Three Easy Pieces:

Searching for the QCD Critical Point,

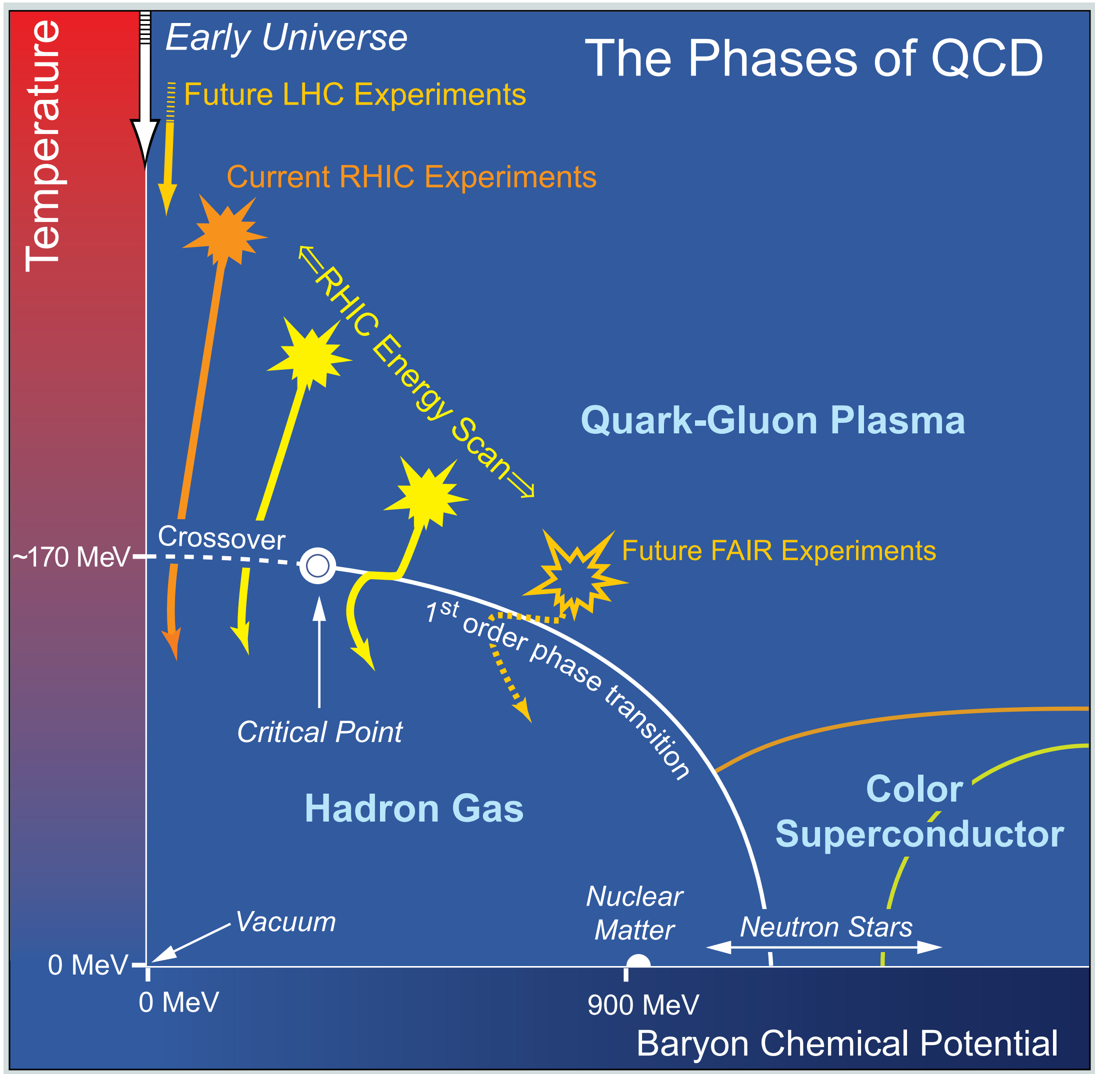
Transverse Momentum Broadening and the Jet Quenching Parameter,

And

Quenching A Lighthouse Beam

Krishna Rajagopal

Searching for the QCD Critical Point



2007 NSAC Long Range Plan

Critical Mode Fluctuations

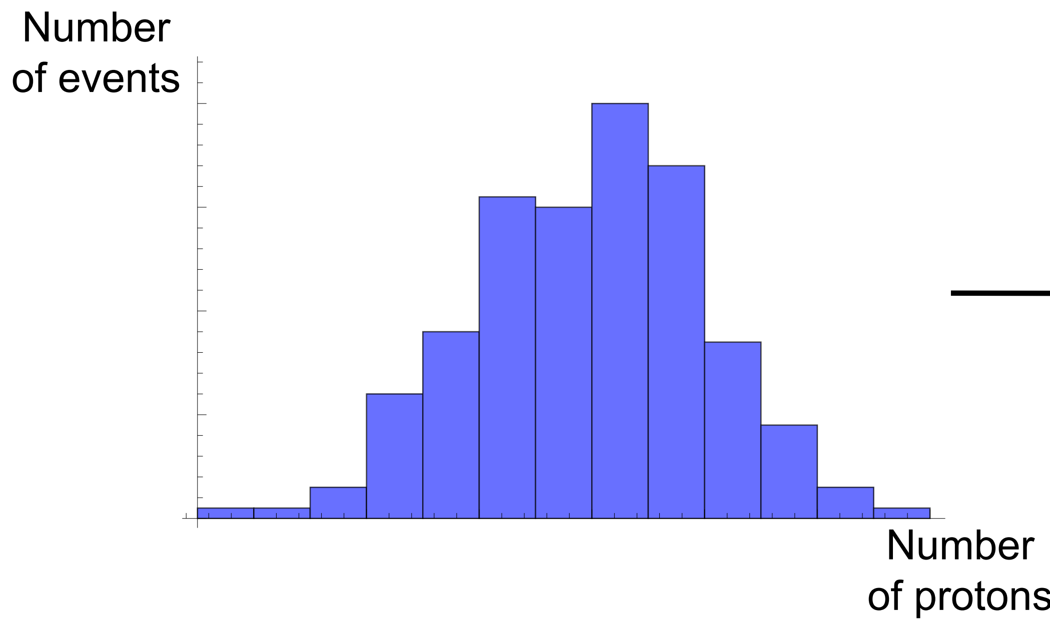
- $\xi \to \infty$ at CP in the thermodynamic limit $(t, V \to \infty)$
- Finite system lifetime ➡ $\xi_{\max} \sim 2\,\mathrm{fm}$ compared to $\sim 0.5\,\mathrm{fm}$ away from the CP (Berdnikov, Rajagopal 00)
- σ couples to pions and protons:

$$\mathcal{L}_{\sigma\pi\pi,\sigma pp} = 2\,G\,\sigma\,\pi^{+}\pi^{-} + g\;\sigma\;\bar{p}\;p$$

- Critical mode fluctuations affect
  - Particle multiplicity fluctuations
  - Momentum distributions
  - Ratios, etc…

of these particles.

Measuring fluctuations in particle multiplicities



→ measure the mean, variance, skewness, etc…

- Can repeat these calculations for pions, net protons, etc
- Want to obtain the critical contribution to these quantities
- We will use cumulants, e.g.:

$$\kappa_2 = \langle N^2 \rangle, \quad \kappa_3 = \langle N^3 \rangle, \quad \kappa_4 \equiv \langle\langle N^4 \rangle\rangle = \langle N^4 \rangle - 3\langle N^2 \rangle^2$$

