Hydrodynamics and the Approach to Equilibrium

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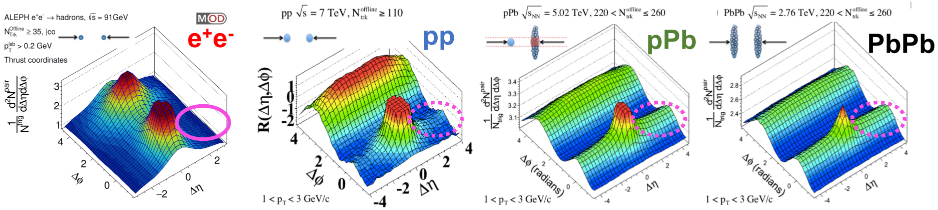
JETSCAPE

Overview

"Elliptic flow" in PbPb, pPb, pp — but not in e^+e^-

Long-range (in rapidity) angular correlations (a.k.a. "elliptic flow") observed in PbPb, pPb, and high- multiplicity pp collisions, but not in e^+e^- collisions:



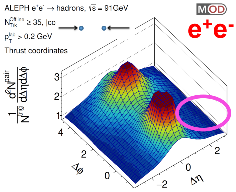
Badea et al., arXiv:1906.00489 CMS, JHEP09 (2010) 091 CMS, PLB 724 (2013) 213

(figure credit: Yen-Jie Lee (MIT))

No sign of the near-side ridge in e^+e^-- collisions up to the highest multiplicities ($\sim$ 55 particles per event)

“Elliptic flow” in PbPb, pPb, pp — but not in e^+e^-

Long-range (in rapidity) angular correlations (a.k.a. “elliptic flow”) observed in PbPb, pPb, and high- multiplicity pp collisions, but not in e^+e^- collisions:



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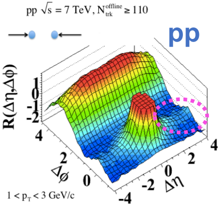
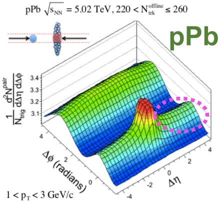
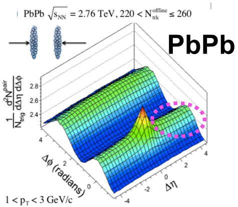


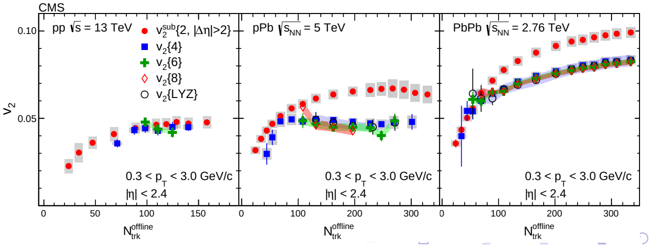
CMS, JHEP09 (2010) 091





CMS, PLB 724 (2013) 213

Whereever the effect is seen, it is a collective phenomenon:



What is the correct description of this feature?

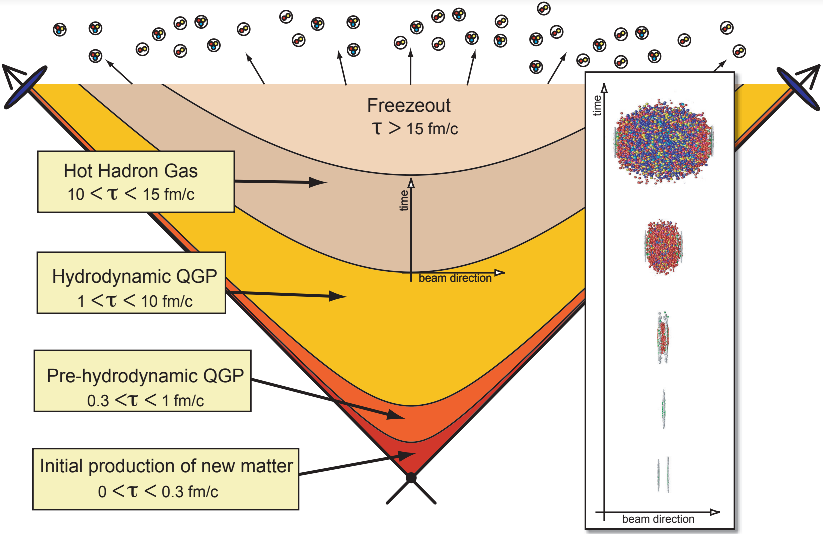
- Is it the same in pp, pPb, and PbPb?
- If not, what changes from pp to PbPb, and how do we identify it?
- If yes, what makes e^+e^- collisions so different?
- Which experimental observables can tell us **before** we do the simulation whether or not hydrodynamics will correctly describe the system?

