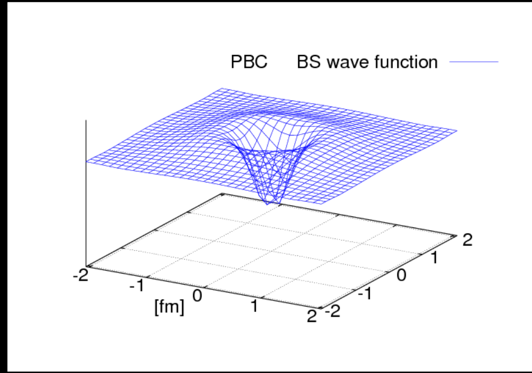
HAL QCD Method

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001
Ishii+ [HAL QCD Coll.], PLB 712 (2012) 437

Spacetime BB correlation, $\mathscr{R}(r,t)$

$$\left\{\frac{1}{4M_B}\frac{\partial^2}{\partial\tau^2}-\frac{\partial}{\partial\tau}-H_0\right\}\mathscr{R}(\mathbf{r},\tau)=\int d^3r' U(\mathbf{r},\mathbf{r}')\mathscr{R}(\mathbf{r}',\tau)$$



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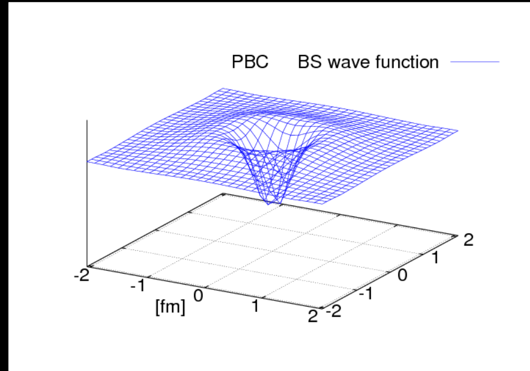
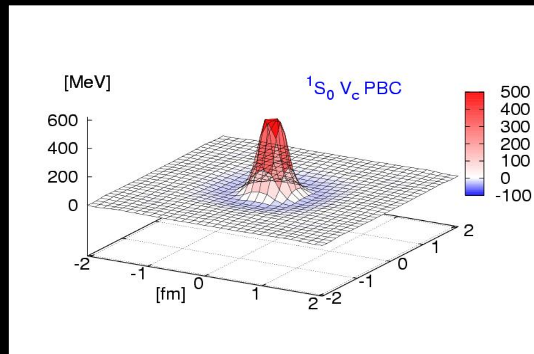


→ Interaction kernel $U(r,r')$

$$U(\mathbf{r},\mathbf{r}')=V(\mathbf{r},\mathbf{v})\delta(\mathbf{r}-\mathbf{r}'),$$

$$V(\mathbf{r},\mathbf{v})=\underbrace{V_\mathrm{C}(r)+V_\mathrm{T}(r)S_{12}}_{\mathrm{LO}}+\underbrace{V_\mathrm{LS}(r)\mathbf{L}\cdot\mathbf{S}}_{\mathrm{NLO}}+\underbrace{O(\mathbf{v}^2)}_{\mathrm{N^2LO}}+\cdots$$



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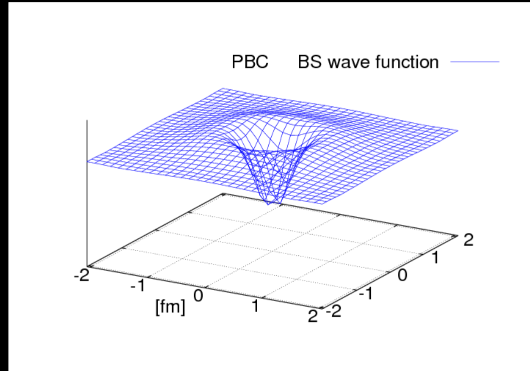

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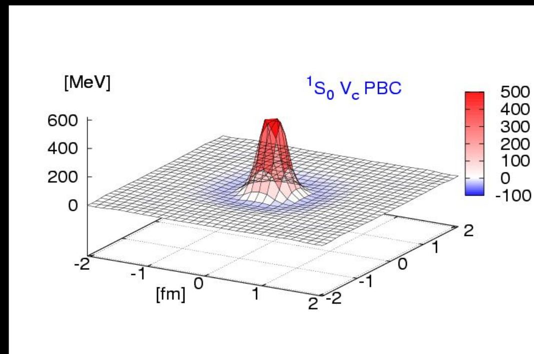
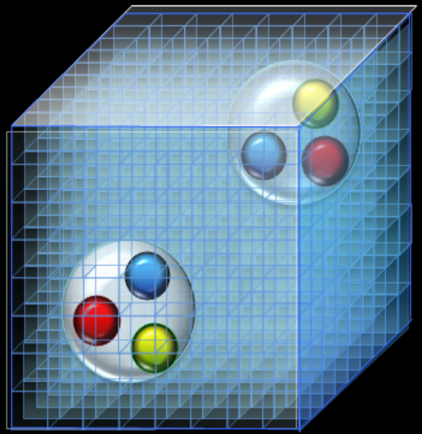


→ Observables (phase shift, binding energy)

HAL QCD Configurations almost at physical pion mass



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$L = 8.1$ fm

$m_\pi = 146$ MeV

$M_K = 525$ MeV

$N_{cfg} = 400$

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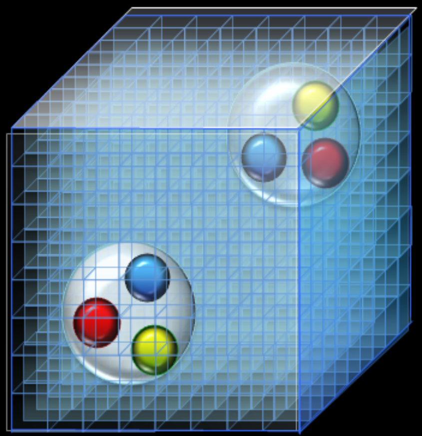

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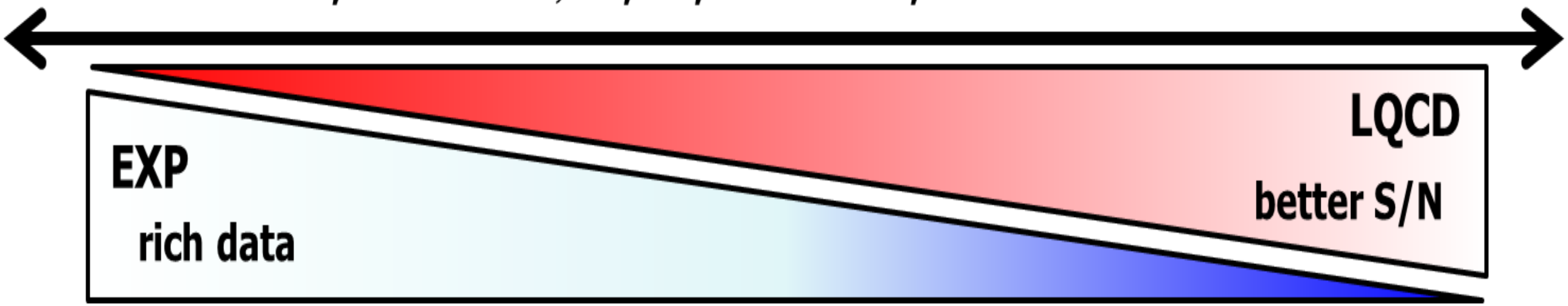
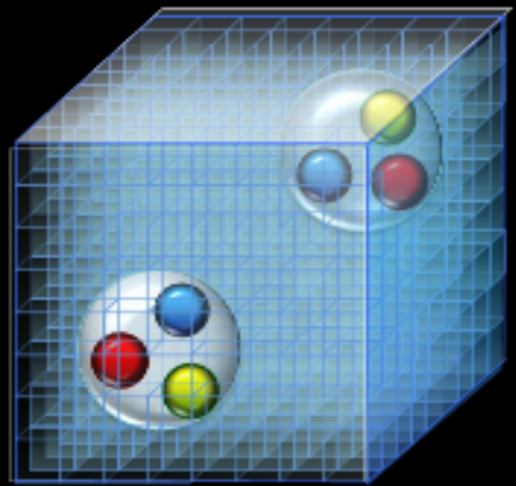
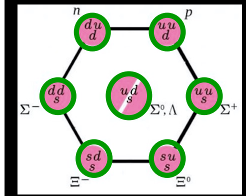
S=0	S=−1	S=−2	S=−3	S=−4	S=−5	S=−6
NN	NΛ, NΣ	ΛΛ, ΛΣ, ΣΣ, NΞ	ΛΞ, ΣΞ	ΞΞ	ΞΩ	ΩΩ



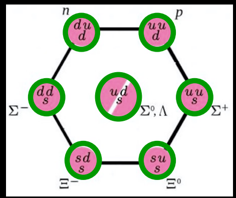
III. Recent YN and YY Results from HAL QCD



$SU(3)_F$ classification of BB system



x



$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$

$NN(^1S_0)$ → 27

$H_{\Lambda\Lambda-N\Xi-\Lambda\Sigma}\,(^1S_0)$ → 1

Jaffe (1977)

$NN(^3S_1)$ → 10*

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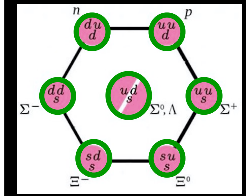

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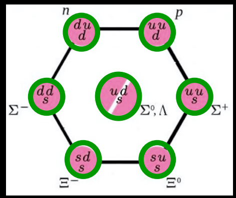

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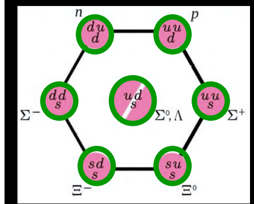

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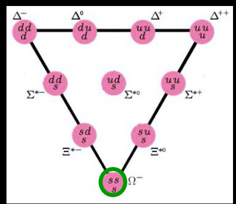
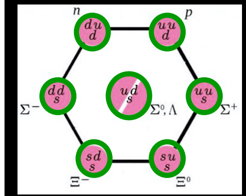
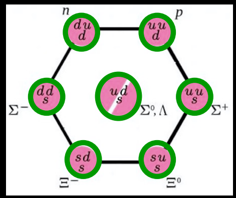


$$8 \times 10 = 35 + 8 + 10 + 27$$

$N\Omega$ ($^5S_2$) → 8

Goldman+ (1987), Oka (1988)

SU(3)$_F$ classification of BB system

 x 

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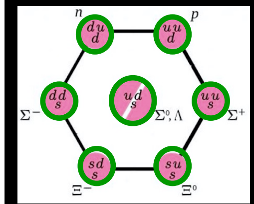
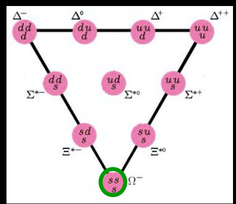
 x 

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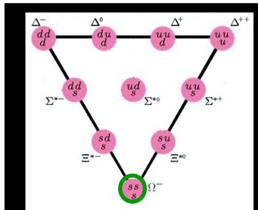
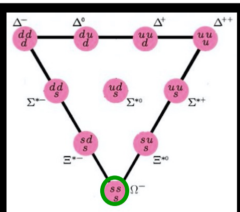
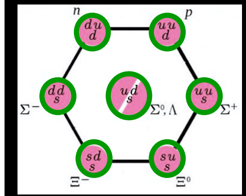
 x 

$$10 \times 10 = 28 + 27 + 35 + 10^*$$

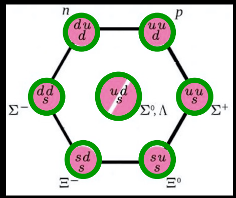
Kopeliovich+ (1990) $\Omega\Omega$($^1S_0$) → 28

$\Delta\Delta$($^7S_3$) → 10* Dyson+ (1964)

$SU(3)_F$ classification of BB system



x



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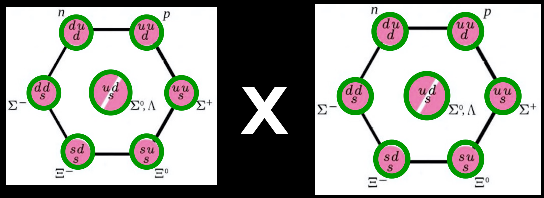
T. Inoue+ [HAL QCD Coll.], arXiv:1809.08932 [hep-lat]



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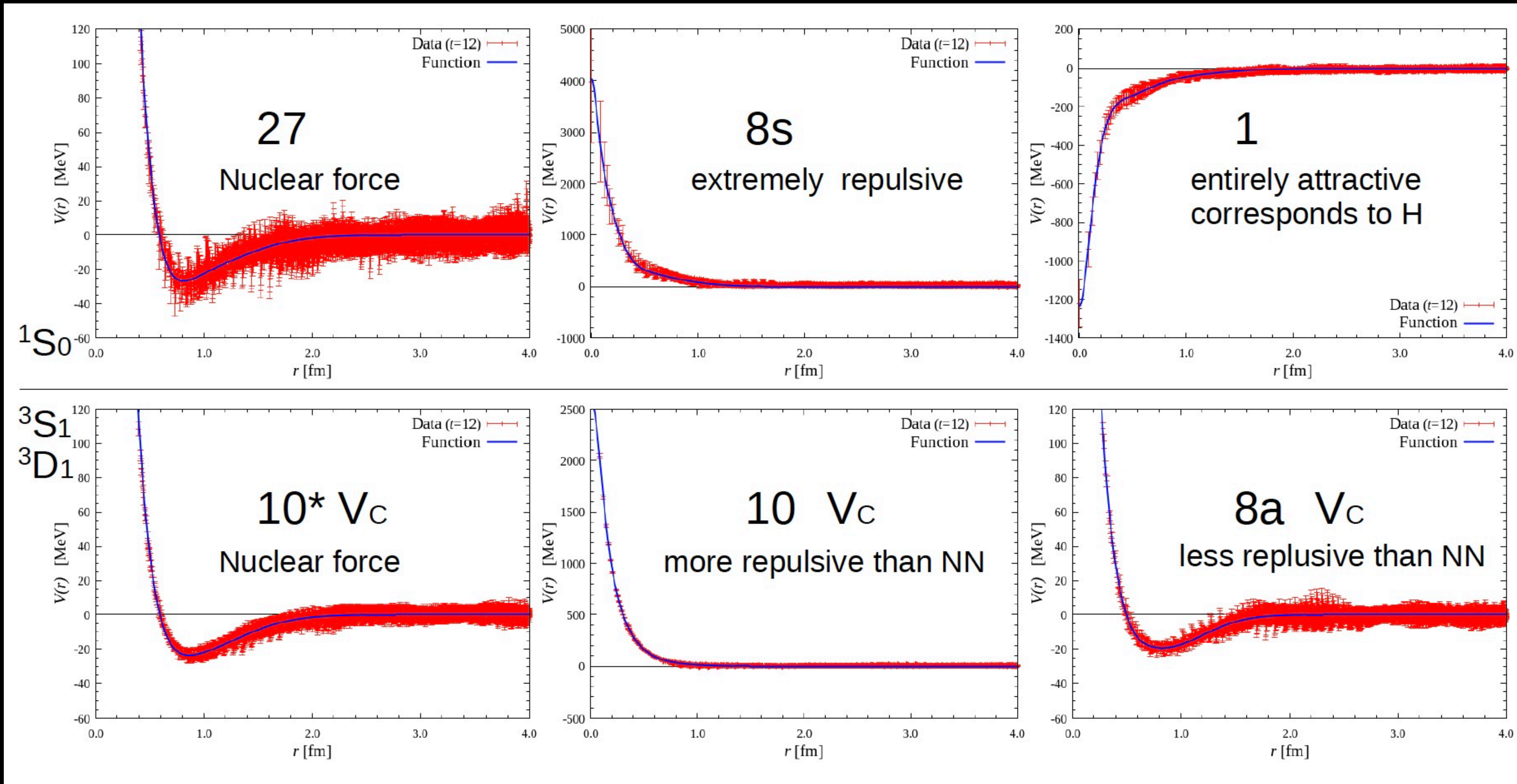

