Recent results on QCD thermodynamics:

Lattice QCD

versus

Hadron Resonance Gas model

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Based on: S. Borsanyi, Z. Fodor, C. Hölbling, S. Katz, S. Krieg, C. R. and K. Szabó, 1005.3508

S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. Katz, S. Krieg, C. R. and K. Szabó 1007.2580

Motivation

- Bielefeld-Brookhaven-Riken-Columbia (+ MILC = hotQCD) Collaboration:

  M. Cheng *et al.* PRD 74 (2006)

  ➠ Chiral susceptibility and Polyakov loop both give $T_c = 192(7)(4)$ MeV

- Wuppertal-Budapest (WB) Collaboration:

  Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, PLB 643 (2006)

  ➠ Chiral susceptibility gives $T_c = 151(3)(3)$ MeV

  ➠ Polyakov loop and strange quark number susceptibilities give $T_c = 175(2)(4)$ MeV

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- 'deconfinement T_c': ~ 15 MeV difference

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- 'chiral T_c': ~ 40 MeV difference

- 'deconfinement T_c': ~ 15 MeV difference

both results are

continuum extrapolated

with physical quark masses

Differences between the two approaches

Wuppertal-Budapest collaboration

- 2006-2009

  Y. Aoki *et al.*, PLB (2006), JHEP (2009)

  - stout action
  - $N_t = 8, 10, 12$
  - $m_s/m_{u,d} = 28.15$

    $\Downarrow$

    $m_\pi = 135$ MeV

hotQCD collaboration

- 2006-2009

  M.Cheng *et al.* PRD(2006),A.Bazavov *et al.* PRD(2009)

  - asqtad, p4 actions
  - $N_t = 6, 8$
  - $m_s/m_{u,d} = 10$

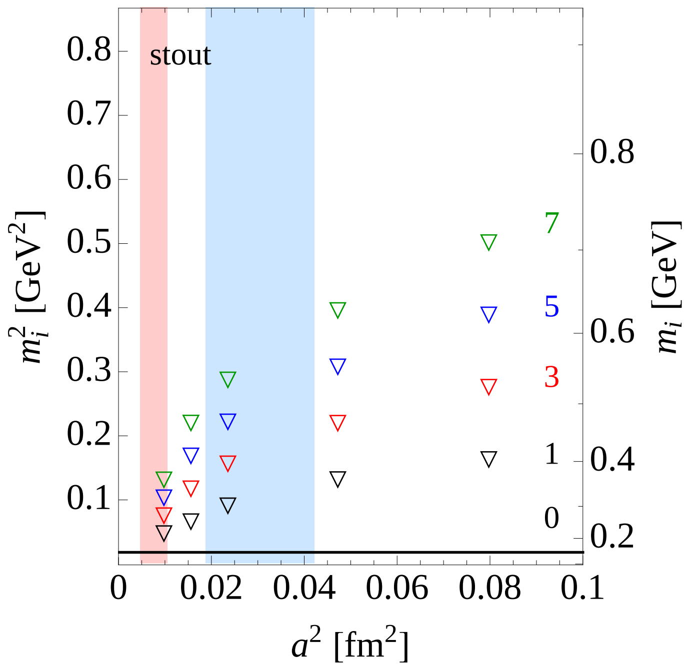
    $\Downarrow$

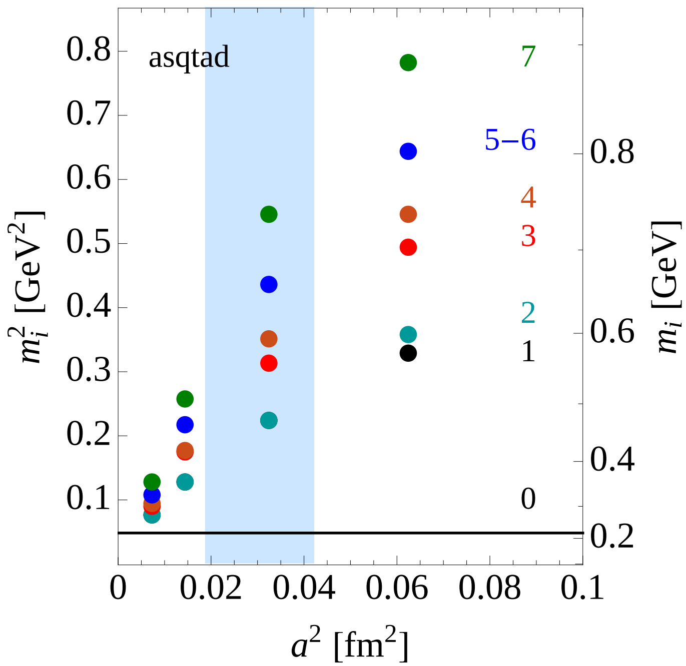
    $m_\pi = 220$ MeV

... but m_π is not unique in the staggered formulation

Pseudo-scalar mesons in staggered formulation

- Staggered formulation: four degenerate quark flavors (‘tastes’) in the continuum limit
- Rooting procedure: replace fermion determinant in the partition function by its fourth root
- At finite lattice spacing the four tastes are not degenerate
  - each pion is split into 16
  - the sixteen pseudo-scalar mesons have unequal masses
  - only one of them has vanishing mass in the chiral limit





New results from the two collaborations

Wuppertal-Budapest collaboration

- 2006-2009

  Y. Aoki *et al.*, PLB (2006), JHEP (2009)

  - stout action
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  - $m_s/m_{u,d} = 28.15$