What is different in e^+e^- from pp?

- No high density of low-x gluons in the initial state
  $\Longrightarrow$ no initial-state gluon radiation
- Single pointlike primary interaction vertex in e^+e^-
- No multiple soft parton interactions in the energy deposition process $e^+e^- \to Z^0$
- Large rapidity gap between the primary produced strongly interacting particles (q and $\bar{q}$)
- Radiation dominated: No medium for the quark jet fragments to rescatter off

Overview

Space-time diagram of a heavy-ion collision



(After M. Strickland, arXiv:1410.5786)

Causal dissipative relativistic fluid dynamics

Israel & Stewart '79; Muronga '02; Denicol, Niemi, Molnár, Rischke '12; & many others ...

Macroscopic evolution of densities and fluid velocity as functions of space and time:

$$\begin{aligned} T^{\mu\nu} &= e\, u^\mu u^\nu - (p+\Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \quad \text{where } \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu = -(x^\mu x^\nu + y^\mu y^\nu + z^\mu z^\nu) \\ j^\mu &= n\, u^\mu + V^\mu \\ p &= p(e,n) \quad (EoS) \end{aligned} \tag{1}$$

- Conservation laws $\partial_\mu T^{\mu\nu} = 0 = \partial_\mu j^\mu \implies$ evolution of e, n, u^μ
- Relaxation equations for the dissipative flows $\pi^{\mu\nu}$, Π, V^μ, e.g.

$$D\pi^{\langle\mu\nu\rangle} = \frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \text{second order terms;}$$

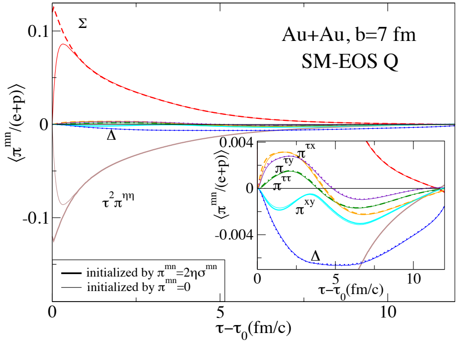
  describe competition between collisions ($\to$ towards equilibrium) and expansion ($\to$ away from equilibrium)

- Large anisotropies in the expansion rate θ ($\theta_L \gg \theta_\perp$) keep the pressure anisotropy

$$P_L - P_\perp = \left(p + \pi^{zz}\right) - \left(p - \tfrac{1}{2}\pi^{zz}\right) = \tfrac{3}{2}\pi^{zz} < 0$$

  big throughout the evolution history:

Large shear stress throughout the QGP phase!



VISH2+1 (from H. Song's PhD thesis (arXiv:0908.3656))

$\Longrightarrow$ "hydrodynamization" $\neq$ "equilibration"

$\Longrightarrow$ hydrodynamics becomes valid well before local momentum isotropy and thermal equilibrium are reached

$\Longrightarrow$ "far-from-equilibrium hydrodynamics" (Romatschke 2018)

Anisotropic hydrodynamics

Martinez & Strickland '10; Florkowski & Ryblewski '11; Bazow et al. '14;
Molnár et al. '16; McNelis et al. '18; and many others ...

$$\begin{aligned} T^{\mu\nu} &= e\, u^\mu u^\nu + P_L z^\mu z^\nu - P_\perp \Xi^{\mu\nu} + \pi_\perp^{\mu\nu} + 2 W_{\perp z}^{(\mu} z^{\nu)} \\ j^\mu &= n\, u^\mu + V_z^\mu + V_\perp^\mu \\ p &= p(e,n) \quad (EoS) \end{aligned} \tag{2}$$

where

$$\begin{aligned} \Xi^{\mu\nu} &= g^{\mu\nu} - u^\mu u^\nu + z^\mu z^\nu = -(x^\mu x^\nu + y^\mu y^\nu) \\ \Pi &= \tfrac{1}{3}(P_L + 2P_\perp) - p(e,n) \\ \pi^{\mu\nu} &= \pi_\perp^{\mu\nu} + 2W_{\perp z}^{(\mu} z^{\nu)} + \tfrac{1}{3}(P_L - P_\perp)(z^\mu z^\nu - \Delta^{\mu\nu}) \end{aligned}$$

