Phenomenological Modeling of the PICOSEC Timing Characteristics

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on behalf of the PICOSEC-RD51 Collaboration

Modeling the Timing Characteristics of the PICOSEC Micromegas Detector

Compiled by

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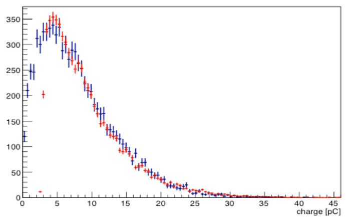
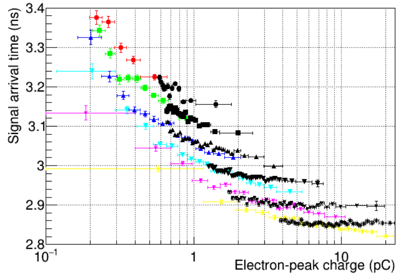
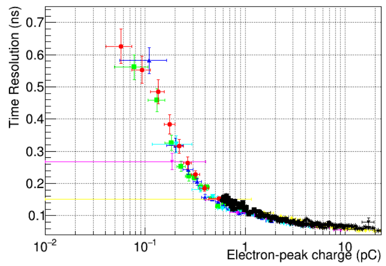
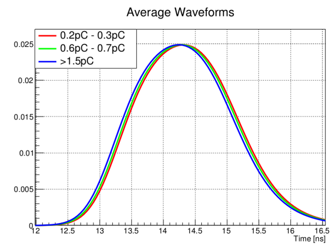
[GARFIELD++] ⊕ [Electronics Response Function]

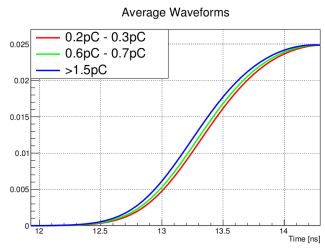
Describes the PICOSEC e-peak response to single photoelectrons and especially the timing characteristics (Signal Arrival Time, Resolution) provided from the leading edge of the e-peak





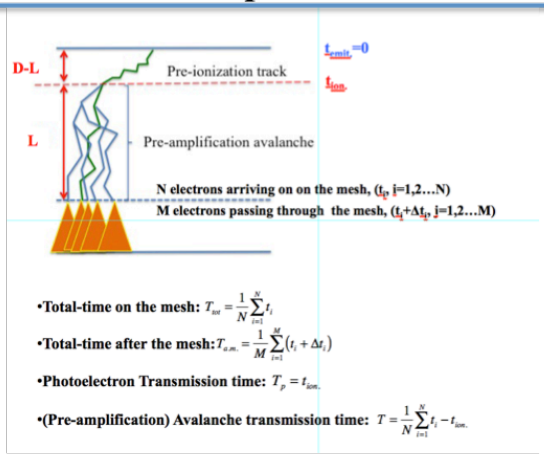




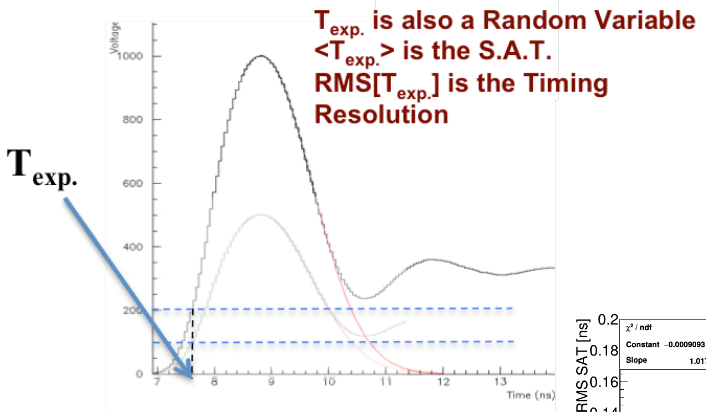


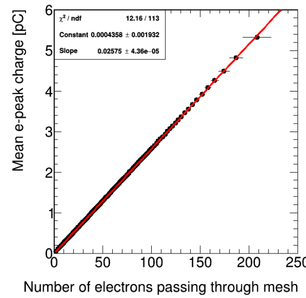
GARFIELD++ simulations have been used to identify and study the behavior of microscopic parameters, which determine the PICOSEC timing characteristics.

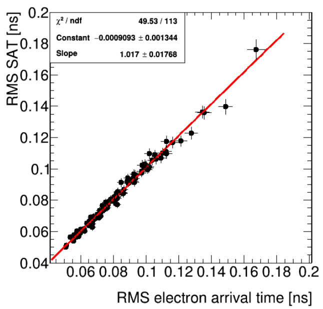
Microscopic Information

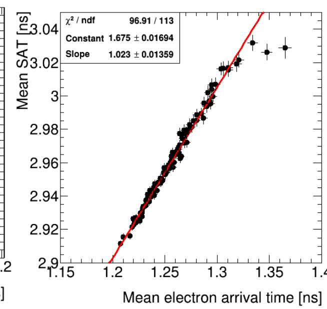


Macroscopic Information –Exp. Observable

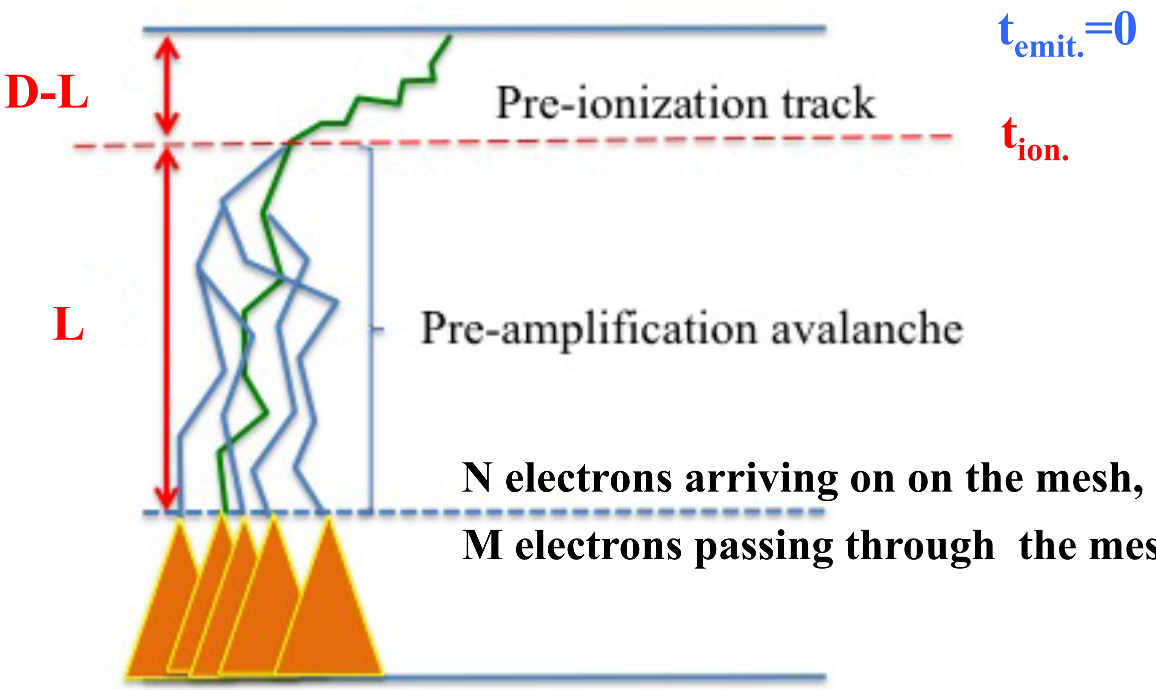








Definitions



N electrons arriving on on the mesh, (t_i, i=1,2…N)

M electrons passing through the mesh, ($t_j+\Delta t_j$, j=1,2…M)

- **Number of pre-amplification electrons after the mesh** ***(determines the e-peak size)***

- **Total-time after the mesh:** $T_m = \sum_{i=1}^{M} [t_i + \Delta t_i]$ ***(determines Resolution and SAT)***

- **Total-time on the mesh:** $T_{tot} = \sum_{i=1}^{N} [t_i]$

- **Photoelectron (Transmission) time:** T_p
- **Avalanche transmission time:** $T=T_{tot}-T_p$
- **Transport time through the mesh**

Random Variables

GARFIELD++ predicts that:

1. **The avalanche drifts faster than the photoelectron before ionization , $V_a > V_p$. *(It explains qualitatively the dependence of the SAT on the e-peak size, i.e. the SAT, relative to the photoelectron emission, decreases linearly with the avalanche length, longer avalanches produce more electrons)***
2. **The avalanche electron drifts faster than the photoelectron, $V_{ea} > V_p$ *(This is due to more frequent inelastic collisions in the avalanche – see Kostas Kordas talk. It can explain 1.)***
3. **The avalanche drifts faster than its constituent avalanche electrons, $V_a > V_{ea}$. *(WHY? and HOW?)***
4. **The avalanche time, i.e. the time taken by an avalanche to drift a certain length L, also depends explicitly on the electron multiplicity at L *(WHY? and HOW?)***
5. **The PICOSEC timing resolution depends strongly on the longitudinal diffusion. The variance of the photoelectron time and of the avalanche electron transmission time depend, as expected, on the drift length. However, the variance of the avalanche time is almost constant, independent of the respective transmission length. *(WHY? and HOW?)***
6. **The photoelectron time and the avalanche time, when they are expressed in terms of the multiplicity of the pre-amplification electrons, are mutually, heavily correlated. *(How this correlation affects the observed dependence of the PICOSEC Timing Resolution? How this correlation comes?)***
7. **The transport of the pre-amplification electrons through the mesh adds to the SAT a constant time delay, which, for any set of voltage settings, is independent of the signal size *(WHY?)***
8. **The passage through the mesh depletes the number of pre-amplification electrons by a factor of 4 (in all the voltage settings considered in this study). At high drift voltages (e.g. 425V) this depletion leaves practically unaffected the PICOSEC resolution. However, at lower drift voltages (e.g <375 V) the timing resolution worsens, even if the mesh transparency remains the same. This effect depends on the pre-amplification avalanche multiplicity (and length) in a specific way. *(WHY? and HOW?)***

In this work, we have developed a phenomenological model, which describes statistically the relevant processes, in a very good quantitative agreement with the GARFIELD++ simulation results, and offers insights to identify the cause of the above behavior of microscopic quantities, which determine the PICOSEC timing characteristics.

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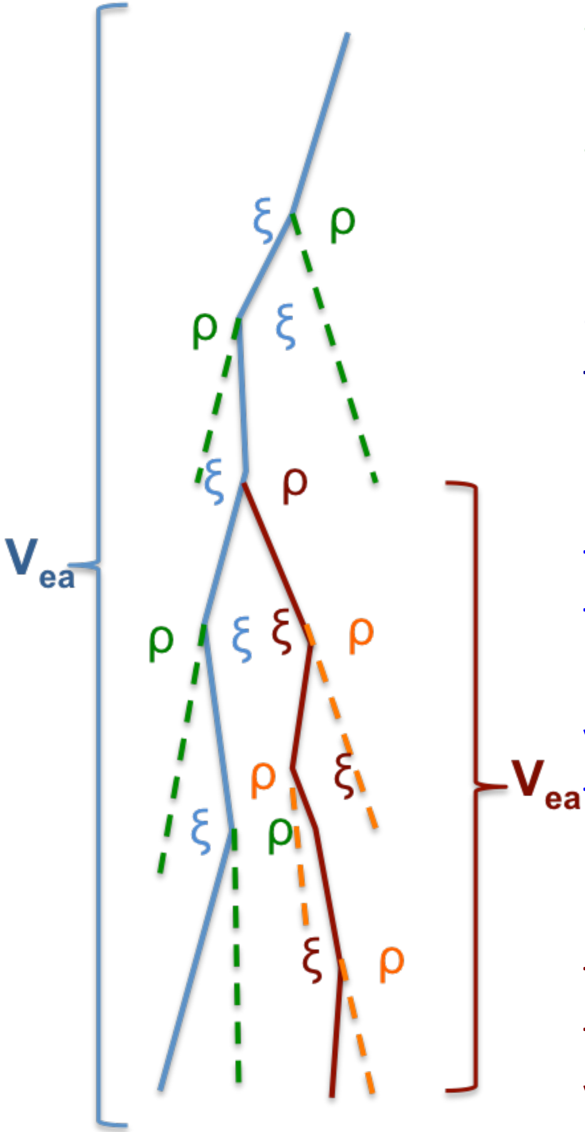
The Model



•An ionizing electron in the avalanche, every time it ionizes, will gain a time ξ_l relative to an electron that undergoes elastic scatterings only.

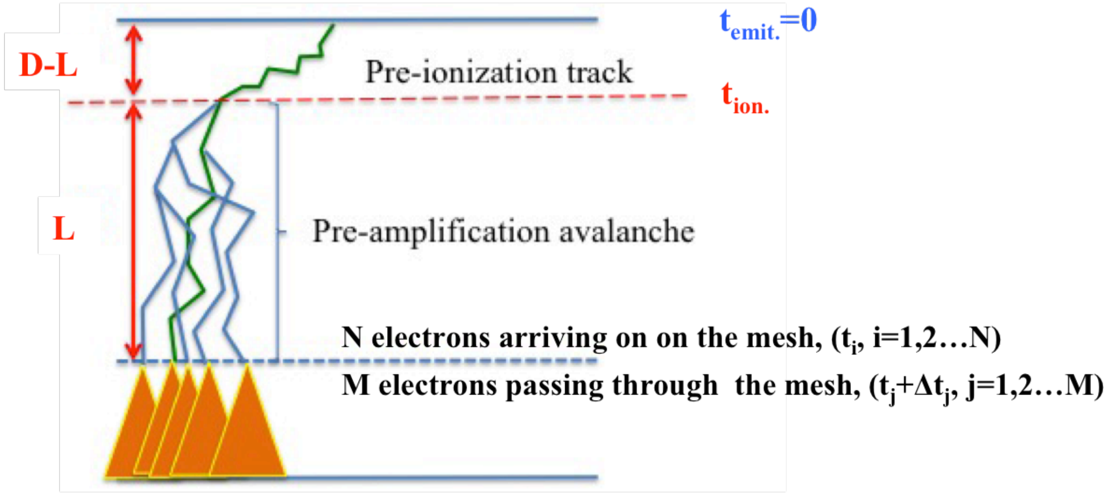
• Any newly produced electron by ionization starts with low energy; at the start of its path, it suffers less delay due to elastic backscattering compared to its parent. Therefore, the model assumes that such a newly produced electron will gain, relative to its parent, a time-gain ρ_l.

• The parameters ξ_l and ρ_l should follow a joint probability distribution determined by the physical process of ionization and the respective properties of interacting molecules.

Model Approximations

•The collective effect of time-gains ξ_l is a change in drift velocity from V_p, which is the photoelectron drift velocity before ionization, to an effective drift velocity V_{ea}, which is the drift velocity of an ionizing electron in the avalanche. By taking V_{ea} to be the drift velocity of any electron in the avalanche, the energy-loss effect on the drift of the parent electron has been taken into account.

•When a new electron is produced in the avalanche, through ionization, it will gain time ρ_l, at its production, which it is assumed to follow a distribution with mean value ρ and variance w^2. From that moment onwards, this new electron propagates with drift velocity V_{ea}, as any other existing electron in the avalanche.



The Modeling

1. **Develop a statistical description of the transmission times up to the mesh**

- Evaluate the mean value of the avalanche-time as a function of the avalanche length and multiplicity
- Evaluate the avalanche drift velocity
- Express the mean values of the transmission times (which contribute the PICOSEC SAT) as functions of the number of pre-amplification electrons (which determine the e-peak size)
- Study the variance of the transmission times (which contribute the PICOSEC timing resolution) and derive formulae which express quantitatively their behavior as function of the avalanche length and/or electron multiplicity

3. **Develop a statistical description of the electron transport through the mesh**

- Derive formulae that express the contribution of the "electron transport through the mesh" to the mean value and variance of the "total time after the mesh" which determines the PICOSEC timing characteristics

4. **Compare every of the above steps with the GARFIELD++ simulation results, for several drift voltage settings and different values of the Penning Transfer Rate**
5. **Extend the Model to predict the p.d.f. followed by the above microscopic variables (instead of just mean values and variances) and compare with GARFIELD++ simulations.**

