Parton showers
with medium-modified splitting functions

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1. **Introduction**
2. **Angular-ordered parton showers in the vacuum (HERWIG)**
3. **Medium-modified Altarelli–Parisi splitting functions**
4. **Phenomenology with modified HERWIG**
5. **Comments on Q-PYTHIA results**
6. **Conclusions**

N. Armesto, G. C., L. Cunqueiro and C.A. Salgado, JHEP 1109 (2009) 122;
G. C., Q-HERWIG release, in progress.

RHIC observations: jet quenching (suppression of high-p_T particles with respect to pp collisions); disappeareance or distortion of particle spectra

Typical explanation: higher radiative energy loss in a dense medium

Analytical calculations are suitable to describe inclusive quantities, but not to yield exclusive final states

Monte Carlo generators are integral part of any experimental analysis: hard-scattering processes, parton showers, hadronization and underlying event

Great interest in having Monte Carlo generators for heavy-ion collisions capable of describing jet quenching

A simple prescription: medium-modified Altarelli–Parisi splitting functions (BDMPS approximation) to enhance branching probability and radiative loss in a medium

Implementation of modified splitting functions in Q-PYTHIA (ordering in virtuality and string hadronization)

Recent work towards Q-HERWIG: modified angular-ordered showers with cluster hadronization

A typical LHC event

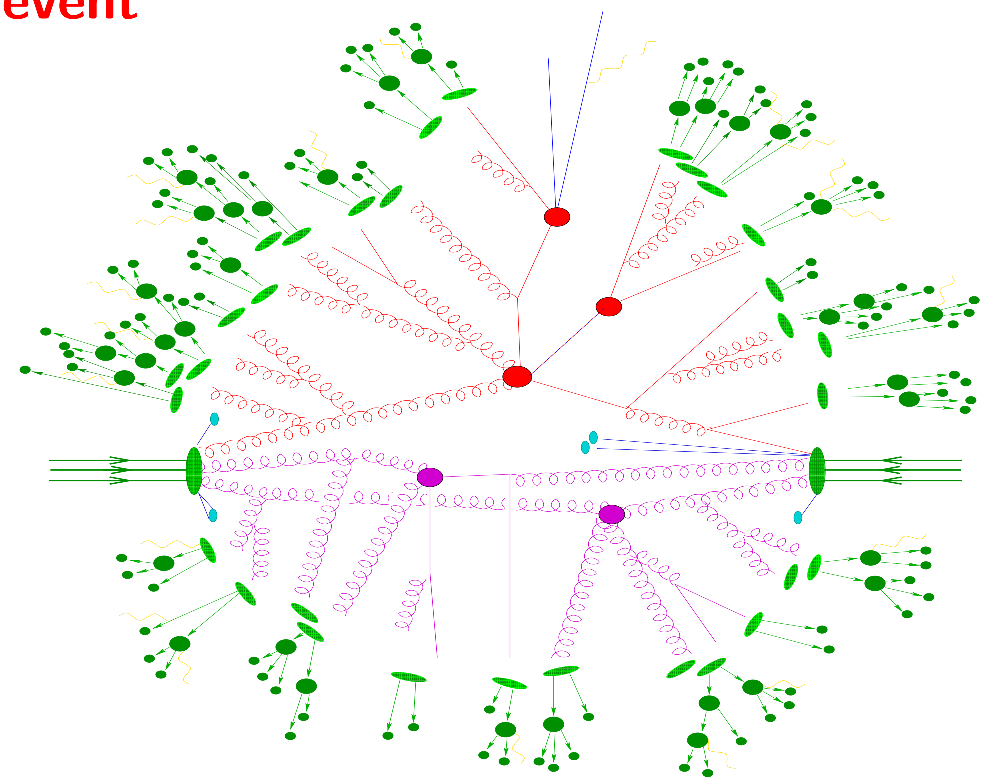


Figure by Frank Krauss

Monte Carlo event generators in the vacuum (HERWIG/PYTHIA):

Hard $2 \to 2$ subprocess: leading-order (LO) matrix element

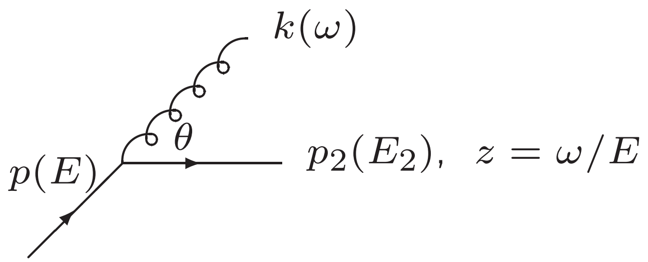
Parton showers in the soft or collinear approximation

Matrix-element corrections for hard and large-angle parton radiation

Phenomenological hadronization models

MC@NLO and POWHEG: NLO+parton showers

Monte Carlo generators (vacuum): radiation in soft/collinear approximation



$$d\mathcal{P} = \frac{\alpha_S}{2\pi}\, P(z)dz\, \frac{dQ^2}{Q^2}\, \Delta_S(Q^2_{\max}, Q^2)$$

Q^2: **ordering variable**

$\Delta_S(Q^2_{\max}, Q^2)$ **Sudakov form factor: no radiation in** $[Q^2, Q^2_{\max}]$

$$\Delta_S(Q^2_{\max}, Q^2) = \exp\left[-\frac{\alpha_S}{2\pi}\int_{Q^2}^{Q^2_{\max}} \frac{dQ'^2}{Q'^2}\int_{z_{\min}}^{z_{\max}} dz P(z)\right]$$

HERWIG : $Q^2 = E^2(1-\cos\theta) \simeq E^2\theta^2/2 \qquad \theta < \pi/2$

Soft approximation: angular ordering

PYTHIA: virtuality evolution $\quad p^2 \simeq 2z(1-z)Q^2$

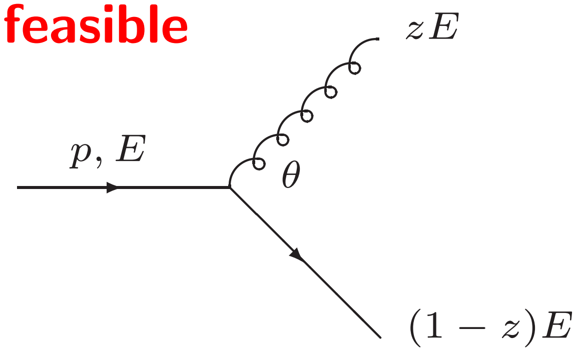
It includes angular ordering by an additional veto

PYTHIA 6.3: transverse-momentum ordering $\quad k_T^2 \simeq 2z^2(1-z)^2Q^2$

HERWIG++ : $Q'^2 = Q^2 + \frac{\max(k^2,p^2)}{z^2} + \frac{k^2}{z^2(1-z)^2}$

PYTHIA 8 (C++): transverse-momentum ordering

Ordering variable to implement probabilistically multiple radiation: in collinear approximation, any $Q^2 \propto \theta^2$ is feasible



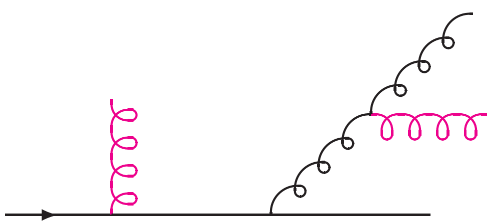
Gluon transverse momentum: $k_T^2 \simeq z^2(1-z)^2E^2\theta^2$

Invariant mass: $p^2 \simeq z(1-z)E^2\theta^2$ **HERWIG:** $Q^2 \simeq E^2\theta^2/2$

