High Energy Hadron Production As Self-Organized Criticality:

An Absorbing State Phase transition

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Paolo Castorina and Helmut Satz arXiv: 1901.10407

High Energy Hadron Production As Self-Organized Criticality:

An absorbing state phase transition

- Statistical model in high energy collisions : a new (?) puzzle
- Self organized criticality (SOC) and Absorbing State phase transition
- Hadronization and Color Absorbing State
- Simple models
- Results

Statistical Model

In modern view, the statistical model is a model of hadronization, describing the process of hadron formation at the scale where QCD is no longer perturbative

arXiv:0901.3643
An introduction to the Statistical Hadronization Model
F. Becattini

In the grand-canonical formulation of the statistical model, the mean hadron multiplicities are defined as

$$\langle N_i \rangle = (2J_i + 1)\frac{V}{(2\pi)^3}\int \mathrm{d}^3p \left\{\frac{1}{\gamma_s^{-s_i}\exp[(E_i - \mu \cdot \mathrm{q_i})/T_{\mathrm{ch}}] \pm 1}\right\}$$

Fireball volume

Strangeness suppression

Number of s or anti-s

chemical potentials

Chemical Freeze –out Temperature = hadronic abundances get frozen

J. Letessier, J. Rafelski and A. Tounsi, Phys. Rev. C64 (1994) 406.

First, a primary hadron yield $\langle n_j \rangle^{\mathrm{primary}}$ is calculated using previous equations.

As a second step, all resonances in the gas which are unstable against strong decays are allowed to decay into lighter stable hadrons, using appropriate branching ratios (B) for the decay k → j published by the PDG. The abundances in the final state are thus determined by

Masses
and widths
At zero temperature

$$\langle n_j \rangle = \langle n_j \rangle^{\mathrm{primary}} + \sum \langle n_k \rangle BR(k \to j).$$

