Heavy-quark Langevin dynamics and single-electron spectra in AA collisions

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University of Torino and CERN - Theory Unit (Centro-Fermi grant)

"The first heavy ion collisions at the LHC",

CERN, 16^{th} **August -** 10^{th} **September 2010**

Work in progress in Torino in collaboration with

W.M. Alberico, A. De Pace, M. Nardi, A. Molinari (DFT and INFN),

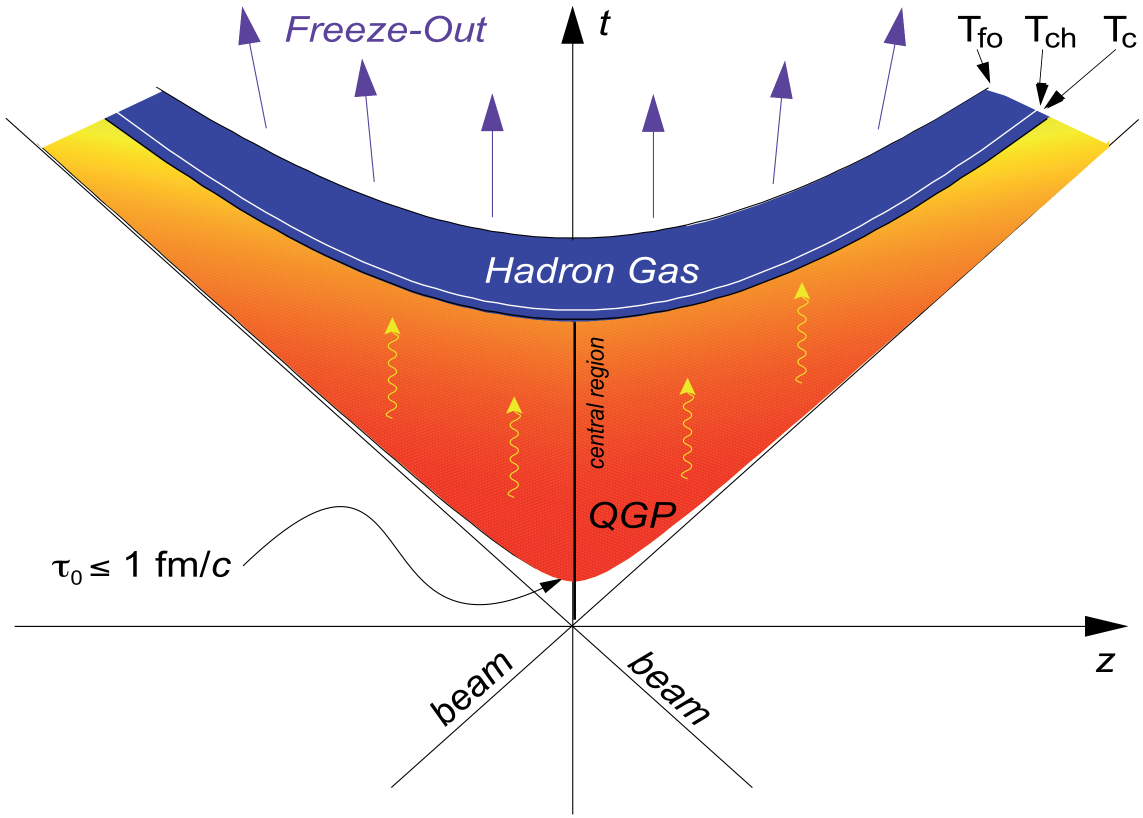
M. Monteno and F. Prino (INFN and ALICE group)

Nucl. Phys. A 831, 59 (2009) and arXiv:1007.4170 [hep-ph]

Outline

- Heavy quarks as probes of the QGP
- Theoretical framework:
  - the relativistic Langevin equation in an expanding medium
  - evaluation of the transport coefficients
- Numerical results: from the initial $Q\overline{Q}$ production to the final e-spectra
  - Nuclear modification factor $R_{AA}(p_T)$
  - Elliptic flow coefficient $v_2(p_T)$

A nucleus-nucleus collision: space-time evolution



- Heavy-quarks produced in the initial stage and before decoupling from the medium...
- ...they cross the plasma, performing a *tomography* of it.

Heavy quarks as probes of the QGP

- Relaxation to thermal equilibrium (cumulated effect of many random collisions):

$$\frac{\langle p^2 \rangle}{2M} = \frac{3}{2} T \quad \Longrightarrow \quad \bar{p}_{\rm heavy} = \sqrt{3MT} \quad (\bar{p}_{\rm light} \sim T)$$

$$\langle p^2_{\rm heavy} \rangle \sim \frac{M}{T} \langle p^2_{\rm light} \rangle \quad \Longrightarrow \quad \tau_{\rm heavy} \sim \frac{M}{T} \tau_{\rm light}$$

  In an expanding fireball with a finite life-time *one would not expect the heavy quarks to reach full equilibrium with the medium.*

- Radiative energy-loss suppressed wrt light parton (*suppression of collinear emission*): collisional energy-loss should play an important role!

The relativistic Langevin equation...

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \ ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\boldsymbol{p}_t)\xi^j(\boldsymbol{p}_{t'})\rangle = b^{ij}(\boldsymbol{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\boldsymbol{p}) \equiv \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

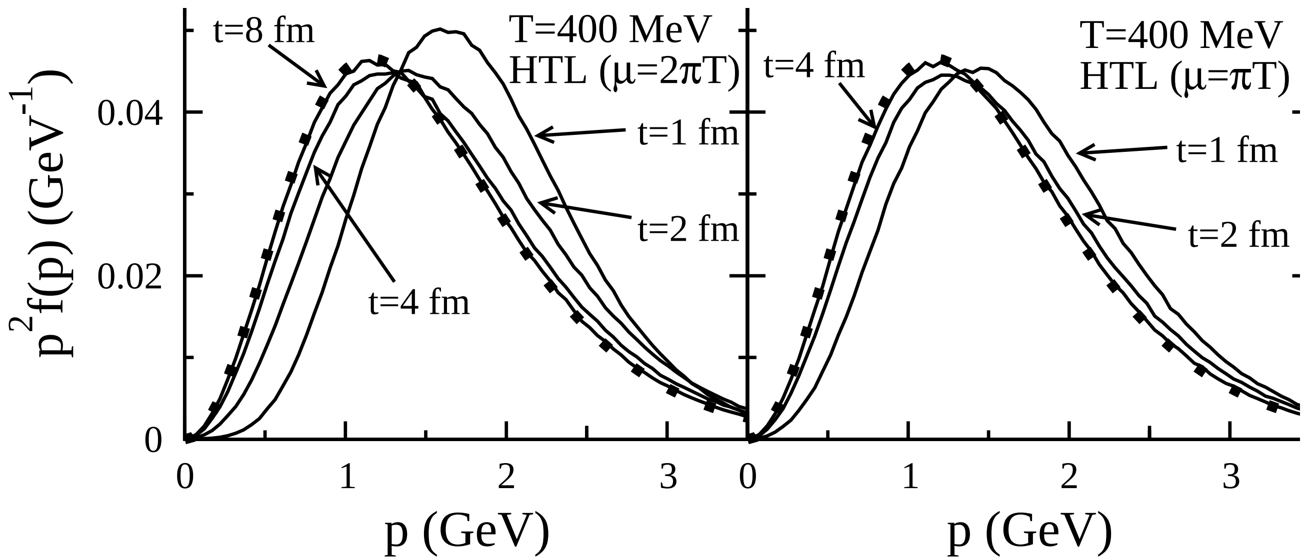
Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_T \equiv \frac{1}{2}\frac{\langle \Delta p_T^2\rangle}{\Delta t}$ and $\kappa_L \equiv \frac{\langle \Delta p_L^2\rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\rm Ito}(p) = \frac{\kappa_L(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_L(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_L(p)-\kappa_T(p)}{v^2}\right]$$

fixed in order to insure the approach to equilibrium (Einstein relation):
Langevin eq. $\Leftrightarrow$ Fokker Planck eq. with steady solution $\exp(-E_p/T)$

...in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[a]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with } \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c)

[a] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

...in an expanding fluid

The fields $u^\mu(x)$ and $T(x)$ are taken from the output of longitudinal boost-invariant

$$x^\mu = (\tau \cosh\eta, \boldsymbol{r}_\perp, \tau \sinh\eta)$$

$$u^\mu = \bar{\gamma}_\perp(\cosh\eta, \bar{\boldsymbol{v}}_\perp, \sinh\eta) \quad \text{with} \quad \bar{\gamma} \equiv \frac{1}{\sqrt{1-\bar{\boldsymbol{v}}_\perp^2}}$$

hydro codes[a]

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[a] P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

The easiest algorithm

Going to the fluid rest-frame:

$$\Delta\bar{p}_n^i = -\eta_D(\bar{p}_n)\bar{p}_n^i\Delta\bar{t} + \xi^i(\bar{t}_n)\Delta\bar{t} \equiv -\eta_D(\bar{p}_n)\bar{p}_n^i\Delta\bar{t} + g^{ij}(\bar{\boldsymbol{p}}_n)\zeta^i(\bar{t}_n)\sqrt{\Delta\bar{t}},$$

$$\Delta\bar{\boldsymbol{x}}_n = \bar{\boldsymbol{p}}_n/\bar{E}_n\Delta\bar{t}$$

with $\Delta\bar{t}=0.02$ fm/c (*in the fluid rest-frame!*) and

$$g^{ij}(\boldsymbol{p}) \equiv \sqrt{\kappa_L(p)}\hat{p}^i\hat{p}^j + \sqrt{\kappa_T(p)}(\delta^{ij} - \hat{p}^i\hat{p}^j) \quad \text{and} \quad \langle\zeta_n^i\zeta_{n'}^j\rangle = \delta^{ij}\delta_{nn'}$$

Hence one needs simply to:

- extract three independent random numbers ζ^i from a gaussian distribution with $\sigma=1$;
- update the momentum and position of the heavy quark;
- go back to the Lab-frame: $\boldsymbol{x}_{n+1}$ and $\boldsymbol{p}_{n+1}$.