    $\Downarrow$

    $m_\pi = 135$ MeV

- 2010

  S. Borsanyi *et al.*, 1005.3508

  - $N_t = 16 \Rightarrow$ continuum

hotQCD collaboration

- 2006-2009

  M.Cheng *et al.* PRD(2006),A.Bazavov *et al.* PRD(2009)

  - asqtad, p4 actions
  - $N_t = 6, 8$
  - $m_s/m_{u,d}=10 \Rightarrow m_\pi=220$ MeV

- 2009-2010

  M. Cheng *et al.* PRD (2010)

  - p4 action
  - $N_t = 8$
  - $m_s/m_{u,d}=20 \Rightarrow m_\pi=160$ MeV

  A. Bazavov, P. Petreczky, 1005.1131 (2010)

  - asqtad, $N_t = 12$, hisq, $N_t = 6, 8$

Purpose of this analysis

Use the Hadron Resonance Model

in order to identify

the origin of the discrepancy

In particular:

$\Rightarrow$ **Discretization effects**

$\Rightarrow$ **Effects due to heavy pions**

Partition function of HRG model

- The pressure can be written as

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \; mesons} \ln \mathcal{Z}^{M}_{m_i}(T, V, \mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \; baryons} \ln \mathcal{Z}^{B}_{m_i}(T, V, \mu_{X^a}) \; ,$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \; .$$

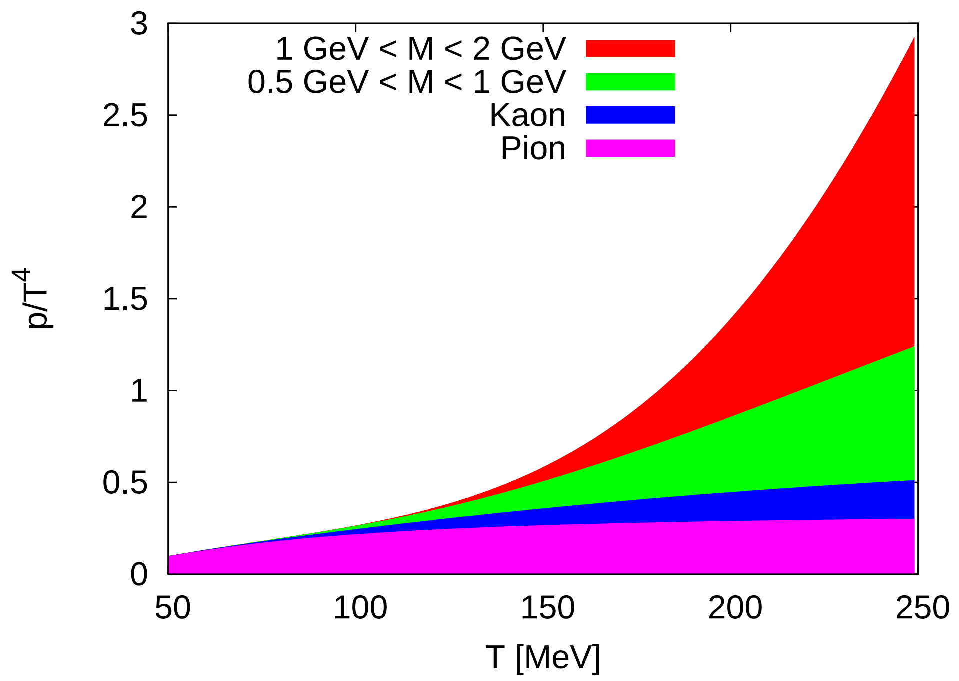
X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

Hadronic species from the Particle Data Book

hadron	m_i (GeV)	d_i	B_i	S_i	I_i	hadron	m_i (GeV)	d_i	B_i	S_i	I_i
π	0.140	3	0	0	1	N (1535)	1.530	4	1	0	1/2
K	0.496	2	0	1	1/2	π_1 (1600)	1.596	9	0	0	1
$\overline{K}$	0.496	2	0	-1	1/2	Δ (1600)	1.600	16	1	0	3/2
η	0.543	1	0	0	0	Λ (1600)	1.600	2	1	-1	0
ρ	0.776	9	0	0	1	Δ (1620)	1.630	8	1	0	3/2
ω	0.782	3	0	0	0	η_2 (1645)	1.617	5	0	0	0
K^*	0.892	6	0	1	1/2	N (1650)	1.655	4	1	0	1/2
$\overline{K}^*$	0.892	6	0	-1	1/2	ω (1650)	1.670	3	0	0	0
N	0.939	4	1	0	1/2	Σ (1660)	1.660	6	1	-1	1
η'	0.958	1	0	0	0	Λ (1670)	1.670	2	1	-1	0
f_0	0.980	1	0	0	0	Σ (1670)	1.670	2	1	-1	1
a_0	0.980	3	0	0	1	ω_3 (1670)	1.667	7	0	0	0
ϕ	1.020	3	0	0	0	π_2 (1670)	1.672	15	0	0	1
Λ	1.116	2	1	-1	0	Ω^-	1.672	4	1	-3	0
h_1	1.170	3	0	0	1	N (1675)	1.675	12	1	0	1/2
Σ	1.189	6	1	-1	1	ϕ (1680)	1.680	3	0	0	0
a_1	1.230	9	0	0	1	K^* (1680)	1.717	6	0	1	1/2
b_1	1.230	9	0	0	1	$\overline{K}^*$ (1680)	1.717	6	0	-1	1/2
Δ	1.232	16	1	0	3/2	N (1680)	1.685	12	1	0	1/2
f_2	1.270	5	0	0	0	ρ_3 (1690)	1.688	21	0	0	1
K_1	1.273	6	0	1	1/2	Λ (1690)	1.690	4	1	-1	0
$\overline{K}_1$	1.273	6	0	-1	1/2	Ξ (1690)	1.690	8	1	-2	1/2
f_1	1.285	3	0	0	1	ρ (1700)	1.720	9	0	0	1
η (1295)	1.295	1	0	0	0	N (1700)	1.700	8	1	0	1/2
π (1300)	1.300	3	0	0	1	$\Delta(1700)$	1.700	16	1	0	3/2