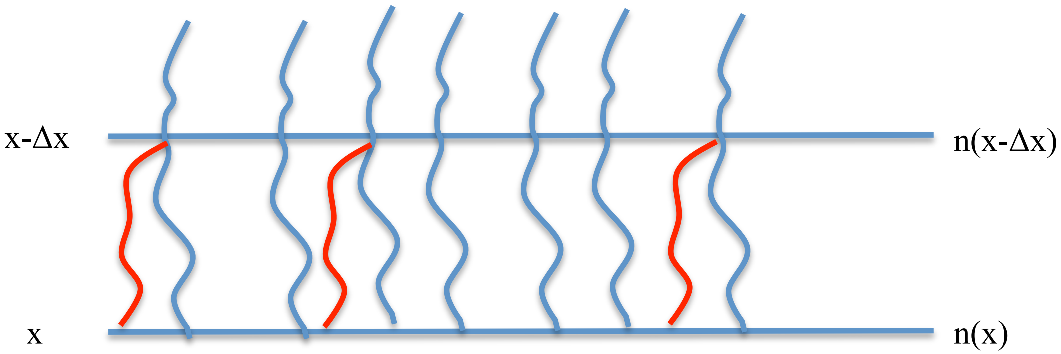
Input Parameters

TABLE A-8: Values of the parameters used by the model

Penning Transfer Rate			**50%**		
Anode Voltage			**450 V**		
Drift Voltage	**325 V**	**350 V**	**375 V**	**400 V**	**425 V**
a ($10^{-2}\mu m^{-1}$)	3.607 ± 0.018	4.400 ± 0.020	5.208 ± 0.027	6.069 ± 0.027	6.950 ± 0.032
a_{eff} ($10^{-2}\mu m^{-1}$)	2.215± 0.001	2.629± 0.001	3.055 ± 0.001	3.484 ± 0.001	3.912 ± 0.001
θ	2.698± 0.142	2.906± 0.154	3.037 ± 0.162	3.313 ± 0.179	3.645 ± 0.191
V_{ea}^{-1} (10^{-3}ns/μm)	7.311± 0.003	6.877± 0.003	6.509 ± 0.002	6.173 ± 0.002	5.866 ± 0.004
V_p^{-1} (10^{-3}ns/μm)	8.065± 0.026	7.678± 0.026	7.266 ± 0.028	6.923 ± 0.028	6.643 ± 0.031
d_{off} (10^{-2}ns)	-3.831 ± 0.084	-3.437 ± 0.082	-2.883 ± 0.075	-2.678 ± 0.068	-2.364 ± 0.079
ρ (10^{-2}ns)	3.570± 0.054	2.919± 0.027	2.489 ± 0.030	2.185 ± 0.028	1.725 ± 0.045
C (10^{-2}ns)	7.555± 0.218	7.511± 0.117	7.668 ± 0.166	7.778 ± 0.196	7.001 ± 0.516
σ_p^2 (10^{-4}ns^2/μm)	2.137± 0.054	1.908± 0.046	1.662± 0.073	1.554± 0.050	1.380± 0.063
Φ (10^{-4}ns^2)	-9.967 ± 2.417	-7.936 ± 1.395	-6.40 ± 1.650	-7.525 ± 1.343	-5.622 ± 1.284
σ_0^2 (10^{-4}ns^2/μm)	2.094 ± 0.005	1.778± 0.003	1.543± 0.004	1.341± 0.003	1.175± 0.004
tr	0.244± 0.009	0.248± 0.044	0.238± 0.011	0.251± 0.009	0.247± 0.009
δ (10^{-2}ns)	7.217 ± 0.034	6.871 ± 0.032	6.607 ± 0.031	6.305 ± 0.030	5.938 ± 0.040
Δt_{mesh} (10^{-1} ns)	1.521 ± 0.005	1.455 ± 0.005	1.400 ± 0.004	1.344 ± 0.003	1.303 ± 0.004

Develop a statistical description of the transmission times up to the mesh

Evaluate the mean value of the avalanche-time as a function of the avalanche length and multiplicity



The average arrival time of the n(x) electrons at a plane on x is expressed as:

$$T\left(x,n(x)\right)=\frac{1}{n(x)}\sum_{k=1}^{n(x)}t_k(x)$$

$$=\frac{1}{n(x)}\left[\sum_{k=1}^{n(x-\Delta x)}\left(t_k(x-\Delta x)+\Delta t_k\right)+\sum_{j=1}^{\Delta n}\left(t_j^f(x-\Delta x)+\Delta\tau_j\right)\right]$$

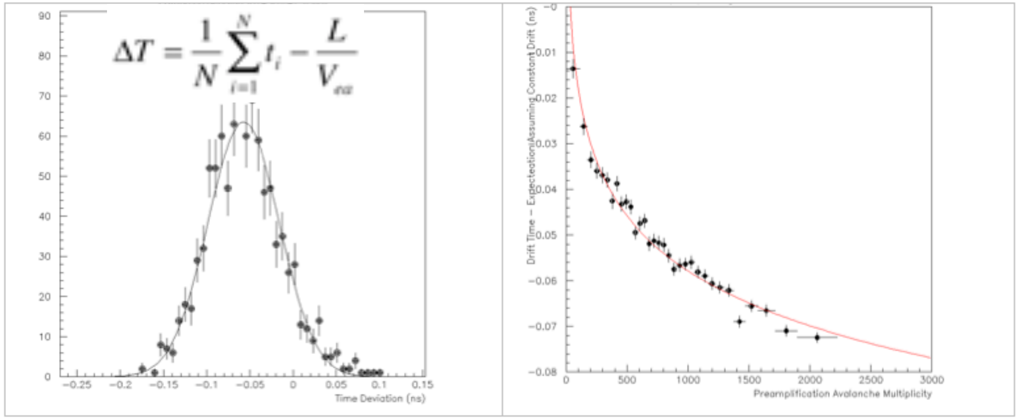
$$=\frac{1}{n(x)}\left[\sum_{k=1}^{n(x-\Delta x)}t_k(x-\Delta x)+\sum_{j=1}^{\Delta n}t_j^f(x-\Delta x)+\sum_{k=1}^{n(x-\Delta x)}\Delta t_k+\sum_{j=1}^{\Delta n}\Delta\tau_j\right]$$

a) each time a new ionization occurs, the newly produced electron gains a certain time, ρ_i ,(i=1,Δn) relative to the parent electron; b) any of the n(x-Δx) pre-existing electrons has the same probability, Δn/n(x-Δx), to produce a new electron; c) averaging for all possible configurations of Δn, newly produced electrons, as well as for all possible values of Δt and ρ, and d) taking the limit that Δx goes to zero

$$d\left\langle T(x),n(x)\right\rangle=\frac{dx}{V_{ea}}-\frac{dn}{n(x)}\rho \quad or \quad \left\langle T\left(L,N_L\right)\right\rangle=\frac{L}{V_{ea}}-\rho\cdot\ln\left(N_L\right)+C$$

$$\langle T(L,N_L)\rangle - \frac{L}{V_{ea}} = -\rho \ln(N_L) + C$$

$$\left\langle \langle T(L,N_L)\rangle - \frac{L}{V_{ea}} \right\rangle_L = -\rho \ln(N_L) + C$$



GARFIELD++ results are described very well (see K. Kordas talk) for a variety of operating parameters, by the above logarithmic expression. Subsequently, by fitting the GARFIELD++ simulation results with this expression, the mean value of the time-gain ρ and the constant term C, were estimated.

Develop a statistical description of the transmission times up to the mesh:

Evaluate the avalanche drift velocity

1

$$\langle T(L,N_L)\rangle = \frac{L}{V_{ea}} - \rho\cdot\ln(N_L) + C$$

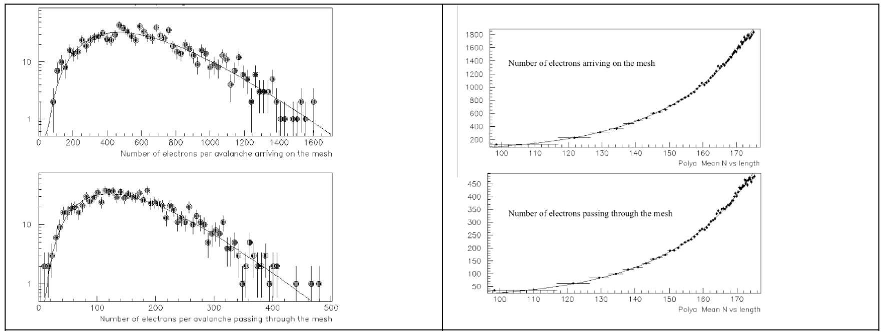
2

$$\langle T(L)\rangle = \int_0^\infty \langle T(L,N_L)\rangle P\left(N_L; q = 2e^{a_{eff}L}, \theta\right) dN_L$$

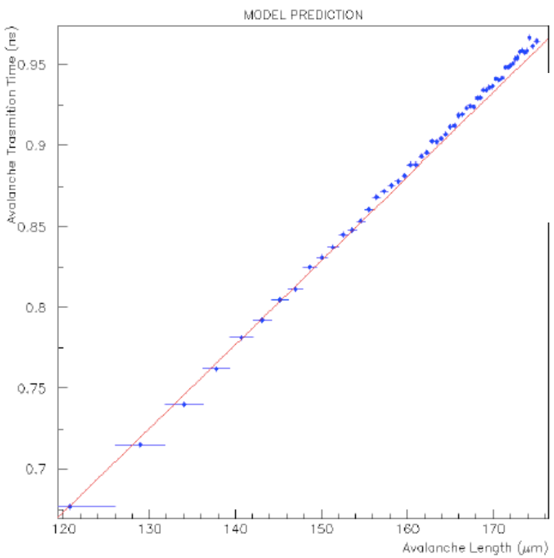
$$= \frac{L}{V_{ea}} - \rho\cdot\int_0^\infty \ln(N_L) P\left(N_L; q = 2e^{a_{eff}L}, \theta\right) dN_L + C$$

3

$$P\left(N_L; q = 2\cdot e^{a_{eff}\cdot L}, \theta\right) = \frac{1}{2\cdot e^{a_{eff}\cdot L}}\cdot\frac{(\theta+1)^{\theta+1}}{\Gamma(\theta+1)}\cdot\left(\frac{N_L}{2\cdot e^{a_{eff}\cdot L}}\right)^{\theta}\cdot\exp\left[-(\theta+1)\cdot\frac{N_L}{2\cdot e^{a_{eff}\cdot L}}\right]$$



K. Kordas talk



4

$$\langle T(L)\rangle = L\left[\frac{1}{V_{ea}} - \rho a_{eff}\right] + \left[-\rho\ln 2 + C + \rho\ln(\theta+1) - \rho\psi(\theta+1)\right]$$

The slope in the above eq. is the inverse of the effective avalanche drift velocity. Due to the fact that both ρ and a_{eff} model-parameters should be positive, the model predicts that the avalanche, as a whole, drifts with higher velocity than the velocity V_{ea} of its constituent electrons. It also describes well the GARFIELD++ results.