- Conservation laws $\partial_\mu T^{\mu\nu} = 0 = \partial_\mu j^\mu \implies$ evolution of e, n, u^μ
- Relaxation equations for P_L, $P_\perp$, $\pi_\perp^{\mu\nu}$, $W_{\perp z}^\mu$, V^μ (McNelis et al. '18)

Multiple (0+1)-d studies (Bjorken, Gubser), presently being implemented in (3+1)-d

- (1) The hydrodynamic approach towards equilibrium

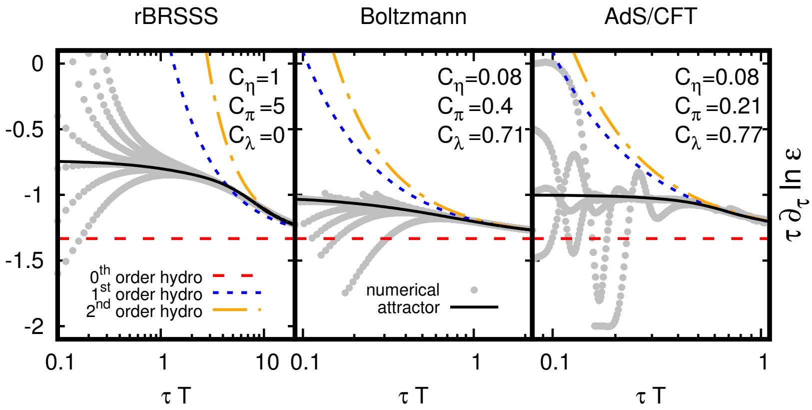
- (2) The pre-hydrodynamic evolution towards hydrodynamization

Overview

Hydrodynamic attractors

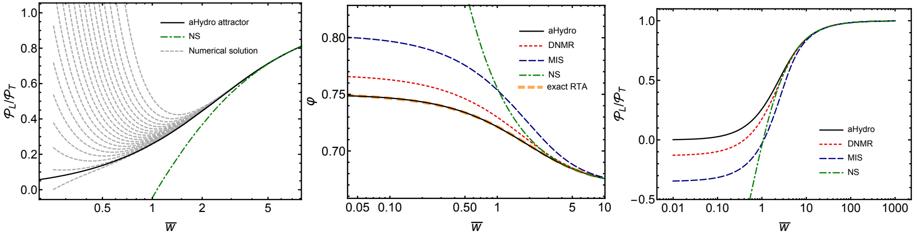
- Hydrodynamics is the effective theory of long-wavelength excitations which can be expressed as a hydrodynamic gradient series (Baier et al. (BRSSS) '08).
- While this gradient series was shown to be asymptotic (i.e. it diverges (Heller et al. '13 –'16, and others)), it can be Borel resummed, yielding an **attractor** (Heller et al. '13) to which the system converges on a microscopic relaxation time scale $\tau_{\rm rel}$. Non-hydrodynamic moments of the underlying phase-space distribution decay on the same time scale $\tau_{\rm rel}$ (Strickland '18). The precise form of this decay depends on the microscopic collision dynamics (Romatschke '17).
- In the limit of small gradients, the attractor reduces to the low-order hydrodynamic gradient series solution. Navier-Stokes theory defines the unique attractor at first order in gradients.
- The existence and properties of the hydrodynamic attractor have been studied in detail for conformal and non-conformal systems undergoing Bjorken flow (Heller et al. '13 –'18; Basar & Dunne '15; Romatschke '17; Denicol & Noronha '16; Strickland '18; . . .). **Some pictures:**

Hydrodynamic attractors for Bjorken flow



Romatschke, PRL 120 (2017) 012301

The anisotropic hydrodynamic attractor for Bjorken flow



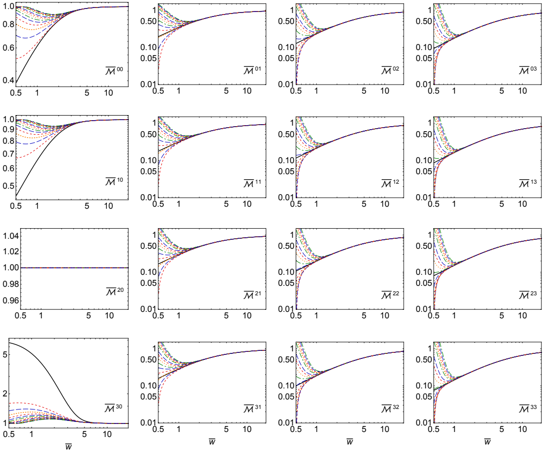
Strickland & Noronha, PRD97 (2018) 036020

$$\varphi = \frac{1}{2}\left(\frac{(P_L/P_T)+3}{(P_L/P_T)+2}\right), \qquad \bar{w} = \frac{\tau}{\tau_{\rm rel}} = \text{inverse Knudsen number}$$

