Patterns and Partners within the QCD Phase Diagram including Strangeness

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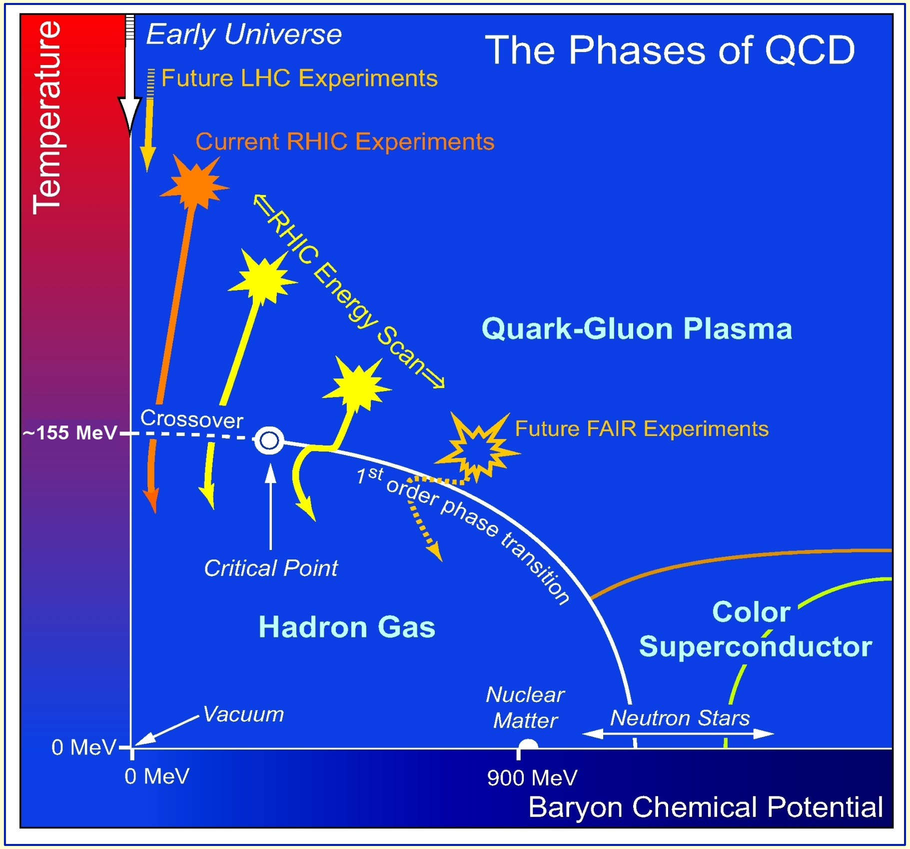
OUTLINE:

- Aspects of the QCD phase diagram
- Chiral and $U(1)_A$ patterns and partners
- Ward Identities
- Effective theory realization

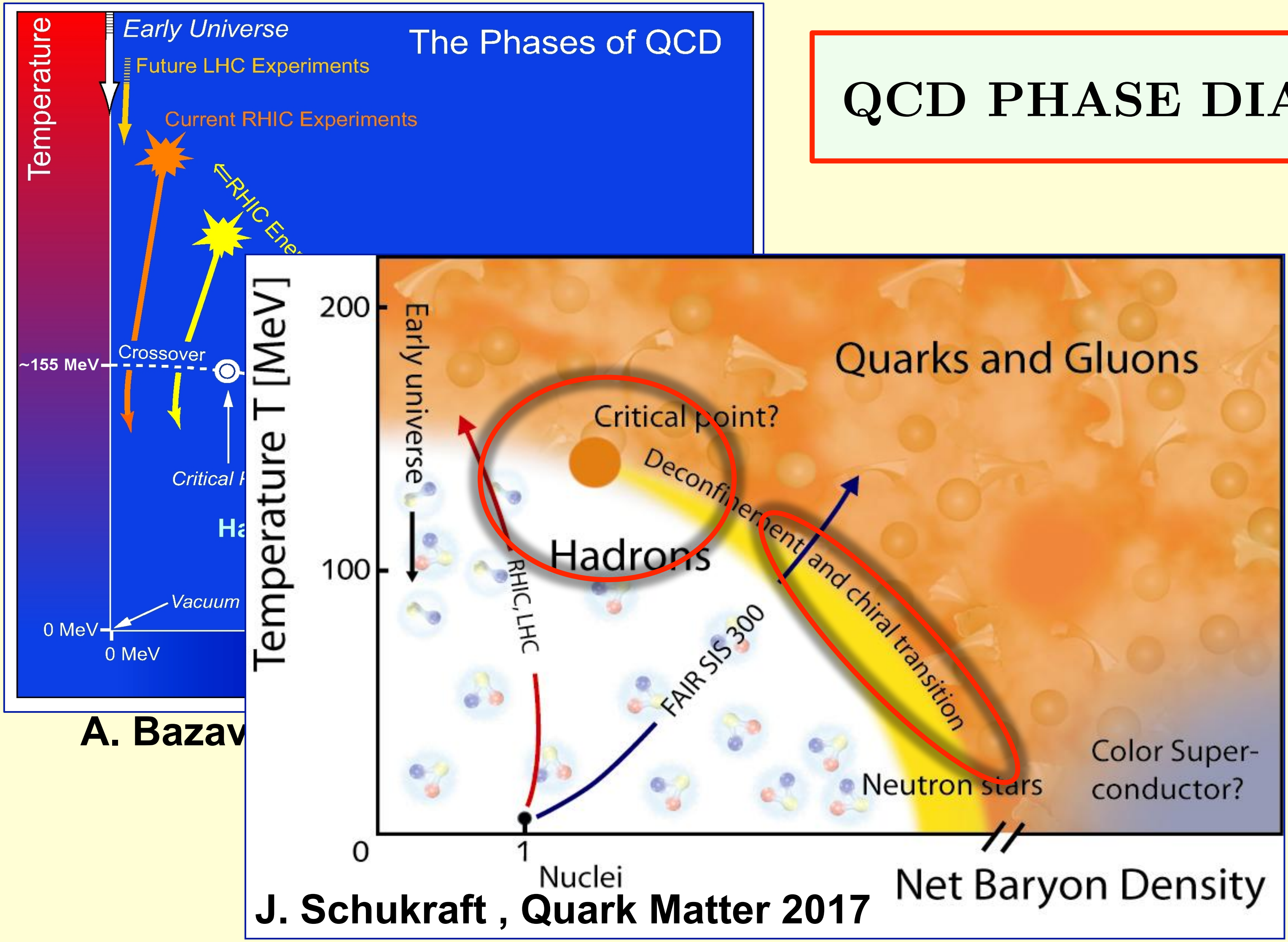
SQM CONFERENCE
BARI 10-15 JUNE 2019

QCD PHASE DIAGRAM



A. Bazavov, Quark Matter 2017

QCD PHASE DIAGRAM



A. Bazav

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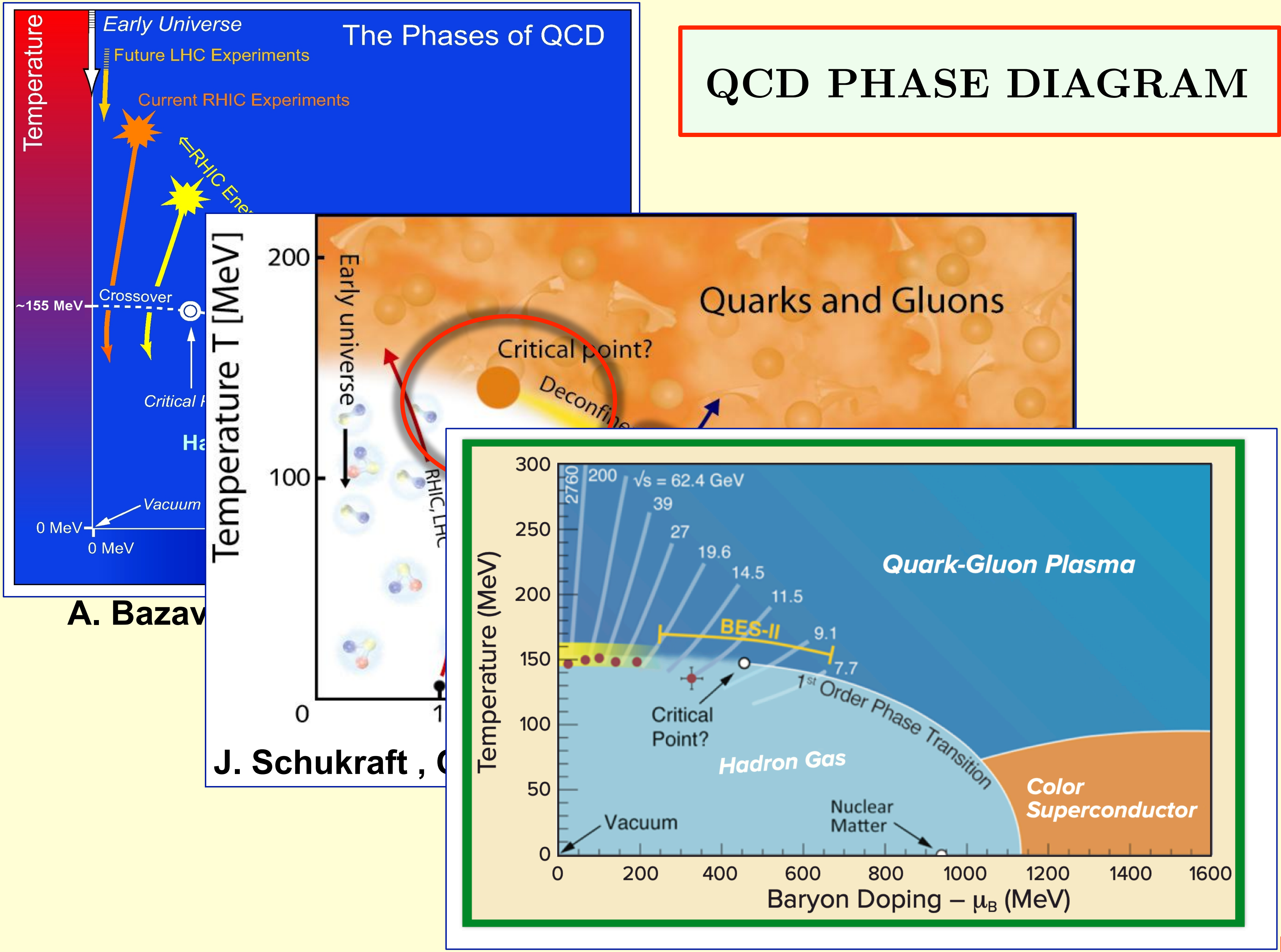

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J. Schukraft , C

S. Pratt, Quark Matter 2017

CROSSOVER transition for $\mu_B = 0$, $N_f = 2 + 1$ and physical masses

Well established in lattice through thermodynamic observables:

$$\Delta_{l,s} = \frac{\langle \bar{q}q \rangle_T - (2m_q/m_s)\,\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_q/m_s)\,\langle \bar{s}s \rangle_0}$$

subtracted quark condensate (inflection point)

$$\chi_S = -\frac{\partial}{\partial m_q}\langle \bar{q}q \rangle_T = \int_x \left[\langle \bar{q}q(x)\bar{q}q(0) \rangle_T - \langle \bar{q}q \rangle_T^2\right]$$

scalar susceptibility (peak)

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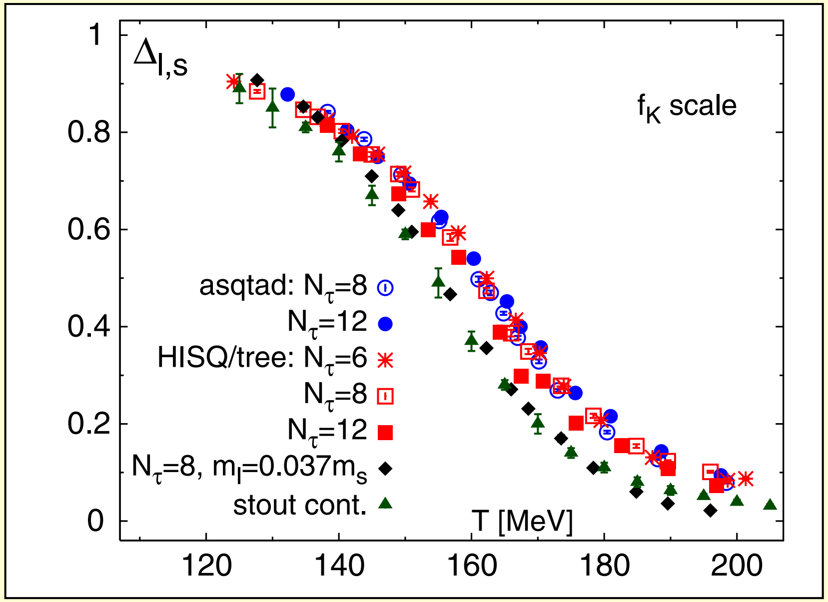
scalar susceptibility (peak)