Develop a statistical description of the transmission times up to the mesh:

Express the mean values of the transmission times as functions of the number of pre-amplification electrons

a) Avalanche Time

1

$$\langle T(L,N_L)\rangle = \frac{L}{V_{ea}} - \rho\cdot\ln(N_L) + C$$

2

$$\langle T(N)\rangle = \int_{x_1}^{x_2}\langle T(N,L)\rangle G(L\mid N)\,dL \qquad G(L|N) = \frac{P(N|L)R(L)}{P(N)}$$

3

$$R(L) = R(L;a) = a\cdot\frac{\exp[a\cdot L]}{\exp[a\cdot x_2]-\exp[a\cdot x_1]} \qquad G(L|N) = \frac{P\left(N;q=2e^{a_{eff}L},\theta\right)R(L;a)}{\int_{x_1}^{x_2}P\left(N;q=2e^{a_{eff}L},\theta\right)R(L;a)\,dL}$$

4

$$\langle T(N)\rangle = \int_{x_1}^{x_2}\left[\frac{L}{V_{ea}} - \rho\ln N + C\right]G(L\mid N)\,dL = \frac{\langle L(N)\rangle}{V_{ea}} - \rho\ln N + C$$

$$\langle L(N)\rangle = \int_{x_1}^{x_2} L\cdot G(L\mid N)\,dL$$

b) Photoelectron Time

1

$$T_p(L) = \frac{D - L}{V_p} + d_{off}$$

2

$$\langle T_p(N) \rangle = \int_{x_1}^{x_2} T_p(L) G(L \mid N) dL = \frac{D - \langle L(N) \rangle}{V_p} + d_{off}$$

c) Total Time on the Mesh

$$\langle T_{tot}(N) \rangle = \langle T_p(N) \rangle + \langle T(N) \rangle = \langle L(N) \rangle \left[\frac{1}{V_{ea}} - \frac{1}{V_p} \right] - \rho \ln N + \left[\frac{D}{V_p} + C + d_{off} \right]$$

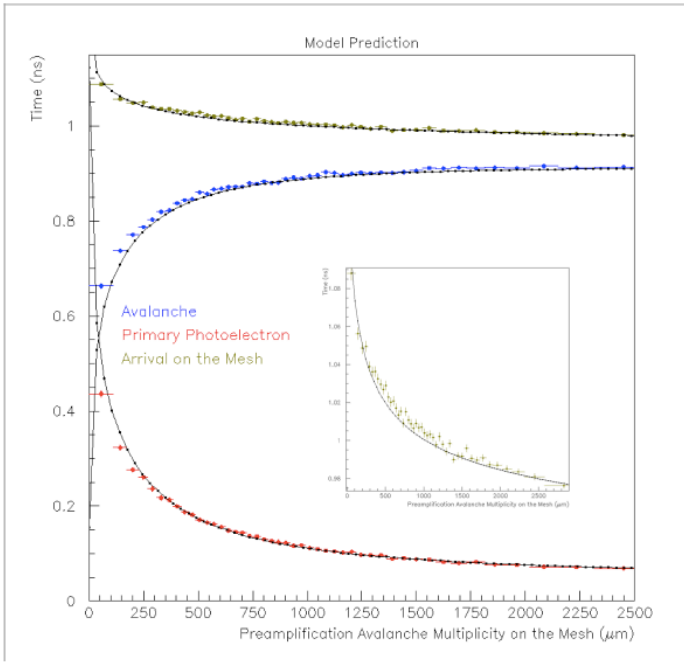


Figure 4: *The points represent results of GARFIELD++ simulations, corresponding to mean transmission times versus the number of avalanche electrons on the mesh, for 50% Penning Transfer Rate and 425 V and 450 V drift and anode voltages respectively:*

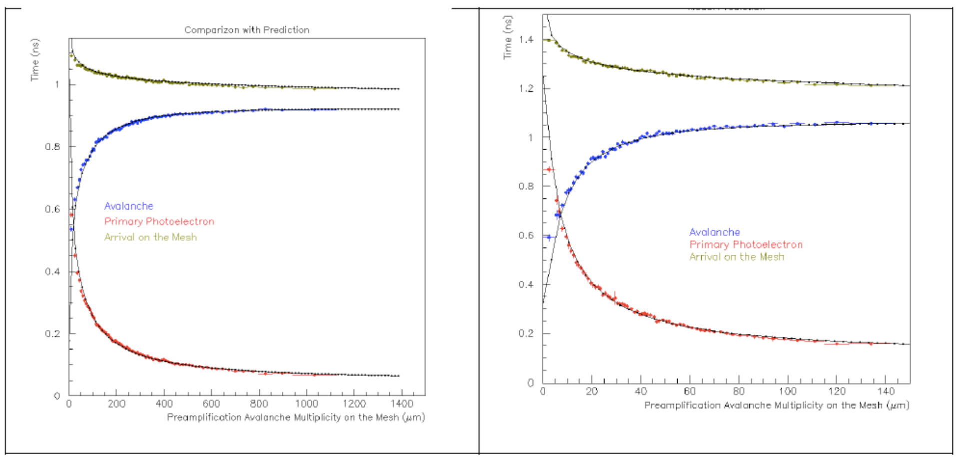


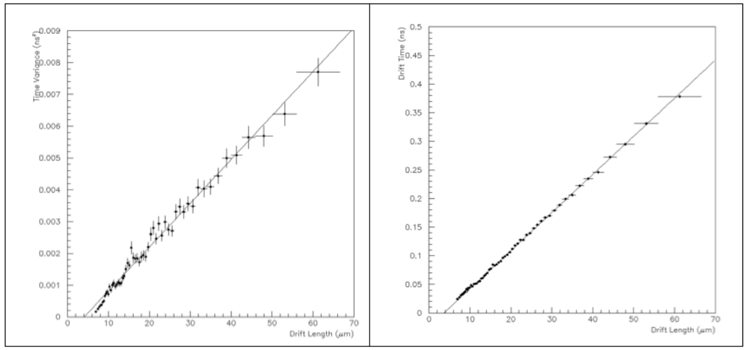
Figure 5: *As in Fig. 4: (left plot) for 0% Penning Transfer Rate and 425 V and 450 V drift and anode voltages respectively; (right plot) for 50% Penning Transfer Rate and 325 V and 450 V drift and anode voltages respectively*

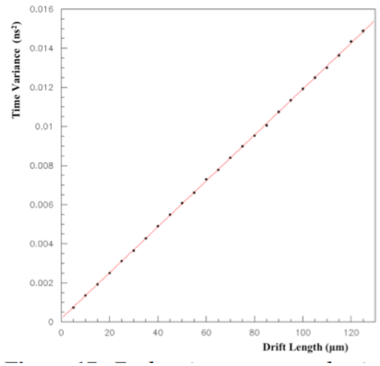
Develop a statistical description of the transmission times up to the mesh:

Study the variance of the transmission times and derive formulae which express quantitatively their behavior as function of the avalanche length

$$V\left[T_p(L)\right] = (D-L)\cdot\sigma_p^2 + \Phi$$

$$V\left[T_{ea}(L)\right] = \sigma_0^2 \cdot L + \varphi \quad (22)$$

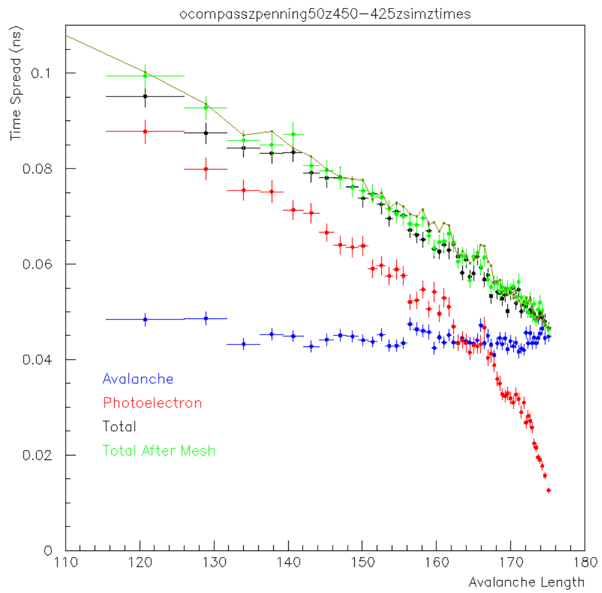


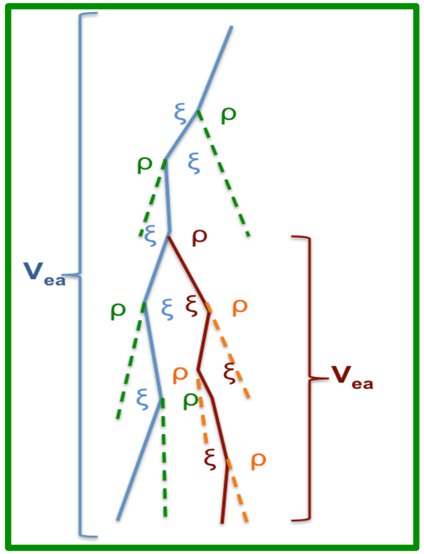


$$T_{tot}(L) = T_P(L) + T(L)$$

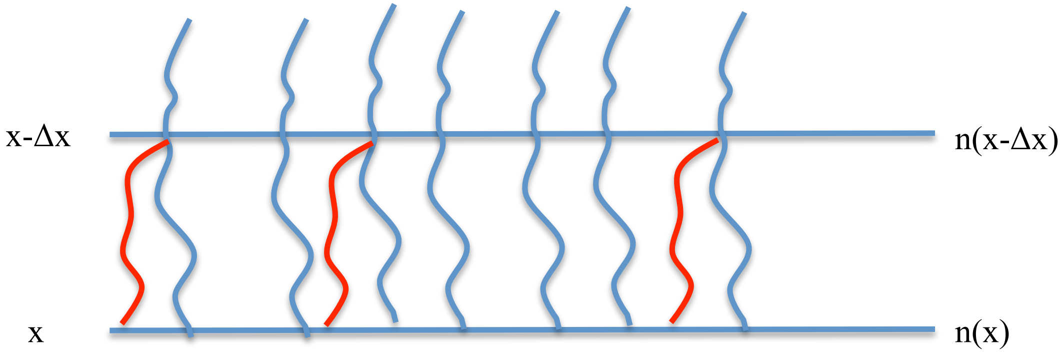
$$V[T_{tot}(L)] = V[T_P(L)] + V[T(L)]$$

The avalanche time and the photoelectron time are mutually uncorrelated





We should express the variance of the avalanche time, V[T(L)], as function of the avalanche length, taking into account the inter-correlation between the avalanche electrons



$$T(x,n(x)) = \frac{1}{n(x)}\left[\underbrace{\sum_{k=1}^{n(x-\Delta x)} t_k(x-\Delta x)}_{A} + \underbrace{\sum_{j=1}^{\Delta n} t_j^f(x-\Delta x)}_{B} + \underbrace{\sum_{k=1}^{n(x-\Delta x)} \Delta t_k}_{C} + \underbrace{\sum_{j=1}^{\Delta n} \Delta t_j^f}_{D} - \underbrace{\sum_{j=1}^{\Delta n} \rho_j}_{E}\right]$$

a) any of the n(x-Δx) pre-existing electrons has the same probability, Δn/n(x-Δx), to produce a new electron; b) only the times t_k(x-Δx) up to the plane on x- are mutually correlated

$$B = \sum_{j=1}^{\Delta n} t_j^f(x-\Delta x) \rightarrow \frac{\Delta n}{n(x-\Delta x)} \sum_{k=1}^{n(x-\Delta x)} t_k(x-\Delta x)$$

$$D = \sum_{j=1}^{\Delta n} \Delta t_j^f \rightarrow \frac{\Delta n}{n(x-\Delta x)} \sum_{k=1}^{n(x-\Delta x)} \Delta t_k$$

$$\begin{aligned} \mathrm{cov}[A,C] = \mathrm{cov}[A,D] &= \mathrm{cov}[A,E] \\ &= \mathrm{cov}[B,C] = \mathrm{cov}[B,D] \\ &= \mathrm{cov}[B,E] = \mathrm{cov}[C,E] \\ &= \mathrm{cov}[D,E] = 0 \end{aligned}$$

$$V[T(x,n(x))] = \frac{1}{n^2(x)}\left(V[A]+V[B]+V[C]+V[D]+V[E]+2\,\mathrm{cov}[A,B]+2\,\mathrm{cov}[C,D]\right)$$

c) evaluate each one of the r.h.s terms and after some algebra

$$V\left[T\left(x,n(x)\right)\right] = \frac{1}{n^2(x-\Delta x)}\left(\sum_{k=1}^{n(x-\Delta x)} \sigma_k^2(x-\Delta x) + \sum_{k=1}^{n(x-\Delta x)} \sum_{l=1,k\neq l}^{n(x-\Delta x)} c_{kl}\right) + \frac{1}{n^2(x-\Delta x)} \sum_{k=1}^{n(x-\Delta x)} \delta_k^2 + \frac{1}{n^2(x)} \sum_{j=1}^{\Delta n} d_j^2$$

C_{kl} stands for COV[t_k(x-Δx), t_l(x-Δx)]

d) Using that $\frac{1}{n^2(x-\Delta x)} \sum_{k=1}^{n(x-\Delta x)} \delta_k^2 = \frac{\sigma_0^2 \cdot \Delta x}{n(x-\Delta x)}$ **and** $\frac{1}{n^2(x)} \sum_{j=1}^{\Delta n} d_j^2 = \frac{\Delta n}{n^2(x)} w^2$

$$V\left[T\left(x,n(x)\right)\right] - V\left[T\left(x-\Delta x, n(x-\Delta x)\right)\right] = \frac{\sigma_0^2 \cdot \Delta x}{n(x-\Delta x)} - w^2\left(\frac{1}{n\ (x)} - \frac{1}{n(x-\Delta x)}\right)$$

The above eq. expresses the increase of the avalanche-time variance as the avalanche grows between two planes, on x-Δx and on x, given that n(x-Δx) electrons reach the first plane and Δn more electrons reach the second plane.

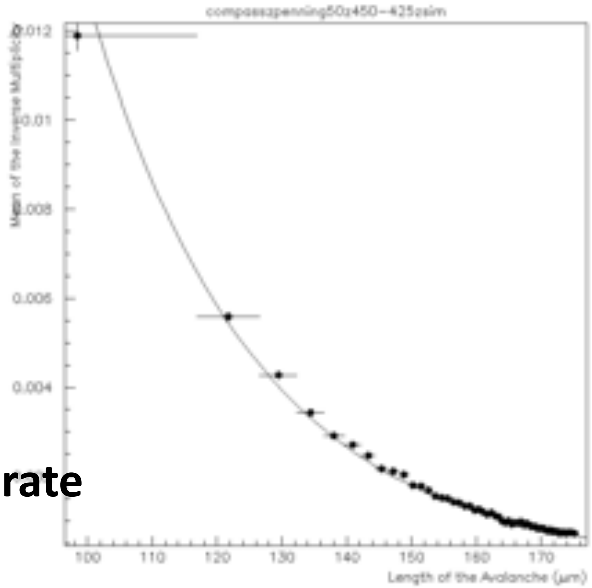
e) For any avalanche evolving up to a length x, the variance of the avalanche-time can be obtained by averaging eq. (38) for all possible values of n(x-Δx) and Δn.

$$\frac{\left\langle V\left[T\left(x,n(x)\right)\right] - V\left[T\left(x-\Delta x, n(x-\Delta x)\right)\right]\right\rangle}{\Delta x} = \sigma_0^2 \left\langle \frac{1}{n(x-\Delta x)} \right\rangle - \frac{w^2}{\Delta x}\left\langle \frac{1}{n\ (x)} - \frac{1}{n(x-\Delta x)} \right\rangle$$

$$\frac{\left\langle V\left[T\left(x,n(x)\right)\right]-V\left[T\left(x-\Delta x,n(x-\Delta x)\right)\right]\right\rangle}{\Delta x}=\sigma_0^2\left\langle\frac{1}{n(x-\Delta x)}\right\rangle-\frac{w^2}{\Delta x}\left\langle\frac{1}{n\ (x)}-\frac{1}{n(x-\Delta x)}\right\rangle$$

f) Using the Gamma distribution to describe the pdf of the avalanche multiplicity

$$\left\langle\frac{1}{n(x)}\right\rangle=\frac{(\theta+1)}{2\theta}\exp\left(-a_{eff}\cdot x\right)$$



