# E&M radiation from QGP with chromo-magnetic monopoles

**Michael Lublinsky**

Ben-Gurion University

based on M.L. , Claudia Ratti, Edward Shuryak, Phys.Rev.D81:014008,2010

# Introduction

The experimentally observed excess in dilepton production at small $p_t$ and small invariant dilepton mass below $m_\rho$ remains a puzzle: the sum of all known contributions fails to explain the data.

noindent Motivated by this, we search for additional, unexplored mechanisms which might contribute to the dilepton production rate. In particular, we focus on the role played by color-magnetic monopoles: we want to estimate the contribution to dilepton production from the process $\mathbf{qM} \to \mathbf{qM}\gamma$.

We employ the static monopole approximation, which is valid for a regime of temperatures $T \gtrsim 2T_c$, where they can be considered heavy, rare objects embedded into matter consisting mostly of the usual electric quasiparticles, quarks and gluons.

This work is rather methodological and is our initial step towards an exploration of the subject:
at the given stage we by no means claim any resolution of the puzzle.

A "naive estimate" for the ratio of emission rates of quark-monopole vs Coulomb scattering of quarks:

$$\frac{I^{qM}}{I^{qq}} \sim \frac{(eg)^2 v^2}{e^4} \frac{n_M}{n_q}.$$

Here $e$ is the usual (electric) QCD coupling, while $g$ is magnetic.

The Dirac quantization condition implies $\mathbf{eg = 1}$.

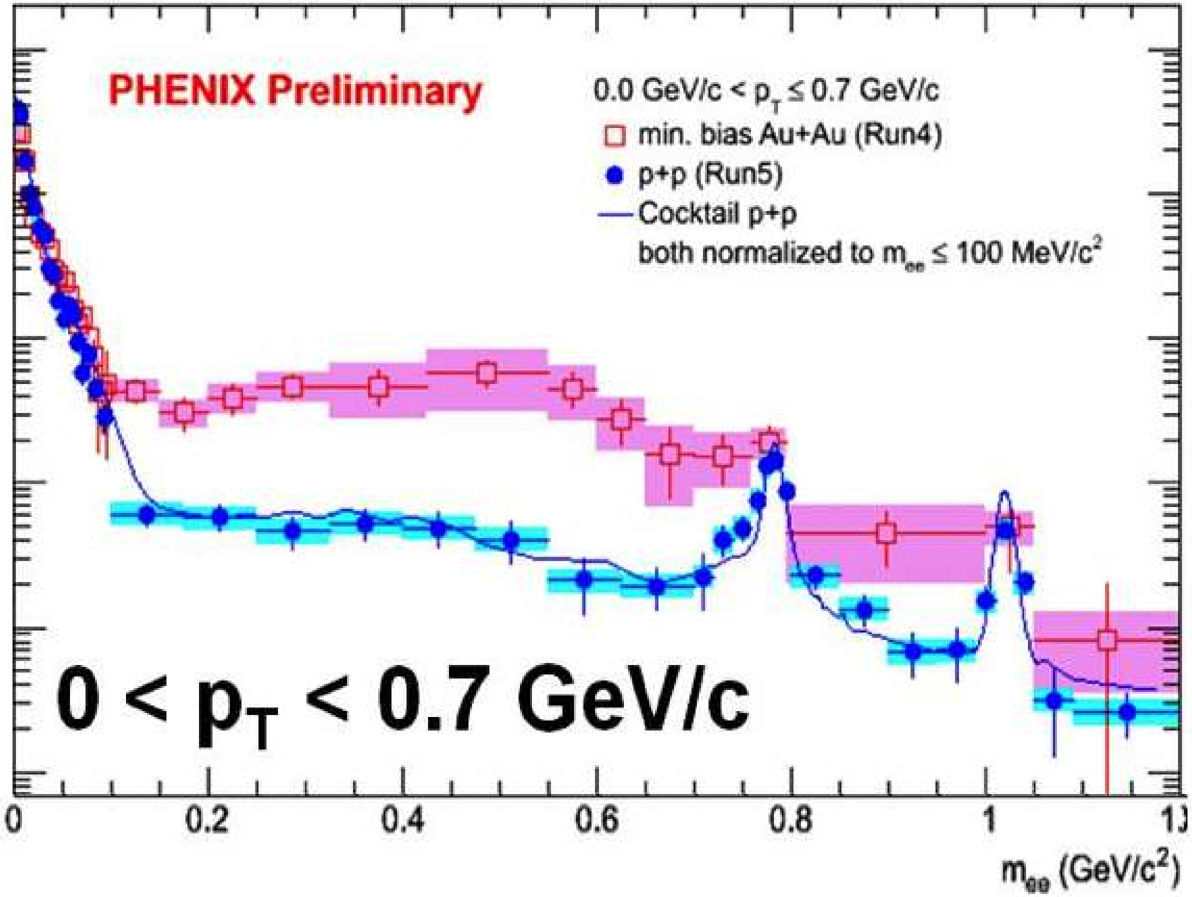
Effectively we are in the strong coupling regime.

