

Overview of experimental critical point search

Tobiasz Czopowicz
Warsaw University of Technology



**Faculty
of Physics**

WARSAW UNIVERSITY OF TECHNOLOGY



Strangeness in Quark Matter
Bari, Italy
June 10 – 15, 2019

Experimental search for the critical point at SQM 2019

- Dmytro Oliinychenko, *Snowballs from hell: light nuclei production in HI collisions*
- Mesut Arslanok, *Higher moments of net-particle fluctuations in Pb-Pb collisions from ALICE*
- Sukanya Sombun, *Higher order net-proton number cumulants dependence on the centrality definition and other spurious effects*
- Ashish Pandav, *Measurement of higher moments of net particle distributions in Au+Au collisions at $\sqrt{s_{NN}} = 54.4$ GeV at RHIC*
- Rene Bellwied, *Fluctuations of net Λ distributions in Au+Au collisions measured as a function of collision energy with the STAR detector at RHIC*
- Alice Ohlson, *Correlations and fluctuations*

Outline

- ① Critical point search strategies
- ② Biasing effects
- ③ Experimental measures
- ④ Experimental results
- ⑤ Summary

① Critical point search strategies

- Critical point of QGP
- Exploring the phase diagram with heavy-ion collisions

② Biasing effects

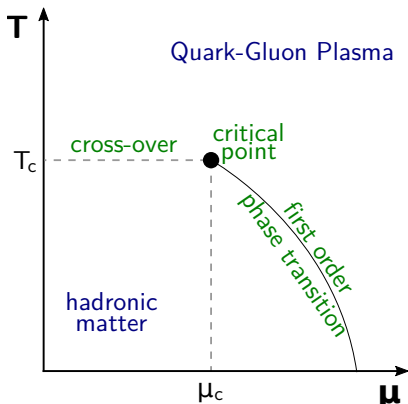
③ Experimental measures

④ Experimental results

⑤ Summary

Critical point of QGP

Critical point search strategies



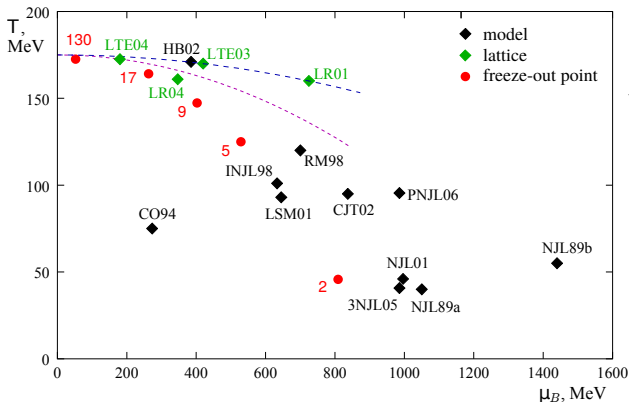
Critical point (CP) – a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition.

2nd order phase transition \rightarrow scale invariance \rightarrow power-law form of correlation function.

These expectations are for fluctuations and correlations in the configuration space which are expected to be projected to the momentum space via quantum statistics and/or collective flow.

Critical point of QGP

Critical point search strategies

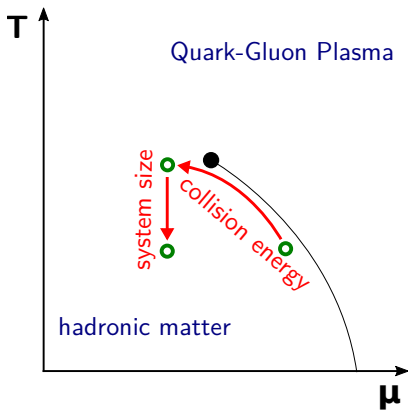


The main signal of the CP is anomaly in fluctuations in a narrow domain of the phase diagram.

However predictions on the CP existence, its location and what and how should fluctuate are model-dependent.

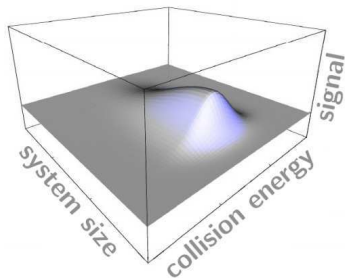
Exploring the phase diagram with heavy-ion collisions

Critical point search strategies



Search for the critical end point in heavy-ion collisions is performed by a scan in the parameters controlled in laboratory (collision energy and nuclear mass number, centrality).