- Numerical solutions join attractor (i.e. lose memory of ICs) after $\tau \gtrsim (1{-}2)\tau_{\rm rel}$. At this point $P_L/P_T \lesssim 0.5$, i.e. shear stress effects are $\mathcal{O}(1)$.
- aHydro reproduces unterlying RTA Boltzmann transport almost perfectly, even for very large shear stress.
- Hydrodynamic attractors merge with Navier-Stokes after $\tau >$ few $\times\, \tau_{\rm rel}$ (not MIS).

Attractors for higher-order (non-hydrodynamic) moments

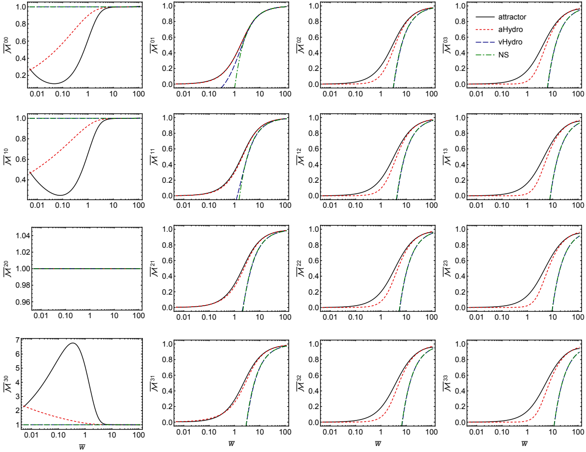
Strickland, JHEP12 (2018) 128 (RTA Boltzmann equation, exact soln. for Bjorken flow)



Attractors exist also for all higher-order (non-hydrodynamic) moments of the distribution function, describing its high-momentum tail. For all anisotropic moments, the solutions of the RTA BE approach the attractors exponentially on time scales $\tau \gtrsim \tau_{\rm rel}$.

Attractors for higher-order (non-hydrodynamic) moments

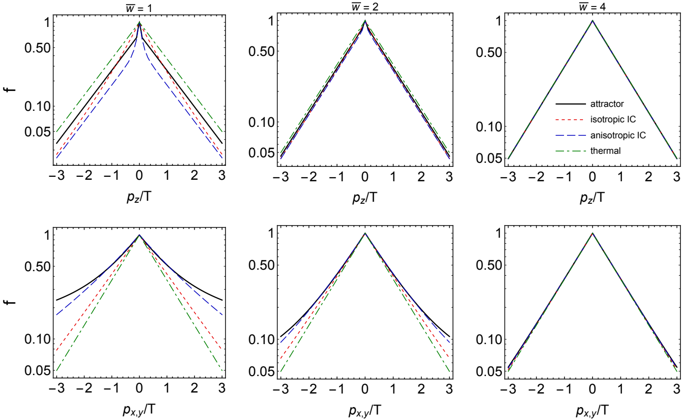
Strickland, JHEP12 (2018) 128 (Bjorken flow)



For aHydro, the hydrodynamic attractors for all anisotropic moments join the exact RTA BE attractors after $\tau \gtrsim \tau_{\rm rel}$. For standard (MIS) vHydro, this happens much later, and long after vHydro has reached the NS limit.

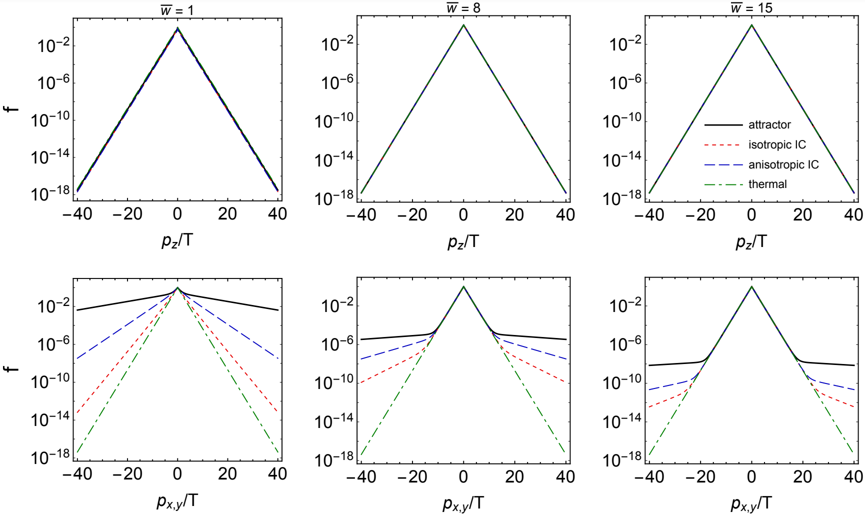
Attractors for the distribution function (Bjorken flow)