For typical (thermal) velocity

$v \sim 0.7, \quad \alpha_s = 0.8, \quad n_M/n_q \sim 0.2$ we end up with

$$\frac{\mathbf{I^{qM}}}{\mathbf{I^{qq}}} \sim \frac{\mathbf{1}}{\mathbf{2}}.$$

# Dilepton Puzzle as the main motivation




hadronic decays in the mixed and hadronic phases of the collision (the hadronic cocktail)

quark-antiquark annihilation in the QGP phase

Compton-like process ($qg \to q\gamma$ and the crossing diagram $q\bar{q} \to \gamma g$),

perturbatively subleading bremsstrahlung diagrams ($qq \to qq\gamma$) and LPM-type resummed effects

# Magnetic Component of QGP

QCD confinement is believed to be related to condensation of monopoles (dual Meissner effect). When deconfinement occurs, monopoles get released into plasma. So, the plasma is not only of quarks and gluons, but also of monopoles.

The Magnetic scenario is strongly advocated by Liao and Shuryak, Phys.Rev.C75:054907,2007

In particular, the magnetic scenario may explain the low viscosity (Liao and Shuryak, Ratti and Shuryak)

We are not advocating this magnetic scenario, but rather assume it is there and use it to estimate monopole's contribution to e/m radiation.

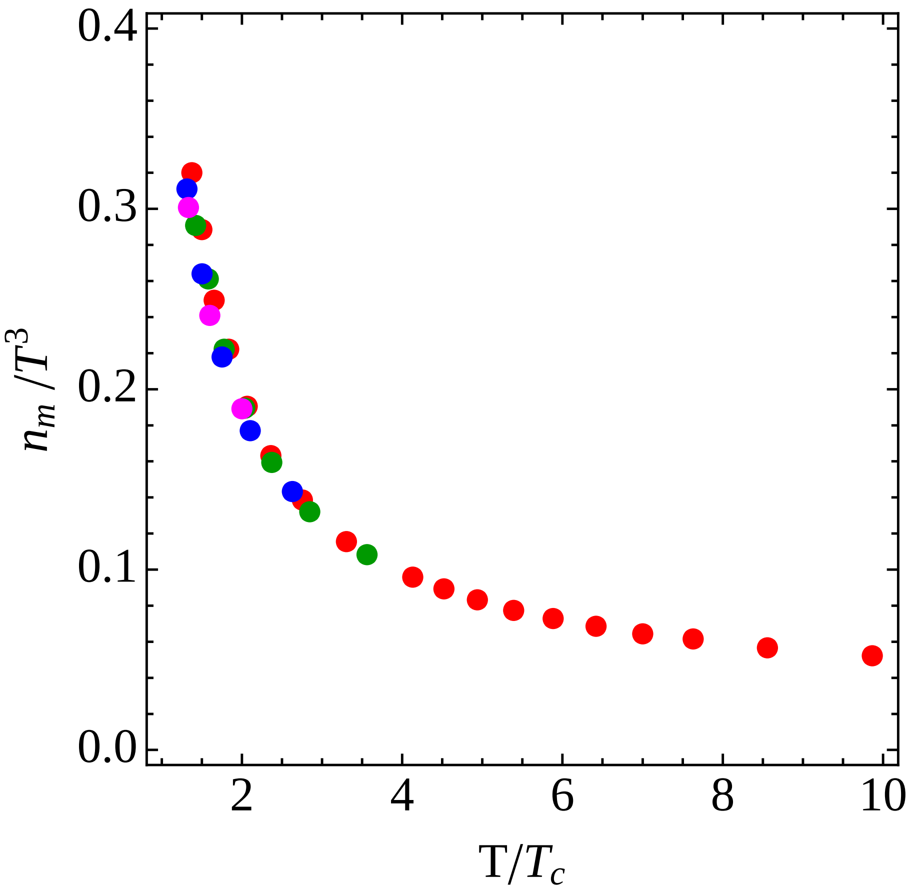
Monopole-like solutions are expected to exist in QCD.
They produce static Coulomb-like (chromo)-magnetic fields

