Correlations and fluctuations

Alice Ohlson
Lund University
Strangeness in Quark Matter, 14 June 2019
Correlations and fluctuations

- Initial state
- Hadrochemistry
- Jet quenching
- Particle production mechanisms
- Freezeout & phase transition
- Final state interactions
- Hydrodynamic flow
- Nucleon/hyperon interactions
- Chiral magnetic effect
- Collectivity in large & small systems
- Event display: ALICE, Pb-Pb $\sqrt{s_{\text{NN}}} = 5.02$ TeV
Correlations and fluctuations

- initial state
- hadrochemistry
- jet quenching
- particle production mechanisms
- freezeout & phase transition
- collectivity in large & small systems
- final state interactions
- nucleon/hyperon interactions
- chiral magnetic effect
- hydrodynamic flow

Event display: ALICE, Pb-Pb $\sqrt{s_{NN}} = 5.02$ TeV
Fluctuations in heavy ion collisions

- Event-by-event fluctuations of particle multiplicities are used to study properties and phase structure of strongly-interacting matter
- Fluctuations grow in the region near a phase transition and/or critical point — can we observe signs of criticality?

**Critical opalescence in CO₂**
Fluctuations in heavy ion collisions

- Event-by-event fluctuations of particle multiplicities are used to study properties and phase structure of strongly-interacting matter.
  - Fluctuations grow in the region near a phase transition and/or critical point — can we observe signs of criticality?
  - Fluctuations of conserved charges can be related to susceptibilities calculable in lattice QCD — precision test of LQCD at $\mu_B \approx 0$
Connecting theory to experiment

- Thermodynamic susceptibilities $\chi$
  - describe the response of a thermalized system to changes in external conditions, fundamental properties of the medium
  - can be calculated within lattice QCD
  - within the Grand Canonical Ensemble, are related to event-by-event fluctuations of the number of conserved charges

Theory: susceptibilities

$$\chi^B_n = \frac{\partial^n (P / T^4)}{\partial (\mu_B / T)^n}$$

Experiment: moments of net-charge, net-strangeness, net-baryon number distributions

$$\Delta N_B = N_B - N_{\bar{B}}$$
Connecting theory to experiment

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  - can be calculated within lattice QCD
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\[ \Delta N_B = V T^3 \chi_1^B \]
\[ \langle (\Delta N_B - \langle \Delta N_B \rangle)^2 \rangle = VT^3 \chi_2^B = \sigma^2 \]
\[ \langle (\Delta N_B - \langle \Delta N_B \rangle)^3 \rangle / \sigma^3 = \frac{VT^3 \chi_3^B}{(VT^3 \chi_2^B)^{3/2}} = \kappa \]
\[ \langle (\Delta N_B - \langle \Delta N_B \rangle)^4 \rangle / \sigma^4 - 3 = \frac{VT^3 \chi_4^B}{(VT^3 \chi_2^B)^2} = \kappa \]

Theory: susceptibilities

Experiment: moments of net-charge, net-strangeness, net-baryon number distributions

$\Delta N_B = N_B - N_{\bar{B}}$
Connecting theory to experiment

- Thermodynamic susceptibilities $\chi$
  - describe the response of a thermalized system to changes in external conditions, fundamental properties of the medium
  - can be calculated within lattice QCD
  - within the Grand Canonical Ensemble, are related to event-by-event fluctuations of the number of conserved charges

\[
\begin{align*}
\langle \Delta N_B \rangle &= VT^3 \chi_1^B \\
\langle (\Delta N_B - \langle \Delta N_B \rangle)^2 \rangle &= \frac{\sigma^2}{\chi_2^B} \\
\langle (\Delta N_B - \langle \Delta N_B \rangle)^4 \rangle &= \frac{\sigma^4}{\chi_2^B} - 3 = \frac{VT^3 \chi_4^B}{(VT^3 \chi_2^B)^2} = \kappa
\end{align*}
\]
Connecting theory to experiment

Theory: susceptibilities calculated in a fixed volume, particle bath in GCE

Experiment: event-by-event multiplicities of identified particles in a detector
Connecting theory to experiment