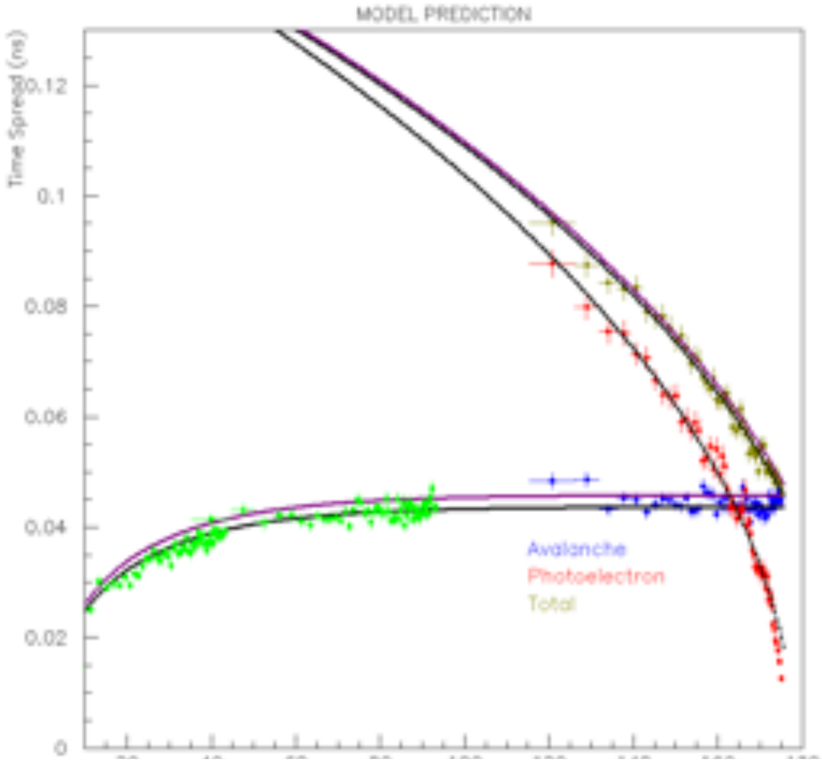
g) Let Δx to go to zero, keep terms proportional to Δx, and integrate

$$\frac{d\left\langle V\left[T(x)\right]\right\rangle}{dx}=\frac{(\theta+1)}{2\theta}\exp\left(-a_{eff}\cdot x\right)\cdot\left[\sigma_0^2+w^2a_{eff}\right]$$

$$V\left[T(L)\right]=\frac{(\theta+1)}{2\theta}\left[\sigma_0^2+w^2a_{eff}\right]\cdot\frac{1-\exp\left(-a_{eff}\cdot L\right)}{a_{eff}}$$

$$V\left[T_{tot}(L)\right]=V\left[T(L)\right]+V\left[T_p(L)\right]$$

$$=\frac{(\theta+1)}{2\theta}\left[\sigma_0^2+w^2a_{eff}\right]\frac{1-\exp\left(-a_{eff}\cdot L\right)}{a_{eff}}+(D-L)\cdot\sigma_p^2+\Phi$$



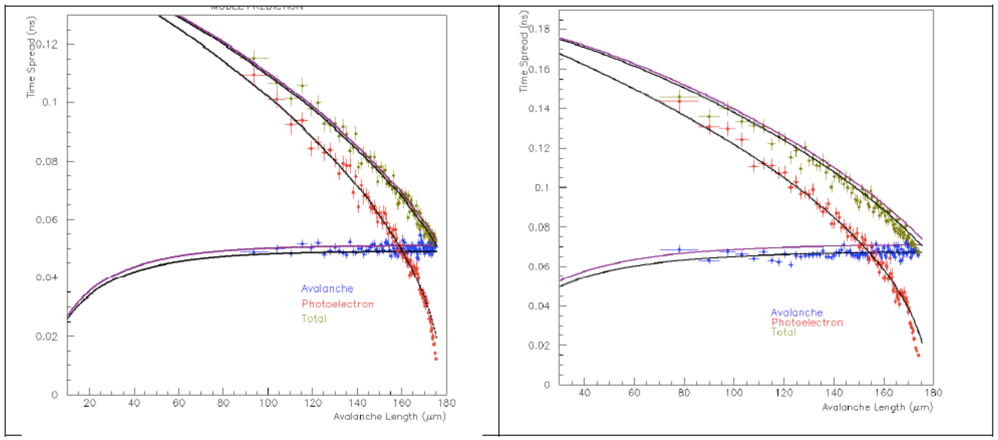


Figure 8: *The points denote results for GARFIELD++ simulations and the solid lines represent the respective predictions of the model, as in Fig 7: (left) with 0% Penning Transfer Rate, 425 V drift and 450 V anode voltages; (right) with 50% Penning Transfer Rate, 350 V drift and 450 V anode voltages.*

Develop a statistical description of the transmission times up to the mesh:

Express the variance of the transmission times as function of the avalanche multiplicity and study the correlation between photoelectron and avalanche

I) The variance of the avalanche time as a function of the avalanche electron multiplicity

a) Recall that, for given n(x-Δx) at x-Δx and n(x) at x, the variance increase is expressed as:

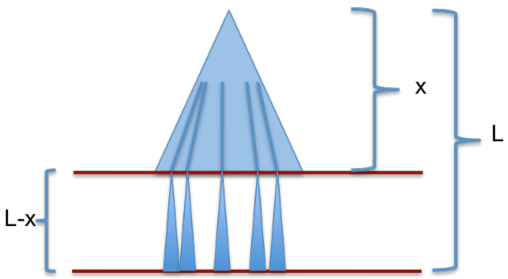
$$V\left[T\left(x,n(x)\right)\right]-V\left[T\left(x-\Delta x,n(x-\Delta x)\right)\right]=\frac{\sigma_0^2\cdot\Delta x}{n(x-\Delta x)}-w^2\left(\frac{1}{n\ (x)}-\frac{1}{n(x-\Delta x)}\right)$$

b) We start by averaging the above eq. over the electron multiplicity n(x), under the condition that at the end of the avalanche development, i.e. at a plane on L longitudinal distance from the start of the avalanche, the observed electron multiplicity, n(L), equals N_L.

$$\left\langle V(x)\right\rangle_{n(L)=N_L}=\int_0^\infty V\left[T\left(x,n(x)\right)\right]\cdot P\left(n(x)\middle|n(L)=N_L\right)dn(x)$$

$$P\left(n(x)\middle|n(L)=N_L\right)=\frac{P\left(n(L)=N_L\middle|n(x)\right)\cdot P\left(n(x)\right)}{P\left(n(L)=N_L\right)}$$

b-1) P(n(x)) is the pdf that an avalanche of length x has n(x) electrons (Gamma distribution); $P(n(L)=N_L|n(x))$ is the pdf that an avalanche has N_L electrons at a plane on L given that it has n(x) electrons at a plane on x. Assuming that each of the n(x) electrons will start an independent avalanche, which will be developed until it reaches the plane on L (i.e. n(x) avalanches, each of length L-x)



$$P\left(n(L)=N_L\middle|n(x)\right)=\underbrace{P_1(n)\otimes P_1(n)\otimes\cdots\otimes P_1(n)}_{n(x)\ times}$$

$$=\frac{1}{q(L-x)}\frac{(\theta+1)^{n(x)\cdot(\theta+1)}}{\Gamma\left(n(x)\cdot(\theta+1)\right)}\cdot\left(\frac{N_L}{q(L-x)}\right)^{n(x)\cdot(\theta+1)-1}\cdot\exp\left[-(\theta+1)\frac{N_L}{q(L-x)}\right]$$

Or

$$P\left(n(L)=N_L\middle|n(x)\right)=\frac{1}{\sqrt{2\pi\cdot n(x)\cdot\sigma^2(L-x)}}\exp\left[-\frac{\left(n(x)\cdot q(L-x)-N_L\right)^2}{2\cdot n(x)\cdot\sigma^2(L-x)}\right]$$

