Nielsen-Olesen Instabilities towards Thermalization

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work in collaboration with K. Itakura, A. Iwazaki

The first heavy ion collisions at LHC
HIC10, CERN, 8-9-2010

Conclusion

- Pre-equilibrium physics of QCD is important
- Gluon is a charged massless vector field, so ...
- **Weibel instability**: anisotropic distribution of charged particles + soft magnetic fluctuation
- **Nielsen-Olesen**: chromo-magnetic field + charged massless vector fluctuation
- N.O. Instability, inherent to non-Abelian theory, could play a role towards thermalization

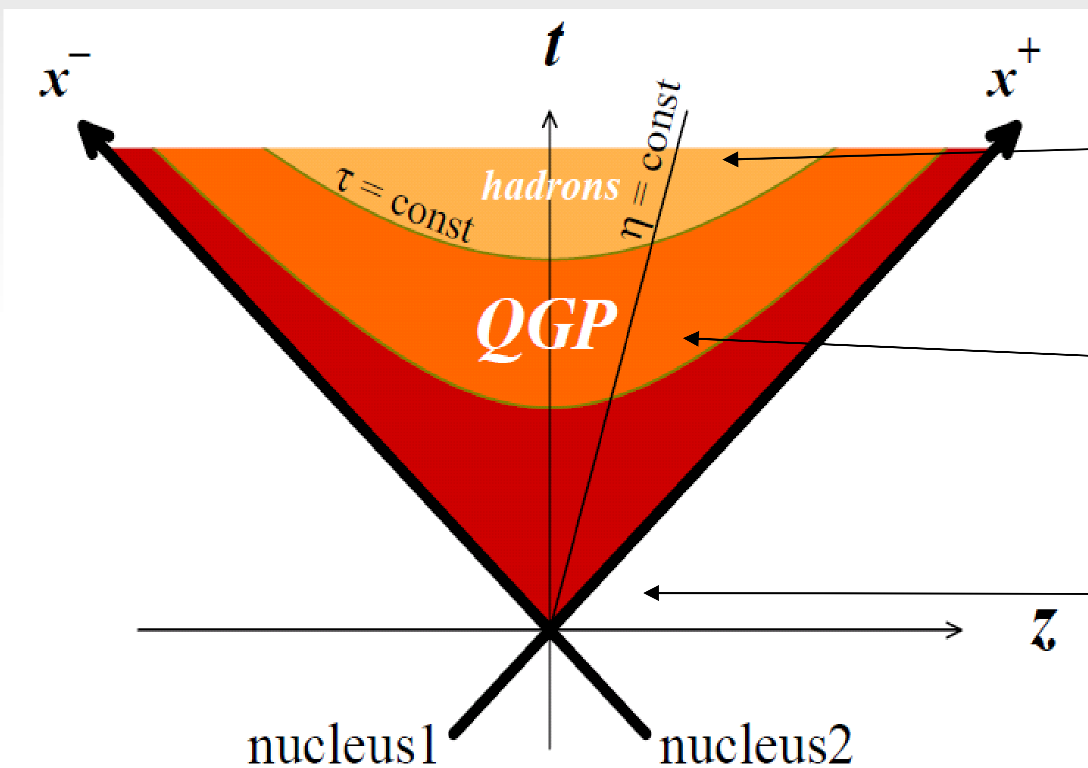
HF, K. Itakura, A. Iwazaki, Nucl. Phys. A **828**, 178 (2009)

A. Iwazaki, Prog. Theor. Phys. **121**, 809 (2009)
HF, K. Itakura, Nucl. Phys. A **809**, 88 (2008)

Outline

- ”Early Thermalization” at RHIC
- Chromo-Weibel instability in brief
- Nielsen-Olesen instability in expanding system
- Nielsen-Olesen instability in a box

event evolution



5. Individual hadrons

   freeze out

4. Hadron gas

   expansion & cooling

3. Quark Gluon Plasma

   thermalization vs expansion

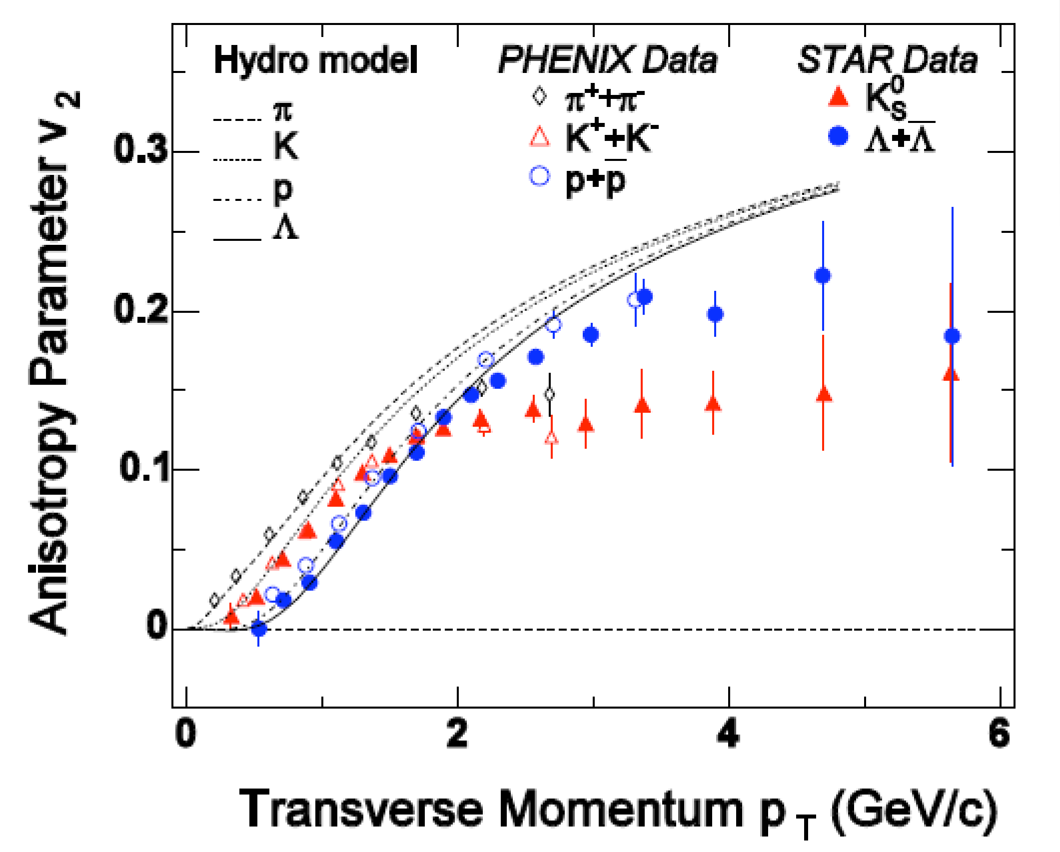
2. Pre-equilibrium stage

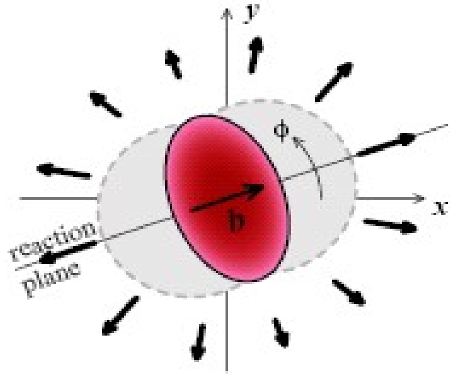
   first contact

1. Nuclei (initial condition)

Large collective motion

- v2=<cos(2ϕ)>, integrated over evolution
- requires ”early thermalization” s.a. ~0.6 fm/c ?