$$\vec{\mathbf{B}}^{\mathbf{a}} = \mathbf{g}\frac{\vec{\mathbf{r}}\,\mathbf{r}^{\mathbf{a}}}{\mathbf{r}^4} \qquad \text{abelian projection} \rightarrow \qquad \vec{\mathbf{B}} = \mathbf{g}\frac{\vec{\mathbf{r}}}{\mathbf{r}^3}$$

QCD electric (quasi) particles $\Longleftrightarrow$ quarks, gluons

Monopoles are QCD magnetic particles

Electric charge $\mathbf{e}$ : $\mathbf{e}^2/4\,\pi = \alpha_s$ Magnetic charge $\mathbf{g}$ : $\mathbf{g}^2/4\,\pi = \alpha_M$




Density of monopoles from the lattice

D'Alessandro and D'Elia (2008)

also Chernodub and Zakharov

# Classical Scattering of an electric particle (quark) off a monopole

When quark scatters off monopole its classical trajectory is a spiral motion on a surface of a cone. To see that we solve for the Lorentz force (Poincare did it first)

$$\mathbf{m}\,\frac{\mathbf{d^2\vec{r}}}{\mathbf{dt^2}} = \mathbf{e}\,\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \frac{\mathbf{eg}}{\mathbf{r^3}}\frac{\mathbf{d\vec{r}}}{\mathbf{dt}} \times \vec{\mathbf{r}}$$

The kinetic energy of the electric charge is conserved

$$\mathbf{E} = \frac{\mathbf{mv^2}}{\mathbf{2}} = \mathrm{const.},$$

The generalized angular momentum

$$\vec{\mathbf{J}} = [\vec{\mathbf{r}} \times \mathbf{m}\vec{\mathbf{v}}] - \mathbf{eg}\frac{\vec{\mathbf{r}}}{\mathbf{r}}$$

is another integral of motion

The trajectory is

$$\mathbf{r} = \sqrt{\mathbf{v^2t^2 + b^2}}.$$

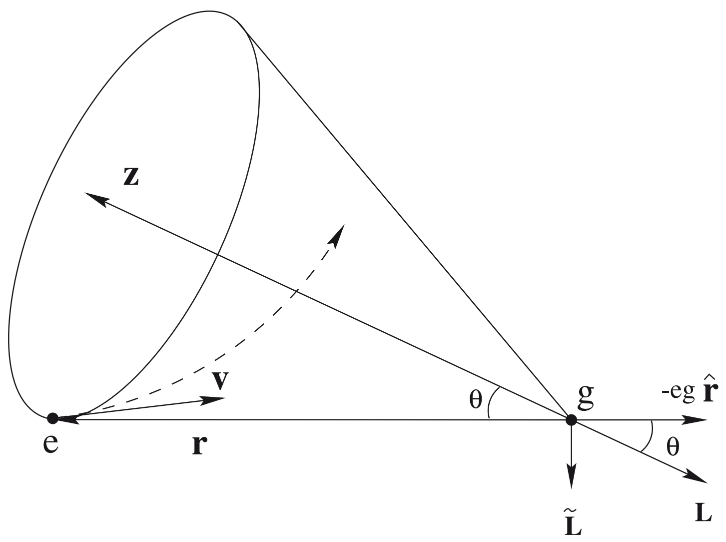
with $b$ being the minimal distance (impact parameter) between the charge and the monopole.