It should be emphasized that above eqs. are strictly valid only in case that n(x) is an integer parameter, but N_L is a continues variable. However, in order to simplify numerical calculations, n(x) is treated as a continues variable

b-2) by averaging $\langle V(x)\rangle_{n(L)=N_L} = \int_0^\infty V\left[T\left(x,n(x)\right)\right]\cdot P\left(n(x)\middle|n(L)=N_L\right)dn(x)$ **, we evaluate the variance increase**

$$\langle V(x)\rangle_{n(L)=N_L} - \langle V(x-\Delta x)\rangle_{n(L)=N_L} = \sigma_0^2\cdot\Delta x\left\langle\frac{1}{n(x-\Delta x)}\right\rangle_{n(L)=N_L} - w^2\left(\left\langle\frac{1}{n(x)}\right\rangle_{n(L)=N_L} - \left\langle\frac{1}{n(x-\Delta x)}\right\rangle_{n(L)=N_L}\right)$$

c) Using the above eq. recursively, letting Δx to go to zero and integrating

$$\langle V(L)\rangle_{n(L)=N_L} = \sigma_0^2\cdot\int_0^L\left\langle\frac{1}{n(x)}\right\rangle_{n(L)=N_L}dx - w^2\left(\frac{1}{N_L}-\frac{1}{2}\right)$$

where

$$\sigma_0^2\cdot\int_0^L\left\langle\frac{1}{n(x)}\right\rangle_{n(L)=N_L}dx = \sigma_0^2\cdot\int_0^L\left[\int_0^\infty\frac{1}{n(x)}\cdot\frac{P\left(n(L)=N_L\middle|n(x)\right)\cdot P\left(n(x)\right)}{P\left(n(L)=N_L\right)}dn(x)\right]dx$$

Evaluated numerically

This the variance of the avalanche-time, for avalanche length L, given than the avalanche electron multiplicity is $N_{L.}$

d) In order to express the time variance as a function of the pre-amplification electron multiplicity N, for any possible avalanche length, the above eq. should be integrated properly, taking into account the contributions of all avalanche lengths, each one with its own weight

Source of Correlations. *Let us consider a sample of avalanches with N electrons on the mesh. Schematically, this sample comprises many (infinite) sets, each with a certain length L, with a population proportional to G(L|N), with an average avalanche time <T(N,L)> and respective variance <V(L)>$_{n(L)=N.}$ In the hypothetical case that the all the above subsets had the same mean avalanche time, the time variance of the whole sample will be given simply by the weighted sum of the respective variances of the subsets. However, due to the fact that each subset has a different mean value, the variance of the avalanche time, for any possible avalanche length, has a more complicated dependence on the pre-amplification electron multiplicity.*

$$\langle V(N)\rangle = \int_{x_1}^{x_2}\langle V(L)\rangle_{n(L)=N}\cdot G(L|N)\,dL + \int_{x_1}^{x_2}\langle T(N,L)\rangle^2\cdot G(L|N)\,dL - \left[\int_{x_1}^{x_2}\langle T(N,L)\rangle\cdot G(L|N)\,dL\right]^2$$

II) The variance of the photoelectron-time as a function of the avalanche electron multiplicity

a) Physically, the variance of the photoelectron time, $V_p(L)$, depends on its drift length, D-L, $V\left[T_p(L)\right]=(D-L)\cdot\sigma_p^2+\Phi$

b) Since the photoelectron drift length is the residual of the respective avalanche length, which determines the mean multiplicity of the avalanche electrons, the variance of the photoelectron time is indirectly connected to the number of the pre-amplification electrons N.

c) Following the same reasoning as in evaluating the variance of the avalanche-time above, the variance of the photoelectron time is expressed as a function of N as:

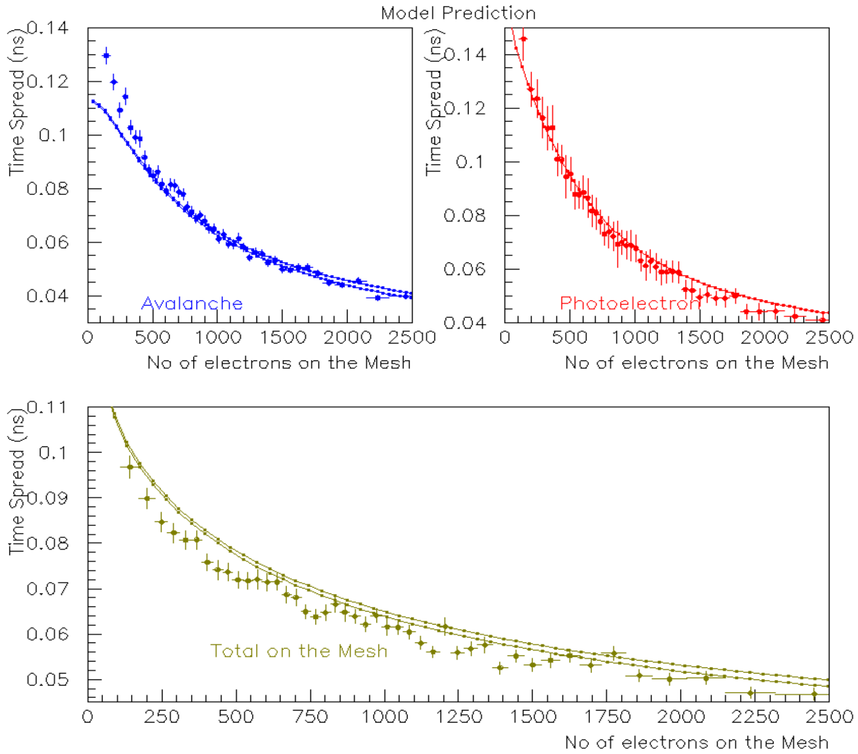
$$\left\langle V_P(N)\right\rangle=\int_{x_1}^{x_2}V_p(L)\cdot G(L|N)\,dL+\int_{x_1}^{x_2}T_p^2(L)\cdot G(L|N)\,dL-\left[\int_{x_1}^{x_2}T_p(L)\cdot G(L|N)\,dL\right]^2$$

III) The variance of the total-time on the mesh as a function of the electron multiplicity

$$\left\langle V_{tot}(N)\right\rangle=\int_{x_1}^{x_2}\left[V_p(L)+\left\langle V(L)\right\rangle_{n(L)=N}\right]\cdot G(L|N)\,dL$$
$$+\int_{x_1}^{x_2}\left[\left\langle T(N,L)\right\rangle+T_p(L)\right]^2\cdot G(L|N)\,dL-\left[\int_{x_1}^{x_2}\left[\left\langle T(N,L)\right\rangle+T_p(L)\right]\cdot G(L|N)\,dL\right]^2$$

The above eq. is not the sum of the the respective photoelectron and avalanche contributions, as it would be the case if the photoelectron and avalanche contributions to the total-time continue to be uncorrelated when expressed as functions of the number of pre-amplification electrons.

This correlation is apparent in the GARFIELD++ simulations (K. Kordas talk) and it is caused by the fact that the same number of pre-amplification electrons can be produced by avalanches of different length and that the mean avalanche transmission time depends on its length.



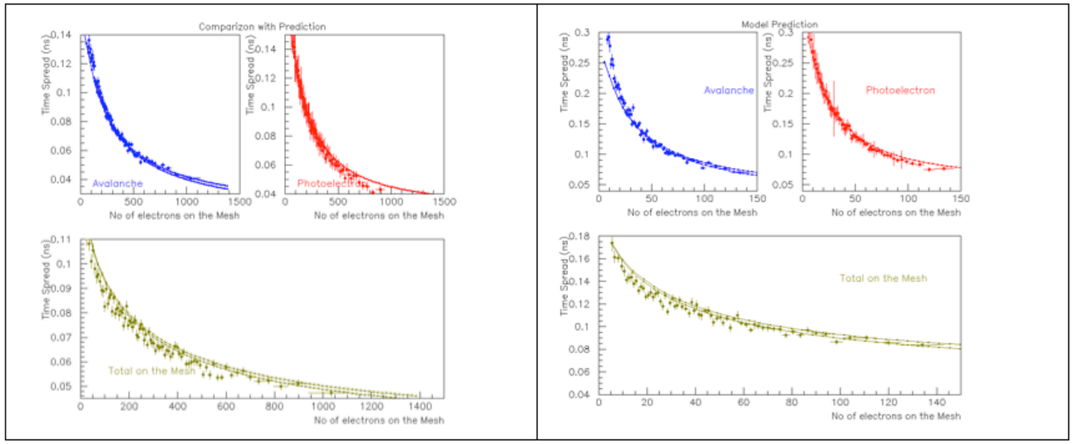


Figure 10: *As in Fig. 9 but with different operational parameters: (left) 0% Penning Transfer Rate, 425 V drift and 450 V anode voltage, (right) 50% Penning Transfer Rate, 325 V drift and 450 V anode voltage.*

Develop a statistical description of the electron transport through the mesh

Derive formulae that express the contribution of the "electron transport through the mesh" to the mean value and variance of the "total time after the mesh"