$$T, V, \gamma_s, \mu_b$$

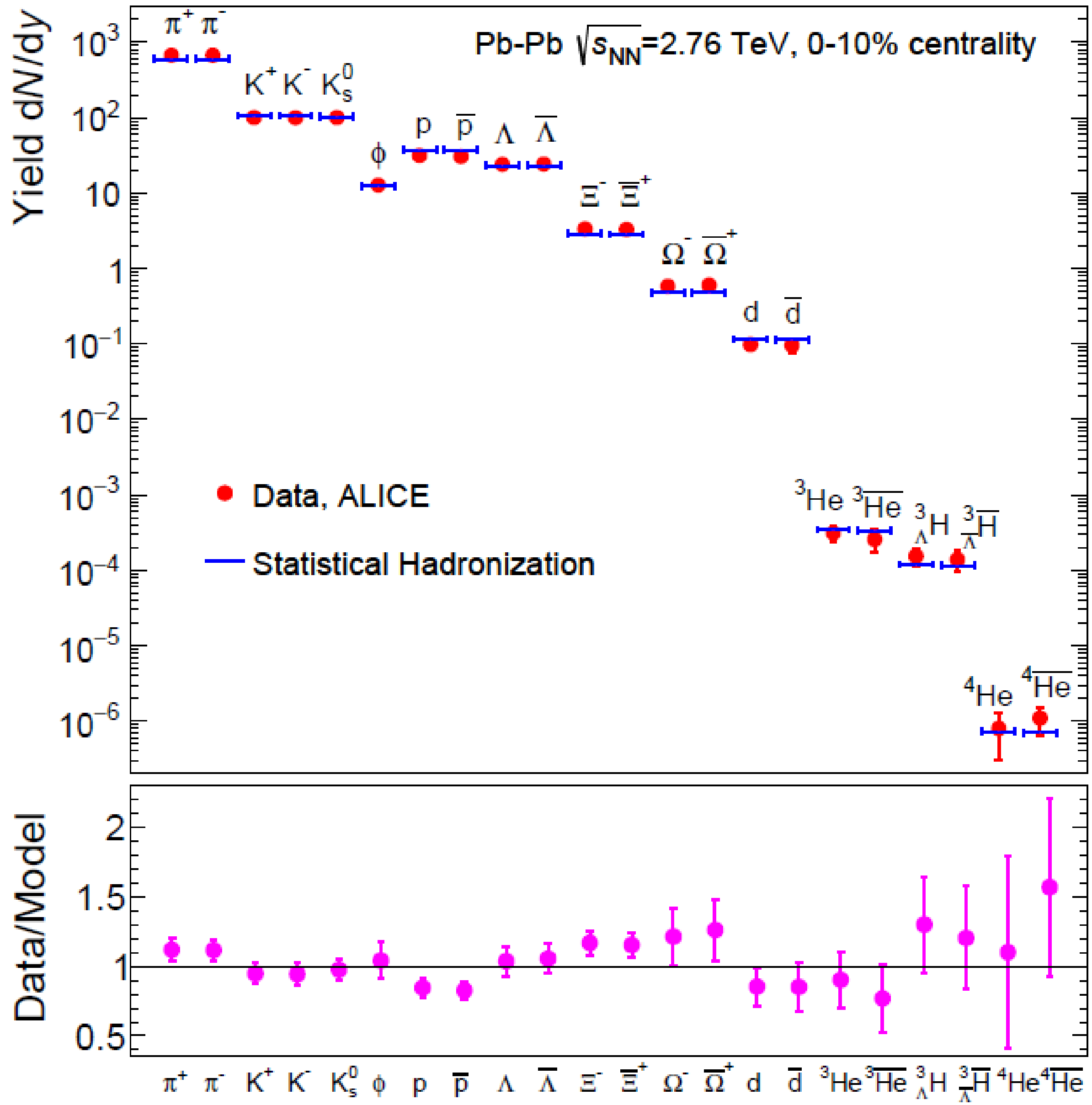
$$\gamma_s = 1$$

T = 155 MeV

9 orders
of magnitude

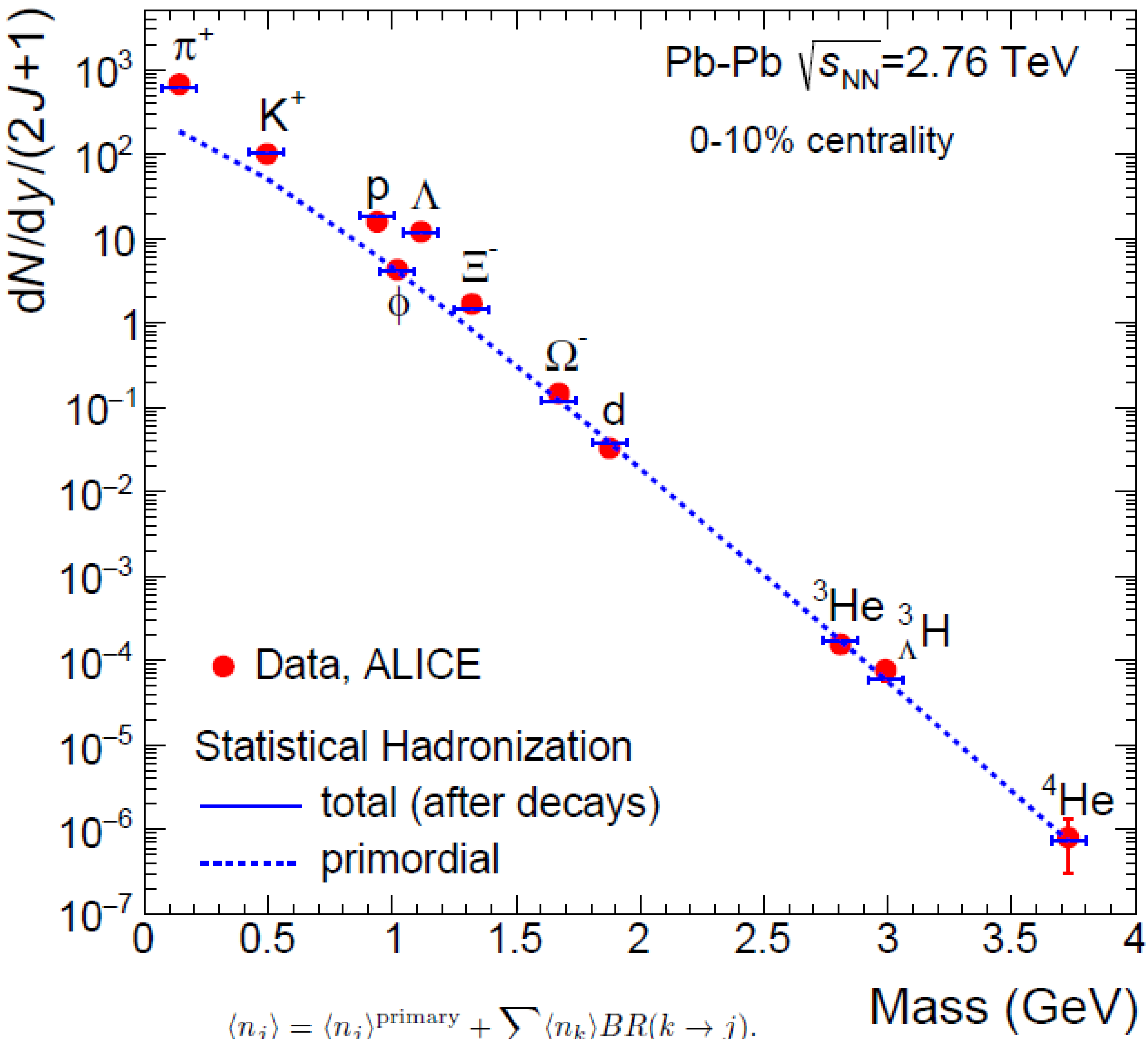
$\mu_B = 0.7 \pm 3.8$ MeV

$R \approx 10.5$ fm



Nature 561 (2018) 321-330

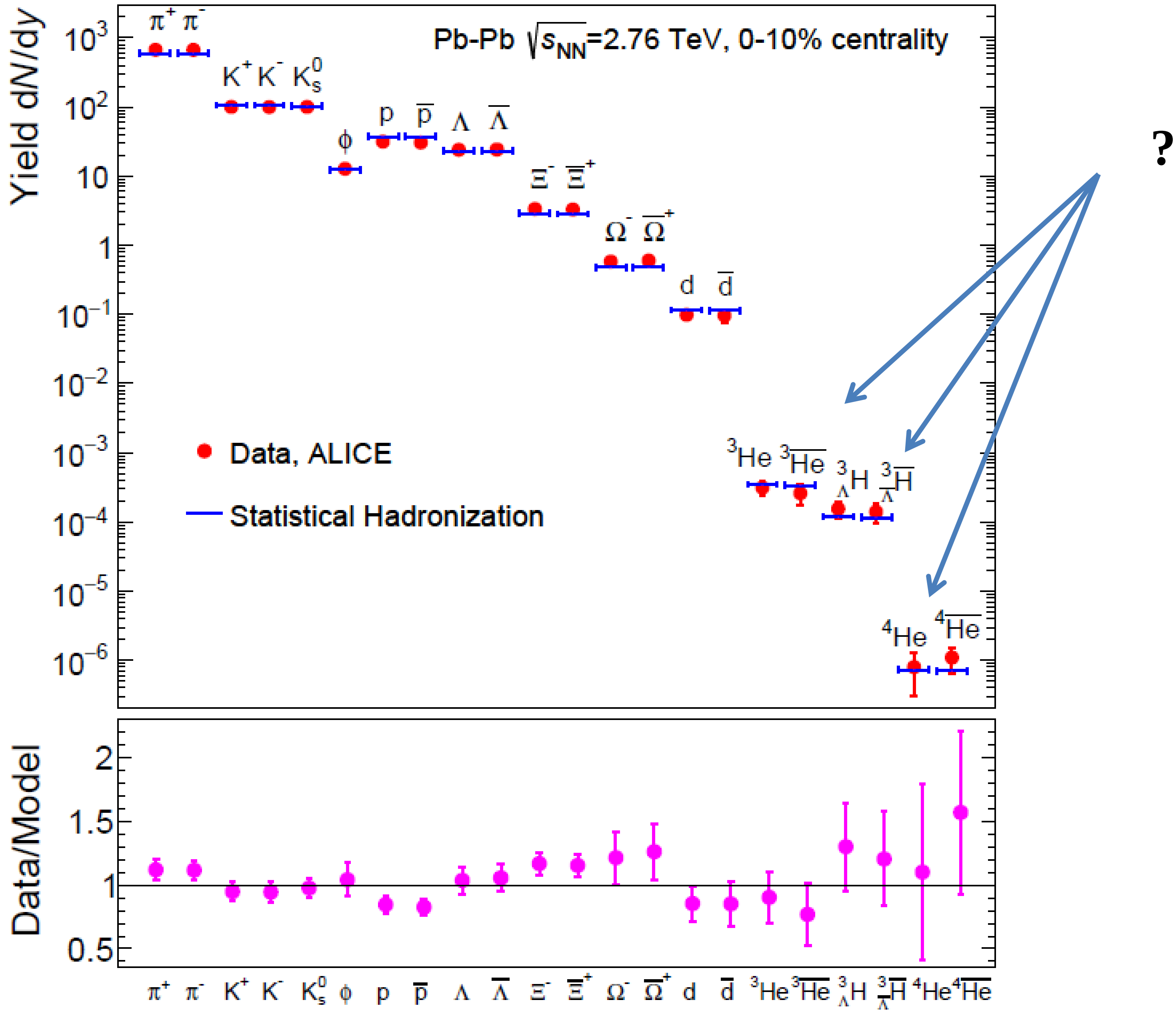
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel,



$$\langle n_j \rangle = \langle n_j \rangle^{\text{primary}} + \sum \langle n_k \rangle BR(k \to j).$$

yields exclusively depend on M and T

For pions, e.g., the resonance decay contribution amounts to 70%.



Pb-Pb $\sqrt{s_{NN}}$=2.76 TeV, 0-10% centrality
Yield dN/dy
Data, ALICE
Statistical Hadronization
Data/Model
?
π^+ π^- K^+ K^- K^0_s ϕ p $\bar{p}$ Λ $\bar{\Lambda}$ Ξ^- $\bar{\Xi}^+$ Ω^- $\bar{\Omega}^+$ d $\bar{d}$ ^3He $^3\overline{He}$ $^3_\Lambda H$ $^3_{\bar{\Lambda}}\bar{H}$ ^4He $^4\overline{He}$

hypertriton

a bound state of (p, n, Λ).

130 ± 30 keV for the energy needed to remove the Λ from it.

Critical temperature

About 160 MeV

This implies that the Λ particle is very weakly bound to a deuteron, resulting in a value for the root-mean-square size for this bound state of close to 10 fm, about the same size as that of the fireball formed in the Pb–Pb collision.

The basic question is:

Why are the yields for the production of light nuclei determined by the rates as specified at the critical hadronization temperature, although in hot hadron gas they would immediately be destroyed?

- Composite objects such are formed at the phase boundary as compact multi-quark

  A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel Nature 561 (2018) 321-330

  D.Oliinychenko – Snowball from hell – this conference

- Coalescence

  Where light nuclei are formed due to final state interactions after a fireball decay.

  Recente review
  P. Braun-Munzinger and B.Donigus

  Loosely-bound objects produced in nuclear collisions at LHC

  Nucl.Phys. A987 (2019) 144-201 and refs. therein

- Analogy with nucleosyntesis

  Vovchenko, Gallmeister,Schaner-Bielich and Greiner, arXiv 1903.10024

A solution to this puzzle can be obtained by abandoning the idea of a thermal hadron medium existing below the confinement point:

The hot quark-gluon system, when it cools down to the hadronization temperature, is effectively quenched by the cold physical vacuum. The relevant basic mechanism for this is self-organized criticality.

P.C. and H.Satz - arXiv: 1901.10407

SELF-ORGANIZED CRITICALITY

The core hypothesis is that systems consisting of many interacting components will, under certain conditions, spontaneously organize into a state with properties akin to the ones observed in a equilibrium thermodynamic system.

As this complex behavior arises spontaneously without the need for external tuning (the **temperature** for example) this phenomena was named Self-Organized Criticality (SOC).

These are systems whose natural dynamics drives them towards, and the maintains them at the *edge* of stability.

WELL KNOWN EXAMPLES IN NATURE – POWER LAWS

SOC refers to the spontaneous organization of a system driven from the outside into a globally stationary state, which is characterized by self-similar distributions of event sizes and fractal geometrical properties

SOC

Sandpile avalanche size

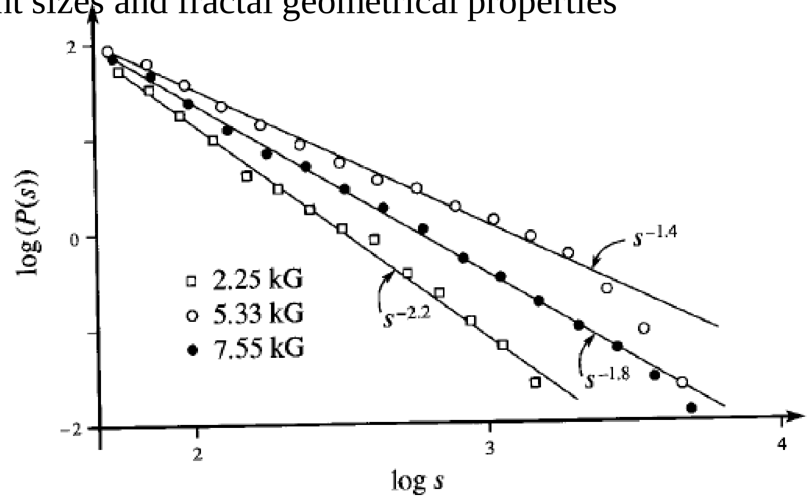


Figure 3.7. Avalanche size probability density.
(Sketch of data presented in Field et al. 1995.)

