Extraction of heavy quark transport coefficients

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Strangeness in Quark Matter 2019, Bari, Italy
June 13, 2019

SUBATECH, IMT Atlantique, Nantes, France
Heavy quarks as probes of the quark gluon plasma

The heavy-quark mass is parametrically large compared to the scales of the QGP:

$$m_{HQ} \gg T_{pc}$$

⇒ promising tool for QGP tomography, because:

- well controlled production in initial hard processes,
- at low $p_T$ well-defined transport coefficient for spatial diffusion $D_s$,
- at high $p_T$, scale separation $T \ll m_{HQ} \ll E$, probe $\hat{q} \sim m_D^2/\lambda$,
- thermalization time is enhanced by $\sim m_{HQ}/T$, preserve memory,
- HQ keep their identity during gluon emission/hadronization.
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Theoretical/phenomenological modeling is a highly complex problem!
Theoretical and phenomenological modeling - in practice

- LO pQCD, e.g. FONLL $\rightarrow$ inclusive spectra, no azimuthal $Q\bar{Q}$ correlations
  M. Cacciari et al. PRL95 (2005), JHEP 1210 (2012)

- NLO pQCD matrix elements plus parton shower, e.g. POWHEG or MC@NLO $\Rightarrow$
  exclusive spectra, like $Q\bar{Q}$ correlations

- Cold nuclear matter effects, i.e. shadowing, $p_T$ broadening, Cronin effect, etc.

- Consistent initialization of HF and LF sectors!
Scattering off light QGP constituents, sampled from fluid dynamics (or given within microscopic transport).

Interaction strength needs to be modeled as a function of $T$ and HQ $p_T$:

- perturbative vs non-perturbative
- collisional vs radiative (coherent/incoherent)
- nature of the light QGP constituents

Proper modeling of the QGP evolution is important! Should be well tested in the light hadron sector!
Theoretical and phenomenological modeling - in practice

- **Coalescence/Recombination** – predominantly at small $p_T$. Parameter-dependent!
  

- **Fragmentation** – predominantly at large $p_T$. Medium-modification?
  
  e.g. M. Cacciari et al., PRL 95 (2005)

- **After hadronization**: final hadronic interactions of $D$ mesons.
  
  M. He et al. PLB701 (2012); L. Tolos et al., PRD88 (2013); J. Torres-Rincon et al., PRD89 (2014)
Conclusions

• Many approaches achieve qualitative/semi-quantitative description of (some of) the available experimental observables.

• No single approach can reproduce all of the existing data.

• Approaches have vastly different ingredients... How to move forward toward (precision) extraction of transport coefficients?

• Large community efforts (theory) to compare their models:

Throughout this talk: for references to models please go to these reviews!
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At low momentum: diffusive “Brownian motion”

Fokker-Planck dynamics:  D. Walton et al., PRL84 (2000); G. Moore et al., PRC71 (2005)

$$\frac{\partial}{\partial t} f_Q(t, \vec{p}) = \frac{\partial}{\partial p^i} \left( A^i(\vec{p}) f_Q(t, \vec{p}) + \frac{\partial}{\partial p^j} \left[ B^{ij}(\vec{p}) f_Q(t, \vec{p}) \right] \right)$$

The friction (drag) and momentum diffusion coefficients depend on the HQ momentum and the medium temperature.

Recast to Langevin equation \( \frac{d}{dt} \vec{p} = -\eta_D(p) \vec{p} + \vec{\xi} \) and \( \langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^i_j \delta(t - t') \)

Transport coefficients connected by fluctuation-dissipation theorem (Einstein relation):

$$\eta_D = \frac{\kappa}{2m_Q T} \hspace{1cm} D_s = \frac{T}{m_Q \eta_D} \quad \text{spatial diffusion coefficient}$$

Fokker-Planck equation is second moment approximation of the Boltzmann equation:

$$\frac{d}{dt} f_Q(t, \vec{x}, \vec{p}) = C[f_Q] \quad \text{with} \quad C[f_Q] = \int d\vec{k} \left[ w(\vec{p} + \vec{k}, \vec{k}) f_Q(\vec{p} + \vec{k}) - w(\vec{p}, \vec{k}) f_Q(\vec{p}) \right]$$

\text{gain term} \hspace{1cm} \text{loss term}
The diffusion coefficient can be defined via the spectral function $\sigma(\omega, \vec{p})$ corresponding to the current-current correlator of heavy quarks:

$$D_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

Lattice QCD at finite $T$ is performed in Euclidean space ⇒ notoriously difficult to calculate dynamical quantities.