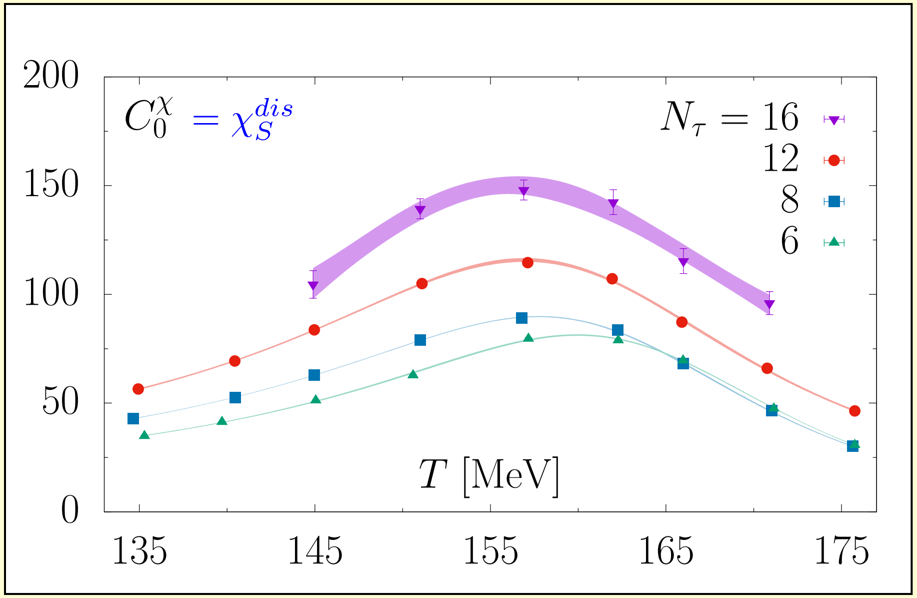
... as well as chiral partners through susceptibilities and screening masses degeneration (see below):

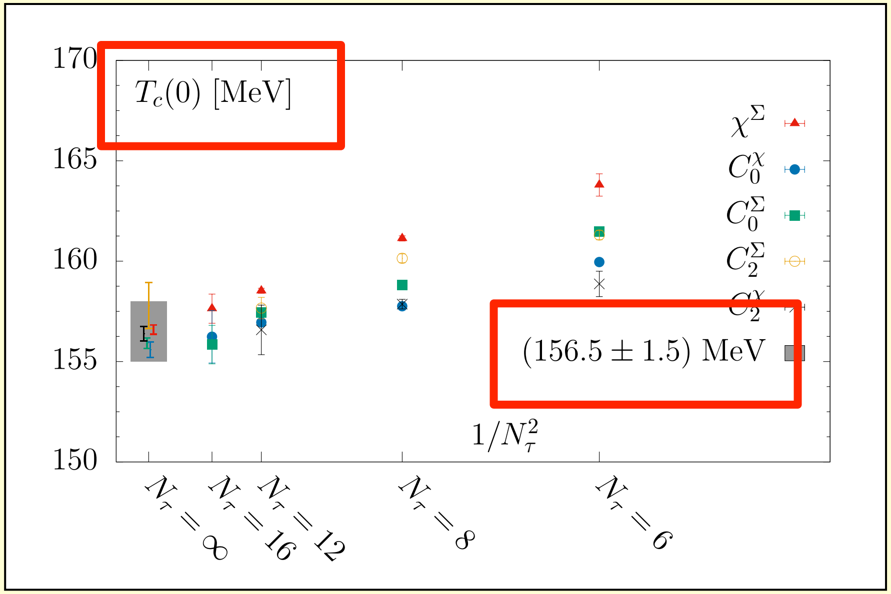
$$\rho/a_1,\ \sigma/\pi,\ \eta/a_0, \ldots$$

CONDENSATE AND SCALAR SUSCEPTIBILITY

A.Bazavov et al (Hot QCD), 2012, 2014, 2018



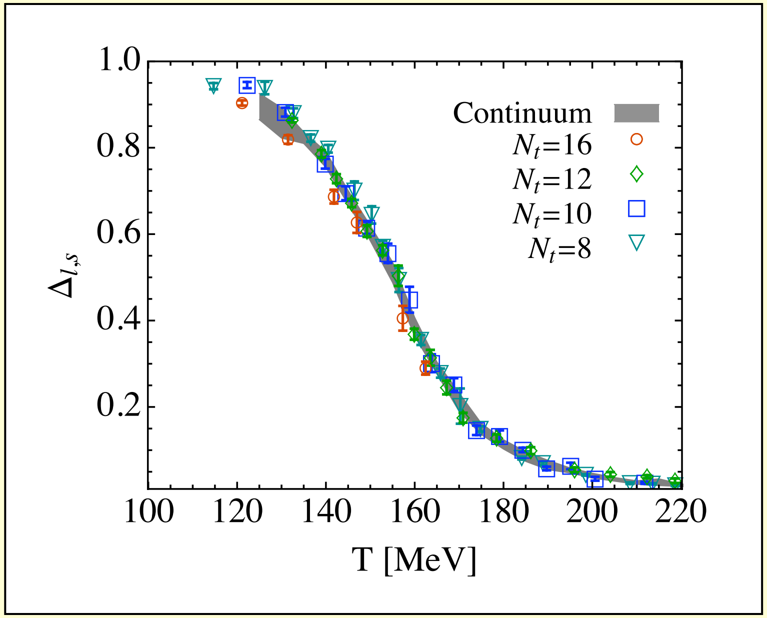


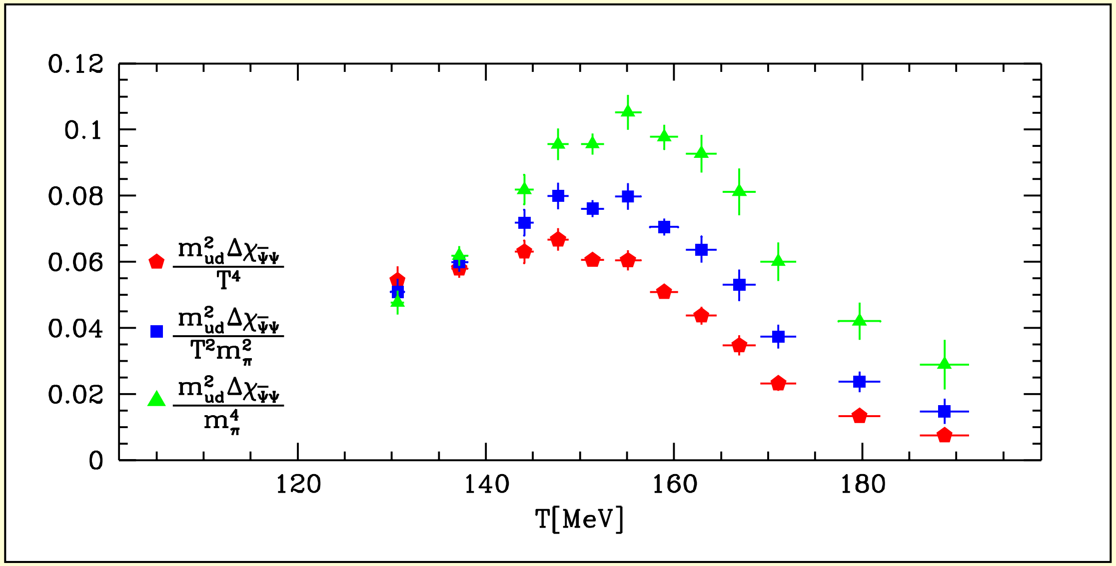


Chiral limit $T_c^0 = 132^{+3}_{-6}$ MeV
with reasonable $O(4)$ scaling
(Ding et al 2019)

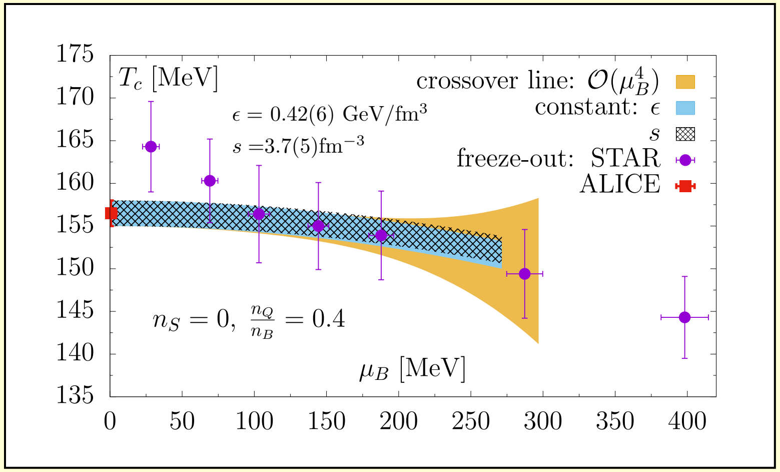
CONDENSATE AND SCALAR SUSCEPTIBILITY

Y.Aoki, S. Borsanyi et al (Budapest-Wuppertal) 2009, 2010





QCD phase diagram explored in HIC → chemical freeze-out close to phase boundary



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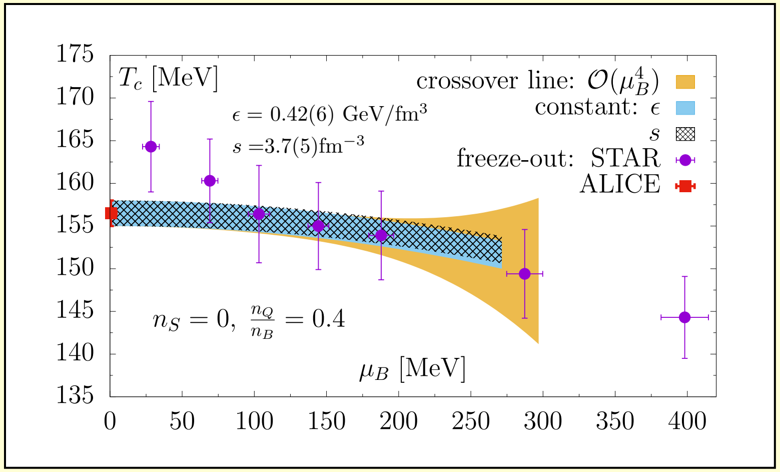

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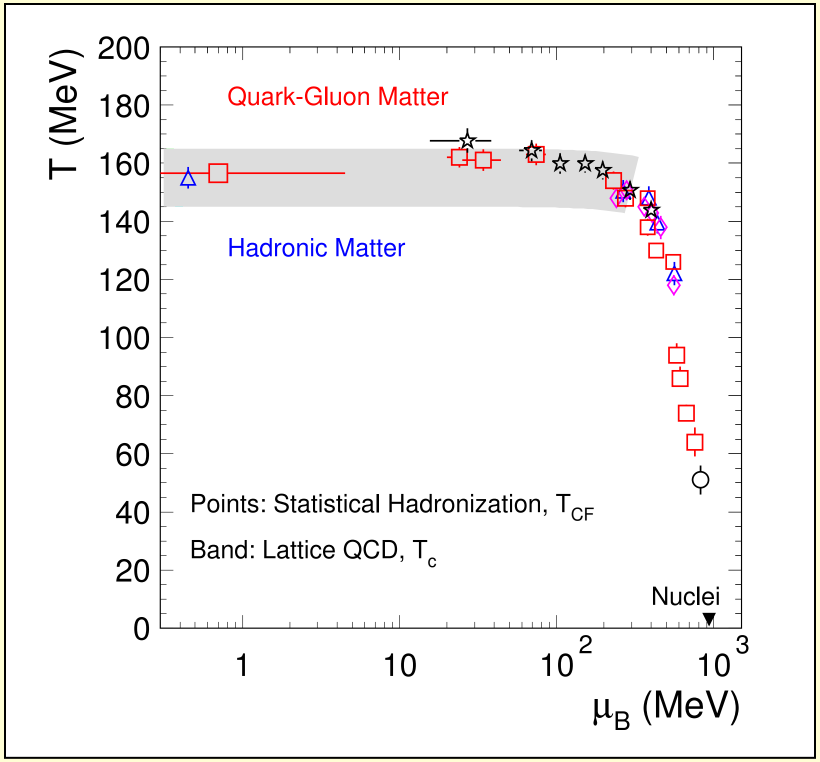
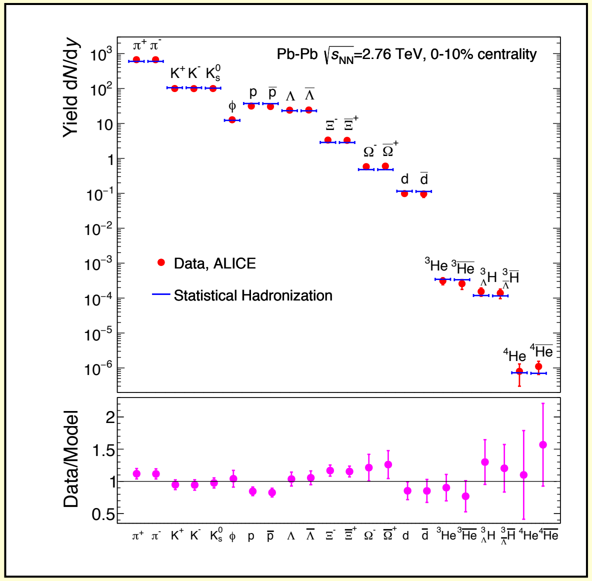