By changing them, we change freeze-out conditions (T, μ_B).



The experimental search for the critical point requires a two-dimensional scan in collision energy and size of the colliding nuclei (centrality).

① Critical point search strategies

② **Biasing effects**

③ Experimental measures

④ Experimental results

⑤ Summary

Biasing effects (and how to minimize them)

Physics:

- volume fluctuations
fix: select central events, use strongly intensive quantities
- conservation laws
fix: limit acceptance, use non-conserved quantities
- formation and decay of resonances
fix: use quantities weakly influenced by resonance decays as suggested by models

Measurement:

- detector efficiency should be large to avoid model-dependent corrections
- incomplete particle identification
fix: use Identity or PSET identification methods
- limited acceptance makes it difficult to compare different results

Kitazawa, PRC 93 (2016) 044911

Gorenstein, PRC 84 (2011) 024902

Bzdak, Koch, PRC 86 (2012) 044904

Bzdak, Koch, PRC 91 (2015) 027901

Rustamov, Gorenstein, PRC 86 (2012) 044906

Nonaka, Kitazawa, Esumi, PRC 95 (2017) 064912

Gazdzicki, Gorenstein, Mackowiak-Pawlowska, Rustamov, arXiv:1903.08103

① Critical point search strategies

② Biasing effects

③ Experimental measures

- Fluctuations in large momentum bins
 - Extensive quantities
 - Intensive quantities
 - Strongly intensive quantities
- Short-range correlations
- Fluctuations as a function of momentum bin size
- Light nuclei production

④ Experimental results

⑤ Summary

Extensive quantities

Experimental measures

A quantity proportional to W (WNM) or V in (IB-GCE) is called an extensive quantity. The most popular are particle number (multiplicity) distribution $P(N)$ cumulants:

- $\kappa_1 = \langle N \rangle$
- $\kappa_2 = \langle (\delta N)^2 \rangle = \sigma^2$
- $\kappa_3 = \langle (\delta N)^3 \rangle = S\sigma^3$
- $\kappa_4 = \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2 = \kappa\sigma^4$

These multiplicity cumulants characterize the shape of multiplicity distribution and quantify fluctuations.

WNM – Wounded Nucleon Model ($\langle N_{A+B} \rangle = \langle W_{A+B} \rangle / 2 \cdot \langle N_{N+N} \rangle$)

IB-GCE – Ideal Boltzmann Grand Canonical Ensemble

Intensive quantities

Experimental measures

Ratio of any two extensive quantities is independent of W (WNM) or V (IB-GCE). It is an intensive quantity.

For example:

$$\langle A \rangle / \langle B \rangle = W \cdot \langle a \rangle / W \cdot \langle b \rangle = \langle a \rangle / \langle b \rangle$$

where A and B are any extensive event quantities, i.e. $\langle A \rangle \sim W$, $\langle B \rangle \sim W$ and $\langle a \rangle = \langle A \rangle$ and $\langle b \rangle = \langle B \rangle$ for $W = 1$.

Popular examples:

- $\frac{\kappa_2}{\kappa_1} = \omega[N] = \frac{\sigma^2[N]}{\langle N \rangle} = \frac{W \cdot \sigma^2[n]}{W \cdot \langle n \rangle} = \omega[n]$ (scaled variance)
- $\frac{\kappa_3}{\kappa_2} = S\sigma$
- $\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$

Strongly intensive quantities

Experimental measures

For an event sample with varying W , cumulants are not extensive quantities any more. For example:

$$\kappa_2 = \sigma^2[N] = \sigma^2[n] \langle W \rangle + \langle n \rangle^2 \sigma^2[W]$$

But having two extensive event quantities, one can construct quantities that are independent of $P(W)$!

Popular examples:

- $\langle K \rangle / \langle \pi \rangle$
- $\Delta[N, P_T] = \frac{1}{C} (\omega[N] \langle P_T \rangle - \omega[P_T] \langle N \rangle)$
- $\Sigma[N, P_T] = \frac{1}{C} (\omega[N] \langle P_T \rangle + \omega[B] \langle N \rangle - 2(\langle NP_T \rangle - \langle P_T \rangle \langle N \rangle))$

where $P_T = \sum_{i=1}^N p_{T,i}$ and C is any extensive quantity (e.g. $\langle N \rangle$).