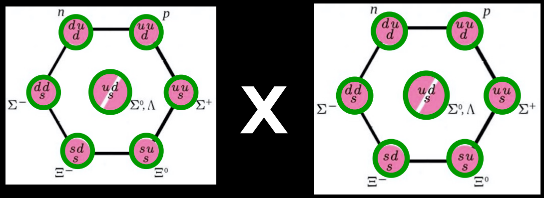

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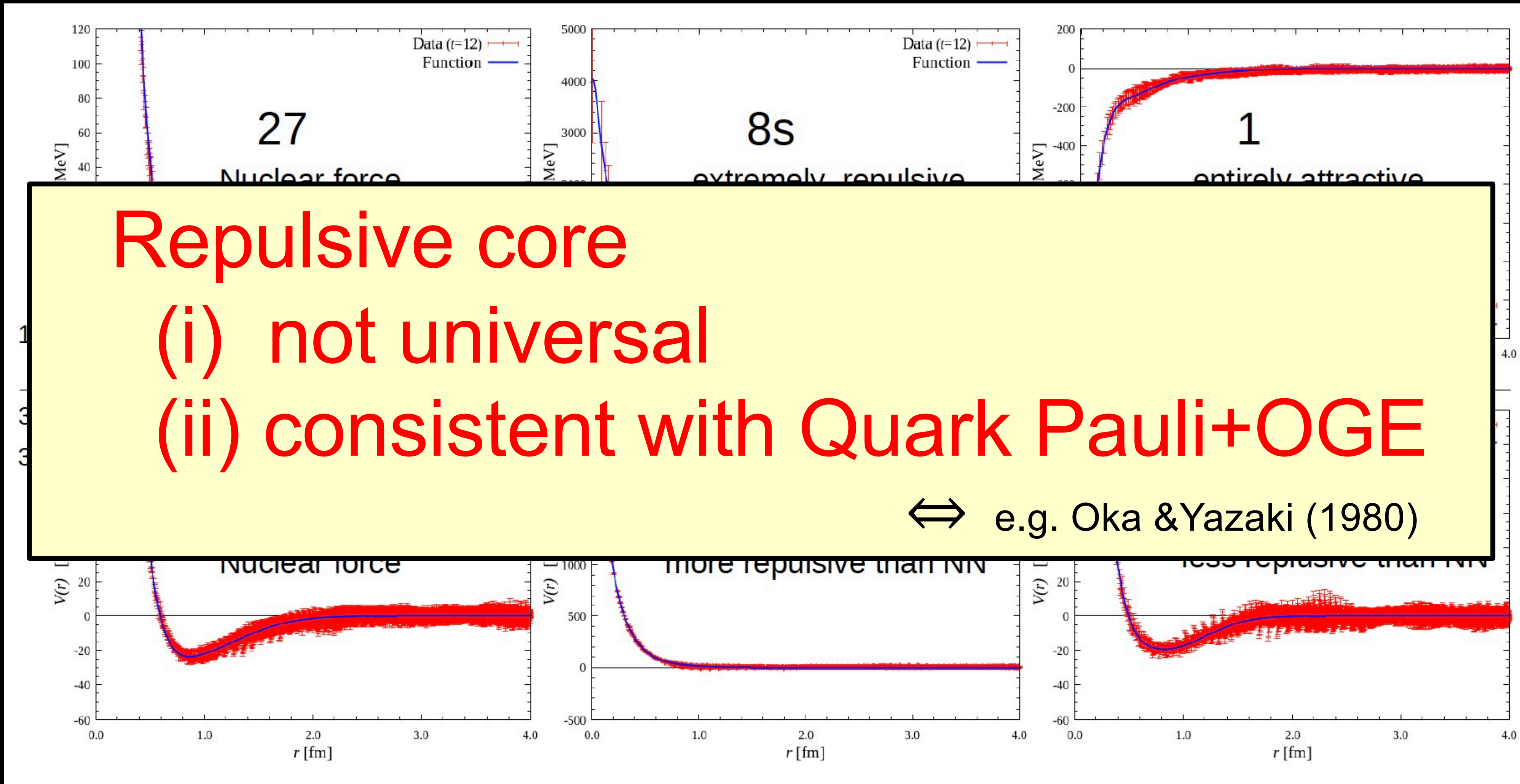


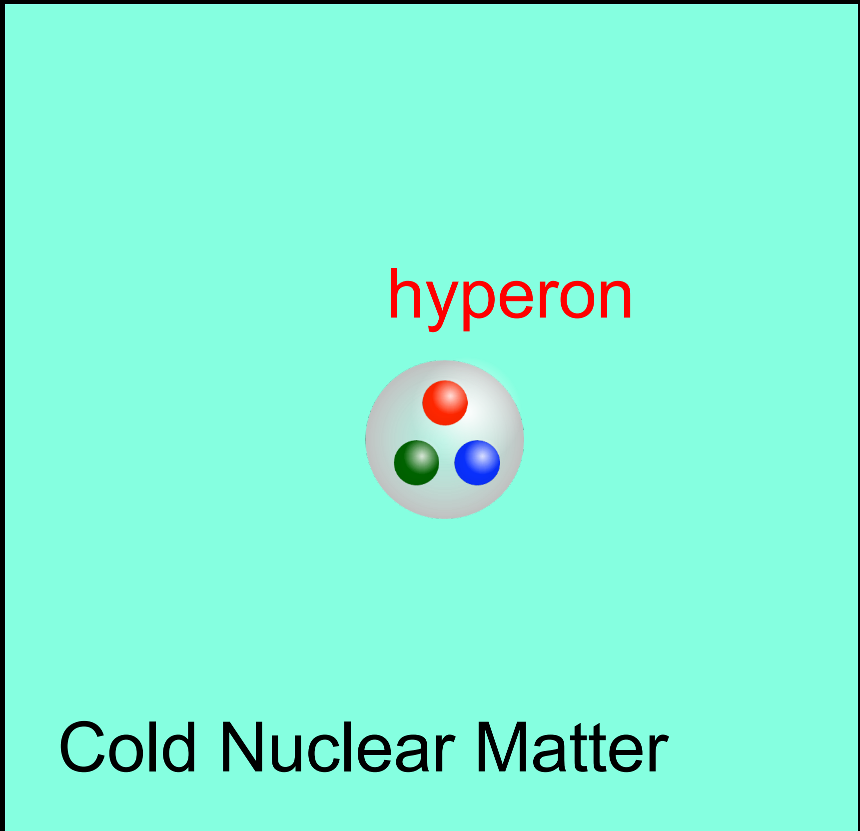
Repulsive core

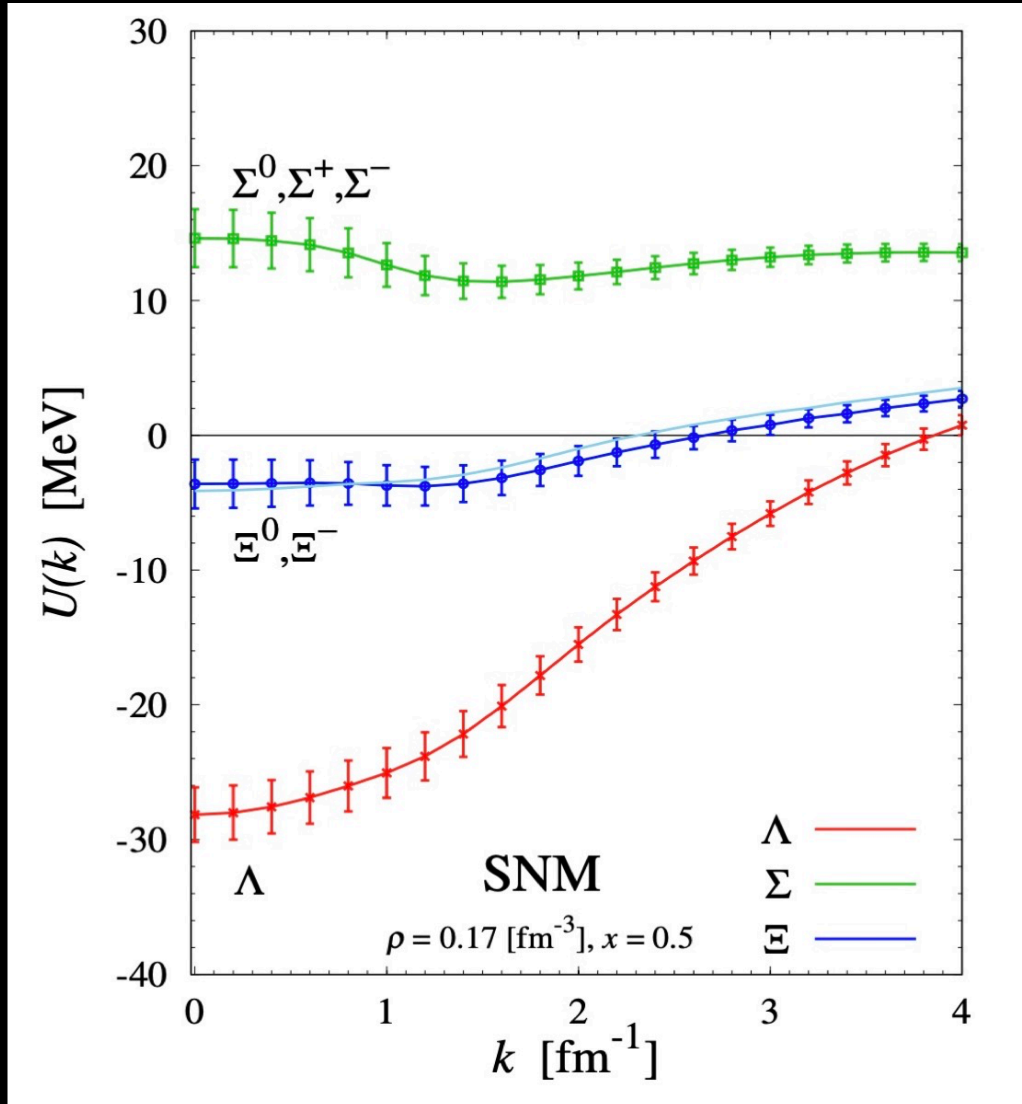
(i) not universal

(ii) consistent with Quark Pauli+OGE

⇔ e.g. Oka &Yazaki (1980)

Hyperon embedded in cold nuclear matter
(HAL QCD + Brueckner-Hartree-Fock)

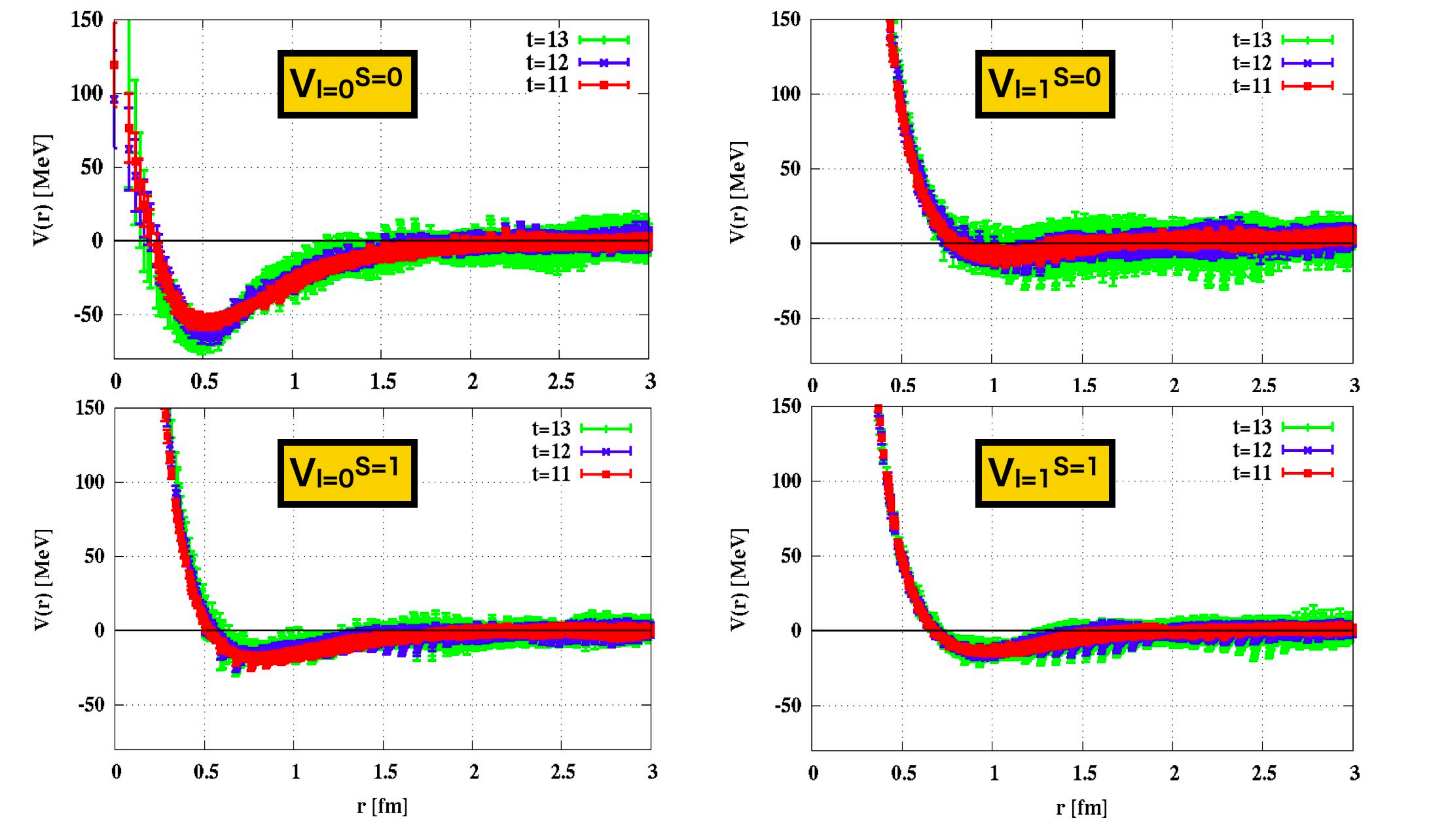




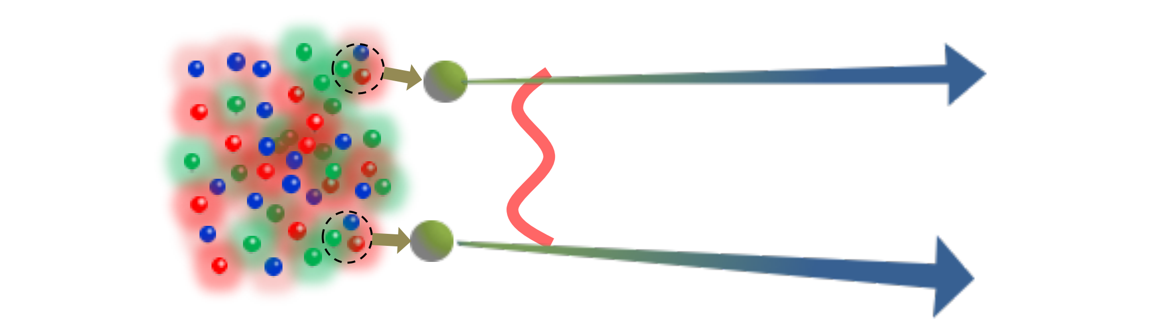
T. Inoue+ [HAL QCD Coll.],
arXiv:1809.08932 [hep-lat]