SOC

Earthquakes

$$N \sim m^{-b}$$

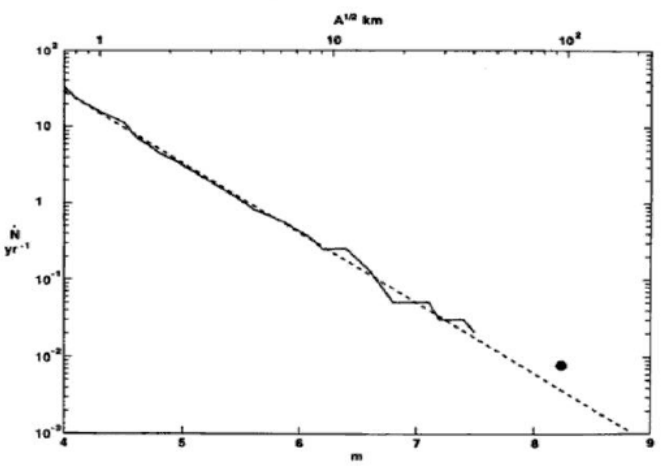
Scale-Free Distributions In Nature:

An Overview of Self-Organized Criticality

Ronald Dickman

Cambridge lecture notes pag 22

Figure 4.2. Number of earthquakes per year $\dot{N}$ occurring in southern California with magnitudes greater than m as a function of m. The solid line is the data from the southern California earthquake network for the period 1932–1994. The straight dashed line is the correlation with (4.1) taking $b = 0.923$ ($D = 1.846$) and $\dot{a} = 1.4 \times 10^5$. The solid circle is the observed rate of occurrence of great earthquakes in southern California (Sieh et al., 1989).



Self-Organized criticality-Theory,Models and Characterization
G.Pruessener, Cambridge U.P.

Power laws and Self-Organized Criticality
in Theory and Nature, D.Markovic and C.Gros, arXiv: 1310.5527

Another example

The perhaps simplest form of self-organized criticality is provided by partitioning integers. Consider the *ordered* partitioning of an integer n into integers. The number $q(n)$ of such partitionings is for $n = 3$ equal to four: 3, 2+1, 1+2, 1+1+1, i.e., $p(3) = 4$.

In general

$$q(n) = 2^{n-1} = \frac{1}{2}\exp\{n \ln 2\}$$

"large integers consist of smaller integers, which in turn consist of still smaller integers, and so on..." This can be formulated as an equation

$$\rho(n) = \delta(n-1) + \sum_{k=2}^{n} \frac{1}{k!} \prod_{i=1}^{k} \rho(n_i)\, \delta(\Sigma_i n_i - n).$$

$$\rho(n) = \text{Constant} * \text{q(n)}$$

In the ordered case considered above, one thus finds that the number of partitions increases exponentially with the size of the integer. Given an initial integer n, we would now like to know the number $N(k,n)$ specifying how often a given integer k occurs in the set of all partitionings of n. To illustrate, in the above case of $n = 3$, we have $N(3,3) = 1$, $N(3,2) = 2$ and $N(3,1) = 5$. To apply the formalism of self-organized criticality, we have to attribute a strength $s(k)$ to each integer. It seems natural use the number of partitions for this, i.e., set

$$s(k) = q(k) = \frac{1}{2} \exp\{k \ln 2\}. \tag{7}$$

The desired number $N(k,n)$ in a scale-free scenario is then given by

$$N(k,n) = \alpha(n)[q(k)]^{-p},$$

leading to

$$\log\ N(k,n) = -[p \log e\ \ \ln 2]\, k + p \log 2 + \log \alpha(n)$$

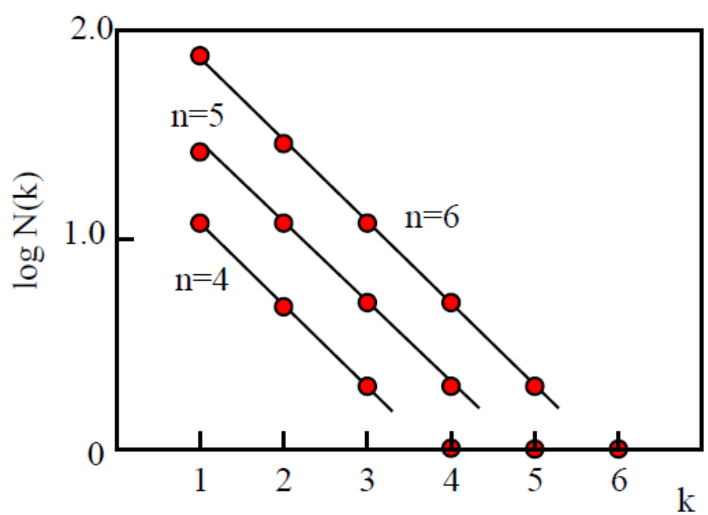


Figure 2: Distributions of the number $N(k,n)$ of integers k for $n = 4, 5$ and 6

Absorbing state

However, for non-equilibrium steady states it is becoming increasingly evident that SOC is related to conventional critical behavior, namely that one of an

absorbing-state phase transition

Haye Hinrichsen
- Non Equilibrium Critical Phenomena and Phase Transitions into Absorbing States, arXiv 0001070.
- LECTURE NOTES : Non-equilibrium phase transitions

Absorbing state : configurations that can be reached by the dynamics but cannot be left

Example: models describing the growth of bacterial colonies or the spreading of an infectious disease among a population

Once an absorbing state, e.g., a state in which all the bacteria are dead, is reached, the system cannot escape from it .

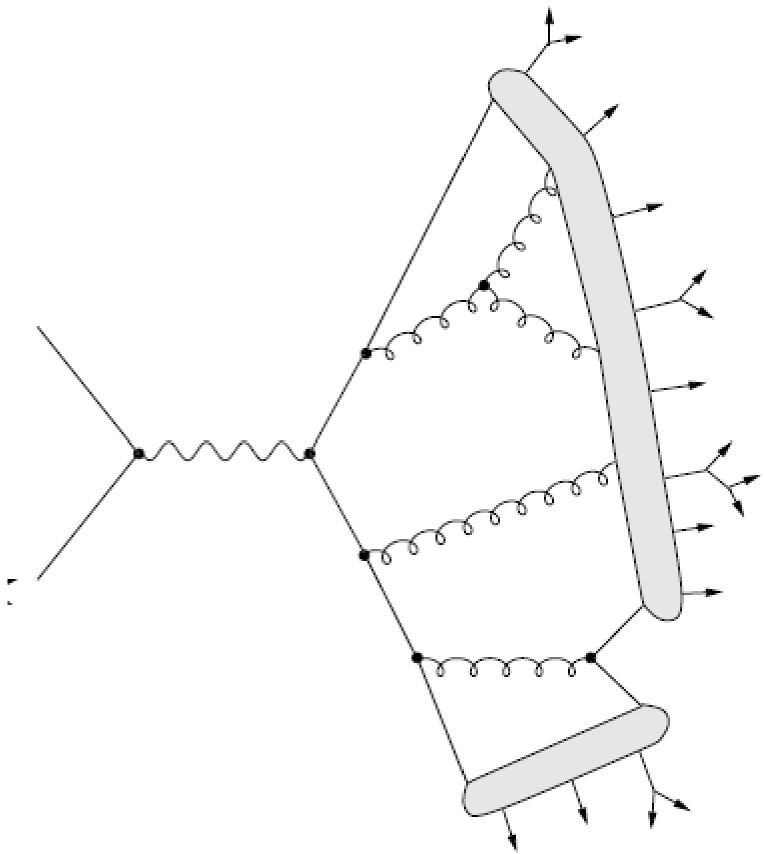
and

Absorbing state phase transitions are among the simplest non-equilibrium phenomena displaying critical behavior and universality.

Gutiérrez, et al – arxive 1611.03288 Experimental signatures of an absorbing-state phase transition in an open driv
many-body quantum system

Barato et al – arxive 0901.451Simplest nonequilibrium phase transition into an absorbing stat

Hadronization and Absorbing State Phase Transition

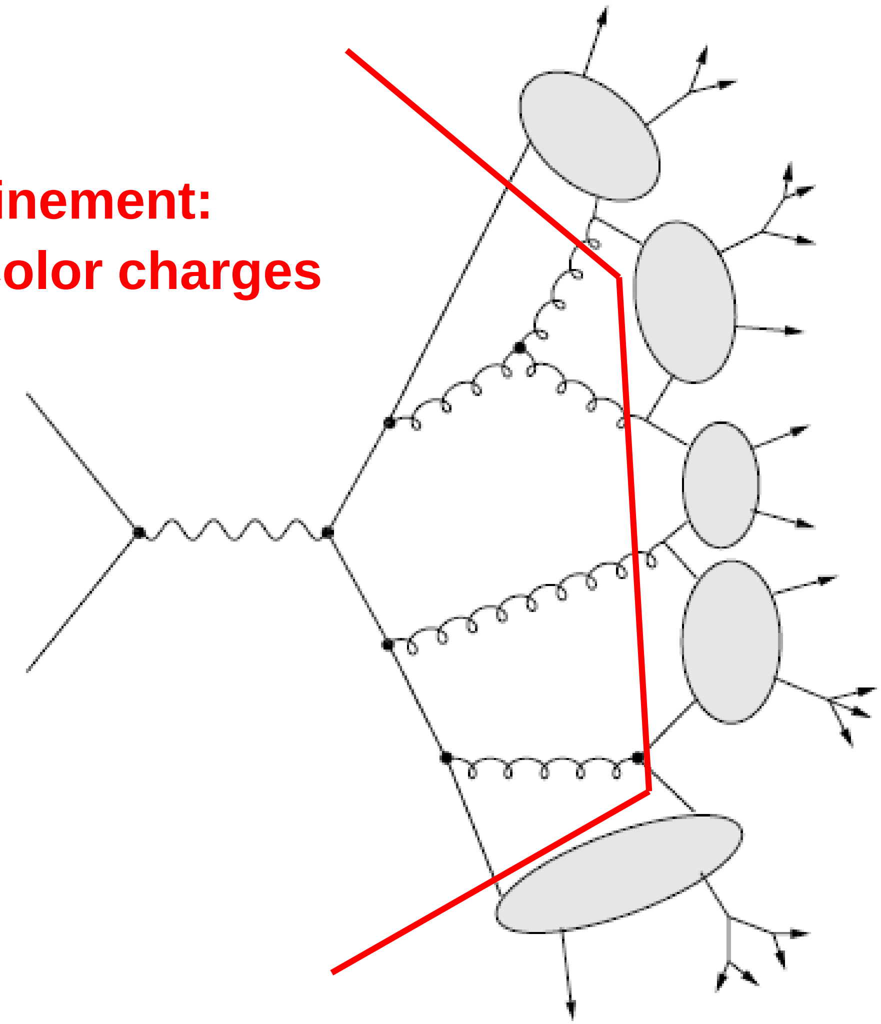


We argue that the evolution of jets produced in hard processes can be computed perturbatively in QCD up to appearance of a "preconfinement" stage consisting of *finite mass* colourless clusters of quarks and gluons.