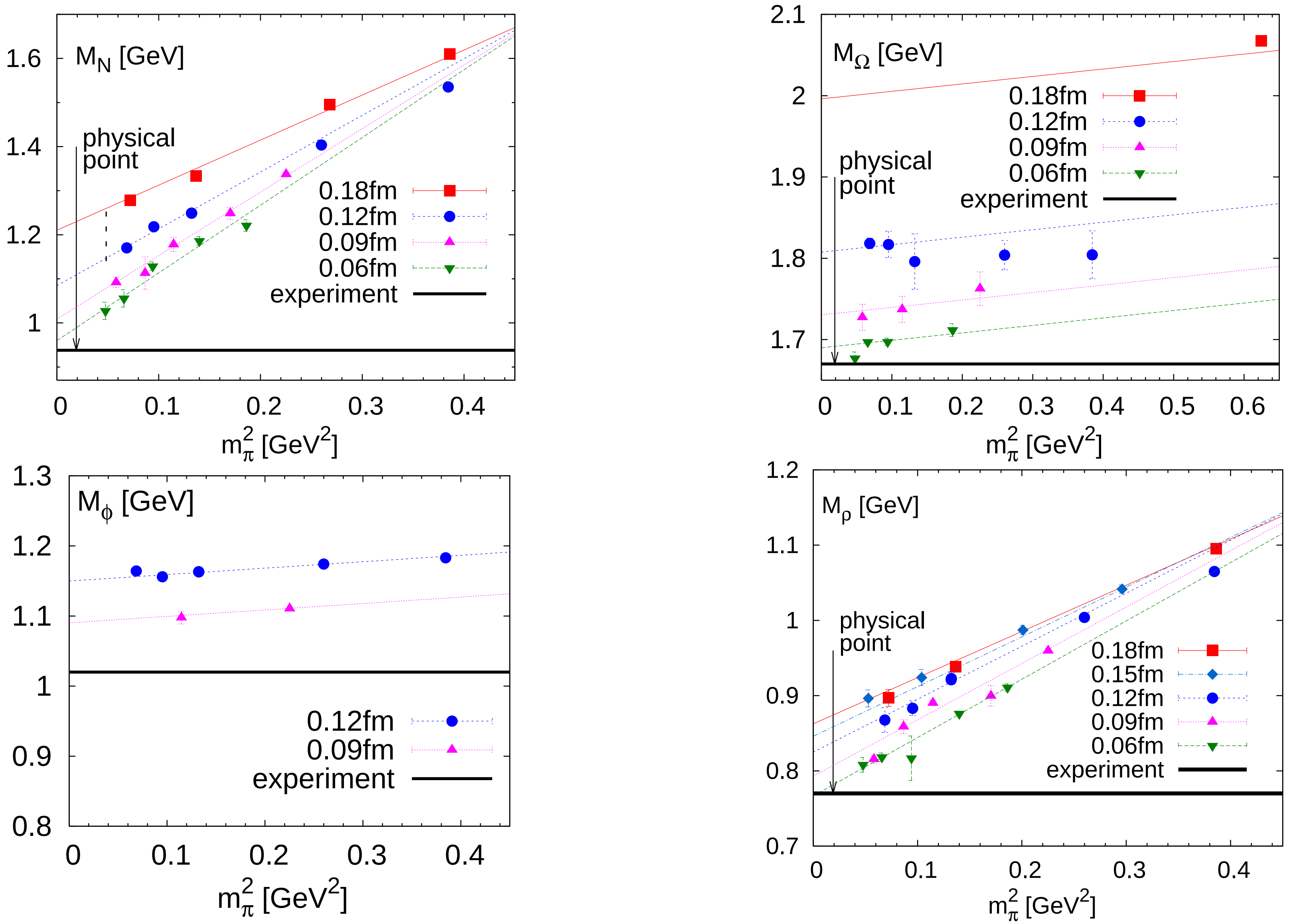
How many resonances do we include?

- With different mass cutoffs we can separate the contributions of different particles



- No visible difference between cuts at 2 GeV and 2.5 GeV in our temperature regime
- We include all resonances with $M \leq$ 2.5 GeV
  - ➡ $\simeq 170$ different masses $\leftrightarrow$ 1500 resonances

Discretization effects



C. W. Bernard *et al.*, PRD (2001), C. Aubin *et al.*, PRD (2004), A. Bazavov *et al.*, 0903.3598.

Hadron masses

- Non-strange baryons and mesons:

$$r_1 m = r_1 m_0 + \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x}, \qquad x = (\frac{a}{r_1})^2$$

- Strange baryons and mesons:

$$r_1 \cdot m_\Lambda(a, m_\pi) = r_1 m_\Lambda^{phys} + \frac{2}{3} \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Lambda^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

$$r_1 \cdot m_\Sigma(a, m_\pi) = r_1 m_\Sigma^{phys} + \frac{1}{3} \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Sigma^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right),$$

$$r_1 \cdot m_\Xi(a, m_\pi) = m_\Xi^{phys} + \frac{1}{3} \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Xi^{phys} - m_p^{phys})}{1 + a_2 x} \left(\frac{m_s}{m_s^{phys}} \right)$$