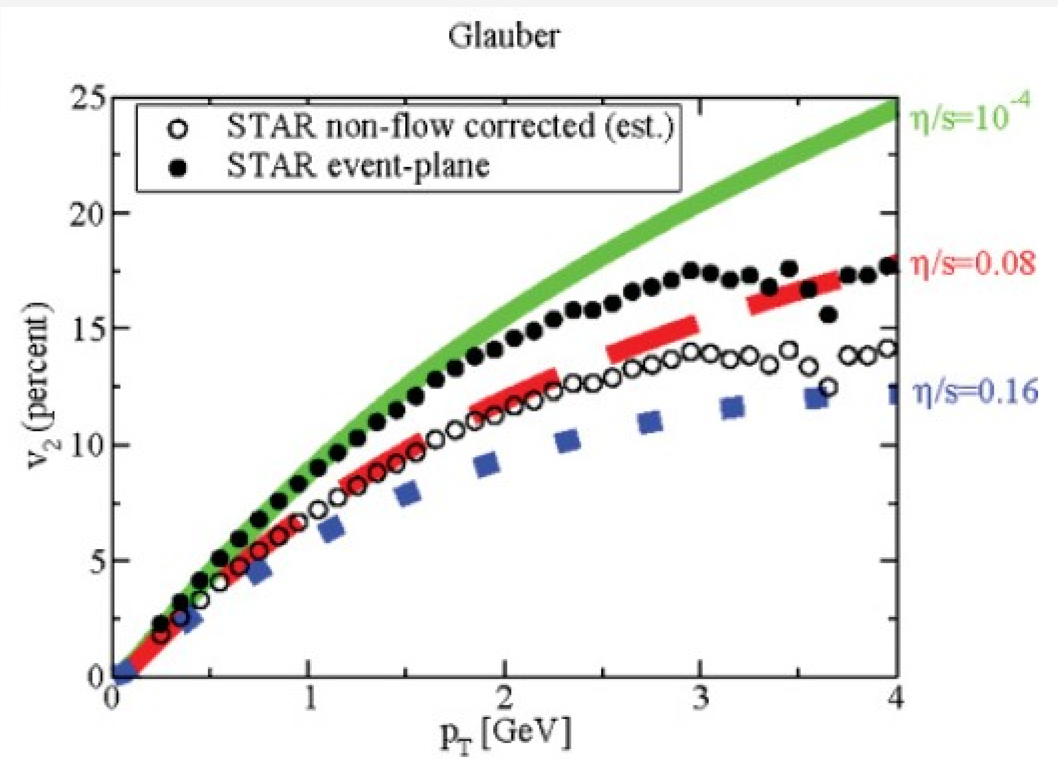


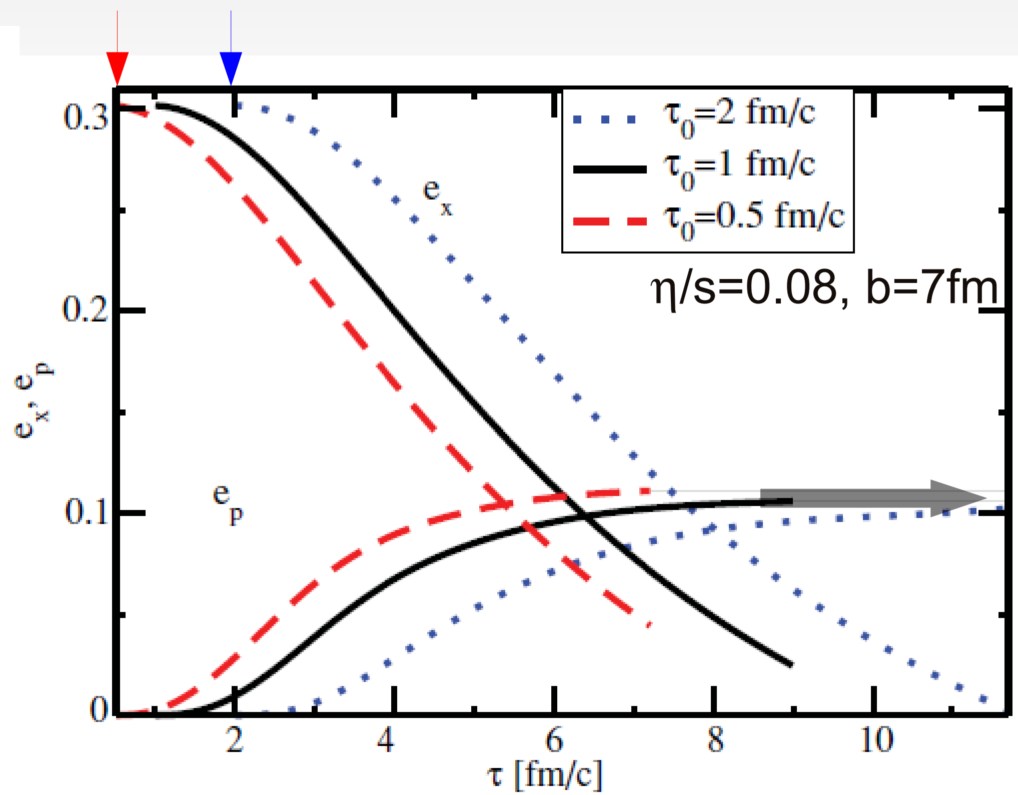


Taken from B.Muller's slide

Large collective motion

- v2=<cos(2ϕ)>, integrated over evolution
- Hydro allows τ_0 = 0.5 - 2 fm/c (cannot fix it)





2nd order shear viscous hydro
Luzum-Romatschke (2008)

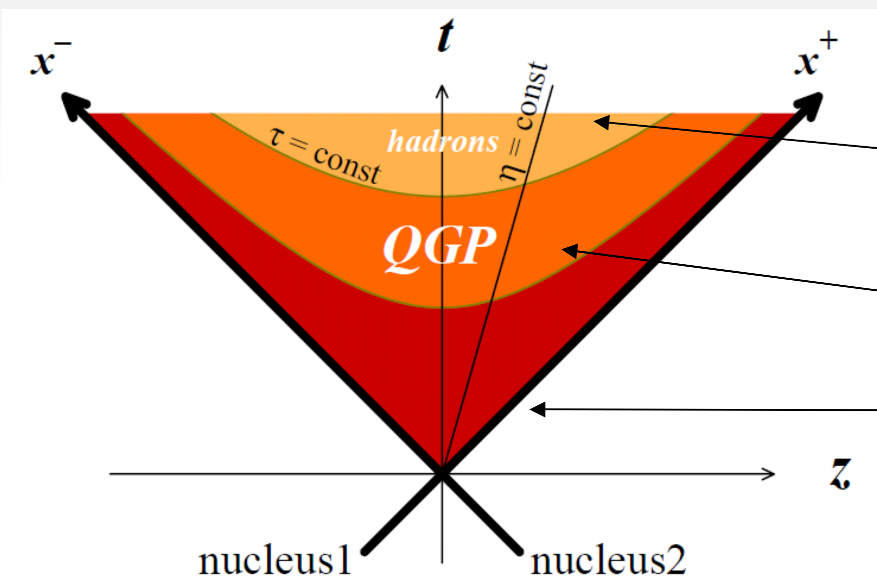
Pre-equilibrium stage

- A missing link between $t = 0$ and $\tau 0$
- A tough and fundamental question on
  - Non-equilibrium dynamics of QCD
- Anyway, a collision starts at $t = 0$

What we discuss here is

- Possible role of Nielsen-Olesen instability in the initial condition given by CGC

Let's forcus on pre-equilibrium stage ...



