Trace anomaly, chiral symmetry breaking and parity doubled nucleons

Chihiro Sasaki
Frankfurt Institute for Advanced Studies

Outline:

1. Introduction: baryons near chiral symmetry restoration
2. Thermodynamics of hadronic matter in a parity doublet model
   Ref. CS and I. Mishustin, arXiv:1005.4811 [hep-ph], to appear in Phys.Rev.C
3. Dynamical χSB, trace anomaly and hadron masses
4. A working hypothesis: anomaly matching

Baryons near chiral symmetry restoration?

- dynamical origin of nucleon mass?
  - Gell-Mann Levy model: spontaneous χSB generates $m_N = g\langle\sigma\rangle$.

$$\mathcal{L}_{\rm GL} = i\bar{N}\partial\!\!\!/ N - g\bar{N}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N + \mathcal{L}_{\rm meson}$$

  - chiral transf. for a nucleon *assumed* to be the same as for a quark
  - in Nature: $N^*N\pi$ interaction?
- m_N at χ-symmetry restoration?
  - standard (naive): DχSB generates masses $m_N \stackrel{\sigma\to 0}{\to} 0$
  - parity doublet (mirror): DχSB generates mass difference $m_{N_+} \stackrel{\sigma\to 0}{\to} m_{N_-} = m_0 \neq 0$ [Detar-Kunihiro (89)]
  - in general linear combinations: $|\alpha\rangle = \sum_J c_J|J\rangle$
- emergence of a scale in QCD:
  - trace anomaly $\Theta^\mu_\mu = \frac{\beta}{2g}G^2 + m(1+\gamma)\bar{q}q$
  - $\langle\frac{\beta}{2g}G^2\rangle^{\rm lattice}_{T_\chi} \sim \frac{1}{2}\langle\frac{\beta}{2g}G^2\rangle_{\rm vac} \neq 0$ [Miller (07)]

- naive vs. mirror
  - axial couplings: $g_A^{++} = g_A^{--}$ (naive) $\quad g_A^{++} = -g_A^{--}$ (mirror)
    as a group-theoretical consequence in a simple LσM for σ and π.

    However,
    cf. other chiral invariant operators allowed [Jaffe-Pirjol-Scardicchio (06)]
    cf. lattice QCD: $g_A^{--} = 0.2 \pm 0.3$ [Takahashi-Kunihiro (07)]
    AdS/QCD: $g_A^{++} = 0.73\,,\ g_A^{--} = 0.38$ [Hashimoto-Sakai-Sugimoto (08)]
    cf. explicit a_1 mesons [Gallas-Giacosa-Rischke (09)]

  - which state is the true chiral partner of $N(940)$?
    if $N(1535)$ then $m_0 = 270$ MeV (from $\Gamma^{(\mathrm{exp})}(N^* \to N\pi) = 70$ MeV)
    $\Leftrightarrow$ cannot reproduce $\Gamma^{(\mathrm{exp})}(N^* \to N\eta) \sim 80$ MeV
    a speculative candidate closer to N? and/or large OZI-violation?

Dense nuclear matter in chiral models

- **nuclear matter: known properties**
  - binding energy: $E/A(\rho_0) - m_N = -16$ MeV
  - saturation density: $\rho_0 = 0.16$ fm^{-3}
  - incompressibility: $K = 9\rho_0^2 \partial^2(E/A)/\partial\rho^2|_{\rho=\rho_0} \simeq 200$ MeV

- **in-medium chiral perturbation theory**

  $\Rightarrow$ in-medium chiral perturbation theory [Lutz, Friman, Kaiser, Meissner, Weise, ...]

- **mean-field models**
  - LSM: no stable ground state corr. to nuclear matter [Kerman-Miller (74)]
  - nucleonic NJL: possible if 4F vector and 8F scalar-vector int. incld.

    [Koch-Biro-Kunz-Mosel (87), Buballa (96), Mishustin-Satarov-Greiner (03)]
  - NJL with diquarks: baryon as a bound state of a quark and a diquak

    [Bentz-Thomas (01), Bentz-Horikawa-Ishii-Thomas (03)]
  - parity doublet model: large $m_0 \sim 800$ MeV needed?

    [Hatsuda-Prakash (89), Zschiesche-Tolos-Schaffner-Bielich-Pisarski (07)]