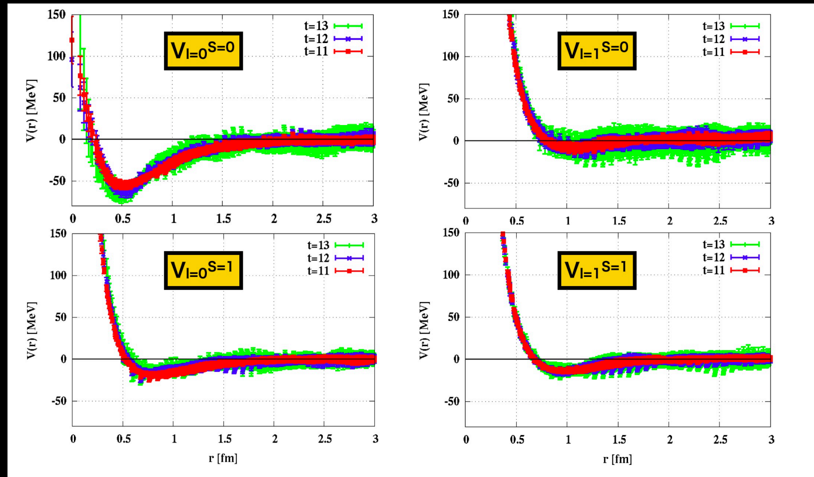
NΞ interaction from HAL QCD

K. Sasaki+
[HAL QCD Coll.]
paper in preparation

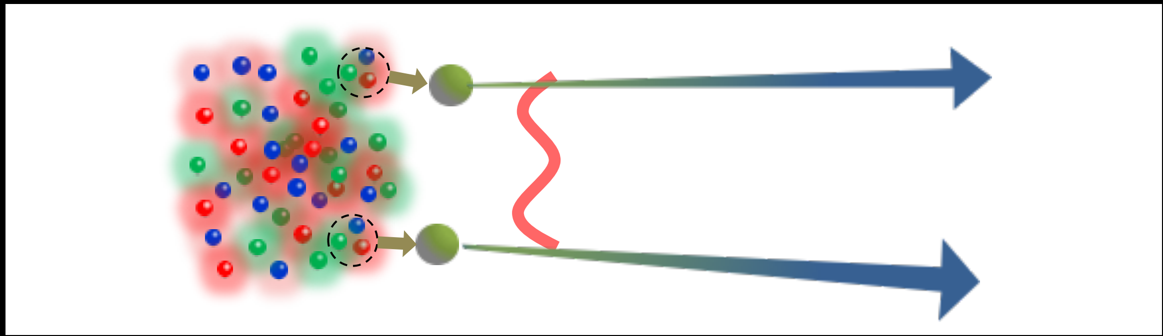


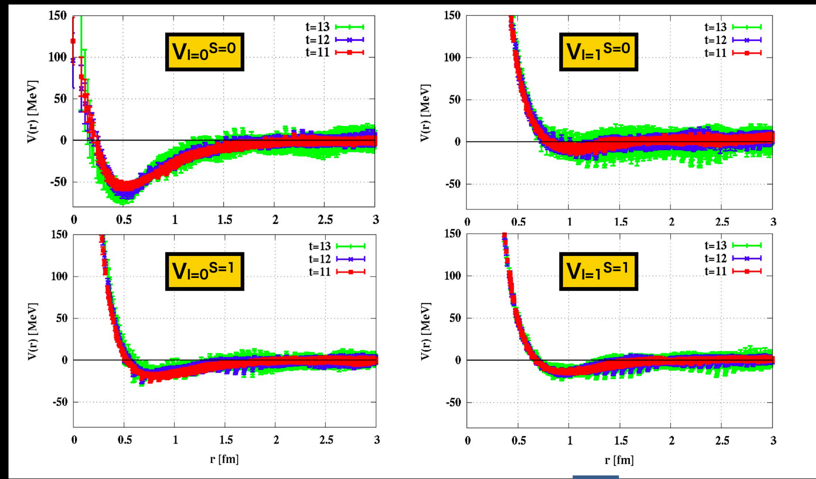
NΞ correlation at LHC(p-Pb) ⇔ HAL QCD



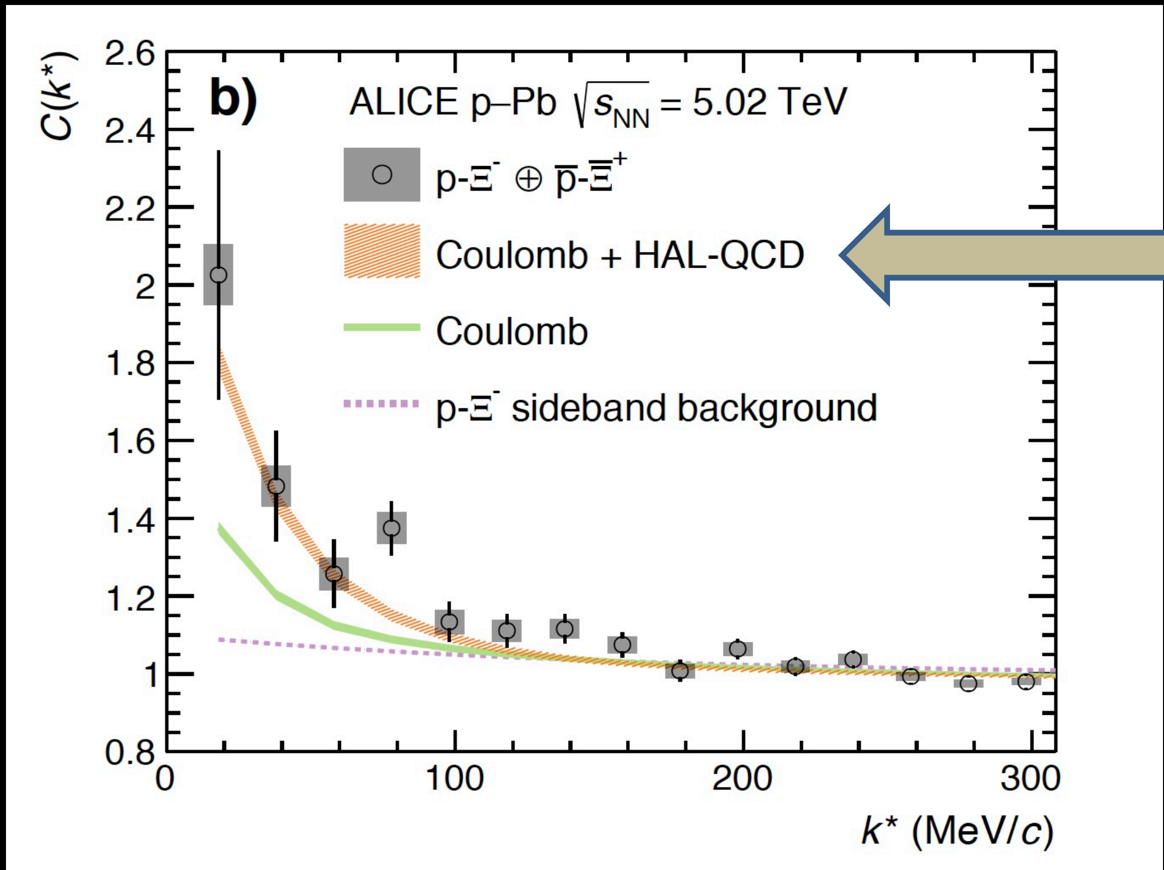


NΞ correlation at LHC(p-Pb) ⇔ HAL QCD





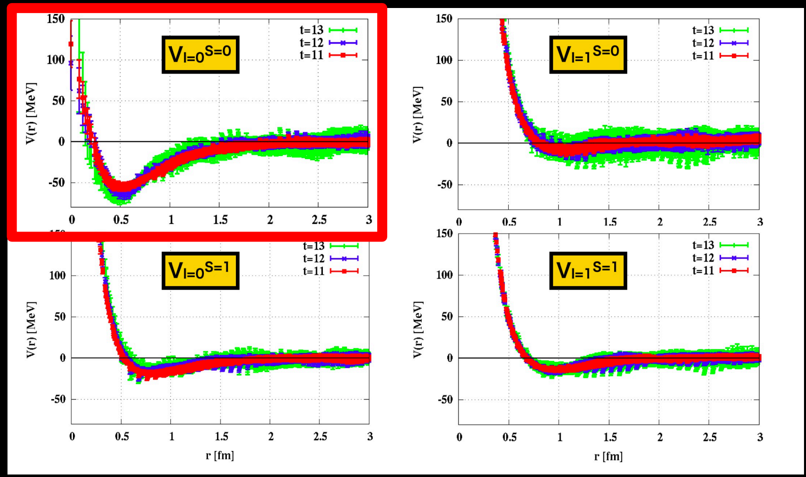
ALICE, Coll., arXiv:1904.12198 [nucl-ex]



$$C(k) = \int S(\vec{r},k)\,|\psi(\vec{r},k)|^2\, d\vec{r}$$

ΛΛ interaction from HAL QCD

K. Sasaki+
[HAL QCD Coll.]
paper in preparation



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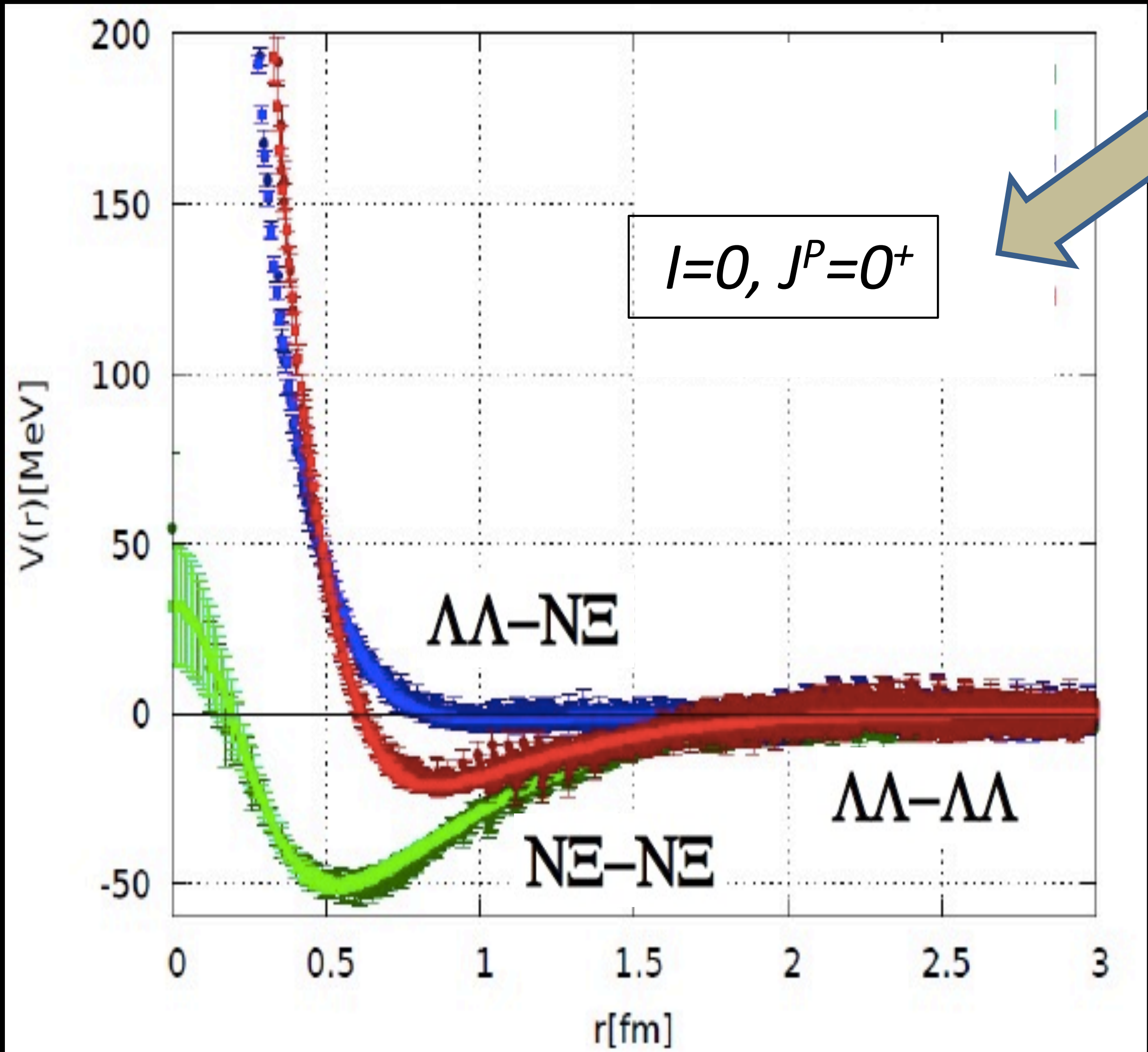
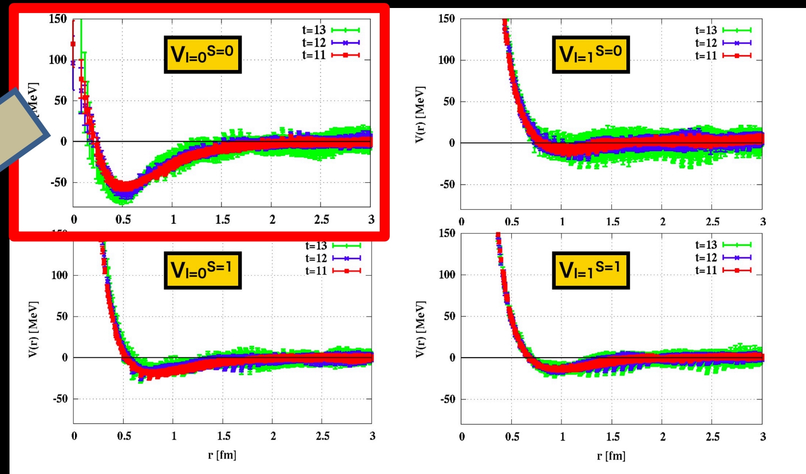
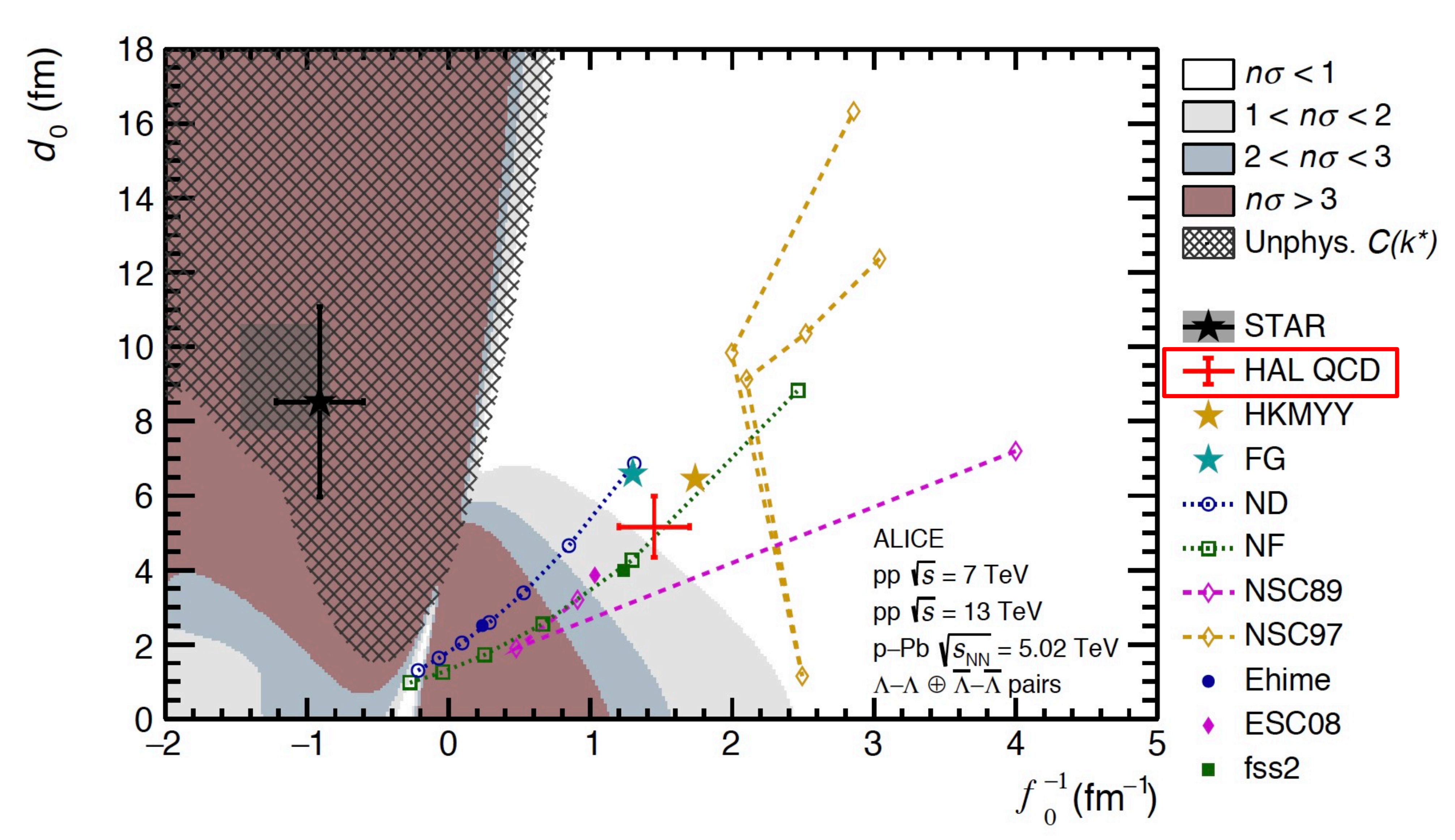