Collinear limit: $\ln k_T^2 \sim \ln p^2 \sim \ln Q^2 \sim \ln \theta^2$

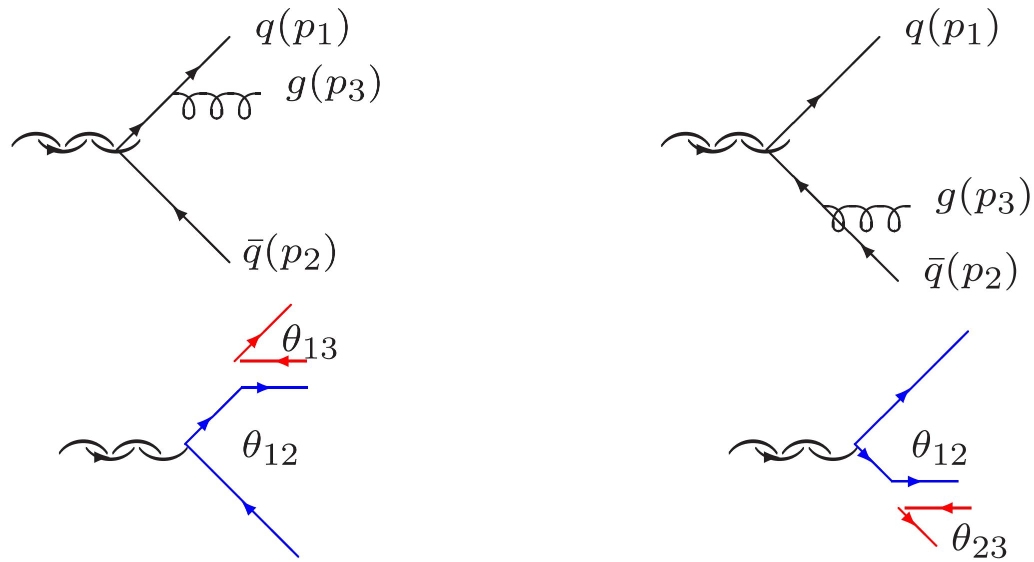
$$\frac{d\theta^2}{\theta^2} \simeq \frac{dk_T^2}{k_T^2} \simeq \frac{dp^2}{p^2}$$

Soft gluons can be emitted anywhere, at any angle



Angular ordering allows one to implement probabilistically multiple soft emissions

Angular ordering



$$|\mathcal{M}|^2 \sim W = \frac{\omega^2}{2}\left(\frac{p_1}{p_1\cdot p_3} - \frac{p_2}{p_2\cdot p_3}\right)^2 = \frac{1-\cos\theta_{12}}{(1-\cos\theta_{13})(1-\cos\theta_{23})} \quad \textbf{(soft limit)}$$

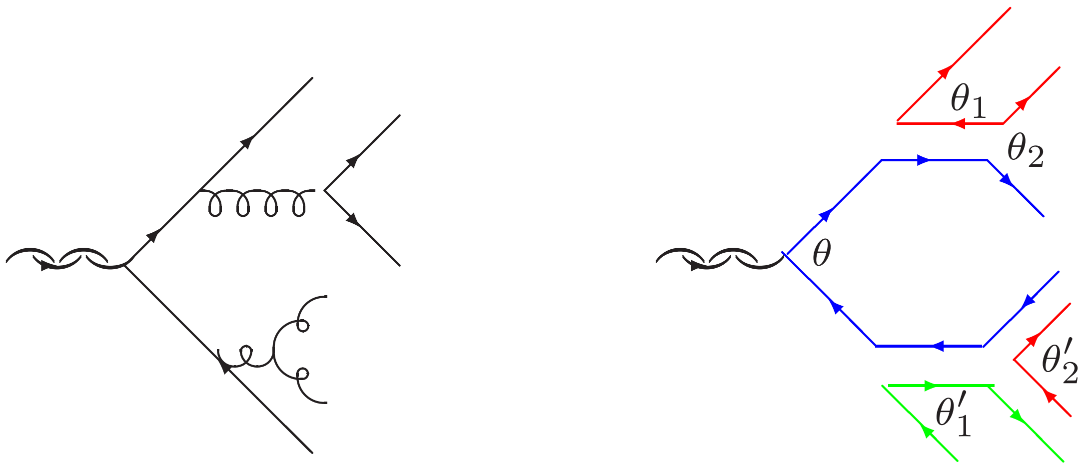
After azimuthal average:

$$W \quad \longrightarrow \quad \frac{1}{1-\cos\theta_{13}}\Theta(\theta_{12}-\theta_{13}) + \frac{1}{1-\cos\theta_{23}}\Theta(\theta_{12}-\theta_{23})$$

q **and** $\bar{q}$ **independent emitters in angular-ordered phase space**

Colour coherence: a parton radiates up to its colour partner

Angular-ordered parton showers



Parton shower $\Rightarrow$ colour flow $\Rightarrow$ angular ordering:

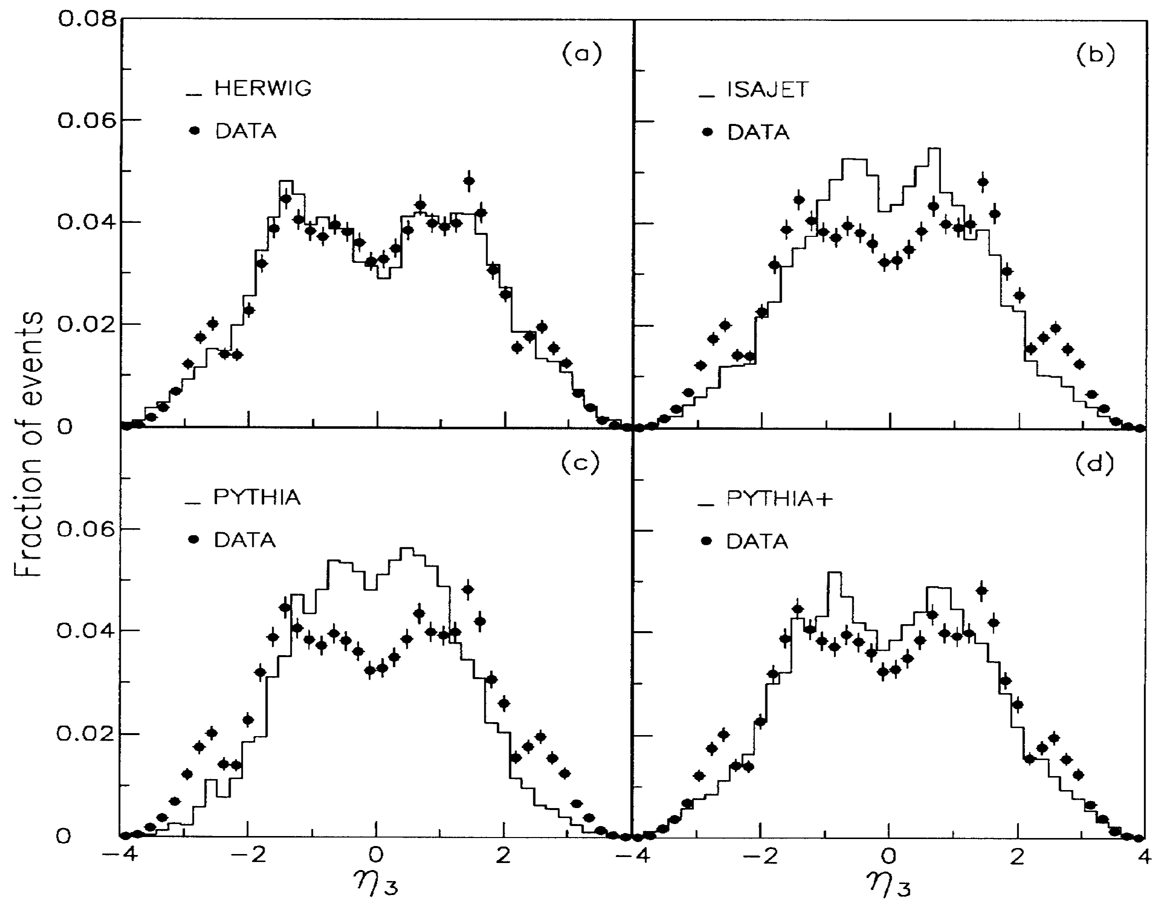
$\theta_1 < \theta$; $\theta_2 < \theta_1$; $\theta'_1 < \theta$; $\theta'_2 < \theta'_1$