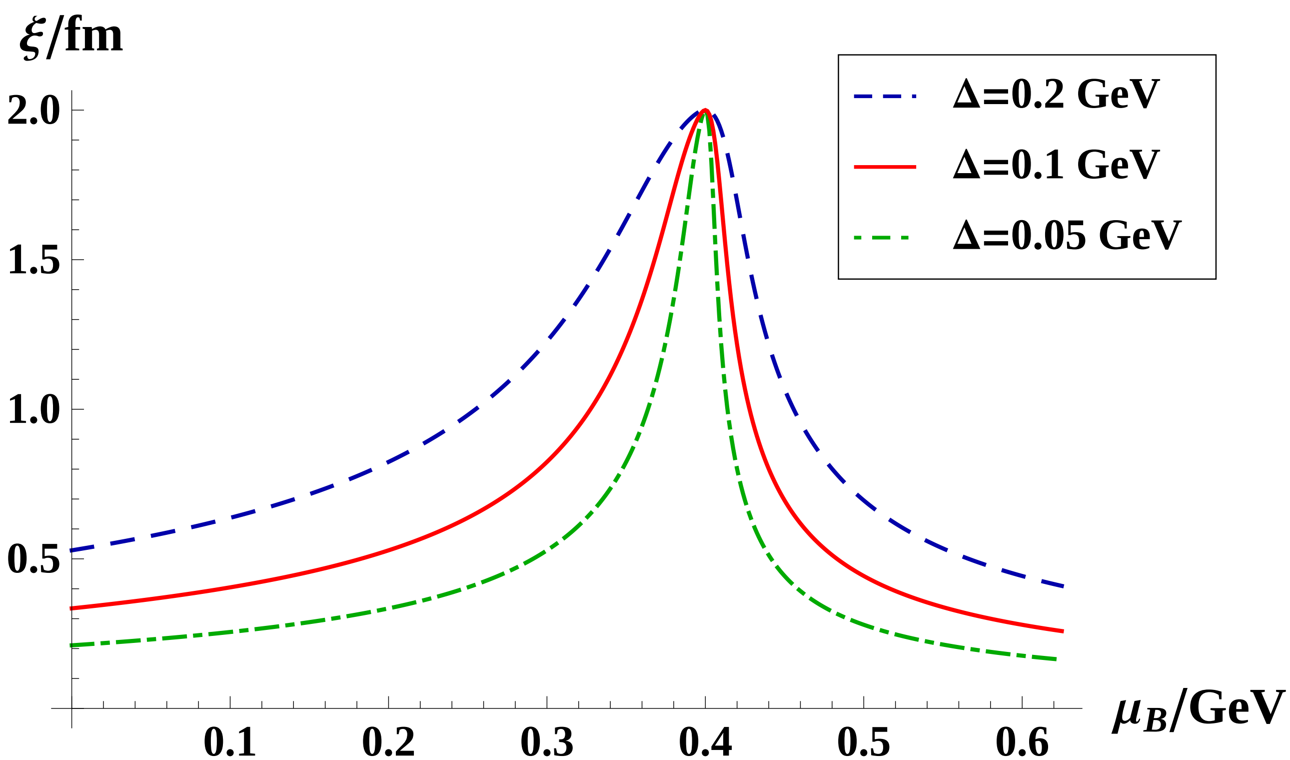
Massachusetts Institute of Technology

Sensitivity of Observables to QCD Critical Point

(Athanasiou, Rajagopal, Stephanov; arXiv:1006.4636)

- Critical fluctuations, with correlation length ξ, couple to $\pi\pi$ and pp. Hence, event-by-event fluctuations in π and p multiplicities & momenta with variance $\propto \xi^2$. (Stephanov, KR, Shuryak, 1999)
- Higher, non-Gaussian, moments of the event-by-event distributions are small in the limit of large number of particles, but are more sensitive to ξ. Skewness $\propto \xi^{4.5}$ and Kurtosis $\propto \xi^7$. (Stephanov, 2008)
- Comprehensive study of 21 observables (skewness and kurtosis of event-by-event histograms of pion number, proton number and net proton number; various pion-proton correlations) that are sensitive to critical fluctuations.
- Predictions for each, as function of few parameters including proximity to QCD critical point.
- Compare their relative sensitivity to critical fluctuations, depending on where on the phase diagram the critical point turns out to be located. (Some better at smaller $\sqrt{s}$; others better at larger $\sqrt{s}$.)
- Completed in time for data from the RHIC beam energy scan begun this spring.

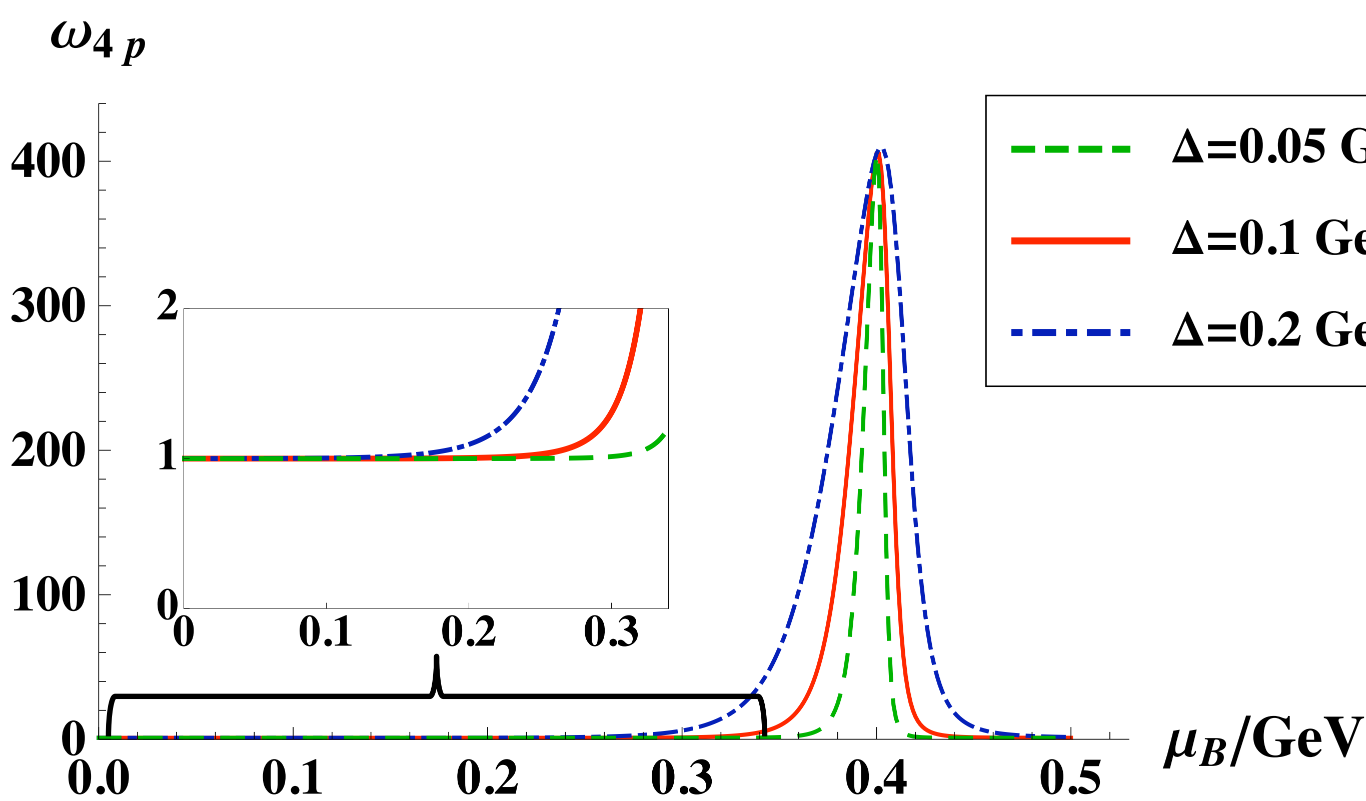
Suppose that $\xi(\mu_B)$ looks like this...



- Guesses (not predictions): freeze out point is closest to critical point in collisions that freezeout at $\mu_B^c = 400$ MeV. $\xi \sim 2$ fm in those collisions. Three choices for width Δ.

- If $\xi(\mu_B)$ looks like this, we can predict...

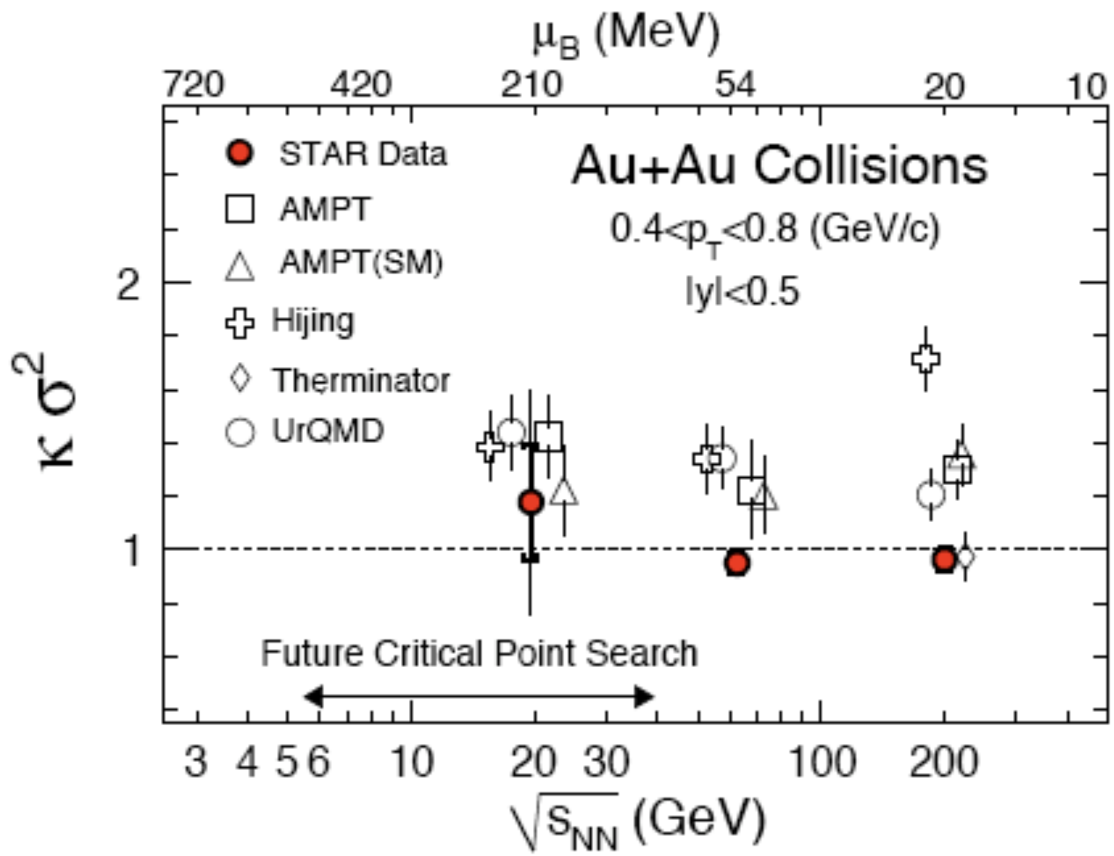
4th moment of event-by-event histogram of number of protons



- $\omega_{4p} = \kappa_{4p}/N_p$. Height of the peak is $\propto \xi^7$. Also depends on μ_B, the effective σpp coupling g, and two other parameters.