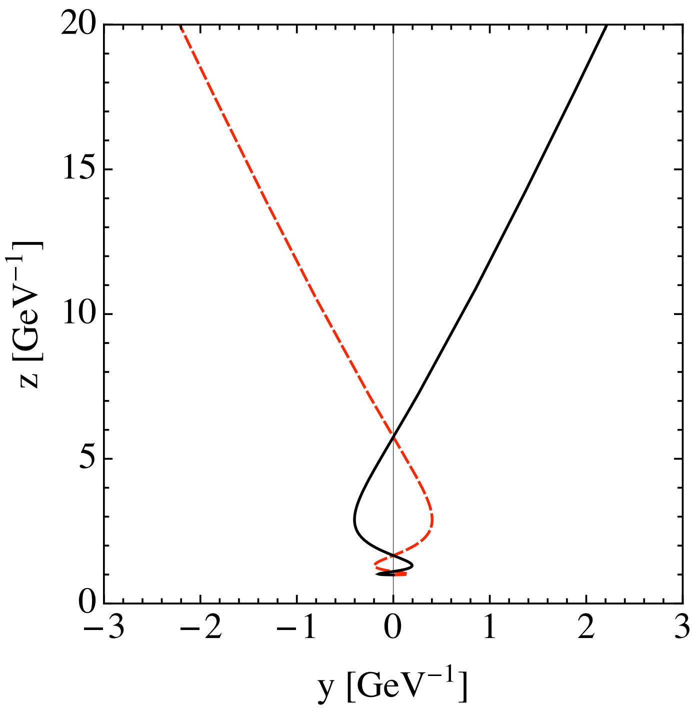
The cone opening angle $\theta$

$$\cos\theta = \frac{\mathbf{eg}}{\sqrt{(\mathbf{mvb})^2 + (\mathbf{eg})^2}}$$

The azimuthal angle $\varphi$ as a function of time:

$$\varphi = \frac{\mathbf{1}}{\sin\theta}\arctan\frac{\mathbf{vt}}{\mathbf{b}}.$$




,

# Dipole radiation from classical trajectory

The intensity of radiation in dipole approximation

$$\mathbf{I} = \frac{2}{3}(\mathbf{e_{em}}\vec{\mathbf{a}})^2.$$

The acceleration of the electric charge,

$$\vec{\mathbf{a}} = \frac{\mathbf{eg}}{\mathbf{m^2 r^3}}\sqrt{(\mathbf{mvb})^2 + (\mathbf{eg})^2}\sin\theta\cos\theta\,[-\cos\varphi, -\sin\varphi, -\tan\theta]\,.$$

The quantity $d\mathcal{E}_\omega$ of energy radiated throughout the time of the collision in the form of waves with frequencies in the interval $d\omega/2\pi$ is

$$\mathbf{d}\mathcal{E}_\omega = \frac{4}{3}\left(\mathbf{e_{em}}\vec{\mathbf{a}}_\omega\right)^2\frac{\mathbf{d}\omega}{\mathbf{2}\pi}.$$

$$\frac{\mathbf{d}\kappa_\omega}{\mathbf{d}\omega} = \int_{\mathbf{b_{min}}}^{\mathbf{b_{max}}} \mathbf{2}\pi\mathbf{bdb}\frac{\mathbf{d}\mathcal{E}_\omega}{\mathbf{d}\omega}.$$

$b_{min}$ can be identified with the size of the monopole core.
$b_{max}$ appears due to finite density of monopoles.

## The Fourier transform of the acceleration $\vec{a}$

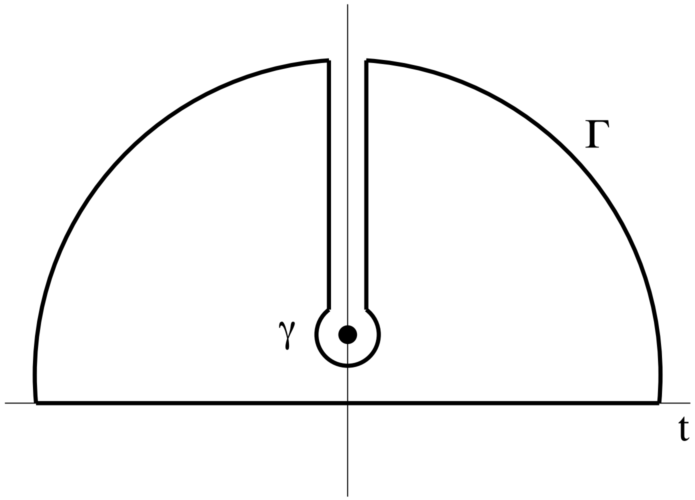
$$(\mathbf{a_x})_\omega = -\frac{(\mathbf{eg})^2}{\mathbf{m^2 b^2 v}\xi}\int_{-\infty}^{\infty}\mathbf{dt}\frac{\exp[\mathbf{i}\bar{\omega}\mathbf{t}]\cos[\xi\arctan\mathbf{t}]}{(\mathbf{t^2+1})^{3/2}}$$