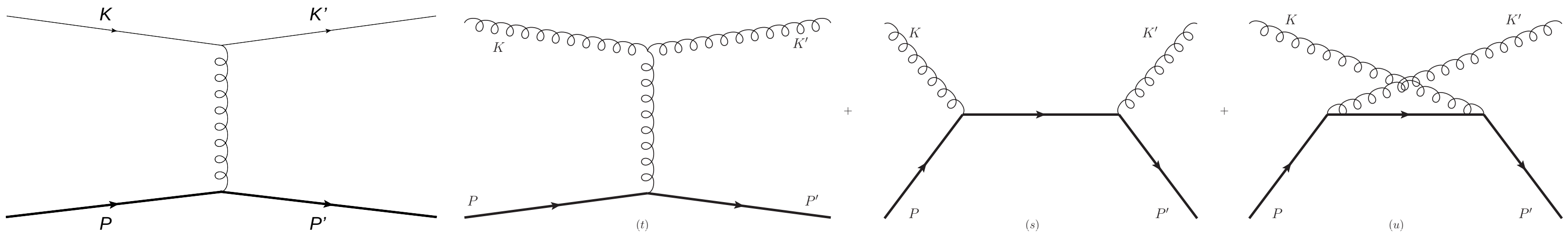
Evaluation $\kappa_{T/L}(p)$

Intermediate cutoff $|t|^* \sim m_D^2$ [a] *separating the contributions of*

- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation
- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation

[a] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

$\kappa_{T/L}(p)$: hard contribution



$$\kappa_T^{g/q(\text{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

$$\kappa_L^{g/q(\text{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

$\kappa_{T/L}(p)$: soft contribution

$$\kappa_T^{\text{soft}} = \frac{1}{2}\frac{C_F g^2}{4\pi^2 v}\int_0^{|t|^*} d|t| \int_0^v dx \frac{|t|^{3/2}}{2(1-x^2)^{5/2}}\,\overline{\rho}(|t|,x)\left(1-\frac{x^2}{v^2}\right)\coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

$$\kappa_L^{\text{soft}} = \frac{C_F g^2}{4\pi^2 v}\int_0^{|t|^*} d|t| \int_0^v dx \frac{|t|^{3/2}}{2(1-x^2)^{5/2}}\,\overline{\rho}(|t|,x)\,\frac{x^2}{v^2}\coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

where

$$\overline{\rho}(|t|,x) \equiv \rho_L(|t|,x) + (v^2 - x^2)\rho_T(|t|,x) \quad (|t| \equiv q^2 - \omega^2,\ x \equiv \omega/q)$$

The result is then expressed in terms of the *spectral functions of the resummed gluons* which are *exchanged* in the collisions *with the plasma particles*:

$$\rho_{L/T}(\omega,q) \equiv 2\text{Im}\Delta_{L/T}(\omega + i\eta, q)$$

The Hard Thermal Loop approximation

It is a one-loop gauge-invariant approximation allowing for the calculation of thermal corrections to vacuum propagators.

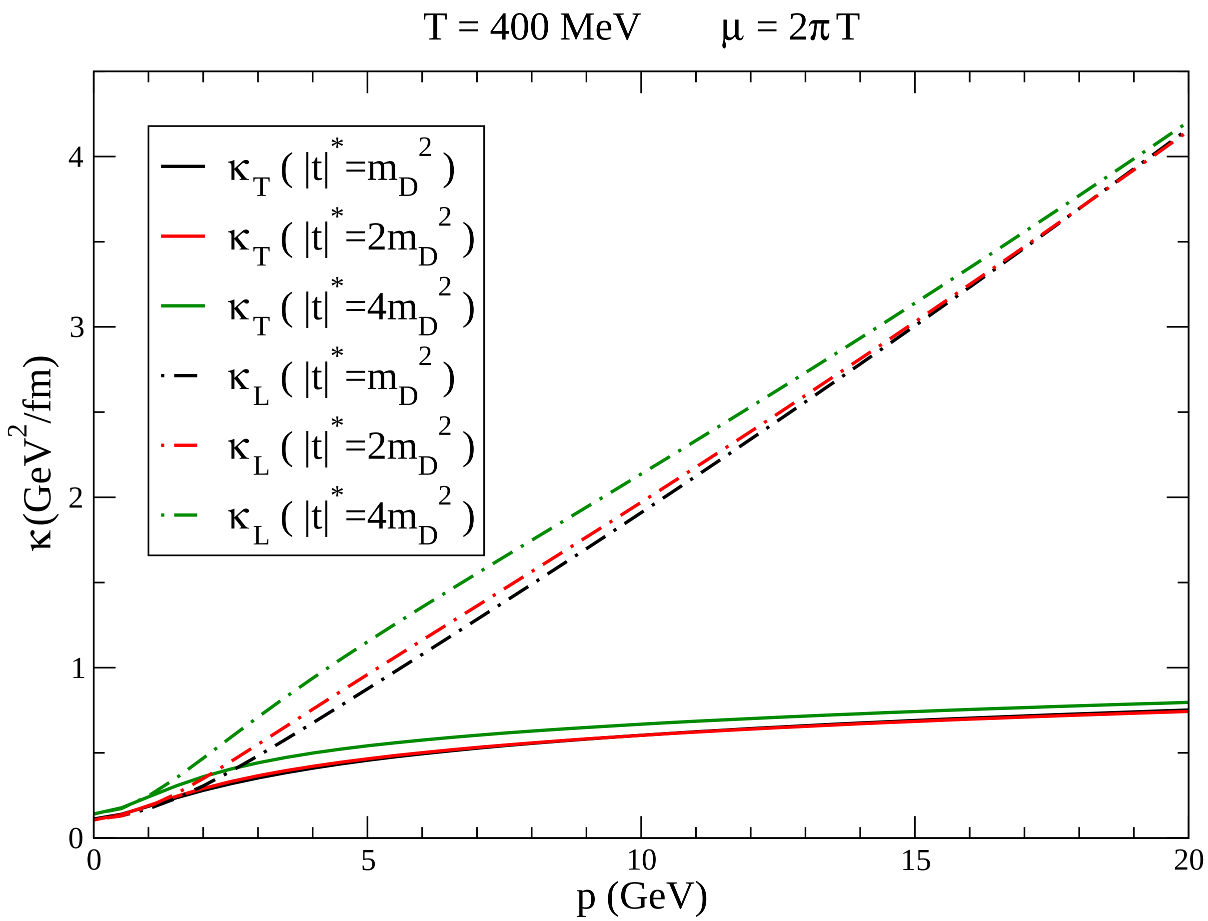
$$\Delta_L(q^0,q) = \frac{-1}{q^2+\Pi_L(x)}, \quad \Delta_T(q^0,q) = \frac{-1}{(q^0)^2-q^2-\Pi_T(x)}$$

with $x \equiv q^0/q$ and

$$\begin{aligned} \Pi_L(x) &= m_D^2\left(1-\frac{x}{2}\ln\frac{x+1}{x-1}\right), \\ \Pi_T(x) &= \frac{m_D^2}{2}\left(x^2+(1-x^2)\frac{x}{2}\ln\frac{x+1}{x-1}\right), \end{aligned}$$

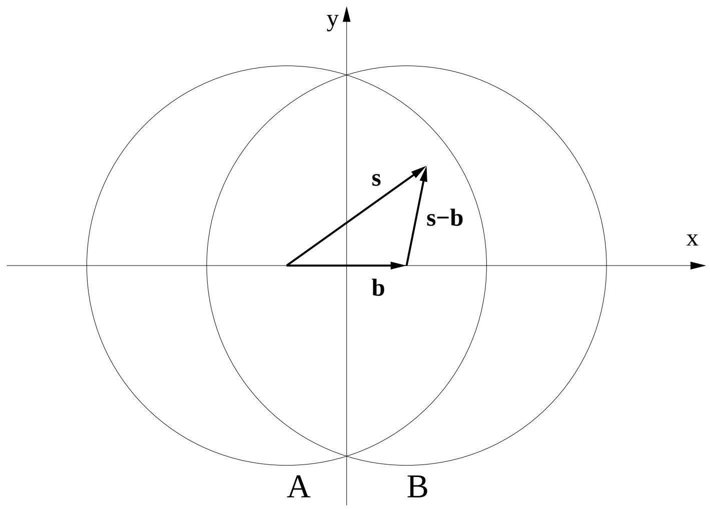
where $m_D \equiv gT\sqrt{\frac{N_c}{3}+\frac{N_f}{6}}$ is the Debye mass, responsible for the screening of electro-static color fields.