Strickland, JHEP12 (2018) 128 (Bjorken flow)



For the RTA BE, the distribution function has also an attractor to which all initial conditions evolve after $\tau >$ few $\times\, \tau_{\rm rel}$. The p_L-distribution thermalizes $\sim$ completely after a few $\tau_{\rm rel}$. After $\tau > 4\,\tau_{\rm rel}$ the p_T-distribution is thermalized up to $p_T \lesssim 3T$ ...

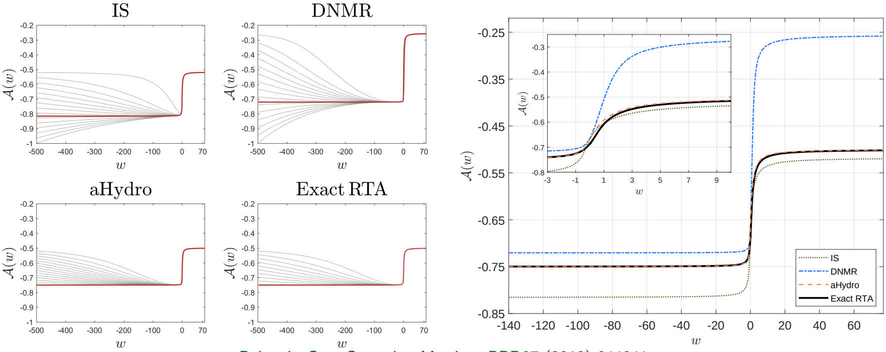
Attractors for the distribution function (Bjorken flow)



. . . and after $\tau > 15\,\tau_{\rm rel}$ up to $p_T \lesssim 3\,\mathrm{GeV}/c$!

Hydrodynamic attractors for Gubser flow

- Gubser flow (Gubser '10) = long. boost-invariant + azimuthally symmetric, strong transverse flow
- Opposite to Bjorken flow, Knudsen number increases with time (exponentially) $\Longrightarrow$ asymptotic free-streaming, shear stress saturates at $\lim_{\rho\to\infty} \pi^\eta_\eta/p_{\rm eq} = 2$.



Behtash, Cruz-Camacho, Martinez PRD97 (2018) 044041

$$w = \tanh\rho/\hat{T} \sim \text{Knudsen number}, \quad \mathcal{A}(w) = d\ln\hat{T}/d\ln\cosh\rho$$

- aHydro attractor and time evolution agree almost perfectly with exact RTA Boltzmann equation even in the free-streaming limit!

Hydrodynamic attractors: Bjorken vs. Gubser

Chattopadhyay, UH, Pal, Vujanovic, PRC97 (2018) 064909

Bjorken:

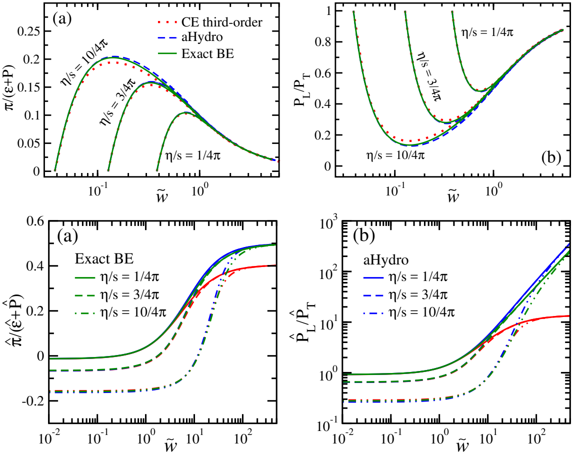
$$\tilde{w} = \frac{\tau\, T(\tau)}{4\pi\eta/s}$$

(inverse Knudsen number)

Gubser:

$$\tilde{w} = \frac{2\tanh\rho}{\hat{T}(\rho)}\,\frac{4\pi\eta}{s}$$

(Knudsen number)



Anisotropic hydrodynamics describes the underlying kinetic theory accurately even well before the evolution trajectory joins the attractor!