- experimental particle misidentification
  cut-based PID or Identity Method

- detector (in)efficiency

- net-$\pi,K,p \leftrightarrow net-Q,S,B$

- global conservation laws

- infinite $\leftrightarrow$ finite volume
  talk by Marcus Bluhm

- volume fluctuations
  talk by Sukanya Sombun

- resonance decays
  talk by Mesut Arslanbek (ALICE)
Precision test of the LQCD baseline at $\mu_B = 0$
-- second moments at the LHC --
Net-proton fluctuations

\[ \kappa_1(p) = \langle N_p \rangle \quad \kappa_2(p) = \left\langle \left( N_p - \langle N_p \rangle \right)^2 \right\rangle \]

\[ \kappa_2(p - \bar{p}) = \left\langle \left( N_p - N_{\bar{p}} - \langle N_p - N_{\bar{p}} \rangle \right)^2 \right\rangle \]

\[ = \kappa_2(p) + \kappa_2(\bar{p}) - 2\left\langle N_p N_{\bar{p}} \right\rangle - \left\langle N_p \right\rangle \left\langle N_{\bar{p}} \right\rangle \]

correlation term

- If multiplicity distributions of protons and anti-protons are Poissonian and uncorrelated
  → Skellam distribution for net-protons

\[ \kappa_2(Skellam) = \kappa_1(p) + \kappa_1(\bar{p}) \]
Net-proton fluctuations

\[ \kappa_1(p) = \langle N_p \rangle \]
\[ \kappa_2(p) = \langle \left( N_p - \langle N_p \rangle \right)^2 \rangle \]
\[ \kappa_2(p - \bar{p}) = \langle \left( N_p - N_{\bar{p}} - \langle N_p - N_{\bar{p}} \rangle \right)^2 \rangle \]
\[ = \kappa_2(p) + \kappa_2(\bar{p}) - 2 \langle N_p N_{\bar{p}} \rangle - \langle N_p \rangle \langle N_{\bar{p}} \rangle \]

- \( \kappa_2(p-\bar{p}) \) shows deviation from Skellam prediction
  - due to correlation term?
  - are protons and anti-protons Poissonian?

\[ \kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p}) \]
Net-proton fluctuations

- Modeling the effects of participant fluctuations and global baryon number conservation

- Inputs to the model: $\kappa_1(p)$, $\kappa_1(\bar{p})$, centrality determination procedure

- Model gives a consistent picture of $\kappa_2(p)$, $\kappa_2(\bar{p})$ and $\kappa_2(p-\bar{p})$ without need of correlations or critical fluctuations
Global conservation laws

- Small $\Delta \eta \rightarrow$ Poissonian fluctuations, ratio to Skellam $\sim 1$
- Large $\Delta \eta \rightarrow$ global baryon number and strangeness conservation effects, ratio to Skellam < 1
- $\Delta \eta$ dependence consistent with effects of baryon number conservation

ALICE Preliminary, Pb-Pb $\sqrt{s_{\text{NN}}} = 2.76$ TeV
$0.6 < p < 1.5$ GeV/$c$, centrality 0-5%


A. Rustamov for ALICE, QM2017

ALI-PREL-122602
Global conservation laws

- Contribution from global baryon number conservation calculated as
  \[ \kappa_2(p - \bar{p}) = 1 - \frac{\langle N_p^{\text{meas}} \rangle}{\langle N_B^{4\pi} \rangle} = 1 - \alpha \]

- Inputs for \(<N_B^{\text{acc}}\) from


  Extrapolation from \(<N_B^{\text{acc}}\) to \(<N_B^{4\pi}\) using AMPT and HIJING

- Deviation from Skellam baseline accounted for by global baryon number conservation

A. Rustamov for ALICE, QM2017
Looking for signs of criticality at $\mu_B = 0$
-- higher moments at the LHC --
Higher moments

• Deviations from unity and signs of criticality are greatly enhanced for the higher moments (4\textsuperscript{th}, 6\textsuperscript{th}, 8\textsuperscript{th},...)