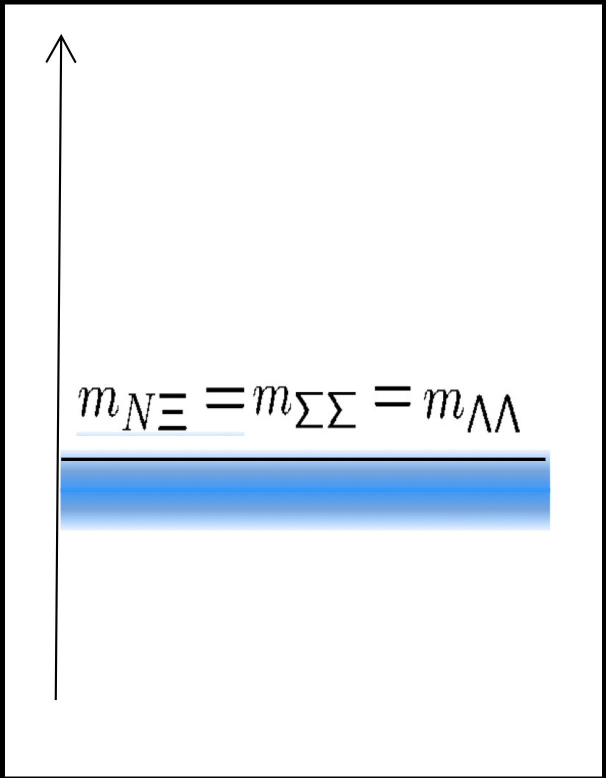
ΛΛ correlation at LHC (p-p, p-Pb) ⇔ HAL QCD

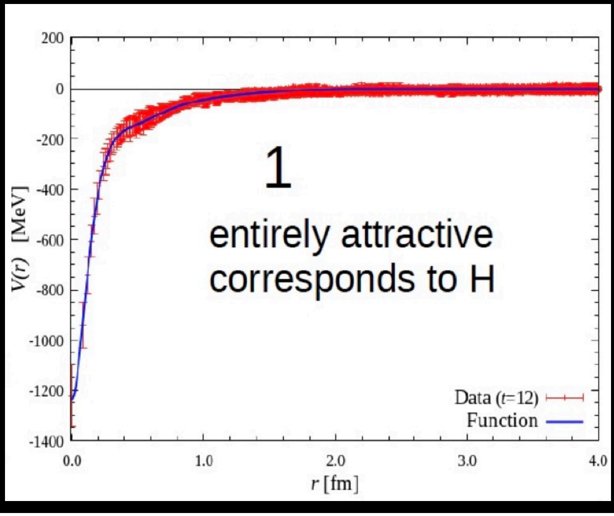
ALICE Coll., arXiv:1905.07209 [nucl-ex]



Fate of "H (uuddss)" dibaryon from HAL QCD

K. Sasaki+
[HAL QCD Coll.]
paper in preparation





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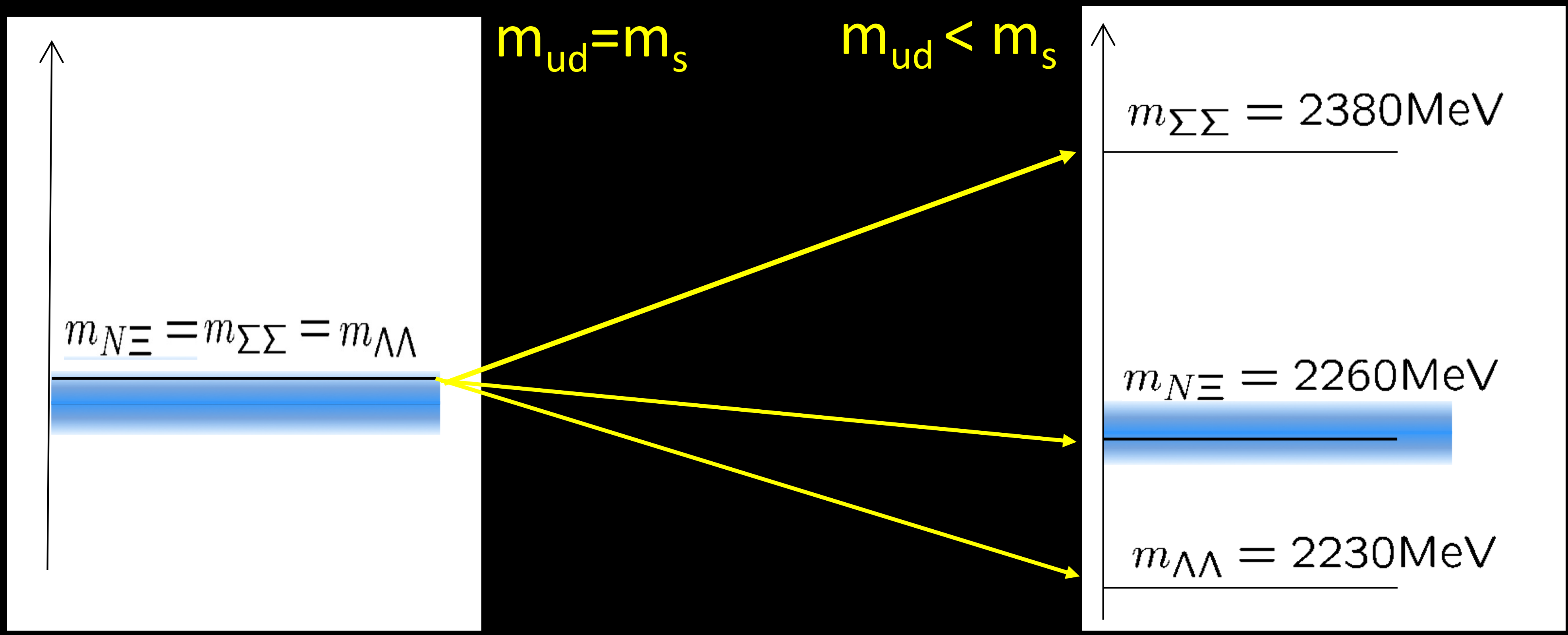

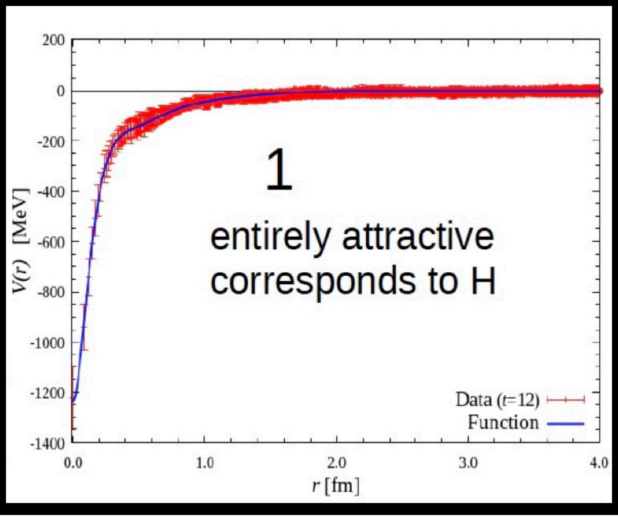

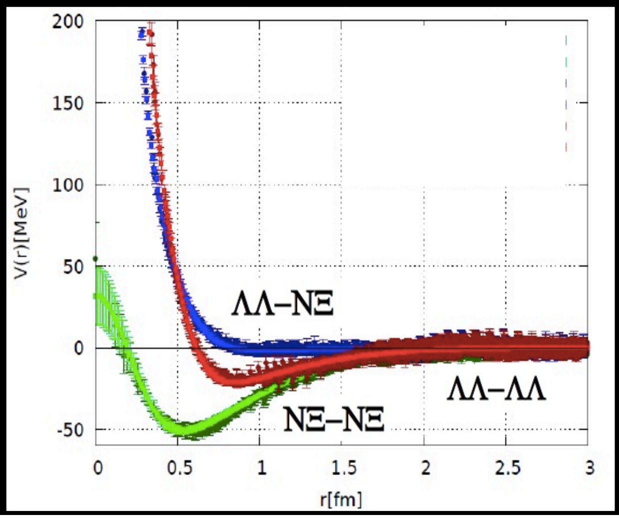

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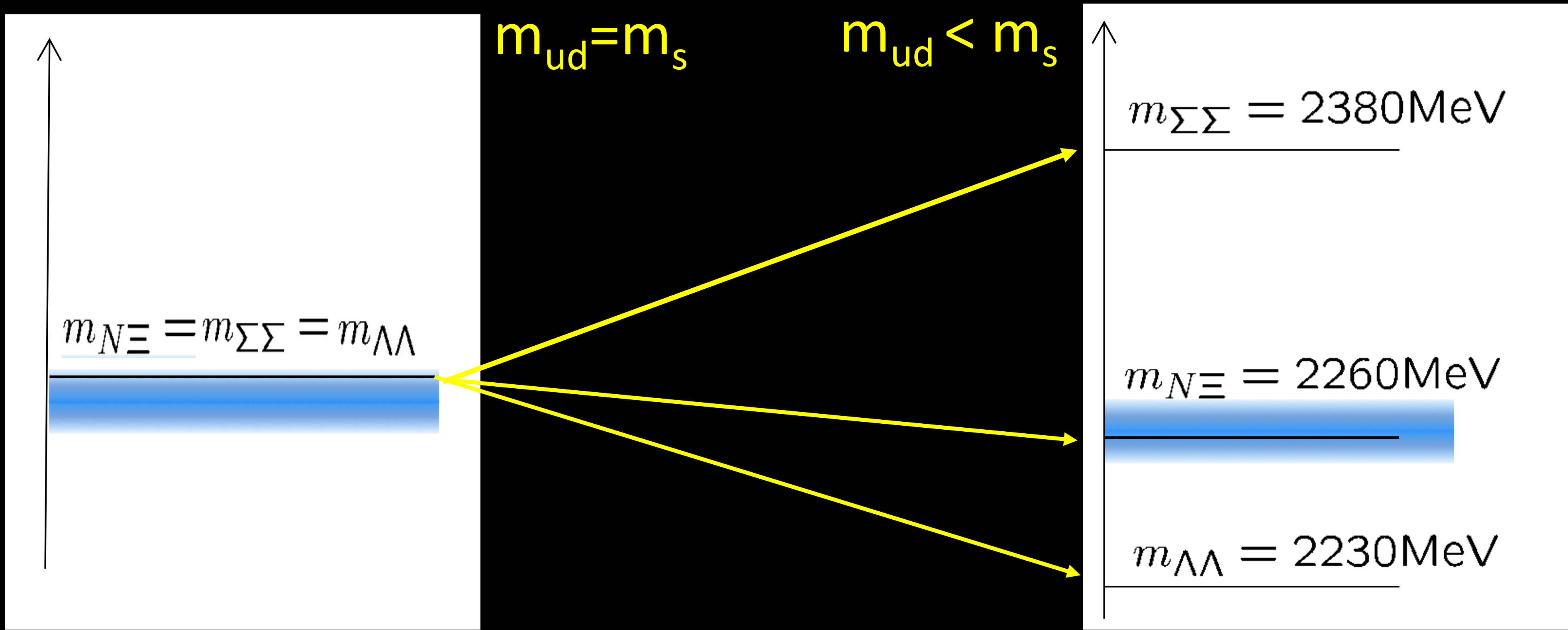
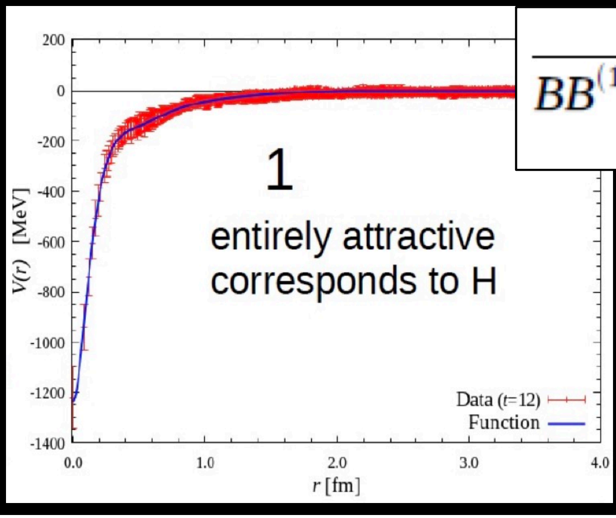
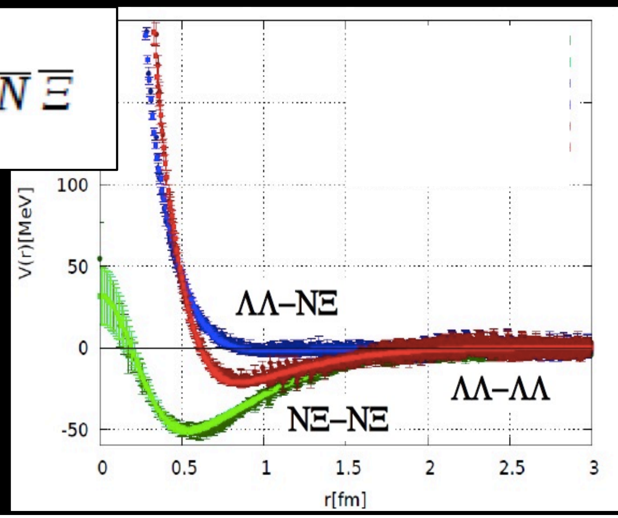
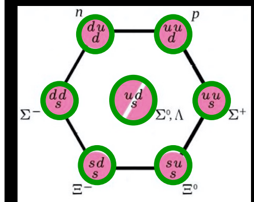




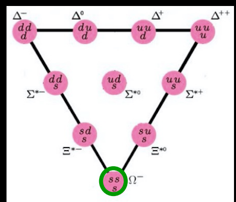
$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$