Summary (1)

Systems that possess a hydrodynamic attractor (this includes both systems that reach asymptotic local thermal equilibrium and others that don't) **hydrodynamize after $\tau \gtrsim \tau_{\mathrm{rel}}$**, even if they are still very far from local thermal equilibrium, and **even if they never reach it**.

This includes heavy-ion collision fireballs with and without transverse expansion.

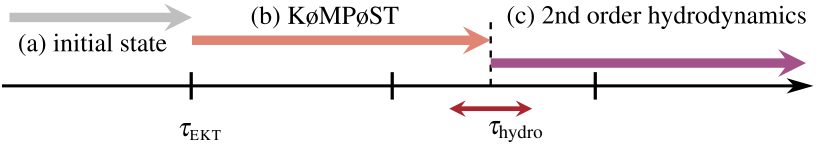
Hydrodynamics fails for heavy-ion collisions if (and only if) $\tau_{\mathrm{rel}} > \tau_{\mathrm{QGP}}$.

Overview

1. Prologue
2. Dissipative relativistic fluid dynamics
3. Hydrodynamic evolution towards equilibrium
4. Pre-hydrodynamic evolution towards hydrodynamization

Initialization of the hydrodynamic stage

To remove uncertainties related to the pre-hydrodynamic stage, much recent work has focused on the correct initialization of hydrodynamics
(see talk by Mazeliauskas Tuesday 15:20):



(from Mazeliauskas PoS HardProbes2018 (2018) 111)

Work has progressed from studying simple (0+1)-d Bjorken flow to (2+1)-d and (3+1)-d expansion with kinetic theory.

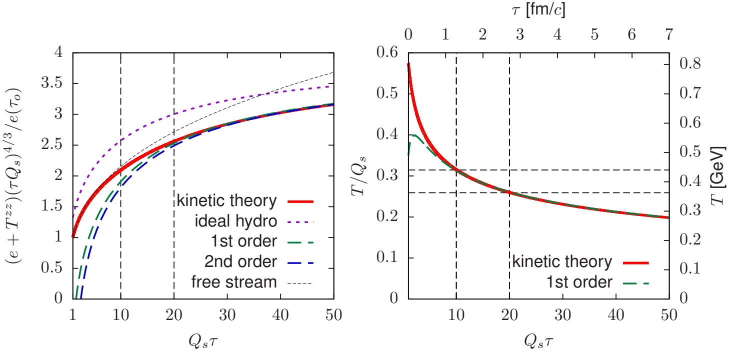
Liu et al. '15; Keegan et al. '16; Kurkela, Mazeliauskas et al. '17–'19, Greif et al. '17; Kurkela, Wiedemann, Wu '18–'19;

and others.

From EKT to hydro: Bjorken expansion

EKT = Effective Kinetic Theory = Boltzmann equation for $m = 0$ partons with pQCD $2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ AMY '03 collision term (Mazeliauskas Tue 15:20):

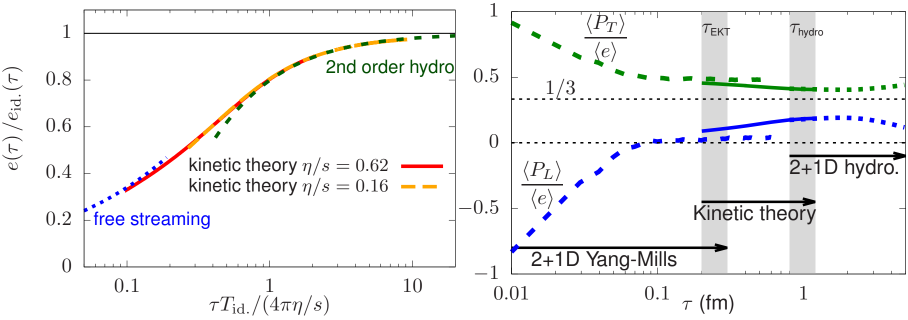
Keegan, Kurkela, Mazeliauskas, Teaney, JEP08 (2016) 171



Smooth transition from ≈ free-streaming at very early times to dissipative hydro at $\tau \sim 10/Q_s \sim 1\,\mathrm{fm}/c$ (probably even earlier to aHydro attractor).

From CYM through EKT to hydro: (2+1)-d KøMPøST

Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, PRL122 (2019) 122302



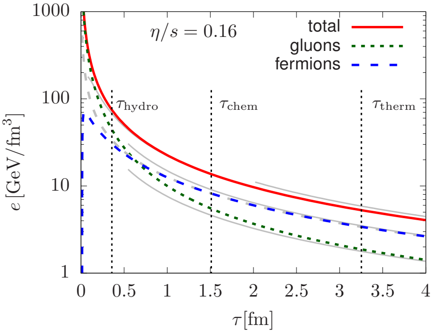
Smooth transition from CYM at very, very early times to EKT at very early times to dissipative hydro at $\tau \sim 1\,\mathrm{fm}/c$ (probably even earlier to aHydro attractor) in PbPb at LHC energies.