- $\omega_{4p} = 1$ for Poisson fluctuations, a background that must be present.

Data on net proton cumulants

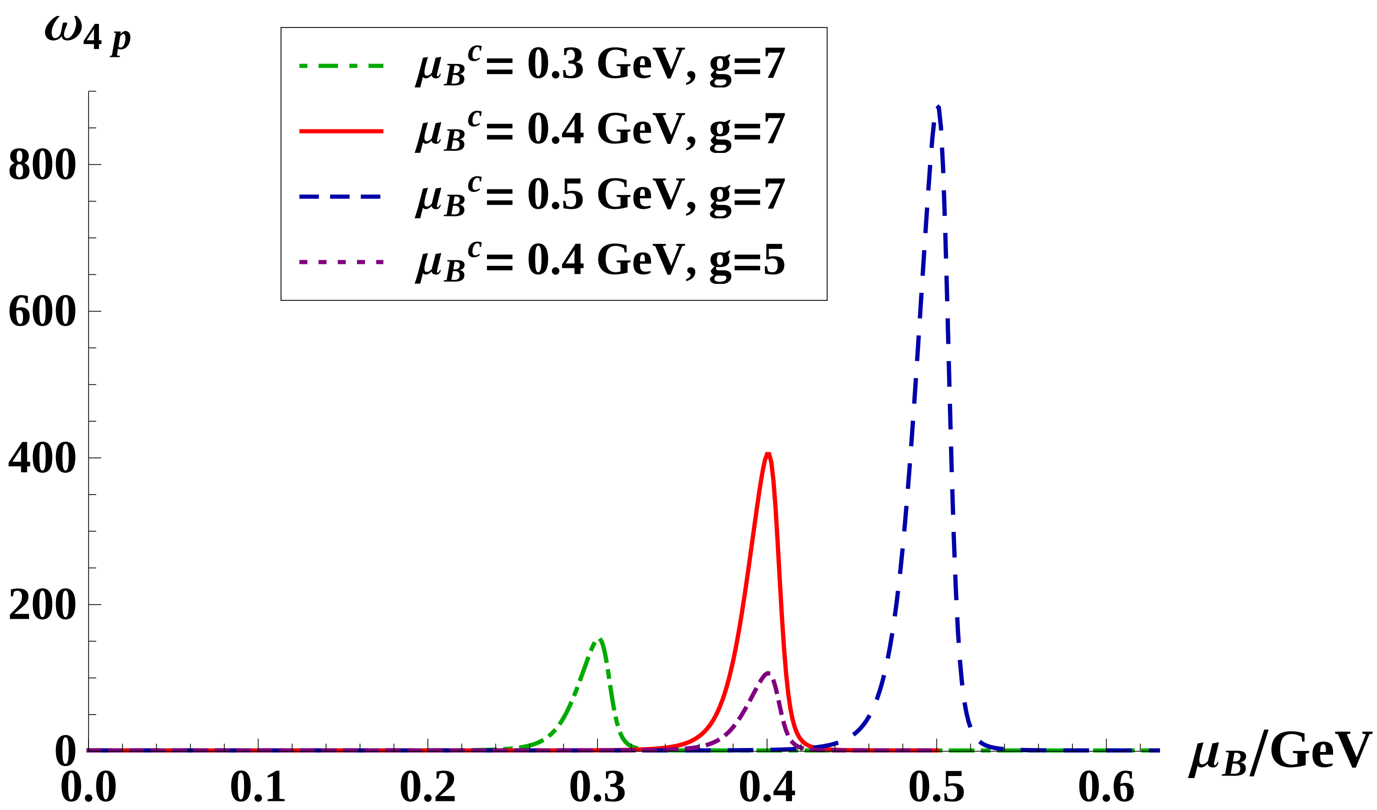


where $\kappa\,\sigma^2 \equiv \frac{\kappa_4}{\kappa_2}$

(STAR Collaboration 2010)

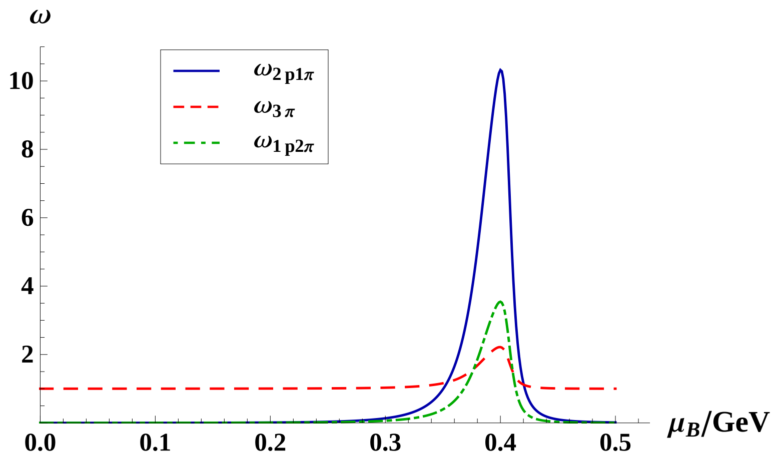
Massachusetts Institute of Technology

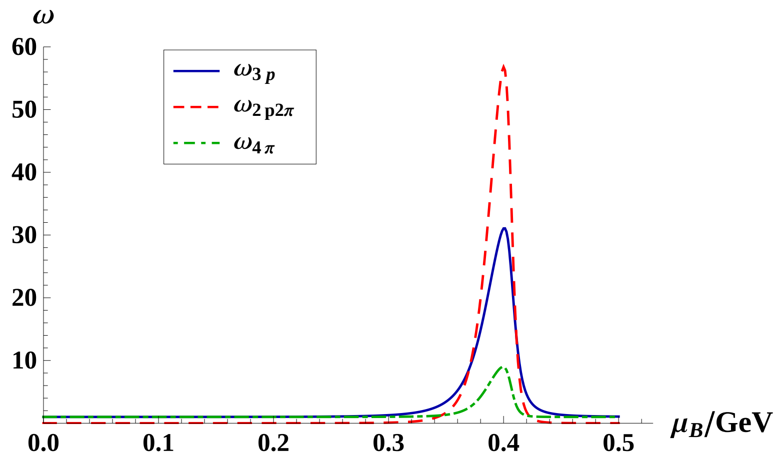
Vary μ_B^c and g....



- Substantial dependence on parameters other than ξ.
- Therefore, need to use more than one observable...

Six more observables





- Each of these 6 observables depends differently on the relevant parameters.
- I could show you more observables.
- Maybe better to think about ratios of observables? Can we use ratios to isolate specific combinations of parameters? Can we find ratios that are independent of the parameters, and hence where we can make robust predictions of their value if the observed fluctuations are indeed dominantly critical fluctuations? Answer: yes and yes ...

Parameter dependence of ratios of observables

ratio	V	$n_p(\mu_B)$	g_p	g_π	$\tilde{\lambda}_3$	$\tilde{\lambda}'_4$	ξ
N_π	1	-	-	-	-	-	-
N_p	1	1	-	-	-	-	-
$\kappa_{ipj\pi}$	1	i	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\omega_{ipj\pi}$	-	$i-\frac{i}{r}$	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\kappa_{ipj\pi}N_\pi^{i-1}/N_p^i$	-	-	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\kappa_{2p2\pi}N_\pi/\kappa_{4\pi}\kappa_{2p}$	-	-	-	-2	-	-	-2
$\kappa_{4p}N_\pi^2/\kappa_{4\pi}\kappa_{2p}^2$	-	-	-	-4	-	-	-4
$\kappa_{2p2\pi}N_p^2/\kappa_{4p}N_\pi^2$	-	-	-2	2	-	-	-
$\kappa_{3p1\pi}N_p/\kappa_{4p}N_\pi$	-	-	-1	1	-	-	-
$\kappa_{3p}N_p^{3/2}/\kappa_{2p}^{9/4}N_\pi^{1/4}$	-	-	$-3/2$	-	1	-	-
$\kappa_{2p}\kappa_{4p}/\kappa_{3p}^2$	-	-	-	-	-2	1	-
$\kappa_{3p}\kappa_{2\pi}^{3/2}/\kappa_{3\pi}\kappa_{2p}^{3/2}$	-	-	-	-	-	-	-
$\kappa_{4p}\kappa_{2\pi}^2/\kappa_{4\pi}\kappa_{2p}^2$	-	-	-	-	-	-	-
$\kappa_{4p}^3\kappa_{3\pi}^4/\kappa_{4\pi}^3\kappa_{3p}^4$	-	-	-	-	-	-	-
$\kappa_{2p2\pi}^2/\kappa_{4\pi}\kappa_{4p}$	-	-	-	-	-	-	-
$\kappa_{2p1\pi}^3/\kappa_{3p}^2\kappa_{3\pi}$	-	-	-	-	-	-	-

- The contribution of critical fluctuations to the five bottom ratios are all independent of ξ and all the other parameters. If they are dominated by critical fluctuations, all five ratios = 1. Very different from Poisson.