Veneziano,Amati PRECONFINEMENT AS A PROPERTY OF PERTURBATIVE QCD PHYSICS LETTERS Volume 83B 87

System evolution

Deconfinement:
Local Color charges



Color singlet
HADRONS

Absorbing
State

One – Way evolution : NO COLOR CHARGE AFTER HADRONIZATION

Hadronization
is an
Absorbing State Phase Transition for Color charge

Non-equilibrium phase transition

In equilibrium, the associated transition rates between two states satisfy detailed balance

In case of absorbing phase transitions the rate out of an absorbing state is zero. Therefore, an absorbing state can not obey detailed balance

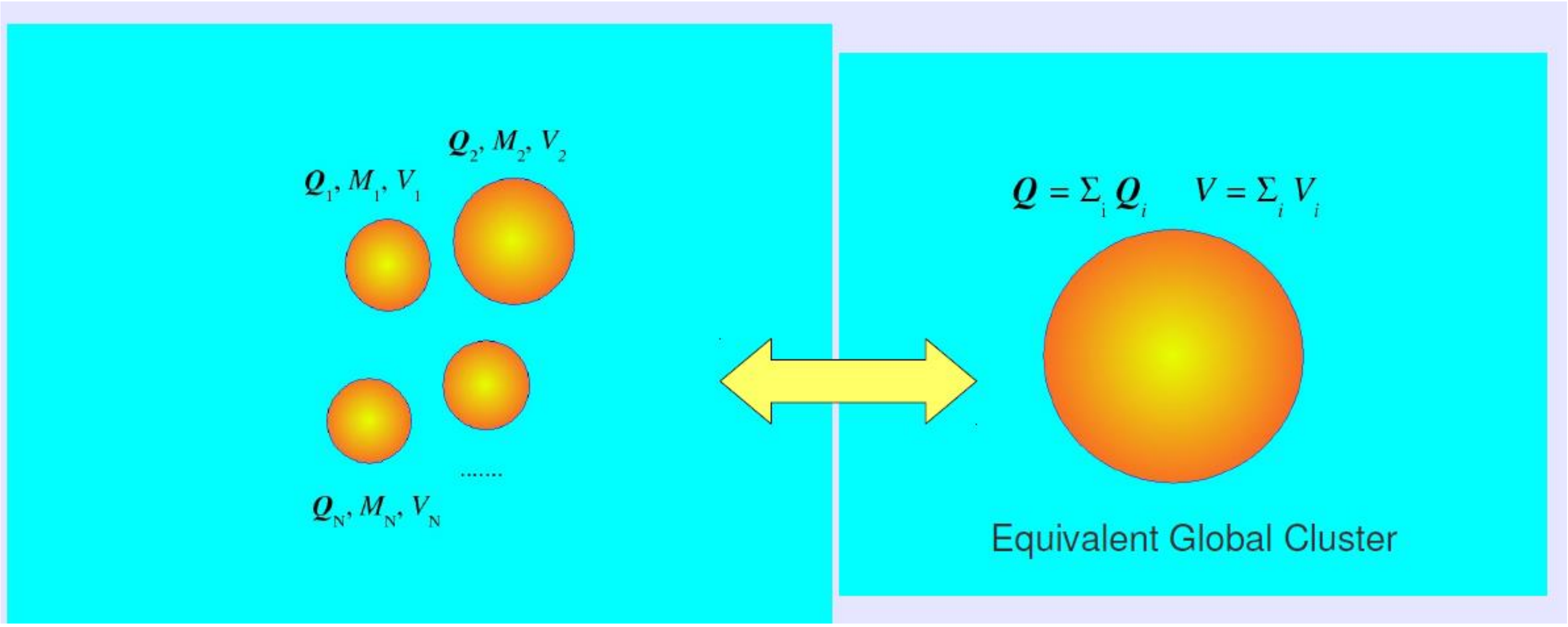
Is there a hidden Absorbing State in the «usual» statitistical hadronization model ?

YES

- massive colorless clusters distributed over rapidities, each decays statistically
- mass and charge distributions of clusters again statistically

$\Rightarrow$ equivalent global cluster

- $V = \Sigma V_i$, $Q = \Sigma Q_i$; large enough for thermodynamics

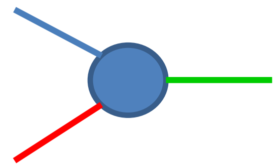


A simple model

Competition between hadronization (color neutralization) and production and/or annihilation of color charge (partons)

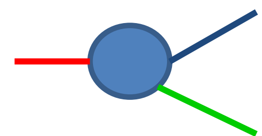
$\rho(t)$ Color charge density / parton number density

Annihilation $P + P \to P$



rate λ

Production $P \to P + P$



rate σ

Color neutralization

$P \to Hadron$

Hadron

rate k.

Mean Field equation

$$\frac{d\rho}{dt} = (\sigma - k)\rho - \lambda\rho^2 = \rho[\sigma - k - \lambda\rho]$$

Production

Color neutralization

Annihilation

$\sigma < k$ then steady state

steady state

$\rho_s = 0$

ABSORBING STATE

If $\sigma > k$ then the steady state is for $\rho_s = (\sigma - k)/\lambda$

CONTINUOUS NON EQUILIBRIUM PHASE TRANSITION

at $\sigma_c = k$

As in thermal equilibrium the critical point is governed by a characteristic power law

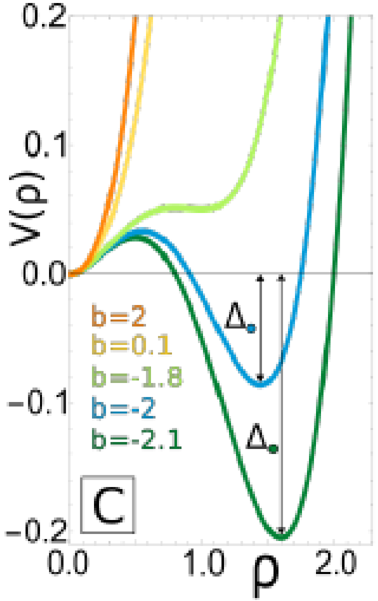
$$\rho_s \simeq (\sigma - \sigma_c)^\beta$$

with $\beta = 1$.

Same model for first order phase transition

$\dot{\rho}(t) = a\rho - b\rho^2 - c\rho^3,$ with $b < 0$ and $c > 0$ $\rho_s = 0$

ABSORBING STATE



Landau-Ginzburg Theory of Self-Organized Criticality
L.Gil and D.Sornette PRL 76 (1996) 3991

Self-Organized Bistability Associated with First-Order Phase Transitions
Serena di Santo, et al. arXiv 1605.05161

Polyakov Loop dynamics - SU(3) gauge theory – no dynamical quark

$$L_3(\boldsymbol{r}) = \mathcal{P}\exp\left[ig\int_0^\beta dx_4\, A_4(\boldsymbol{r}, x_4)\right]$$

Polyakov loop modeling for hot QCD

Kenji Fukushima,[1] and Vladimir Skokov[2]

arXiv 1705.00718

$$\langle \mathrm{tr}_c\, L_3(\boldsymbol{r})\rangle = e^{-\beta F_q(\boldsymbol{r})}$$

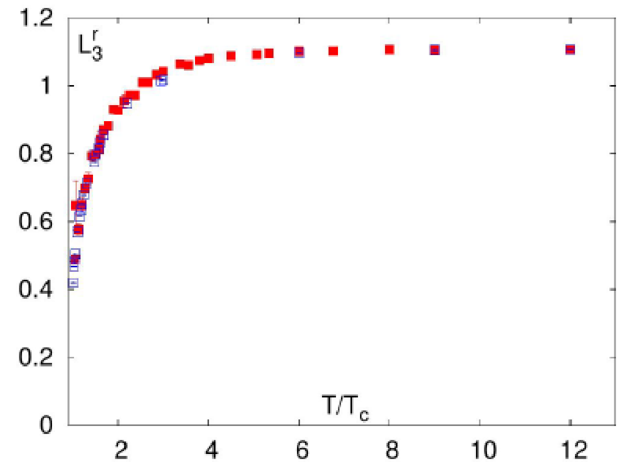
trace is taken in color space only.

$$\langle \mathrm{tr}_c\, L_3(\boldsymbol{r})\rangle \;=\; 0$$

$$F_Q = \infty$$

Confinement in pure gauge theory

The Polyakov loop in pure YM theory



S. Gupta et. al., PRD 77, 034503 (2008)

The renormalized Polyakov loop is an **order parameter** of the transition in pure YM theory.