Short-range correlations

Experimental measures

Quantum statistics leads to short-range correlations in momentum space, which are sensitive to particle correlations in configuration space (e.g. of CP origin).

Popular measure:

Momentum difference in LCMS, \mathbf{q} , is decomposed into three components:

q_{long} – momentum difference along the beam

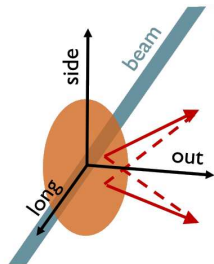
q_{out} – parallel to the pair transverse momentum vector $\mathbf{k}_t = (\mathbf{p}_{T,1} + \mathbf{p}_{T,2})/2$

q_{side} – perpendicular to q_{out} and q_{long}

The two-particle correlation function C is often approximated by a three-dimensional Gauss function:

$$C(\mathbf{q}) \cong 1 + \lambda \cdot \exp(-R_{\text{long}}^2 q_{\text{long}}^2 - R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2)$$

where λ describes the correlation strength and $R_{\text{out}}, R_{\text{side}}, R_{\text{long}}$ denote Gaussian HBT radii.



Short-range correlations

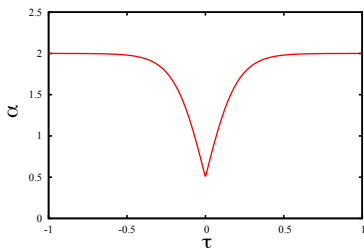
Experimental measures

A more general idea – Lévy-shaped source (1-D):

$$C(q) \cong 1 + \lambda \cdot e^{(-qR)^\alpha}$$

where $q = |p_1 - p_2|_{\text{LCMS}}$, λ describes correlation length, R determines the length of homogeneity and Lévy exponent α determines source shape:

- $\alpha = 2$
Gaussian, predicted from simple hydro
- $\alpha < 2$
anomalous diffusion, generalized central limit theorem
- $\alpha = 0.5$
conjectured value at the critical point



Fluctuations as a function of momentum bin size

Experimental measures

Momentum space is partitioned into M bins.

Second factorial moment is calculated as a function of cell size (number of cells, M):

$$F_2(M) \equiv \frac{\left\langle \frac{1}{M} \sum_{i=1}^M n_i (n_i - 1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^M n_i \right\rangle}$$

At the second order phase transition the system is a simple fractal and the factorial moment exhibits a power law dependence on M :

$$F_2(M) \sim (M)^{\varphi_2}$$

Prediction for critical point: $\varphi_2 = 5/6$.

Wosiek, APPB 19 (1988) 863

Bialas, Hwa, PLB 253 (1991) 436

Bialas, Peschanski, NPB 273 (1986) 703

Antoniou, Diakonou, Kapoyannis, Kousouris, PRL 97 (2006) 032002

Fluctuations as a function of momentum bin size

Experimental measures

However, to cancel the $F_2(M)$ dependence on the single particle inclusive momentum distribution, one needs a uniform distribution of particles in bins. One can either subtract $F_2(M)$ for mixed events:

$$\Delta F_2(M) = F_2^{\text{data}}(M) - F_2^{\text{mixed}}(M)$$

or use cumulative quantities, e.g.

$$Q_x = \int_{x_{\min}}^x \rho(x) dx \bigg/ \int_{x_{\min}}^{x_{\max}} \rho(x) dx$$

Light nuclei production

Experimental measures

Based on coalescence model, particle ratios of light nuclei are sensitive to the nucleon density fluctuations at kinetic freeze-out and thus to CP.

In the vicinity of the critical point or the first order phase transition, density fluctuation becomes larger.