$SU(3)_F$ classification of BB system

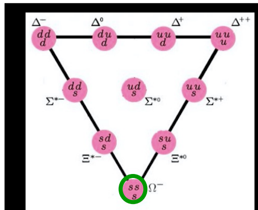


x

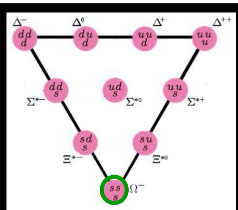


$8 \times 10 = 35 + 8 + 10 + 27$

$N\Omega\ (^5S_2)$ Goldman+ (1987), Oka (1988)



x



$10 \times 10 = 28 + 27 + 35 + 10^*$

Kopeliovich+ (1990) $\Omega\Omega(^1S_0)$

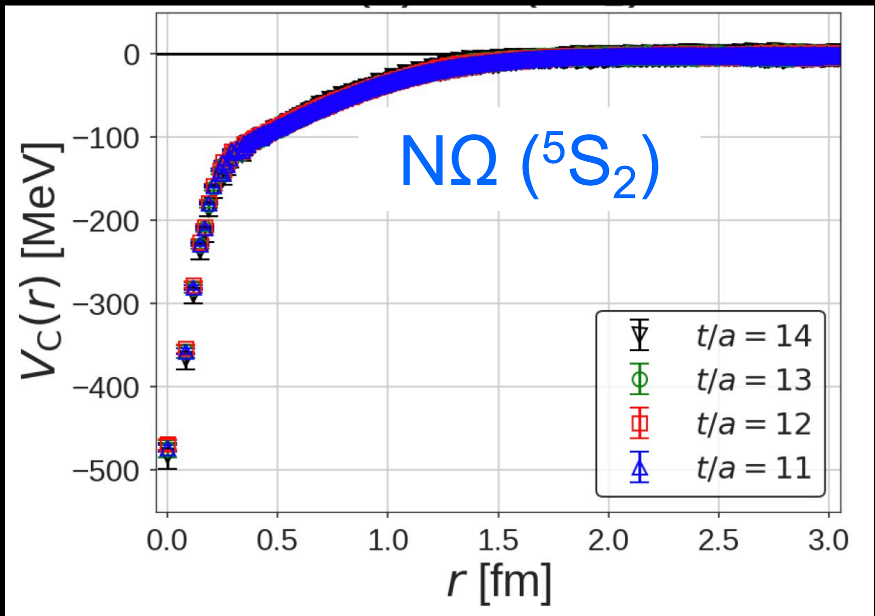
$\Delta\Delta(^7S_3)$ Dyson+ (1964)

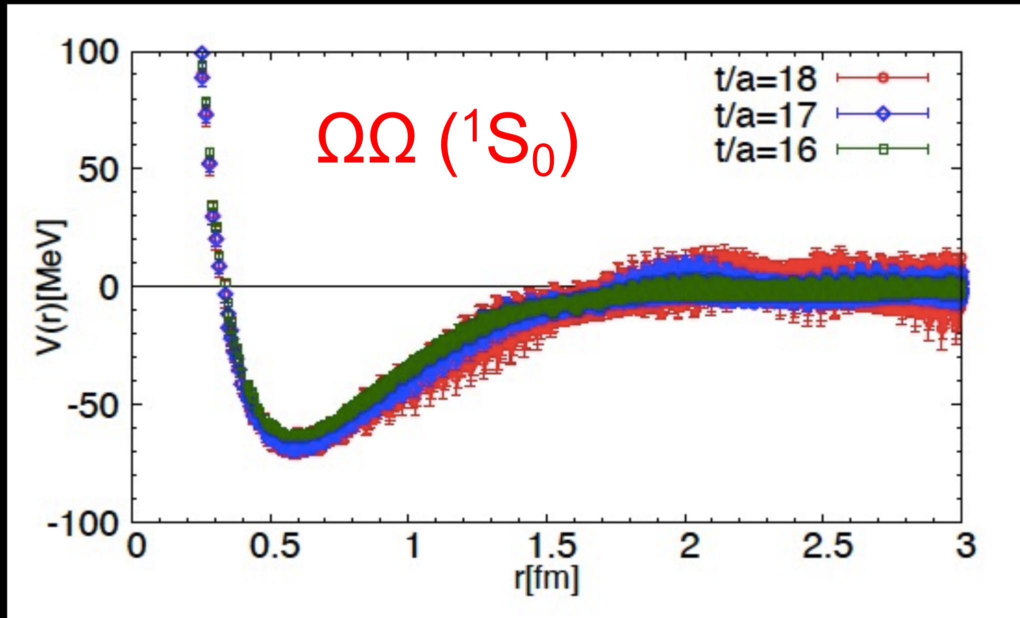
S=-3

Iritani+ [HAL QCD Coll.], Phys.Lett. B792(2019) 284

S=-6

Gongyo+ [HAL QCD Coll.], PRL 120 (2018) 212001





Both are Pauli allowed

S=-3

Iritani+ [HAL QCD Coll.], Phys.Lett. B792(2019) 284

S=-6

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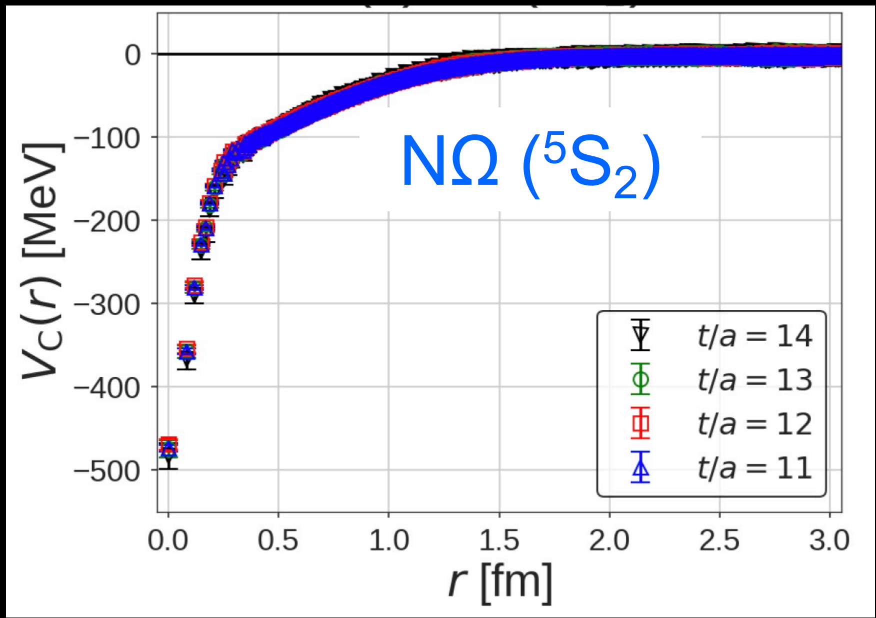
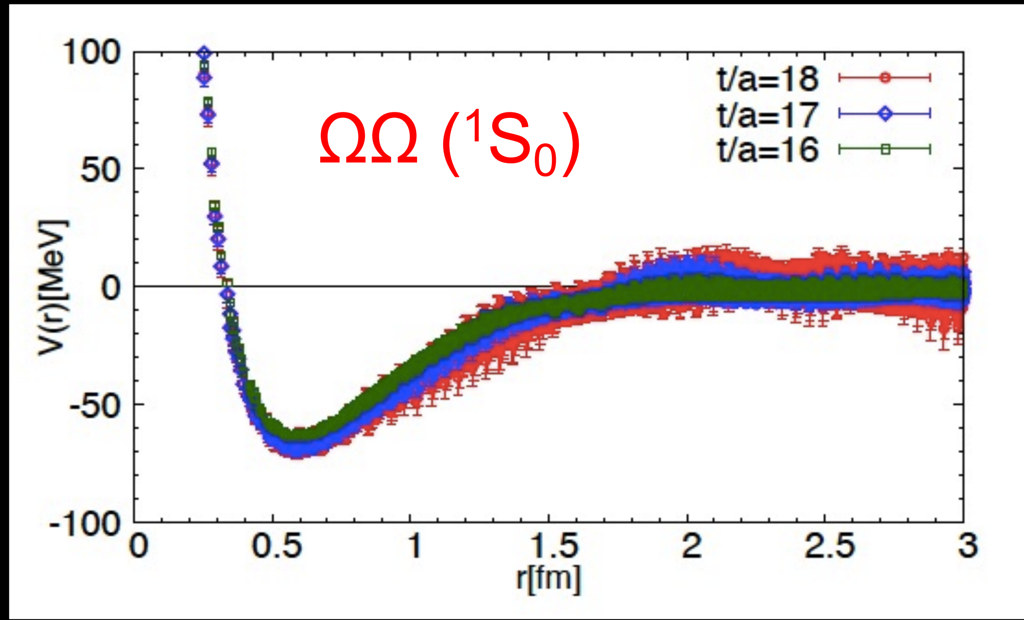
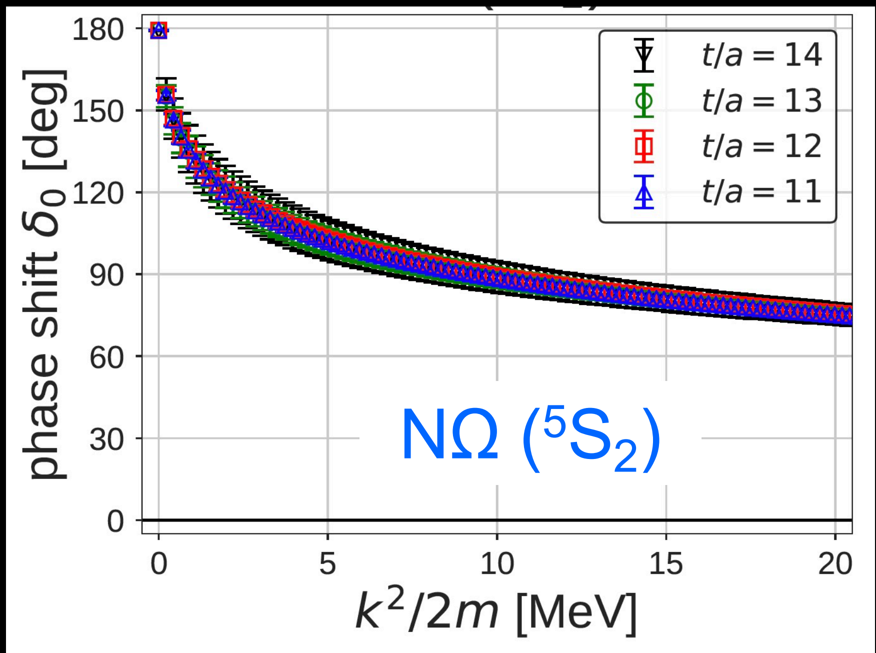
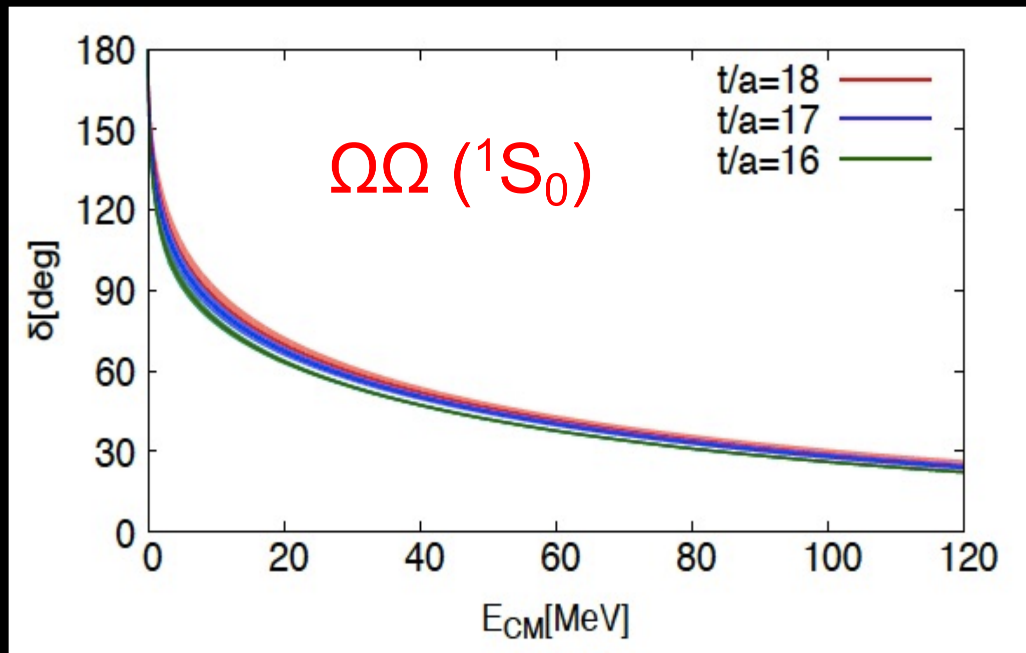
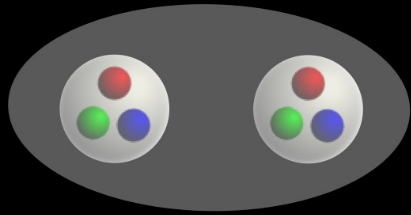