- Nontrivial, and robust, predictions. If large, non-Gaussian, fluctuations are seen in data, these will allow to confirm whether they are due to the proximity of the critical point.

- Other ratios constrain values of parameters, including ξ.

RHIC Beam Energy Scan

- Data was taken this spring at $\sqrt{s} = 39$, 11.5 and 7.7 GeV, corresponding to $\mu_B \simeq 110$, 320 and 420 MeV. First results expected in months ...

- Data to be taken next spring at $\sqrt{s} = 27$ and 18 GeV, corresponding to $\mu_B \simeq 160$ and 220 MeV.

- If the critical point is located at $\mu_B^c < 450$ MeV, we should start to see signs of its presence ...

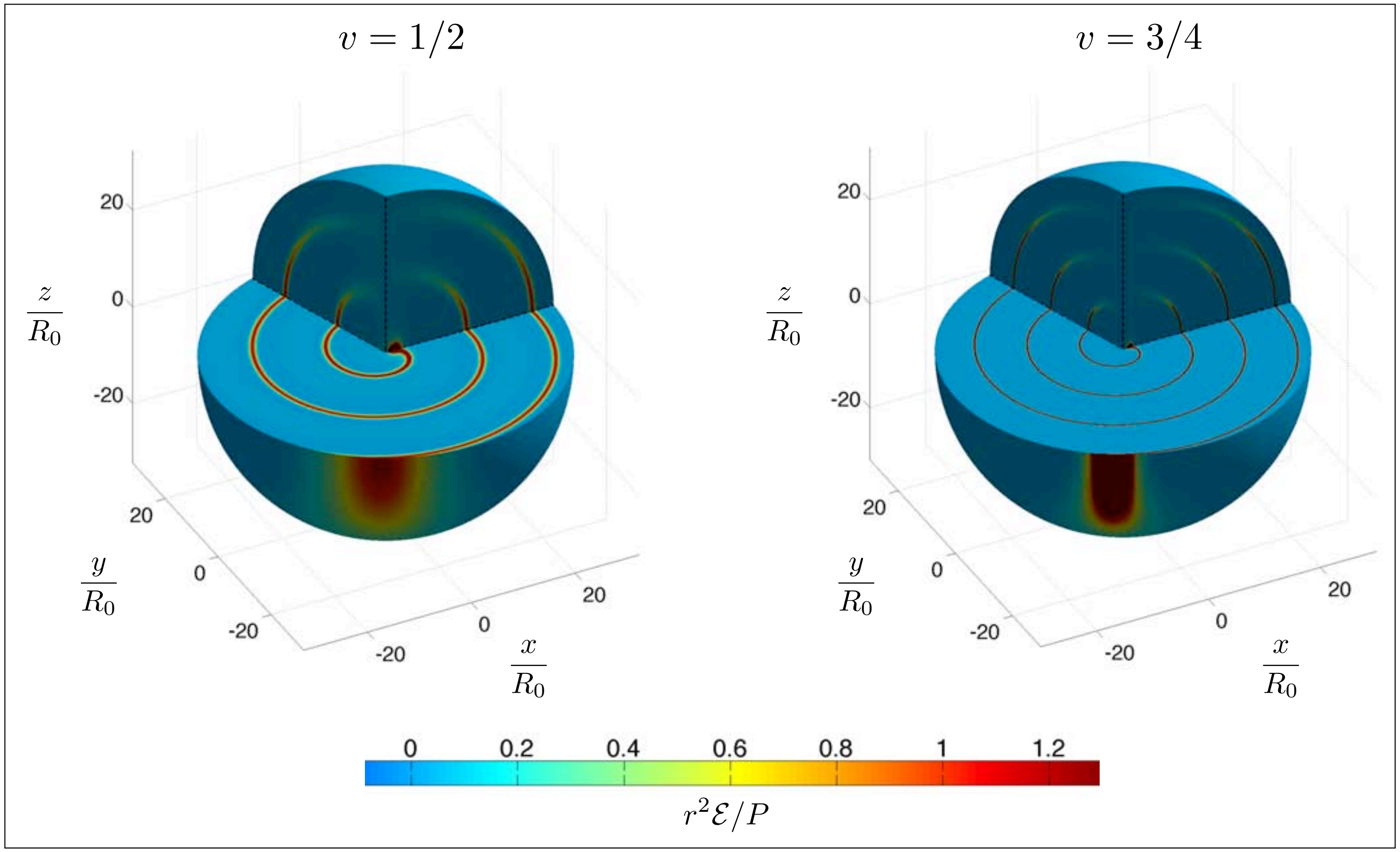
Transverse Momentum Broadening and the Jet Quenching Parameter, Redux

(D‘Eramo, Liu, Rajagopal; arXiv:1006.1367)

- “Semi-controlled”* QCD calculation, using SCET, of the momentum broadening of a fast quark propagating through a medium — namely $P(k_\perp)$, the probability that the fast quark picks up transverse momentum $k_\perp$ — and the jet quenching parameter $\hat{q} \equiv \frac{1}{L}\int k_\perp^2 P(k_\perp) d^2k_\perp$, with L the thickness of the medium.
- *“Semi-controlled”: radiation artificially turned off; other than that, controlled in the $T \ll E^{\text{jet}}$ limit.
- Clarifies rigorous definition of $\hat{q}$ in QCD or any other gauge theory and calculation of $\hat{q}$ in theories with a gravity dual.
- Previous calculation of $\hat{q}$ in the strongly coupled plasma of $\mathcal{N} = 4$ SYM theory, done using gauge/gravity duality, needed to be redone, became simpler, and the result is unchanged.
- Lekaveckas and D’Eramo now calculating $P(k_\perp)$ in a weakly coupled QCD plasma.
- The next step is the inclusion of radiation. Goal is to use SCET framework to set up a calculation of radiative energy loss and jet modification that is fully controlled in the high jet energy limit ($T \ll k_\perp^{\text{radiated gluon}} \ll E^{\text{jet}}$; a problem with three scales) relevant at the LHC, even if the plasma is strongly coupled at scales of order T. D’Eramo has begun the calculation and we look forward to joining forces with Chris Lee.

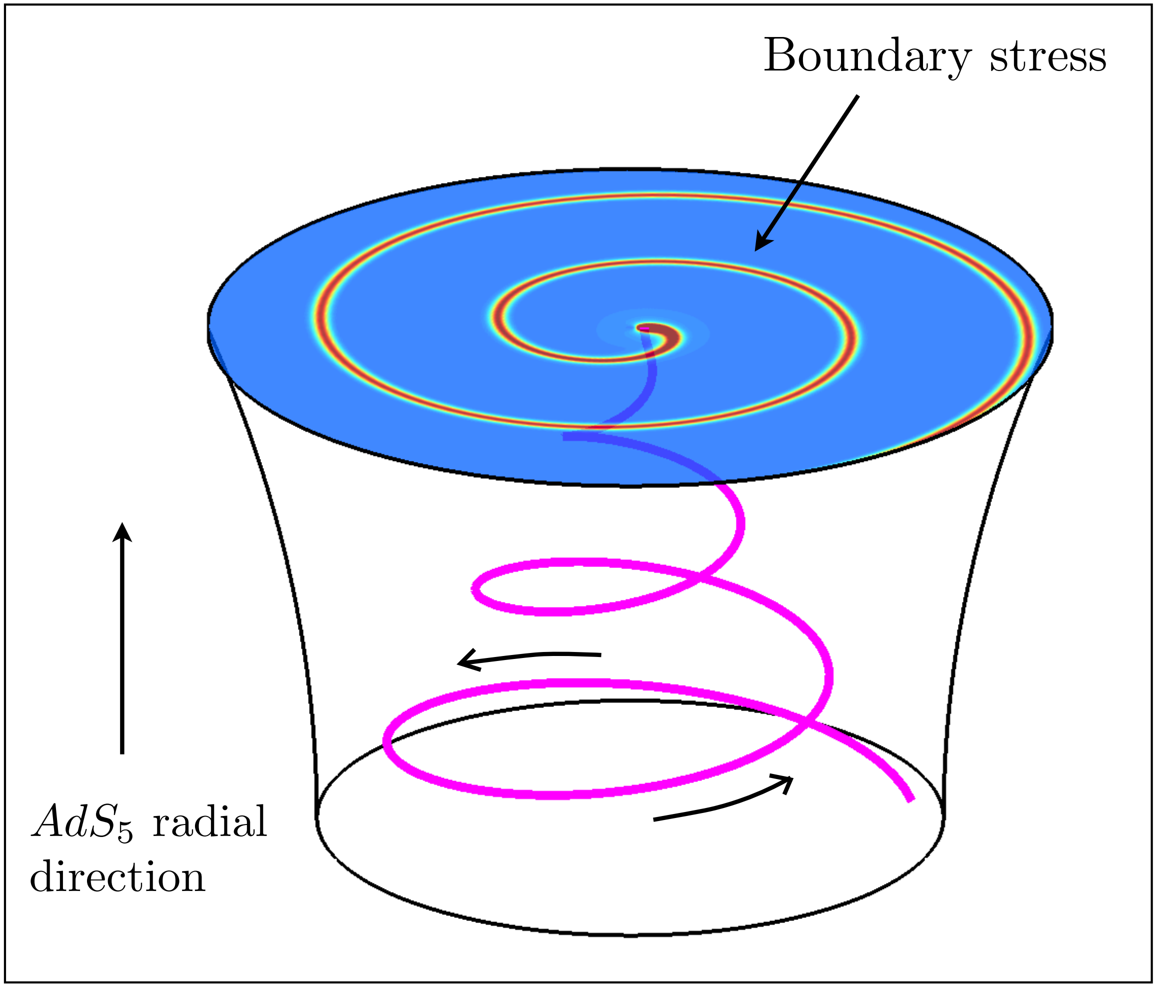
Synchrotron Radiation in Strongly Coupled Gauge Theories