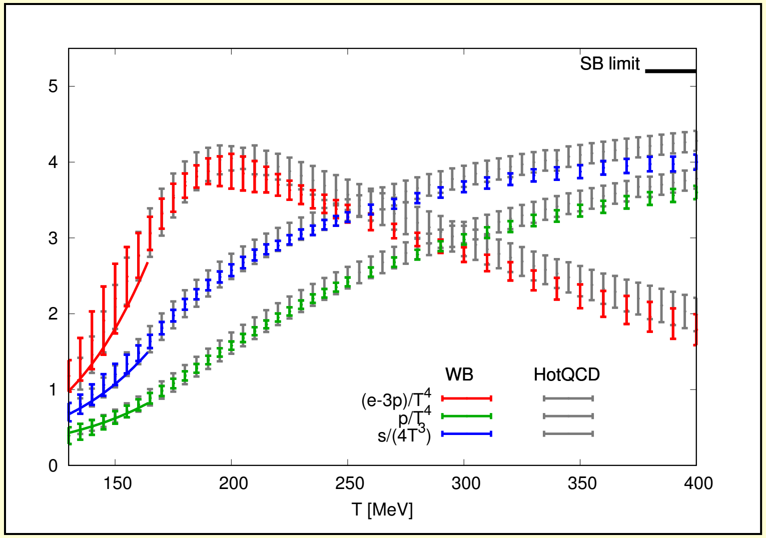
Andronic et al 2018



Chemical FO from Hadron Statistical Model
fit to hadron yields
(central ALICE data)

OTHER HIGHLIGHTS OF QCD TRANSITION

Pressure, entropy, trace anomaly



From C.Ratti 2018 (2014 WB, HotQCD data)

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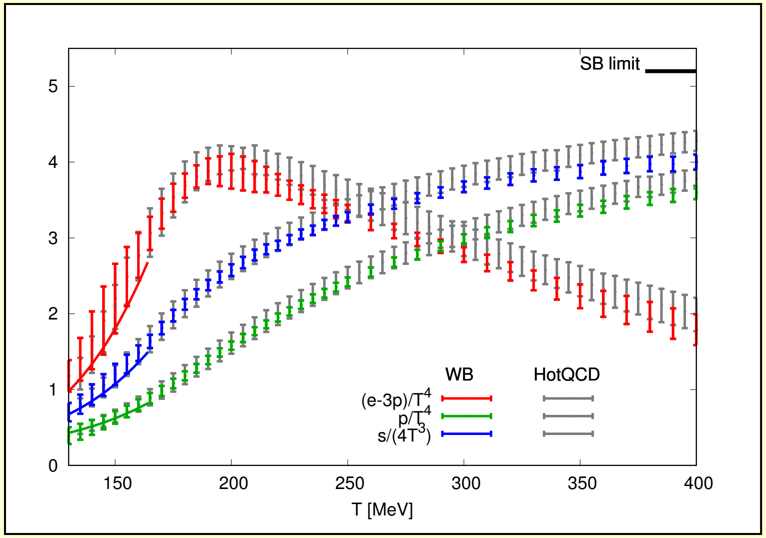

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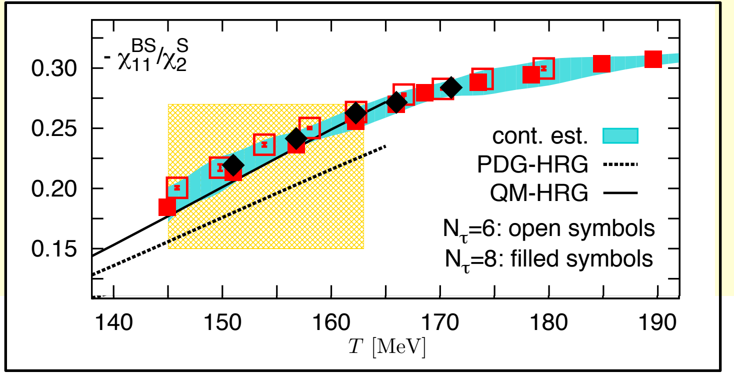
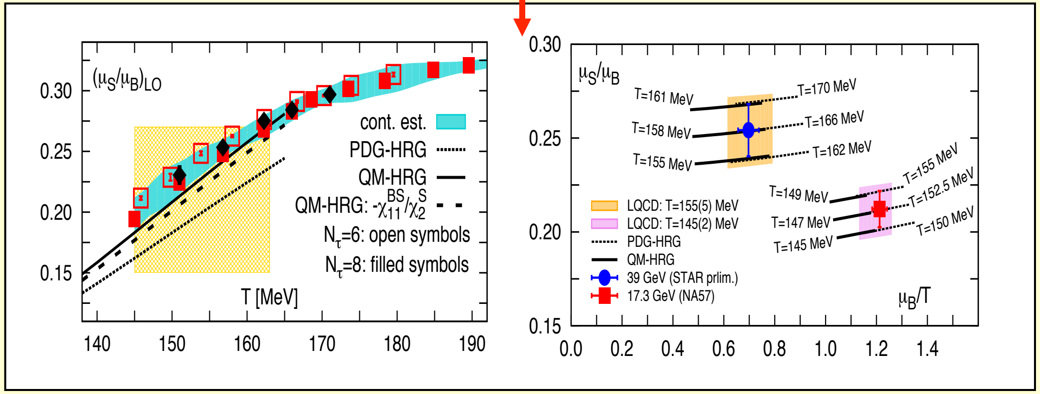
Fluctuations of conserved charges (Q,B,S)

Bazavov et al 2014



related to strangeness freeze-out conditions



$$n_S = 0,\ n_Q/n_B = 0.4$$

CHIRAL PATTERN AND PARTNERS

$\Rightarrow$ **What is the nature of the transition (chiral pattern)?**

CHIRAL PATTERN AND PARTNERS

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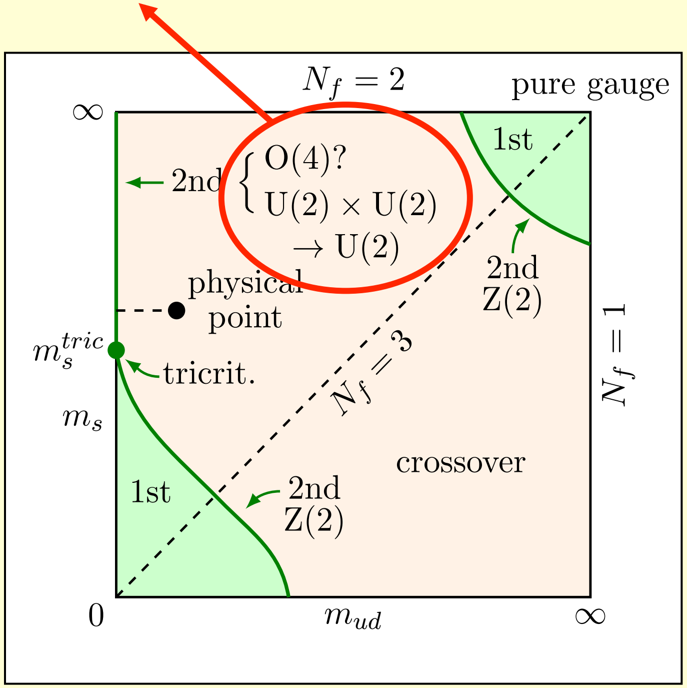
Universality class depends on the strenght of $U(1)_A$ breaking @T_c



B.B. Brandt et al 2019

Gross, Pisarski, Yaffe 1981, Shuryak 1994. Cohen 1996. Lee-Hatsuda 1996

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CHIRAL PATTERN AND PARTNERS

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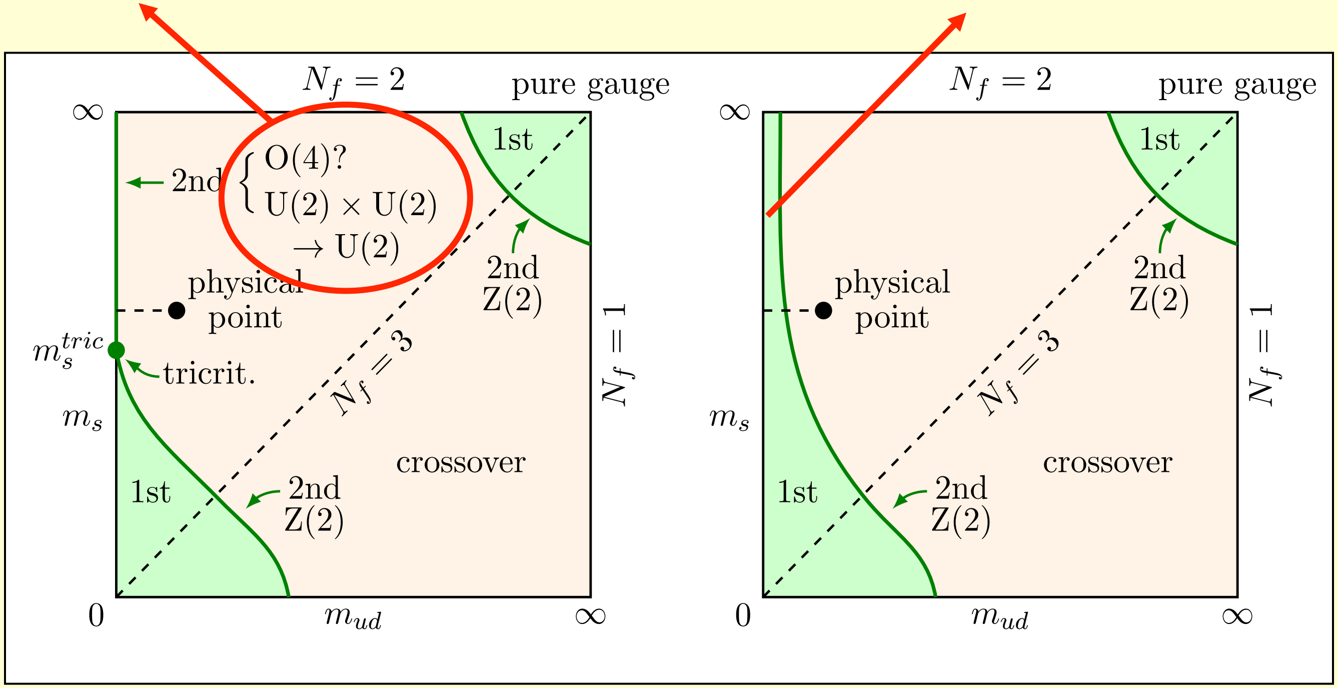
Transition order can even change if $U(1)_A$ is sufficiently restored



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CHIRAL PATTERN AND PARTNERS

$\Rightarrow$ **Is $U(1)_A$ asymptotic restoration effective already at T_c?**

(specially near chiral limit)

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$\Rightarrow$ Is $U(1)_A$ asymptotic restoration effective already at T_c?

(specially near chiral limit)

- One of the (few) issues still under debate

- $U(1)_A$ @T_c affects also critical point at $\mu_B \neq 0$ **Mitter, Schaefer 2014**

- $U(1)_A$ restoration shows in $M_{\eta'}$ reduction in effective theories, lattice and experiment (increase of η' production in dileptons&photons)

Ishii et al 2017. Gu et al 2018. AGN, J.R.Elvira 2018. Kotov, Lombardo et al 2019

Kapusta, Kharzeev, McLerran 1996. Csorgo, Vertesi, Sziklai 2010

$O(4)$ and $U(1)_A$ partners for scalar/pseudoscalar nonets: $I = 0, 1$

$$\begin{tikzcd}
\pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l \arrow[r, leftrightarrow, "SU_A(2)"] \arrow[d, leftrightarrow, "U_A(1)"] & \sigma_l = \bar{\psi}_l\psi_l \arrow[d, leftrightarrow, "U_A(1)"] \\
\delta^a = \bar{\psi}_l\tau^a\psi_l \arrow[r, leftrightarrow, "SU_A(2)"] & \eta_l = i\bar{\psi}_l\gamma_5\psi_l
\end{tikzcd}$$

$$\pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi, \quad \delta^a = \bar{\psi}_l\tau^a\psi_l \sim a_0(980)$$

$$\sigma_l = \bar{\psi}_l\psi_l,\ \sigma_s = \bar{s}s \to f_0(500), f_0(980)\ \textbf{(mixed)}$$

$$\eta_l = i\bar{\psi}_l\gamma_5\psi_l,\ \eta_s = i\bar{s}\gamma_5 s \to \eta, \eta'\ \textbf{(mixed)}$$