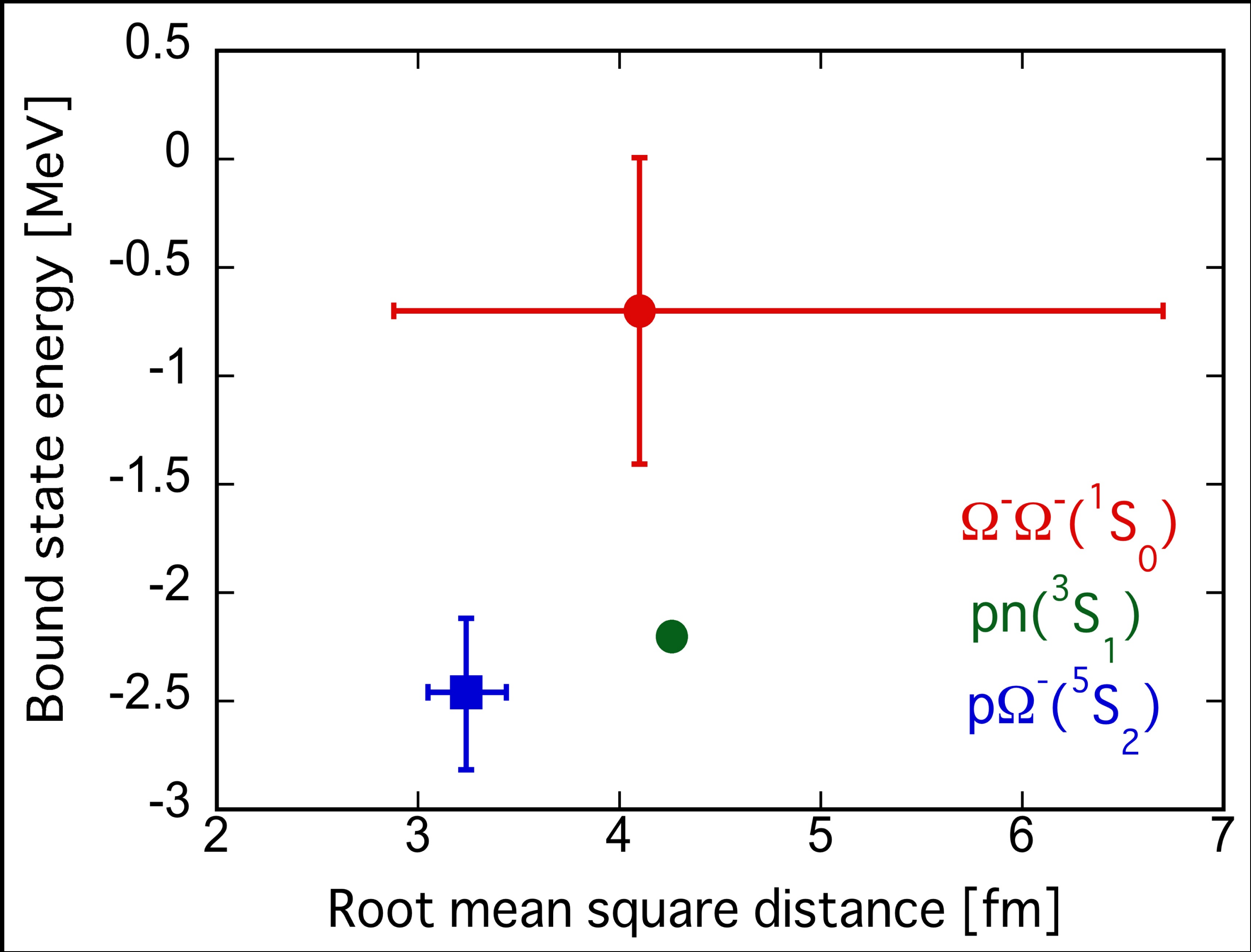






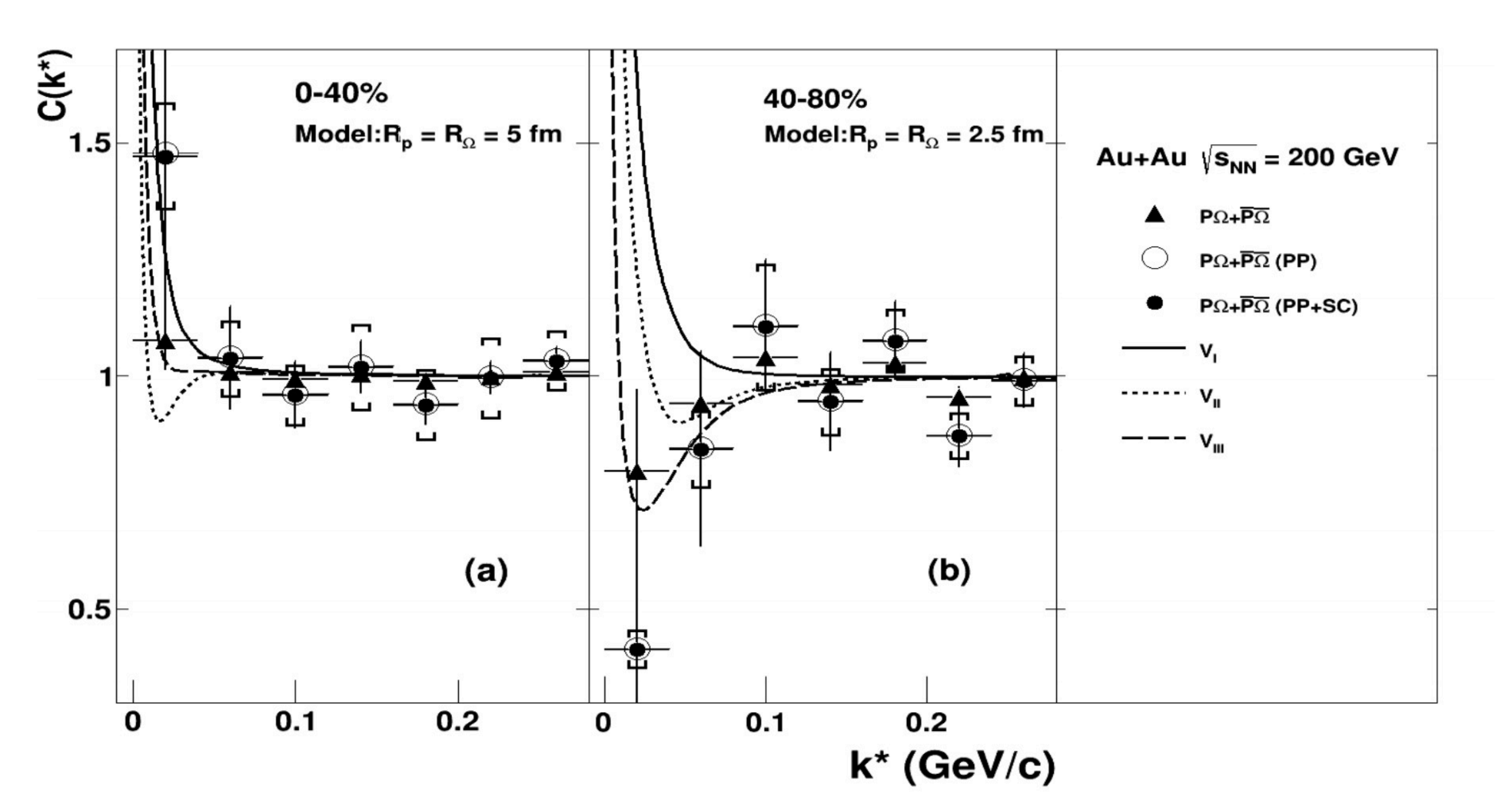
New dibaryons near unitarity ?





pΩ correlation at RHIC (Au-Au)

STAR Coll., Phys. Lett. B790 (2019) 490



Y=2 States in Su(6) Theory

Freeman J. Dyson and Nguyen-Huu Xuong
Phys. Rev. Lett. **13**, 815 – Published 28 December 1964; Erratum Phys. Rev. Lett. **14**, 339 (1965)

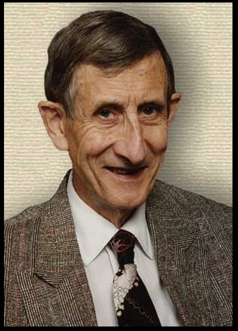
[From F. Dyson, Sep. 13, 2017]

Thank you very much for sending me your paper on the Omega-Omega calculation. This is a beautiful piece of work, and it will be a big step forward if the experimenters are able to confirm it.

Now I wish you the joy of seeing it confirmed.

Yours sincerely,

Freeman Dyson



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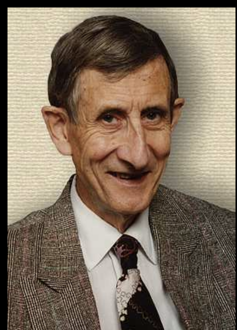
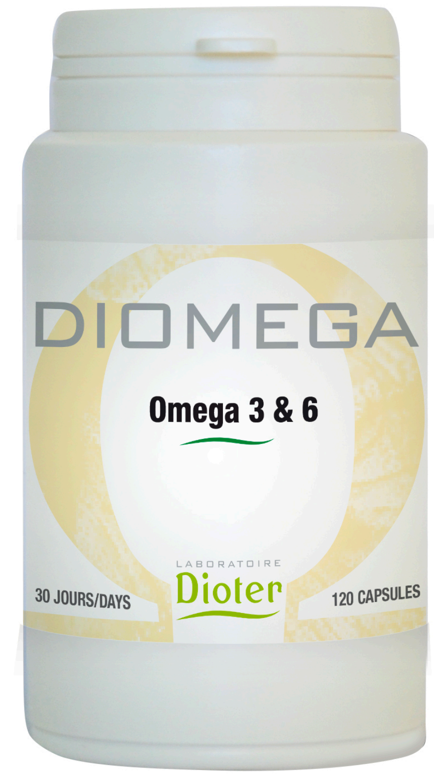


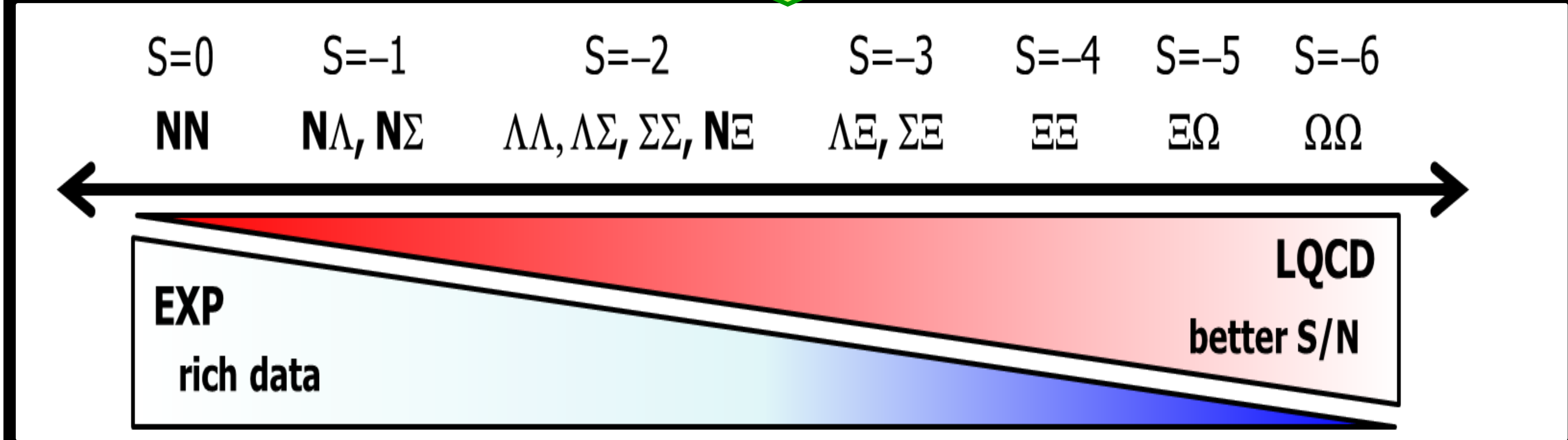


€23

https://ods.od.nih.gov/factsheets/Omega3FattyAcidsandHealth-HealthProfessional/

Summary

YN & YY interactions from LQCD
- Prediction era has come -



Summary

2011-2019
K (10 PFlops)

2021-
Fugaku (1000 PFlops)

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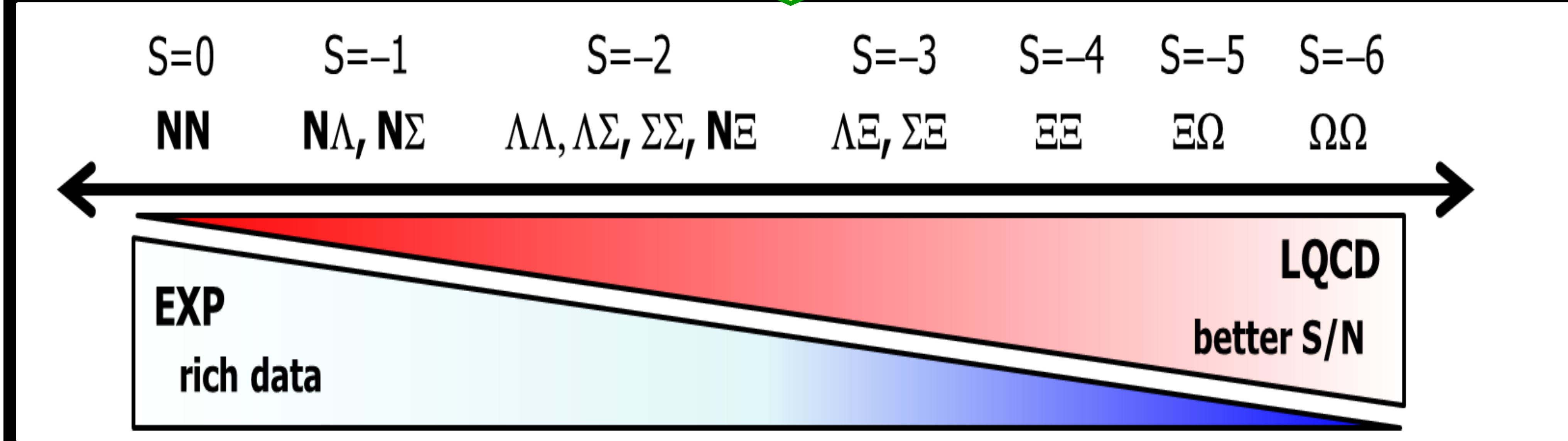

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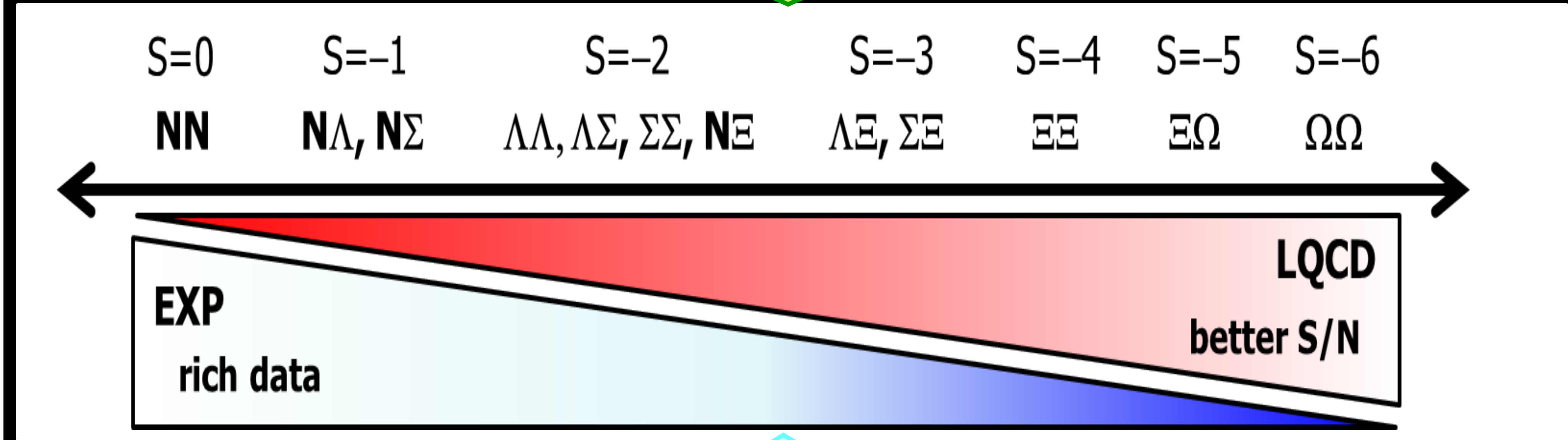


Femtoscopy at RHIC, LHC
Hypernuclei at J-PARC, FAIR

Backup slides

Quantum Chromodynamics (QCD)

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

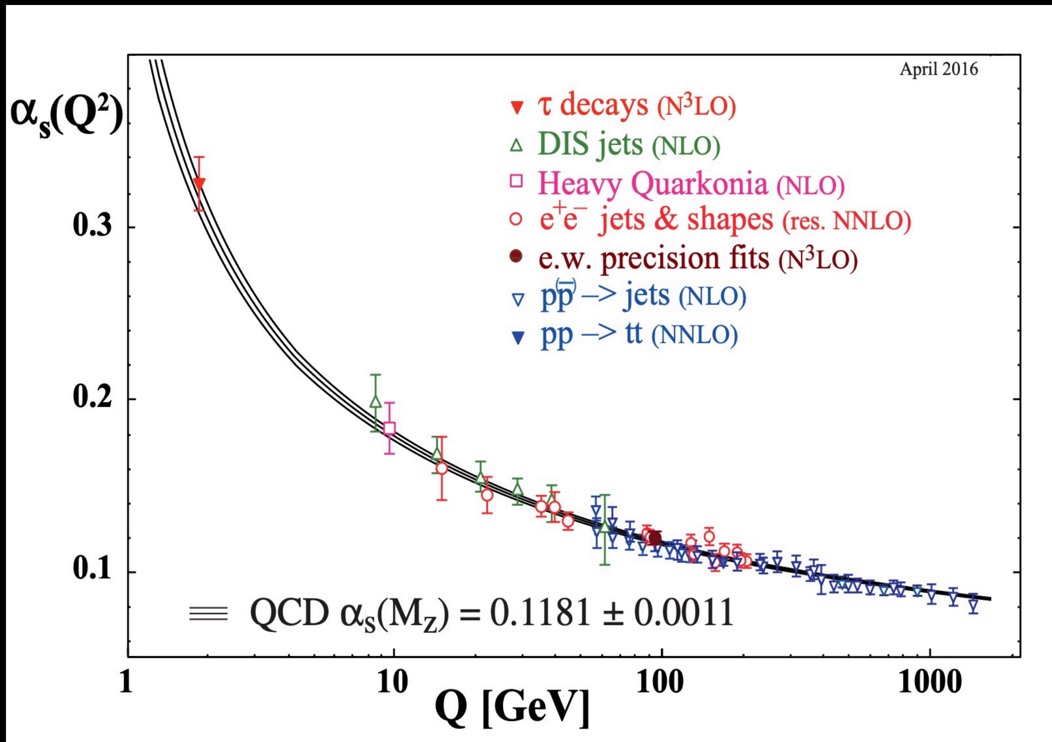
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