Neutron density fluctuation can be expressed by proton, triton and deuteron yields:

$$\Delta n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle} \approx \frac{1}{2\sqrt{3}} \frac{N_p \cdot N_t}{N_d^2} - 1$$

① Critical point search strategies

② Biasing effects

③ Experimental measures

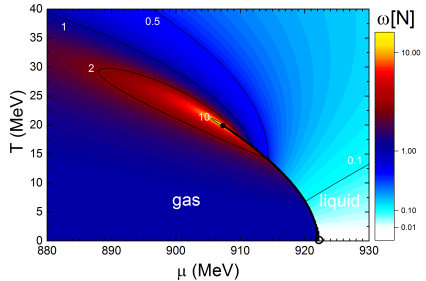
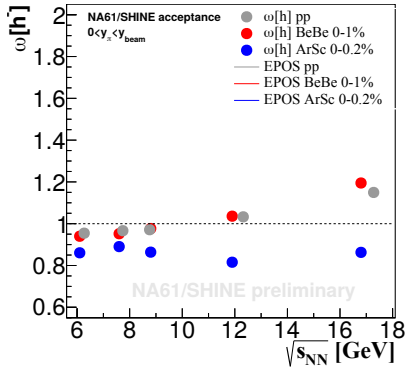
④ Experimental results

- Fluctuations in large momentum bins
 - Multiplicity fluctuations
 - Multiplicity-transverse momentum fluctuations
 - Net-proton fluctuations
 - Net-kaon and net-charge fluctuations
- Short-range correlations
- Fluctuations as a function of momentum bin size
- Light nuclei production

⑤ Summary

Multiplicity fluctuations

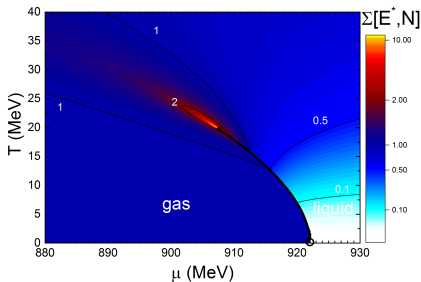
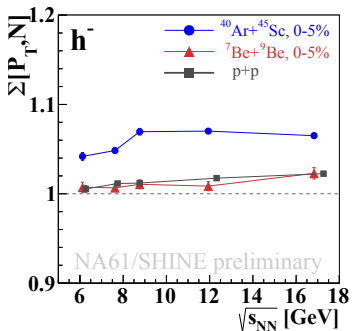
Experimental results: Fluctuations in large momentum bins



No prominent structures that could be related to the critical point are observed so far...

Multiplicity-transverse momentum fluctuations

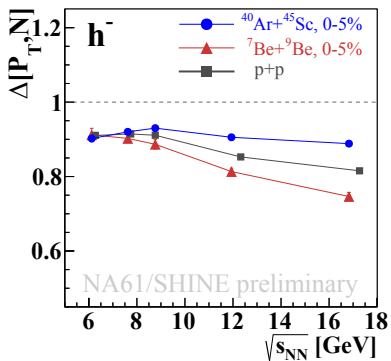
Experimental results: Fluctuations in large momentum bins



No prominent structures that could be related to the critical point are observed so far...

Multiplicity-transverse momentum fluctuations

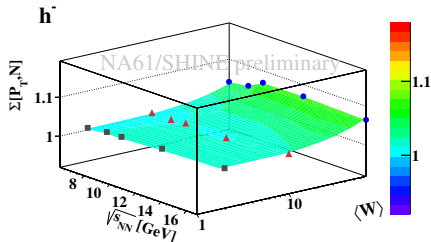
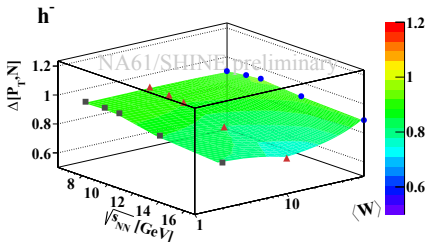
Experimental results: Fluctuations in large momentum bins



No prominent structures that could be related to the critical point are observed so far...