Polyakov Loop dynamics - SU(3)

A. Dumitru, R. D. Pisarski arXiv:0209001

O. Scavenius, A. Dumitru, J.T. Lenaghan arXiv:0201079

$$\ell(\vec{x}) \equiv \frac{1}{3}\,\mathrm{tr}\,L(\vec{x}) = \frac{1}{3}\,\mathrm{tr}\,\mathcal{P}\exp\left(ig\int_0^{1/T} A_0(\vec{x},\tau)\,d\tau\right)$$

$$L = \frac{N}{g^2}|\partial_\mu l|^2 T^2 - V(l)$$

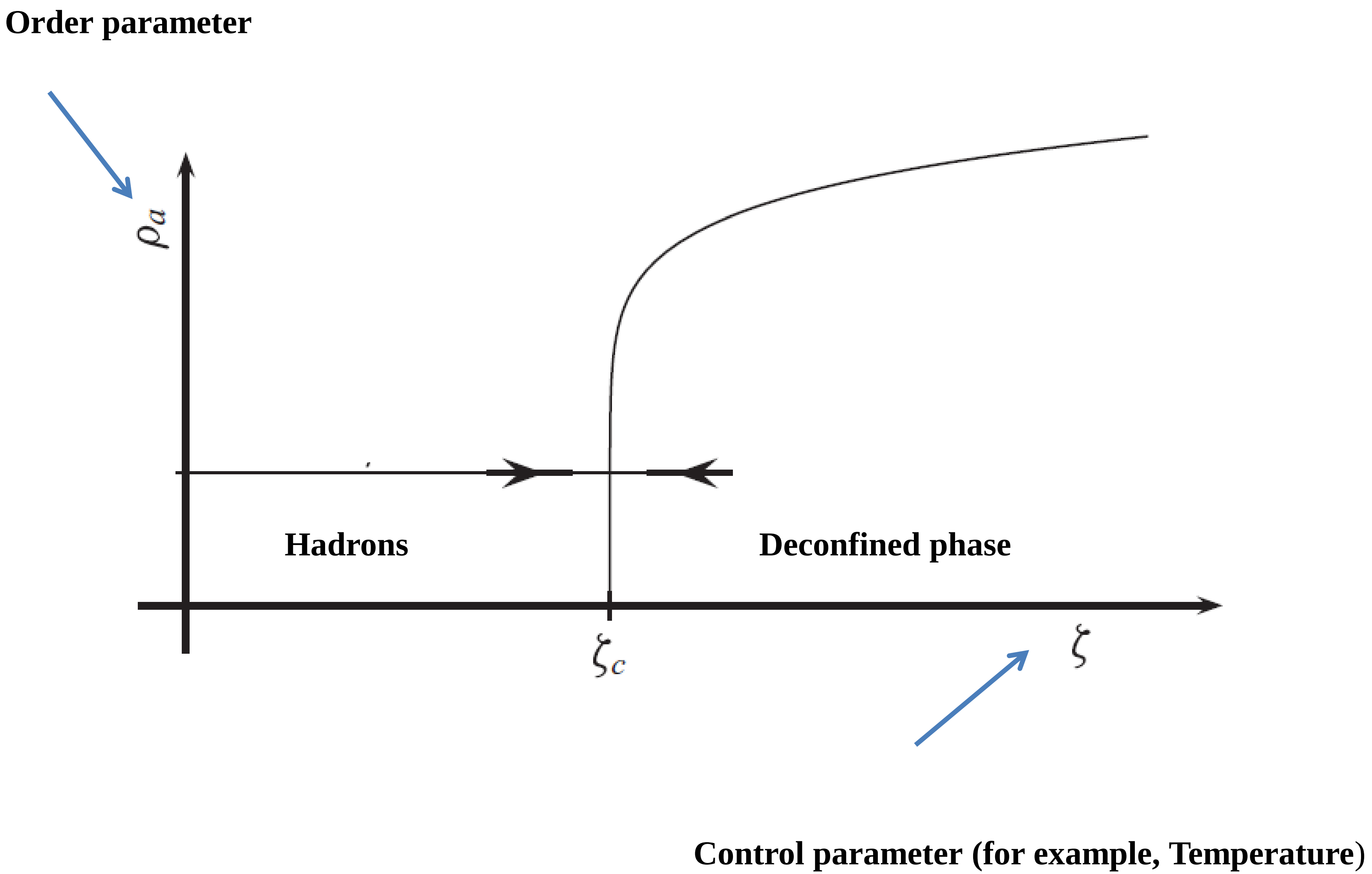
$$V(l) = (\frac{-b_2}{2}|l|^2 - \frac{b_3}{6}(l^3 + (l^*)^3) + \frac{1}{4}(|l|^2)^2)b_4 T^4$$

- neglect the second order time derivative

$$x \to gT\sqrt{\frac{b_4}{2N}}x, \qquad \dot{\phi} = \nabla^2\phi + \phi(b_2 + b_3\phi - \phi^2) \qquad l(x) \text{ as } \phi(x)$$

$$\tau \to \frac{b_4 g^2 T^2}{2\eta N}\tau$$

$\bar{l} = 0$ Absorbing state


Order parameter
ρ_a
Hadrons
Deconfined phase
ζ_c
ζ
Control parameter (for example, Temperature)

1) The Hadronization mechanism
is a non equilibrium phase transition
to an Absorbing State

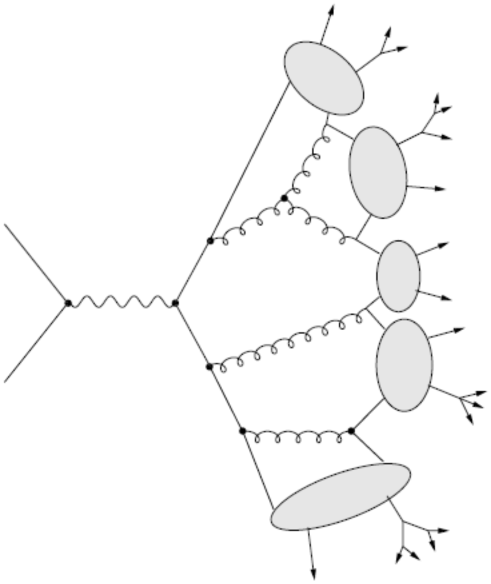
2) The dynamical evolution is driven by
color d.o.f. up to the hadronization
time/temperature

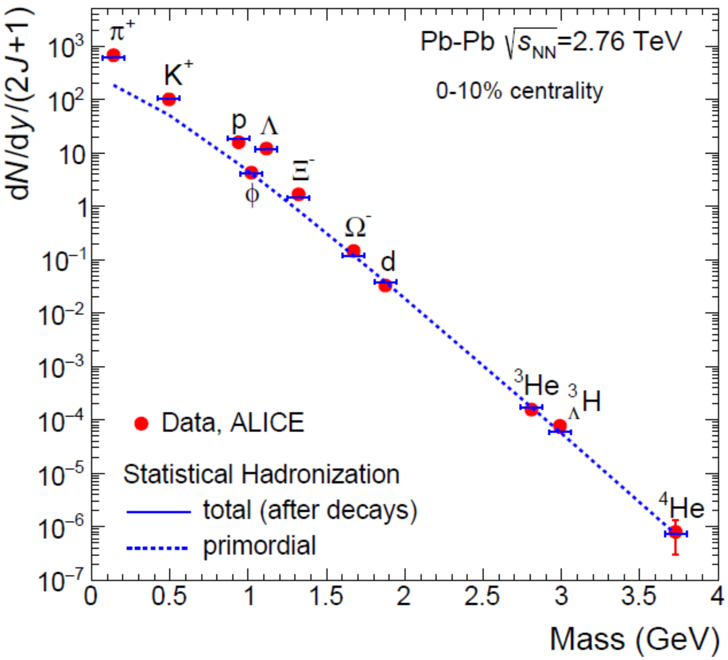
3) Due to the absorbing state → the system
is essentially «frozen» at the values of the
parameters at the transition

Universal scaling behavior of non-equilibrium phase transitions, Sven Lubeck,
International Journal of Modern Physics B 18, 3977 (2004)

Critical Dynamics , U.C. Tauber, Cambrideg University Press

Particle Yields in high energy collisions – non equilibrium





Statistical model

In SOC avalanche dynamics , the size distribution follows a power law

$$P(S) \propto S^{-\gamma}$$

What is the statistical «size» associated to a hadron of mass M

«size» density of state of mass M

The yield of a hadron of mass M is therefore determined by the density of state of mass M at the absorbing state , i.e. at the hadronization/deconfinement temperature T_c

More precisely…

the mass degeneration → particle i carrying the quantum numbers denoted by $\vec{C}_i = (B_i, S_i, Q_i)$ with four-momentum between p_i and $p_i + dp_i$ might also take on different masses, whose distribution is given by the function

$$\rho(m_i)$$

Hagedorn just imagined that a heavy particle was somehow composed out of lighter ones, and these again in turn of still lighter ones, and so on, until one reached the pion as the lightest hadron. By combining heavy ones, you would get still heavier ones, again: and so on. The crucial input was that the composition law should be the same at each stage. Today we call that self-similarity

→ Bootstrap equation

The Legacy of Rolf Hagedorn: Statistical Bootstrap and Ultimate Temperature – K.Redlich and H.Satz arxiv: 1501.07523

Cluster can be considered as clusters of colorless clusters done by clusters of colorless clusters…..

$$\rho(m) = \delta(m - m_0) + \sum_N \frac{1}{N!} \left[\frac{4\pi}{3(2\pi m_0)^3} \right]^{N-1} \int \prod_{i=1}^{N} [dm_i \; \rho(m_i) \; d^3 p_i] \; \delta^4(\Sigma_i p_i - p),$$

with m_0 denoting the lowest possible mass (the single grain of sand). The equation can be solved analytically

$$\rho(m) \sim m^{-a} \epsilon^{m/T_H} \;\rightarrow\; \ln \rho \sim \frac{m}{T_H} - a \ln m, \qquad a = 3,$$

and T_H as solution of

$$\left(\frac{2}{3\pi}\right) \left(\frac{T_H}{m_0}\right) K_2(m_0/T_H) = 2 \ln 2 - 1,$$

where $K_2(x)$ is a Hankel function of pure imaginary argument.