$$(\mathbf{a_y})_\omega = -\frac{(\mathbf{eg})^2}{\mathbf{m^2 b^2 v}\xi}\int_{-\infty}^{\infty}\mathbf{dt}\frac{\exp[\mathbf{i}\bar{\omega}\mathbf{t}]\sin[\xi\arctan\mathbf{t}]}{(\mathbf{t^2+1})^{3/2}}$$

$$(\mathbf{a_z})_\omega = -\frac{(\mathbf{eg})^2}{\mathbf{mb}\xi}\int_{-\infty}^{\infty}\mathbf{dt}\frac{\exp[\mathbf{i}\bar{\omega}\mathbf{t}]}{(\mathbf{t^2+1})^{3/2}}$$

$$\xi = \frac{\sqrt{(\mathbf{mvb})^2+(\mathbf{eg})^2}}{\mathbf{mvb}}$$

$$\bar{\omega} = \omega\frac{\mathbf{b}}{\mathbf{v}}.$$



$$(\mathbf{a_z})_\omega = -\frac{2\mathbf{eg}\bar{\omega}}{\mathbf{mb}\xi}\mathbf{K}_1(\bar{\omega}),$$

$$(\mathbf{a_x})_\omega = \frac{(\mathbf{eg})^2}{\mathbf{m^2b^2v}\xi}\left\{\exp(-\bar{\omega})\cos\left(\frac{\pi\xi}{2}\right)\left[\frac{1}{4}\Gamma\left(\frac{1}{2}(\xi-1)\right)\mathbf{U}\left(\frac{1}{2}(\xi-1),-1,2\bar{\omega}\right)\right.\right.$$

$$\frac{4\mathbf{p}!\Gamma\left(-\mathbf{p}+\frac{\xi+3}{2}\right)}{\xi^2-1}\sum_{\mathbf{k}=0}^{\mathbf{p}}\frac{(-\bar{\omega})^{\mathbf{k}}}{\mathbf{k}!(\mathbf{p}-\mathbf{k})!\Gamma\left(\frac{\xi-3}{2}-\mathbf{p}+\mathbf{k}+1\right)}\times$$

$$\left.\left.\times\ 2^{\mathbf{k}-2}\Gamma\left(\mathbf{p}-\frac{\xi+1}{2}\right)\mathbf{U}\left(\mathbf{p}-\frac{\xi+1}{2},\mathbf{k}-1,2\bar{\omega}\right)\right]\right\}$$

$$(\mathbf{a_y})_\omega = -\frac{(\mathbf{eg})^2}{\mathbf{m^2b^2v}\xi}\left\{\mathbf{i}\exp(-\bar{\omega})\cos\left(\frac{\pi\xi}{2}\right)\left[\frac{1}{4}\Gamma\left(\frac{1}{2}(\xi-1)\right)\mathbf{U}\left(\frac{1}{2}(\xi-1),-1,2\bar{\omega}\right)\right.\right.$$

$$\frac{4\mathbf{p}!\Gamma\left(-\mathbf{p}+\frac{\xi+3}{2}\right)}{\xi^2-1}\sum_{\mathbf{k}=0}^{\mathbf{p}}\frac{(-\bar{\omega})^{\mathbf{k}}}{\mathbf{k}!(\mathbf{p}-\mathbf{k})!\Gamma\left(\frac{\xi-3}{2}-\mathbf{p}+\mathbf{k}+1\right)}\times$$

$$\left.\left.\times\ 2^{\mathbf{k}-2}\Gamma\left(\mathbf{p}-\frac{\xi+1}{2}\right)\mathbf{U}\left(\mathbf{p}-\frac{\xi+1}{2},\mathbf{k}-1,2\bar{\omega}\right)\right]\right\}$$

where $p$ is the smallest integer number larger than $(\xi+3)/2$.
$p-2$ is the number of full rotations around $z$ axis. $U(a,b,z)$ is the confluent hypergeometric function.