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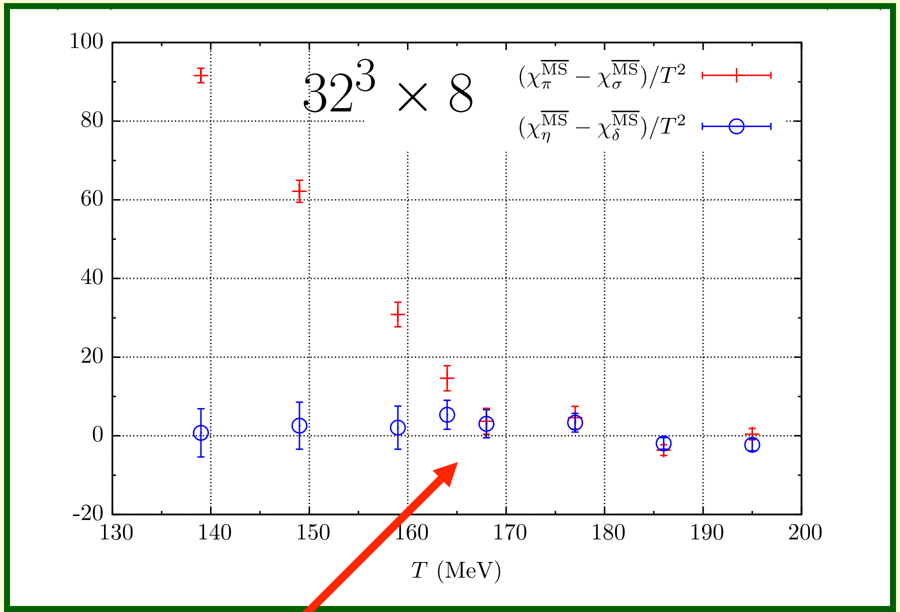
Buchoff et al (LLNL/RBC coll) PRD89 (2014)

$N_f = 2 + 1$ susceptibilities



$O(4)$ **OK** (with large uncertainties in $\chi_\eta - \chi_\delta$)

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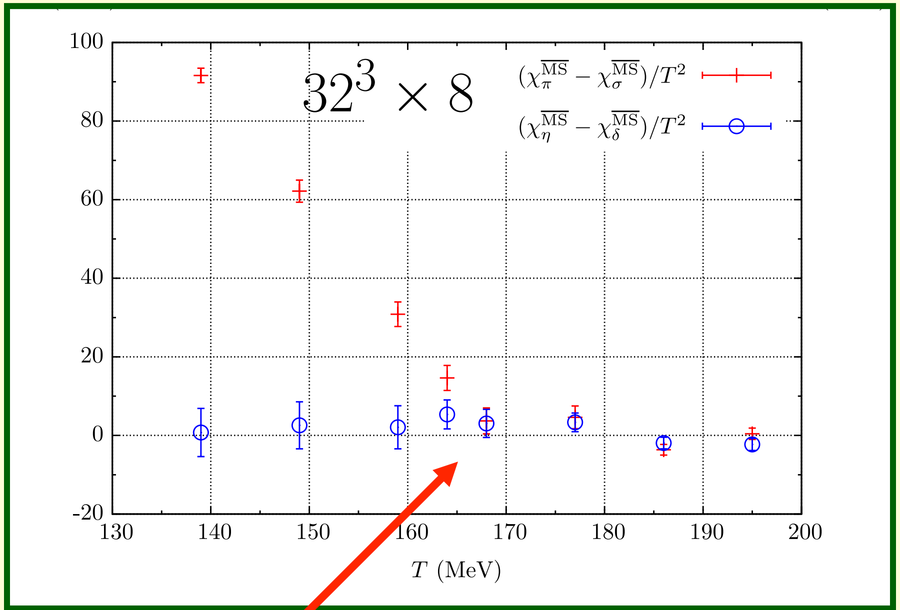
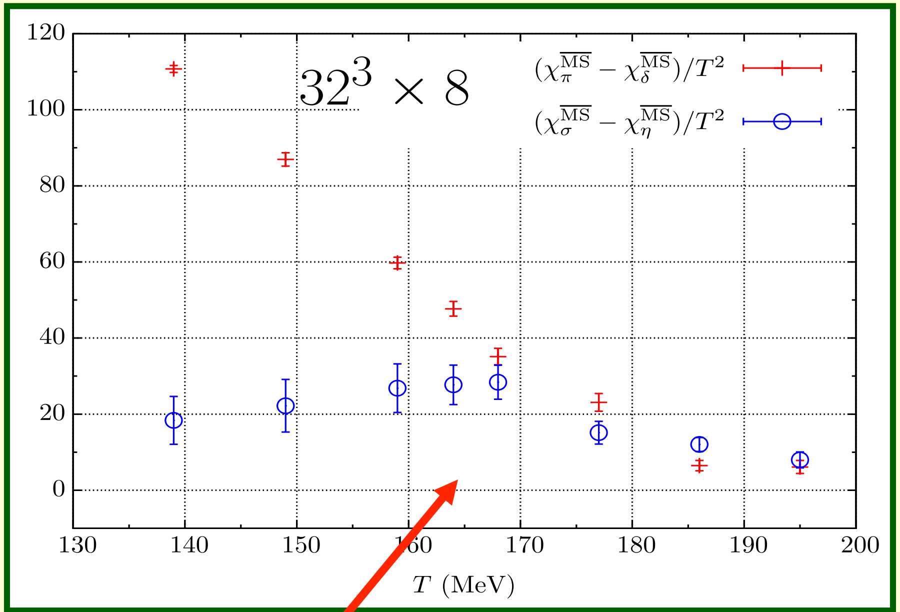
Buchoff et al (LLNL/RBC coll) PRD89 (2014)

$N_f = 2 + 1$ susceptibilities (strangeness at work)





$O(4)$ **OK** (with large uncertainties in $\chi_\eta - \chi_\delta$)

significant $U(1)_A$ breaking @ T_c for physical masses

Aoki et al, 2012, Cossu et al, 2013 ($N_f = 2$, $\hat{m} \to 0$)

Brandt et al 2016, ($N_f = 2$, $\hat{m} \neq 0$)
2019 ($N_f = 2$, incl. $\hat{m} \to 0$ screening masses)

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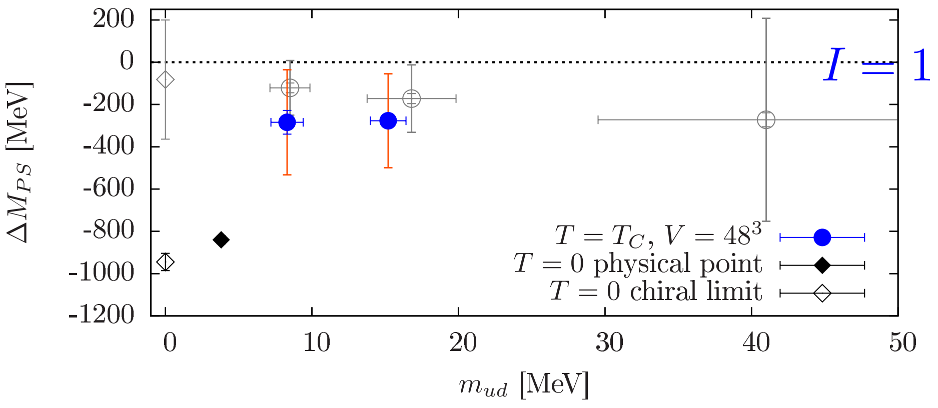

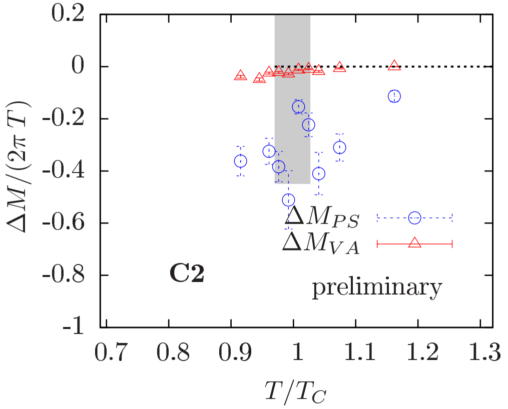

Aoki et al, 2012, Cossu et al, 2013 $(N_f = 2,\ \hat{m} \to 0)$

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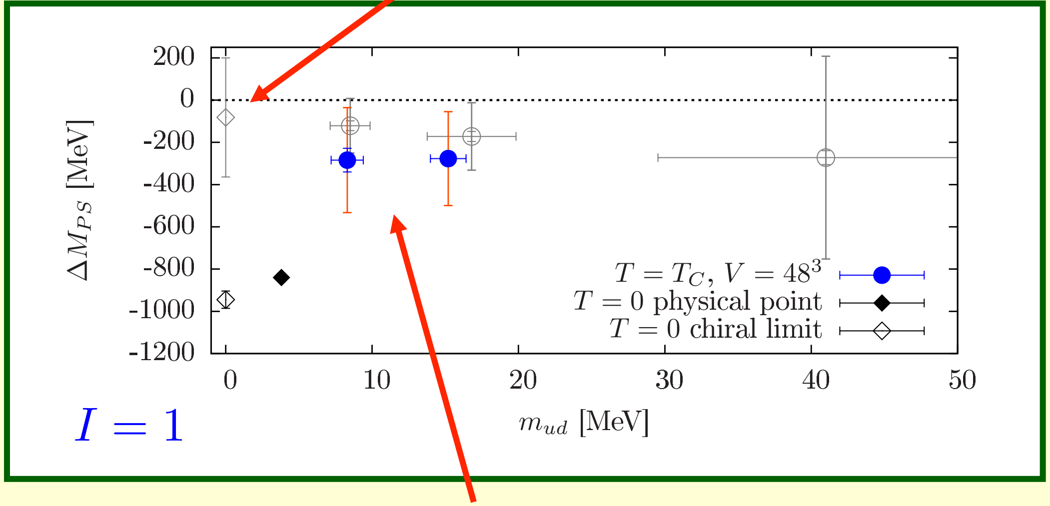
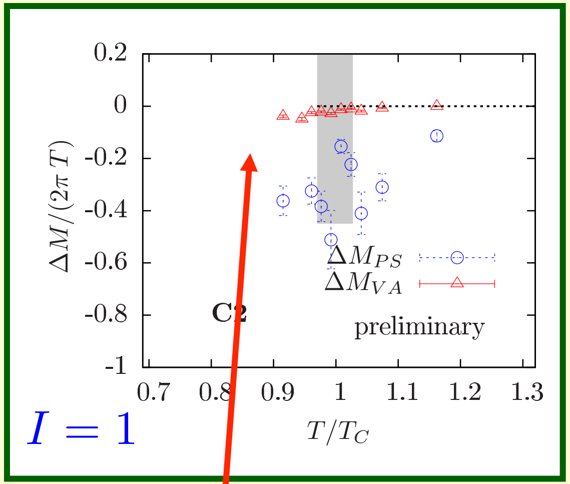
Compatible with $U(1)_A$ restoration @T_c in chiral limit





Small $U(1)_A$ breaking in phys.limit, increasing with larger volumes

ρ/a_1 $O(4)$ more efficient

WARD IDENTITIES obtained from the QCD generating functional may shed light on chiral patterns and partners

AGN, J.Ruiz de Elvira, 2016, 2018

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- π **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) = -\hat{m}\chi_P^\pi(T)$
- K **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) + 2\langle \bar{s}s \rangle (T) = -(\hat{m} + m_s)\,\chi_P^K(T)$
- η, A **SECTOR** $\rightarrow$ η/η' **mixing &** $U_A(1)$ **anomaly enter:**

$$\begin{aligned}
\chi_P^{\eta_l}(T) &= -\frac{\langle \bar{q}q \rangle_l (T)}{\hat{m}} - \frac{4}{\hat{m}^2}\chi_{top}(T)\\
\chi_P^{\eta_s}(T) &= -\frac{\langle \bar{s}s \rangle (T)}{m_s} - \frac{1}{m_s^2}\chi_{top}(T)\\
\chi_P^{ls} &= -\frac{\hat{m}}{2m_s}\left[\chi_P^\pi(T) - \chi_P^{\eta_l}(T)\right] = -\frac{2}{\hat{m}m_s}\chi_{top}(T)
\end{aligned}$$

$\chi_P^{ab} \equiv \int_T dx \left\langle P^a(x)P^b(0)\right\rangle, \ \langle \bar{q}q \rangle_l = \left\langle \bar{u}u + \bar{d}d \right\rangle, \ \hat{m} = m_u = m_d$ $\qquad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x)A(0)\rangle$ **TOPOLOGICAL SUSCEPTIBILITY**

Crossed ls correlator nonzero due to η/η' mixing. From WI:

$$\boxed{\chi_P^{ls}(T) = -2\frac{\hat{m}}{m_s}\chi_{5,disc}(T) = -\frac{2}{\hat{m}m_s}\chi_{top}(T)}$$

where $\chi_{5,disc} = \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right)$ **measures** $O(4) \times U(1)_A$ **restoration**

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$\Rightarrow$ $SU(2)_A$ **transforms (**η_s **invariant)** $P_{ls} \xrightarrow{(*)} \langle \delta\eta_s \rangle = 0$ **by parity**

$$\boxed{\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta} \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \chi_{5,disc} \overset{O(4)}{\sim} 0, \ \chi_{top} \overset{O(4)}{\sim} 0}$$