Chemical vs. thermal equilibration vs. hydrodynamization

Kurkela & Mazeliauskas, PRL122 and PRD99 (2019); from Aleksas' talk Tue 15:20:

Physical equilibration time-scales in hadronic collisions

$$\tilde{\tau} = \underbrace{(\tau/\tau_R)^{3/2}}_{\text{scaled time variable}} \times \underbrace{(4\pi\eta/s)^{3/2} \times \langle s\tau\rangle^{-1/2} \times (4\pi^2\nu_{\text{eff}}/90)^{1/2}}_{\text{phenomenological input}}$$



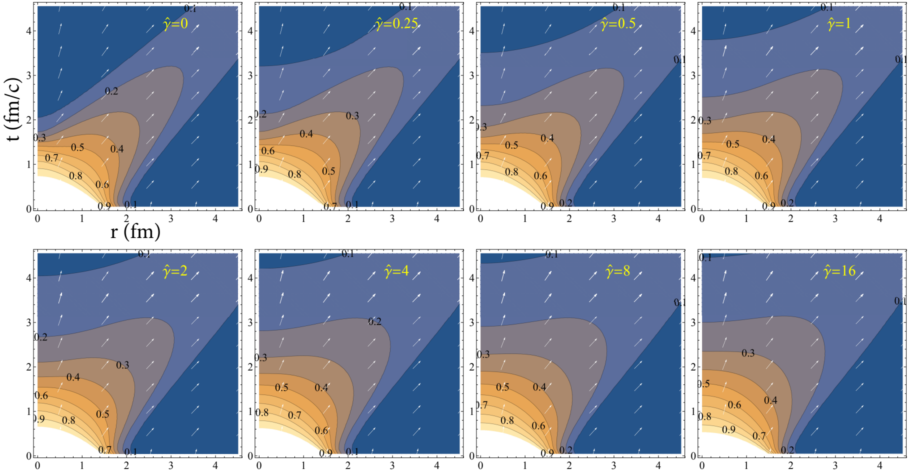
Input:
$\eta/s \approx 0.16$
$\langle s\tau\rangle \approx 4.1\ \mathrm{GeV}^2$
$\nu_{\text{eff}} \approx 40$

$$\underbrace{\tau_{\text{hydro}}}_{\pm 10\%\ \text{viscous}\ e(\tau)} < \underbrace{\tau_{\text{chem}}}_{\pm 10\%\ \text{fermion eq.}\ e(\tau)} < \underbrace{\tau_{\text{therm}}}_{\pm 10\%\ \text{ideal}\ e(\tau)}$$

The first two ($\tau_{\rm hydro}$ and $\tau_{\rm chem}$) scale with the relaxation time $\tau_{\rm rel}(\tau) \propto 1/T(\tau)$!

From kinetic theory to hydrodynamics: ITA BE in (2+1)-d

Kurkela, Wiedemann, Wu arXiv:1905.05139; Wiedemann Mo 9:30



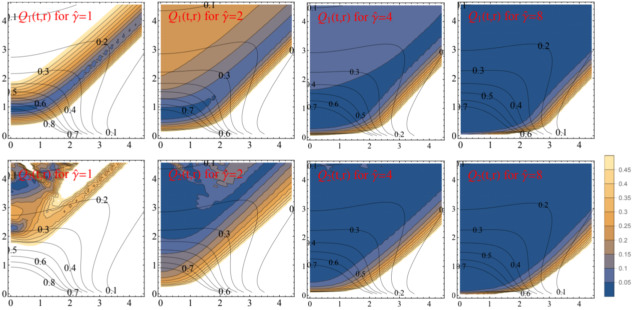
$$\hat{\gamma} = \gamma R^{3/4}(e_0\tau_0)^{1/4} \sim \left(R/\tau_{\rm rel}(\tau{=}R)\right)^{9/8} = \left(\mathrm{Kn}(\tau{=}R)\right)^{-9/8}$$

Hydrodynamization of (2+1)-d ITA BE

Wiedemann Mo 9:30

particle-like: $\hat{\gamma} \lesssim 2$ | transition: $2 \lesssim \hat{\gamma} \lesssim 4$ | hydro-like: $4 \lesssim \hat{\gamma}$

(Kurkela, Wiedemann & Wu, arXiv:1905.05139)



Upper (lower) panels: Q_1 (Q_2) measured up to 1st (2nd) order.