# The benchmark process: $qq \to qq\gamma$

Landau & Lifshitz V2

For the sake of comparison we estimate of the radiation produced in the scattering of $qq$, $q\bar{q}$ and $\bar{q}\bar{q}$ pairs in the plasma.

In the case of an attractive interaction between particles (namely in the singlet channel for the $q\bar{q}$ scattering and the antitriplet channel for the $qq$ and $\bar{q}\bar{q}$ scatterings):

$$\frac{\mathbf{d}\mathcal{E}_\omega}{\mathbf{d}\omega} = \frac{\mathbf{2}\pi\alpha^2\omega^2}{\mathbf{3v^4}}\left(\frac{(\mathbf{e_{em}})_1}{\mathbf{m_1}} - \frac{(\mathbf{e_{em}})_2}{\mathbf{m_2}}\right)^2 \left\{\left[\mathbf{H}^{(1)\prime}_{\mathbf{i}\nu}(\mathbf{i}\nu\epsilon)\right]^2 + \frac{\epsilon^2 - \mathbf{1}}{\epsilon^2}\left|\mathbf{H}^{(1)}_{\mathbf{i}\nu}(\mathbf{i}\nu\epsilon)\right|^2\right\}$$

where:

$$\nu = \frac{\omega\alpha}{\mu\mathbf{v^3}}, \qquad \epsilon = \sqrt{\mathbf{1} + \frac{\mu^2\mathbf{b^2v^4}}{\alpha^2}}, \qquad \mu = \frac{\mathbf{m_1m_2}}{\mathbf{m_1+m_2}}$$

and $\mathbf{H}^{(1)}_{\mathbf{i}\nu}(\mathbf{i}\nu\epsilon)$ is the Hankel function of the first kind:

$(e_{em})_1,\ m_1$ and $(e_{em})_2,\ m_2$ are the electric charge and mass of the two colliding particles, $\alpha = \mathbf{C^R}\,\alpha_\mathrm{s}$ is the strong coupling constant multiplied by the corresponding Casimir factor $C^R$ for the channel under study:

$$\mathbf{C^8} = \mathbf{4/3}\,; \qquad \mathbf{C^1} = \mathbf{1/6}\,; \qquad \mathbf{C^6} = \mathbf{2/3}\,; \qquad \mathbf{C^{\bar{3}}} = \mathbf{1/3}\,.$$

When the interaction is repulsive (namely in the octet channel for the $q\bar{q}$ scattering, and in the sextet channel for the $qq$ and $\bar{q}\bar{q}$ scatterings),

$$\frac{d\mathcal{E}_\omega}{d\omega} = \frac{2\pi\alpha^2\omega^2}{3v^4}\left(\frac{(e_{em})_1}{m_1} - \frac{(e_{em})_2}{m_2}\right)^2 \left\{ \left[H_{i\nu}^{(1)\prime}(i\nu\epsilon)\right]^2 + \frac{\epsilon^2 - 1}{\epsilon^2}\left|H_{i\nu}^{(1)}(i\nu\epsilon)\right|^2 \right\} \exp\left[-2\pi\nu\right].$$

We consider quark matter with three equal mass and density light flavors

The total energy radiated in unit volume and per frequency interval $d\omega$ throughout the time of the collision in the case of Coulomb scattering:

$$\frac{d\Sigma}{d\omega} = \frac{4\pi^2\alpha_{em}\omega^2}{3v^4}\frac{n_q^2}{18^2}\frac{4}{m^2}\int_0^{bmax}\left[\left(\frac{4}{3}\alpha_s\right)^2 f\left(\frac{4}{3}\alpha_s\right) + 8\left(\frac{1}{6}\alpha_s\right)^2 f\left(\frac{1}{6}\alpha_s\right)\exp[-2\pi\nu\alpha_s/6]\right.$$

$$\left. + 3\left(\frac{2}{3}\alpha_s\right)^2 f\left(\frac{2}{3}\alpha_s\right) + 6\left(\frac{1}{3}\alpha_s\right)^2 f\left(\frac{1}{3}\alpha_s\right)\exp[-2\pi\nu\alpha_s/3]\right] bdb$$

$$f(\alpha) = \left[H_{i\nu}^{(1)\prime}(i\nu\epsilon)\right]^2 + \frac{\epsilon^2 - 1}{\epsilon^2}\left|H_{i\nu}^{(1)}(i\nu\epsilon)\right|^2.$$