$$dP_1 = \frac{\alpha_S}{2\pi}\, P(z_1)dz_1\, \frac{d\cos\theta_1}{1-\cos\theta_1}\Delta_S(\theta,\theta_1)$$

$$dP_2 = \frac{\alpha_S}{2\pi}\, P(z_2)dz_2\, \frac{d\cos\theta_2}{1-\cos\theta_2}\, \Delta_S(\theta_1,\theta_2)\; dP_1$$

Iterating dP one constructs the multiple-radiation algorithm

CDF, PRD50 (1994) 5562 – 3-jets events $E_T(\text{leading}) > 100$ GeV, $E_{T3} > 10$ GeV

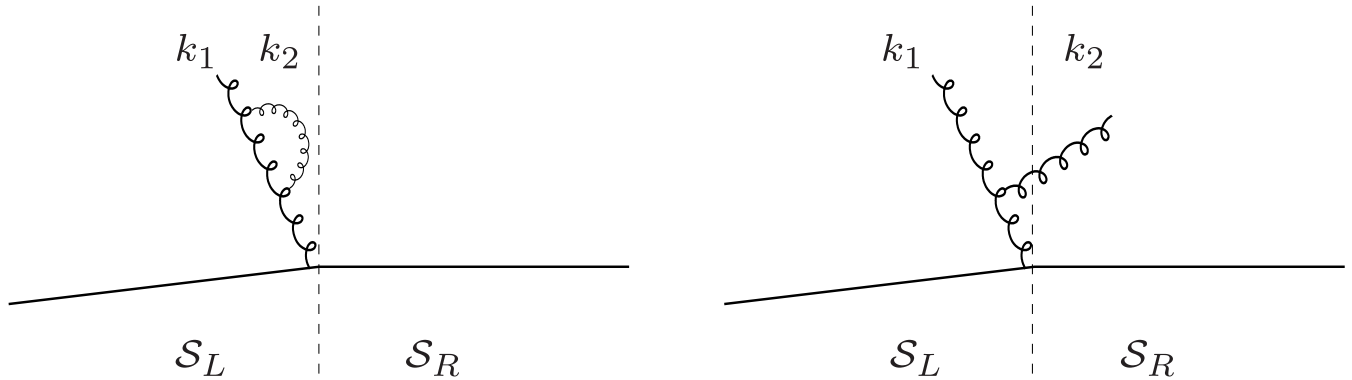


(a)
HERWIG
DATA
(b)
ISAJET
DATA
(c)
PYTHIA
DATA
(d)
PYTHIA+
DATA
Fraction of events
0.08
0.06
0.04
0.02
0
−4
−2
0
2
4
η_3

Figure 10:

Non-global observables: radiation in a limited region of the phase space

M. Dasgupta and G. Salam, PLB (2001) 323; JHEP 0203 (2002) 017; M. Dasgupta, Pramana 62 (2004) 675



Two soft gluons $|k_2| \ll |k_1| \ll Q = \sqrt{s}$

Incomplete cancellation of real and virtual diagrams in $\mathcal{S}_R$

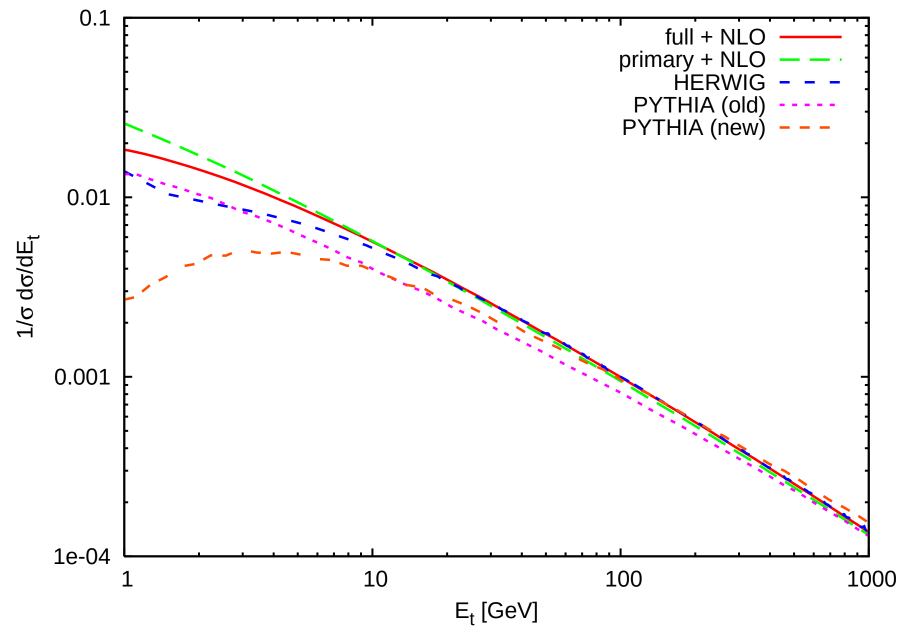
$\alpha_S^2 L^2$: **non-global logarithm due to soft and large-angle radiation in $\mathcal{S}_R$**