$\kappa_{T/L}(p)$: numerical results



Dependence on the intermediate cutoff $|t|^*$ very mild!

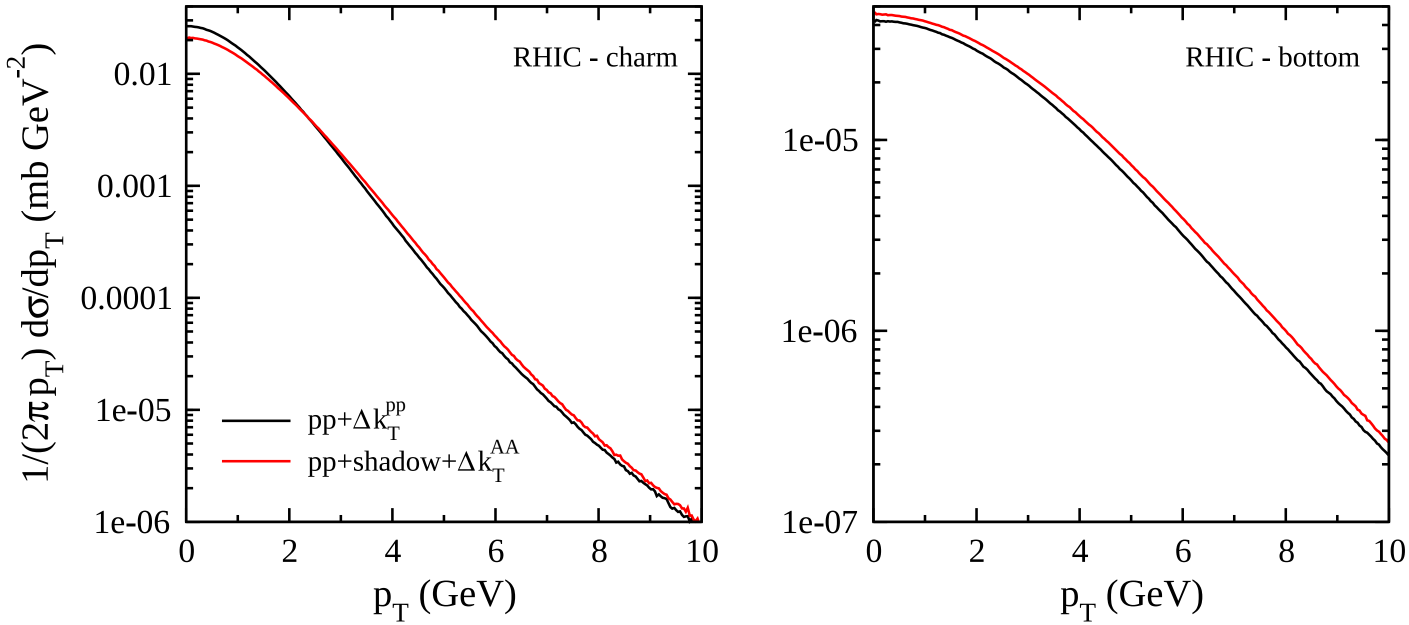
Initial heavy-quark spectra



- Single inclusive HQ spectra $E(dN/d^3p) \equiv dN/dyd\boldsymbol{p}_\perp$ generated with POWHEG (implementing pQCD @ NLO) using the PDF set CTEQ6M including the nuclear modifications given by EPS09 for AA
- HQ distributed in the transverse plane according to the nuclear overlap $dN/d\boldsymbol{x}_\perp \sim T_{AB}(x,y) \equiv T_A(x+b/2,y)T_B(x-b/2,y)$, with

$$T_{A/B}(\boldsymbol{x}_\perp) \equiv \int_{-\infty}^{+\infty} dz\,\rho_{A/B}(\boldsymbol{x}_\perp, z)$$

Initial heavy-quark spectra: results



Each quark is given a random $\boldsymbol{k}_\perp$ broadening extracted from

$$g(\boldsymbol{k}_\perp) = \frac{1}{\pi\langle k_\perp^2\rangle}\exp(-k_\perp^2/\langle k_\perp^2\rangle),$$

with[a] $\langle k_\perp^2\rangle = \langle k_\perp^2\rangle_{pp} + \langle \delta k_\perp^2\rangle_{AB}(\vec{b}, \vec{s})$.

[a] J. Hüfner and P. Zhuang, PLB 515, 115 (2001) and PLB 559, 193 (2003).

Final spectra : nuclear modification factor

$$R_{AA}(p_T) = \frac{1}{\langle N_{\rm coll}^{AA} \rangle} \frac{dN/dp_T|_{AA}}{dN/dp_T|_{pp}}$$

and elliptic flow coefficient

$$\frac{dN}{d\phi\, p_T dp_T} = \frac{dN}{2\pi\, p_T dp_T} \big(1 + 2 v_2(p_T) \cos[2(\phi - \psi_{\rm RP})] + ...\big)$$

Minimum bias Au-Au collisions @ RHIC: b=8.44 fm

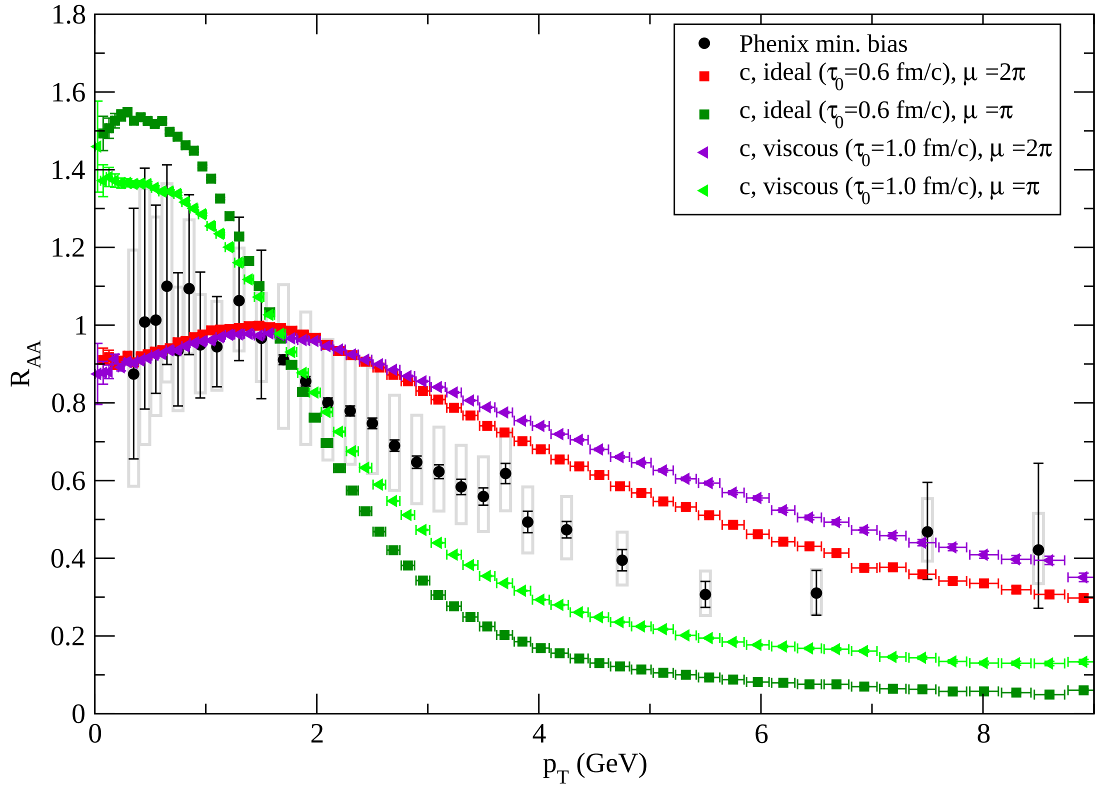
Hydro scenario

Hydro code	τ_0 (fm/c)	s_0 (fm^{-3})	T_0 (MeV)
ideal	0.6	110	357
viscous	1.0	83.8	333