Multiplicity-transverse momentum fluctuations

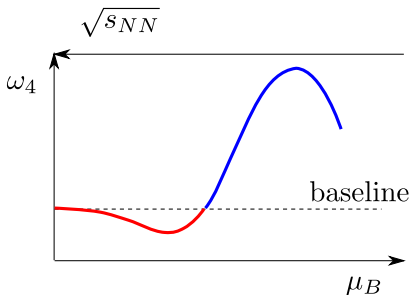
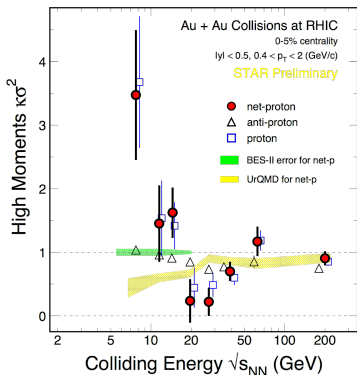
Experimental results: Fluctuations in large momentum bins



No prominent structures that could be related to the critical point are observed so far...

Net-proton fluctuations

Experimental results: Fluctuations in large momentum bins

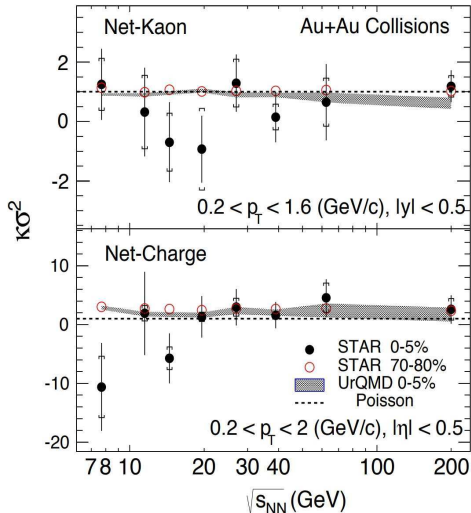


Observation of non-monotonic energy dependence of fourth order net-proton fluctuation.

STAR: PRL 112 (2014) 032302
Stephanov, PRL 107, 052301

Net-kaon and net-charge fluctuations

Experimental results: Fluctuations in large momentum bins



Within errors, the results on net-kaon and net-charge show flat energy dependence.

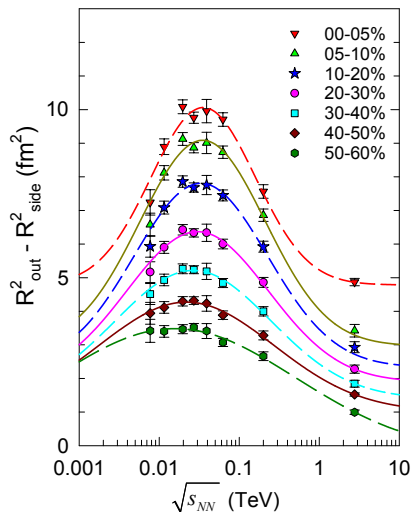
No prominent structures that could be related to the critical point are observed so far...

STAR: PLB 785 (2018) 551
PRL 113 (2014) 092301

T. Czopowicz (WUT)

Short-range correlations

Experimental results



Data taken from:

STAR: Phys. Rev. C 92, 014904 (2015)