Relate the current-current correlators (calculated on the lattice) to spectral functions

$$G(\tau; T) = \int_0^{\infty} \frac{d\omega}{2\pi} \sigma(\omega; T) K(\tau, \omega; T)$$

Approximations/limitations: quenched QCD, heavy quark vs. charm quark, (no) continuum extrapolation, ...
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\]

Approximations/limitations:
quenched QCD, heavy quark vs. charm quark, (no) continuum extrapolation, ...

\[
D_s \sim 2 - 7(2\pi T)
\]
Most model approaches achieve a qualitative/semi-quantitative agreement with the available data - for very different interaction strengths and concepts!

- basic: no parameter tuning
- tune 1: tuning to data

After tuning to data, the quantitative differences in the drag coefficient become smaller but there is still no convergence.
Tuning to the data?

- Every model relies on some explicit or implicit tuning/choice of parameters.
- There is no common strategy on how to do this...
- Values of $\chi^2$/dof from model to data comparison:

<table>
<thead>
<tr>
<th>Models</th>
<th>2.76 ATeV Pb-Pb</th>
<th>200 AGeV Au-Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duke</td>
<td>0.769</td>
<td>2.819</td>
</tr>
<tr>
<td>CCNU-LBNL</td>
<td>0.132</td>
<td>1.49</td>
</tr>
<tr>
<td>Catania</td>
<td>0.113</td>
<td>1.01</td>
</tr>
<tr>
<td>TAMU</td>
<td>0.178</td>
<td>2.40</td>
</tr>
<tr>
<td>Frankfurt PHSD</td>
<td>0.637</td>
<td>1.59</td>
</tr>
<tr>
<td>Nantes col. + rad.</td>
<td>0.629</td>
<td>17.3</td>
</tr>
<tr>
<td>Nantes col. only</td>
<td>0.524</td>
<td>17.9</td>
</tr>
</tbody>
</table>

- Need to adjust parameters systematically to the experimental data!
Heavy quark evolution in a static QGP

- If very different transport coefficients can describe the same experimental data sufficiently well - other ingredients must balance the discrepancy: initial spectra, medium evolution, hadronization...

- Take them out by looking at HQ propagation through a static QGP medium at $T = 300$ MeV.

- Use a common parametrized initial spectra.

- Large differences in the $R_{AA}$ observed if only in-medium interaction is compared!

JET Collaboration
Heavy quark evolution in a static QGP - new tune

- New tune proposed: fixed value of $R_{AA} = 0.3$ at $p_T = 15$ GeV and $t = 3$ fm.

Three groups of models emerge approaches including

1. elastic and inelastic processes
2. pQCD-based elastic scattering of partons with small quasi-particle masses and TAMU
3. elastic scatterings driven by quasi-particle models (large thermal masses in the transition region)
Bayesian model-to-data statistical analysis

- HQ Langevin dynamics + 2+1d fluid dynamics + UrQMD
  \((\eta/s(T), \zeta/s(T))\) constraint by bulk observables via Bayesian analysis


Experimental data

Momentum

Temperature

Assume parametrization:

\[
D_s(T, p) = \frac{1}{1+(\gamma^2 p)^2} \left( D_s 2\pi T \right)^{\text{lin}}(T; \alpha, \beta) + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2} \left( D_s 2\pi T \right)^{\text{pQCD}}(T, p)
\]

Know the probability distributions of all parameters and correlations
⇒ momentum and temperature dependence of charm quark diffusion coefficient!

Y. Xu, J. Bernhard, S. Bass, MN, S. Cao, PRC97 (2018)
Bayesian model-to-data statistical analysis

- HQ Langevin dynamics + 2+1d fluid dynamics + UrQMD

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Y. Xu, J. Bernhard, S. Bass, MN, S. Cao, PRC97 (2018)
Effect of transport coefficients

- Next step in the comparison: common transport equation and common model of the QGP evolution, here the 2 + 1D OSU hydro code with PHSD initial conditions.

- Notable differences appear for $p_t \gtrsim 2$ GeV in the charm quark $R_{AA}$ and $v_2$.
- QP models (PHSD, Catania) and LBT/Duke (same radiative eloss) group together.
Effect of initial spectra and shadowing

- Test different parameter sets of FONLL initial HQ spectra and different degrees of shadowing in the Nantes model.