5. Individual hadrons

freeze out

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thermalization vs expansion

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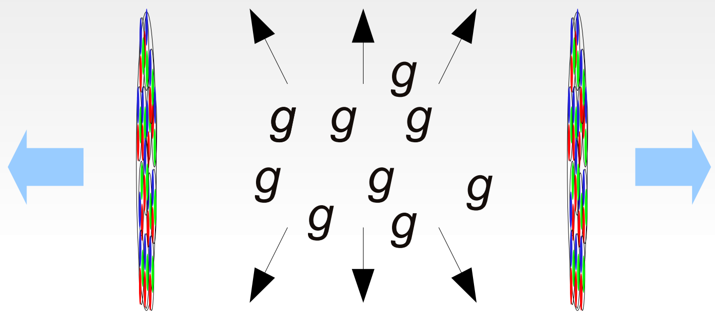
first contact

1. Nuclei (initial condition)

Initial condition given by CGC

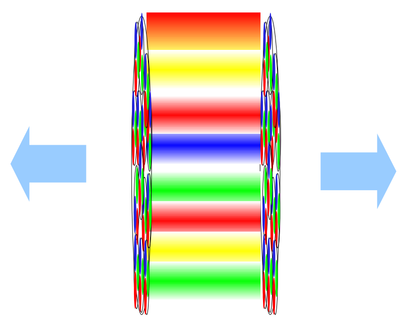
- Small-x gluons characterized by saturation scale Qs
- Shortly after a collision, system is so dense that classical field description applies

$$T^{\mu\nu} = \begin{bmatrix} e & & & \\ & e/2 & & \\ & & e/2 & \\ & & & 0 \end{bmatrix}$$

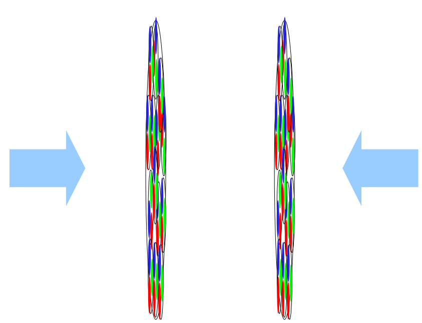


t > 1/Qs; kinetic description will become valid

$$T^{\mu\nu} = \begin{bmatrix} e & & & \\ & e & & \\ & & e & \\ & & & -e \end{bmatrix}$$



0< *t* ~1/Qs; long chromo E&B formed btw receding nuclei due to non-abelian interactions



t <0; (trans) color fields within incoming nuclei

Chromo-Weibel Instability

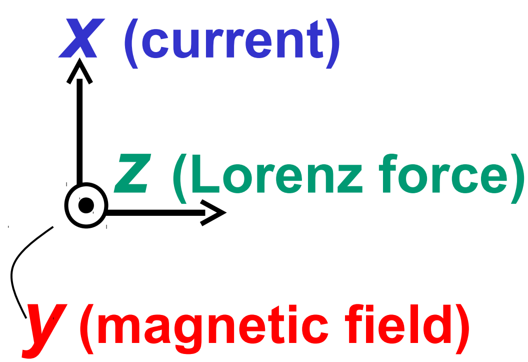
Mrowczynski, Arnold-Lenaghan-Moore,
Dumitru-Nara-Strickland, ⋯

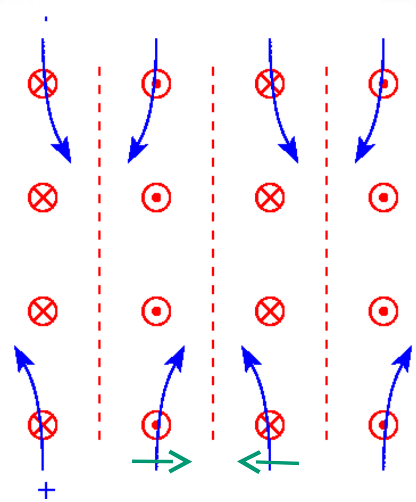
An anisotropic momentum distribution of **charged particles**
is unstable against inhomogeneous magnetic perturbation

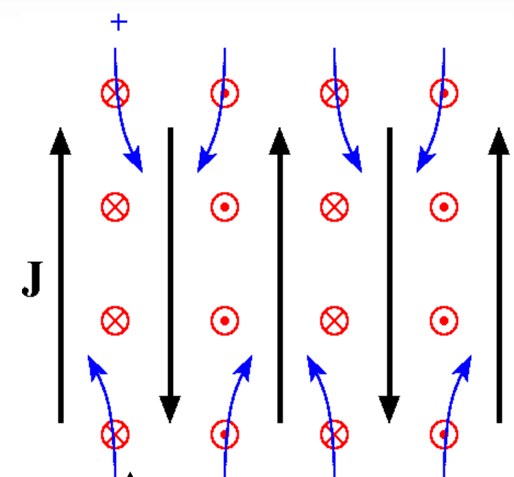
$$p^\mu[\partial_\mu - g q^a F^a_{\mu\nu}\partial^\nu_p - g f_{abc} A^b_\mu q^c \partial_{q^a}] f(x,p,q) = 0$$