For $m_0 = m_\pi \simeq$ 140 MeV $\Rightarrow$ $T_H \simeq 150$ MeV,

$$\rho(m) \sim m^{-a} \epsilon^{m/T_H} \rightarrow \ln \rho \sim \frac{m}{T_H} - a \ln m,$$

is an asymptotic solution of the bootstrap equation; it evidently diverges for $m \rightarrow 0$ and must be modified for small masses. Using a similar result for $\rho(m)$ obtained in the dual resonance model Hagedorn proposed

$$\rho(m) = \text{const.}(1 + (m/\mu_0))^{-a} \exp(m/T_H)$$

where $\mu_0 \simeq 1 - 2$ GeV is a normalization constant.

At this point we should emphasize that previous equations are due to the self-organized nature of the components, with an integer consisting of integers, a fireball of fireballs. They are in no way a result of thermal behavior. We have expressed the slope coefficient of m in terms of the Hagedorn "temperature" only in reference to subsequent applications. In itself, it is totally of combinatorical origin.

The emergent picture is a sudden quench of the partonic medium at the absorbing state

- The initial hot system of deconfined quarks and gluons rapidly expands and cools;

- the difference between transverse and longitudinal motion implies a global non-equilibrium behavior

- The longitudinal expansion quickly drives the system to the hadronisation point, and it is now suddenly thrown into the cold physical vacuum.

quenching process, the system freezes out into the degrees of freedom presented by the system at the transition point and subsequently remains as such, apart from possible hadron or resonance decays.

<< there is no hot interacting hadron gas. >>

RESULTS

In such a scenario, high energy nuclear collisions lead to a system which at the critical point breaks up into components of different masses m, subject to self-similar composition and hence of a strength $\rho(m)$ as given by

$$\rho(m) = \text{const.}(1 + (m/\mu_0))^{-a} \exp(m/T_H)$$

Self-Organized Criticality suggests

$$N(m) = \alpha[\rho(m)]^{-p}$$

for the yield of a hadron of mass m

$$\downarrow$$

$$\log N(m) = -m\left(\frac{p \log e}{T_H}\right)\left[1 - \left(\frac{aT_H}{m}\right)\ln(1 + \frac{m}{\mu_0})\right] + \text{const.}$$

$$\log N(m) = -m\left(\frac{p\log e}{T_H}\right)\left[1-\left(\frac{aT_H}{m}\right)\ln(1+\frac{m}{\mu_0})\right] + \text{const.}$$

obtained assuming only scale-free behavior

and a mass weight determined by the number of partitions.

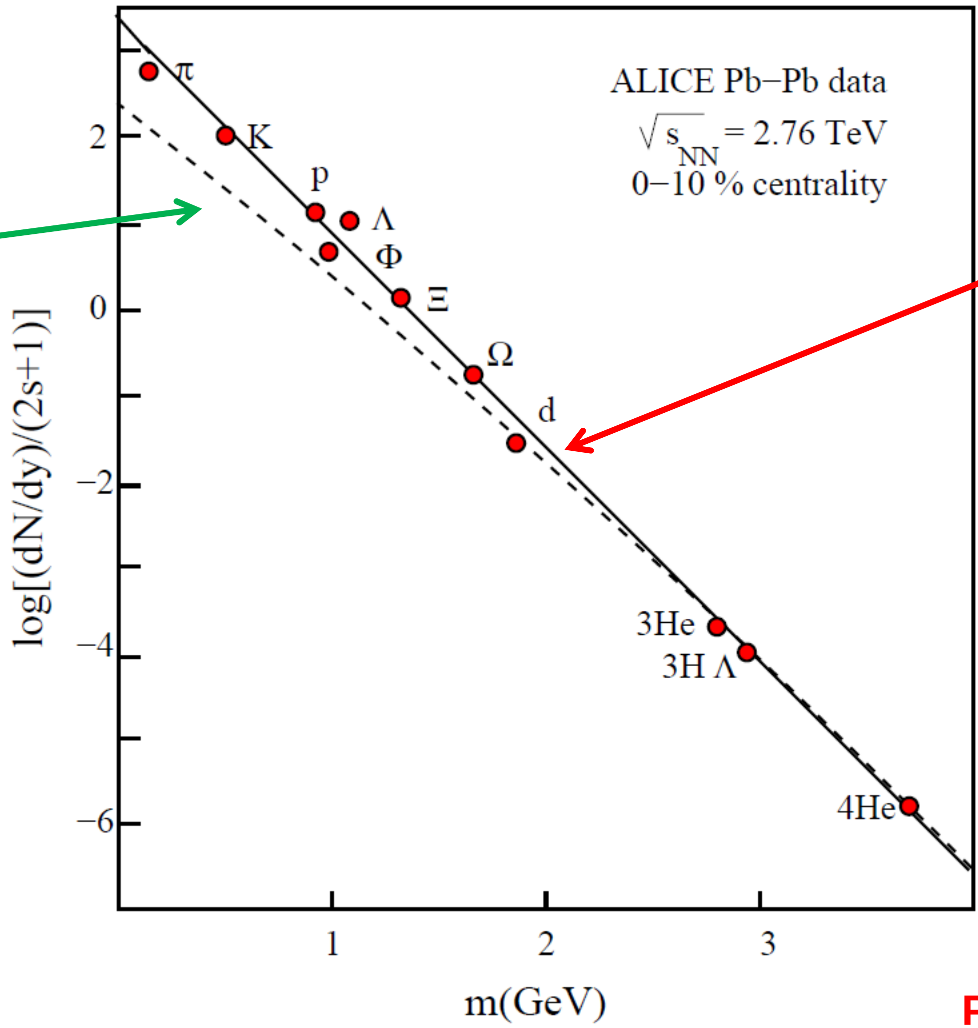
NO EQUILIBRIUM THERMAL SYSTEM OF ANY KIND IS ASSUMED

COMPARISON WITH ALICE DATA

$$\log[(dN/dy)/(2s+1)] \simeq -m\left(\frac{0.43\,p}{T_H}\right) + +p\,a\log[1+(m/\mu)] + A,$$

$a = 3$

$a = 0$



PRIMARY PRODUCTION

$\mu = 2$ GeV $\quad p = 0.9,\ A = 3.4.$

Comment 1 - Nuclear Physics – SOC in multifragmentation?

comment, we recall the behavior of proton-nucleus or nucleus-nucleus collisions at low energy, leading to what is denoted as nuclear multifragmentation * The result of such collisions will be nuclear fragments of size A, and the distribution of these fragments is found to obey the so-called Fisher law *

$$P(A) = \text{const.} A^{-\tau}, \tag{21}$$

corresponding to the critical point of droplet condensation, with $\tau = 2.33$. It thus constitutes another instance of self-organized criticality, with all fragments governed by the same law. The difference between this form and the one for high energy collisions is that at low energy, the the break-up is simply into mass fragments, whereas at high energy it produces all possible excitation states.

* J. Randrup and S. E. Koonin, Nucl. Phys. A356 (1981) 223

For a survey, see e.g.

B. Borderie and M. F. Rivet, Prog. Part. Nucl. Phys. 61 (2008) 551

M. E. Fisher, Physics 3 (1967) 255

Conclusions and outlook

1) The Hadronization mechanism
is a non equilibrium phase transition
to an Absorbing State

2) The dynamical evolution is driven by
color d.o.f. up to the hadronization
time/temperature

Light nuclei /hypernuclei
are produced at T_c and
survive

3) Due to the absorbing state → the system
is «frozen» at the values of the parameters at
the transition

There is no hadron resonance gas after
deconfinement transition or a very short-lived one *

4) Is there some peculiar behaviour related to the presence of the absorbing state?

5) SOC in nuclear fragmentation?

* P. Braun-Munzinger, J. Stachel and Ch. Wetterich, Phys. Lett. B 596 (1994) 61.

Auxiliary material

1. Thermal Hadron Production

what is "thermal"?