We have taken into account all possible color channels for $qq$, $q\bar{q}$ and $\bar{q}\bar{q}$ scatterings, and all possible flavor combinations.

# Estimates for the QGP

A quark moving in a deconfined medium acquires a thermal mass due to its interaction with the other particles of the medium. From the lattice results (Karrsch and Kitazawa) we take a value of $m \simeq 0.8T$:

$$\mathbf{m(T\ =\ 2\,T_c)\ \ \simeq\ \ 0.3GeV}$$

we vary the quark's (thermal) velocity

$$v\ \sim\ 0.5 - 0.7$$

.

The total density of quarks and antiquarks can be obtained for example from the PNJL model.

$$\mathbf{n_q \simeq 2.8T^3 \simeq 0.12}\ \mathrm{GeV}^{\mathbf{3}}.$$

$$\alpha_{\mathrm{s}}\ =\ \mathbf{0.8}$$

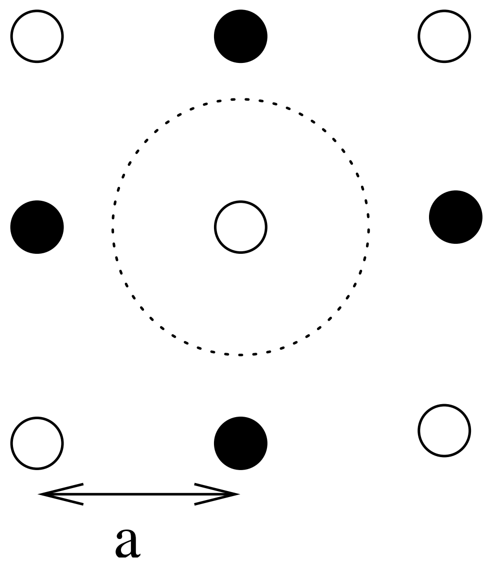
A "sphere of influence of one monopole" (the dotted circle) gives the maximal impact parameter