ALICE: Phys.Lett. B696 (2011) 328

Clear non-monotonic energy
dependence in Au+Au/Pb+Pb

Short-range correlations

Experimental results

Finite size scaling analysis

$$L = \bar{R}$$

$$1/\bar{R} = \sqrt{(1/\sigma_x)^2 + (1/\sigma_y)^2}$$

σ_x, σ_y - widths of source

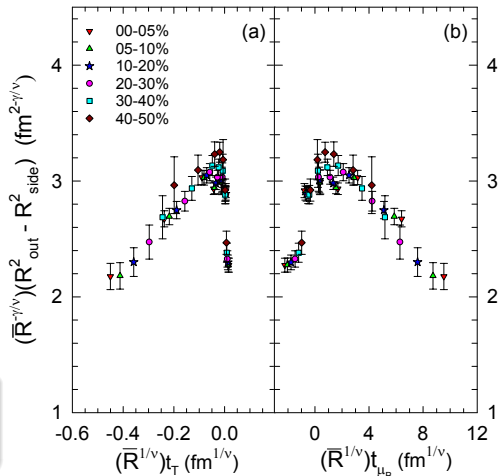
$$t_T = \frac{T - T_C}{T_C}$$

$$t_{\mu_B} = \frac{\mu - \mu_C}{\mu_C}$$

CP in Au+Au at $\sqrt{s_{NN}} = 47.5$ GeV

$T_C = 165$ MeV

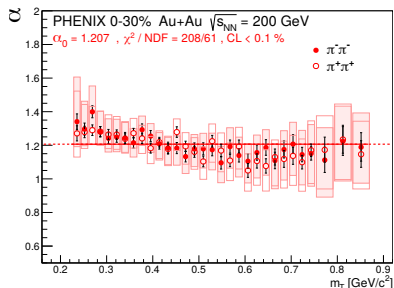
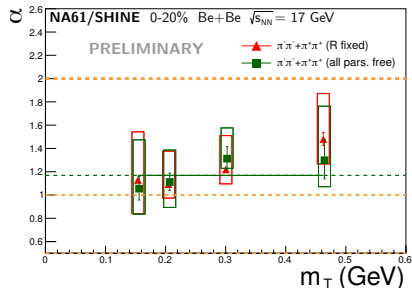
$\mu_C = 95$ MeV



Short-range correlations

Experimental results

Transverse mass dependence of Lévy exponent α



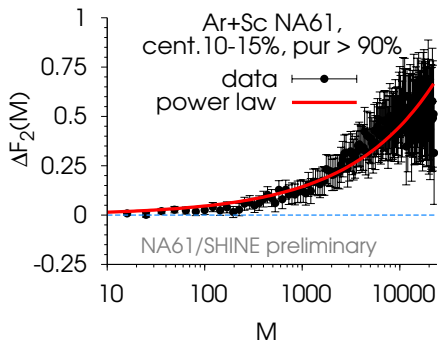
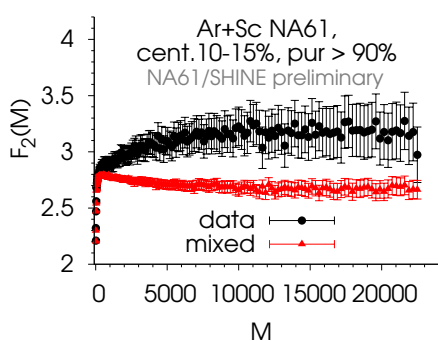
$$\alpha(\text{Be+Be at } 17 \text{ GeV}) \approx \alpha(\text{Au+Au at } 200 \text{ GeV}) \approx 1.2$$

No indication of the critical point so far...

Fluctuations as a function of momentum bin size

Experimental results

Mid-rapidity protons at 17 GeV

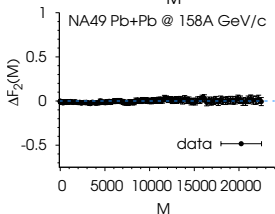
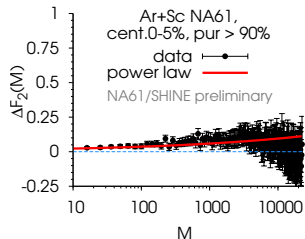
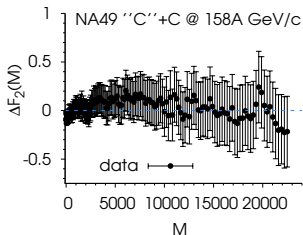
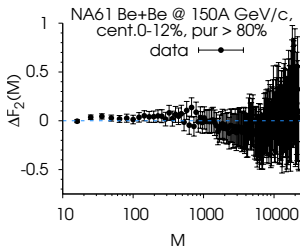


Note that points are strongly correlated.

Fluctuations as a function of momentum bin size

Experimental results

ΔF_2 for mid-rapidity protons at 17 GeV



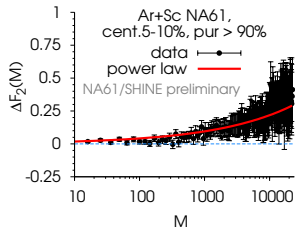
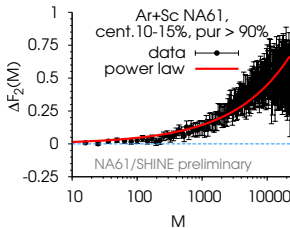
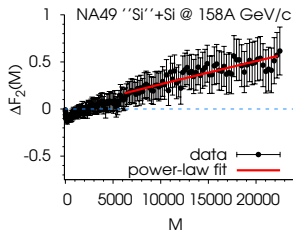
No signal visible in central Be+Be,
C+C, Ar+Sc and Pb+Pb