$$D_\mu F^{\mu\nu} = J^\nu = g\int \frac{d^3p}{(2\pi)^3} dq q v^\nu f(x,p,q)$$

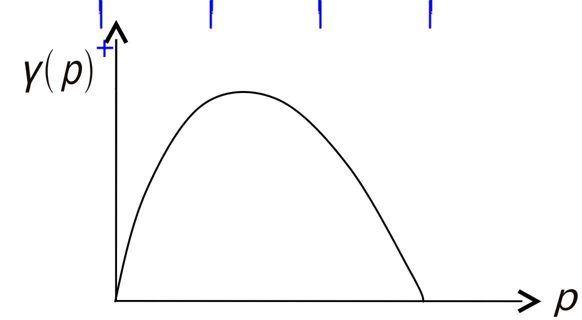
Separation: particles+fields







$$B \propto e^{\gamma t}$$



Chromo-Weibel Instability

Mrowczynski, Arnold-Lenaghan-Moore,
Dumitru-Nara-Strickland, …

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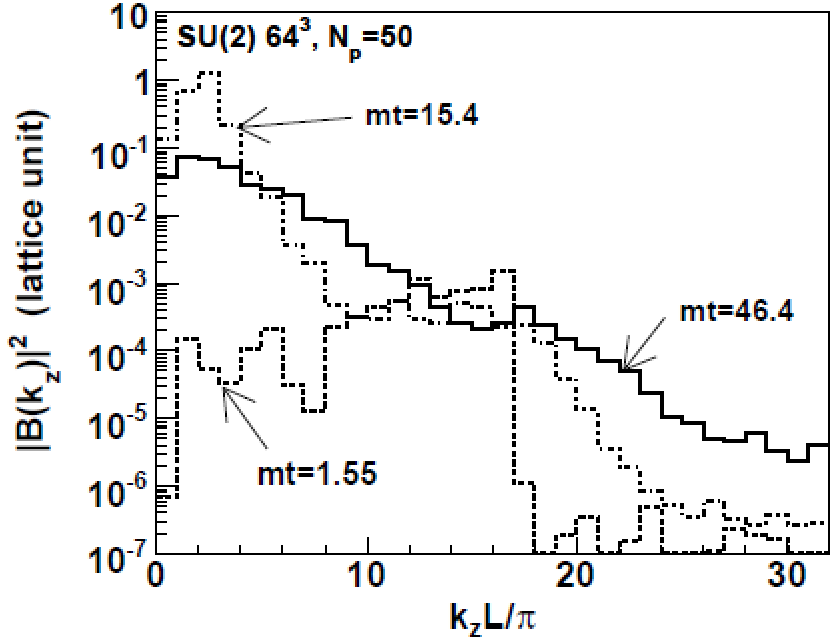
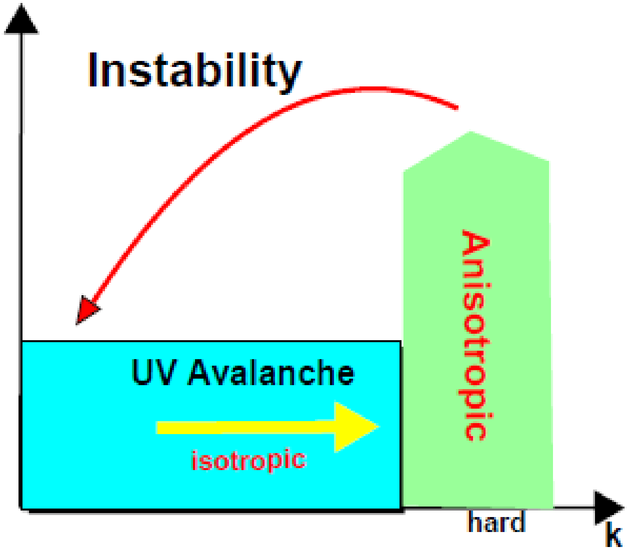
Separation: particles+fields

Finding: Non-abelian coupling causes “Avalanche”





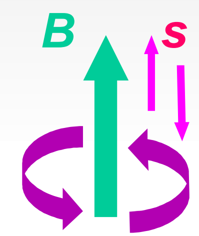
Dumitru-Nara-Strickland (2007)

Nielsen-Olesen Instability

Nielsen-Olesen 1978, Chang-Weiss 1979, Sikivie 1979, …
Turdon 1980, ..., in QCD ground state problem

A uniform magnetic field configuration in **non-Abelian** gauge theory is unstable

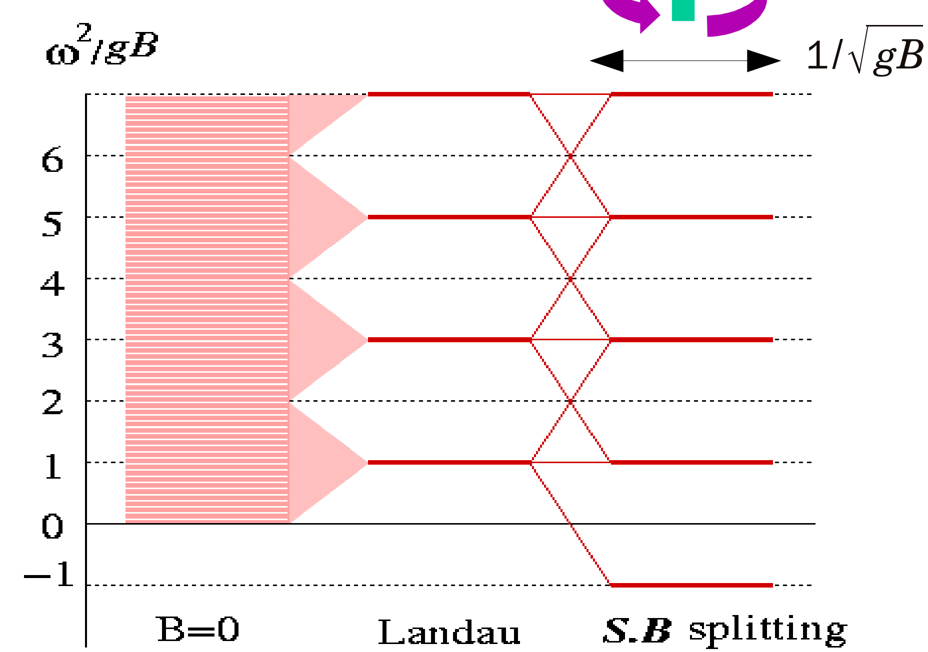
- Off-diagonal gauge fields= "**charged massless vector fields**", which become unstable because of ***S.B* coupling**!



- Take SU(2) and choose B_z^3 as the background field

$$A_\mu = \begin{pmatrix} \frac{1}{2}A_\mu^3 & \frac{1}{\sqrt{2}}\phi_\mu^* \\ \frac{1}{\sqrt{2}}\phi_\mu & -\frac{1}{2}A_\mu^3 \end{pmatrix}$$

- Lowest level: $\omega^2 = -gB + p_z^2$



Why N.-O. Instability relevant?

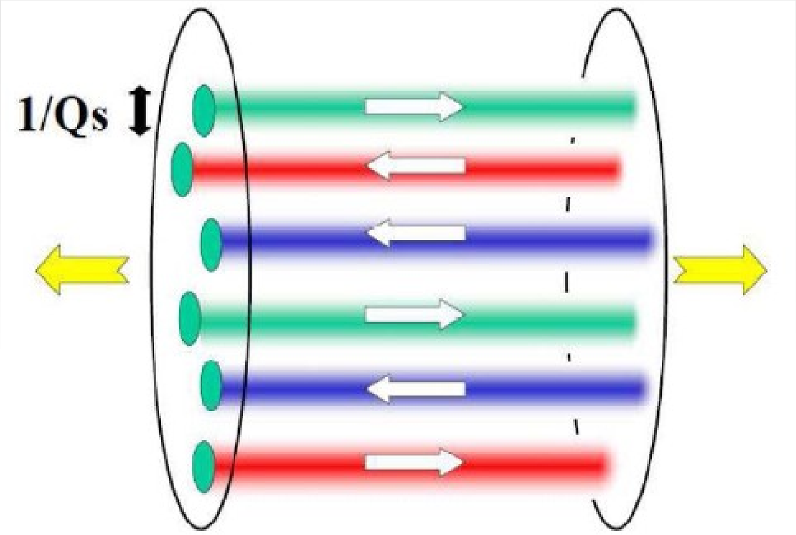
[Fries, Kapusta, Li; Lappi, McLerran]

- Because CGC predicts longitudinal chromo-*E* &*B* of O(gE, gB)=Q_s^2 at $t = 0$

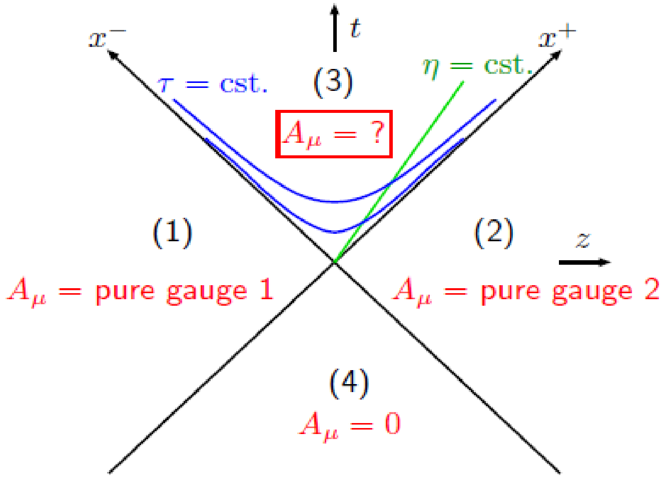
$$E^z\big|_{\tau=0^+} = -ig\left[\alpha_1^i, \alpha_2^i\right],$$
$$B^z\big|_{\tau=0^+} = ig\epsilon_{ij}\left[\alpha_1^i, \alpha_2^j\right].$$