• But huge statistics are needed and experimental effects must be carefully controlled
First higher moments from ALICE

- Consistent results between $\sqrt{s_{NN}} = 2.76$ TeV and 5.02 TeV within statistical and systematic uncertainties
- In central events, consistency with Skellam baseline ($C_4/C_2 = 1$)
- Higher statistics and improved understanding of systematics are needed to obtain the precision needed for LQCD comparisons
Exploring the phase diagram and searching for the critical point
-- higher moments at RHIC and SIS --
STAR results: net-charge, net-K, net-p

**Net-Proton**

<table>
<thead>
<tr>
<th>Skellam Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net-proton</td>
</tr>
<tr>
<td>0.4&lt;p_T&lt;0.8 (GeV/c),</td>
</tr>
</tbody>
</table>

| Colliding Energy (|s_NN| (GeV)) |
|-------------------|
| 5 6 7 8 10 20 30  |
| 40 50 60 70 80 100 |
| 200                |

**Net-Charge**

<table>
<thead>
<tr>
<th>a) Net-charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au+Au</td>
</tr>
<tr>
<td>h_y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>M/σ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
</tbody>
</table>

| 0.03 |
| 0.1  |
| 0.2  |
| 0.3  |

<table>
<thead>
<tr>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

**Net-Kaon**

<table>
<thead>
<tr>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>


STAR results: net-protons in the BES

- non-monotonic behavior observed below $\sqrt{s_{NN}} = 39$ GeV

STAR, PRL 112 (2014) 032302
STAR results: net-protons in the BES

- non-monotonic behavior observed below \( \sqrt{s_{NN}} = 39 \text{ GeV} \)

**talk by Ashish Pandav (STAR)**

STAR, PRL 112 (2014) 032302
• At RHIC, proton and anti-proton multiplicities not equal

\[
\frac{\kappa_3(n_p - n_{\bar{p}})}{\kappa_2(n_p - n_{\bar{p}})} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}} (1 - 2\alpha)
\]

\[
\frac{\kappa_4(n_B - n_{\bar{B}})}{\kappa_2(n_B - n_{\bar{B}})} = 1 - 6\alpha(1 - \alpha) \left[ 1 - \frac{2}{\langle N_B + N_{\bar{B}} \rangle_{CE}} \left( \langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_{\bar{B}} \rangle_{CE} \right) \right]
\]

• Above \( \sqrt{s_{NN}} = 11.5 \) GeV: deviation from unity can be described by global baryon number conservation
experimental effects and systematic uncertainties must be very precisely controlled before drawing conclusions, but sixth moments appear to be within the statistical reach → stay tuned for updates!

talk by Ashish Pandav (STAR)
STAR + HADES: net-protons vs $\sqrt{s_{NN}}$

- different correction methods:
  - unfolding + volume fluctuation correction
  - E-by-E correction of factorial moments + vol. fluct. corr.

$\frac{\kappa_4}{\kappa_2}$

- HADES preliminary
- (Net-)protons
  - HADES 0-10 %
  - HADES 30-40 % ($\Delta y = 0.2$)
  - STAR 0-5 %
  - STAR 30-40 % ($\Delta y = 0.5$)

$\sqrt{s_{NN}}$

- large differences in results (still under investigation)

T. Galatyuk, CPOD 2018

8 June 2019

Correlations and fluctuations
A. Ohlson (Lund U.)
Correlated fluctuations of conserved quantities
-- net-Λ fluctuations and mixed cumulants --
Correlated fluctuations

- Probe correlated fluctuations of net-charge, net-strangeness, net-baryon number
- Access off-diagonal elements, mixed derivatives $\chi^{BS}$, $\chi^{BQ}$, $\chi^{QS}$
- Provides a more complete set of comparisons to LQCD predictions

- Experimental observables:
  - Net-$\Lambda$ fluctuations
  - Cross-cumulants

F. Karsch, EMMI Workshop on Fluctuations, Wuhan, October 2017
**Net-$$\Lambda$$ fluctuations**

\[ C_1(\Lambda) = \langle N_\Lambda \rangle \quad C_2(\Lambda) = \left\langle \left( N_\Lambda - \langle N_\Lambda \rangle \right)^2 \right\rangle \]

\[ C_2(\Lambda - \bar{\Lambda}) = \left\langle \left( N_\Lambda - N_{\bar{\Lambda}} - \langle N_\Lambda - N_{\bar{\Lambda}} \rangle \right)^2 \right\rangle \]

\[ = C_2(\Lambda) + C_2(\bar{\Lambda}) - 2\left( \langle N_\Lambda N_{\bar{\Lambda}} \rangle - \langle N_\Lambda \rangle \langle N_{\bar{\Lambda}} \rangle \right) \]