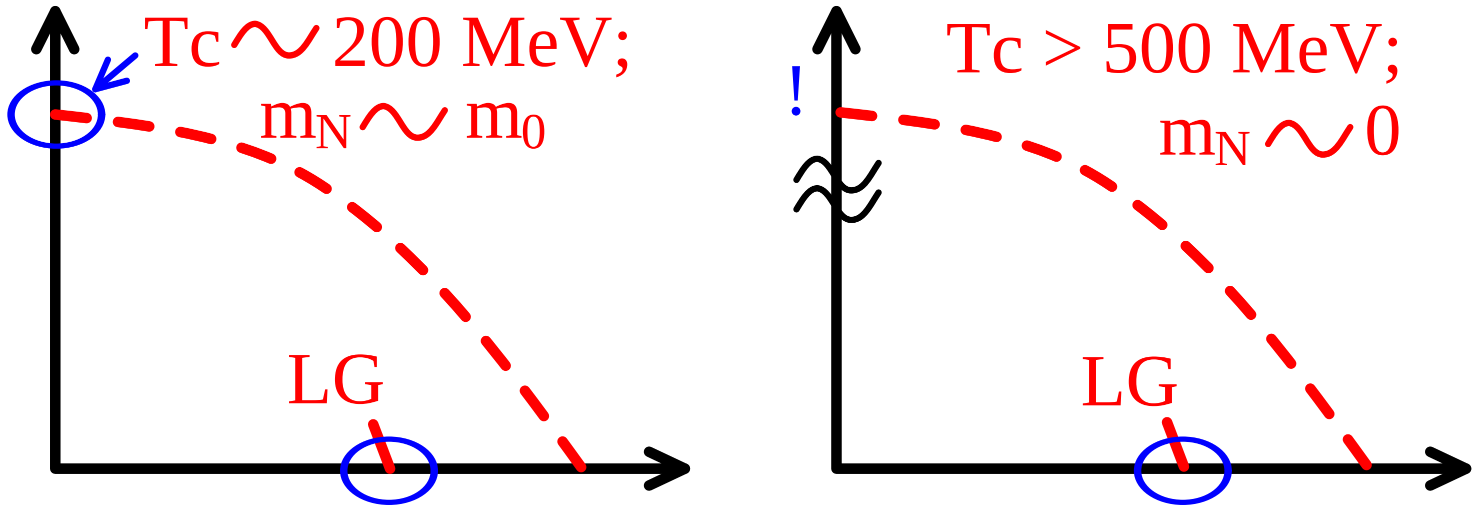
- thermodynamics on top of the nuclear matter ground state [CS-Mishustin (2010)]

  e.g. extrapolation of nucleonic NJL at $\mu_B = 0$??

$$m_N = m_N^0 + \gamma_N G_S \int \frac{d^3p}{(2\pi)^3} \frac{m_N}{E} \left[1 - 2n_f(m_N; T)\right]$$

parameters fixed to reproduce nuclear matter properties [Mishustin et al. (03)]

$\Lambda = 0.4$ GeV , $G_S = 1.7$ GeV fm^3 , $m_N^0 = 41$ MeV $\Rightarrow$ $T_c \sim 500$ MeV



$T_c^{\rm lattice}(\mu \sim 0)$ & nuclear matter ground state

$\Rightarrow$ a minimal set of constraint on modeling

- $N_f = 2$ **parity doublet model** [Zschiesche et al. (07)]
  - 2 nucleon fields

$$\psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2)$$
$$\psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0)$$