$\alpha_{1,2}$: gauge field of each nucleus generated by random color charge before collision

  - ✓ transverse scale should be ~ 1/Qs
  - ✓ A tube structure is our assumption



T. Lappi's fig.

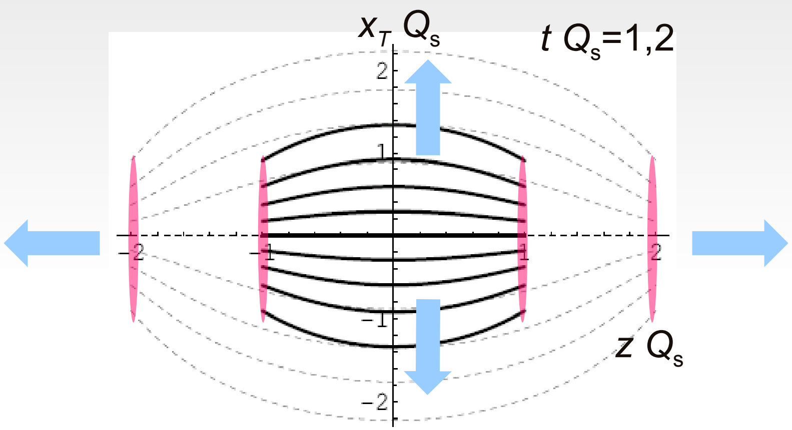


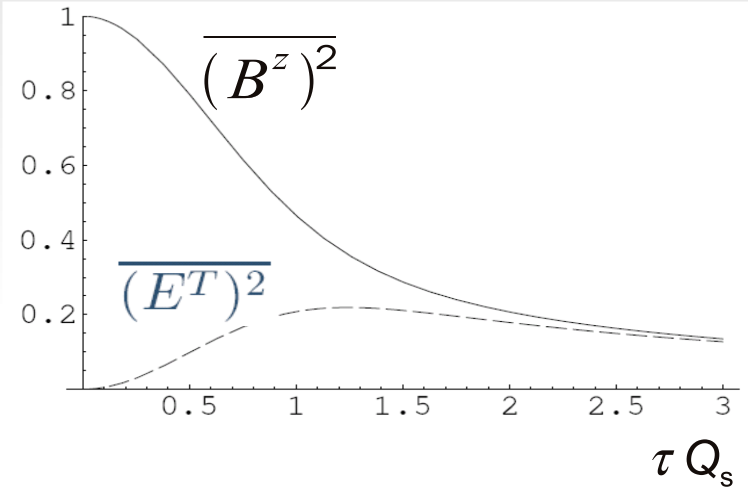
- Because field description suits for initial dense stage with no separation btween particles and fields
  - – Numerical simulations done by Lappi, Romatschke-Venugopalan, Berges-Scheffler-Sexty

A flux tube model in τ–η coordinates

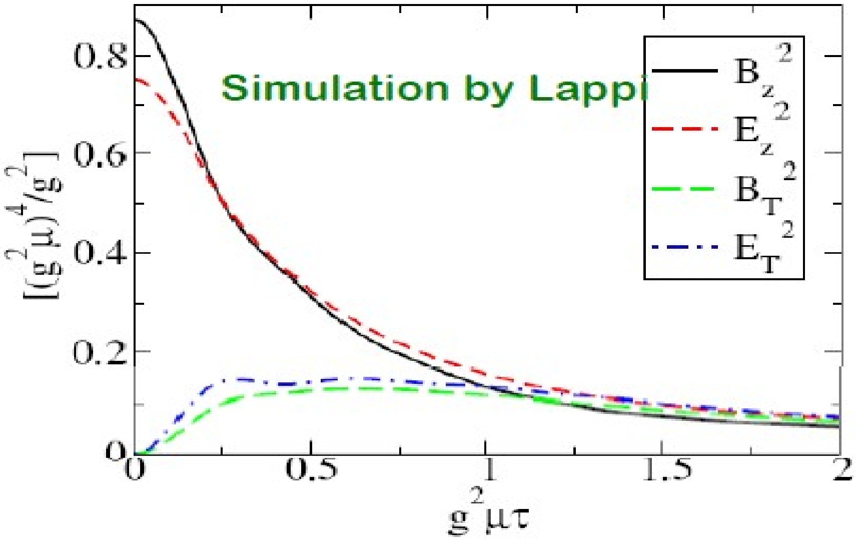
[HF, K. Itakura, 2008]

- Take a single purely magnetic flux tube pointing to 3rd color direction and w/ Gaussian profile (Abelianised and solved by Bessel fn)





- Looks similar to boost invariant numerical simulation
- Longitudinal pressure never become positive

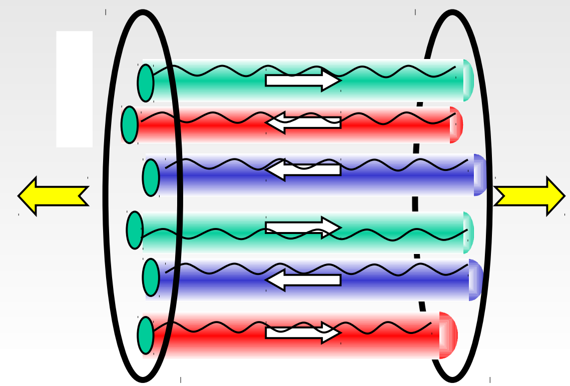


Fluctuation around the background: stability analysis

[HF, K. Itakura, 2008]

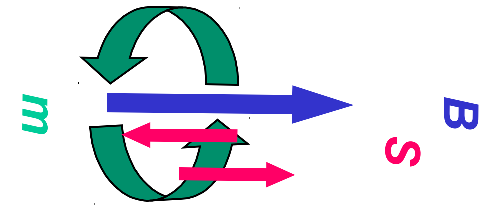
- Thermalization requires generation of long pressure from η-depndent fluctuations of quantum / finite thickness origin

$$A_\mu = \begin{pmatrix} \frac{1}{2}A^3_\mu & \frac{1}{\sqrt{2}}\phi^*_\mu \\ \frac{1}{\sqrt{2}}\phi_\mu & -\frac{1}{2}A^3_\mu \end{pmatrix}$$

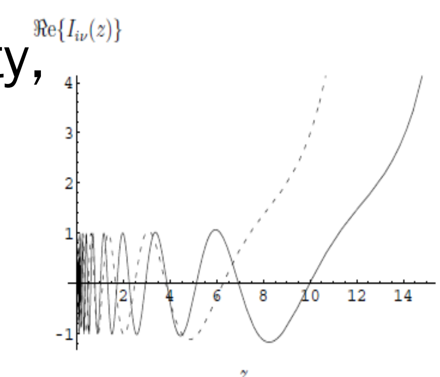


$$\phi_\mu \equiv \frac{1}{\sqrt{2}}(a^1_\mu + i\, a^2_\mu)$$ … charge eigenmode

$$\phi^\pm \equiv \frac{1}{\sqrt{2}}(\phi^x \pm i\, \phi^y)$$ … spin eigenmode



- Nielsen-Olesen Instability comes into play!
  - ✓ Assume a constant background for further simplicity,