$(*)\ \eta_l \to i\bar{\psi}_l\gamma_5 e^{i\frac{\pi}{2}\gamma_5\tau^b}\psi_l = -\delta^b$

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where $\chi_{5,disc} = \frac{1}{4}\left(\chi_P^\pi - \chi_P^{\eta_l}\right)$ measures $O(4)\times U(1)_A$ restoration

Also Azcoiti 2016 with similar WI

$$\boxed{\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^\delta \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \ \chi_{5,disc} \overset{O(4)}{\sim} 0,\ \chi_{top} \overset{O(4)}{\sim} 0}$$

$\Longrightarrow O(4)\times U(1)_A$ pattern at *exact* chiral restoration (for above partners)

(hence consistent with Cossu, Aoki, Brandt et al $N_f = 2$)

efficient $\eta_l - \delta$ degeneration $@T_c$

$I = 1/2$ SECTOR $K - K_0^*(700)(\kappa)$ DEGENERATION

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2 - \hat{m}^2}\underbrace{\left[\langle \bar{q}q\rangle_l(T) - 2\frac{\hat{m}}{m_s}\langle \bar{s}s\rangle(T)\right]}$$

$\Rightarrow$ **dictated by subtracted condensate** $\Delta_{l,s}$

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$\Rightarrow$ **dictated by subtracted condensate $\Delta_{l,s}$**

$\Rightarrow$ $\chi_S^\kappa \overset{O(4)}{\sim} \chi_P^K$ **degeneration for exact chiral rest.** $(\hat{m}, \langle\bar{q}q\rangle_l \to 0^+)$

$\Rightarrow$ K/κ **also** $U(1)_A$ **degenerated, hence in physical case WI relates** $O(4)\times U(1)_A$ **partners with chiral** $\Delta_{l,s}$ **(well determined in lattice)**

WI AND LATTICE SCREENING MASSES

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow \textbf{measured in lattice}$$

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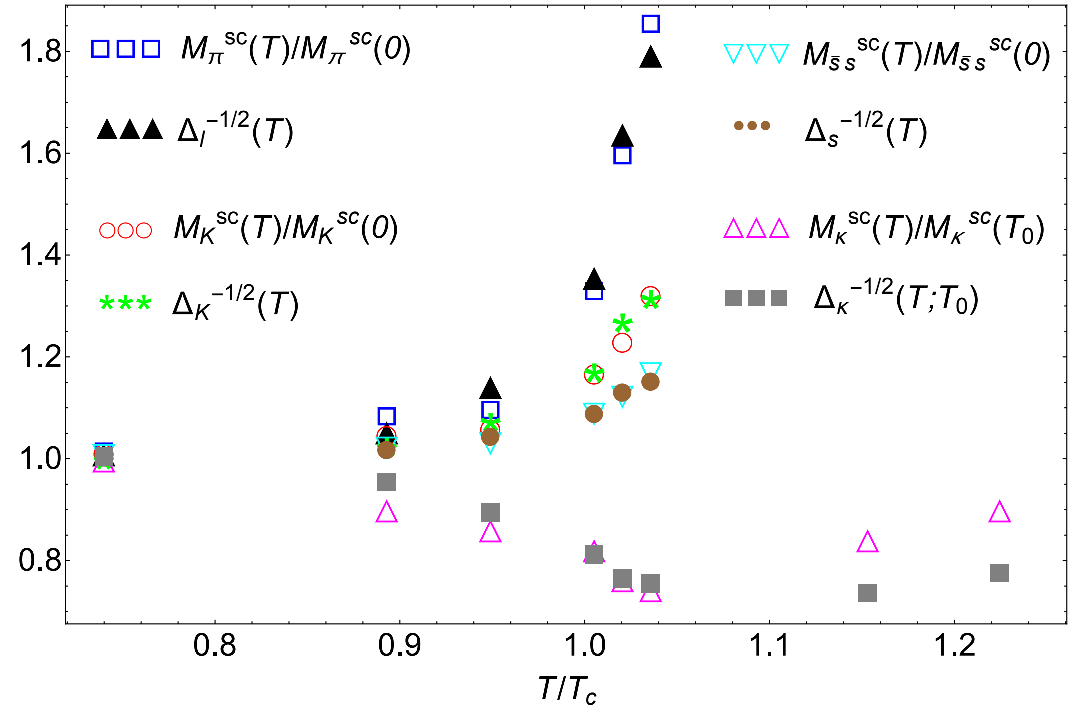
$$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow \textbf{measured in lattice}$$

$$\begin{aligned}
\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} &\sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)}{\langle\bar{q}q\rangle_l(T)}\right]^{1/2}\\
\frac{M_K^{sc}(T)}{M_K^{sc}(0)} &\sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)+2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)+2\langle\bar{s}s\rangle(T)}\right]^{1/2}\\
\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} &\sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle\bar{s}s\rangle(0)}{\langle\bar{s}s\rangle(T)}\right]^{1/2}\\
\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} &\sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)-2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)-2\langle\bar{s}s\rangle(T)}\right]^{1/2}
\end{aligned}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

WI AND LATTICE SCREENING MASSES

Same lattice setup for masses (Cheng et al EPJC'11) and condensates (PRD'08)



- $<$ 5% deviations below T_c from predicted WI scaling
- Δ_i subtracted condensates with two fit parameters
- Rapid T_c increase in $M_\pi^{sc} \sim \langle \bar{q}q \rangle_l^{-1/2}$.

  Softer $M_K^{sc} \sim (\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle)^{-1/2}$. Even softer $M_{\bar{s}s}^{sc} \sim \langle \bar{s}s \rangle^{-1/2}$
- κ minimum from condensate diff. (last two points not fitted)

EFFECTIVE THEORY REALIZATION

- **Effective Hadron Theories** needed for systematic analysis below the transition

- **U(3) ChPT** model-independent framework for light mesons (π, K, η, η') within $\delta \sim 1/N_c \sim m_q \sim T^2$ counting. (1)

- Light meson scattering dominant **interactions** in the thermal bath. **Unitarized scattering** generates (thermal) resonances (2)

- **HRG** approach includes heavier states and describes very well most observables for $T \lesssim T_c$ (3)

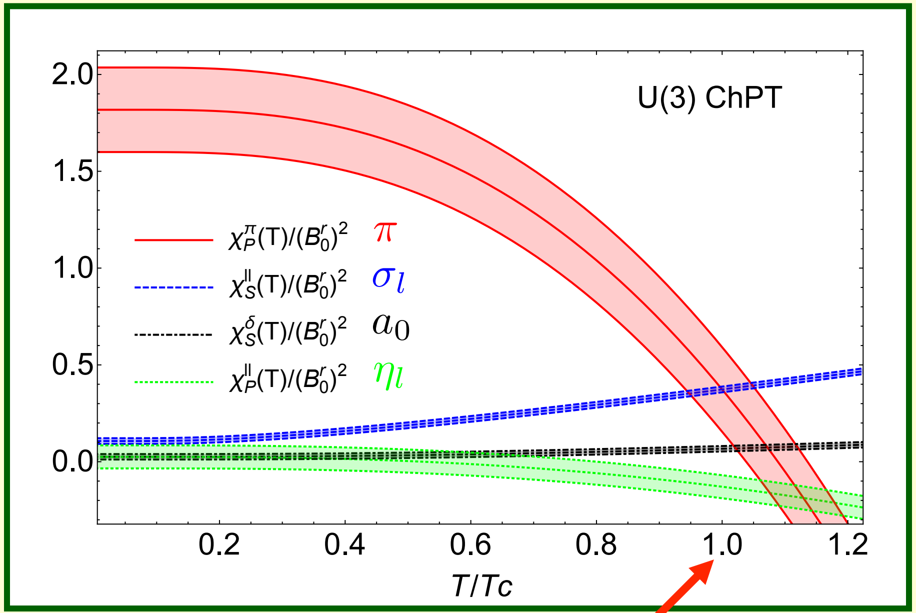
- **Notable exceptions** where (U)ChPT OK near T_c: χ_S, χ_{top}

(1) Gasser, Leutwyler, Gerber, Kaiser, Herrera-Siklody et al, AGN, Ruiz de Elvira

(2) Cabrera, Dobado, Fernández-Fraile, AGN, Llanes-Estrada, Peláez, Ruiz de Elvira, Torres-Andrés

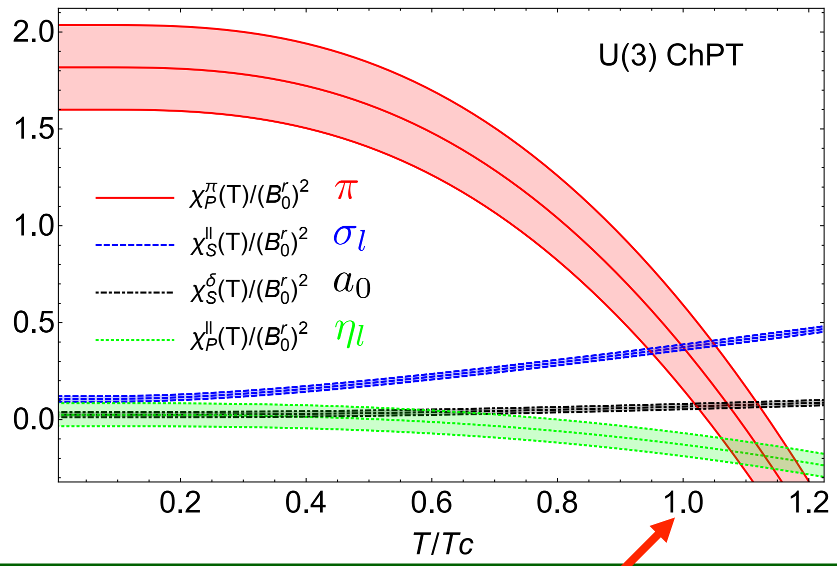
(3) Karsch, Tawfik, Redlich, Tawfik-Toublan, Huovinen, Petreczcky, Jankowski, Blaschke, Spalinski, ...

Partners in $U(3)$ ChPT



Differences within ChPT uncertainty in massive case.
(degeneration $T_{U(1)_A} \sim 1.1 T_{chiral}$)

Partners in $U(3)$ ChPT



Differences within ChPT uncertainty in massive case.
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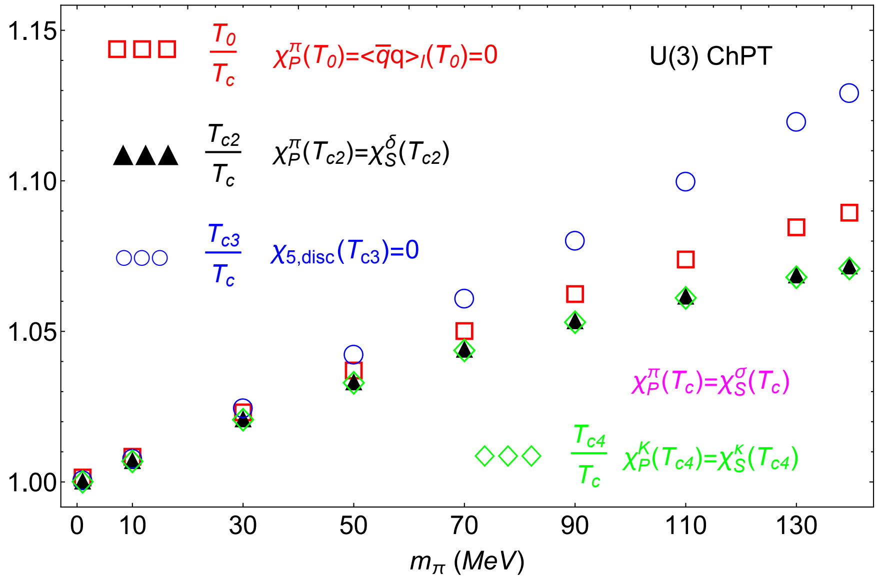


$\rightarrow O(4) \times U_A(1)$ **in chiral limit**

with $\boxed{\dfrac{\chi_{5,disc}(T)}{\chi_{5,disc}(0)} \sim \dfrac{\langle\bar{q}q\rangle_l(T)}{\langle\bar{q}q\rangle_l(0)}}$ **(holds reasonably also in $N_f = 2+1$ lattice)**

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018

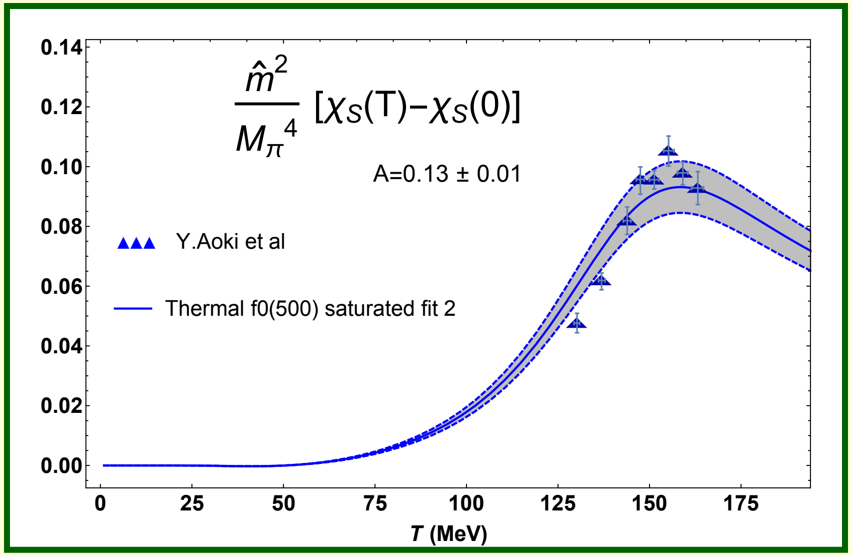
⇒ χ_S **saturated by lightest scalar pole $f_0(500)$ (from unitarized thermal $I = J = 0$ $\pi\pi$ scattering) :**