NA49: EPJC 75 (2015) 587
NA61/SHINE: PoS(CPOD2017) 054

Fluctuations as a function of momentum bin size

Experimental results

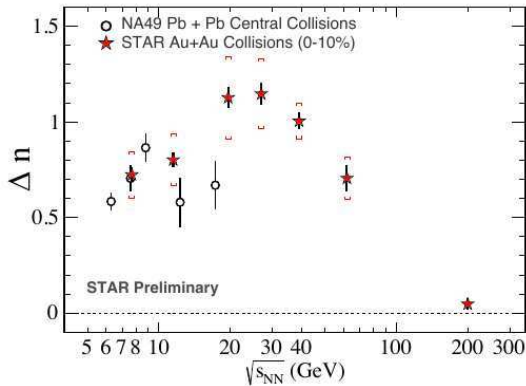
ΔF_2 for mid-rapidity protons at 17 GeV



A deviation of ΔF_2 from zero seems apparent in central Si+Si and mid-central Ar+Sc

Light nuclei production

Experimental results



$$\Delta n \approx \frac{1}{2\sqrt{3}} \frac{N_p \cdot N_t}{N_d^2} - 1$$

Data taken from:

NA49: PRC 94, 044906 (2016)

STAR: PRL 121, 032301 (2018)

NPA 967, 788

Δn shows a non-monotonic behavior on collision energy with a peak $\sqrt{s_{NN}} \approx 20$ GeV

① Critical point search strategies

② Biasing effects

③ Experimental measures

④ Experimental results

⑤ Summary

Summary

4th moment of
net-proton dist.:
 $\approx 7 \text{ GeV}$
(Au+Au)

Proton intermittency:
 $\approx 17 \text{ GeV}$
(Si+Si and Ar+Sc)

Light ion production:
 $\approx 20 \text{ GeV}$
(Au+Au)

Pion interferometry:
 $\approx 47 \text{ GeV}$
(Au+Au)

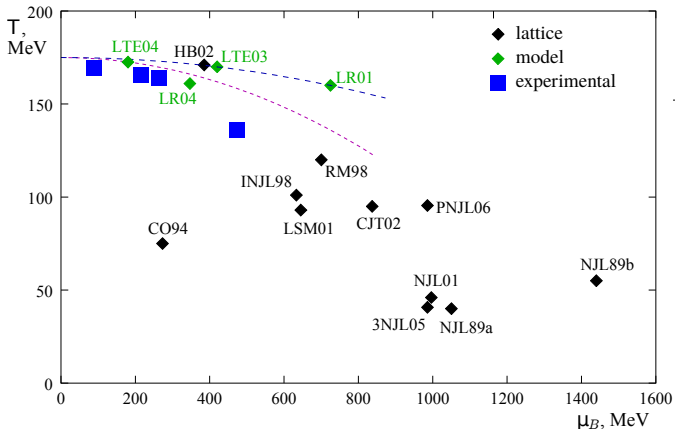
Summary

4th moment of
net-proton dist.:
 ≈ 7 GeV
(Au+Au)

Proton intermittency:
 ≈ 17 GeV
(Si+Si and Ar+Sc)

Light ion production:
 ≈ 20 GeV
(Au+Au)

Pion interferometry:
 ≈ 47 GeV
(Au+Au)



Summary

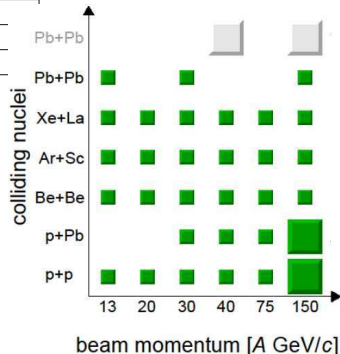
4th moment of
net-proton dist.:
 ≈ 7 GeV
(Au+Au)

Proton intermittency:
 ≈ 17 GeV
(Si+Si and Ar+Sc)

Light ion production:
 ≈ 20 GeV
(Au+Au)

Pion interferometry:
 ≈ 47 GeV
(Au+Au)

\sqrt{s} (GeV)	Statistics(Millions) - BES-I	Statistics(Millions) - BES-II
7.7	~4	~ 100
9.1	-	~ 160
11.5	~12	~ 230
14.5	~20	~ 300
19.6	~36	~ 400
27	~70	~ 500



**New, high-quality
data is coming
soon...**

Thank You!