$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau\phi^\pm) + \left(E_N \mp 2gB + \frac{\nu^2}{\tau^2} \right)\phi^\pm = 0$$

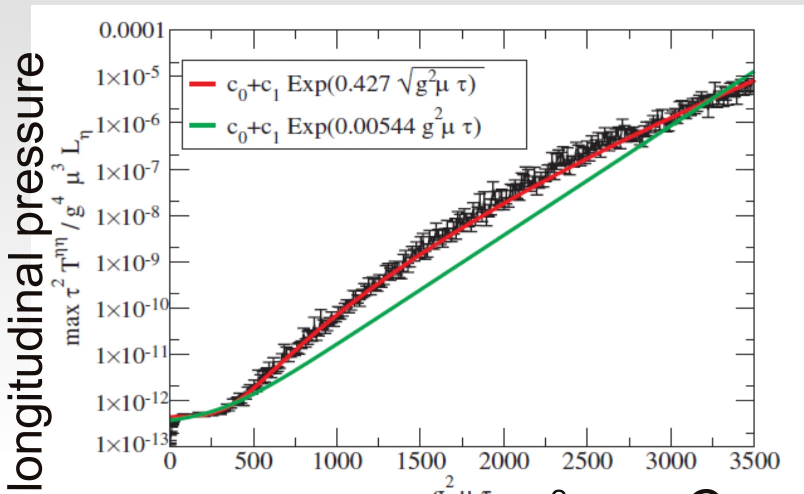
→ **– *gB*** <0 for the lowest mode

- **Spring constant (…) vanishes when <u>ν = sqrt(gB) τ</u>**

Simulation in τ–η coordinates

P.Romatschke & R. Venugopalan, 2006

Small rapidity dep fluctuations generate long pressure as time proceeds

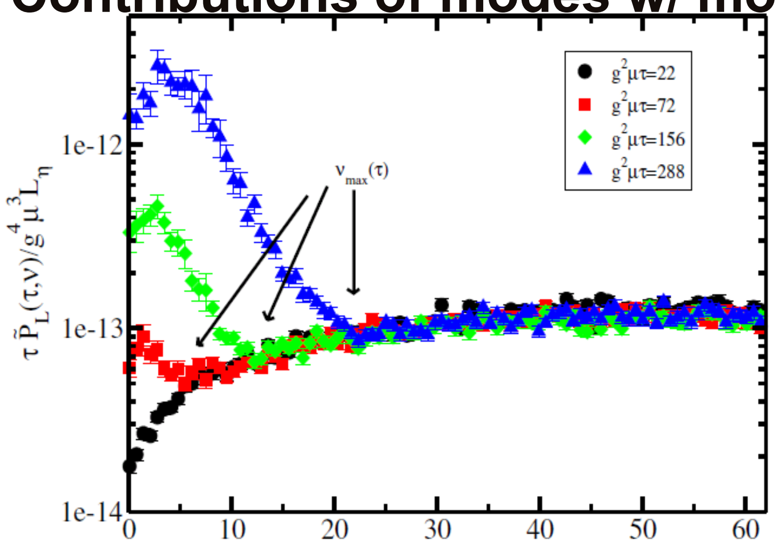


$g^2\mu\tau \sim Q_s\tau$

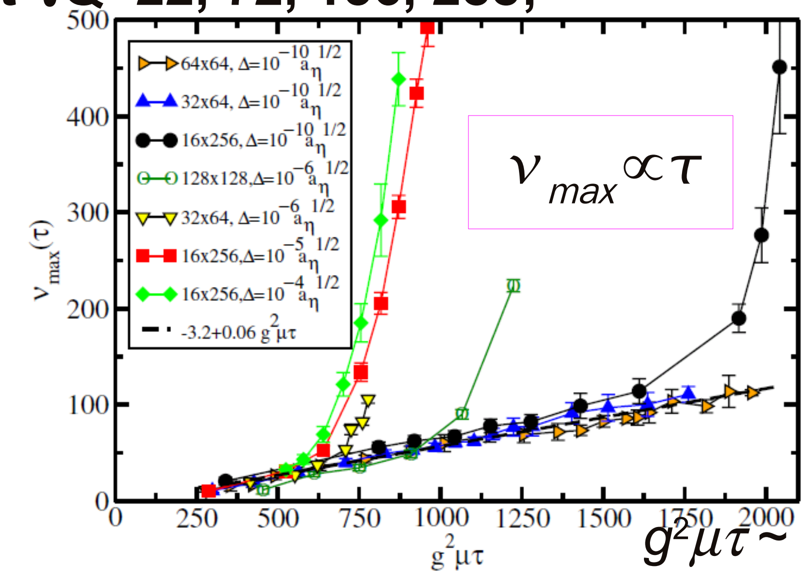
3+1D numerical simulation

$$P_L \sim \exp\left(C\sqrt{g^2\mu\tau}\right)$$

Contributions of modes w/ mom ν at τQ=22, 72, 156, 288;