- Effect from uncertainties in the initial $pp$ reference spectra relatively small.
- Large uncertainty from error bands in shadowing descriptions (here EPS09) at $p_T < 4$ GeV, negligible for $p_T > 6$ GeV.
Effect of medium evolution: equilibrium vs nonequilibrium

- Use a common transport equation of HQ: Duke-Langevin implementation with two sets of transport coefficients (Duke and PHSD).
- Models for medium evolution: 2 + 1D OSU Hydro code, 3 + 1D vHLLE Hydro code and local temperature and velocity profiles from PHSD transport.

- Dimensionality matters for the elliptic flow.
- For each set of transport coefficients: large deviations induced by hydro vs transport evolution!
- This deviation is more pronounced for the Duke set of transport coefficients.
Effect of medium evolution

- A common transport coefficient $D_s$ for Langevin codes, pQCD Born cross section for Boltzmann transport codes, rescale by $K = 5$.
- Applied in the various models for medium evolution.

- Difference in the charm quark $R_{AA}$ can be largely understood by the difference of the total entropies in the QGP evolution models.
- Outliers: LBL-CCNU Boltzmann (massless partons sampled), PHSD (implementation of pQCD cross sections via $K(T, p)$ rescaling)
Effect of medium evolution - HQ as probes of the QGP

- Different initial conditions: PHSD vs. Trento; same 2 + 1D OSU Hydro code; same HQ transport model (Duke LV).
- Trento in Hydro and PHSD transport describe well the flow of light hadrons.

- HQ pick up substantially more flow for the Trento IC from $t > 4$ fm/c on. ⇒ HQ are indeed a good probe of the QGP evolution history!
Hadronization via recombination/coalescence

\[
\frac{dN_M}{d^3 P} = g_M \int \{ dx dp \} f_q(x_1, p_1) f_q(x_2, p_2) |\psi_M(y, P)|^2 \delta(P - p_1 - p_2)
\]

R. Fries et al. PRL 90 (2003); V. Greco et al. PRC 68 (2003)

- Recombination prob. large at low \(p_T\) \(\Rightarrow\) bump in \(R_{AA}\), adds 25 – 50% to \(v_n\).
- What is the meson/baryon wavefunction? Dependence on parameters, like the constituent quark mass, etc... \(\Rightarrow\) \(\Lambda_c/D^0\) and \(D_s/D^0\) can help constrain!
- Implementation on a fluid dynamical background not trivial. What is the probability for inhomogeneous fluid velocities, etc...?
- RRM M. He, R. Fries, R. Rapp PRC 82 (2010), PRC 86 (2012) recovers equilibrium limit for the hadron spectra and conserves energy.
- RRM gives smaller and softer \(D\) spectra than instantaneous recombination.

R. Fries et al. PRL 90 (2003); V. Greco et al. PRC 68 (2003)

Plenary talk by F. Prino, Wed 12:00
Effect of hadronization

- Take charm quark results from the common pQCD*5 transport coefficient and hadronize into $D$ mesons.

- Common fall-off region around $p_T \sim 2$ GeV gets fanned out. **EMMI-RRTF**

- No flow bump develops for UrQMD (ICM Wigner functions with $m_q > 370$ MeV) and the PHSD (ICM Wigner functions with finite times).

- Strong coalescence effect in the Nantes model also leads to an enhanced contribution to flow.
Effect of hadronization - $H_{AA}$

- Take the ratio of the $D$ meson over the charm quark spectra to highlight the effect of hadronization.

$$H_{AA}(p_T, p_t = p_T) = \frac{dN_D/dp_T}{dN_c/dp_t}$$

- Dominant structure: flow bump at low/intermediate $p_T$ (TAMU, Nantes, POWLANG).

- Levels off in the high $p_T$ region, where fragmentation is dominant.
Einstein’s relation (for a Boltzmann-Jüttner eq. distr.) \( \eta_D = \frac{\kappa_L}{2ET} - \frac{\kappa_L - \kappa_T}{p^2} - \frac{\partial \kappa_L}{\partial p^2} \)

- Calculated separately from the Boltzmann collision integral, \( \eta_D, \kappa_L, \kappa_T \) do not fulfill Einstein’s relation \( \Rightarrow \) system does not equilibrate in a Langevin approach.