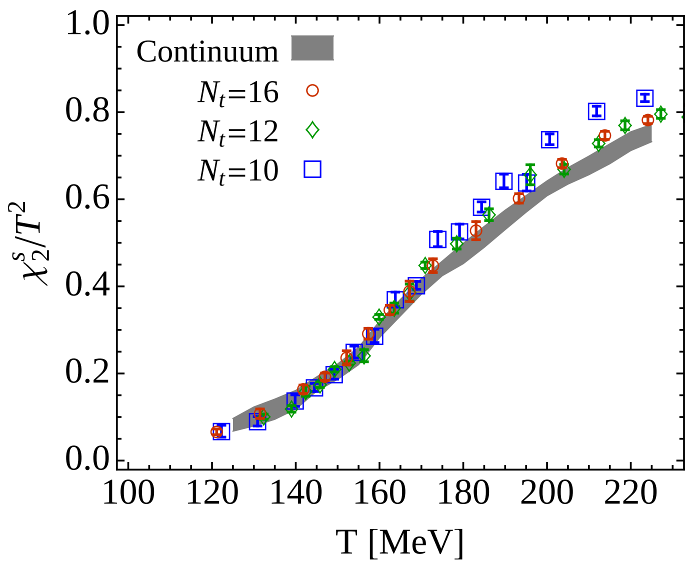
$$r_1 m_\Omega(a, m_\pi) = r_1 m_\Omega^{phys} + a_1 (r_1 m_\pi)^2 - a_1 (r_1 m_\pi^{phys})^2 + b_1 x + (m_\Omega^{phys} - m_\Delta^{phys}) \cdot 1.02 x$$

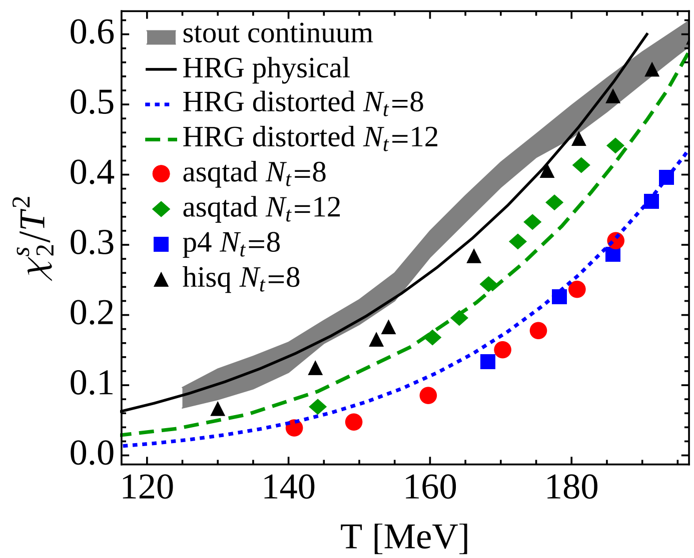
- Distorted spectrum implemented in the HRG model
- Assumption: all resonances behave as their fundamental states

P. Huovinen and P. Petreczky (2009).

Results: strangeness susceptibilities

$$\chi_n^S = T^n \frac{\partial^n p(T,\mu_B,\mu_S,\mu_I)}{\partial \mu_S^n}\Big|_{\mu_X=0}$$



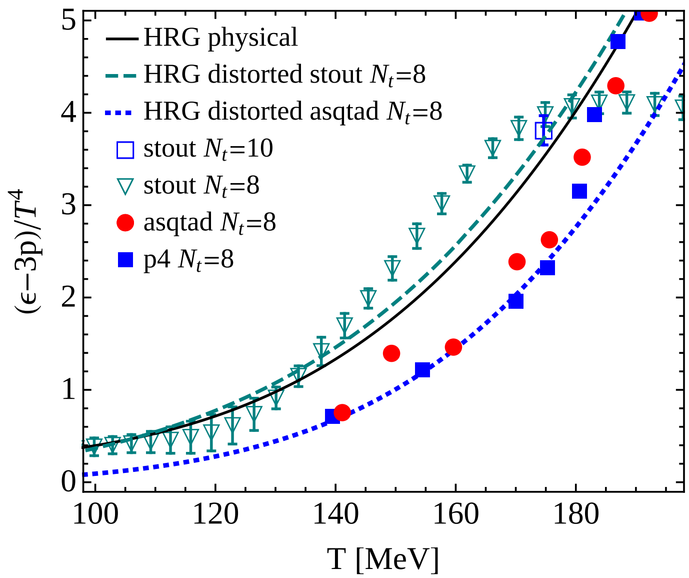


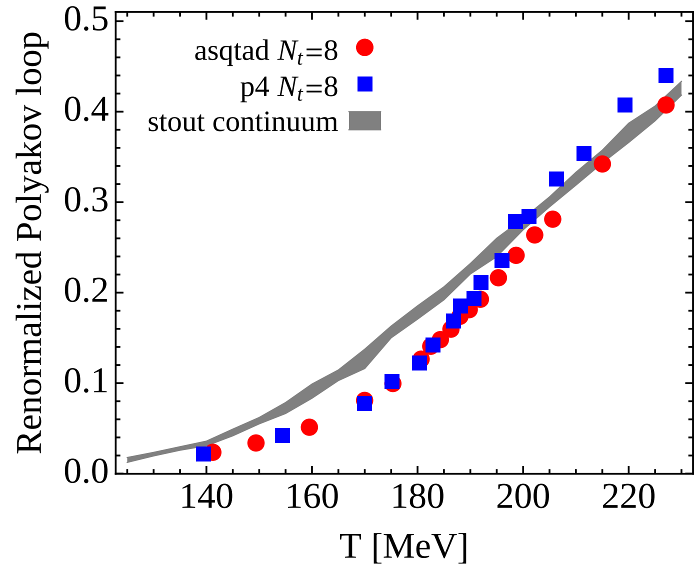
- HRG results in good agreement with stout action
- asqtad and p4 results show similar shape but shift in temperature
  - ➡ HRG results with corresponding distorted spectrum reproduce asqtad and p4 results