Initial production (from POWHEG)

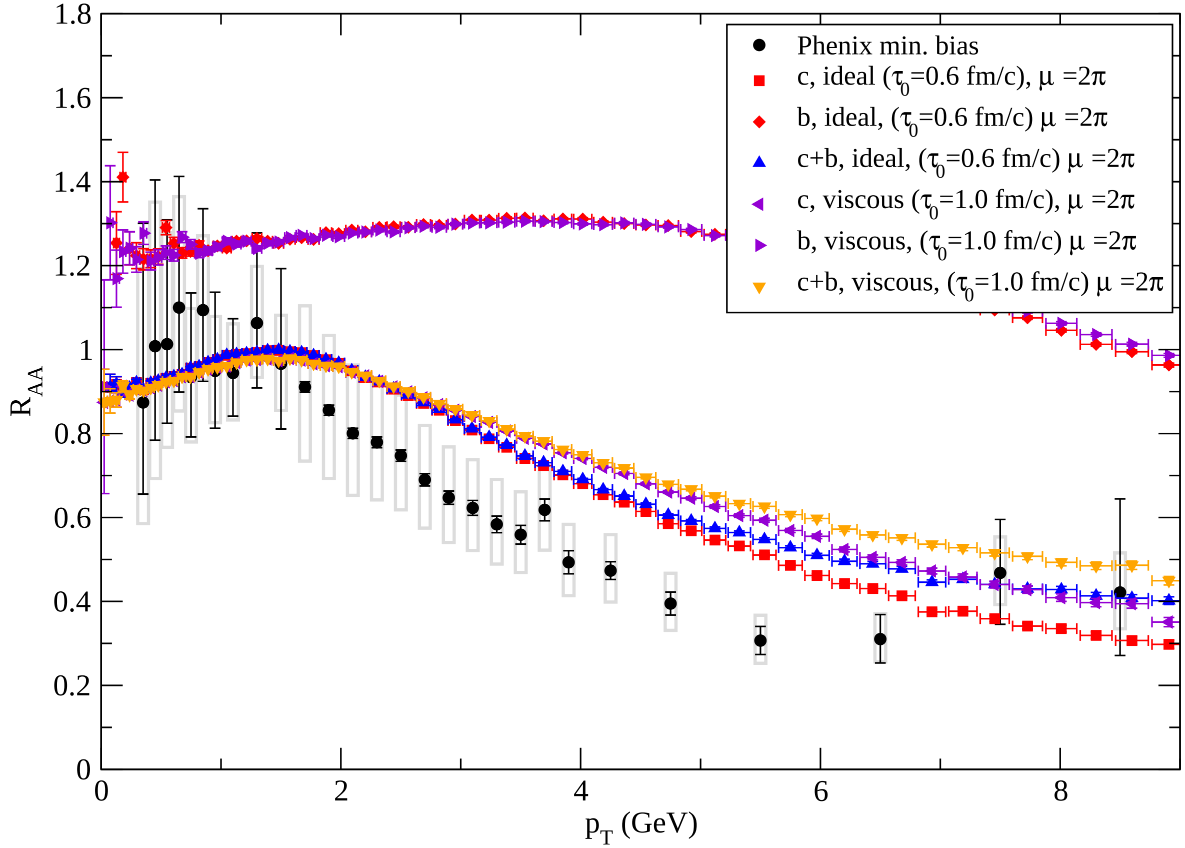
$\sqrt{s}_{NN}=200$ GeV	$\sigma_{c\bar{c}}$ (μb)	$\sigma_{b\bar{b}}$ (μb)
pp	254.14	1.769
AA	236.11	2.033

Final heavy-quark spectra: charm



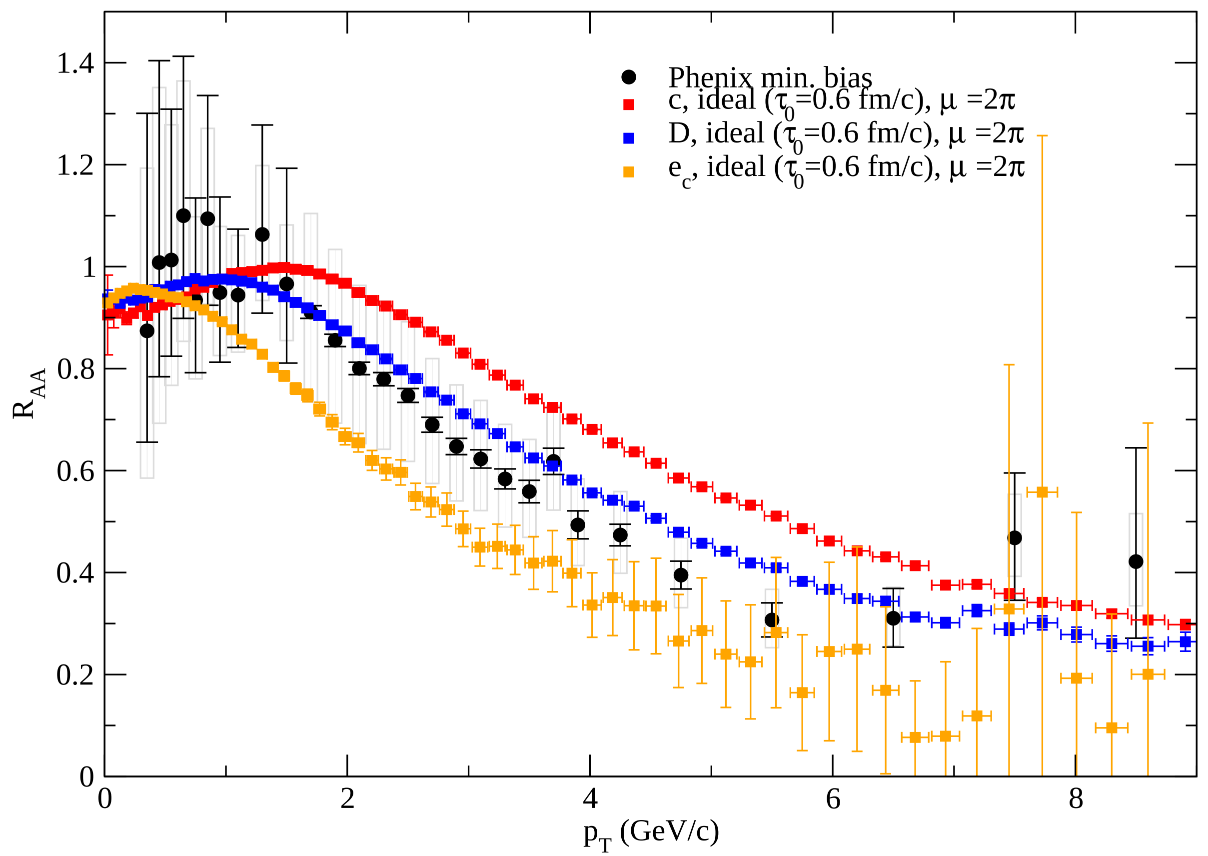
- Mild dependence on the hydro scenario (ideal vs viscous)
- Strong dependence on the scale at which the coupling $\alpha_s(\mu)$ is evaluated ($\mu = 2\pi T$ and $\mu = \pi T$): at $T = 200$ MeV $\alpha_s \approx 0.34$ and 0.63

Final heavy-quark spectra: charm + bottom



- R_{AA} of b strongly affected by initial-state effects (*anti-shadowing*)
- Very mild quenching of initial b spectrum in the medium

Effects of fragmentation and decays: h_c and e_c



Fragmentation and semileptonic decays ($D \to X \nu e$) lead to a *quenching of* R_{AA}!