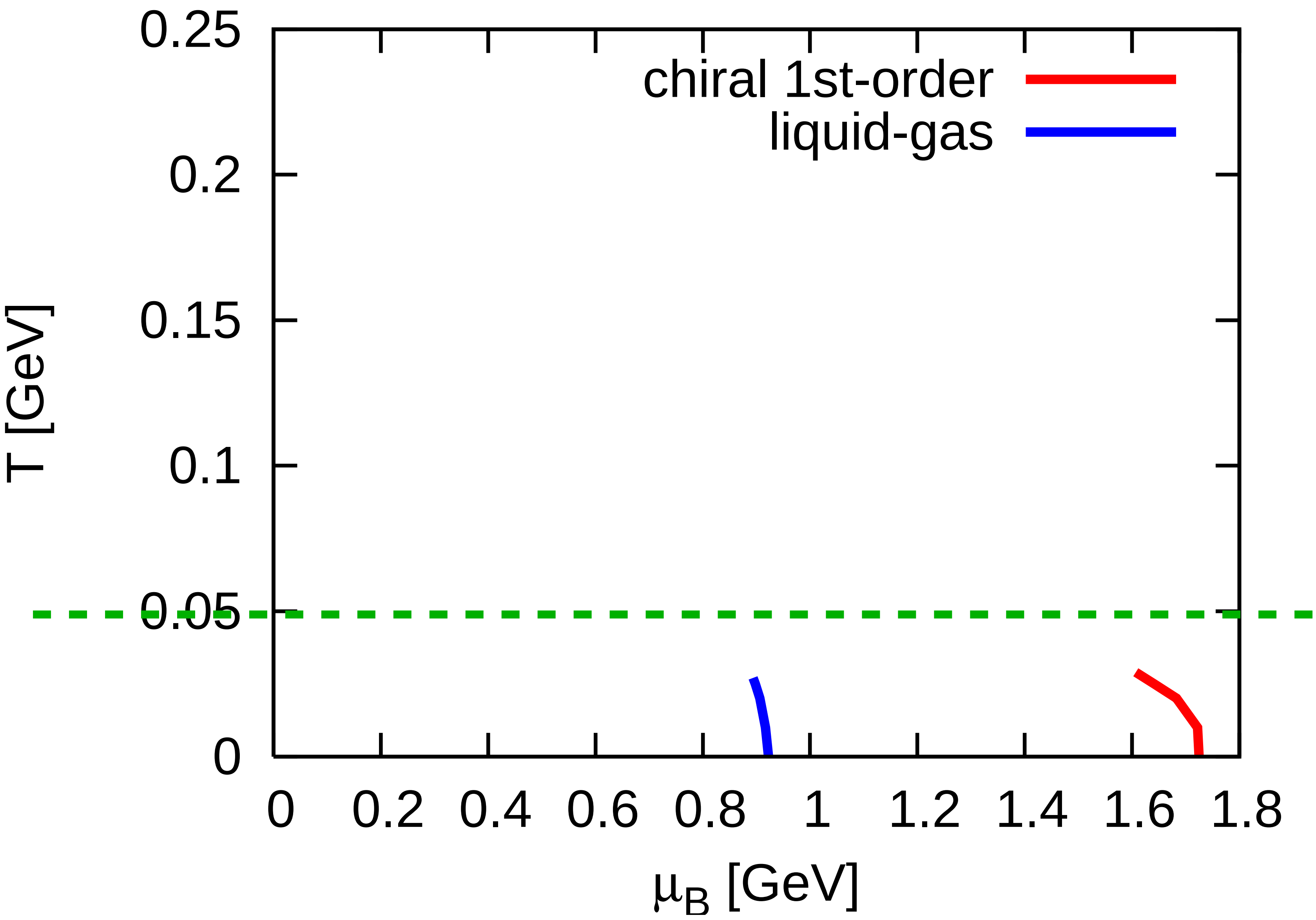
  - Lagrangian

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2\right) \\
&\quad + a\bar{\psi}_1(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 - g_\omega \bar{\psi}_1 \omega\!\!\!/ \psi_1 - g_\omega \bar{\psi}_2 \omega\!\!\!/ \psi_2 + \mathcal{L}_M \,, \\
\mathcal{L}_M &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\bar{\mu}^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \epsilon\sigma \\
&\quad - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu\omega^\mu + (g_4)^4(\omega_\mu\omega^\mu)^2
\end{aligned}$$

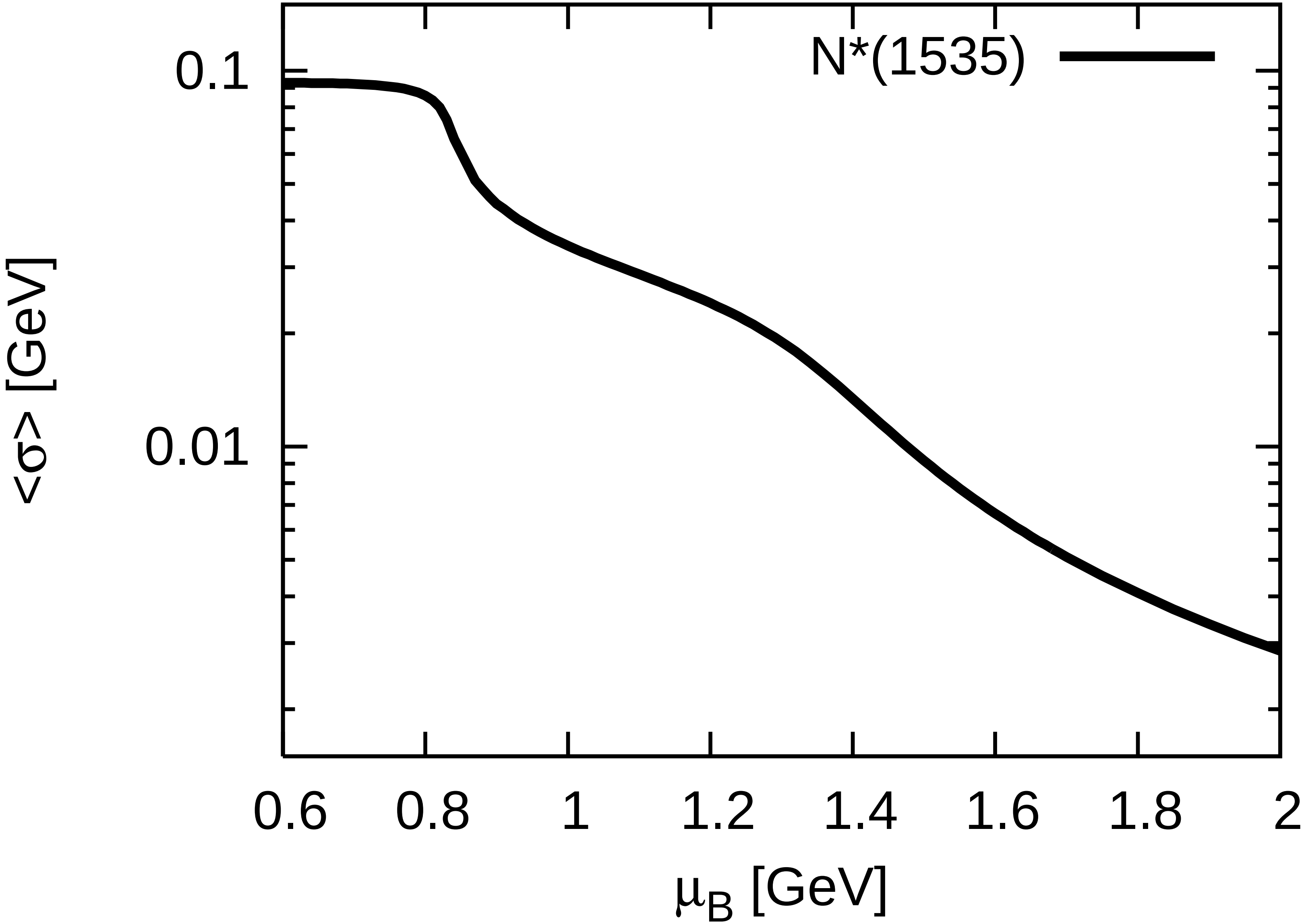
  - masses: $m_\pm = \frac{1}{2}\left[\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right] \stackrel{\sigma\to 0}{\longrightarrow} m_0$
  - "transition" from meson-rich to baryon-rich matter in hadronic phase