S. Borsanyi *et al.*, 1005.3508

Results: trace anomaly and Polyakov loop

$$\frac{\theta(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$





- No significant difference between curves with and without splitting
- Very good agreement between physical HRG and stout results
- Very good agreement between distorted HRG and asqtad/p4 results

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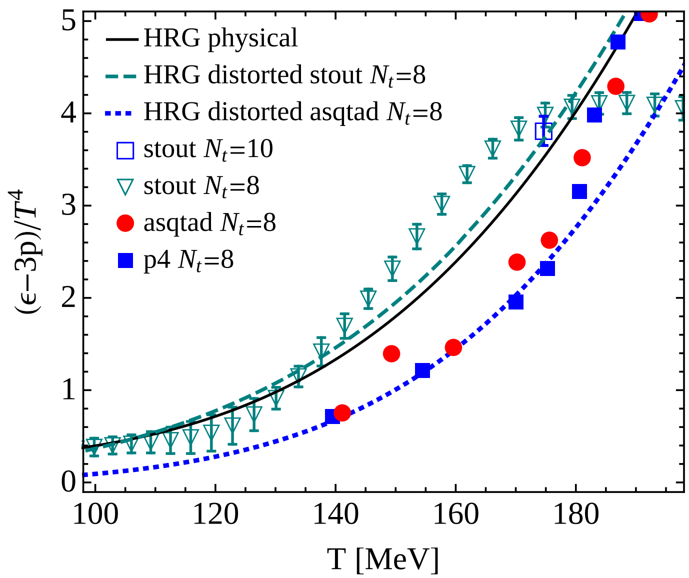
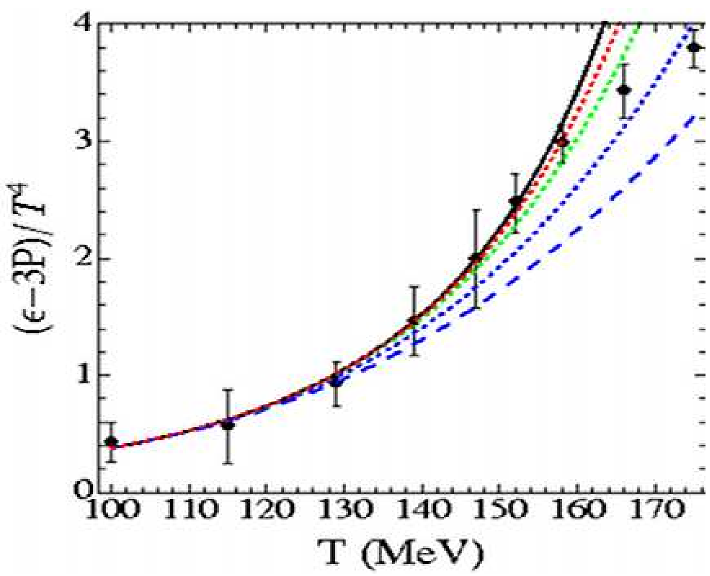
Inclusion of exponential spectrum in HRG model

$$\frac{\theta(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$

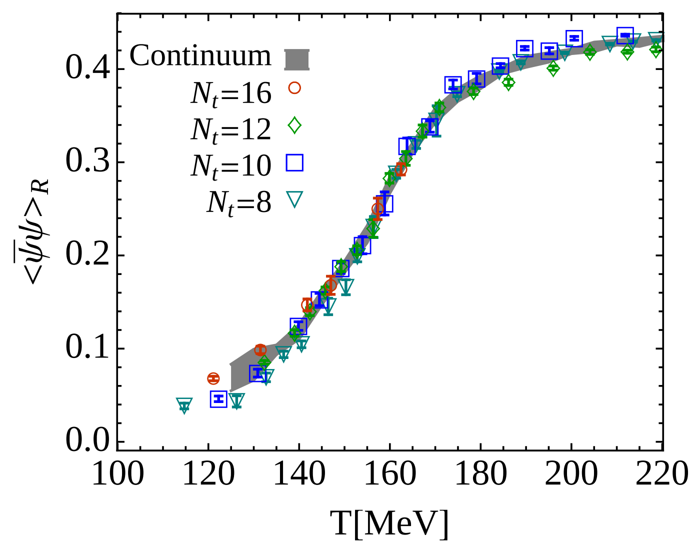


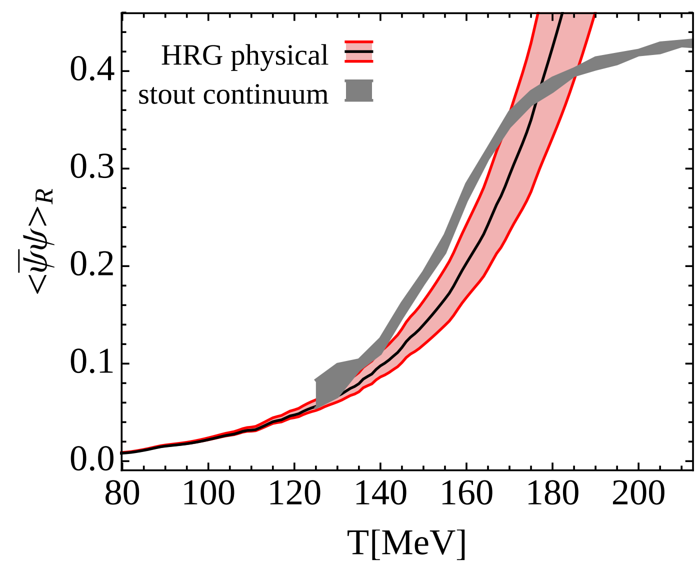


- For large masses few states are known experimentally
- Inclusion of exponentially growing hadron mass spectrum (A. Majumder, B. Müller: 1008.1747)
- Agreement between lattice and HRG improved up to $T \sim 155$ MeV