Transverse-energy flow in a rapidity gap

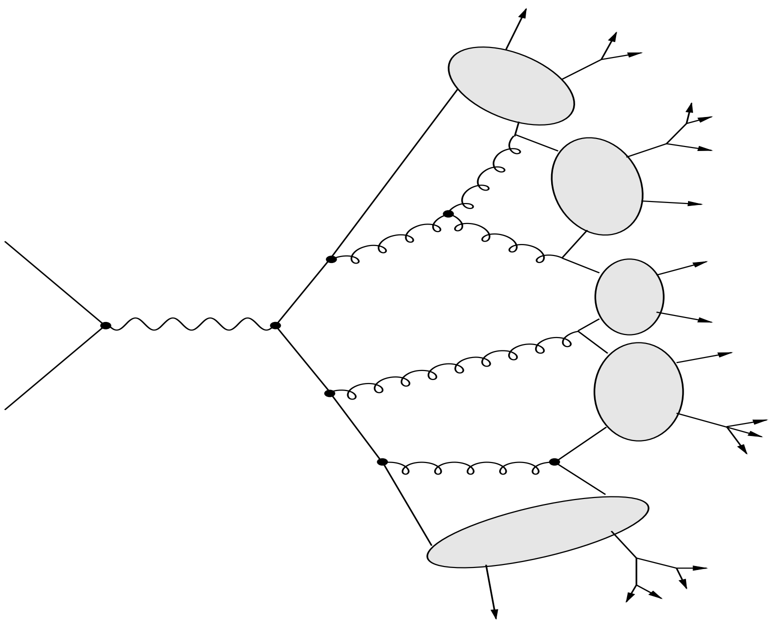
e^+e^- **annihilation**

$\Delta\eta = 3$, $\sqrt{s} = 100$ **TeV**

A. Banfi, G.C. and M. Dasgupta, JHEP 0703:050,2007



Hadronization models



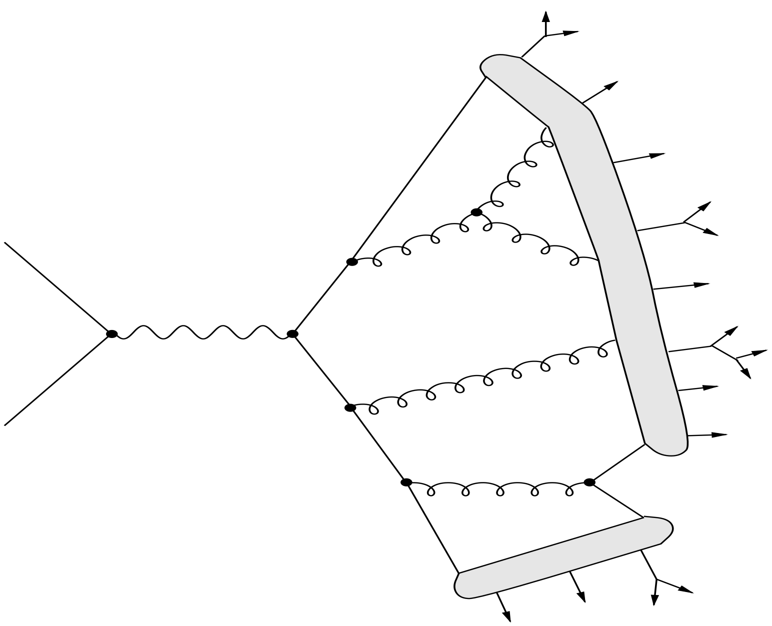
Cluster model (HERWIG)

Perturbative evolution ends at $Q^2 = Q_0^2$

Angular ordering $\Rightarrow$ colour preconfinement

Forced gluon splitting $(g \to q\bar{q})$

Colour-singlet clusters decay into the observed hadrons



String model (PYTHIA)

q and $\bar{q}$ move in opposite direction

The colour field collapses into a string, with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Figures from R.K. Ellis, W.J. Stirling and B.R. Webber, QCD and Collider Physics

Medium-induced radiation

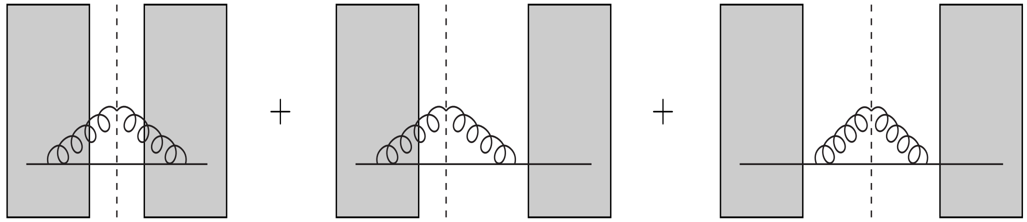


Figure from J. Calderrey-Solana, C.A. Salgado, Acta Phys.Polon.B38:3731-3794,2007

Three cases according to whether the gluon is emitted inside or outside the medium in the matrix element and its complex conjugate

$$\frac{d^2 I}{dz dp_T^2} = \frac{\alpha_S}{2\pi}\frac{P(z)}{p_T^2} + \frac{d^2 I_{\rm med}}{dz dp_T^2}$$

Medium contribution by means of the BDMPS approximation

R. Baier, Yu.L. Dokshitzer, S. Peigné, D. Schiff, NPB484 (1997) 265, JHEP09 (2001) 033

Static scattering centres in a screened potential: $V(x) = \frac{g_S}{4\pi}\,\frac{e^{-\mu|\vec{x}-\vec{x}_i|}}{|\vec{x}-\vec{x}_i|}\quad \frac{1}{\mu} \ll \lambda$

Independent multiple scatterings: parton formation time is smaller than time between two subsequent interactions (mean free path) $\Delta t < \lambda$

Hadronization occurs outside the medium

Soft-enhanced radiation (prescription needed to include hard radiation)

Medium-modified splitting functions

Soft/collinear factorization, ordering variable and colour flow as in the vacuum

Decoherence and destruction of in-medium angular ordering $\Rightarrow$ talk by A.Leonidov

$$\frac{d^2 I_{\rm med}}{dz dp_T^2} = \frac{\alpha_S}{2\pi p_T^2}\,\Delta P(z,E,p^2,\hat{q},L) \quad \Longrightarrow \quad \Delta P(z,E,p^2,\hat{q},L) = \frac{2\pi p_T^2}{\alpha_S}\,\frac{d^2 I_{\rm med}}{dz dp_T^2}$$

$$P(z) \to P(z) + \Delta P(z,E,p^2,\hat{q},L)$$

Branching algorithm in a dense medium:

$$d\mathcal{P} = \frac{\alpha_S}{2\pi}\,[P(z) + \Delta P(z,E,p^2,\hat{q},L)]dz\,\frac{dQ^2}{Q^2}\,\Delta_S(Q^2_{\rm max},Q^2,E,p^2,\hat{q},L)$$

$\hat{q} = \langle k_T^2\rangle/\lambda$**: transport coefficient;** L**: medium length;** p^2: **virtuality**

$\hat{q}L$: **accumulated transverse momentum;** $\omega_c = \hat{q}L^2/2$: **frequency**

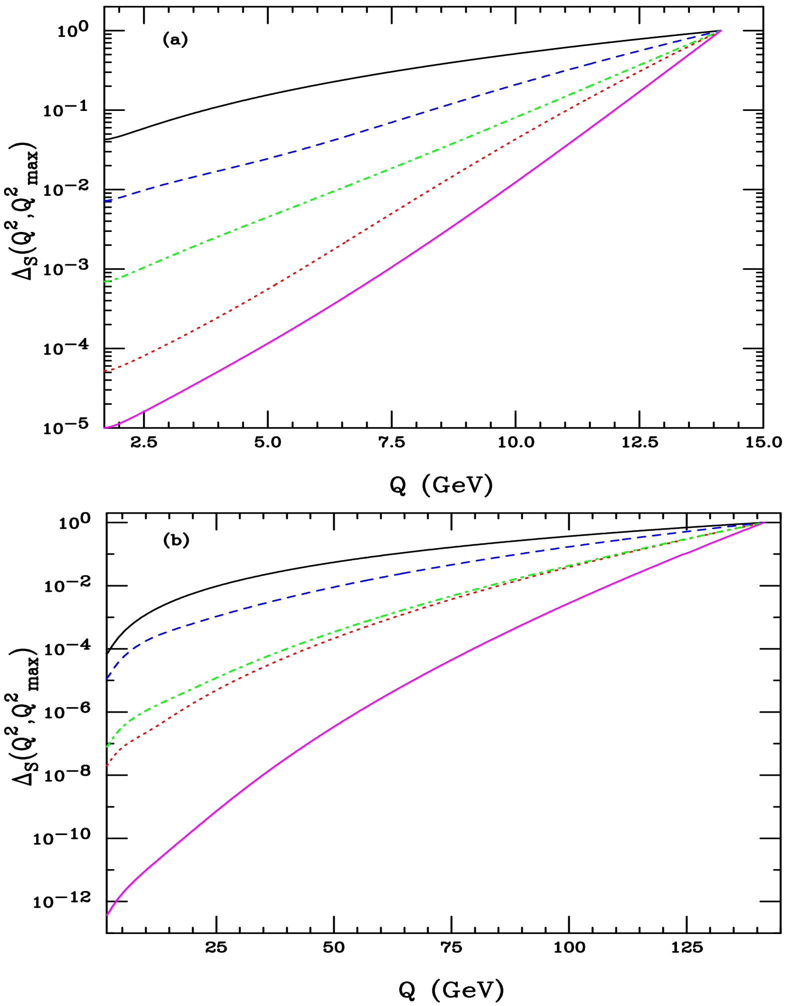
Effective length to account for parton formation time:

$$L = L_0 - \frac{2zE}{k_T^2}$$

Gluon Sudakov form factors for $E = 10$ GeV (top) and 100 GeV (bottom)