ν : conjugate to rapidity η

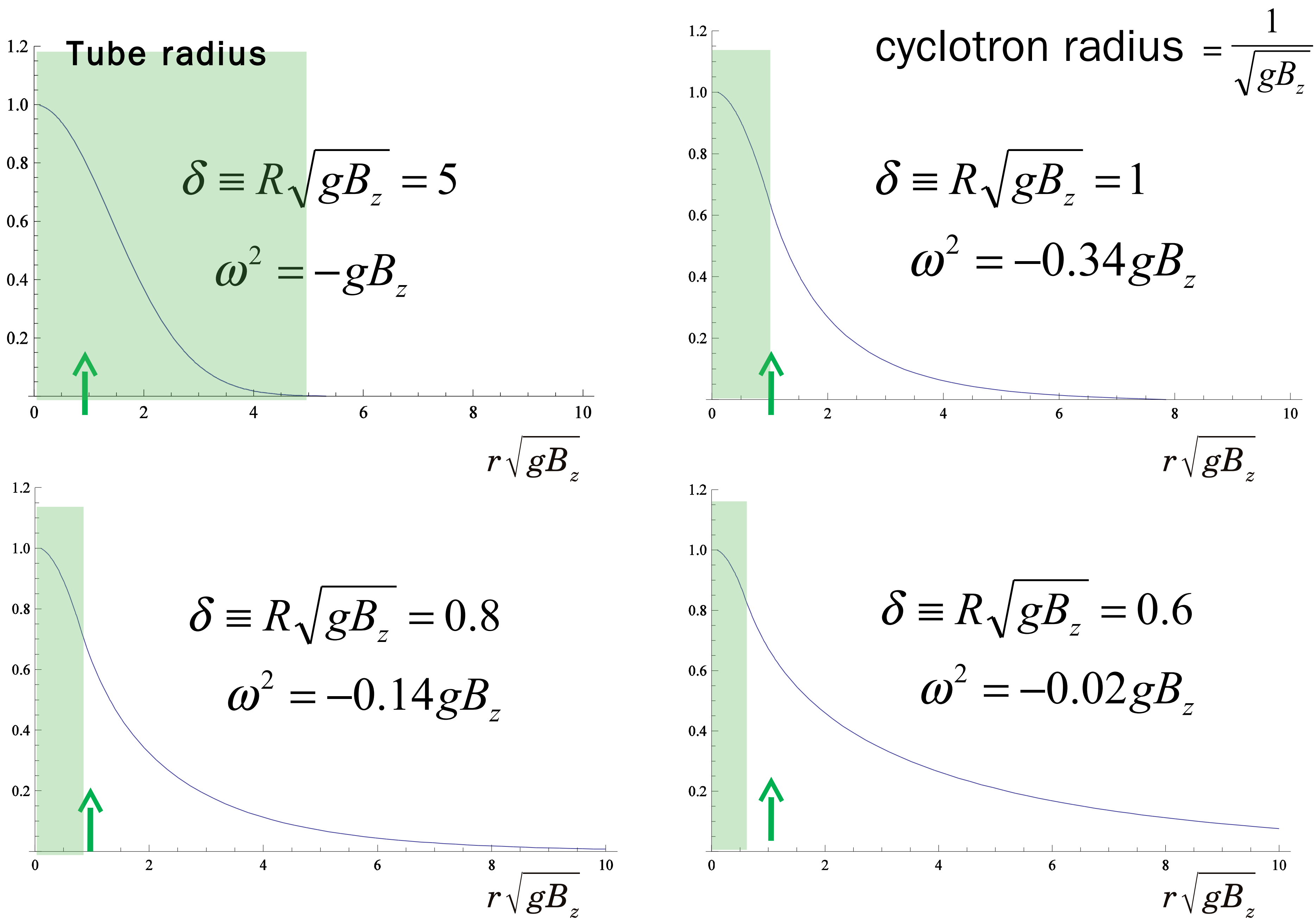


$g^2\mu\tau \sim Q_s\tau$

Tube radious vs $\Phi_-(r)$ at pz=0

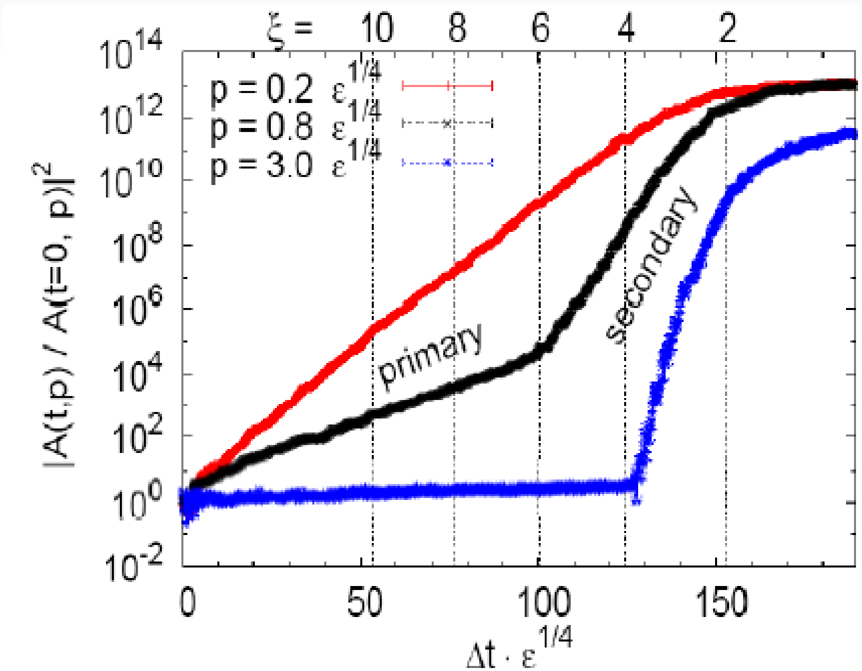
Slide by A. Iwazaki

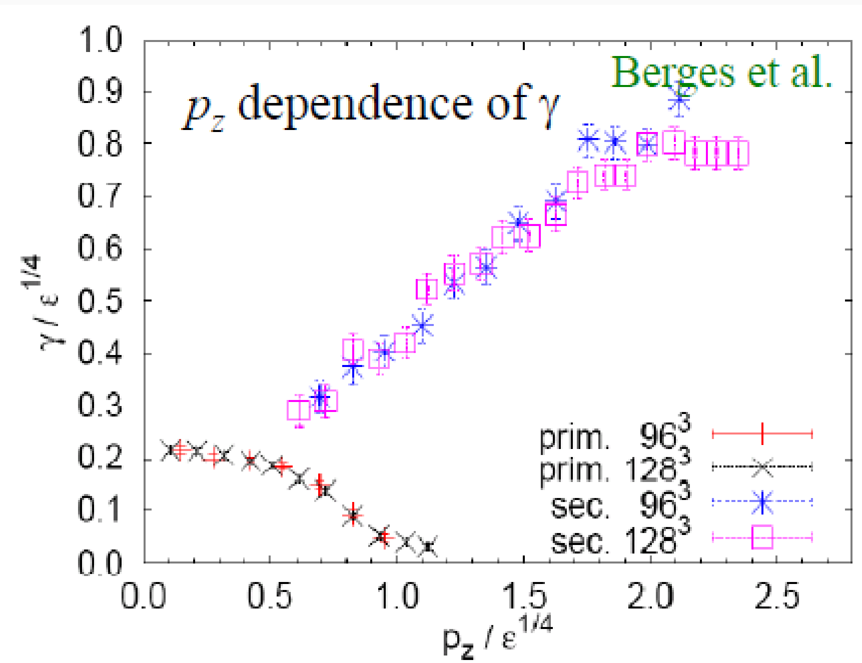
The narrower the tube, the smaller the binding and the growth rate



Plasma instability in a box

- A classical statistical field simulation of SU(2) in a box with anisotropic initial condition Berges-Scheffler-Sexty
  - <pT>~Qs, <pz>~0
- The primary and secondary instabilities are observed





Plasma instability in a box

- N.O. Instability may be relevant because …

Initial config is generated stochastically but is purely magnetic

$$\langle |A_j^b(t=0,\mathbf{p})|^2 \rangle = \frac{C}{(2\pi)^{3/2}\Delta^2\Delta_z} \exp\left\{-\frac{p_x^2+p_y^2}{2\Delta^2} - \frac{p_z^2}{2\Delta_z^2}\right\},$$

$$E_j^a = -\dot{A}_j^a = 0$$ at t = 0

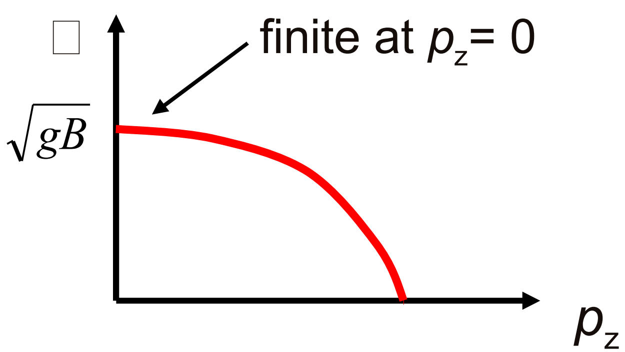
Magnetic fields *B* is

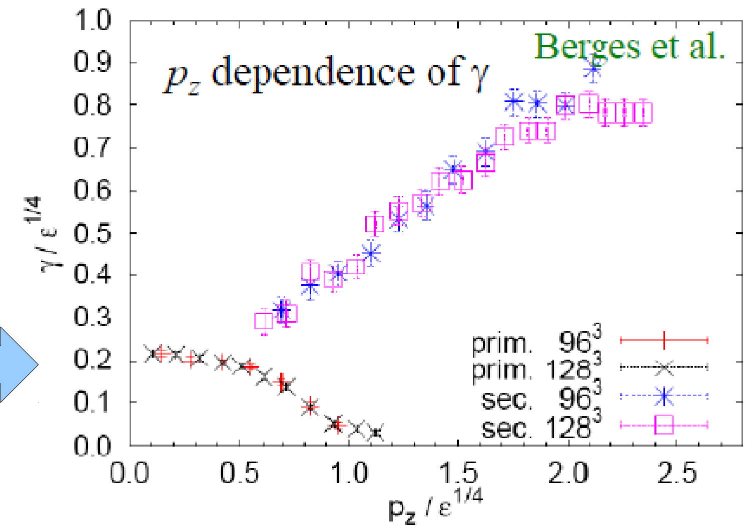
homogeneous in the z direction

inhomogeneous in the transverse plane (*Δ* ~ Qs)