Results: chiral condensate

$$\langle\bar{\psi}\psi\rangle_R = -\left[\langle\bar{\psi}\psi\rangle_{l,T} - \langle\bar{\psi}\psi\rangle_{l,0}\right]\frac{m_l}{X^4} \quad \text{with} \quad \langle\bar{\psi}\psi\rangle_l = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$





$$\langle\bar{\psi}\psi\rangle_l = \langle\bar{\psi}\psi\rangle_{l,0} + \langle\bar{\psi}\psi\rangle_\pi + \sum_{i\in mesons}\frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l} + \sum_{i\in baryons}\frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l}.$$

- Contribution of pions from Chiral Perturbation Theory Gerber and Leutwyler (1989)
- $\frac{\partial m_i}{\partial m_\pi^2}$ from fit to lattice data Camalich, Geng and Vacas (2010)

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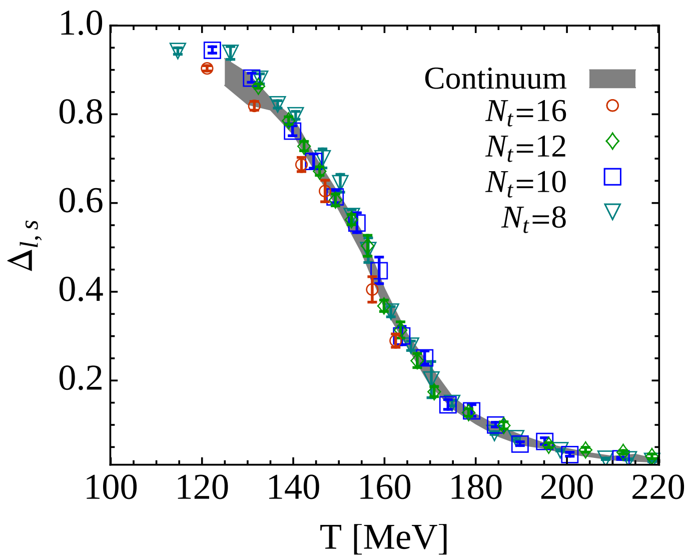
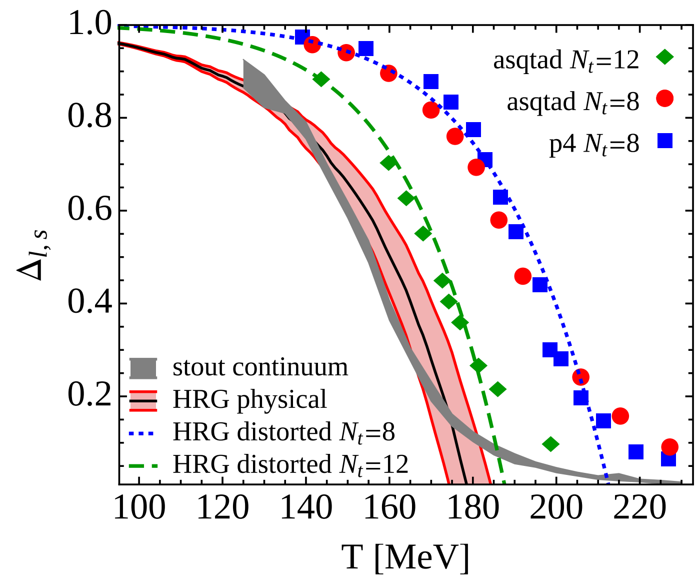
Results: subtracted chiral condensate

$$\Delta_{l,s} = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,0}}$$





$$\langle\bar\psi\psi\rangle_s = \langle\bar\psi\psi\rangle_{s,0} + \langle\bar\psi\psi\rangle_K + \sum_{i\in mesons} \frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s} + \sum_{i\in baryons} \frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s}.$$

- $\frac{\partial m_i}{\partial m_s}$ from fit to lattice data Camalich, Geng and Vacas (2010)