Additional slides

Limiting volume fluctuations with event selection

Additional slides

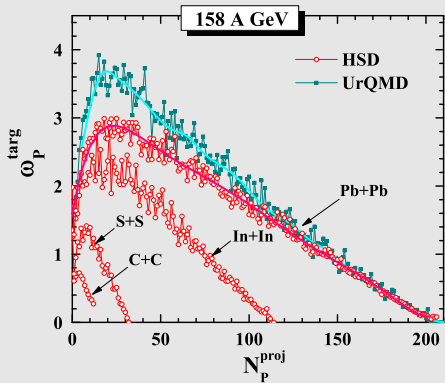
- In collider experiments' measurements of 'target' and 'projectile' spectators is possible under the same experimental conditions, but only free nucleon spectators can be measured. Fragments are lost in the beam pipe.
- In fixed-target experiments, all 'projectile' spectators can be measured but 'target' spectators are absorbed in the target material – can't be measured.

Possible strategy for fixed-target experiment:

- collisions of equal size nuclei
- select events with no projectile spectators

Limiting volume fluctuations with event selection

Additional slides



In symmetrical system, selecting events, in which all nucleons took part in the collisions ($N_P^{\text{proj}} = A$) reduces almost to zero target participant fluctuations (ω_P^{targ}).

Strongly intensive quantities

Additional slides

$$\left. \begin{aligned} \langle A \rangle &= \langle a \rangle \cdot \langle W \rangle \\ \langle B \rangle &= \langle b \rangle \cdot \langle W \rangle \end{aligned} \right\} \Rightarrow \langle A \rangle / \langle B \rangle = \langle a \rangle / \langle b \rangle$$

If one subtract two variances:

$$\begin{aligned} \text{Var}[A] &= \text{Var}[A] \langle W \rangle + \langle a \rangle^2 \text{Var}[W] & | \cdot \langle b \rangle / \langle a \rangle \\ \text{Var}[B] &= \text{Var}[B] \langle W \rangle + \langle b \rangle^2 \text{Var}[W] & | \cdot \langle a \rangle / \langle b \rangle \end{aligned}$$

The dependence on $\text{Var}[W]$ cancels out:

$$\text{Var}[A] \langle b \rangle / \langle a \rangle - \text{Var}[B] \langle a \rangle / \langle b \rangle = \text{Var}[A] \langle W \rangle \langle b \rangle / \langle a \rangle - \text{Var}[B] \langle W \rangle \langle a \rangle / \langle b \rangle$$

And that gives:

$$\omega[A] \langle B \rangle - \omega[B] \langle A \rangle = \langle W \rangle (\omega[a] \langle b \rangle - \omega[b] \langle a \rangle)$$

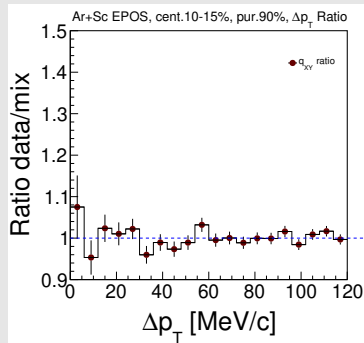
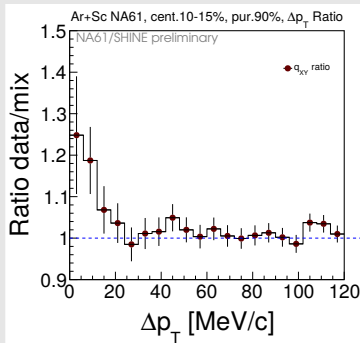
If one used covariance of A and B:

$$\omega[A] \langle B \rangle + \omega[B] \langle A \rangle - 2\text{cov}[A, B] = \langle W \rangle (\omega[a] \langle b \rangle + \omega[b] \langle a \rangle - 2\text{cov}[a, b])$$

Intermittency

Additional slides

Distributions of Δp_T for protons selected for intermittency analysis from Ar+Sc collisions at 150A GeV/c.



Strong correlations in $\Delta p_T \rightarrow 0$ in NA61/SHINE data may indicate intermittent behavior.