Single-electron spectra from the decays of charm

$$D \to X\nu e$$

and bottom hadrons:

$$\begin{aligned} B &\to D\nu e \\ B &\to D\nu e \to X\nu e\nu e \\ B &\to DY \to X\nu e Y \end{aligned}$$

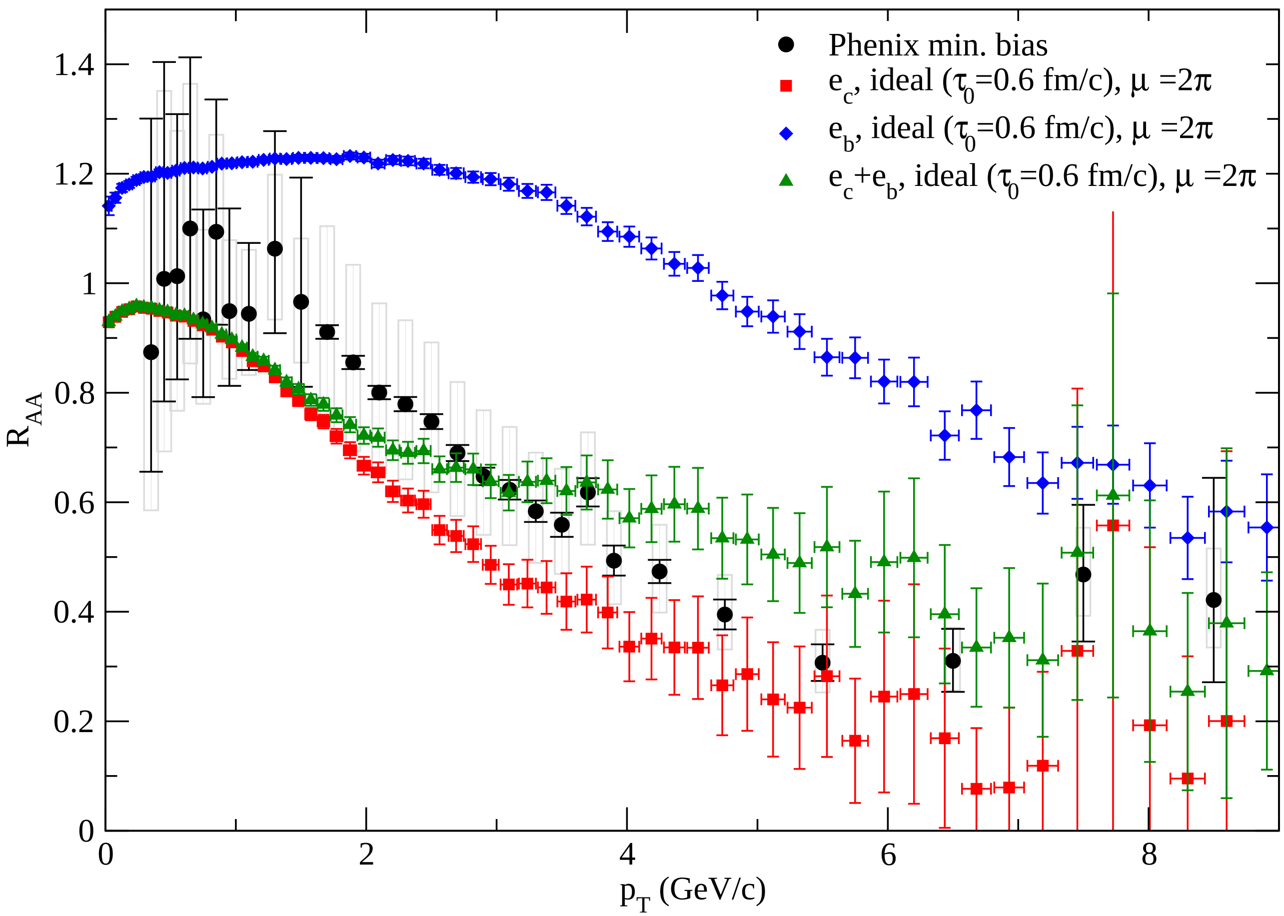
We plot the electrons falling into the PHENIX acceptance ($|\eta| < 0.35$)

Single-electron spectra: procedure

- One starts with two samples ($90 \cdot 10^6$) of charm and bottom quarks;
- They are made fragment with Peterson FFs with the branching fractions given by the PDG;
- Each hadron is sent to the PYTHIA decayer (with updated PDG decay tables) till producing *at least* one electron in the final state;
- The two sources must be combined with their appropriate weight:

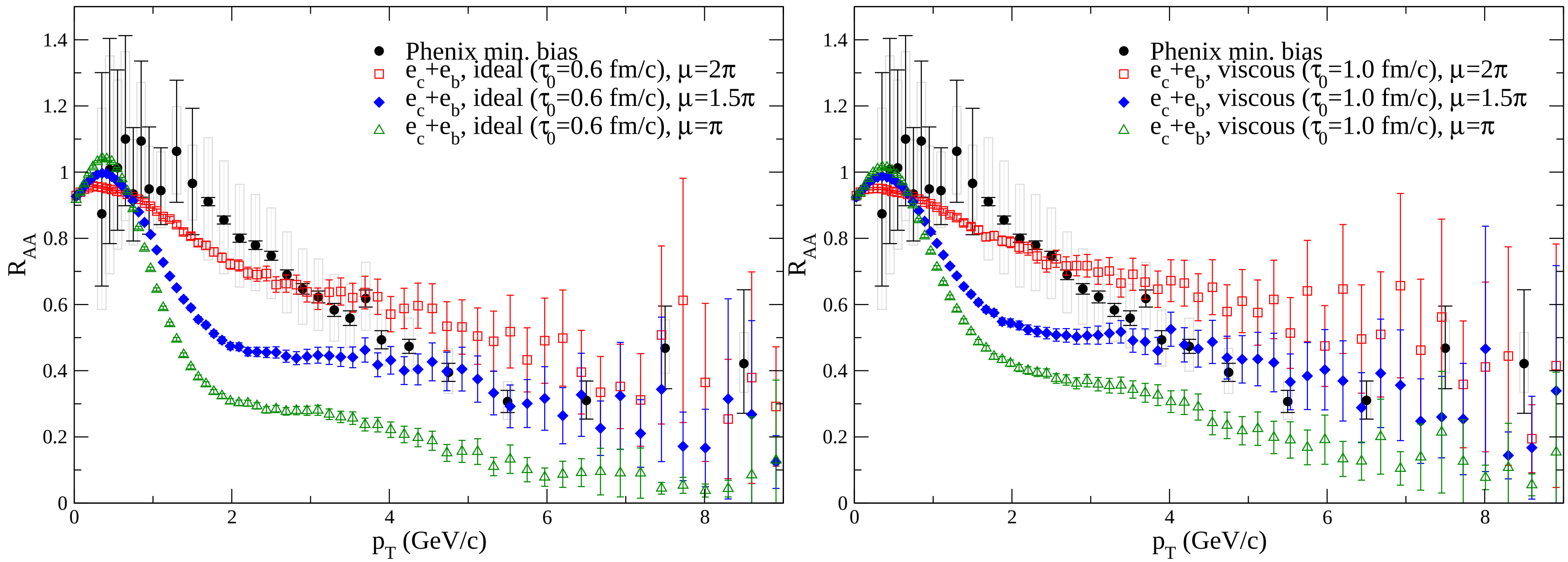
$$R_{AA}(p_T) = \frac{\sigma_{c\bar{c}}^{AA} \sum_{h_c} \frac{N_{h_c}^{\rm init}}{N_{h_c}^{\rm sampl}} \frac{dN_e}{dp_T}\Big|_{AA} + \sigma_{b\bar{b}}^{AA} \sum_{h_b} \frac{N_{h_b}^{\rm init}}{N_{h_b}^{\rm sampl}} \frac{dN_e}{dp_T}\Big|_{AA}}{\sigma_{c\bar{c}}^{pp} \sum_{h_c} \frac{N_{h_c}^{\rm init}}{N_{h_c}^{\rm sampl}} \frac{dN_e}{dp_T}\Big|_{pp} + \sigma_{b\bar{b}}^{pp} \sum_{h_b} \frac{N_{h_b}^{\rm init}}{N_{h_b}^{\rm sampl}} \frac{dN_e}{dp_T}\Big|_{pp}}$$

Single-electron spectra: $e_c + e_b$



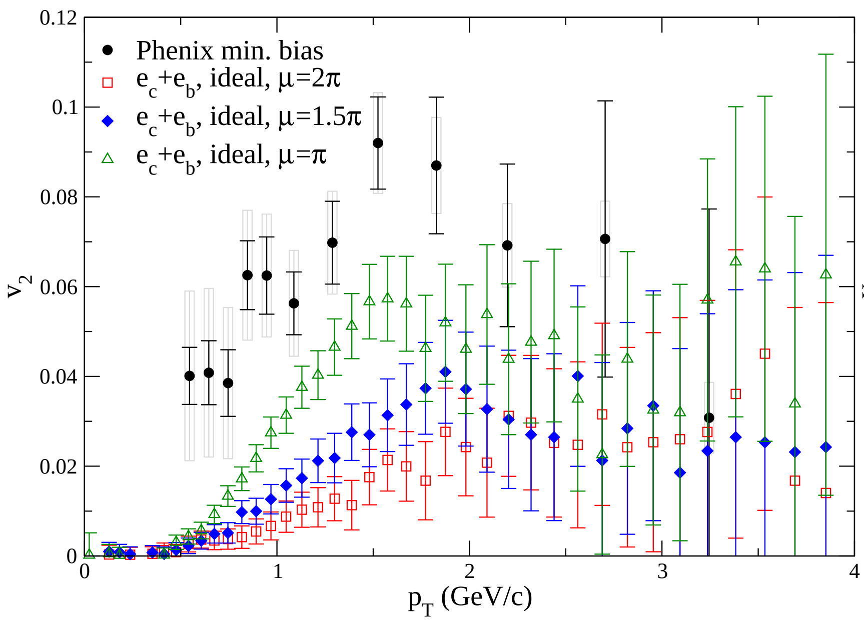
Electrons from b-decays start to contribute for $p_T \gtrsim 2$ GeV/c