$$\rho_{\rm meson}/\rho_{\rm baryon} \sim 1 \,, \quad \rho_i = \gamma_i \int \frac{d^3p}{(2\pi)^3} n_i(T, \mu; m_i)$$

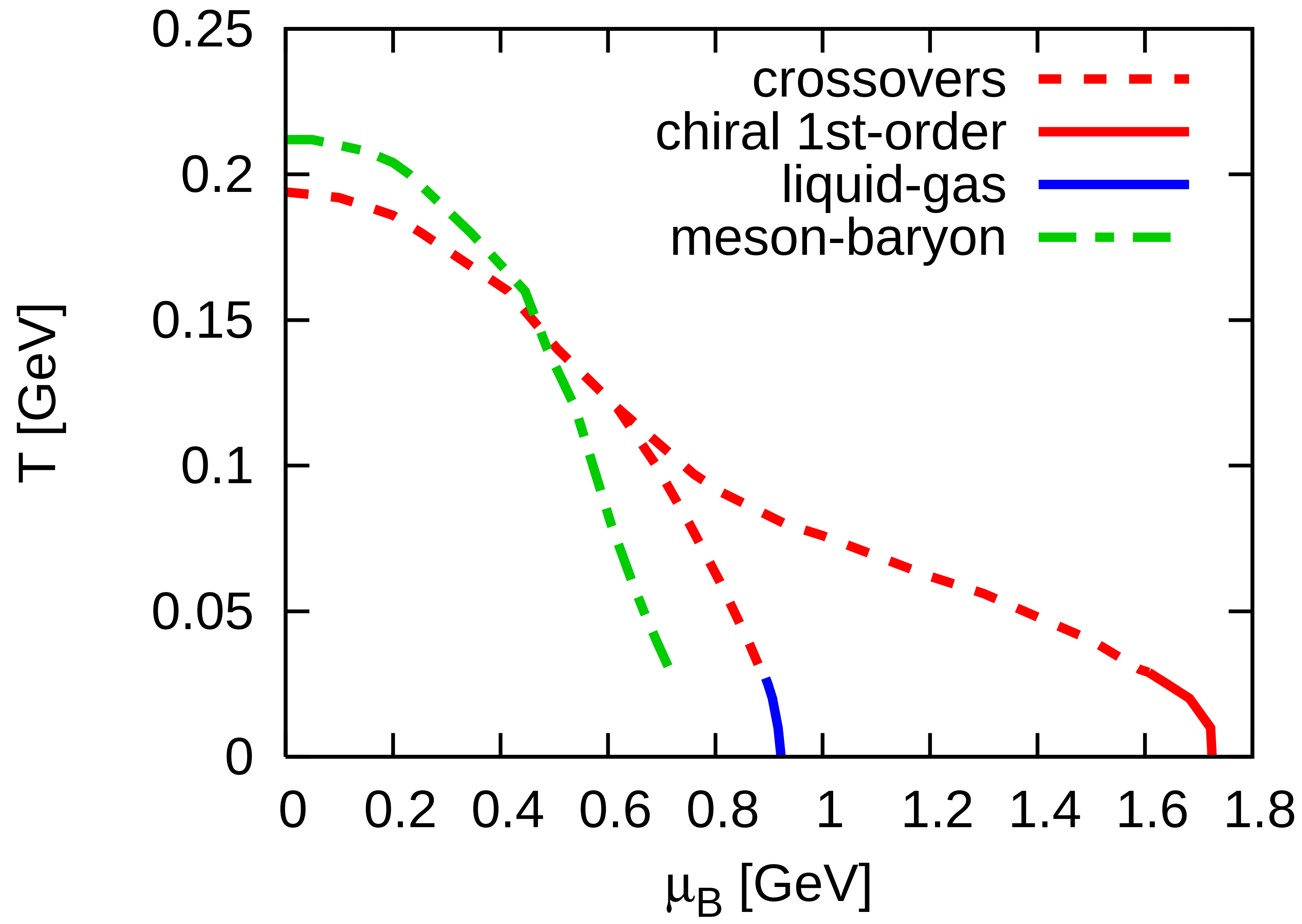
- **phase diagram in PDM**: $m_{N_-} = 1.5$ GeV