Quark Masses

quark masses (from lattice QCD)	**[MeV]** (MS-bar @ 2GeV)
m_u	2.16 (9)(7)
m_d	4.68 (14)(7)
m_s	92.0 (2.1)

FLAG Coll.(2016) http://flag.unibe.ch/

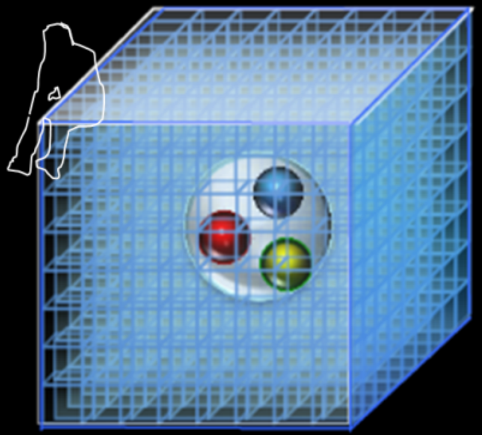
Gauge Coupling



PDG (2018) http://pdg.lbl.gov/

Lattice QCD with isospin breaking

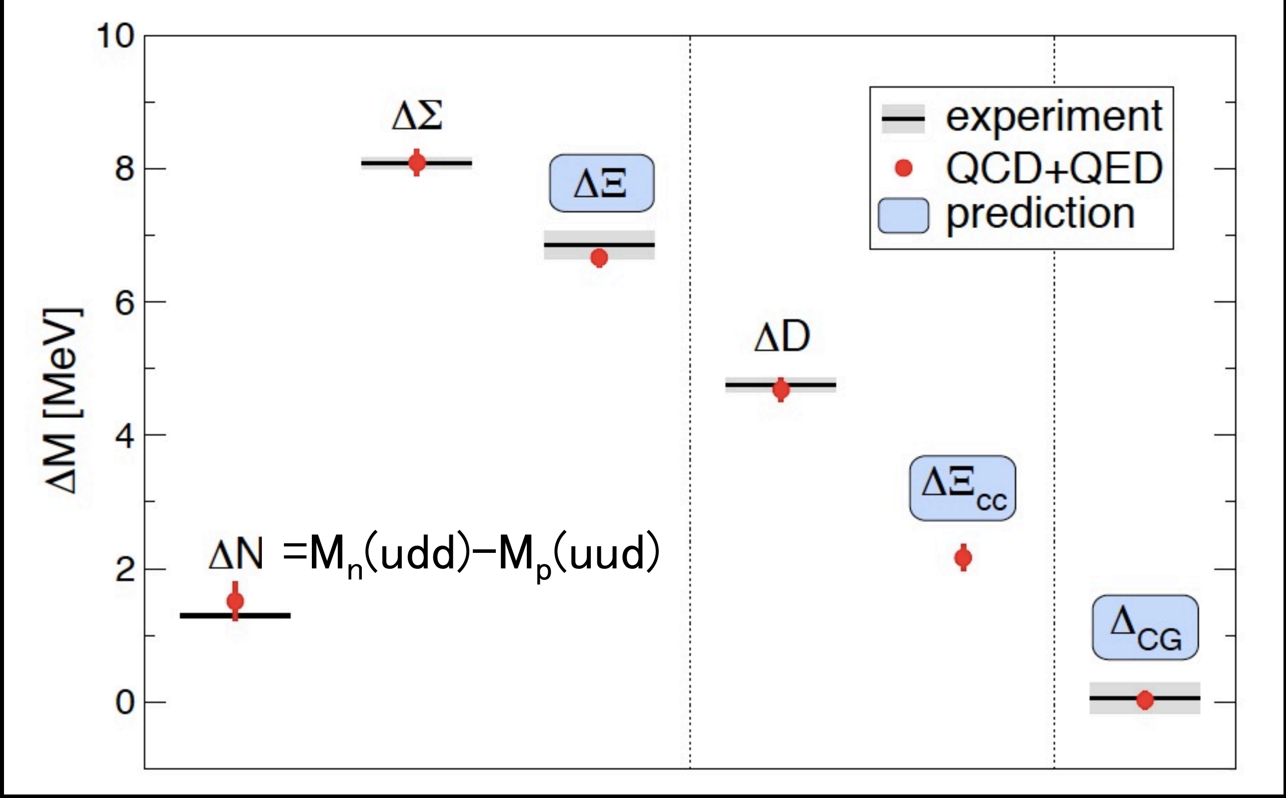
BMW Coll., Science 347 (2015) 1452



a_{min} = 0.054 fm

L_{max} = 8 fm

$m_{\pi,min}$ =190 MeV



$(M_n - M_p)_{lat}$ = 1.51(16)(23) MeV

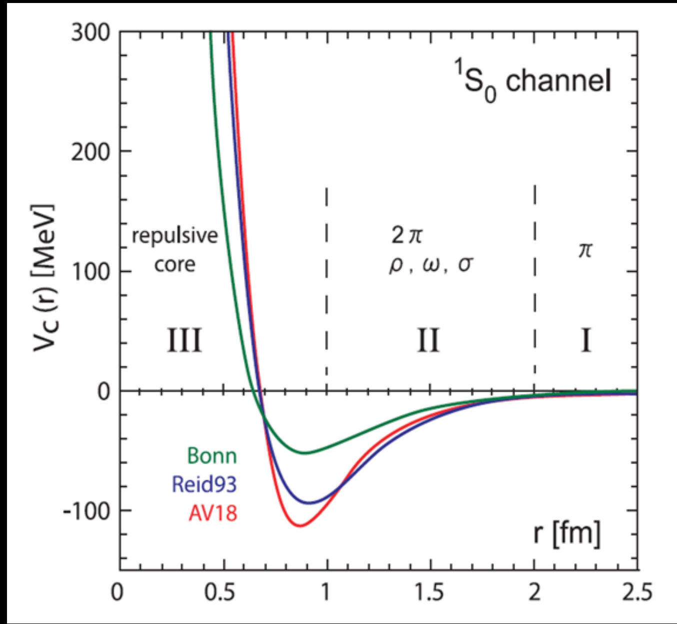
$(M_n - M_p)_{exp}$ = 1.29 MeV

Baryon interactions: Phenomenology vs. LQCD

- NN int.: about 4500 np and pp scattering data

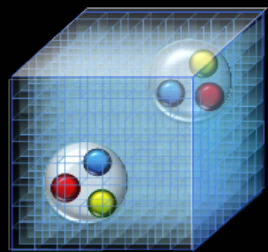
phenomenological NN interactions	# of parameters
CD Bonn (p space)	38
AV18 (r space)	40
EFT in N^3LO (nπ+contact)	24

R. Machleidt, arXiv:0704.0807 [nucl-th]

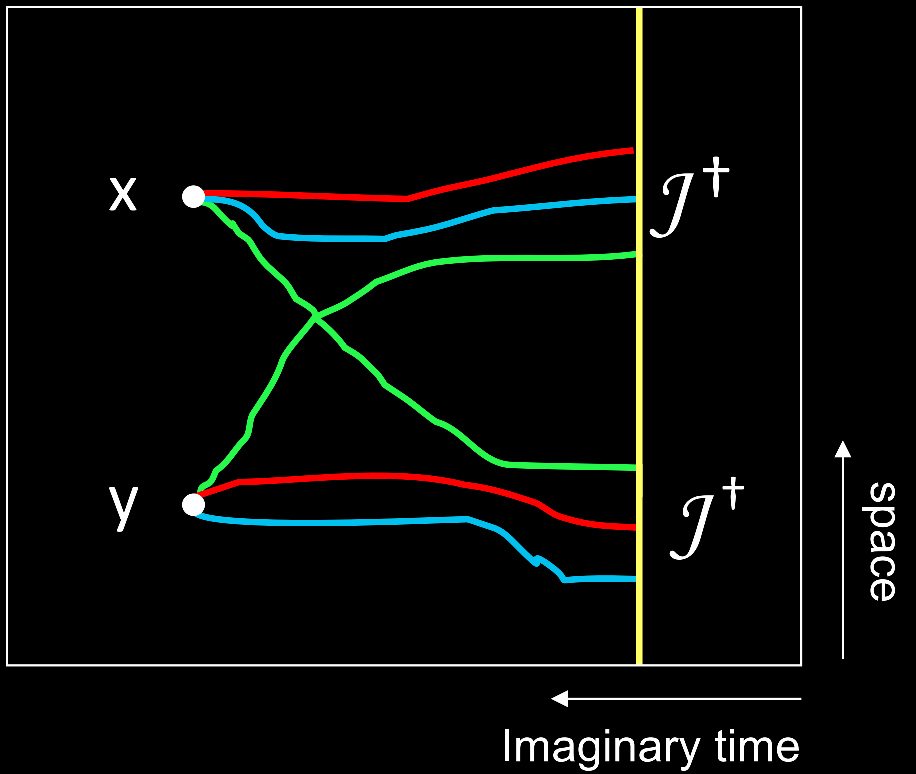


- Hyperon int. : data limited

QCD has only 4 parameters:
$(g, m_{u,d,s})$ at low energies



Scattering problem in LQCD



$$\langle N_1(\mathbf{x},t) N_2(\mathbf{y},t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle$$
$$= \sum_n \langle 0 | N_1(\mathbf{x}) N_2(\mathbf{y}) | n \rangle a_n e^{-E_n t}$$
$$\xrightarrow{t>t^*} \phi(\mathbf{r},t) = \sum_{n<n^*} b_n \phi_n(\mathbf{r}) e^{-E_n t}$$

HAL QCD Method

ϕ(r,t) → 2PI kernel (T=U+GUT) → phase shift, binding energy

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001
Ishii+ [HAL QCD Coll.], PLB 712 (2012) 437

irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} + \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^+}\,\overline{\Sigma^-} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^0}\,\overline{\Sigma^0} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^-}\,\overline{\Sigma^+}$$

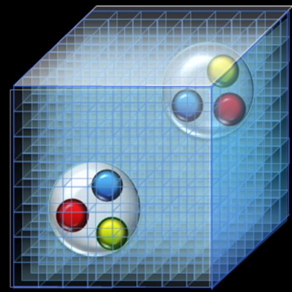
$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} + \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} - \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^+} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^+}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} - \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} + \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

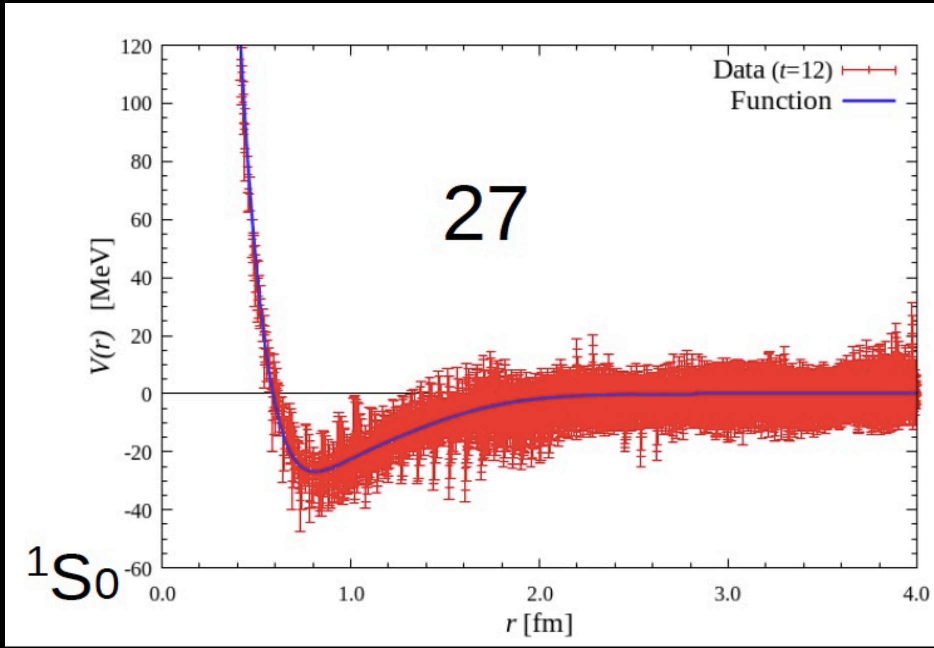
Nuclear Force: $V_C(r)$ and $V_T(r)$

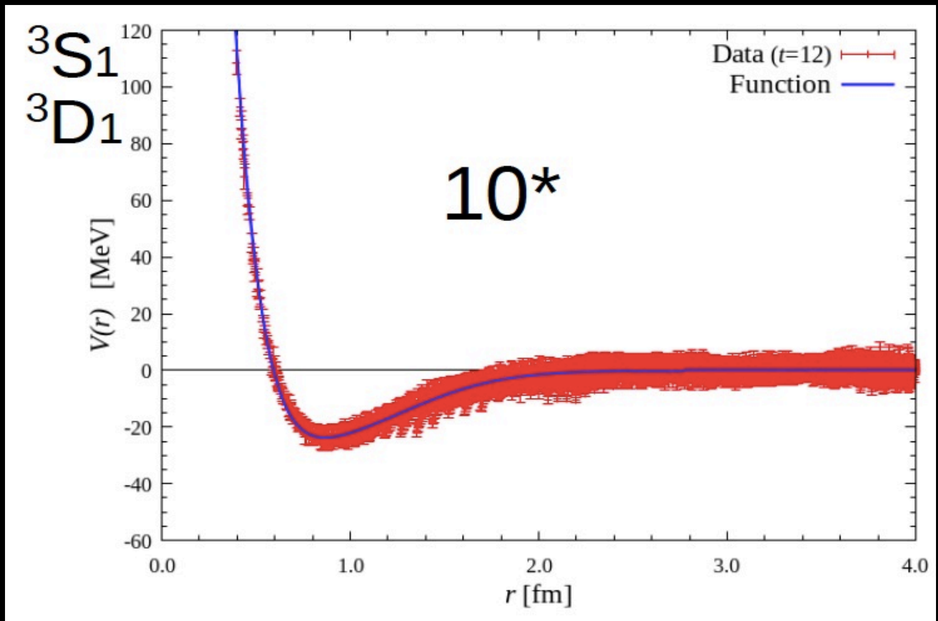


a = 0.085 fm
L = 8.1 fm
m_π = 146 MeV
M_K = 525 MeV

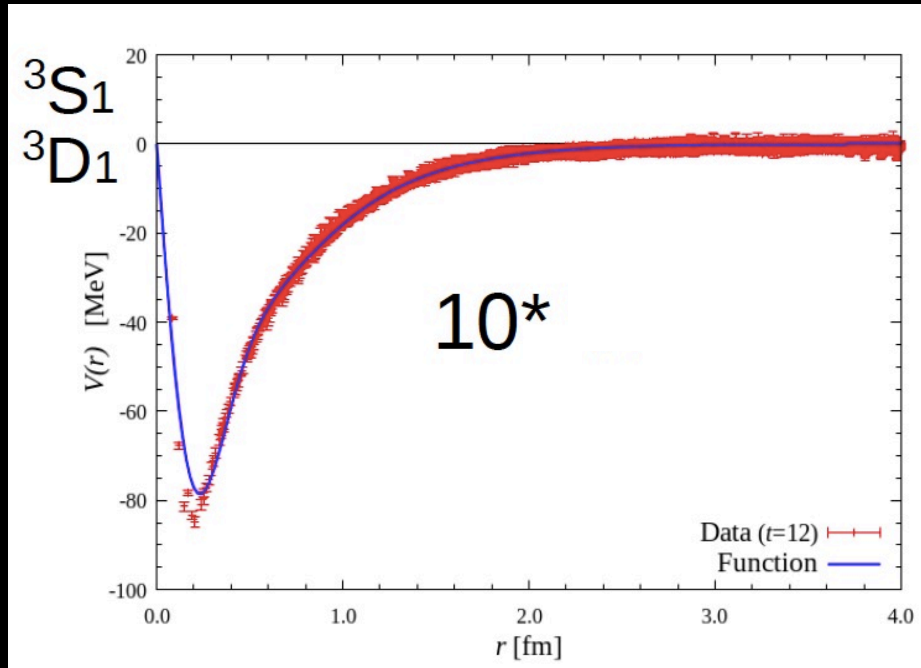
T. Inoue et al.
[HAL QCD Coll.]