$$\mathbf{b_{max} = n_M^{-1/3}/2.}$$

$$\mathbf{n_M(T \simeq 2T_c) \ \simeq \ 0.02} \ \text{GeV}^3$$

$$\mathbf{b_{max} \simeq 1.8} \ \text{GeV}^{-1}$$

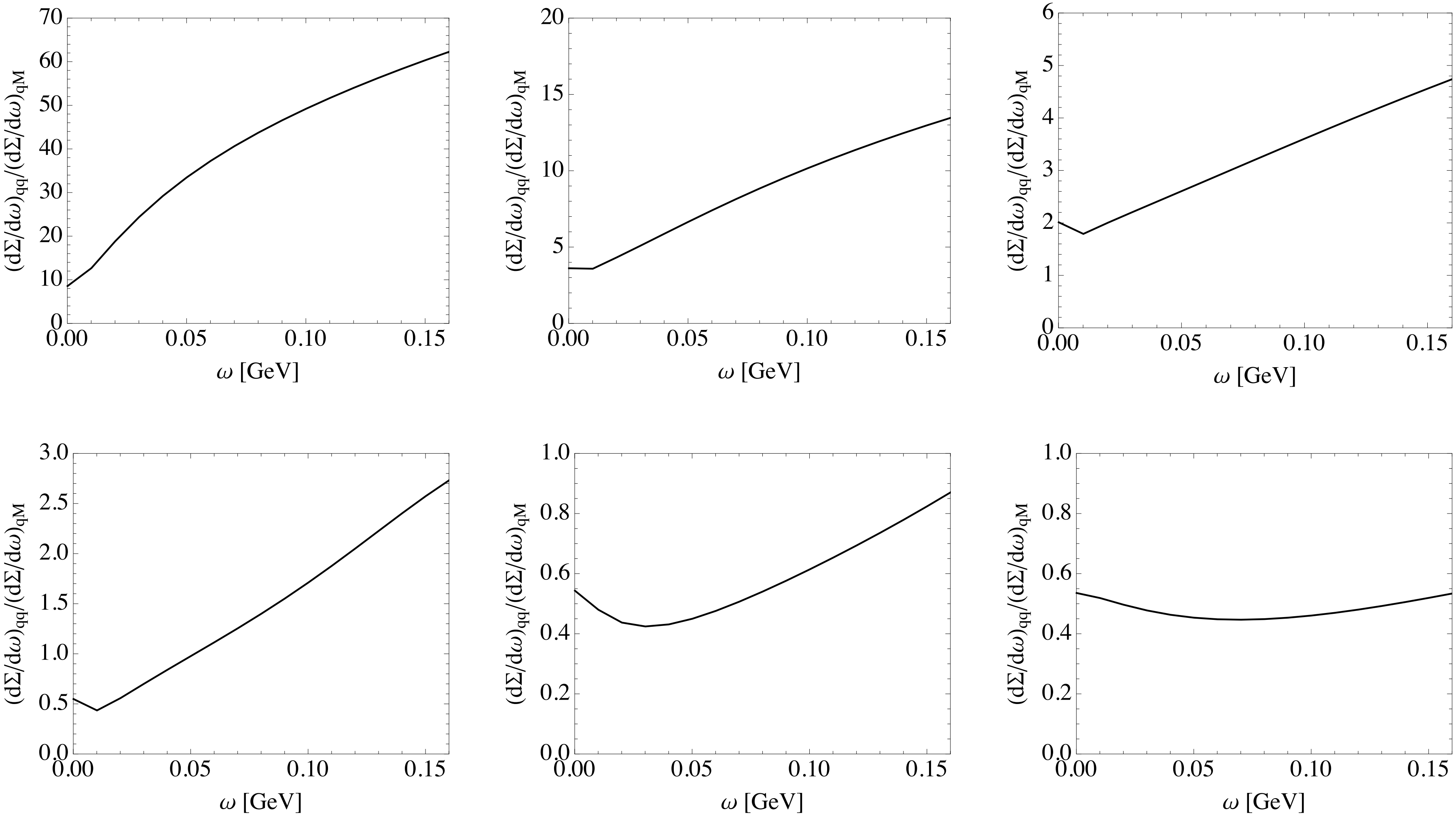
$$\mathbf{b_{min} \ \simeq \ 0.15} \ \text{fm=0.78 GeV}^{-1}$$




for the total energy radiated in unit volume throughout the time of the collision in a given frequency interval:

$$\mathbf{\frac{d\Sigma}{d\omega} = \frac{2}{9}\frac{d\kappa_\omega}{d\omega}n_q n_M = \frac{2}{9}n_q n_M \frac{2}{3\pi}\alpha_{em} 2\pi \int_{b_{min}}^{b_{max}} b|\vec{a}_\omega|^2 db}$$

where the factor $\frac{2}{9}$ comes from the different electric charges fur $u$, $d$ and $s$ quarks.




Upper raw: ratio of $d\Sigma/d\omega$ for Coulomb and quark-monopole scattering. The integral over $b$ is taken up to $\infty$. For these plots we use $v = 0.3$, $v = 0.5$, $v = 0.7$.

Bottom raw: the same ratio, but the integral over $b$ is taken up to the corresponding $b_{max}$.

# Outlook

So far, we have written a missing chapter in Landau & Lifshitz V2

This is only a beginning, but our initial results are quite encouraging.

- non-relativistic approximation $\longrightarrow$ relativistic treatment
- dipole approximation $\longrightarrow$ retarded potentials
- classical trajectory $\longrightarrow$ back reaction effects due to energy loss
- classical $\longrightarrow$ quantum description.
- two body $\longrightarrow$ many body scattering.
- We need to compute in the region close to $T_c$, where the monopoles are light, but quarks are heavy, and include fireball evolution.
- If the magnetic scenario is correct, monopoles should also contribute to jet quenching.