- Small deviations from Skellam baseline \(\rightarrow\) correlation term?
- non-Poissonian $$\Lambda$$ or $$\bar{\Lambda}$$ distributions?
- critical fluctuations?
- effects of volume fluctuations and global conservation laws?

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**ALICE Preliminary**

Pb–Pb, \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \)

\( 1 < p_{T,\Lambda} < 4 \text{ GeV}/c, |\eta_\Lambda| < 0.5 \)
Comparison to net-protons

Qualitatively similar results for net-protons

- different kinematic range,
- different contributions from resonance decays
Δη dependence of net-Λ fluctuations

- Small Δη → Poissonian fluctuations, ratio to Skellam ~1
- Large Δη → global baryon number and strangeness conservation effects, ratio to Skellam < 1
- Systematic uncertainties are highly correlated point-to-point
- Δη dependence consistent with effects of baryon number conservation → strangeness conservation should also be considered
- Consistency also with HIJING
Net-Λ fluctuations at STAR

STAR Preliminary

Au + Au collisions at STAR
0.9 < p_T (GeV/c) < 2.0
|y| < 0.5

Correlations and fluctuations
A. Ohlson (Lund U.)

8 June 2019

talk by Rene Bellwied (STAR)
talk by Rene Bellwied (STAR)

• Effect of net-baryon number conservation → can describe the data if B and S conservation treated additively
Cross-cumulants at STAR

- Full matrix of cross-cumulants of Q, p, K, up to second order measured in the BES

\[ \sigma_{\alpha,\beta}^{1,1} = \langle (\delta N_\alpha - \langle \delta N_\alpha \rangle) (\delta N_\beta - \langle \delta N_\beta \rangle) \rangle \]

Diagram showing the cross-cumulants for different energies and nuclear models.
Cross-cumulants in the BES

- Take ratios to remove volume dependence and self-correlations
  - Effects of resonance decays must be quantitatively assessed
- Measurements do not agree with Poisson baseline, UrQMD or HRG predictions

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STAR, arXiv:1903.05370 [nucl-ex]
What have we learned so far?
Where do we go from here?
Conclusions

• Event-by-event fluctuations of identified particles
  – yield information on properties of the QGP medium
  – test lattice QCD predictions at $\mu_B = 0$
  – allow us to look for effects of criticality

• Progress in bringing experimental and theoretical effects under control in order to perform quantitative comparison

• A wealth of data from ALICE, STAR, and HADES
  – Net-proton and net-$\Lambda$ fluctuations at LHC energies: no deviations from Skellam baseline observed after accounting for baryon number conservation, agreement with LQCD predictions
  – Net-proton fluctuations at RHIC energies: can be described above $\sqrt{s_{NN}} = 11.5$ GeV by baryon number conservation
  – Net-$\Lambda$ fluctuations and cross-cumulants allow large set of comparisons to theory including LQCD
Outlook

- Runs 3+4 at the LHC will allow us to measure the fourth and sixth moments of the net-proton distribution with unprecedented precision

- BES-II + detector upgrades at RHIC will allow us to probe fluctuations across a wide range of the phase diagram


Grazie!
backup
Net-pion and net-kaon fluctuations

- Pions show good agreement with HIJING
- Production of pions and kaons from resonance decays contributes significantly to the measurement
- Skellam distribution is not a proper baseline for net-pions and net-kaons
• Deviations from Skellam can be attributed to global baryon number conservation, more significant in more peripheral collisions
• Disagreement with HIJING
First higher moments from ALICE!