(Athanasiou, Chesler, Liu, Nickel, Rajagopal; arXiv:1001.3880)



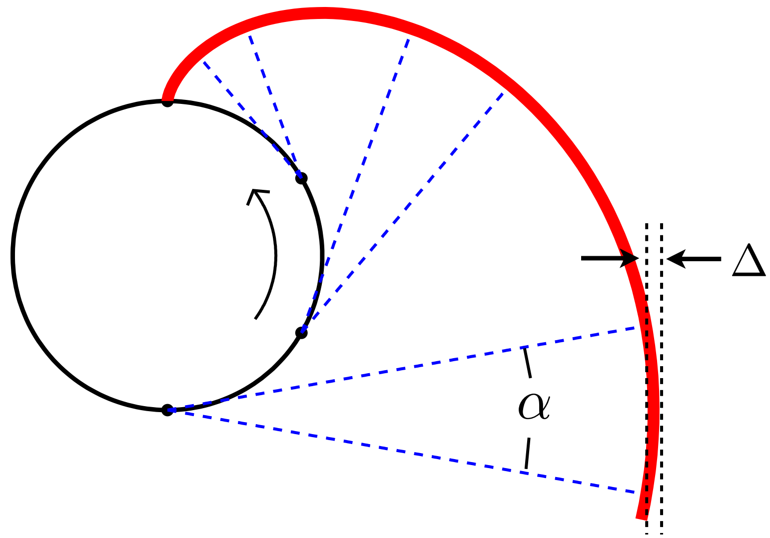
- Fully **quantum mechanical** calculation of gluon radiation from a rotating quark in a **strongly coupled** large N_c **non abelian** gauge theory, done via gauge/gravity duality. "Lighthouse beam" of synchrotron radiation. Surprisingly similar to classical electrodynamics.

How the Calculation Was Done....



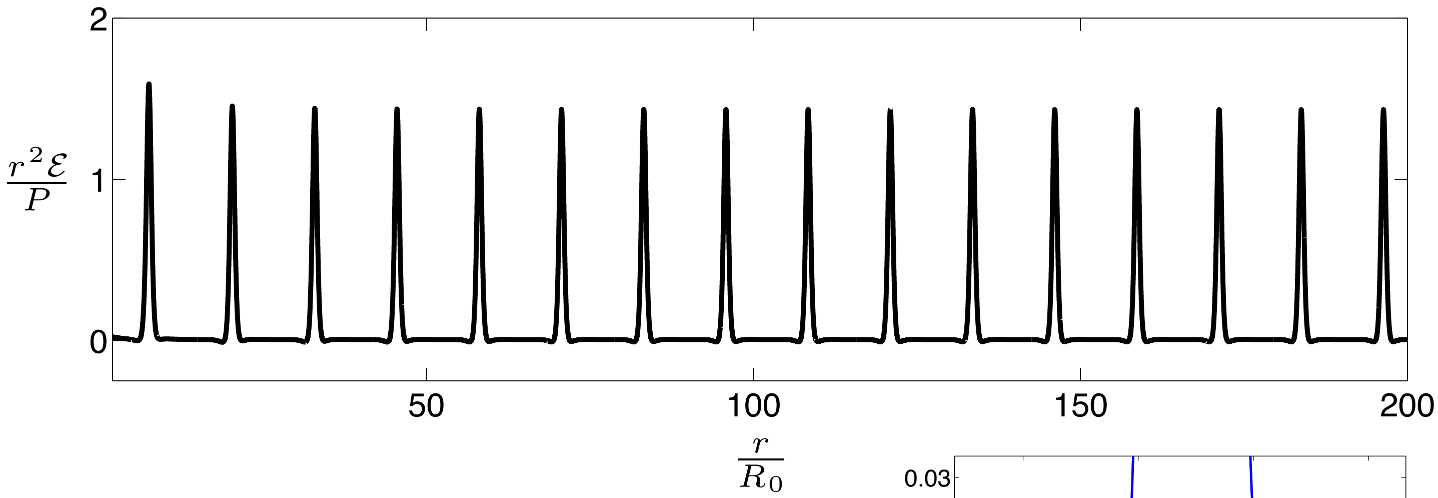
- String profile determined analytically. So is the synchrotron radiation. I'm showing you pictures, not formulae, but everything is analytical.

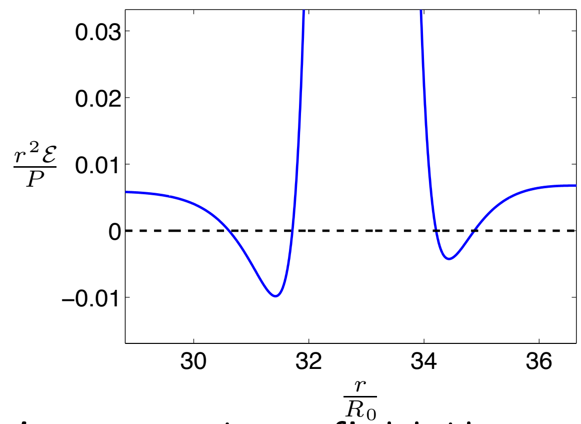
Synchrotron radiation, at weak or strong coupling



- For synchrotron radiation in classical electrodynamics, or in strongly coupled nonabelian $\mathcal{N} = 4$ SYM, there are two characteristic angular widths: $\alpha \propto 1/\gamma$ and $\Delta \propto 1/\gamma^3$. ($\gamma = 1/\sqrt{1 - v^2}$ where $v =$ speed of rotating quark.)
- These dependences, and indeed the shape of the spiral, follow directly from:
  - The radiation is emitted forward.
  - Pulse of radiation propagates at the speed of light *without spreading*.
- The surprise is that the pulses propagate outwards without spreading in a nonabelian gauge theory in the infinite coupling limit!

Almost Classic Synchrotron Pulses

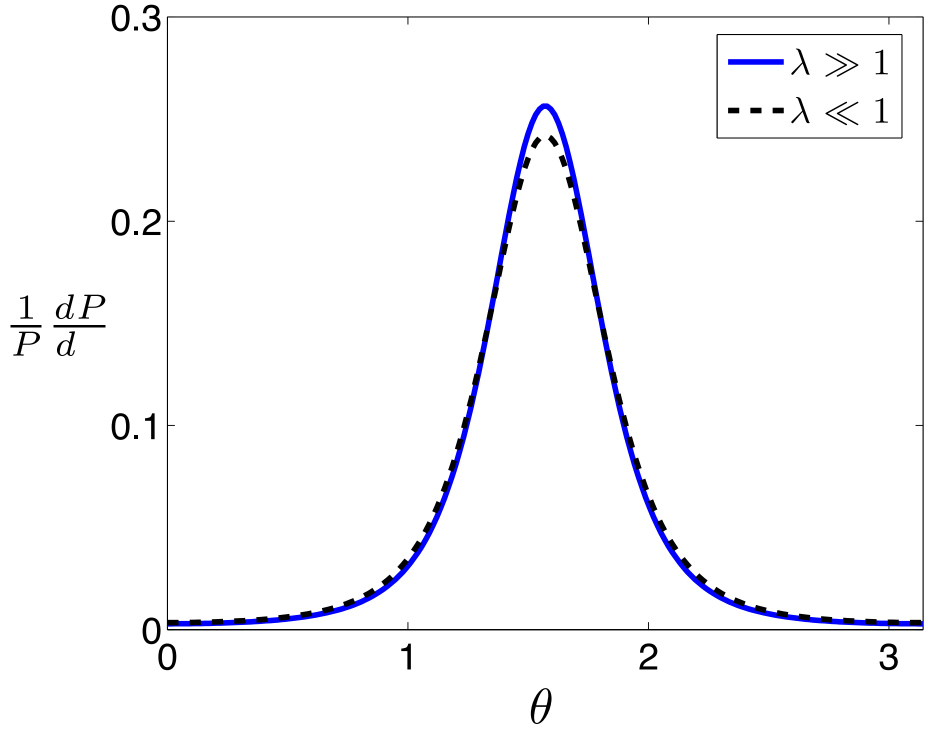




- Width of pulses $\propto 1/\gamma^3$. Here, $v = 0.5$.
- Energy density can go negative — this *is* a quantum field theoretical calculation — but quantum effects turn out to be small.
- And, the pulses really do propagate without spreading. Even though radiation is nonabelian and theory is infinitely strongly coupled.
- No isotropization. No sign of any analogue of "parton showering", à la Hatta, Iancu and Mueller.