- Charm quark \( R_{AA} \) decreases when ER is enforced.
- Large ambiguities introduced by the arbitrary choice, which transport coefficient to calculate explicitly \( \rightarrow \) is LV dynamics a good choice?

Duke-Frankfurt

see also: S. Das et al, PRC90 (2014); W. Horowitz, PRD91 (2015); S. Li et al, PRC99 (2019)
And at high transverse momentum?

- Comparison of the radiative energy loss component in a static QGP: Djordjevic et al: DGLV (with dynamical scattering centers); Vitev: SCET; LBL-CCNU: HT; Nantes: GB+LPM

- For \( L < 5 \text{ fm} \) relevant for HIC: DGLV, SCET and HT are consistent.

- Strongly dependent on QGP evolution! (Vitev: 1D Bjorken expansion, transverse Glauber geometry; Djordjevic: static, spherically symmetric fireball)

Similar \( \Delta E(L) \), but different \( R_{AA} \)!

Talk by M. Djordjevic, Tue 14:00
New observables

- High sensitivity of **heavy-quark correlations** to collisional vs. radiative energy loss \(MN, J. Aichelin, P.B. Gossiaux, K. Werner, PRC90 (2014), 1305.3823\) or strong coupling vs. pQCD.

![Graph](image)

- Higher-order flow coefficients more sensitive to the transport coefficient than \(v_2\).

- Consistent coupling from HQ and light flavor sector allows us to study correlations between the light and the heavy flavor sector → **event engineering**:

  \[\Delta \phi \mid \Delta \phi = \lambda = 5.5, D(p)\]

  \[\alpha = 0.3, D(p)\]

  \[\lambda = 5.5, D_{\text{const}}\]

  \[\alpha = 0.3, D_{\text{const}}\]


**Talks by R. Katz, Tue 14:40; S. Plumari, Tue 16:50**
Charm quarks in baryon-rich matter

- Charm quarks become accessible at lower beam energies thanks to large luminosities, e.g. HADES/CBM at GSI/FAIR.
- Correlated $c\bar{c}$ pairs are an important contribution to dilepton invariant mass spectra.

G. Inghirami et al, EPJC79 (2019)

- Strong medium modification at FAIR energies due to slow medium evolution and soft initial spectra.
- Fugacity factors lead to substantial differences in $D$ vs $\bar{D}$ meson $R_{AA}$.

Interesting HF physics to explore in baryon-rich matter!
Conclusions

- Many models can qualitatively/semi-quantitatively describe the HQ $R_{AA}$ and $v_2$.
- A **systematic effort in the theory community** has revealed a couple of important ingredients:
  - A common **systematic**, e.g. via Bayesian analysis, **tuning of parameters** to the experimental data could reduce spread in the transport coefficients.
  - Uncertainties in **nuclear shadowing** $\rightarrow$ large uncertainties in the HQ $R_{AA}$.
  - Only couple HQ to medium evolution models that are well-tested in the light flavor sector!
  - HQ observables are sensitive to the **medium evolution**, equilibrium vs nonequilibrium, or even the IC for hydro.
  - Hadronization via **recombination** introduces large effects and differences between the various models. Needs more work.
- Great potential in **new observables**, like higher-order flow, azimuthal and momentum correlations, heavy-light correlations, to explore theoretically and experimentally, and baryon-rich matter!

Thanks to J. Aichelin, S. Bass, E. Bratkovskaya, S. Cao, P.B. Gossiaux, T. Song, K. Werner, Y. Xu for fruitful collaborations!
extra
Partial thermalization of heavy quarks with the medium

Need a good quantitative measure - or at least some common understanding - of what we mean by “(partial) thermalization”!

- HQ seem to pick up almost all of the anisotropy of the medium.
- But equilibration times are much larger, $\tau_{\text{relax}} \sim \tau_{\text{QGP}}$.
- Hydro-like flow coefficients $\neq$ thermalization/equilibration.
- Other observables do not agree with thermalized charm.

S. Cao, S. Bass PRC 84 (2011)

Calculation by S. Cao
• Under which conditions should Brownian motion be a valid approximation for relativistic particles?
• Calculations of transport coefficients from the underlying theory do not necessarily fulfil FDT ⇒ calculate one and adapt the others via the FDT.
• For charm quarks: Langevin leads to Gaussian momentum distribution, Boltzmann very different.
• Does this affect the $R_{AA}$ and the $v_2$ or do we need correlation observables to see a difference?