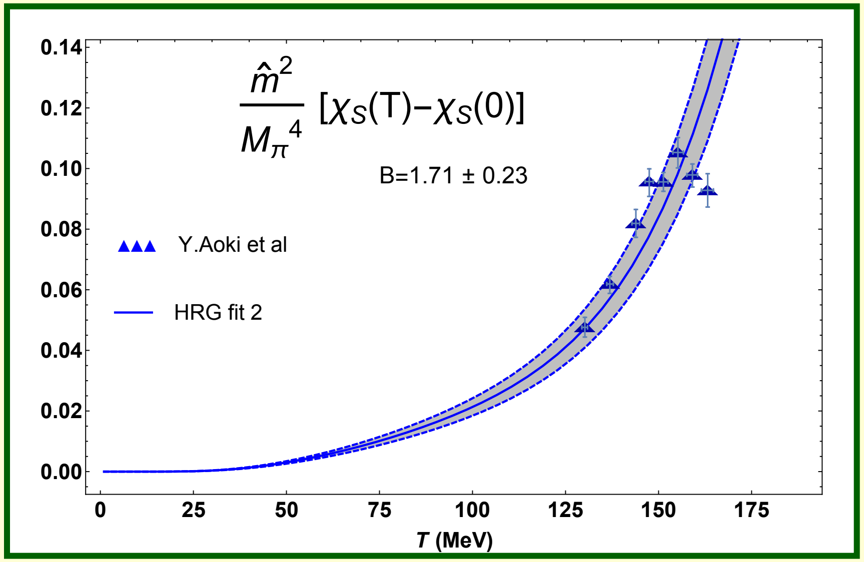
$$\boxed{\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}}$$

$$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}$$

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018





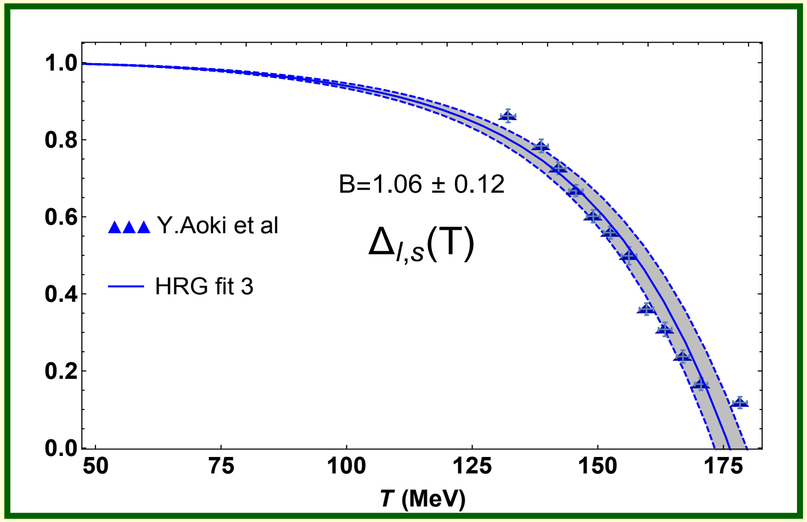
HRG Jankowski et al 2013

free energy density normalization B fitted.

$$\chi_S^U(T) = A\frac{m_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)}$$

$(A_{ChPT} \simeq 0.14)$

Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155



- **Thermal f_0 approach better around T_c**
- **HRG fits of $\Delta_{l,s}$ and χ_S at conflict**

Topological Susceptibility in $U(3)$ ChPT

AGN, J.R.Elvira, A.Vioque, 2019 (preliminary)

$$\epsilon_{vac}(\theta) = \epsilon_{vac}(0) + \frac{1}{2}\chi_{top}\theta^2 + \frac{1}{24}c_4\theta^4 + \dots$$

$\sim$ **Axion mass** $\sim$ **Axion coupling**

M_0 anomalous part of $M_{\eta'}$ $(m_{u,d,s} = 0)$

$(-\Sigma)$ LO quark condensate

$$\chi_{top}^{U(3),LO} = \Sigma \frac{M_0^2 \bar{m}}{M_0^2 + 6B_0\bar{m}}$$

$$\bar{m} = \left[\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right]^{-1}$$

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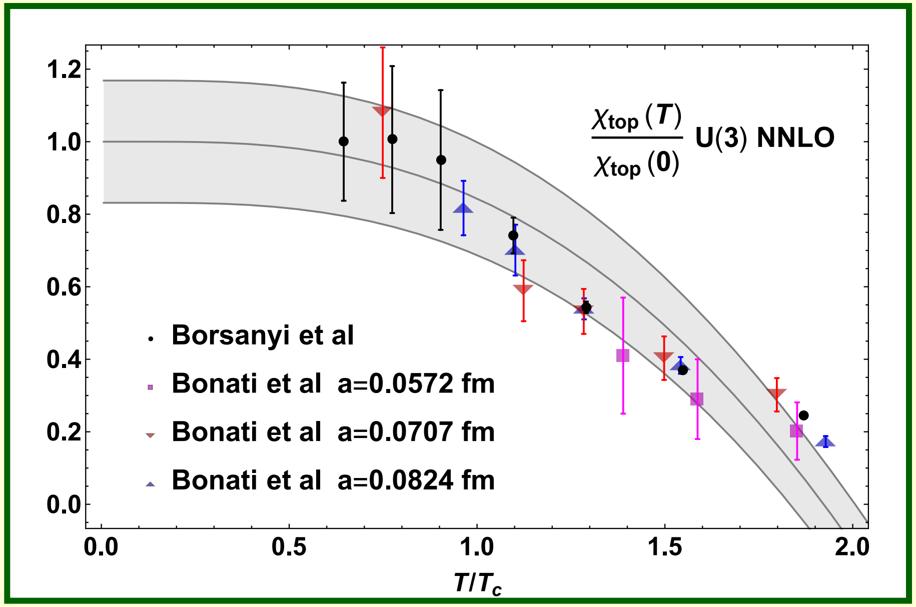
$$\bar{m} = \left[\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right]^{-1}$$

- **Well described by ChPT: vanishes** $m_q \to 0$
  $SU(2)$ **dominates**
- $SU(3)$ **for** $M_0 \to \infty$
- **Quenched** $m_q \to \infty$: $\chi_{top}^{LO} = F^2 M_0^2/6$
  (Witten-Veneziano 1979) → **mesons crucial**
- T**-dependence dominated by** $\langle \bar{q}q \rangle_l^{ChPT}$
  → **2nd term in WI** $\chi_{top} = -\frac{1}{4}\left[m_{ud}\langle \bar{q}q\rangle_l + m_{ud}^2 \chi^{\eta_l}\right]$
  relevant near T_c

$\left[\chi_{top}^{latt}\right]^{1/4} = 73(9)$ (Bonati et al 2016)

$\chi_{top}^{1/4}$ [MeV]	$U(3)$	$SU(2)$	$SU(3)$
LO	74(3)	75(3)	75(3)
NLO	74(3)	78(3)	83(2)
NNLO	81(2)		

$(m_u = m_d)$



Leutwyler,Smilga 1992: $SU(3)$ **LO**
Mao et al 2009; Bernard et al: 2012: $SU(3)$ **NLO**
Grilli et al 2016: $T \neq 0$ $SU(2)$ **NLO**

CONCLUSIONS

$\star$ **Understanding of QCD phase diagram has clearly improved. Open problems include phases and properties of baryon-rich region, nature (pattern) of transition, ...**

$\star$ **WI help** $\Rightarrow O(4)\times U(1)_A$ **for exact chiral restoration of S/P nonet. OK with** $N_f=2$ **lattice and ChPT. Also explain scr.masses**

$\star$ **In physical** $N_f=2+1$ **case, stronger** $U(1)_A @ T_c$

$\Rightarrow$ **strangeness matters! (other effects need to be understood)**

$\star$ **Strange** K/κ **interesting channel** $\rightarrow O(4)\times U(1)_A$ **degen.** $\sim \Delta_{l,s}$

$\star$ **Eff.Theo: HRG mostly OK below** T_c**. Saturated** χ_S **with thermal** $f_0(500)$ **OK with lattice.** χ_{top} **well described by ChPT.**

BACKUP SLIDES

Ward Identities

Formally from QCD through A/V transformations:

$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}_P(y)\bar{\psi}(x)\left\{\frac{\lambda^a}{2},\mathcal{M}\right\}\gamma_5\psi(x)\right\rangle + i\frac{\delta_{a0}}{\sqrt{6}}\langle \mathcal{O}_P(y)A(x)\rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y)\bar{\psi}(x)\left[\frac{\lambda^a}{2},\mathcal{M}\right]\psi(x)\right\rangle$$

$$\lambda^0 = \sqrt{2/3}\,\mathbb{1}, \quad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Ward Identities

Formally from QCD through A/V transformations:

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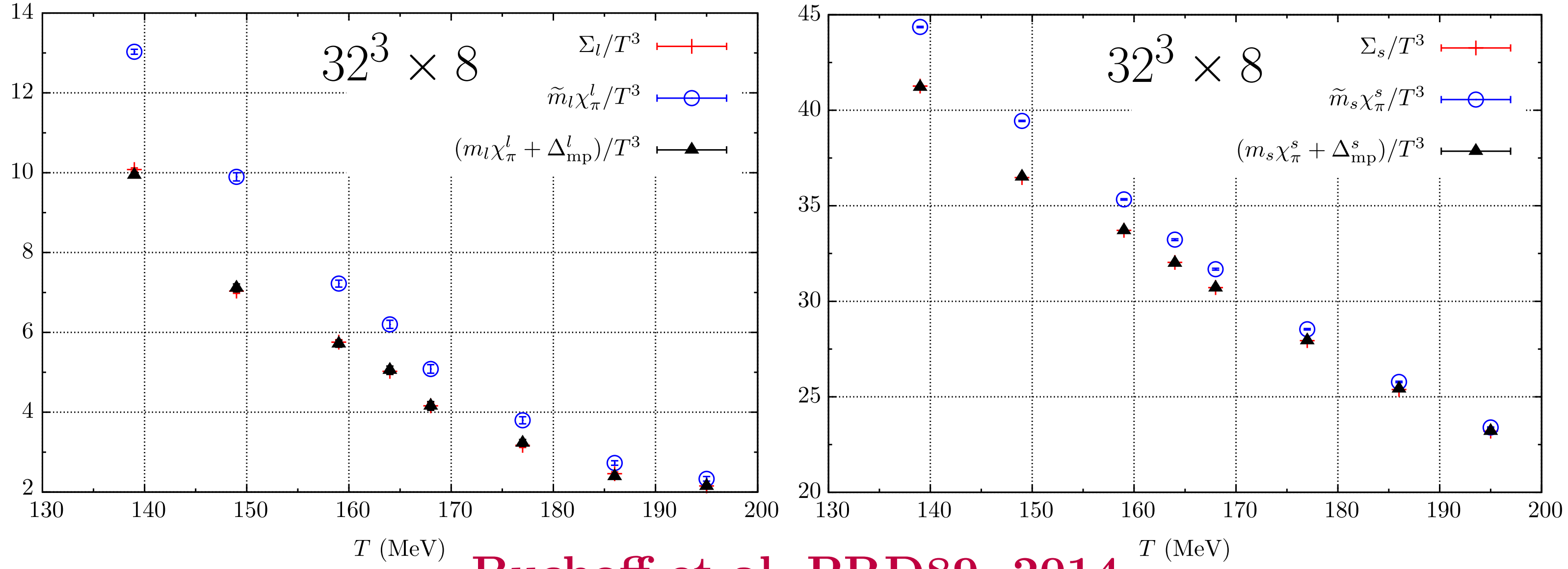
$$\lambda^0 = \sqrt{2/3}\,\mathbb{1}, \quad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$\mathcal{O}_P^b = i\bar{\psi}\gamma_5\lambda^b\psi \equiv P^b \to$ **1p vs 2p fns** $\to \langle \bar{q}q \rangle$ **vs** χ_P

$\mathcal{O}_P^{bc} = P^b S^c \to$ **2p vs 3p** $\to$ **ch.partners vs meson vertices**
(e.g. $\chi^\sigma - \chi^\pi \sim \sigma\pi\pi, \ldots$**)**