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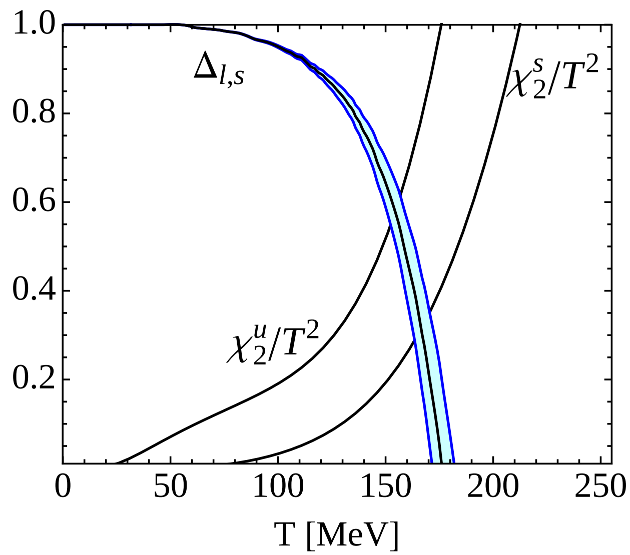
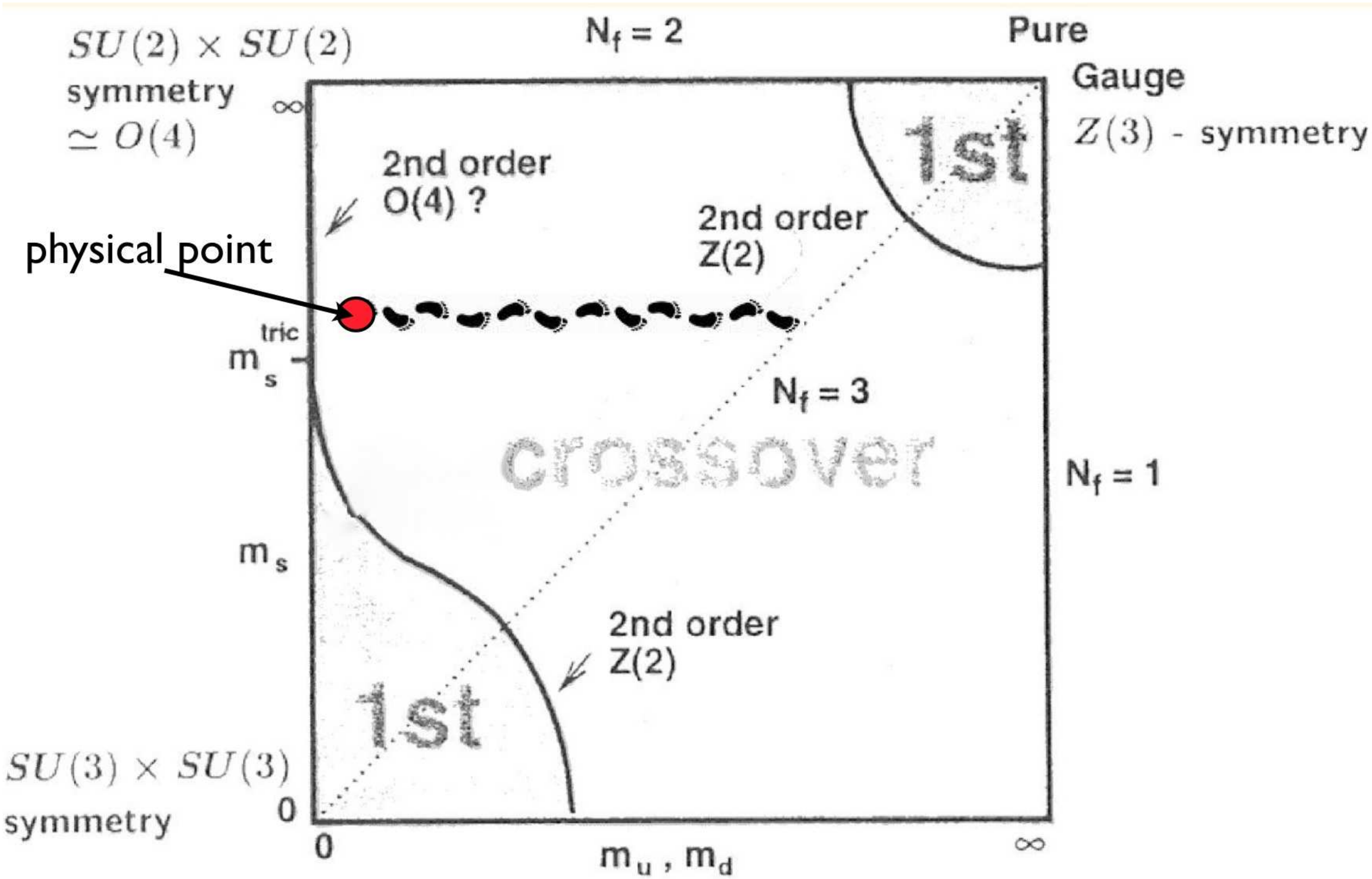
T_c summary from Wuppertal-Budapest collaboration

	$\chi_{\bar{\psi}\psi}/T^4$	$\Delta_{l,s}$	$\langle\bar{\psi}\psi\rangle_R$	χ_2^s/T^2	ϵ/T^4	$(\epsilon-3p)/T^4$
WB'10	147(2)(3)	157(3)(3)	155(3)(3)	165(5)(3)	157(4)(3)	154(4)(3)
WB'09	146(2)(3)	155(2)(3)	-	169(3)(3)	-	-
WB'06	151(3)(3)	-	-	175(2)(4)	-	-