S. Das et al, PRC90 (2014)
Radiative energy loss

- LO pQCD matrix element for $2 \rightarrow 3$ process Kunszt et al. PRD21 (1980)
- Gunion-Bertsch approximation derived in the high-energy limit, where the radiated gluon $k_\perp$ and the momentum transfer $q_\perp$ are soft $\ll \sqrt{s}$.
- Incoherent radiation off a massless parton, mid-rapidity
- Extension beyond mid-rapidity and to finite mass $m_Q$ (heavy quarks!)
  $\Rightarrow$ distribution of induced gluon radiation:

$$P_g(x, k_\perp, q_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1-x}{x} \left( \frac{k_\perp}{k_\perp^2 + x^2 m_Q^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + x^2 m_Q^2} \right)^2$$

$\Rightarrow E_{\text{rad}}^{\text{loss}} \propto E L$

J. Gunion, PRD25 (1982); O. Fochler et al. PRD88 (2013); J. Aichelin et al. PRD89 (2014)
Coherent emission - LPM

- coherent emission if \( \tau_{\text{form}} = \sqrt{\frac{3}{q}} > l_{\text{mfp}} \)
- QCD analogon to the Landau-Pomeranchuk-Migdal (LPM) effect
- Important in QCD: rescattering of the forming gluon with medium partons ⇒ less suppression than in QED
- At large energies in BDMPS-Z: ⇒ \( E_{\text{rad}}^\text{loss} \propto \sqrt{E L} \)
- For very energetic partons \( \tau_{\text{form}} > L \), then \( E_{\text{rad}}^\text{loss} \propto L^2 \), estimate for the LHC (\( L \sim 2 \text{ fm}, \hat{q} \sim 2 \text{ GeV/fm} \) ⇒ \( \omega_c \sim 20 \text{ GeV} \))


Baier et al. PLB 345 (1995); NPB 483 (1997); ibid. 484 (1997); B. G. Zakharov, JETP Lett. 63 (1996) 952
suppression of high-energetic (small angle) gluon emission by the heavy quark mass:

\[
\frac{d\sigma_{\text{rad}}}{d\theta} \propto \frac{\theta^2}{\left(\theta^2 + \frac{M_Q^2}{E_Q^2}\right)}
\]

- Suppresses gluon emission in the dead cone \( \theta_D = \frac{M_Q}{E_Q} \)
- Introduces a mass hierarchy in the radiative energy loss.
- But: assumes hard scatterings!

When the hard scattering assumption is relaxed, emission at low \( k_{\perp} \) is significantly less suppressed:

\[
\frac{P_g(x,k_{\perp};M)}{P_g(x,k_{\perp};0)}
\]

\[\text{hard-scattering approximation}\]

\[\text{all scatterings}\]
Charm production (and diffusion?) in pPb collisions

- $3 + 1d$ fluid dynamical evolution + Langevin dynamics, initial shadowing.

- Centrality dependence of $R_{pPb}$ expected due to energy loss. (Note, that experimentally $Q_{pPb}$!)

- Indications that $v_2$ of $D$ mesons decouples from medium flow - unlike in AA collisions - and decreases with centrality.

- Can HF measurements in pPb help answering the question of initial vs final state effects?

Y. Xu et al, Duke University, in preparation

see also: A. Beraudo, JHEP1603 (2016)
Modeling of heavy-quark dynamics in the QGP

production interaction with the medium hadronization

medium description coupling medium - HF sector

- Model the QGP: a locally thermalized medium provides the scattering partners.
- Input from a fluid dynamical description of the bulk QGP medium: temperatures and fluid velocities.
- Use a fluid dynamical description which describes well the bulk observables!

smooth initial conditions fluctuating initial conditions

plot by V. Ozvenchuk, Nantes
• Due to the radial flow of the matter low-$p_T$ $c\bar{c}$-pairs are pushed into the same direction.

• Initial correlations at $\Delta \phi \sim \pi$ are washed out but additional correlations at small opening angles appear.

• This happens only in the purely collisional interaction mechanism!

• No “partonic wind” effect observed in collisional+radiative(+LPM) interaction mechanism!
centrality dependence:

- increase of initial eccentricities
- decrease of interaction rate and medium size

⇒ expectation: heavy-flavor flow shows a weaker dependence on centrality, especially for $v_3$

MN et al. PRC91 (2015)
**At small $p_T$: relative enhancement of flow in purely collisional scenario over collisional+radiative(+LPM) larger for $v_3$ than for $v_2$**
Charm flow: hadronization and energy loss

collisional+radiative(+LPM), $K = 0.8$

- Contribution to the flow from hadronization.
- For low $p_T$ the charm flow is predominantly due to the flow of the bulk.