- **pion decay constant at an intermediate temperature**



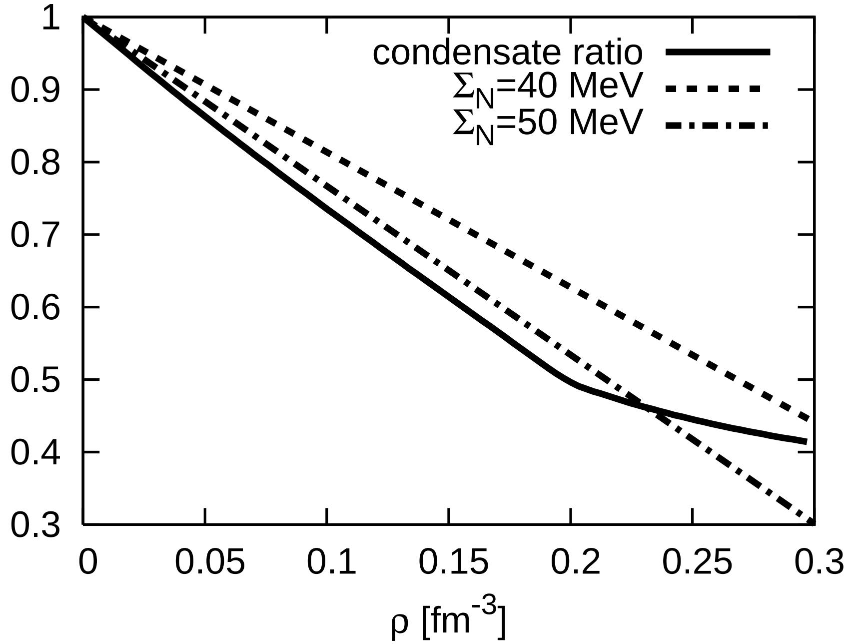
- **phase diagram in parity doublet model**



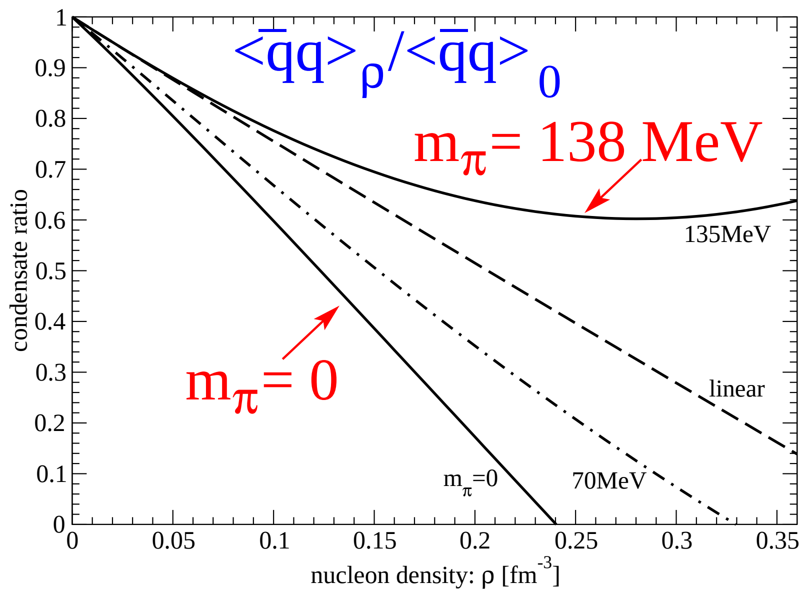
broken phase (meson-rich & baryon-rich) and restored phase

- **in-medium quark condensate and the low-energy theorem**

present MF model



ChPT [Kaiser-de Homont-Weise (08)]



up to the leading order in ρ:

$$R(\rho) = \frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{\Sigma_N}{m_\pi^2 f_\pi^2}\rho\,, \quad \Sigma_N = 45 \pm 8\,\text{MeV}$$

beyond linear-density-approximation:

$$R^{(\rm PDM)}(\rho \sim 0.2\,\text{fm}^{-3}) \sim 0.5\,, \quad R^{(\rm ChPT)}(\rho \sim 0.2\,\text{fm}^{-3}) \sim 0.7$$

$\Rightarrow$ importance of two-pion exchange correlations with $\Delta(1232)$

$*$ explicit breaking term in baryonic sector too

The origin of hadron masses: DχSB vs. trace anomaly

- **trace anomaly in QCD**
  $\mathcal{L}_{\rm QCD}$ is invariant **at classical level** under scale transf. $x^\mu \to e^\tau x^\mu$ but not **at quantum level**.