Strong vs. Weak Coupling Radiation Pattern

(Time-averaged power distribution at infinity, as a function of polar angle)



- Angular distribution of radiation at weak and strong coupling very similar.
- Width in polar angle of the beam is $\propto 1/\gamma$. Here, $v = 0.9$.
- At weak coupling, radiation contains gluons and color-adjoint scalars.
- Strong coupling radiation pattern is precisely as if it contained twice as much scalar radiation relative to gluon radiation as at weak coupling. There is no way to say whether this is actually so, since the radiation is nonabelian and strongly coupled.

Quenching a Lighthouse Beam

- The next step is to repeat this calculation at nonzero temperature. Shine this "lighthouse beam" of gluons into the strongly coupled plasma.

- Make $R_0 \ll 1/T$ and $\gamma \gg 1$, namely a tightly collimated lighthouse beam that propagates for several turns of its synchrotron spiral before hitting length scales $\sim 1/T$.

- We know that on length scales $\gg 1/T$ the lighthouse beam will thermalize, becoming outgoing hydrodynamic waves which then dissipate.

- We will be able to answer a question like: A beam whose angular width in vacuum (and initially in the plasma) is θ propagates outward for a distance $2/T$ (or $1/T$, or $3/T$, or $\ldots$); what is its angular width? How rapidly does the strongly coupled plasma succeed in broadening the lighthouse beam?

- Measuring the broadening of jets produced in heavy ion collisions relative to those produced in $p-p$ collisions is currently one of the high priority goals of the RHIC program, and will soon be investigated also in heavy ion collisions at the LHC.

- We hope to get at this question using our SCET calculation (i.e. in QCD, assuming separation of scales — as at LHC?) once we have included radiative processes in that calculation.

- Watching the broadening of a lighthouse beam is an alternative (entirely strongly coupled — as at RHIC?) approach to the same question.

Au+Au: cross-section ratio R=0.2/R=0.4

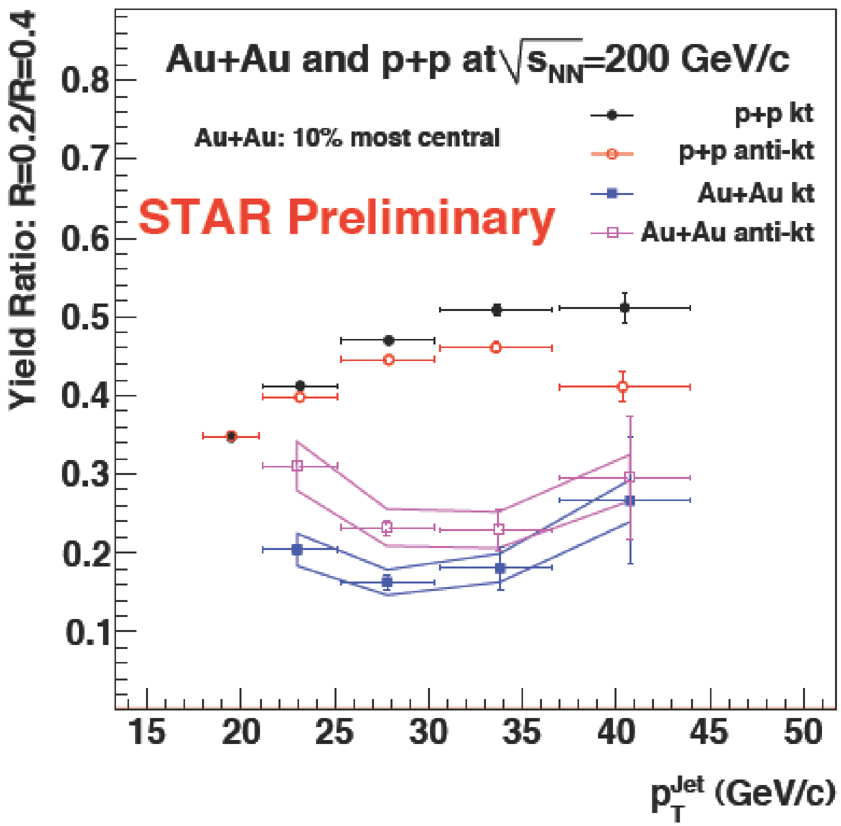
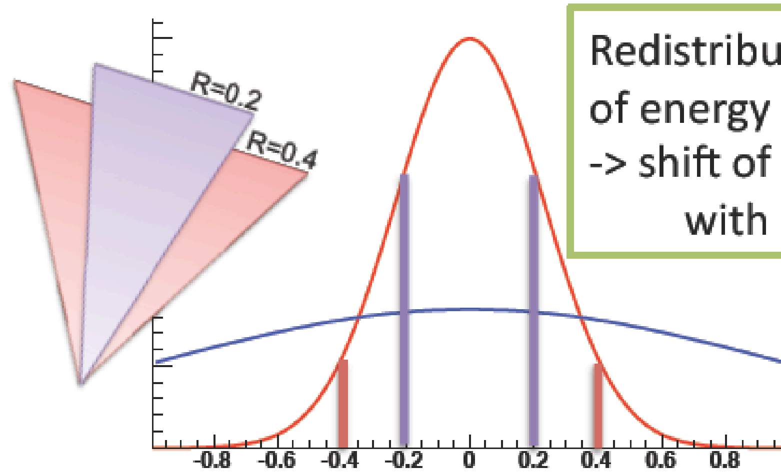


Illustration: Gaussian 1D profile



Redistribution of energy results in -> shift of jets p_T with fixed R

RED: p+p
BLUE: Au+Au

Strong broadening of the jet energy profile seen in measurement

Jet quenching parameter via Soft Collinear Effective Theory

based on: Francesco D'Eramo, H. Liu and KR, arXiv:1006.1367
slides largely borrowed from Francesco's talk in Prague last week

Krishna Rajagopal

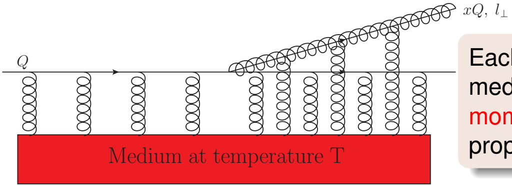
Massachusetts Institute of Technology

CERN, 18 August, 2010

Energy loss in the high energy limit

The medium has two main effects on the propagating hard parton:

- parton energy loss via QCD analogue of bremsstrahlung
- changing the direction of the parton's momentum.



Each hard parton continually kicked by the medium: each is subject to transverse momentum broadening, picking up $k_\perp^2$ as it propagates through the medium.

The jet quenching parameter $\hat{q}$

$$\boxed{\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L}}$$

$\hat{q}$ plays central role in the energy loss calculation, but it is *defined* via transverse momentum broadening only, i.e. by looking at just one hard parton in the absence of radiation.

Toward a factorized description

Separation of scales

Energy loss and transverse momentum broadening involve widely separated scales

$$E^{\rm jet} \equiv Q \gg l_\perp \gg T$$

Can we find a factorized description? Physics at each scale cleanly separated at lowest nontrivial order, corrections to factorization systematically calculable, order by order in the small ratio between the scales.

First step

Formulation of the momentum broadening in the language of Soft Collinear Effective Theory (SCET). (Radiation turned off by hand.)

Our focus

Non-radiative $k_\perp$ broadening in the $Q \to \infty$ limit:

- easiest case to handle;
- natural context in which the jet quenching parameter arises.

Our language: SCET

In the $T \ll Q$ limit:

- natural separation of scales
- natural organization of the modes into kinematic regimes

Set-up of the problem

Energy scales

Hard parton with initial four momentum: $q_0 \equiv (q_0^+, q_0^-, q_{0\perp}) = (0, Q, 0)$

propagating through some form of QCD matter.

Example: QGP in equilibrium at temperature T
(our analysis would apply to other forms of matter).

We assume $Q \gg T$, we have a small dimensionless ratio $\lambda \equiv \frac{T}{Q} \ll 1$.

Goal: compute $P(k_\perp)$

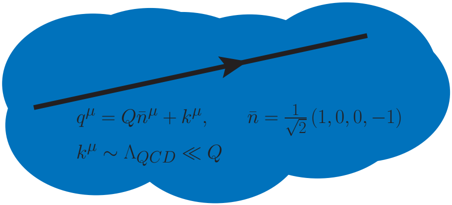
$P(k_\perp)$: probability distribution for the hard parton to acquire transverse momentum $k_\perp$ after traversing the medium.

$P(k_\perp)$ depends on the medium length L.

Normalization convention $\int \frac{d^2k_\perp}{(2\pi)^2} P(k_\perp) = 1$

Jet quenching parameter: $\hat{q} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 P(k_\perp)$

Soft Collinear Effective Theory (SCET)



SCET

Effective theory of highly energetic, approximately massless particles interacting with "soft stuff".