MN et al. PRC91 (2015)
Low versus high momentum

CMS
Preliminary

$R_{AA}$

$T_{AA} +$ lumi. uncertainty

Centrality 0-100%

charged hadrons $|y| < 1.0$

$D^0$ $|y| < 1.0$

$B^+$ $|y| < 2.4$

Low $p_T \sim m_Q$

• Very different from light partons.

• Nonperturbative!

• Partial thermalization with the light partons in the QGP?

• Diffusion $D$ mainly via collisional processes?

• Hadronization via coalescence/recombination?

• Initial shadowing and cold nuclear matter effects?

High $p_T \gg m_Q$

• Strong mass dependence expected.

• Perturbative regime...

• Rare processes, probe the opacity of the matter.

• Energy loss $dE/dx$ via collisional and radiative processes?

• Coherent energy loss $\rightarrow$ jet-quenching parameter $\hat{q}$?

• Hadronization via (medium-modified?) fragmentation.
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- Coherent energy loss $\rightarrow$ jet-quenching parameter $\hat{q}$?
- Hadronization via (medium-modified?) fragmentation.
Open versus hidden heavy-flavor

- Represent most of the HF production cross section
- Effects in cold nuclear matter: shadowing, energy loss
- Single particle
- \( Q \) is always a color charged object
- Energy/momentum loss due to interactions with the medium \( \Rightarrow \) HF diffusion coefficient

Hidden heavy flavor

- Small amount of total cross section
- Effects in cold nuclear matter: shadowing, energy loss, break-up
- Bound state with non-zero extension
- Can propagate as a color-neutral bound state
- Melting due to screening of \( Q\bar{Q} \) potential in the medium \( \Rightarrow \) formation/temperature of QGP

J. Bjorken, 1982

Illustration by Alex Doig; Agnes Moscy QM 2011

\[ \text{dissociation} \quad \leftrightarrow \quad \text{regeneration} \quad \Rightarrow \]

T. Matsui and H. Satz, PLB 178 (1986)
Fluid dynamical description and heavy quarks

Formation of QGP, which evolves fluid dynamically as a nearly perfect fluid.

Energy-momentum conservation

\[ \partial_\mu T^{\mu\nu} = 0 \]

For conserved charge densities

\[ \partial_\mu N^\mu = 0 \]

equation of state

\[ p = p(e, n) \rightarrow (T, \mu_B) \]

→ Coupling heavy quarks to the medium (\( T \) and \( u^\mu \)) via Fokker-Planck dynamics!

Observable: Fourier coefficients of

\[ \frac{d^2 N}{dp_T dy} \propto \sum_n v_n \cos(n\phi) \]

Sensitive to viscosity \( \eta/s \)
Formation of QGP, which evolves fluid dynamically as a nearly perfect fluid.

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sensitive to viscosity \(\eta/s\)
Fokker-Planck dynamics

Assumption: scatterings in the medium are dominated by small momentum transfer
⇒ Description of the HQ momentum distribution \( f_Q \) as a function of time via
Fokker-Planck dynamics: \( \text{D. Walton et al., PRL84 (2000); G. Moore et al., PRC71 (2005)} \)

\[
\frac{\partial}{\partial t} f_Q(t, \vec{p}) = \frac{\partial}{\partial p^i} \left( A^i(\vec{p}) f_Q(t, \vec{p}) + \frac{\partial}{\partial p^j} \left[ B^{ij}(\vec{p}) f_Q(t, \vec{p}) \right] \right)
\]

The friction (drag) and momentum diffusion coefficients depend on the HQ momentum and the medium temperature.