$\mathcal{O}_S^b = \bar{\psi}\lambda^b\psi \equiv S^b \to \langle \bar{q}q \rangle$ **vs** χ_S **for** κ **sector** $b = 4, \ldots, 7$

Check of WI in lattice



Buchoff et al, PRD89, 2014

- Both π and $\bar{s}s$ channel need compensating lattice current to reduce finite-size effects
- Small deviations in $\bar{s}s$ channel compatible with anomaly suppression
- No results for K channel (so far) which would test $\langle\bar{q}q\rangle_l + 2\langle\bar{s}s\rangle$ combination

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_l(y) \pi(x) \pi(0) \rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \delta(y) \pi(x) \eta_l(0) \rangle$$

$$P_{ls}(y) = \frac{1}{3} \hat{m} \int_T dx \, \langle \mathcal{T} \eta_s(y) \pi(x) \delta(0) \rangle$$

$$S_{ls}(y) = -\frac{1}{3} \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_s(y) \pi(x) \pi(0) \rangle$$

$$d^{abc} \left[P_{KK}(y) - S_{\kappa\kappa}(y) \right] = \hat{m} \int_T dx \, \left\langle \mathcal{T} K^b(y) \kappa^c(x) \pi^a(0) \right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$\sigma\pi\pi$ **vertex**

$\rightarrow$ $\pi\pi$ **scattering** $I = J = 0$

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \sigma_l(y) \pi(x) \pi(0) \rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m} \int_T dx \, \langle \mathcal{T} \delta(y) \pi(x) \eta_l(0) \rangle$$

$$P_{ls}(y) = \frac{1}{3} \hat{m} \int_T dx \, \langle \mathcal{T} \eta_s(y) \pi(x) \delta(0) \rangle$$

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$$d^{abc} \left[P_{KK}(y) - S_{\kappa\kappa}(y) \right] = \hat{m} \int_T dx \, \left\langle \mathcal{T} K^b(y) \kappa^c(x) \pi^a(0) \right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$U(1)_A$ **partners**

$$
\begin{aligned}
P_{\pi\pi}(y) - S_{\delta\delta}(y) &= \int_T dx\, \langle \mathcal{T} \pi(y)\delta(0)\tilde{\eta}(x)\rangle \\
P_{ll}(y) - S_{ll}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y)\sigma_l(0)\tilde{\eta}(x)\rangle \\
P_{ls}(y) - S_{ls}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y)\sigma_s(0)\tilde{\eta}(x)\rangle \\
P_{ss}(y) - S_{ss}(y) &= \int_T dx\, \langle \mathcal{T} \eta_s(y)\sigma_s(0)\tilde{\eta}(x)\rangle \\
P_{KK}(y) - S_{\kappa\kappa}(y) &= \int_T dx\, \langle \mathcal{T} K(y)\kappa(0)\tilde{\eta}(x)\rangle
\end{aligned}
$$

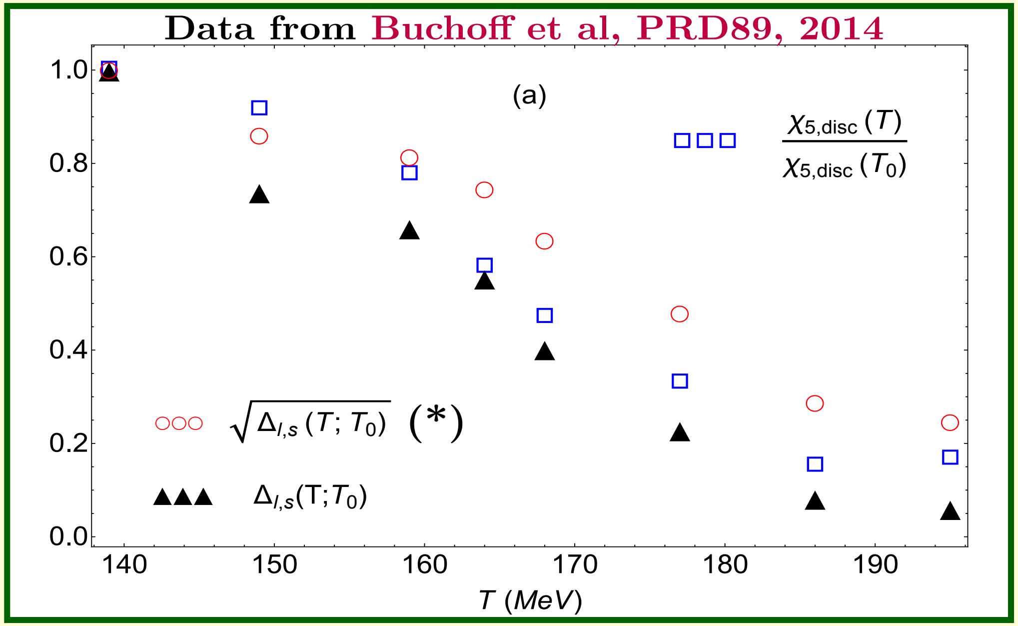
$\tilde{\eta}(x) = \hat{m}\eta_l(x) + m_s\eta_s(x) + \frac{1}{2}A(x)$ **three sources of** $U(1)_A$ **breaking**

Chiral Patterns and Partners from WI

Physical case ($N_f = 2+1$, $\hat{m} \neq 0$):

- **m_s distortion.**
- **Worse $\chi_P^{\eta_l} - \chi_S^{\delta}$ degeneration in lattice.**
- **$\chi_{5,disc}$ would scale dictated by quark condensate:** **(1)**

Data from Buchoff et al, PRD89, 2014



$\Delta_{l,s}(T;T_o)$ relative to $T_0 = 139$ MeV

$32^3 \times 8$ lattice size

$\hat{m}/m_s = 0.088$

(1) $\chi_{top} \sim \hat{m}\, \langle \bar{q}q \rangle_l$ in ch. limit
V.Azcoiti, PRD94 (2016)

(*) from χ_P^{ls} WI and normalization $\pi \sim \sqrt{-\langle \bar{q}q \rangle_l\, G_\pi^{-1}(p^2=0)}$
compatible with $\chi_P^{\pi} = -\langle \bar{q}q \rangle_l / \hat{m}$

WI and Screening Masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T = 0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

Connected/Disconnected susceptibilities

$$\boxed{\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^{\pi}(T) - \chi_S^{\sigma}(T)\right] + \frac{1}{4}\left[\chi_S^{\delta}(T) - \chi_P^{\eta_l}(T)\right]}$$

$\Rightarrow$ **Is the vanishing of $\chi_{5,disc}$ in conflict with χ_S^{dis} peaking at the chiral transition?**

Connected/Disconnected susceptibilities

$$\boxed{\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis}} \quad (m_u = m_d)$$

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From ChPT in the chiral limit $M_\pi \to 0^+$(IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_{5,disc}(T_c) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \chi_S^{dis}(T_c)$$

"peak" with same coeff.

$$\chi_{5,disc}(T_c) = \tilde{\chi}_S^{dis}(T_c) + \cancel{\frac{1}{4}\left[\chi_P^{\pi}(T_c) - \chi_S^{\sigma}(T_c)\right]} + \underbrace{\frac{1}{4}\left[\chi_S^{\delta}(T_c) - \chi_P^{\eta_l}(T_c)\right]}_{IR\ regular}$$

Connected/Disconnected susceptibilities

$$\boxed{\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

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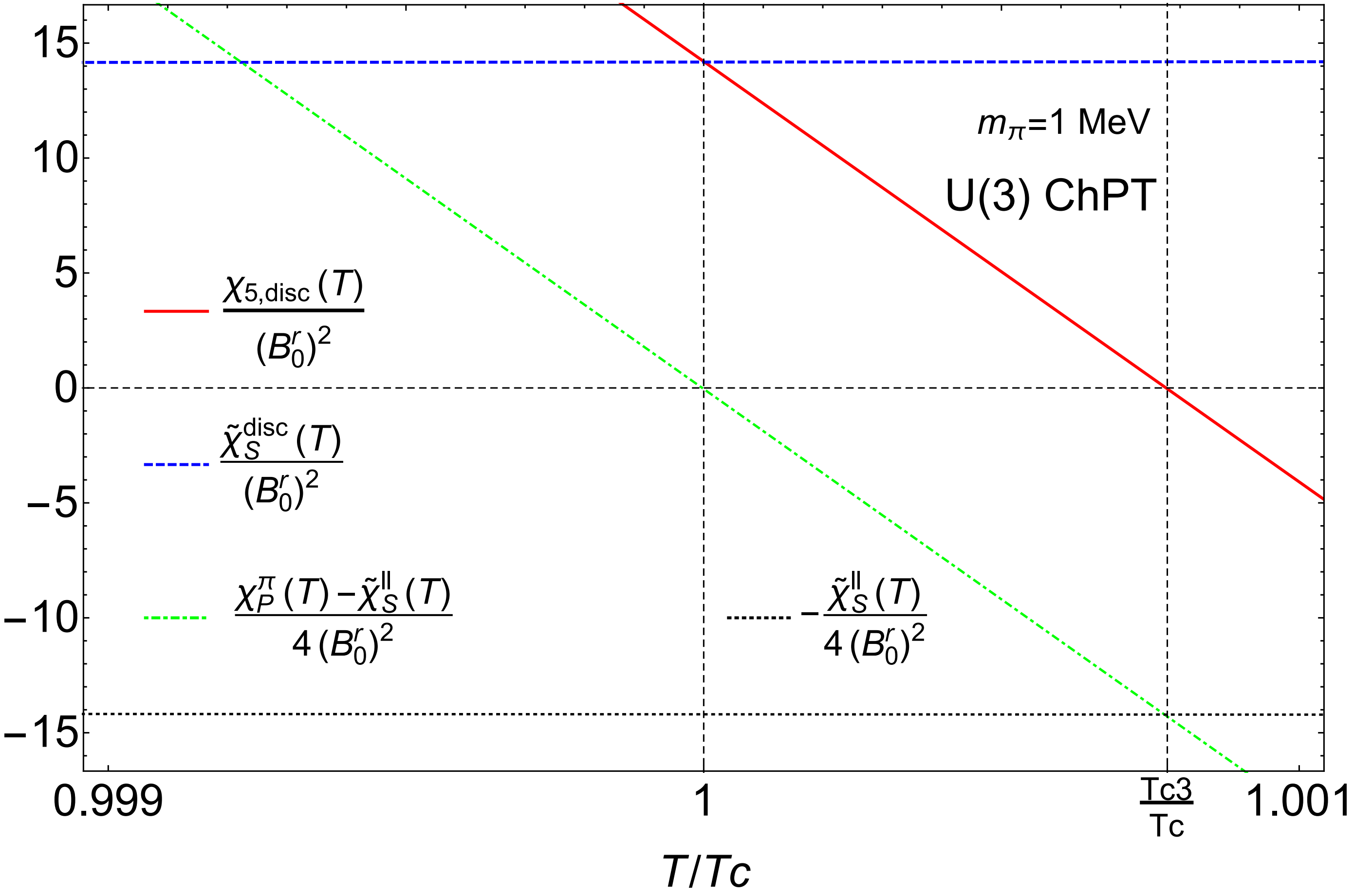
From ChPT in the chiral limit $M_\pi \to 0^+$(IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_S^{disc}(T_{c3}) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \frac{1}{4}\chi_S^{\sigma}(T_{c3})$$ **"peak" with same coff.**

$$\chi_{5,disc}(T_{c3}) = 0 = \tilde{\chi}_S^{dis}(T_{c3}) + \frac{1}{4}\left[\cancel{\chi_P^{\pi}(T_{c3})} - \chi_S^{\sigma}(T_{c3})\right] + \frac{1}{4}\left[\underbrace{\chi_S^{\delta}(T_{c3})}_{IR\ regular} - \cancel{\chi_P^{\eta_l}(T_{c3})}\right]$$

Connected/Disconnected susceptibilities



Connected/Disconnected susceptibilities

$$\chi_S^{ll} \equiv \chi_S^{\sigma} = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^{\delta} + 4\chi_S^{dis} \qquad (m_u = m_d)$$

- **In general, only the total** $\chi_S^{\sigma} \sim \frac{\partial}{\partial \hat{m}} \langle \bar{q}q \rangle_l$ **expected to peak**

A. V. Smilga and J. J. M. Verbaarschot, PRD54 1996

$\Rightarrow$ χ_S^{δ} **could peak at** $U(1)_A$ **restoration**

Actually χ_S^{δ} **grows for** $T < T_c$ **and should vanish asymptotically if** $\chi_S^{\delta} \sim \chi_P^{\pi} = -\langle \bar{q}q \rangle_l / \hat{m} \to 0$

From Bazavov et al, PRD85, 2012

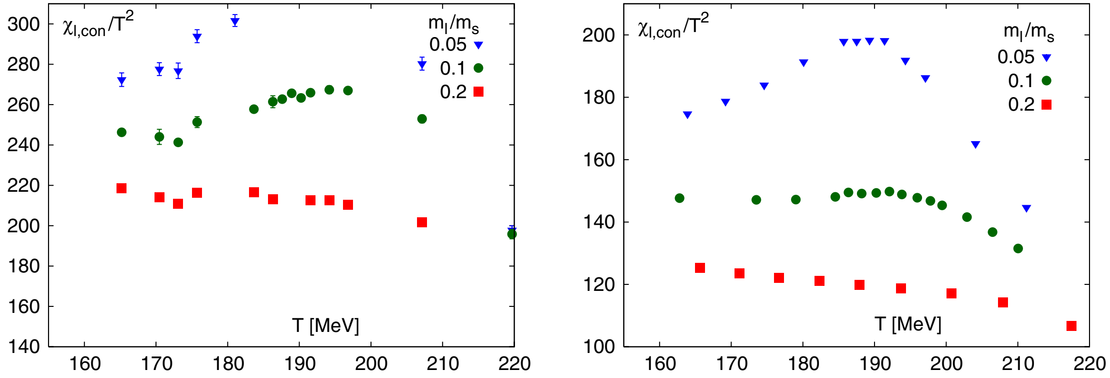


FIG. 12 (color online). The connected part of the chiral susceptibility for the p4 (left) and asqtad (right) actions for different quark masses on $N_\tau = 8$ lattices.