C. Bauer et al., Phys. Rev. D 63: 014006, 2001 . . .

Egs: $B \to \pi\pi$; $e^+e^- \to 2$ jets; . . .
Eg: "soft stuff" = QGP Idilbi, Majumder, Phys.Rev.D80:054022,2009. [arXiv:0808.1087]

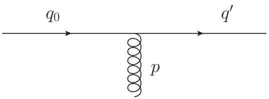
SCET degrees of freedom

Introduce fields for infrared degrees of freedom. Conventional notation:

modes	$q^\mu = (q^+, q^-, q_\perp)$	q^2	fields
collinear	$Q\,(\lambda^2, 1, \lambda)$	$\lambda^2 Q^2$	$\xi_{\bar{n}},\ A^\mu_{\bar{n}}$
soft	$Q\,(\lambda, \lambda, \lambda)$	$\lambda^2 Q^2$	$\xi_s,\ A^\mu_s$
ultra-soft	$Q\,(\lambda^2, \lambda^2, \lambda^2)$	$\lambda^4 Q^2$	$\xi_{us},\ A^\mu_{us}$

where Q is the high scale in the problem. (For us, $Q = E^{\rm jet}$, and $\lambda \equiv T/Q$.)
Offshell modes with $q^2 \gg \lambda^2 Q^2$ are integrated out (in coefficients).

$k_\perp$ broadening in the high energy limit



$$q_0 = (0, Q, 0)$$

$$q' = q_0 + p$$

Soft gluon: $p = (\lambda, \lambda, \lambda)Q$

Final state $\sim Q(\lambda, 1, \lambda)$ not collinear
Kicked off-shell by $q'^2 \sim \lambda Q^2$
Process suppressed by $\alpha_s(\sqrt{TQ})$
Subsequent radiation induced.

Glauber gluon: $p = (\lambda^2, \lambda^2, \lambda)Q$

Final state is collinear
Further Glaubers keep parton collinear
No induced radiation. Interaction vertex: $\alpha_s(T)$

Relevance of Glauber gluons Idilbi and Majumder

Both processes yield momentum broadening of order $\lambda Q \sim T$.

Soft suppressed by $\alpha_s(\sqrt{TQ})$, Glaubers dominate $k_\perp$-broadening for $Q \to \infty$.

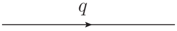
Glauber gluons less numerous than soft, so both may be relevant at the Q values accessible at RHIC and the LHC.

Radiation must be included. We've turned it off by hand.

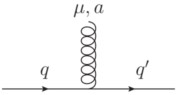
SCET + Glauber Lagrangian

Effective Lagrangian describing the interaction between

- Collinear partons: $q = Q(\lambda^2, 1, \lambda)Q$;
- Glauber gluons: $p = (\lambda^2, \lambda^2, \lambda)Q$.



$$= i\, \not{\bar{n}} \frac{Q}{2q^+Q - q_\perp^2 + i\epsilon}$$



$$= igt^a n_\mu \not{\bar{n}}$$

SCET + Glauber Lagrangian at LO in λ

Start from the QCD Lagrangian.

Keep only the relevant degrees of freedom.

Power counting in λ at the level of the Lagrangian. At the leading order in λ

$$\mathcal{L}_{\bar{n}} = \sum_{q_\perp, q'_\perp} e^{i(q_\perp - q'_\perp)\cdot x_\perp}\, \bar{\xi}_{\bar{n}, q'_\perp} \left[i\bar{n}\cdot D + \frac{q_\perp^2}{2Q} \right] \not{n}\, \xi_{\bar{n}, q_\perp}$$

Idilbi, Majumder, Phys.Rev.D80:054022,2009. [arXiv:0808.1087]

So, what do we want to calculate using this Lagrangian? . . .

Optical theorem

Use the optical theorem to relate $P(k_\perp)$ to a matrix element that we can then calculate using the SCET Lagrangian ...

Probability amplitude for the process $\alpha \to \beta$: $S_{\beta\alpha} = \delta_{\beta\alpha} + iM_{\beta\alpha}$.

The S-matrix is unitary: $\sum_\beta |S_{\beta\alpha}|^2 = 1 \Rightarrow 2\,\mathrm{Im}\; M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2$.

With radiation turned off, single particle in final and inital state, β differs from α only on its value of $k_\perp$: in cubic box of side L, $\sum_\beta = L^2 \int \frac{d^2 k_\perp}{(2\pi)^2}$.
Unitarity of S-matrix $\leftrightarrow$ $P(k_\perp)$ is normalized.

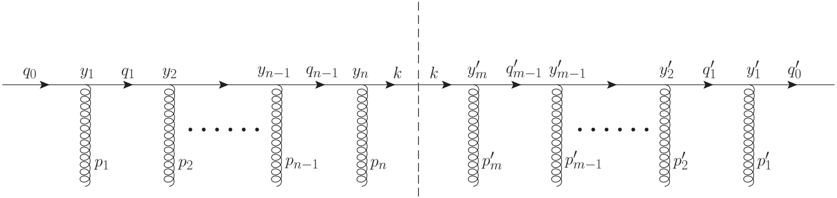
We identify: $P(k_\perp) = L^2 \begin{cases} |M_{\beta\alpha}|^2 & \beta \neq \alpha \\ 1 - 2\mathrm{Im}\, M_{\alpha\alpha} + |M_{\alpha\alpha}|^2 & \beta = \alpha \end{cases}$

Plan: Calculate $\mathrm{Im}\; M_{\alpha\alpha}$, which gives $\sum_\beta |M_{\beta\alpha}|^2$, which gives $P(k_\perp \neq 0)$. Normalization fixes $P(0)$.

Forward scattering amplitude

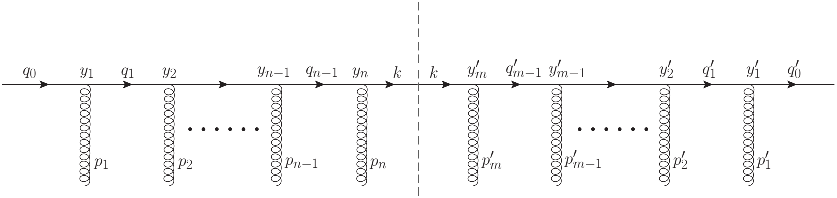
Strategy

- Compute $2\,\mathrm{Im}\ M_{\alpha\alpha}$ by cutting the appropriate diagrams;
- Use the unitarity relation to identify $|M_{\beta\alpha}|^2$;
- Evaluate $P(k_\perp)$.



$$2\,\mathrm{Im}\,M_{\alpha\alpha} = \sum_{m=1,n=1}^{\infty} \mathcal{A}_{mn} = \sum_{m=1,n=1}^{\infty} \int \frac{d^2k_\perp}{(2\pi)^2}\,\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$$

Forward scattering amplitude evaluation



SCET Lagrangian Feynman rules give:

$$
\begin{aligned}
\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp} = & \frac{1}{\sqrt{2}QL^3}\int \frac{dk^+dk^-}{(2\pi)^2}\prod_{i=1}^{n-1}\frac{d^4q_i}{(2\pi)^4}\prod_{j=1}^{m-1}\frac{d^4q'_j}{(2\pi)^4} \\
& \times \bar{\xi}_{\bar{n}}(q'_0)\prod_{j=m-1}^{1}\left[(-ig)A^+(-p'_j)\,\not{n}\,\frac{-iQ}{2Qq'^{+}_j - q'^{2}_{j\perp} - i\epsilon}\,\not{\bar{n}}\right](-ig)A^+(-p'_m)\not{n} \\
& \times 2\pi Q\delta\left(2k^+Q - k_\perp^2\right)\not{\bar{n}}\; igA^+(p_n)\not{n}\prod_{i=1}^{n-1}\left[\frac{iQ}{2Qq_i^+ - q^2_{i\perp} + i\epsilon}\not{\bar{n}}\; igA^+(p_i)\not{n}\right]\xi_{\bar{n}}(q_0)
\end{aligned}
$$

(Analysis is analogous if hard parton is a collinear gluon.)

Evaluating $\frac{d^2\mathcal{A}_{mn}}{d^2k_\perp}$ in the $Q \to \infty$ limit: $Q \gg k_\perp^2 L \sim \hat{q}L^2$

If this condition is satisfied, the propagators of the internal quarks are $\propto \delta^2(z_\perp)$. It requires L is short enough that the hard parton trajectory in position space remains well-approximated as a straight line, even though it picks up transverse momentum.

In this limit, using $A^+(p_i) = \int d^4y_i e^{ip_i y_i} A^+(y_i)$, evaluating, summing over m and n, and taking $\langle \ldots \rangle$ at the end of the calculation, we find

$$\sum_{m=1,n=1}^{\infty} \frac{d^2\mathcal{A}_{nm}}{d^2k_\perp} = \frac{\sqrt{2}}{L^3 N_c} \int dy^+ d^2y_\perp d^2y'_\perp \, e^{-ik_\perp \cdot (y_\perp - y'_\perp)} \left\langle \mathrm{Tr}\left[\left(W_F^\dagger[y^+, y'_\perp] - 1 \right) \left(W_F[y^+, y_\perp] - 1 \right) \right] \right\rangle$$

where we have introduced the fundamental Wilson line along the lightcone (with $L^- \equiv \sqrt{2}L$)

$$W_F\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ig \int_0^{L^-} dy^- \, A^+(y^+, y^-, y_\perp) \right] \right\}$$

Cleaning up the result, and an important point

- Because medium is translation invariant, result is independent of y^+ and depends only on $x_\perp \equiv y_\perp - y'_\perp$.
- Incident flux is $1/L^3$. t/L particles going through box in time t. Divide result by t/L, to obtain probability distribution for a single particle to acquire $k_\perp$.