The most characteristict feature

$$\omega^2 = -gB + p_z^2$$



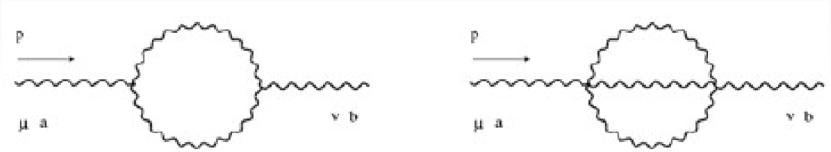


A scenario for the secondary

Berges-Schaffler-Sexty

- What causes the secondary?
  - ✓ Diagramatic interpretation: non-linearity generates the secondary

$$\exp(\gamma_0 t) \longrightarrow \exp(2\gamma_0 t),\ \ \exp(3\gamma_0 t)$$



A scenario for the secondary

HF, K.Itakura, A.Iwazaki

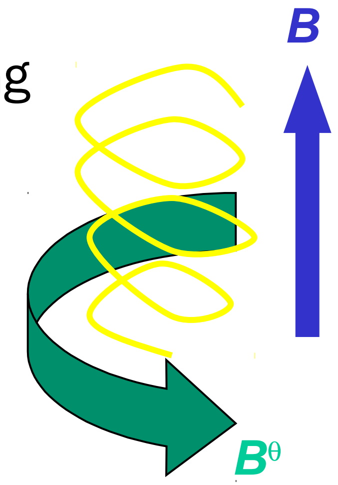
- What causes the secondary?
  - ✓ If the primary N.O. instability generates a color current, ...

- The color charge can move along B field
  - ✓ Parametric estimate for the current when $\phi^2 \sim Qs^2/g^2 \sim gB/g^2$

$$J^z = \sum_{\alpha=\pm} ig\left\{\phi^{\alpha *} D^z \phi^\alpha - (D^z\phi^\alpha)^* \phi^\alpha\right\}$$
$$\sim \mathcal{O}(g \cdot \sqrt{gB} \cdot B/g) = \mathcal{O}((gB)^{3/2}/g) \sim \mathcal{O}(Q_s^3/g)$$

  - ✓ Ampere's law, ***J*** = rot***B*** , and dimensionality imply a strong azimuthal field

$$B^\theta = \mathcal{O}(Q_s^2/g)$$



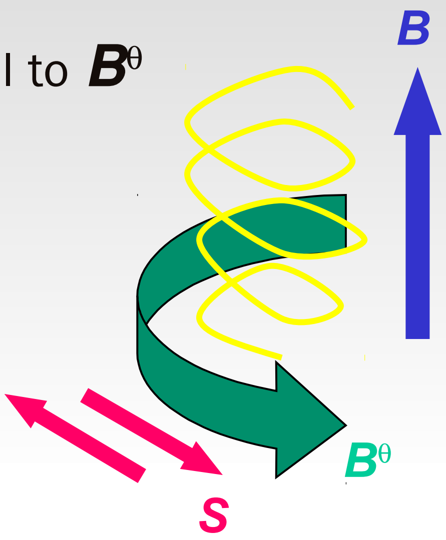
- Let us examine the consequence of this $\boldsymbol{B}^\theta$

A scenario for the secondary

HF, K.Itakura, A.Iwazaki

- Stability analysis around $\boldsymbol{B}^{\theta}$
  - ✓ Combine the fluctuation field so as to polarise in (anti)parallel to $\boldsymbol{B}^{\theta}$

$$\varphi^{\pm} = (\phi^z \pm i\phi^r)/\sqrt{2} \qquad A_z = -rB_\theta$$

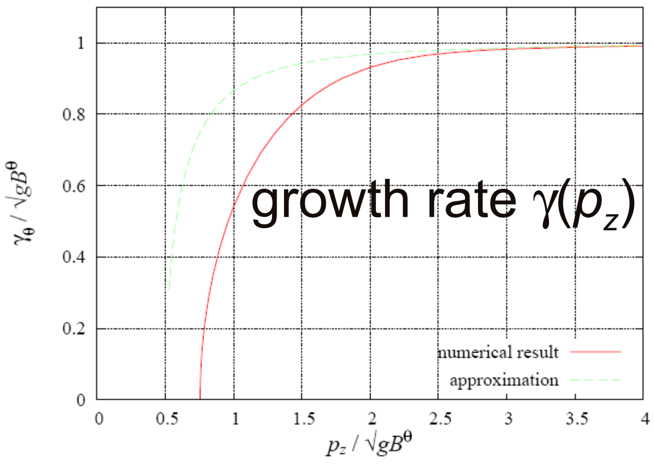


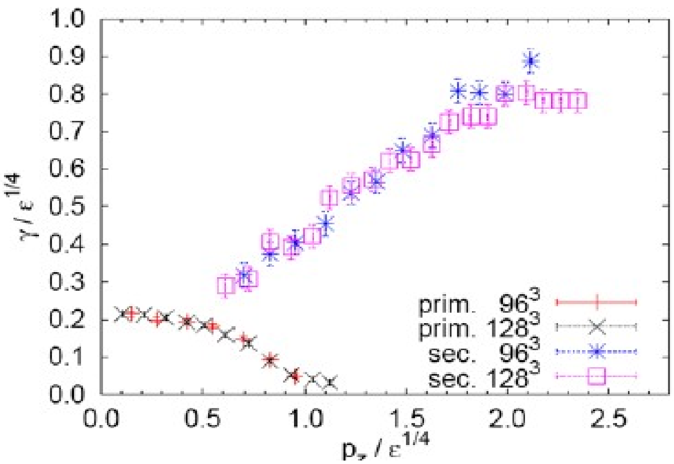
  - ✓ We find a "Schroedinger" equation for harmonic fluctuation

$$\left\{ -\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} + \left(p_z - gB^\theta r\right)^2 + \frac{1}{2r^2} - 2gB^\theta \right\} \tilde{\varphi}^-(r) = \omega^2\, \tilde{\varphi}^-(r)$$

spring constant

- The growth rate γ increases with p_z





Remarks

- Nielsen-Olesen instability has been considered here in a dynamic situation
- N.O. Instability gives a new aspect in non-Abelian plasma evolution
- More detailed study requires a systematic test with numerical simulations
  - Weibel-type instability in gauge field simulation w/o particles?
  - Weibel and N.O. co-exist? Or more complication?
  - How does EOS build up?

- Pre-equilibrium evolution must be challenged in order to
  - understand the non-equilibrium dynamics of QCD
  - construct the whole picture of heavy ion physics at RHIC & LHC

- At LHC, field description is expected more appropriate

Conclusion

- Pre-equilibrium physics of QCD is important
- Gluon is a charged massless vector field, so ...
- **Weibel instability**: anisotropic distribution of charged particles + soft magnetic fluctuation
- **Nielsen-Olesen**: chromo-magnetic field + charged massless vector fluctuation
- N.O. Instability inherent to non-Abelian theory, could play a role towards thermalization

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