Screening vs pole masses

Lattice parametrization for inverse pseudo correlator (Karsch et al 2003):

$$K_P^{-1}(\omega,\vec{p}) \sim -\omega^2 + A^2(T)|\vec{p}|^2 + M^{pole}(T)^2$$

$$A(T) = \frac{M^{pole}(T)}{M^{sc}(T)}$$

Pseudoscalar susceptibility: $\chi_P = \dfrac{N_\chi}{M^2 + \Sigma_T(0,0)}$

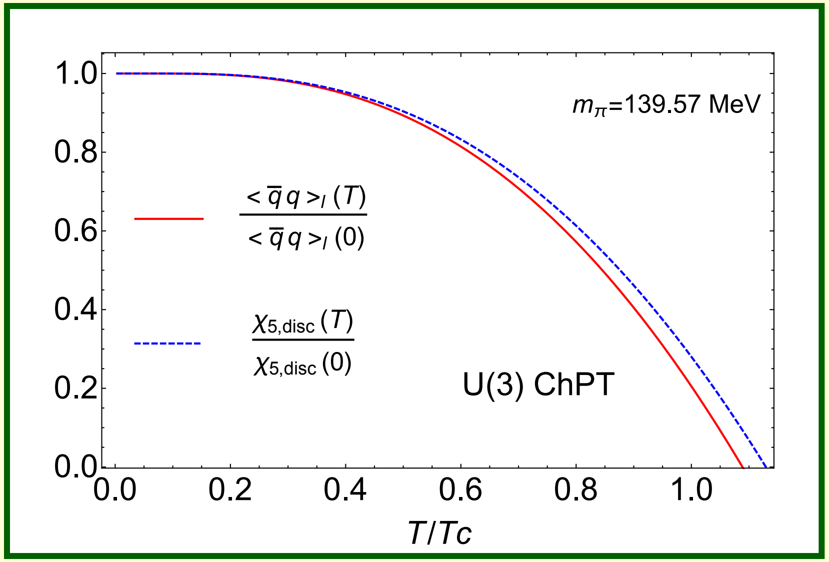
$p = 0$ **expansion:** $\Sigma(\omega,\vec{p};T) = \Sigma_T(0,0) + \alpha(T)\omega^2 - \beta(T)|\vec{p}|^2 + \mathcal{O}(p^4)$

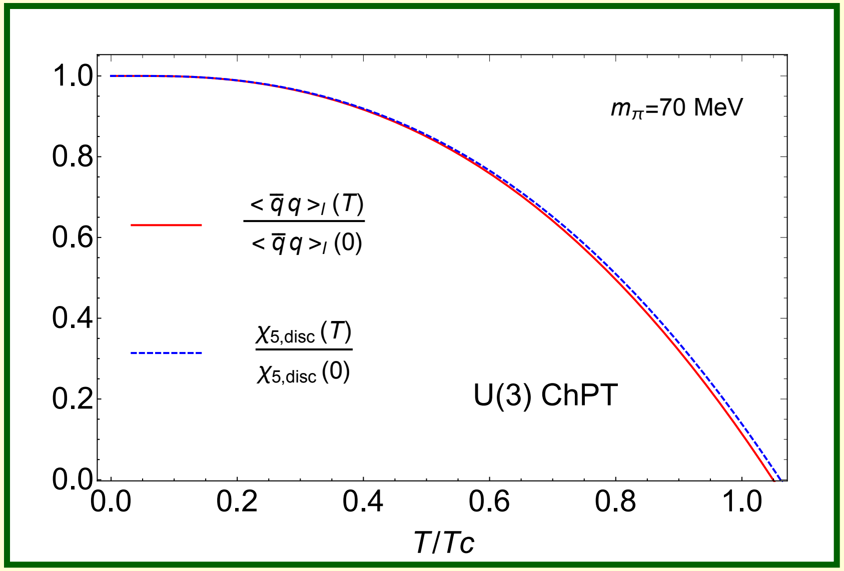
$$A^2(T) = \frac{1+\beta(T)}{1+\alpha(T)}$$

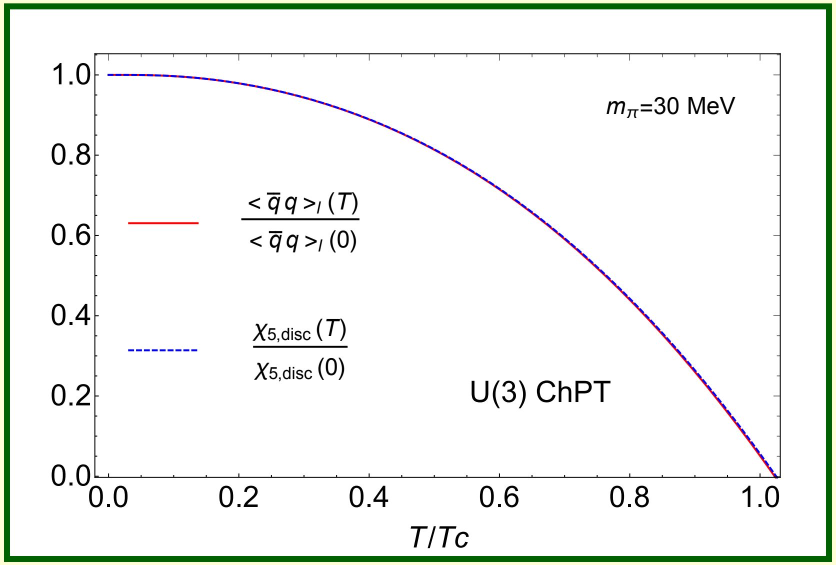
$$\left[M^{pole}(T)\right]^2 = \frac{M^2 + \Sigma_T(0,0)}{1+\alpha(T)}$$

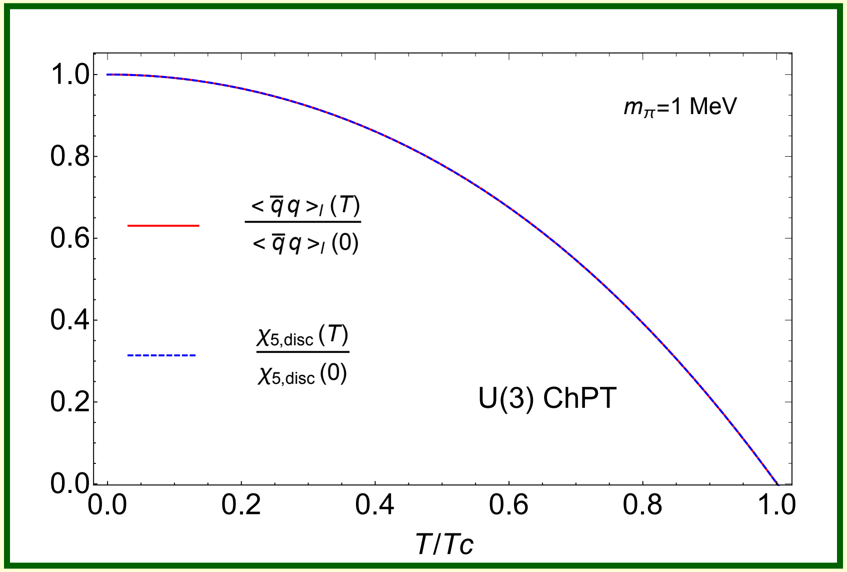
Therefore, $N_\chi \chi_P^{-1}(T) = [1+\alpha(T)]\, A^2(T)\, [M^{sc}(T)]^2$

$\chi_{5,disc}$ vs $\langle \bar{q}q \rangle_l$ scaling in $U(3)$ ChPT

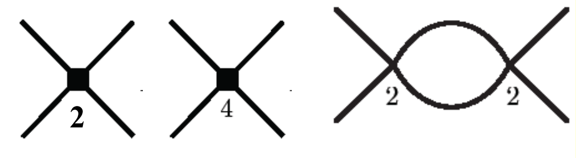








Unitarizing scattering: resonances



ChPT Partial waves $t^{IJ} = t_2^{IJ} + t_4^{IJ} + \dots$

Unitarity $\rightarrow$ **Im** $t(s) = \sigma(s)|t(s)|^2 \;\; (s \geq 4M^2) \Rightarrow$ **Im** $t^{-1} = -\sigma$

$\sigma(s) = \sqrt{1 - \dfrac{4M^2}{s}} = \dfrac{p_{CM}}{\sqrt{s}}$ **two-particle phase space**

$$\boxed{t^U(s;T) = \frac{\left[t_2(s)\right]^2}{t_2(s) - t_4(s;T)}}$$

(IAM)

Exactly proven for large NGB and chiral limits:

S.Cortés, AGN, J.Morales '16

FINITE TEMPERATURE:

$$t_4(s) \;\rightarrow\; t_4(s;T)$$

$$\sigma \;\rightarrow\; \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T$$

Thermal phase Space.

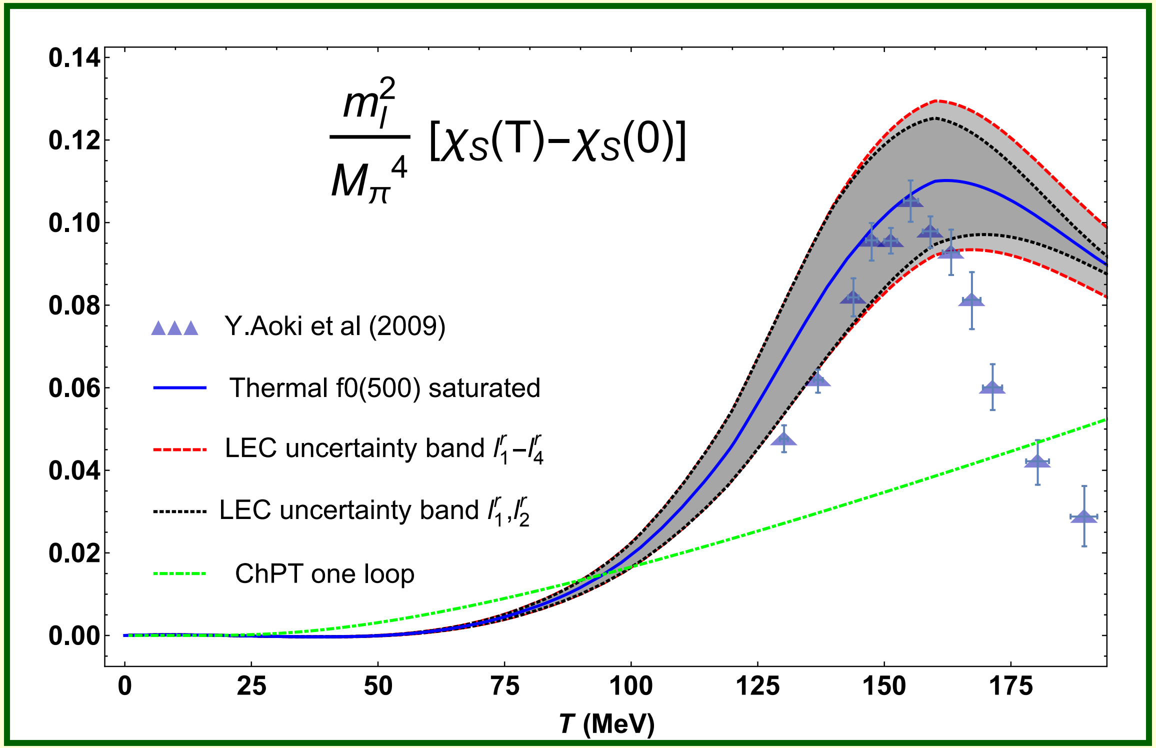
Bose net enhancement $(1+n)^2 - n^2$

A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez,E.Tomás-Herruzo, '02 '05 '07

Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ state

S.Ferreres, AGN, A.Vioque, 2018

$$\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}$$



- **Consistent with lattice transition peak.**
- **LECs and uncertaintities from unitarized $T=0$ fit in Hanhart, Peláez, Ríos PRL100 (2008)**

$$s_p = 446.5 - i220.4 \text{ MeV}$$

- **Consistent T_c reduction and χ_S growth near chiral limit**