Result (in a font such that you can read it):

$$\sum_{m=1,n=1}^{\infty} \frac{d^2\mathcal{A}_{nm}}{d^2k_\perp} = \frac{1}{N_c}\int d^2x_\perp\, e^{-ik_\perp\cdot x_\perp}\left\langle \mathrm{Tr}\left[\left(W_F^\dagger[0,x_\perp]-1\right)\left(W_F[0,0]-1\right)\right]\right\rangle$$

A_μ as a background field

We first described a hard parton propagating in a specified field configuration $A_\mu(p)$. Valid no matter whether the medium is strongly coupled or weakly coupled. This question only arises at the end, when you try to actually evaluate the $\langle \ldots \rangle$.

Final result for $P(k_\perp)$

Unitarity: $\int \frac{d^2 k_\perp}{(2\pi)^2} \sum_{n=1,m=1}^{\infty} \frac{d^2 \mathcal{A}_{nm}}{d^2 k_\perp} = 2\,\mathrm{Im}\, M_{\alpha\alpha} = \sum_\beta |M_{\beta\alpha}|^2 = L^2 \int \frac{d^2 k_\perp}{(2\pi)^2} |M_{\beta\alpha}|^2$

Expression for $P(k_\perp)$ Casalderrey-Solana and Salgado

$$P(k_\perp) = \int d^2 x_\perp\, e^{-ik_\perp \cdot x_\perp}\, \mathcal{W}_{\mathcal{R}}(x_\perp), \qquad \mathcal{W}_{\mathcal{R}}(x_\perp) = \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr}\left[W_{\mathcal{R}}^\dagger[0, x_\perp]\, W_{\mathcal{R}}[0,0] \right] \right\rangle$$

$$W_{\mathcal{R}}\left[y^+, y_\perp\right] \equiv P\left\{ \exp\left[ig \int_0^{L^-} dy^-\, A_{\mathcal{R}}^+(y^+, y^-, y_\perp) \right] \right\}$$

for a collinear particle in the $SU(N)$ representation $\mathcal{R}$, with dimension $d(\mathcal{R})$.

- $P(k_\perp)$ is what SCET folks call a "soft function". It depends only on the medium, namely physics at the soft scales $\sim T$. And, thus, so does $\hat{q}$.
- Upon turning radiation off by hand, we have a nice factorization: transverse momentum broadening governed by a field theoretically well-defined property of the medium. Could calculate corrections to factorization (i.e. non-infinite Q) that are suppressed by powers of T/Q. Better first to include radiation, and see whether and how factorization arises then.

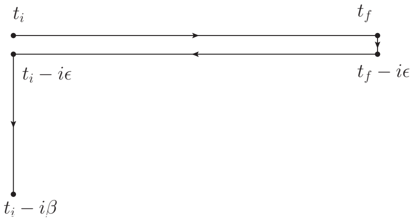
Evaluating $\hat{q}$ and operator ordering

$\hat{q}$ from light-like Wilson lines

$$\hat{q} \equiv \frac{\langle k_\perp^2 \rangle}{L} = \frac{\sqrt{2}}{L^-} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \int d^2 x_\perp \, e^{-i k_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

The operator ordering in the expectation value of the two Wilson lines, $\mathcal{W}_\mathcal{R}(x_\perp)$, is not that of a standard Wilson loop. Recall $A^+ = (A^+)^a t^a$.

Standard Wilson loop: $(A^+)^a$ time ordered, t^a path ordered.
Wilson lines in $\mathcal{W}_\mathcal{R}(x_\perp)$: $(A^+)^a$ and t^a both path ordered.



Means $\mathcal{W}_\mathcal{R}(x_\perp)$ should be described using the Schwinger-Keldysh contour

- one of the light-like Wilson lines on the $\mathrm{Im}\, t = 0$ segment
- the other light-like Wilson line on the $\mathrm{Im}\, t = -i\epsilon$ segment

$\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM theory revisited

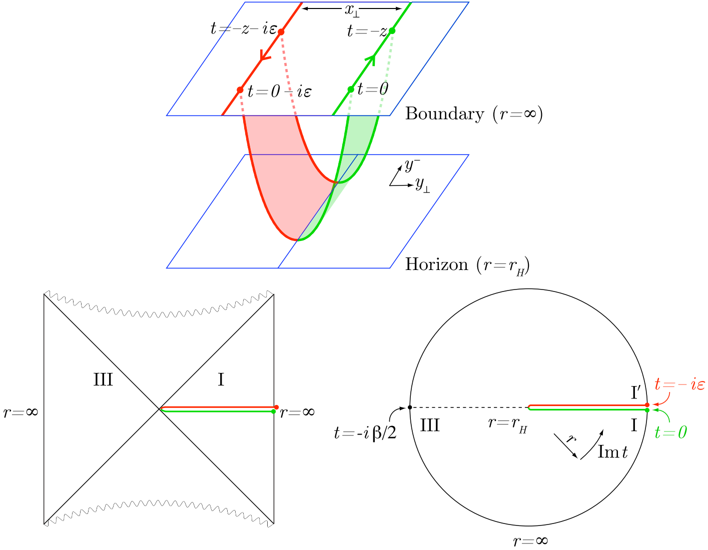
At RHIC, physics of the QGP at scales $\sim T$ is not weakly coupled. Insights can be obtained by calculating $\hat{q}$ in strongly coupled $\mathcal{N} = 4$ SYM — calculation can be done because the theory has a dual gravitational description. (Liu, Rajagopal, Wiedemann, 2006)

LRW used gauge/gravity duality to evaluate $\mathcal{W}$ with the standard, i.e. wrong, operator ordering. Must be redone.

AdS/CFT basics

- $\mathcal{N} = 4$ $SU(N_c)$ gauge theory
- large N_c and $g^2_{YM} N_c$ limit
- Gravity dual: 4+1 dimensional AdS Schwarzschild black hole with Hawking temperature T
- $\langle W(\mathcal{C}) \rangle = \exp\left[i\left\{S(\mathcal{C}) - S_0\right\}\right]$ where $S(\mathcal{C})$ is the action of an extremized string worldsheet, bounded by the Wilson lines along the contour $\mathcal{C}$ located at the 3+1 dimensional boundary, "hanging" into the AdS black hole spacetime.
- Operator ordering? Value of the $x_\perp$-independent subtraction S_0?

Taking the ordering into account Skenderis and van Rees, 2009



Any string world sheet connecting the Wilson lines at $\mathrm{Im}\,t=0$ and $\mathrm{Im}\,t=-i\epsilon$, as required by the Schwinger-Keldysh implementation of the operator ordering, must touch the horizon.

$P(k_\perp)$ in strongly coupled $\mathcal{N} = 4$ SYM theory

- $\mathcal{W} = \exp[i(S - S_0)]$. S is action of the connected world sheet. S_0 is twice the action of a disconnected world sheet hanging straight down from one Wilson line to the horizon.
- The connected world sheet that was identified on physical grounds by LRW is the only one that enters the calculation. LRW result for S unchanged. Subtlety resolved.
- S and S_0 are both imaginary, making $\mathcal{W}$ real. S_0 must be such that $P(k_\perp)$ is normalized. Provides a check of the LRW result for S_0.

For a hard parton in the adjoint representation, we find

$$\mathcal{W}(x_\perp) = \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L^- x_\perp^2\right] \qquad P(k_\perp) = \frac{4\sqrt{2}\pi}{\hat{q}L^-}\exp\left[-\frac{\sqrt{2}k_\perp^2}{\hat{q}L^-}\right]$$

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{g^2 N_c}\,T^3 \,.$$

- $P(k_\perp)$ describes diffusion in $k_\perp$-space.
- $\hat{q}$ not proportional to entropy density s, which is proportional to N_c^2. In fact, in a large class of conformal theories with gravity duals, $\hat{q}_{\rm CFT}/\hat{q}_{\mathcal{N}=4} = \sqrt{\hat{s}_{\rm CFT}/\hat{s}_{\mathcal{N}=4}}$
- No quasiparticles in this strongly coupled plasma, so $\hat{q}$ doesn't count scattering centers.

Summary and future directions

Summary

- Probability distribution $P(k_\perp)$ evaluated within SCET formalism;
- Glauber gluons responsible for $k_\perp$ broadening in the absence of radiation;
- $P(k_\perp)$ and $\hat{q}$ are properties of the medium (factorization);
- Subtleties about the operator ordering: strong coupling $\hat{q}$ evaluation more straightforward, previous result unchanged.

Future directions

- Evaluate $P(k_\perp)$ for weakly coupled QCD plasma at high enough T.

  [in progress, with Mindaugas Lekaveckas.]

  - $P(k_\perp)$ very different than at strong coupling.
  - Falls off slowly $\sim k_\perp^{-4}$ at large $k_\perp$.
  - Has a $\delta^2(k_\perp)$ at $k_\perp = 0$, describing the probability of no scattering.
- Include radiation, and interaction with soft gluons in addition to Glauber gluons. Three scale problem. See how $\hat{q}$ enters in the spectrum of the radiated gluons.
- Include corrections suppressed by powers of T/Q.