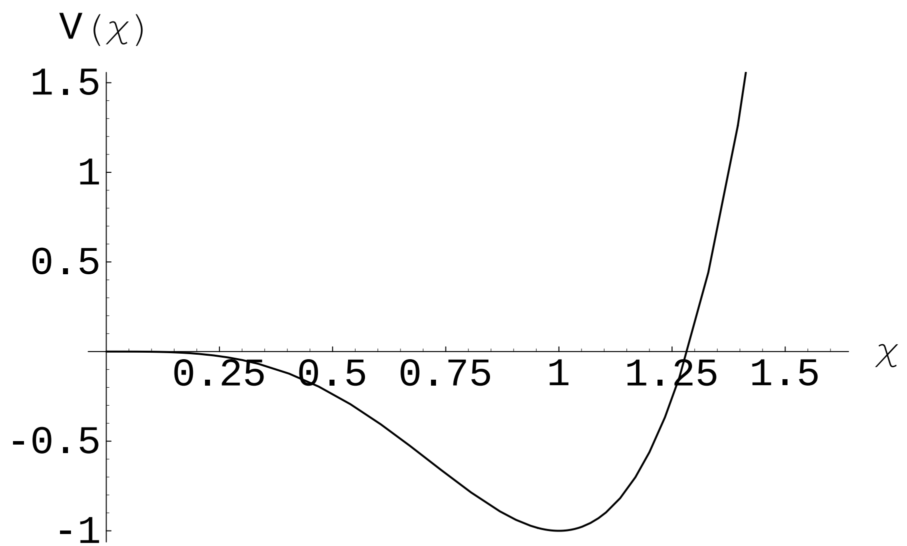
$$\partial_\mu J^\mu = T^\mu_\mu = -\left(\frac{11}{24}N_c - \frac{1}{12}N_f\right)\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu} + \sum_f m_f \bar{q}_f q_f$$

- **trace anomaly plus chiral Lagrangian** [Schechter (80)]
  a dilaton $\chi \sim$ composite operator $G_{\mu\nu}G^{\mu\nu}$: scalar glueball states

$$\mathcal{L}_\chi = \frac{1}{2}(\partial\chi)^2 - V(\chi)\,,$$

$$V(\chi) = \frac{1}{4}B\left(\frac{\chi}{\chi_0}\right)^4\left[\ln\left(\frac{\chi}{\chi_0}\right)^4 - 1\right]$$

minimum at $\chi = \chi_0$.
vacuum energy $E = -\frac{1}{4}B\,,$
glueball mass $m_G = \dfrac{2\sqrt{B}}{\chi_0}$



- **how to determine m_0?**

dilatons in a parity doublet model: **chiral plus scale invariance**

$$\mathcal{L} = \mathcal{L}_{\rm BM}(N_+, N_-, \sigma, \vec{\pi}, \chi) + \mathcal{L}_\chi(\chi)$$

its thermodynamics: mean field approximation & taking $\mu = 0$

$$V(\chi, T) = \frac{1}{4} B \left(\frac{\chi}{\chi_0}\right)^4 \left[\ln\left(\frac{\chi}{\chi_0}\right)^4 - 1\right] + V_T(\chi, T)$$

the gap equation assuming $\sigma \sim 0$:

$$4\gamma m_0^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{E(p;m_0)} \frac{1}{e^{E(p;m_0)/T} + 1} + \chi_0 \frac{\partial V}{\partial \chi} \sim 0$$

determines

$$m_0 = m_0(\langle\chi\rangle; T)\,, \quad T \sim T_{\rm chiral}$$

$\Rightarrow$ how to relate $\langle\chi\rangle$ with $\langle G_{\mu\nu}G^{\mu\nu}\rangle$?

- **lessons from a NJL model w/ dilatons** [Kusaka-Weise (92,94)]

  matching condition $\partial_\mu J^\mu_{\rm QCD} = \partial_\mu J^\mu_{\chi{\rm NJL}}$ determines:

$$\left(\frac{11}{24}N_c - \frac{1}{12}N_f\right)\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle = \frac{B}{4}\left(\frac{\chi}{\chi_0}\right)^4 - \left(4 + \beta\frac{\partial}{\partial\beta} - \chi\frac{\partial}{\partial\chi}\right)V_T\Big|_{\chi=\langle\chi\rangle}$$

- **lessons from Lattice QCD** [Miller (07)]

$$\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle = 0.012\,\text{GeV}^4 \quad \stackrel{T\sim T_{\rm chiral}}{\longrightarrow} \quad 0.006\,\text{GeV}^4$$

- **thermal expectation value of gluon operator eventually determines** $m_0 = m_0(\langle G^2\rangle_T)$**.**