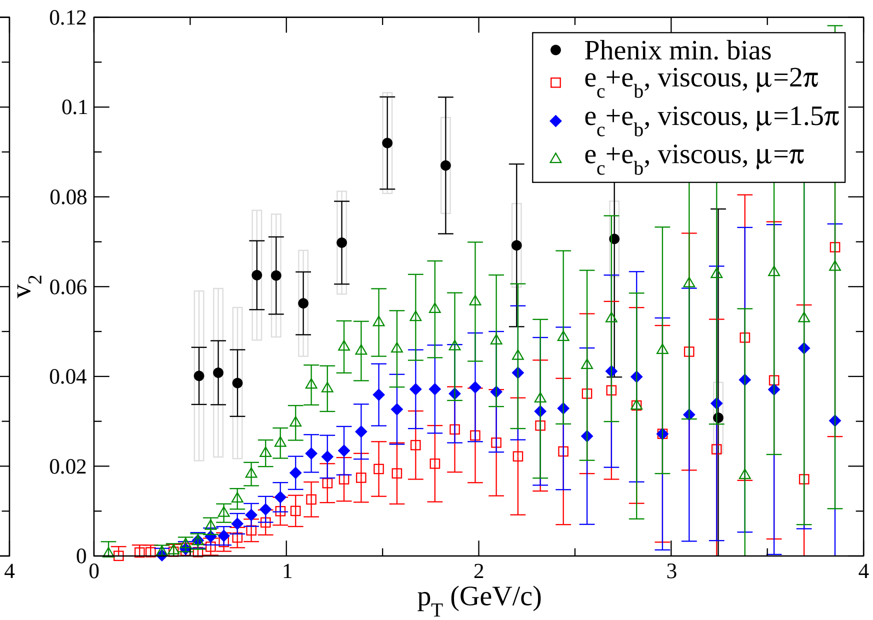
Single-electron spectra: $R_{AA}(p_T)$



- mild dependence on the hydro scenario (ideal/viscous)
- high-p_T reproduced better with $\mu\sim 1.5$ $\Rightarrow$ $\alpha = 0.32$ at $T = 300$ MeV (compare with $\alpha_{\rm AMY} = 0.28$ used for jet quenching)
- intermediate-p_T spectra could get increased by coalescence

Single-electron spectra: elliptic flow

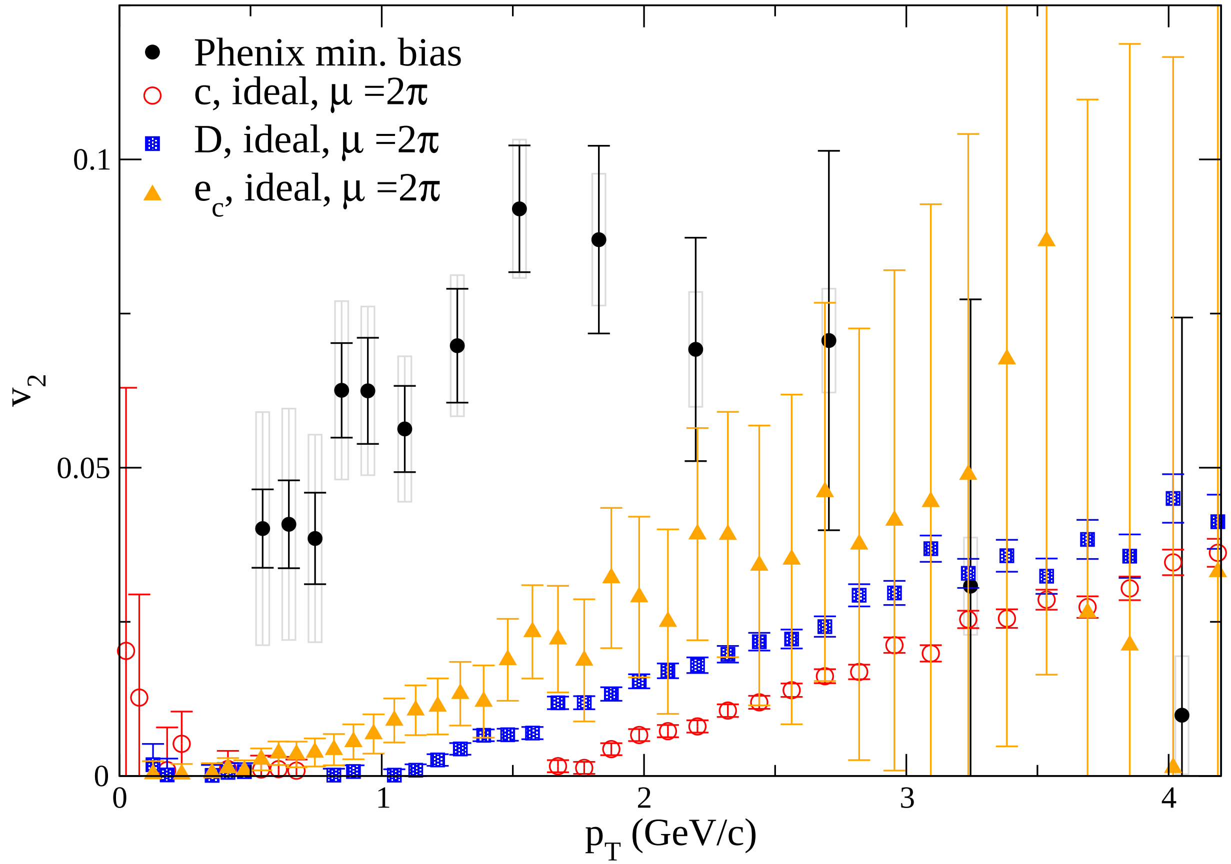




- v_2 with hot-QCD + fragmentation results underestimated
- v_2 could be increased by processes not (yet) included (resonant scattering, coalescensce)[a]

[a] H. van Hees, V. Greco and R. Rapp, Phys. Rev. C 73, 034913 (2006)

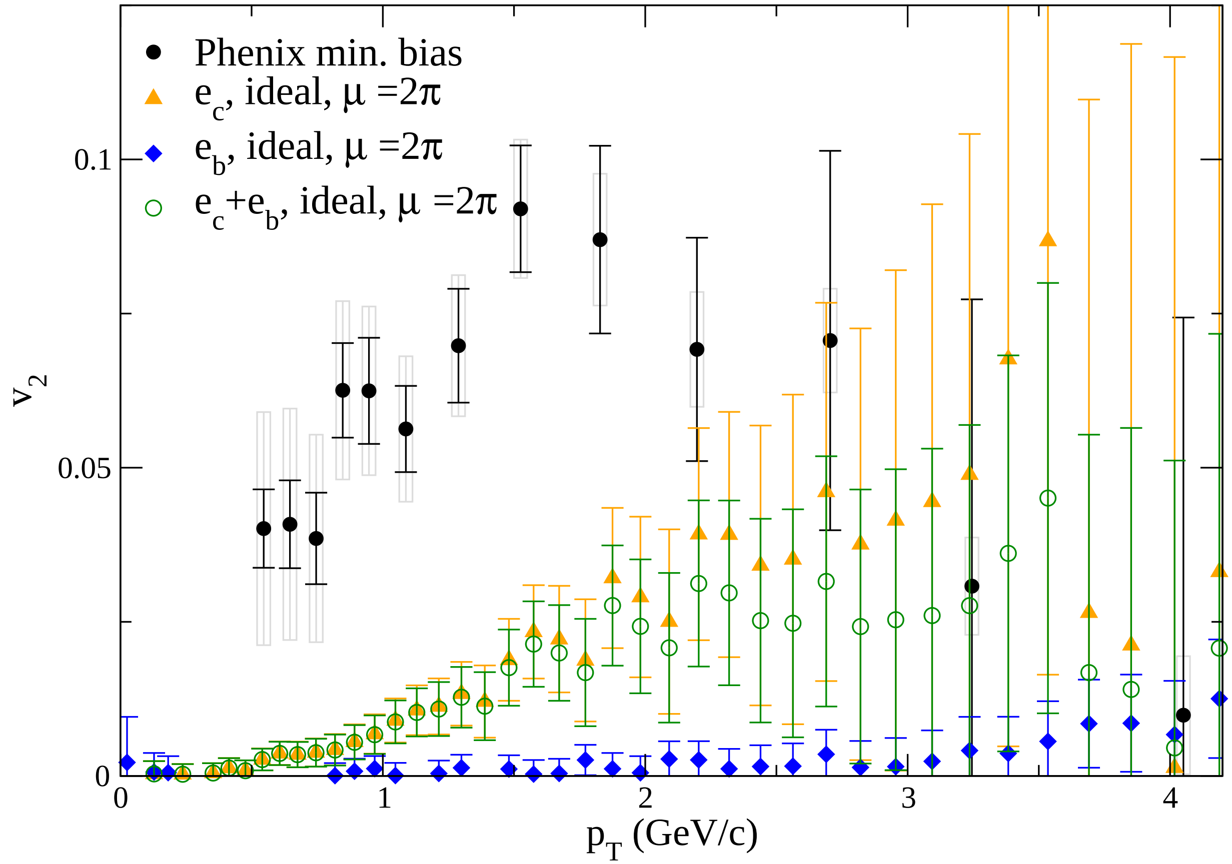
Heavy-flavor v_2: from quarks to electrons