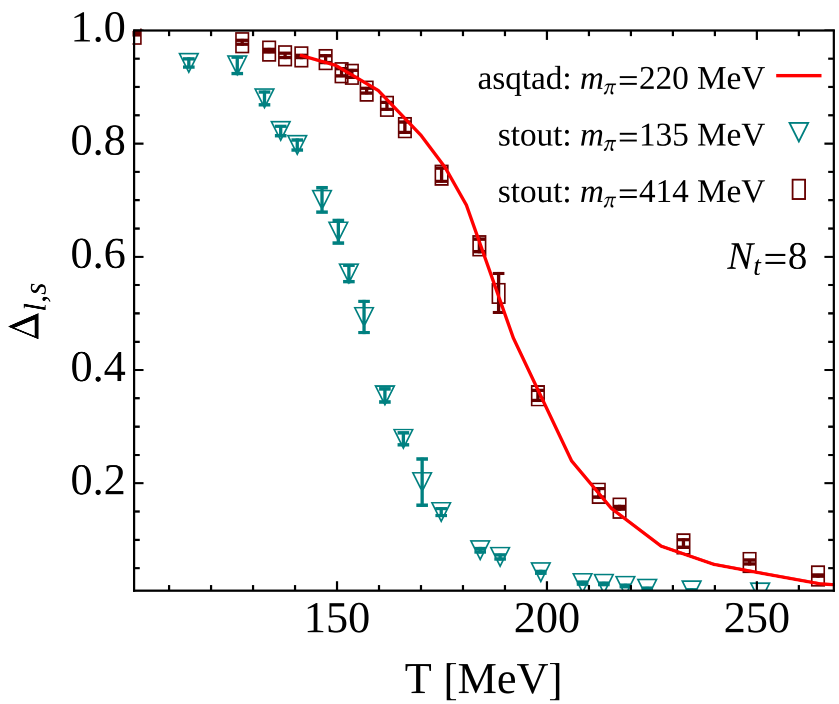
- Different variables give different T_c values: the transition is broad S. Borsanyi *et al.*, 1005.3508
- HRG model predicts the same feature



Pion mass dependence



Transition temperature and m_π



- WB reproduces the hotQCD transition temperature by using a heavier pion mass
  - ➟ corresponds to average pion mass $m_\pi = 587$ MeV at $T = 135$ MeV

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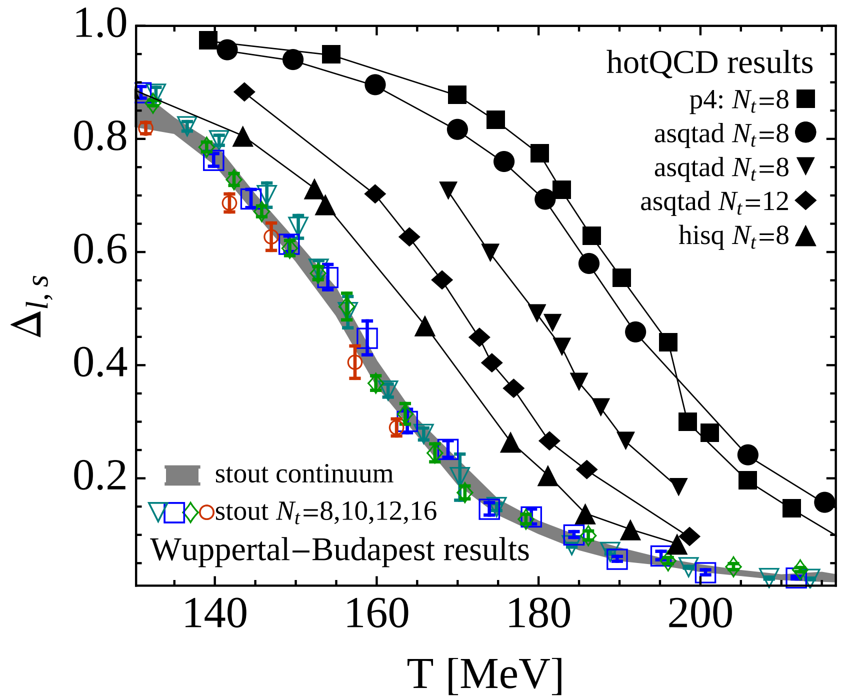
Progress in T dependence (chiral condensate)



- Wuppertal-Budapest: $m_s/m_{u,d} \simeq 28$
  - 2006: $N_t = 8,\ 10$
  - 2009: $N_t = 12$
  - 2010: $N_t = 16$
- hotQCD
  - 2006: $N_t = 4,\ 6$, p4 action
  - 2009: $N_t = 6,\ 8,\ 12,\ m_s/m_{u,d} = 10,\ 20$, asqtad and p4 actions
  - 2010: $N_t = 6,\ 8,\ m_s/m_{u,d} = 20,$ hisq action

Conclusions

- The present analysis concludes the WB investigation of T_c with stout action
- Results from 2006 and 2009 are improved:
  - physical quark masses used in simulations also at $T = 0$
  - smaller lattice spacings $N_t = 16$
  - continuum limit provided for all observables
- The new results are in perfect agreement with those from 2006 and 2009
- The QCD transition is a broad analytic crossover
- Good agreement between HRG model predictions and WB continuum results
- hotQCD results can be reproduced in HRG model with distorted spectrum
- new hotQCD results are getting closer to WB ones

Backup slides

What happens below T_c?

- At low T and $\mu = 0$, QCD thermodynamics is dominated by pions
- The interaction between pions is suppressed
  - chiral perturbation theory: pion contribution to the thermodynamic potential
  - the energy density of pions from 3-loop ChPT differs only less than 15% from the ideal gas value

    P. Gerber and H. Leutwyler (1989)
- as T increases, heavier hadrons start to contribute
- for $T \geq 120$ MeV heavy states dominate the energy density
- their mutual interactions are proportional to $n_i n_k \sim \exp[-(M_i + M_k)/T]$: they are suppressed
  - the virial expansion can be used to calculate the effect of the interaction

Why HRG?

- In the virial expansion, the partition function can be split into a non-interacting piece and a piece which includes all interactions Dashen, Ma and Bernstein (1969)
- virial expansion and experimental information on scattering phase shift Prakash and Venugopalan (1992)
  - ➡ interplay between attractive and repulsive interaction

> Interacting hadronic matter
>
> can be well approximated by
>
> a non-interacting gas of resonances