Central force





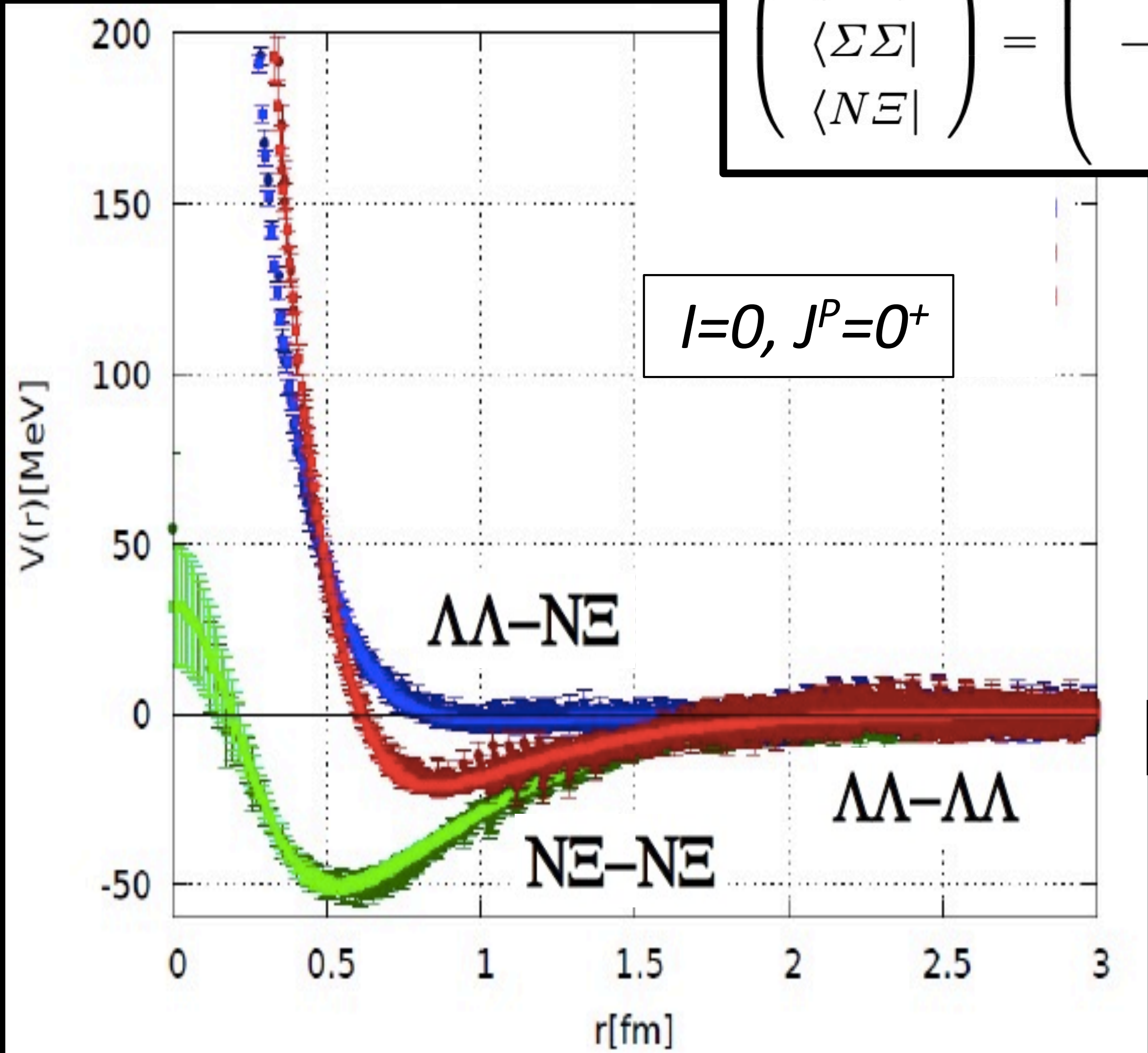
Tensor force

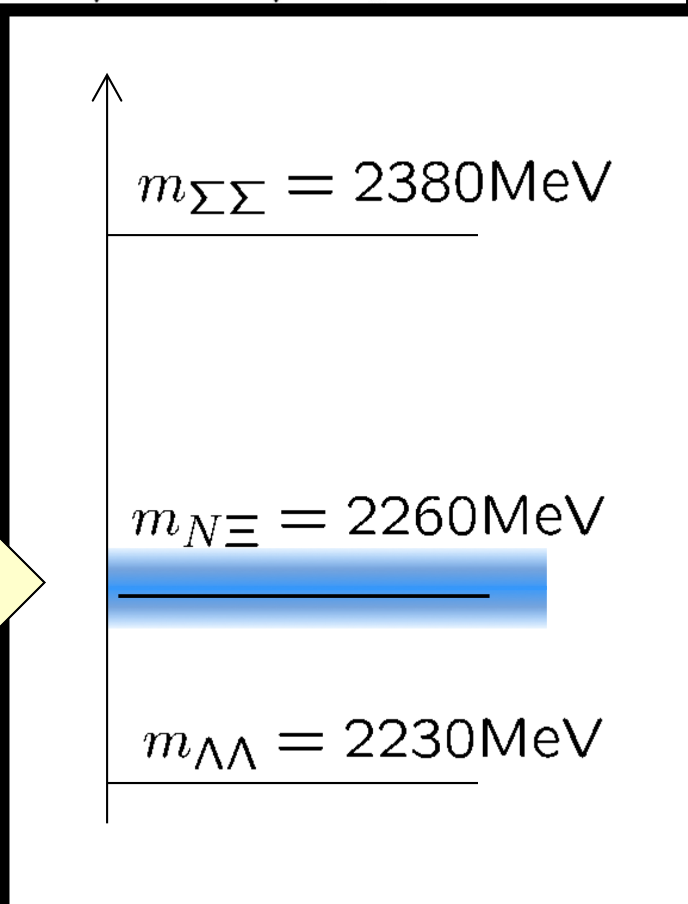


Fate of "H (uuddss)" dibaryon from HAL QCD

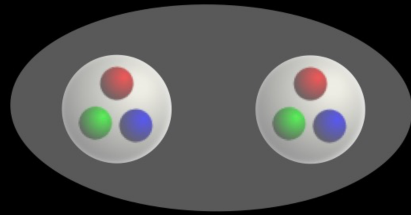
K. Sasaki+
[HAL QCD Coll.]
paper in preparation

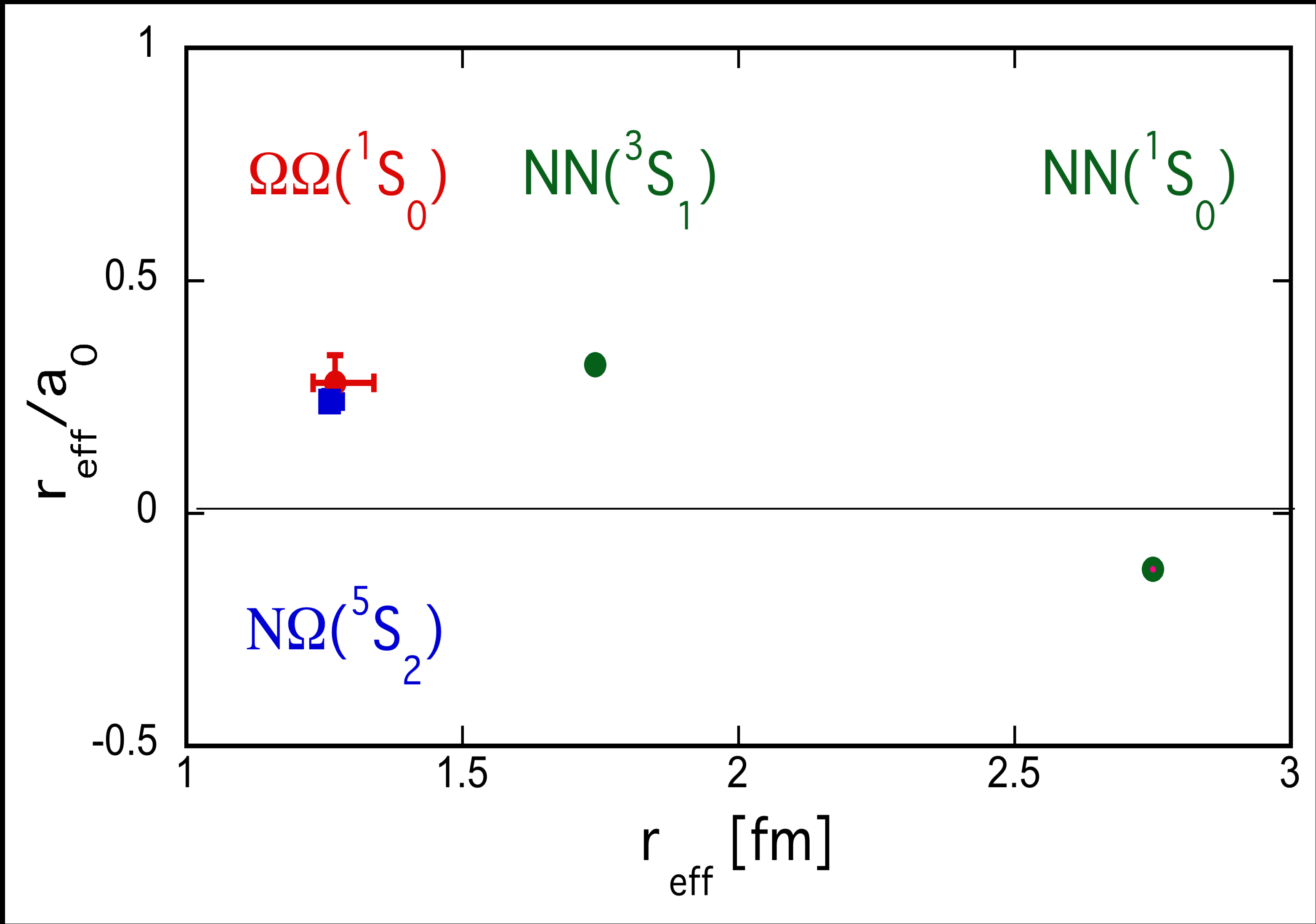
$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle N\Xi | \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{27} | \\ \langle \mathbf{8}_s | \\ \langle \mathbf{1} | \end{pmatrix}$$





BB systems near unitarity?

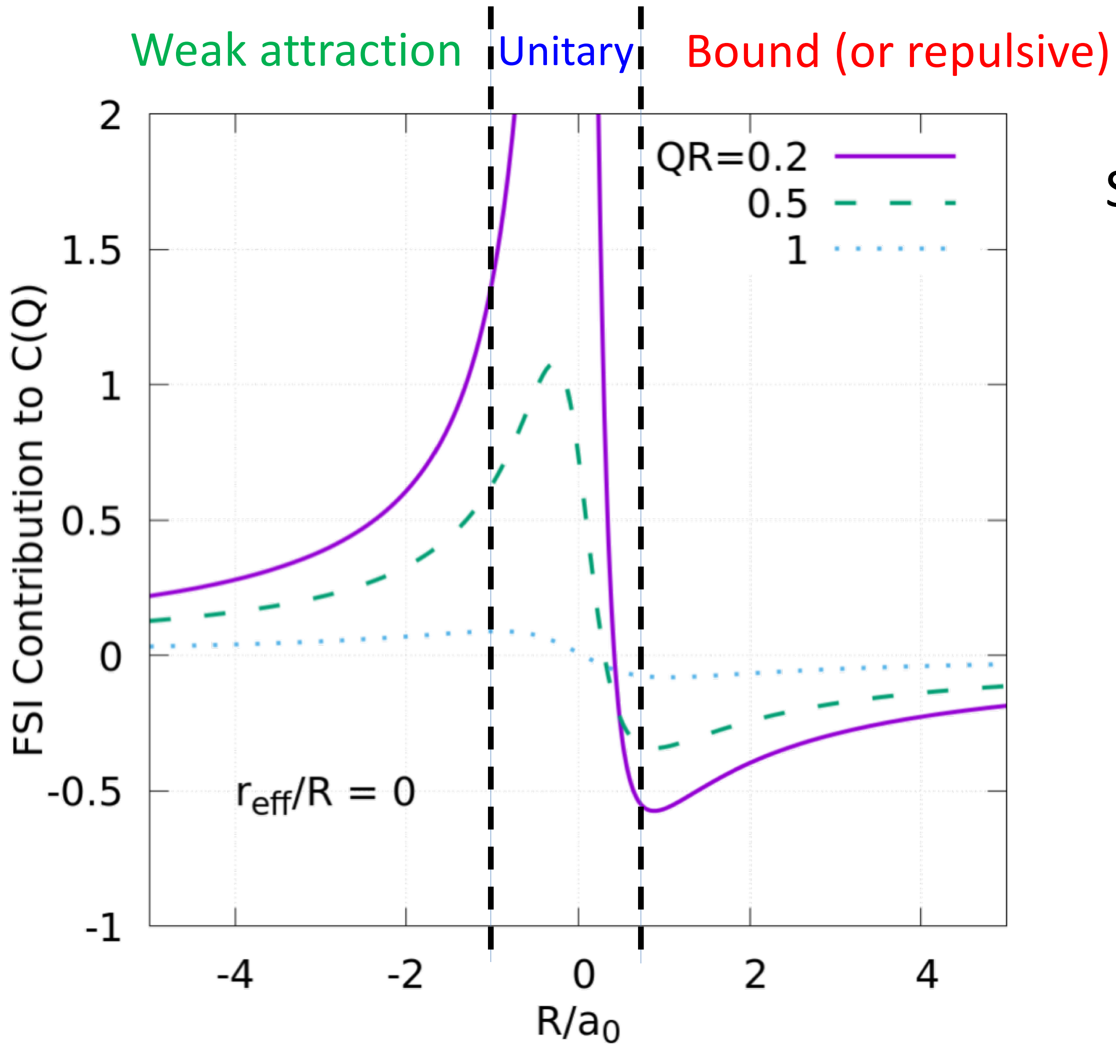




Correlation from FSI

$$C_{AB}(Q) - 1 = \frac{4\pi}{(2\pi R^2)^3} \int dr r^2 S^{\rm rel}(r)[|\chi_Q(r)|^2 - |j_0(Qr)|^2]$$

Lednicky+ '82



Static/Spherical Source

$$S^{\rm rel}(r) = (\pi R^2)^{3/2} \exp\left(-\frac{r^2}{4R^2}\right)$$

Asymptotic S-wave scattering w.f.

$$\chi_Q(r) = \frac{\sin(Qr + \delta)}{Qr}$$

$$Q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} Q^2$$

By Kenji Morita (QST/RIKEN)