Fragmentation and semi-leptonic decays shift the points to lower p_T[a]

[a]NB The plot refers to the weakest coupling explored

Single-electron v_2: charm and bottom



The *flow of bottom* *is* *negligible* and leads to a *quenching of the inclusive* v_2[a]

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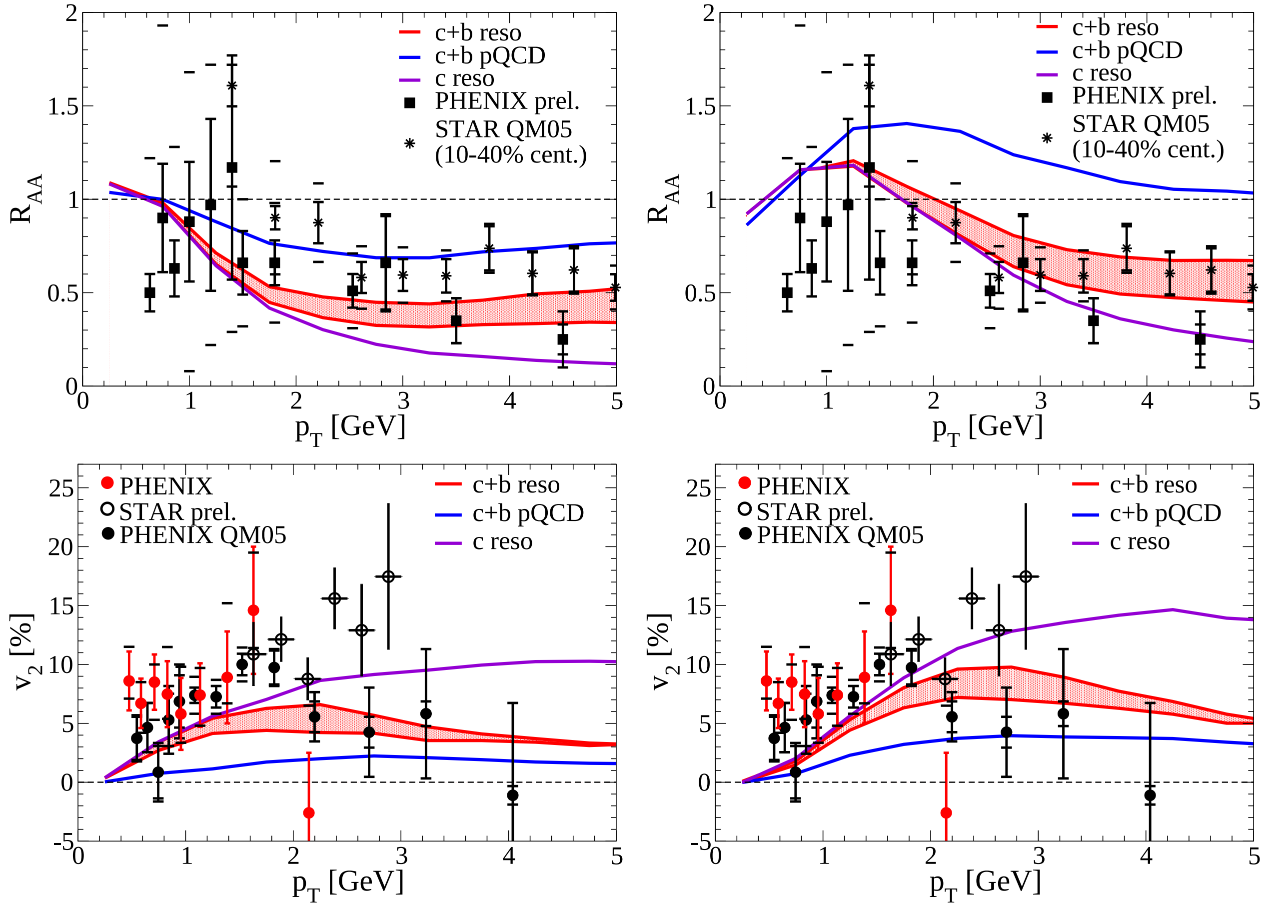
[a]NB The plot refers to the weakest coupling explored

Which role could be played by coalescence?

Fragm. (left panels) vs Coalescence + Fragm.[a] (right panels)



[a]H. van Hees, V. Greco and R. Rapp, PRC 73, 034913 (2006)

Central collisions @ LHC: b=3.45 fm

Hydro scenario

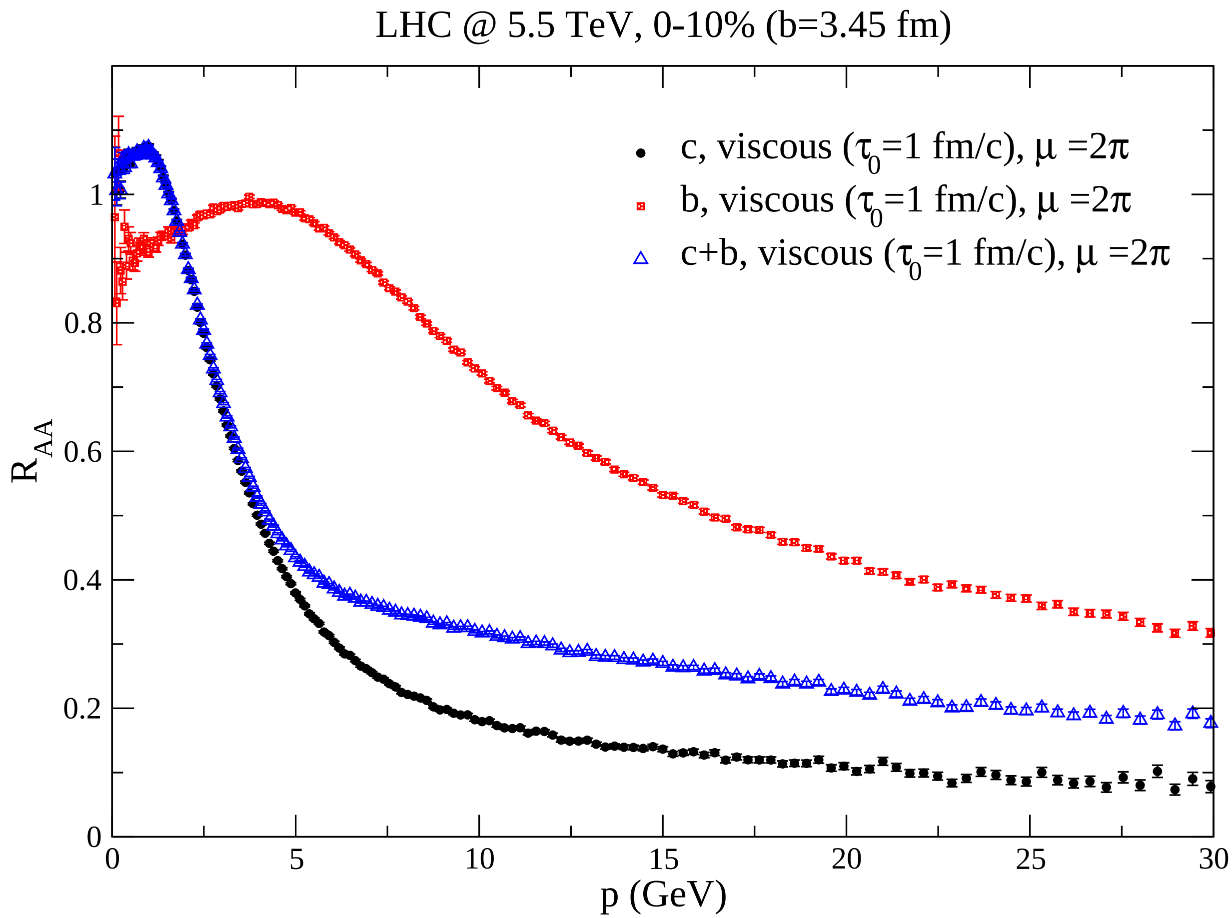
Hydro code	τ_0 (fm/c)	s_0 (fm^{-3})	T_0 (MeV)
viscous	1.0	184	420

Initial production (from POWHEG):