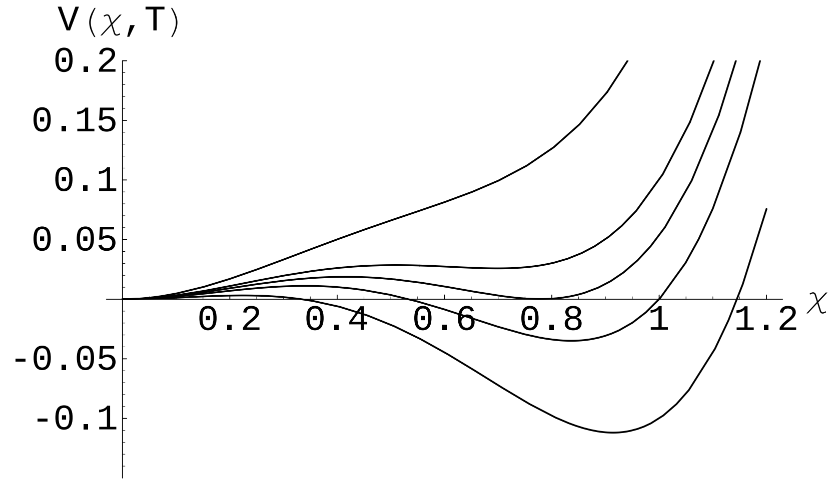
- **... actual number?**: V_T should be known.

e.g. $V_T = \frac{1}{2}AT^2(\chi/\chi_0)^2$

$\Rightarrow$ 1st-order transition

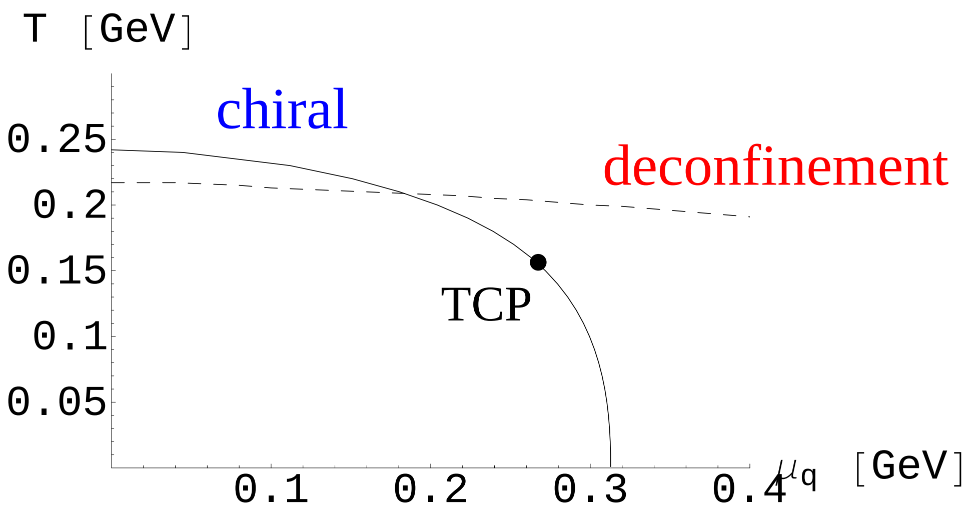
A, B, χ_0 from m_G, bag const., T_c

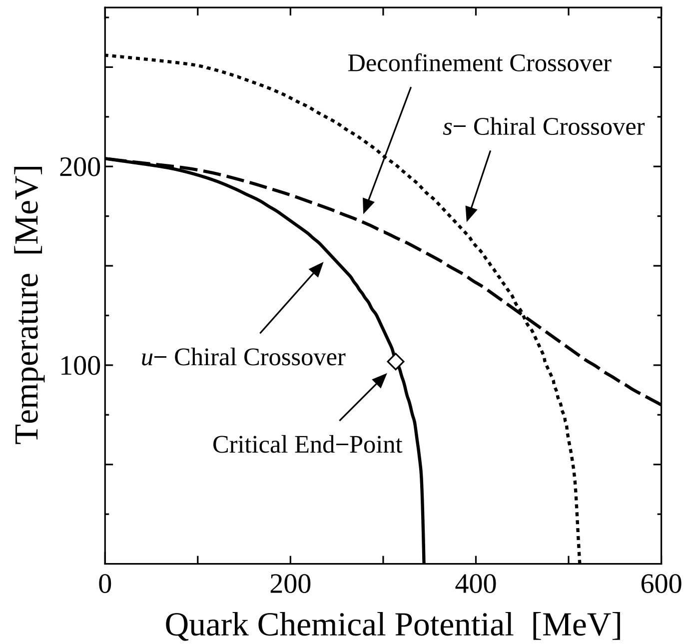
$\Rightarrow m_0(T_{\rm chiral}) \sim 0.5$ GeV

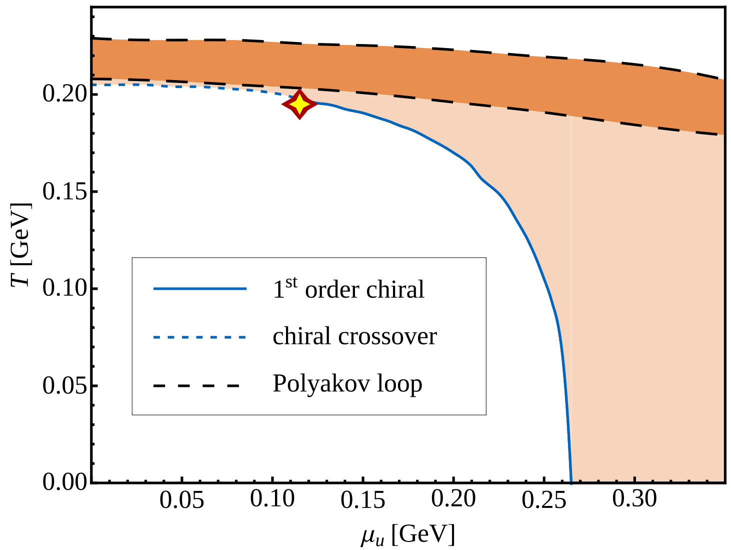


Dynamical chiral symmetry breaking vs. confinement

- **phase diagram from PNJL models: 3 regions**







[upper-left] CS-Friman-Redlich (06); [right] Fukushima (08)