- equal *a priori* probabilities for all states in accord with given overall average energy $\Rightarrow$ temperature T;

- partition function of ideal resonance gas

$$\ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T)$$

  Boltzmann factor $\phi(m_i, T) = 4\pi m_i^2 T K_2(m_i/T) \sim \epsilon^{-m_i/T}$;

- relative abundances $\dfrac{N_i}{N_j} = \dfrac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)} \sim \epsilon^{-(m_i - m_j)/T}$

  predicted in terms of temperature T

basic observation in all high energy multihadron production

thermal production pattern

Fermi, Landau, Pomeranchuk, Hagedorn

- species abundances ~ ideal resonance gas at T_H
- universal $T_H \simeq 160 \pm 10 Mev$ for all (large) $\sqrt{s}$

caveats

- strangeness suppression in elementary collisions
- strangeness suppression weakened/removed in nuclear collisions

F. Becattini, Z. Phys. C69 (1996) 485.

F. Becattini, *Universality of thermal hadron production in pp, $p\bar{p}$ and e^+e^- collisions*, in *Universality features in multihadron production and the leading effect*, Erice 1966, World Scientific, Singapore (1998) 74-104; arXiv:hep-ph/9701275.

F. Becattini and G. Passaleva, Eur. Phys. J. C23 (2002) 551.

F. Becattini and U. Heinz, Z. Phys. C76 (1997) 268.

J. Cleymans et al., Phys. Lett. B 242 (1990) 111.

J. Cleymans and H. Satz, Z. Phys. C57 (1993) 135.

K. Redlich et al., Nucl. Phys. A 566 (1994) 391.

P. Braun-Munzinger et al., Phys. Lett. B344 (1995) 43.

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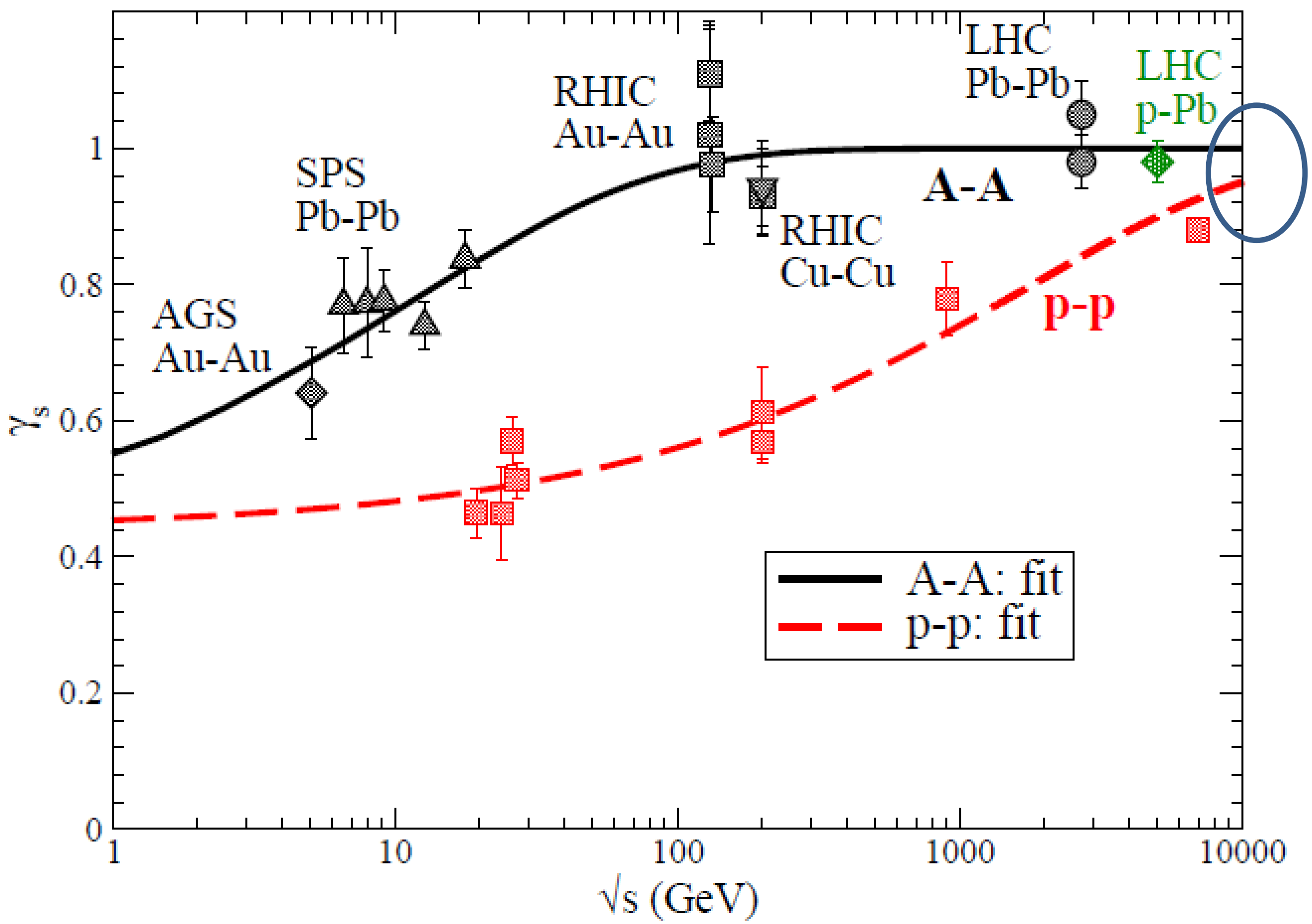


All high energy collisions

$$\gamma_s = 1$$

Universal Strangeness Production in Hadronic and Nuclear Collisions
P. Castorina, S. Plumari, H. Satz Int.J.Mod.Phys. E25 (2016) no.08, 1650058

RECALL - EQUILIBRIUM LATTICE QCD

QCD numerical lattice studies have shown that the deconfinement/confinement transition is in fact a rapid cross-over rather than a genuine thermodynamic phase transition

But for m_q=0

In the chiral limit of two flavor QCD one does recover critical behavior

$$M(T) \sim (T_c - T)^{\beta},\ T \leq T_c$$

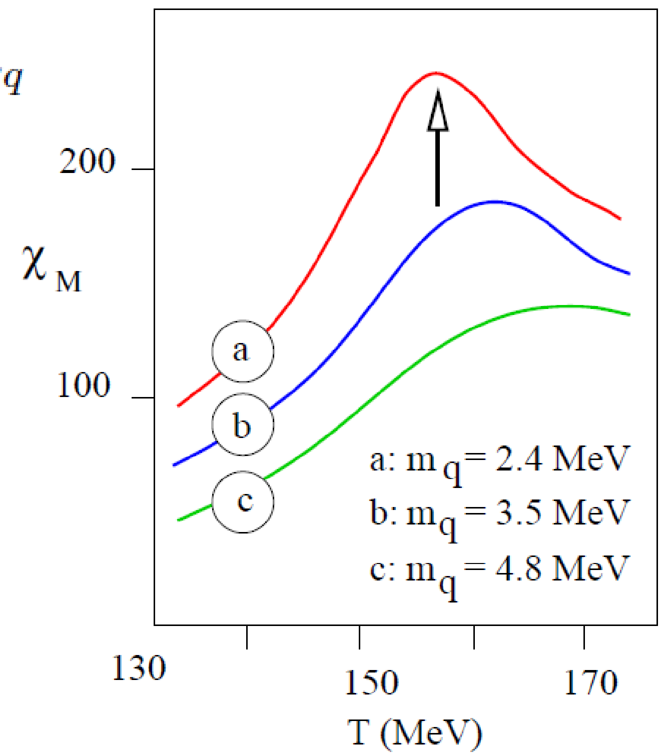
chiral condensate $M = <\psi\bar{\psi}>$ as order paramenter

$$M(T = T_c, m_q = 0) = 0$$

$$\xi(T, m_q = 0) \sim |T - T_c|^{-\nu},$$

The small but finite u and d quark masses m_q in physical QCD act like a weak external field in spin system preventing genuine singular behavior * The behavior of the system for the actual small quark mass values is nevertheless thought to be strongly influenced by the near-by singularity. As a result, specific thermodynamic variables are sharply peaked, defining a pseudo-critical point $T_{pc}(m_q)$ close to $T_c(m_q = 0)$.