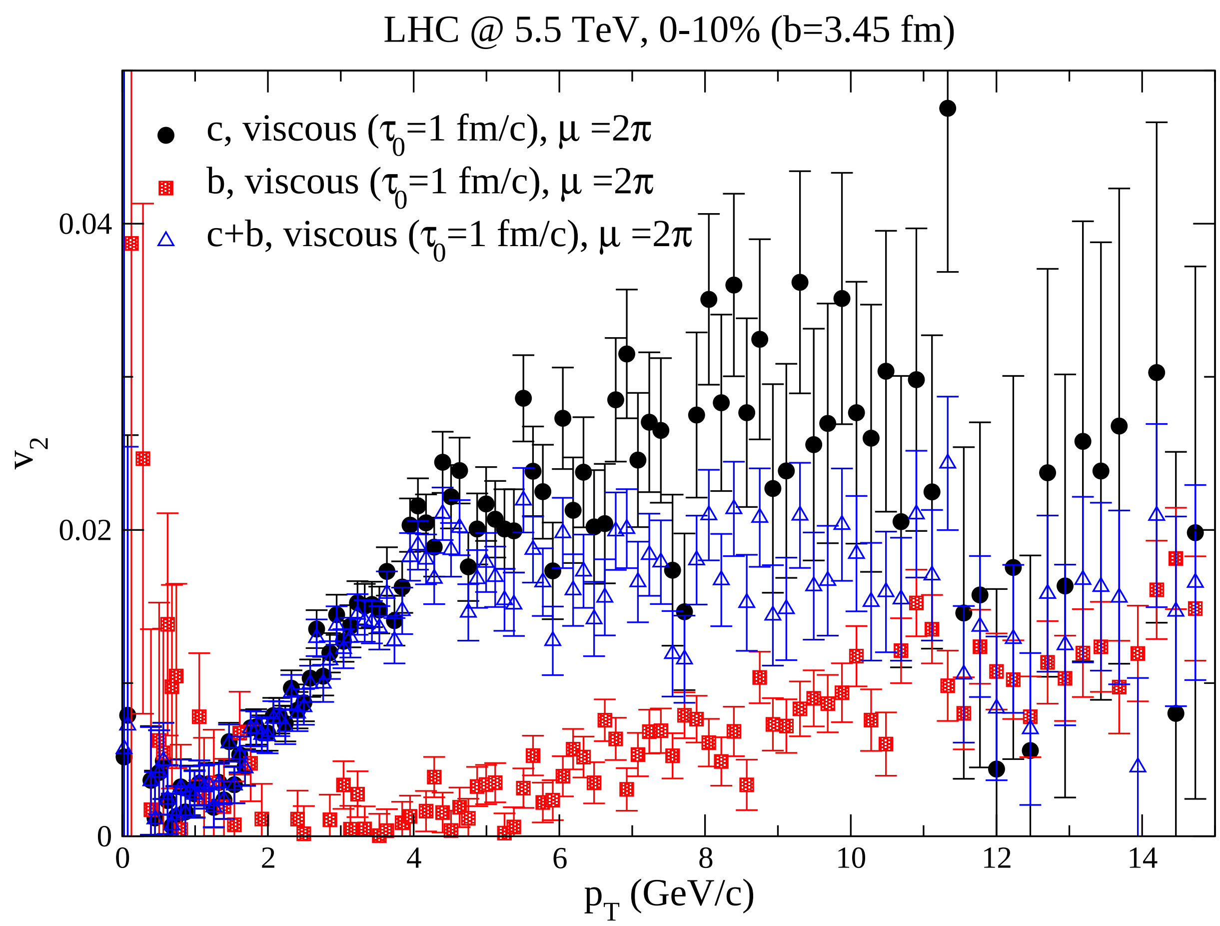
$\sqrt{s}_{NN}$=5.5 TeV	$\sigma_{c\bar{c}}\,(mb)$	$\sigma_{b\bar{b}}\,(mb)$
pp	3.0146	0.1872
AA	2.2877	0.1686

NB *nPDFs give an important initial-state effect*!

Heavy-quark R_{AA} @ LHC



Heavy-quark v_2 @ LHC



Other cases (centrality, hydro scenario, coupling) are currently running...

Conclusions and perspectives

- The relativistic Langevin equation is a powerful tool to study the HQ dynamics in the the QGP: it is an effective theory completely determined by the coefficients $\kappa_{T/L}(p)$ (*no matter their microscopic origin!*)
- $\kappa_{T/L}(p)$ have been evaluated considering only collisional energy loss and distinguishing soft and hard scatterings
- For large p_T ($p_T \gtrsim 4$ GeV/c) it is possible to accommodate RHIC data for the single-electron spectra
- Coalescence could improve the description at lower p_T, raising R_{AA} and v_2
- Preliminary predictions for LHC were attempted

Back-up slides

Glauber and $k_\perp$ broadening

Each HQ is given a $\boldsymbol{k}_\perp$-kick extracted from a gaussian distribution with

$$\langle k_\perp^2 \rangle_{AB}(\vec{b},\vec{s}) = \langle k_\perp^2 \rangle_{pp} + \frac{a_{gN}}{2}\left[\frac{\int dz_A\, \rho_A(\vec{s},z_A) l_A(\vec{s},z_A)}{T_A(\vec{s})} + \frac{\int dz_B\, \rho_B(\vec{s}-\vec{b},z_B) l_B(\vec{s}-\vec{b},z_B)}{T_B(\vec{s}-\vec{b})}\right]$$

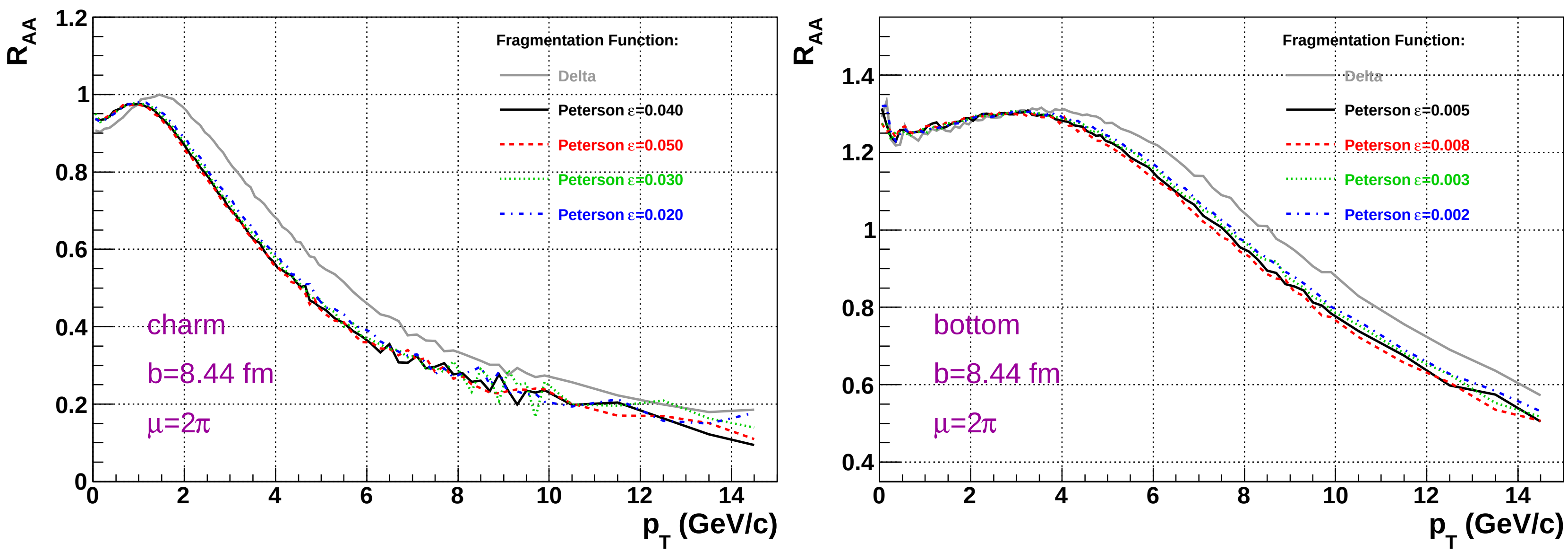
due to the length crossed by the incoming partons in nucleus A/B before the hard event:

$$l_A(\vec{s},z_A) \equiv \int_{-\infty}^{z_A} dz\, \rho_A(\vec{s},z)/\rho_0 \quad \text{and} \quad l_B(\vec{s}-\vec{b},z_b) \equiv \int_{z_B}^{+\infty} dz\, \rho_B(\vec{s}-\vec{b},z)/\rho_0$$

We choose

a_{gN} (GeV^2/fm)	SPS	RHIC	LHC
c	0.072	0.092	0.153
b	0.197	0.252	0.420

Effects of fragmentation



Fragmentation performed with Peterson FF tends to slightly suppress R_{AA}

- Mild dependence on the parameter ϵ
- $\epsilon = 0.04$ and 0.005 (for c and b) fixed in order to reproduce HQET FFs[a]

Fragmentation fractions taken from DESY results and PDG_2009

[a] E. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. D 48, 5049 (1993)