- Measured with traditional (cut-based) PID method
- Consistent results between $\sqrt{s_{NN}} = 2.76$ TeV and 5.02 TeV within statistical and systematic uncertainties
- In central events, consistency with Skellam baseline ($C_4/C_2 = 1$) at LHC energies

8 June 2019

Correlations and fluctuations
A. Ohlson (Lund U.)
Comparison to HIJING

- HIJING does not describe strangeness production well
  - underestimates $C_1$ and $C_2$ by factor $\sim 4$
- However, $C_2(\Lambda-\bar{\Lambda})/C_2(\text{Skellam})$ ratio agrees with data
  - coincidence? or due to description of fluctuations and resonance contributions in HIJING?
The challenge: event-by-event PID

- Traditional method:
  - count number of pions ($N_\pi$), kaons ($N_K$), protons ($N_p$) in each event
    $$N_p = \sum_{i}^{\text{tracks}} \begin{cases} 1 & \text{particle } i \text{ is a proton} \\ 0 & \text{particle } i \text{ is not a proton} \end{cases}$$
  - find moments of distributions of $N_\pi$, $N_K$, $N_p$, ....
Traditional method

- What if PID is unclear?
  - use other detector information or reject phase space bin
  - results in lower efficiency
• As a function of the PID variable \( m \), determine probability \( w \) that particle is of a given species
• Calculate event-by-event sum of weights \( W_\pi, W_K, W_p, \ldots \)
\[
W_p = \sum_i w_p(m_i)
\]
• Using knowledge of inclusive \( m \) distributions, unfold moments of \( W \) distributions to get moments of \( N \)
• Contamination is accounted for, full phase space can be used
Identity method for invariant mass

- Net-$\Lambda$ fluctuations: explore correlated fluctuations of baryon number and strangeness
- For any value of $m_{\text{inv}}$, the probability that a $\pi p$ pair comes from the decay of a $\Lambda$ baryon is known
- Apply Identity Method for four “species”: $\Lambda$, $\bar{\Lambda}$, combinatoric $\pi^- p$, combinatoric $\pi^+ p$
Efficiency corrections: several ideas

- Simple scaling of moments using HIJING and/or AMPT
- Correction of factorial moments assuming binomial track loss
  - extension to Identity Method
- Correction using moments of detector response matrix
- Full unfolding of moments

All correction methods rely on different assumptions, which must be assessed and tested carefully!


Net-Λ fluctuations at STAR

0-5% central collisions
- Net L, data
- Net kaon, data
- Net proton, data

STAR Preliminary

(b)
Net L : 0.9 < p_T (GeV/c) < 2.0, lyl < 0.5
Net proton : 0.4 < p_T (GeV/c) < 0.8, lyl < 0.5
Net kaon : 0.2 < p_T (GeV/c) < 1.6, lyl < 0.5

STAR Preliminary
What is the correlation length?

• Balance functions of identified particles

\[
B(\Delta y) = \frac{1}{2} \left[ C_2^{+-} - C_2^{++} + C_2^{--} - C_2^{+} \right]
\]

Bass, Danielewicz, Pratt PRL. 85, 2689 (2000)

\[
C_{2}^{a,b}(\Delta y) = \frac{\langle N_{a,b}^{a,b}(\Delta y) \rangle}{\langle N_{a}^{a}(\Delta y) \rangle} \quad a, b \in \{+, -\}
\]

same for \( \Delta \phi \) and \( \Delta p_T \)

J. Pan for ALICE, QM2018

General charges:
• e: (±)electric charge
• S: (anti)strangeness
• B: (anti)baryon number

• Sensitive to physics of balancing charges:
  – Charge production mechanisms, two-wave production scenario, radial flow, quantum statistics, ...

Bass, Danielewicz, Pratt PRL. 85, 2689 (2000)

ALICE Preliminary Pb-Pb \( \sqrt{s} = 2.76 \text{ TeV} \)

0.2<p<2 GeV/c \( \Delta y \): 30-40%

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Bass, Danielewicz, Pratt PRL. 85, 2689 (2000)
• Width of $\pi$ balance function shows centrality dependence, K balance function width is independence of centrality → consistent with two-wave production model, but other physical effects (e.g. radial flow) must be considered
• Full species matrix (including protons) coming soon!