$$\chi_M \sim \partial M(T)/\partial m_q$$



pseudocritical temperature of QCD

* S. Digal, E. Laermann and H. Satz, Europ. Phys. J. C18 (2001) 583

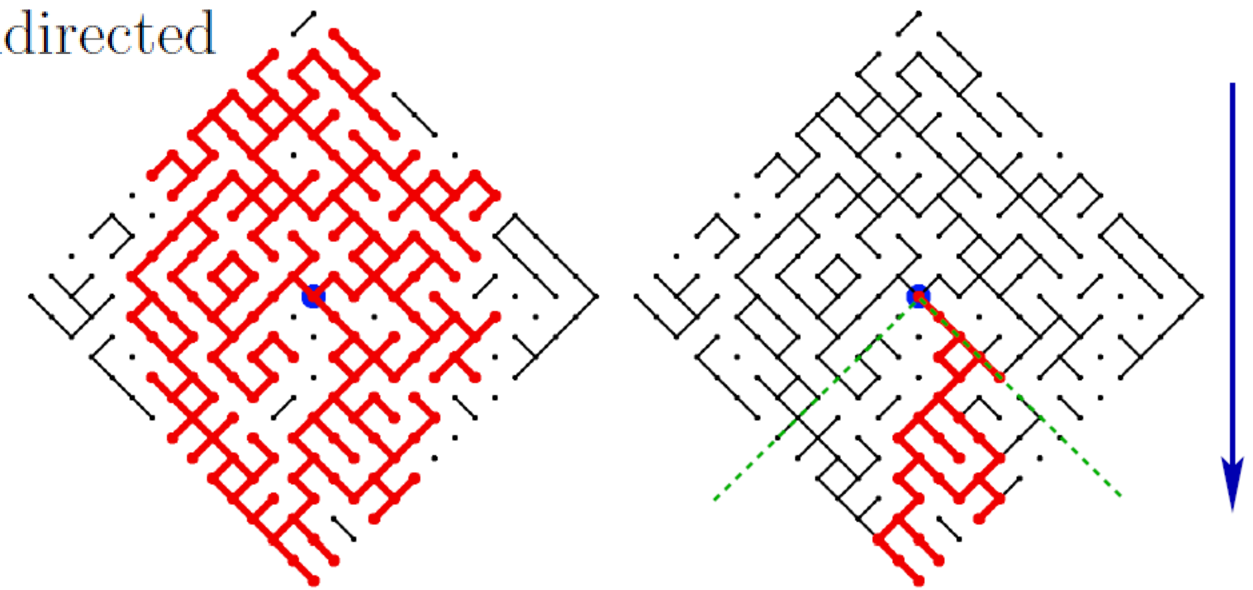
Direct Percolation

Non-equilibrium phase transitions

Haye Hinrichsen

is often used as a simple model for water percolating through a porous medium. The model is defined on a tilted square lattice whose sites represent the pores of the medium. The pores are connected by small channels (bonds) which are open with probability p and closed otherwise. As shown in Fig. 2, water injected into one of the pores will

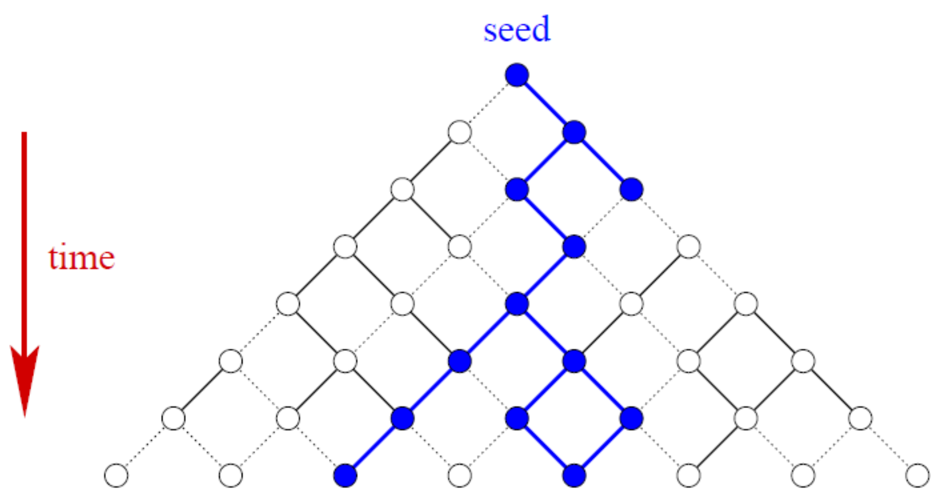
flow is undirected



isotropic percolation | directed percolation

percolate along open channels, giving rise to a percolation cluster of wetted pores whose average size will depend on p.

DP the water is restricted to flow along a preferred direction in space,



t	$N(t)$
0	1
1	1
2	2
3	1
4	2
5	2
6	3
7	2

Figure 3. Directed bond percolation. The process shown here starts with a single active seed at the origin. It then evolves through a sequence of configurations along horizontal lines (called states) which can be labeled by a time-like index t. An important quantity to study would be the total number of active sites $N(t)$ at time t.

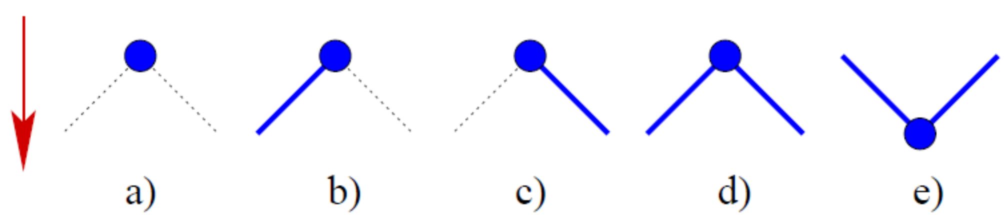


Figure 4. Interpretation of the dynamical rules of directed bond percolation as a reaction-diffusion process: a) death process, b)-c) diffusion, d) offspring production, and e) coagulation.

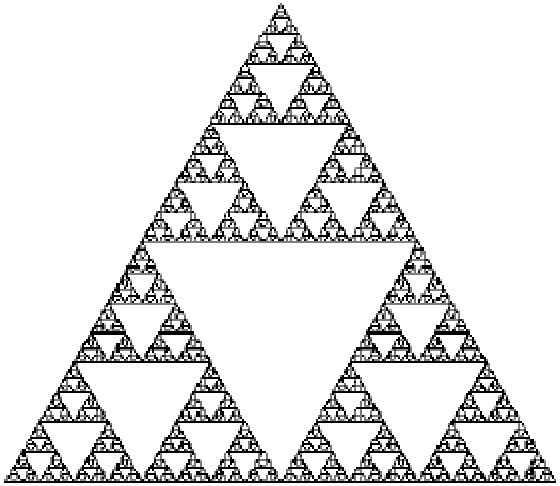
As only active sites at time t can activate sites at time $t+1$, the configuration *without* active sites plays a special role. Obviously, such a state can be reached by the dynamics but it cannot be left. In the literature such states are referred to as *absorbing*.

The mere existence of an absorbing state demonstrates that DP is a dynamical process far from thermal equilibrium. As explained in the Introduction, equilibrium statistical mechanics deals with stationary equilibrium ensembles that can be generated by a dynamics obeying detailed balance, meaning that probability currents between pairs of sites cancel each other. As the absorbing state can only be reached but not be left, there is always a non-zero current of probability into the absorbing state that violates detailed balance. Consequently the temporal evolution before reaching the absorbing state cannot be described in terms of thermodynamic ensembles, proving that DP is a non-equilibrium process.

Bootstrap

Hagedorn proposed that “a fireball consists of fireballs, which in turn consist of fireballs, and so on...” The concept later reappeared in various forms in geometry; in 1915, it led to the celebrated triangle devised by the Polish mathematician Wacław Sierpinski: “ a triangle consists of triangles, which in turn consist of triangles, and so on...”, in the words of Hagedorn.

The Sierpinski Triangle



The problem Hagedorn had in mind was, of course, considerably more complex. His heavy resonance was not simply a sum of lighter ones at rest, but it was a system of lighter resonances in motion, with the requirement that the total energy of this system added up to the mass of the heavy one. And similarly, the masses of the lighter ones were the result of still ligher ones in motion. The bootstrap equation for such a situation becomes

$$\rho(m,V_0) = \delta(m-m_0) + \sum_N \frac{1}{N!}\left[\frac{V_0}{(2\pi)^3}\right]^{N-1} \int \prod_{i=1}^{N} [dm_i\, \rho(m_i)\, d^3p_i]\, \delta^4(\Sigma_i p_i - p), \qquad (4)$$

where the first term corresponds to the case of just one lightest possible particle, a "pion". The factor V_0, the so-called composition volume, specified the size of the overall system, an intrinsic fireball size. Since the mass of any resonance in the composition chain is thus determined by the sum over phase spaces containing lighter ones, whose mass is specified in the same way, Hagedorn called this form of bootstrap "statistical".

Comment 2 : Transverse momentum distribution

Description of the RHIC $p_\perp$-spectra in a thermal model with expansion*

Wojciech Broniowski and Wojciech Florkowski

Arxiv: nucl-th/0106050

In this Letter we offer a very simple explanation of the $p_\perp$-spectra recently measured at RHIC [1–3]. Our approach has the following ingredients: i) simultaneous *chemical* and *thermal* freeze-outs, with the hadron distributions given by the thermal model; in other words, hadrons decouple completely when the thermodynamic parameters reach the freezing conditions, and no particle rescattering after freeze-out is present, ii) these thermal distributions are folded with a suitably parameterized hydrodynamic expansion, involving longitudinal and transverse flow, finally, iii) feeding from resonances, including cascades, is incorporated in a complete way.

SOC in transverse momentum distribution?

The most important universality class of absorbing-state transitions is directed percolation (DP)

Critical Dynamics , U.C. Tauber, Cambrideg University Press

A Field Theory Approach to Equilibrium and Non-Equilibrium Scaling Behavior

Universal scaling behavior of non-equilibrium phase transitions, Sven Lubeck, International Journal of Modern Physics B 18, 3977 (2004)

The effective action describing the critical behavior of DP universality class is the Reggeon Field Theory -

Directed percolation and Reggeon field theory J L Cardy and R L Sugar 1980 *J. Phys. A: Math. Gen.* **13** L423