For $\hat{\gamma} \gtrsim 4$ the kinetic theory hydrodynamizes very early and stays "liquid";
for $2 \lesssim \hat{\gamma} \lesssim 4$ the theory first hydrodynamizes but soon exits the fluid regime again;
for $\hat{\gamma} \lesssim 2$, standard viscous hydrodynamics is not a good approximation. aHydro?

Summary (2)

- Kinetic theory smoothly joins early CYM or free-streaming dynamics to viscous hydrodynamics, at $\boldsymbol{T\tau/(4\pi\eta/s) \lesssim 1}$ (corresponding to $\boldsymbol{\tau \lesssim 1\,\mathrm{fm}/c}$ in PbPb @ LHC). Perhaps even earlier when using aHydro (to be checked).
- **Equilibration hierarchy** $\boldsymbol{\tau_{\mathrm{hydro}} < \tau_{\mathrm{chem}} < \tau_{\mathrm{therm}}}$. The first two scales are $\boldsymbol{\propto \tau_{\mathrm{rel}}}$. Momentum isotropization and thermal equilibration are delayed by strong anisotropies in the expansion rate in heavy-ion collisions. Chemical equilibration is not affected by these.
- (Grand canonical) **chemical equilibrium** in the QGP is reached in collisions with $\boldsymbol{dN_{\mathrm{ch}}/d\eta \gtrsim 110}$, for all collision systems (Mazeliauskas, Tue 15:20). Canonical equilibrium in small collision systems may be reached at even smaller $dN_{\mathrm{ch}}/d\eta$ (to be checked).
- **Hydrodynamization** happens at significantly smaller $dN_{\mathrm{ch}}/d\eta \ll 110$.
- For **fixed collision system**, increasing $dN_{\mathrm{ch}}/d\eta$ reduces the Knudsen number and thus accelerates hydrodynamization and chemical equilibration.
- For **fixed $\boldsymbol{dN_{\mathrm{ch}}/d\eta}$**, changing the system size neither improves nor degrades hydrodynamization, to zeroth order. First-order corrections (stronger viscous heating and faster radial expansion) somewhat increase the effective Knudsen number in small collision systems, to the detriment of hydrodynamization.

Thank you!

Extras

vHydro vs. aHydro for (0+1)-d Bjorken flow with bulk visc.

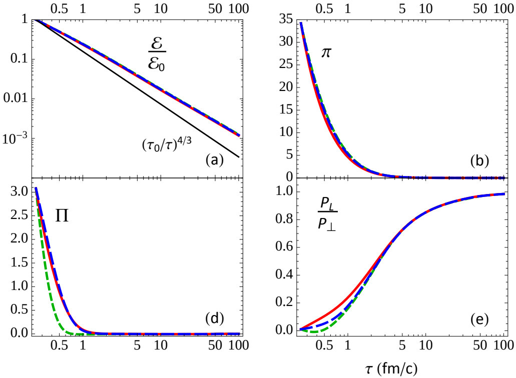
McNelis, Bazow, UH, PRC97 (2018) 054912

Glasma-like IC:

red solid: aHydro

blue long-dashed: vHydro

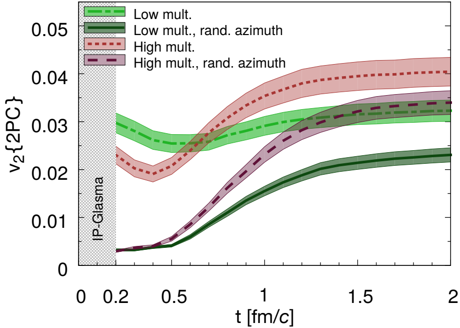
green short-dashed: vHydro with different transport coefficients



Using transport coefficients from the same microscopic theory, standard viscous and anisotropic hydrodynamic evolutions are surprisingly similar in (0+1)-d, even for large viscous stresses.

(3+1)-d BAMPS: loss of IC memory and hydro in pPb

Greif, Greiner, Schenke, Schlichting, Xu, PRD96 (2017) 091504



Low-multiplicity pPb: Initial elliptic-flow-like momentum correlations survive to the end; similarly strong collectivity cannot be generated from nothing by FSI scattering in BAMPS.

High-multiplicity pPb: Initial elliptic-flow-like momentum correlations get partially erased and replaced by stronger signal from FSI scattering; similar-strength final v_2 can also be created from nothing by FSI.