Recast to Langevin equation \( \frac{d}{dt} \vec{p} = -\eta_D(p) \vec{p} + \vec{\xi} \) and \( \langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t') \)

Transport coefficients connected by fluctuation-dissipation theorem (Einstein relation):

\[
\eta_D = \frac{\kappa}{2m_Q T} \quad \quad D_s = \frac{T}{m_Q \eta_D} \quad \quad \text{spatial diffusion coefficient}
\]

Fokker-Planck equation is second moment approximation of the Boltzmann equation:

\[
\frac{d}{dt} f_Q(t, \vec{x}, \vec{p}) = C[f_Q] \quad \text{with} \quad C[f_Q] = \int d\vec{k} \left[ w(\vec{p} + \vec{k}, \vec{k}) f_Q(\vec{p} + \vec{k}) - w(\vec{p}, \vec{k}) f_Q(\vec{p}) \right]
\]

Do both dynamics lead to similar phenomenological results? Probably yes for \( R_{AA}, \nu_2 \), probably no for more differential observables... \( \text{S. Das et al, PRC90 (2014)} \)
- One-gluon exchange model: reduced IR regulator $\lambda m_D^2$ in the hard propagator

- Running coupling $\alpha_{\text{eff}}(t)$ and self-consistent
  
  \[ m_D^2 = (1 + 6n_f)4\pi\alpha_s(m_D^2)T^2 \]


- Extension of Gunion-Bertsch approximation beyond mid-rapidity and to finite mass $m_Q$ ⇒ distribution of induced gluon radiation ($E_{\text{rad}}^\text{loss} \propto E L$):

  \[
  P_g(x, \vec{k}_\perp, \vec{q}_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1-x}{x} \left( \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + x^2m_Q^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2 + x^2m_Q^2} \right)^2
  \]

  J. Gunion, PRD25 (1982); B. Zakharov, JETPL 63/65 (1996/7); O. Fochler et al. PRD88 (2013); J. Aichelin et al. PRD89 (2014)

T-matrix approach - TAMU

- Thermodynamic T-matrix approach, $T = V + VGT$, given by a two-body driving kernel $V$, estimated from the IQCD internal/free energy for a static $Q\bar{Q}$ pair.

  D. Cabrera, R. Rapp PRD 76 (2007); H. van Hees, M. Mannarelli, V. Greco, R. Rapp PRL 100 (2008)

- Comprehensive sQGP approach for the EoS, light quark & gluon spectral functions, quarkonium correlators and HQ diffusion.

  F. Riek, R. Rapp PRC 82 (2010); S. Liu, R. Rapp arxiv:1612.09138

- Resonance correlations in the T-matrix naturally lead to recombination (resonance recombination model) near $T_c$ from the same underlying interactions!

  M. He, R. Fries, R. Rapp PRC 82 (2010), PRC 86 (2012)

low-$p$ light quarks/gluons
no good quasiparticles!

inelastic reaction rates for strongly suppressed $Y(2S)$

charm spatial diffusion coefficient
Quasiparticles or AdS/CFT

Quasiparticles:

- Nonperturbative effects near $T_c$ are captured by $\alpha_s(T)$, leading to thermal masses/widths, determined from fits to lQCD EoS.
  - Implemented for HF dynamics in e.g. PHSD (full off-equilibrium transport). H. Berrehrah et al. 1604.02343, T. Song et al. PRC 92 (2015), PRC 93 (2016)

Strong coupling - AdS/CFT:

- HF string moves at constant $v$ over BH: fluctuations grow with $v$, $D(p)$.
- New derivation parametrizes between LF/HF $\Rightarrow$ string falling onto the BH horizon, $D_{\text{const}}$. R. Moerman, W. Horowitz, arxiv:1605.09285
• Consistent picture of quarkonium suppression and regeneration in the QGP.
• Spectral functions contain information about masses, binding energies, reaction rates of the $Q \bar{Q}$.
• Interpretation of melting peaks in the spectral function via modeling of $Q \bar{Q}$ dynamics in HIC.
• SHM describes $N_{Q\bar{Q}}^{\text{eq}}(T, \gamma)$ based on thermal values.

$\frac{dN_{Q\bar{Q}}}{d\tau} = -\Gamma [N_{Q\bar{Q}} - N_{Q\bar{Q}}^{\text{eq}}]$ reaction rate $\Gamma$: (gluo + quasi-free dissociation) governs suppression and regeneration.

• Toward the inclusion of quantum effects into transport.

Help from hidden heavy flavor?

P. Braun-Munzinger et al. arxiv:0901.2500; M. Gazdzicki et al. PRL 83 (1999); A. Andronic et al. NPQ 789 (2007)

G. Bhanot et al., NPB 156 (1979); D. Kharzeev et al. PLB 334 (1994)
N. Brambilla et al., PRD 78 (2008); X. Zhao et al. NPA 859 (2011);
T. Song et al. PRC 84 (2011); M. Strickland et al. NPA 879 (2012);
E. Ferreiro, PLB 731 (2014); K. Zhou et al. PRC 89 (2014)

JP. Blaizot et al., NPA946 (2016); PB. Gossiaux et al., arxiv:1611.06499
Help from hidden heavy flavor?