$Q_{\max} = \sqrt{2}\ E$, $Q_0 \simeq 1.7$ **GeV**, $z_{\min} = Q_0/Q$, $z_{\max} = 1 - z_{\min}$

$\hat{q} = 1,\ 10$ **GeV^2/fm;** $L_0 = 2$, 5 **fm;** $\hat{q}L_0 = 2$, 5, 20, 50 **GeV^2**

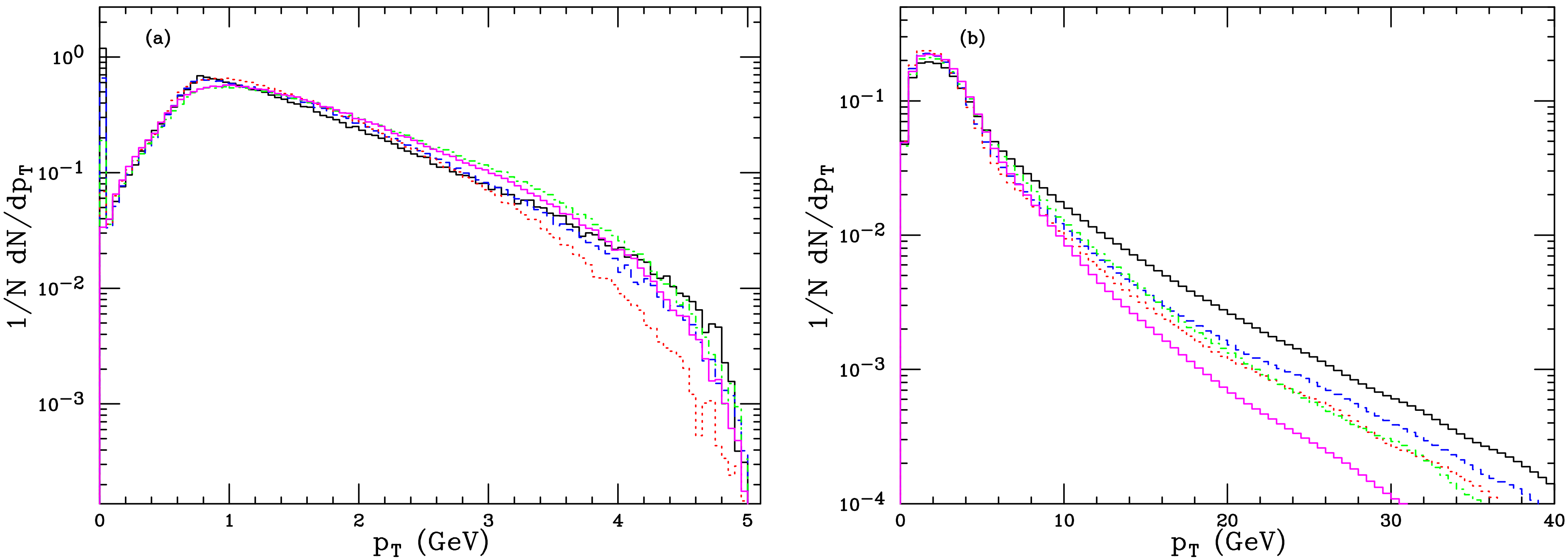


Solid: vacuum; **Dashes:** $\hat{q}L_0 = 2$ **GeV^2**; **Dots:** $\hat{q}L_0 = 5$ **GeV^2**; **Dot-dashes:** $\hat{q}L_0 = 20$ **GeV^2** **Solid:** $\hat{q}L_0 = 50$ **GeV^2**

Showers initiated by gluons of 10 and 100 GeV - average parton multiplicities:

E	$\hat{q}L_0 = 0$	$\hat{q}L_0 = 2$ GeV^2	$\hat{q}L_0 = 5$ GeV^2	$\hat{q}L_0 = 20$ GeV^2	$\hat{q}L_0 = 50$ GeV^2
10 GeV	2.56	3.05	4.14	3.60	4.56
100 GeV	6.95	7.41	8.79	8.93	11.70

Transverse momentum distributions:

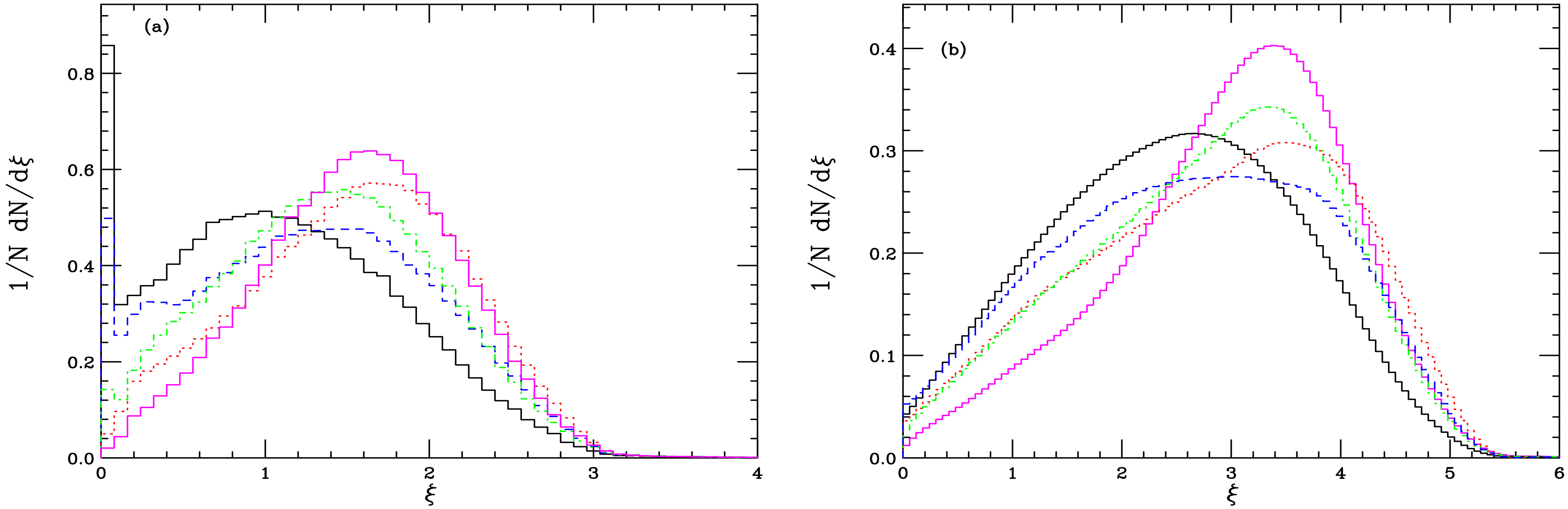


(a): $E = 10$ **GeV; (b):** $E = 100$ **GeV**

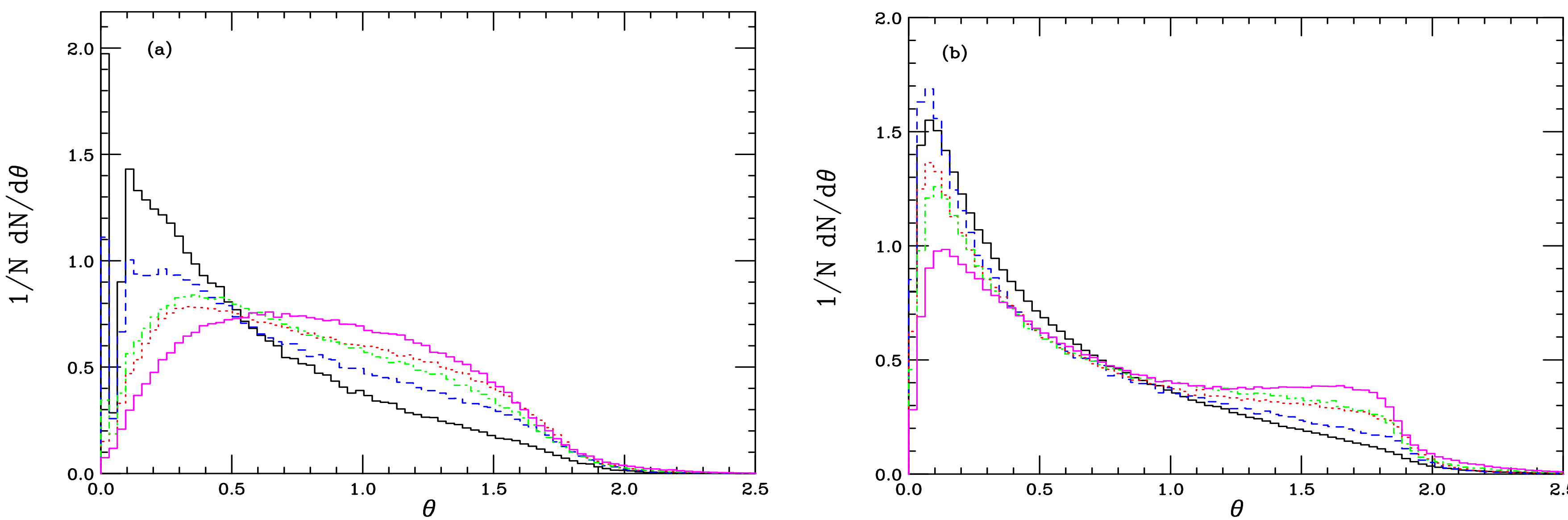
Solid: vacuum; **Dashes:** $\hat{q}L_0 = 2$ **GeV^2**; **Dots:** $\hat{q}L_0 = 5$ **GeV^2**; **Dot-dashes:** $\hat{q}L_0 = 20$ **GeV^2** **Solid:** $\hat{q}L_0 = 50$ **GeV^2**

Peak at $p_T = 0$**: events with no branchings, depending on** $\Delta_S(Q_0^2, Q_{\max}^2)$

Logarithmic energy fraction $\xi = \ln(E_g/|p|)$

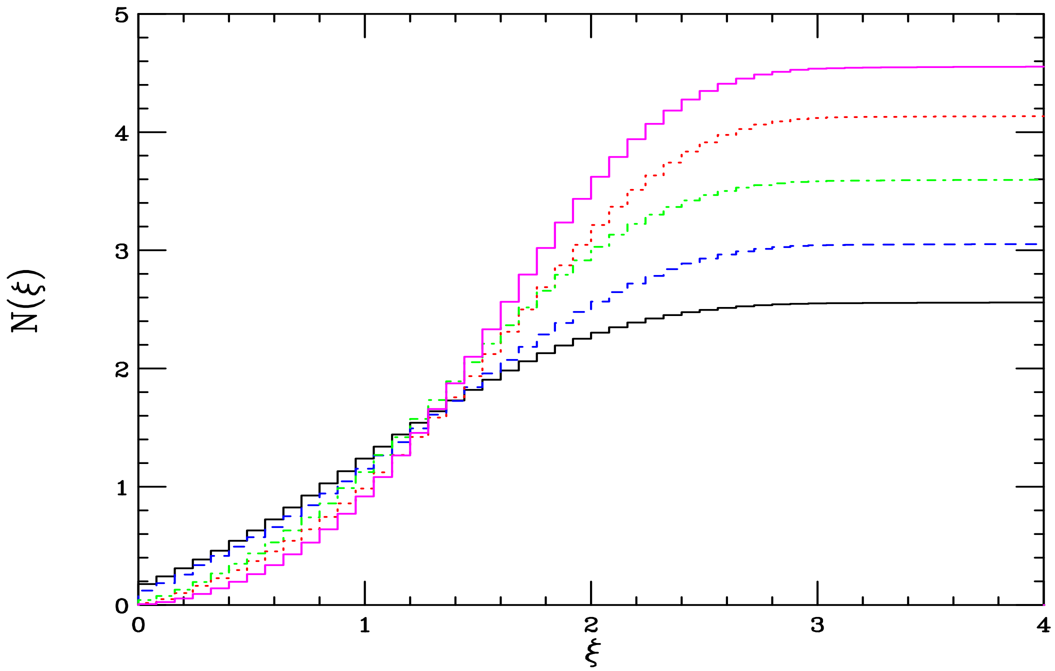


Emission angle θ (dead zone for $\theta > \pi/2$ in the showering frame)



Integrated ξ spectrum with $E = 10$ GeV to remove the spike:

$$N(\xi) = \int_0^{\xi} d\xi' \frac{dN}{d\xi'}$$

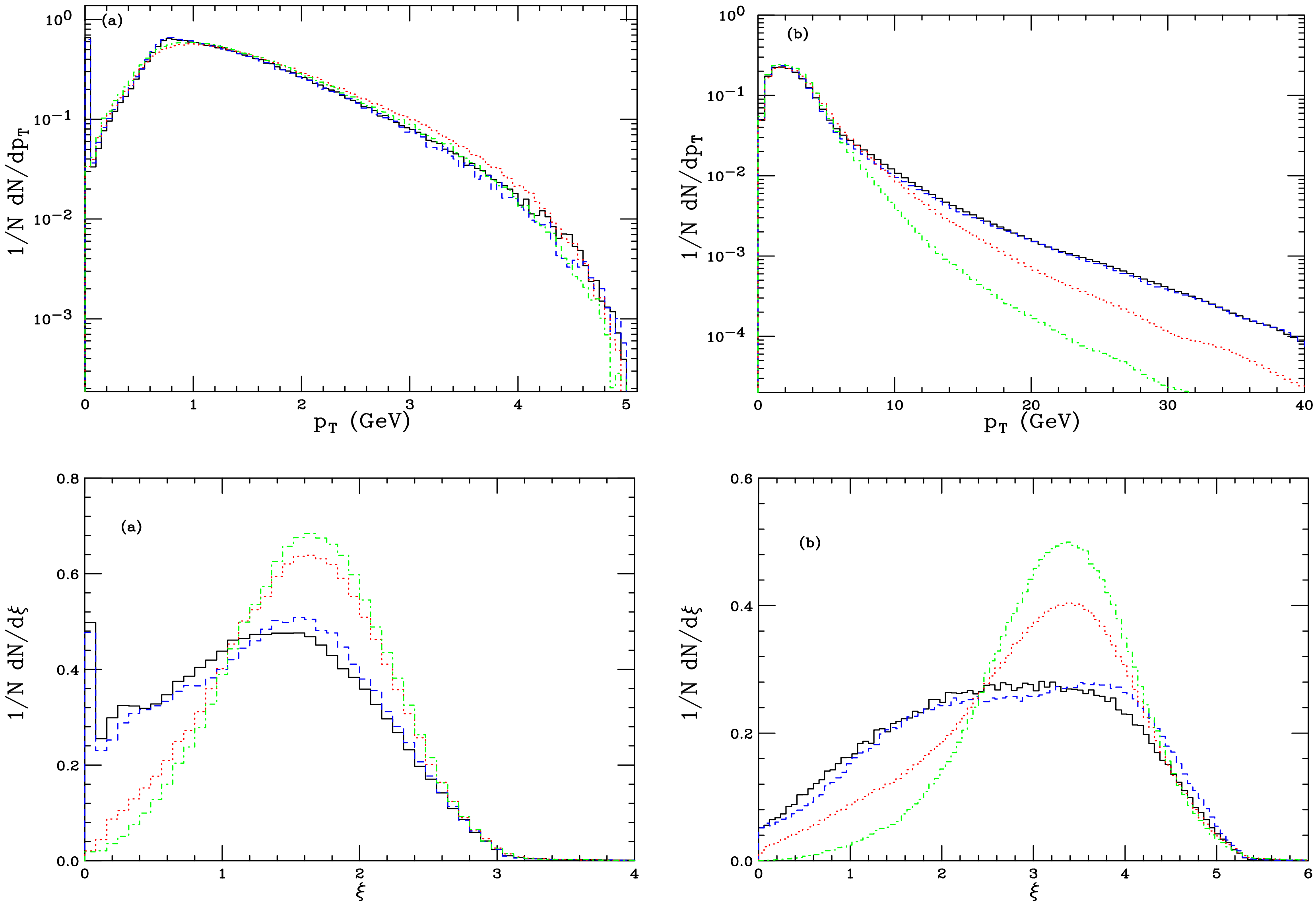


Solid: vacuum; **Dashes:** $\hat{q}L_0 = 2$ **GeV^2**; **Dots:** $\hat{q}L_0 = 5$ **GeV^2**;

Dot-dashes: $\hat{q}L_0 = 20$ **GeV^2**; **Solid:** $\hat{q}L_0 = 50$ **GeV^2**

$\xi = 0.5$: **suppression from** 10% ($\hat{q}L_0 = 2$ GeV^2) **to** 60% ($\hat{q}L_0 = 50$ GeV^2)

Spectra for fixed $L = L_0$: larger medium-induced effects

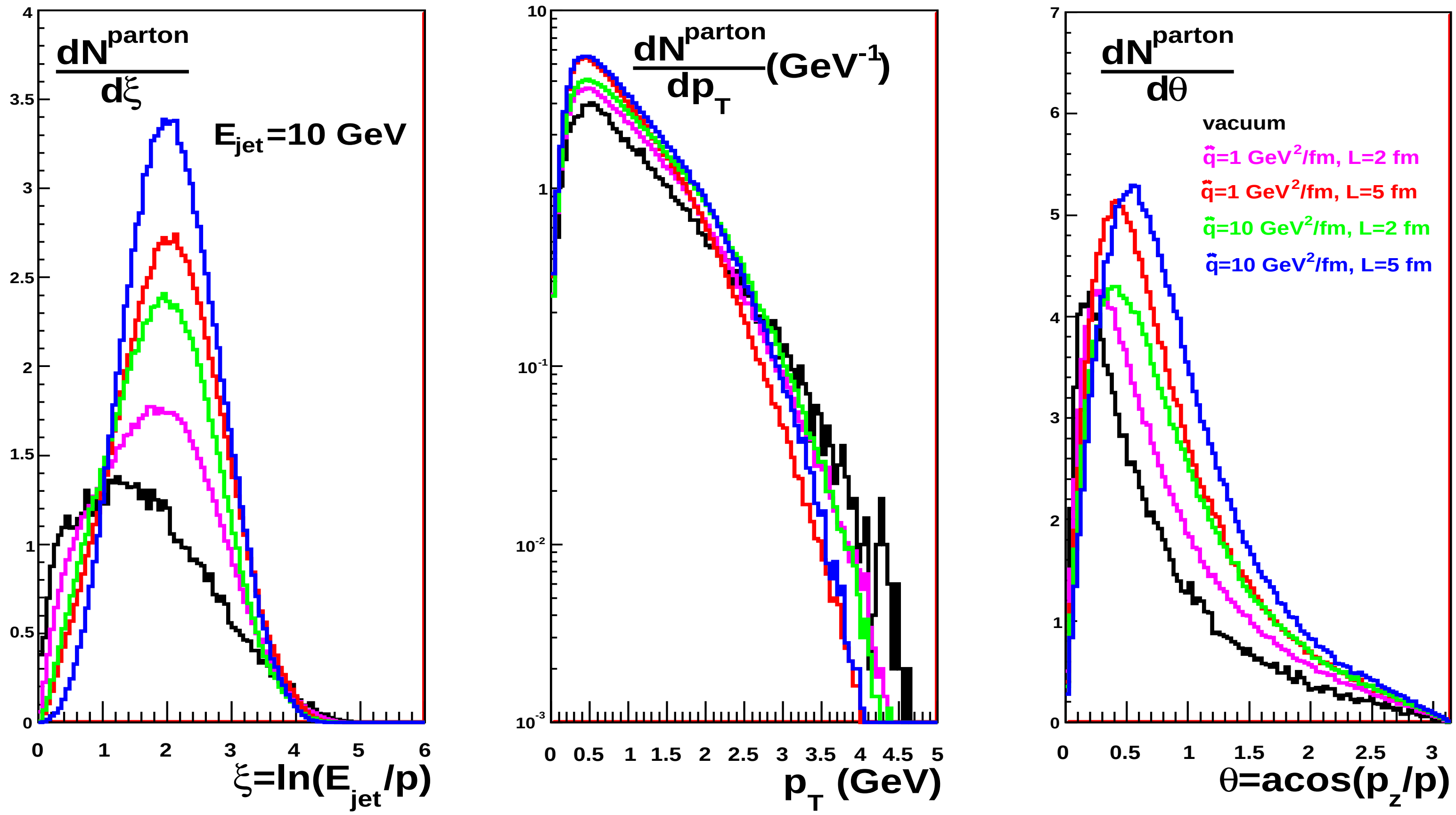


Solid: $\hat{q}L_0 = 2$ **GeV^2, variable** L; **Dashes:** $\hat{q}L_0 = 2$ **GeV^2, fixed** L;
Dots: $\hat{q}L_0 = 50$ **GeV^2, variable** L; **Dot-dashes:** $\hat{q}L_0 = 50$ **GeV^2, fixed** L