[lower] Hell-Roessner-Cristoforetti-Weise (09)

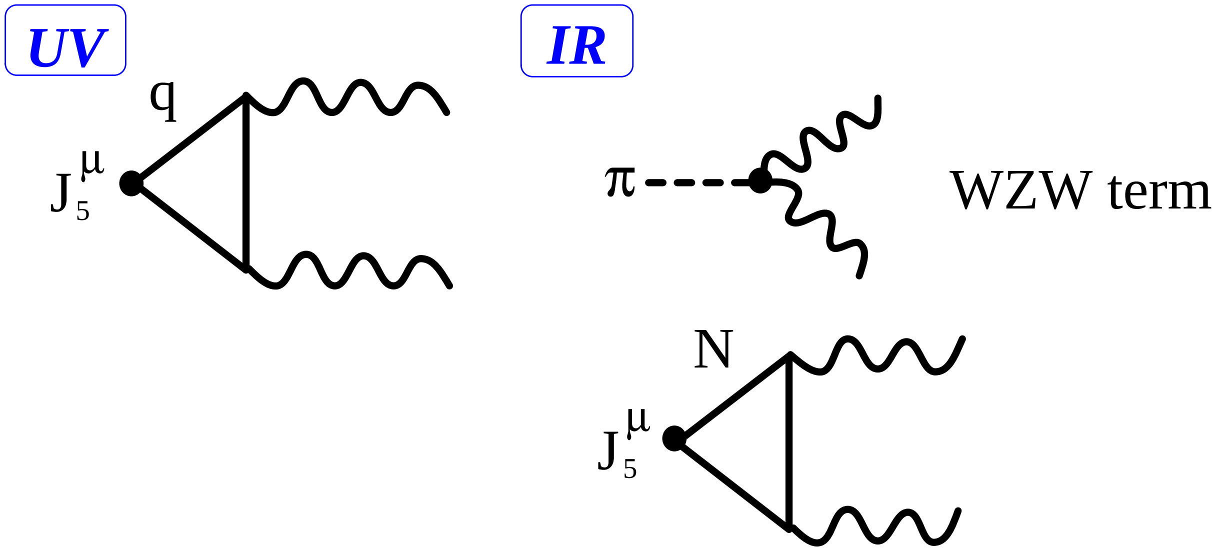
confinement in Wigner-Weyl phase?

– back reaction from quarks to $\mathcal{U}(\Phi)$

[Schaefer et al. (07), Fukushima (10)]

– anomaly matching: $N_f = 2$ or 3

• **anomaly matching between UV and IR theories**



– chirally restored phase: no NG boson thus no WZW term

– triangle diagrams with baryon:
matched for $N_f = 2$ but not for $N_f = 3$ ['t Hooft (80)]
good symmetries at which T and μ?

– parity doubled nucleons:
anomalies are partly cancelled but partly can survive ($g_A^{+-} \neq 0$).

- **how are anomalies saturated in matter?**
  - **at high temperature & low density:**
    chirally-restored phase with confinement with known hadrons
    * allowed for $N_f = 2 \quad \longrightarrow \quad T_{\rm chiral} \leq T_{\rm deconf}$
    * not allowed for $N_f = 3 \quad \longrightarrow \quad T_{\rm deconf} \leq T_{\rm chiral}$
    * possible in mirror baryon scenario when axial-couplings are properly adjusted. $\quad \longrightarrow \quad T_{\rm chiral} \leq T_{\rm deconf}$
  - **at high density & low temperature:**
    new gapless excitation might appear on the Fermi surface? (either "boson" or "fermion") **what are they? dynamical origin?**
  - unless lack of Lorentz covariance spoils them totally,
    * if relevant low-energy excitations known,
      then anomaly matching constrains phases.
    * if relevant low-energy excitations unknown,
      then they should be constrained by anomaly matching.

anomaly matching as a working hypothesis

Summary and remarks

- **study of hot & dense matter composed of hadrons**
  - parity doublet model (LG and chiral transitions)
  - saturation properties & pseudo critical temperature from LQCD
  - meson-baryon "transition": a trace of LG transition
- **effective Lagrangians/low-energy excitations constrained by anomaly matching** $\Rightarrow$ reliable access to QCD
- **origin of** m_0
  - chiral & scale invariance in effective Lagrangians
  - m_0 from gluon condensate
  - further effort: identifying low-lying scalar modes, tetra-quark state, dilaton potential at finite T etc.