- Initially produced $J/\psi$, which survives the QGP phase, has little $v_2$ due to low elastic interaction rate.
- Charmonium regeneration concentrated at low $p_T \lesssim m_{J/\psi} \Rightarrow$ leads to a non-zero elliptic flow for $J/\psi$, with similar values as for D mesons.

TAMU: X. Zhao et al., NPA 904-905 (2013); Tsinghua: K. Zhou et al., PRC 89 (2014); ALICE QM2017

- Reasonable theoretical description at low $p_T$, additional effects at intermediate $p_T$ for $\sqrt{s} = 5.02$ TeV!
- Need to address open heavy-flavor dynamics and quarkonium regeneration within the same dynamical framework!

$\Rightarrow$ Information about charm quark thermalization and recombination/regeneration mechanism!
Resonance recombination model (RRM) - $D_s$ enhancement

- Resonance correlations in the T-matrix naturally lead to recombination (resonance recombination model) near $T_c$ from the same underlying interactions! M. He, R. Fries, R. Rapp PRC 82 (2010), PRC 86 (2012)

- RRM reproduces long-time limit of thermal equilibration as given by a fluid dynamical calculation.

- Finite recombination time window around $T_{pc} = 170$ MeV.

- Thermal weights from SHM, $\gamma_s = 0.85$


$\Rightarrow$ Significantly enhanced $D_s$ $R_{AA}$ over non-strange $D$ $R_{AA}$!

M. He, R. Fries, R. Rapp, PRL 110 (2013); PLB 735 (2014)
What can we learn from the v3?

- Most models give a $\tau_{\text{relax}}$ for charm quarks much longer than the evolution of the QGP, but $v_2(\text{HF}) \lesssim v_2(\text{LF}) \rightarrow$ indication for “partial” thermalization?

- Higher-order Fourier coefficients were important for understanding charged hadron flow ⇒ What about heavy-flavor $v_3$, $v_4$, ...?

- Expectation: $v_3$ and higher-order coefficients (and centrality dependence) show the incomplete coupling of HQ to the medium!


- Could heavy-flavor flow, $v_2$ and $v_3$, have a different origin? Escape mechanism?

  Z.W. Lin NPA956 (2016)
New observables?

- High sensitivity of **heavy-quark correlations** to collisional vs. radiative energy loss.  
  MN, J. Aichelin, P.B. Gossiaux, K. Werner, PRC90 (2014), 1305.3823

**different view on HQ flow coefficients:**

\[
\lambda = \frac{\alpha_s}{\pi} \quad (\pi)
\]

\[
\alpha_s = \frac{\lambda}{\pi} \quad (\pi)
\]

\[
\lambda = \frac{\alpha_s}{\pi} \quad (\pi)
\]

\[
\alpha_s = \frac{\lambda}{\pi} \quad (\pi)
\]

- **Flow coefficients** from \(DD\) correlations agree well with \(v_n\) from event plane/participant plane.
- Low \(p_T\) pairs more likely to remain correlated for strong than for weak coupling.
- Challenge: the \(c\bar{c}\) proton-proton baseline is not well understood theoretically + experimental feasibility?
New observables?

- High sensitivity of **heavy-quark correlations** to collisional vs. radiative energy loss.  
  MN, J. Aichelin, P.B. Gossiaux, K. Werner, PRC90 (2014), 1305.3823

**Different view on HQ flow coefficients:**

![Graph showing flow coefficients from DD correlations](image)

- Flow coefficients from **DD** correlations agree well with $v_n$ from event plane/participant plane.
- Low $p_T$ pairs more likely to remain correlated for **strong** than for weak coupling.
- Challenge: the $c\bar{c}$ proton-proton baseline is not well understood theoretically + experimental feasibility?
Going from MC@sHQ+EPOS2 to EPOS-HQ allows us to study correlations between the light and the heavy flavor sector! PB. Gossiaux, J. Aichelin, MN, V. Ozvenchuk, K. Werner, arXiv:1705.02271

Nucleon - pion $v_n$ correlate strongly (same production mechanism/hypersurface).

Event-by-event D meson - pion flow is less correlated, especially $v_3$.

Select light flavor events with high and low $q$-vector and look at heavy flavor response in these event classes!