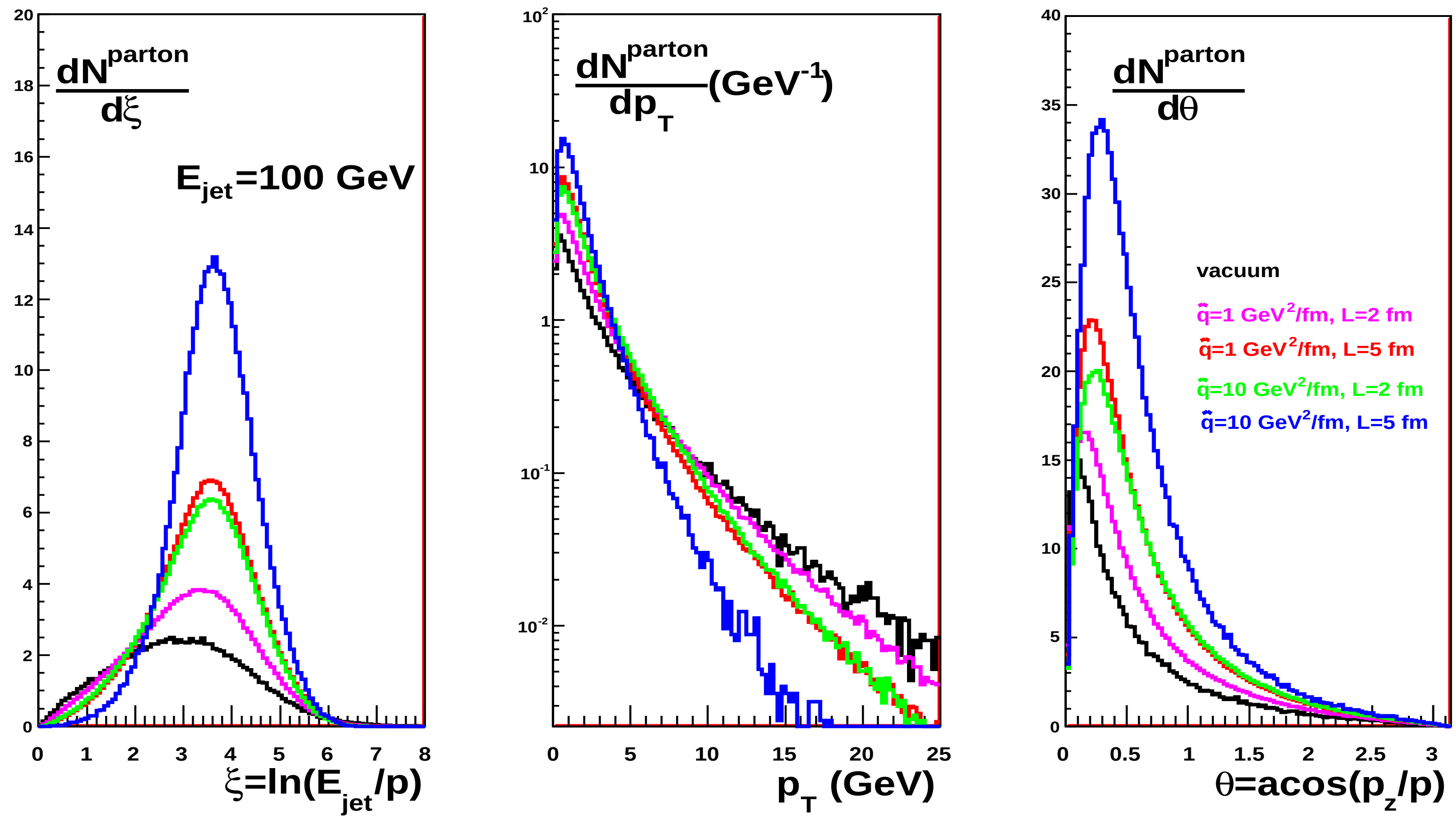
Results using Q-PYTHIA – $E_g = 10$ GeV

N. Armesto, L. Cunqueiro, C.A. Salgado, Eur.Phys.J.C63 (2009) 679

Different showering variables and evolution range i.e. $Q_{\max} = 2E$, $z_{\min} = p_0^2/p^2$



Results using Q-PYTHIA – $E_g = 100$ GeV



Larger medium effects in Q-PYTHIA – no peak at $p_T = \xi = \theta = 0$

Conclusions and outlook

Medium-modified splitting functions in the HERWIG angular-ordered parton shower algorithm in the BDMPS approximation

Larger energy loss (suppression in the Sudakov form factor)

Remarkable medium effects: higher parton multiplicity, suppression at large p_T, wider angular distributions, small-ξ suppression

In progress:

Analysis at hadron level and comparison with Q-PYTHIA (tuned codes)

Investigation of angular ordering and structure of medium-induced interference

Studies of jet events at RHIC and LHC (FastJet)

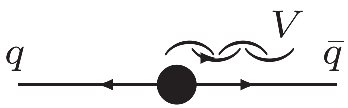
Implementation of medium geometry and release of Q-HERWIG event generator

Strategies to include hard and large-angle radiation in medium-induced showers

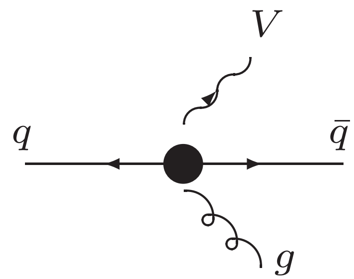
HERWIG++ for jet quenching?

Outside angular-ordered phase space: exact matrix-element corrections

Vector boson production $(V = W, Z)$**:** $q\bar{q} \to V$ **(LO)** $q\bar{q} \to Vg$ **. . . (NLO)**

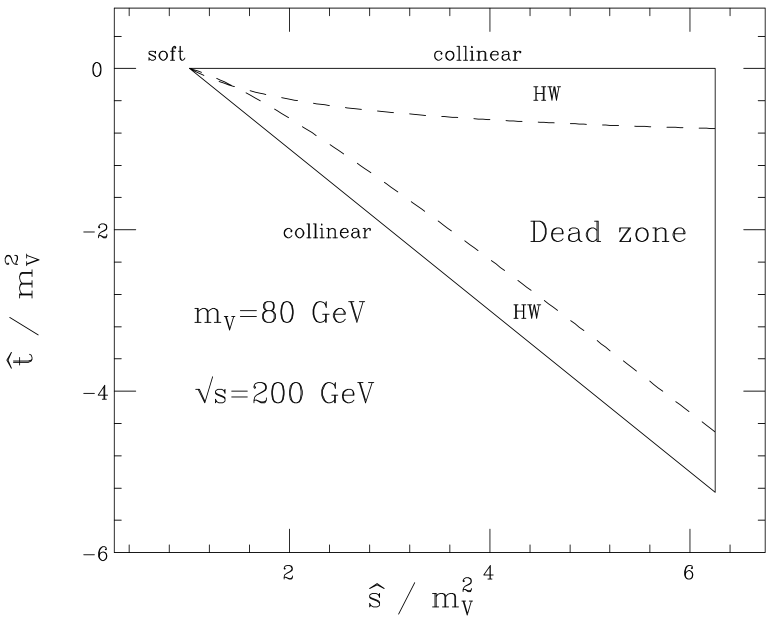


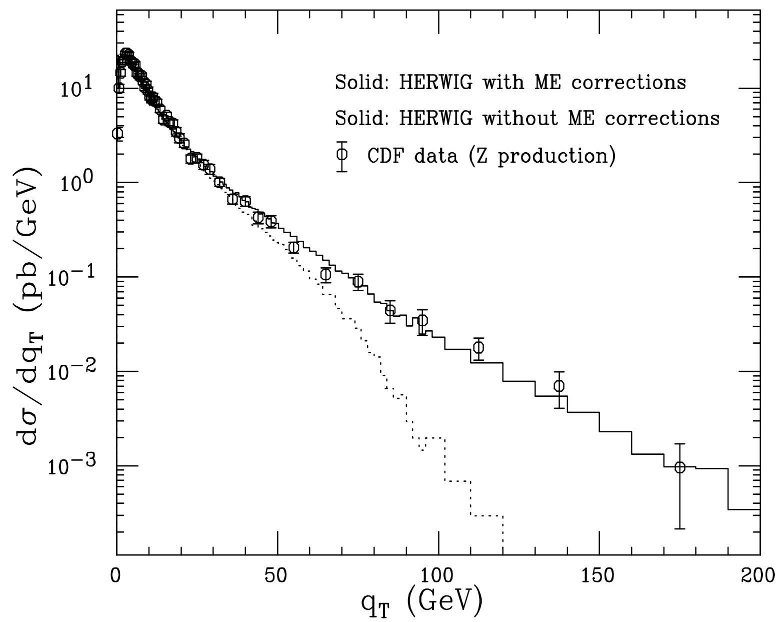
LO: $q_{T,V} = 0$



NLO: $q_{T,V} = q_{T,g} \neq 0$

Soft radiation (AO): small q_T; hard/large-angle emission: large q_T





$\hat{s} = (p_q + p_{\bar{q}})^2$, $\hat{t} = (p_q + p_g)^2$, $\hat{u} = (p_q - p_g)^2$ G.C. and M.H. Seymour, NPB 565 (2000) 227