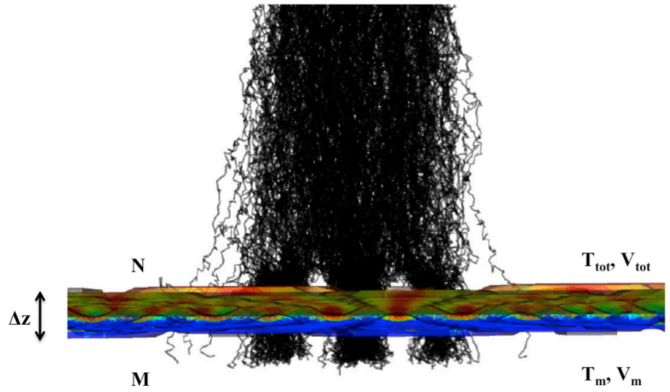


Figure 11: Pictorial representation of electron tracks passing through a micromegas detector's mesh (from "New Approach to 3D Electrostatic Calculations for Micro-pattern Detectors", P. Lazi et al, Journal of Instrumentation 6, no. 12 (December 1, 2011): P12003-P12003)

$$T_{tot}(L,N) = T(L,N) + T_p(L) = \frac{1}{N}\sum_{k=1}^{N} t_k + T_p(L)$$

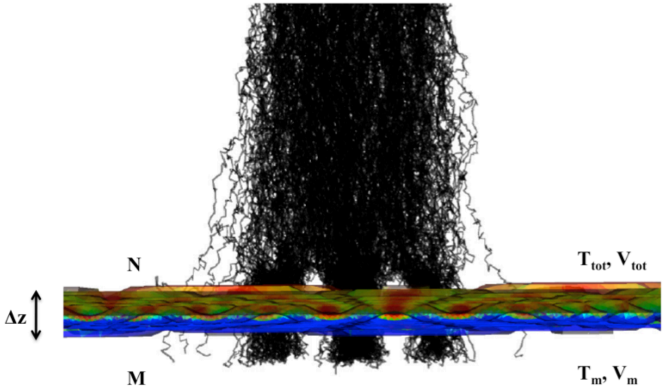
$$T_m(L,N) = \frac{1}{M}\sum_{k=1}^{M}(t_k + \Delta t_k) + T_p(L) = \frac{1}{M}\sum_{k=1}^{M} t_k + \frac{1}{M}\sum_{k=1}^{M}\Delta t_k + T_p(L)$$

a) Assuming that each of the N electrons arriving on the mesh has the same probability, M/N, to pass through the mesh and that <Δt> is the mean time needed by an electron to pass through the mesh

$$T_m(L,N) = \frac{1}{M}\frac{M}{N}\sum_{k=1}^{N} t_k + \frac{1}{M}\frac{M}{N}\sum_{k=1}^{N}\Delta t_k + T_p(L)$$
$$= \frac{1}{N}\sum_{k=1}^{N} t_k + \frac{1}{N}\sum_{k=1}^{N}\Delta t_k + T_p(L) = T_{tot}(L,N) + \langle \Delta t \rangle$$

the total arrival time after the mesh is the total arrival time on the mesh delayed by a constant time, which is independent of the avalanche characteristics, as observed in the detailed GARFIELD++ simulation

b) Recall that the variance of the total time on the mesh is written as:



$$V\left[\frac{1}{N}\sum_{k=1}^{N} t_k\right] = \frac{1}{N^2}\left(\sum_{k=1}^{N} V_k^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\left(\langle t_i t_j\rangle - \langle t_i\rangle\langle t_j\rangle\right)\right)$$

$$= \frac{1}{N^2}\left(N\sigma_0^2 \cdot L + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij}\right) = \frac{\sigma_0^2 \cdot L}{N} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij} \qquad \text{(b-1)}$$

Write the variance of the total time after the mesh as:

$$V\left[\frac{1}{M}\sum_{k=1}^{M} t_k\right] = \frac{\sigma_0^2 \cdot L}{M} + \frac{1}{M^2}\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} C_{ij} \qquad \text{(b-2)}$$

c) Notice that, each pre-amplification electron has the same probability, M/N, to pass through the mesh; ii) the covariance term, $\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} C_{ij}$ **, in eq. (b-2) has M(M-1) elements whilst the covariance term in eq. (b-1) has N(N-1) elements.**

Using (b-1) to "absorb" the correlation terms

$$V_m(L,N) = \frac{\sigma_0^2 \cdot L}{M} + \left(\frac{\sigma_0^2 \cdot L}{N} - \frac{\sigma_0^2 \cdot L}{N}\right) + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij} + \frac{\delta^2}{M} + V_p[L]$$

$$= \frac{\sigma_0^2 \cdot L}{M} - \frac{\sigma_0^2 \cdot L}{N} + \frac{\delta^2}{M} + \left\{\frac{\sigma_0^2 \cdot L}{N} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} C_{ij} + V_p[L]\right\}$$

$$= \sigma_0^2 \cdot L\left(\frac{1}{M} - \frac{1}{N}\right) + \frac{\delta^2}{M} + V_{tot}(L,N)$$

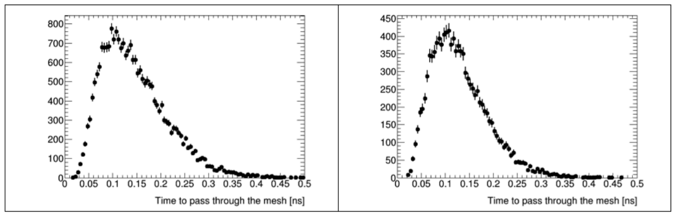


Figure G-2: The distribution of times needed by the avalanche's electrons to pass through the mesh, when the drift voltage is 350 V (left) and 425 V (right). Both plots correspond to an anode voltage of 450 V. The RMS values of the distribution shown at the left plot is 68.7 ps whilst at the right plot are 59.4 ps.

The variance of the total time after the mesh, which determines the PICOSEC timing resolution, for e-peaks corresponding to avalanche length L and avalanche multiplicity N, is given by:

$$V_m(L,N) = \frac{1}{N}\left[\sigma_0^2 \cdot L\left(\frac{1}{tr} - 1\right) + \frac{\delta^2}{tr}\right] + V_{tot}(L,N)$$

where tr=M/N is the "mesh transparency"

c) evaluate the variance of the total time after the mesh for events with avalanches of a certain length, L, by averaging properly over all possible N:

$$V\left[T_m(L)\right]=\left\langle V_m(L,N)\right\rangle_N=\frac{\theta+1}{2\theta}\exp\left[-a_{eff}L\right]\cdot\left[\sigma_0^2\cdot L\left(\frac{1}{tr}-1\right)+\frac{\delta^2}{tr}\right]+V\left[T_{tot}(L)\right]$$

d) evaluate the variance of the total time after the mesh for events with avalanches of a certain multiplicity, by averaging properly over all possible L:

$$V\left[T_m(N)\right]=\left\langle V_m(N)\right\rangle_L=\frac{1}{N}\left[\sigma_0^2\cdot\left\langle L(N)\right\rangle\left(\frac{1}{tr}-1\right)+\frac{\delta^2}{tr}\right]+\left\langle V_{tot}(N)\right\rangle$$

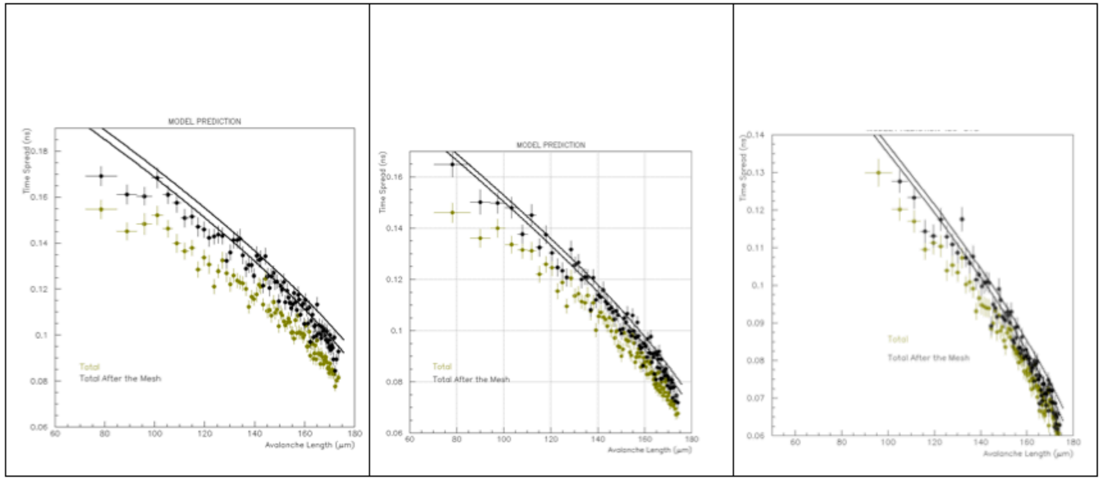


Figure 11: *The points represent GARFIELD++ simulation results for the spread of the total time on the mesh (golden points) and after the mesh (black points) versus the avalanche length. The solid lines represent the model predictions for the spread of the total time after the mesh. The double lines indicate a 100% uncertainty in the RMS of the time-gain variable. The voltage settings considered in these comparisons were 450 V at the anode and 325 V (left plot), 350 V (center plot) and 400 V (right plot) and the Penning Transfer Rate was 50%.*

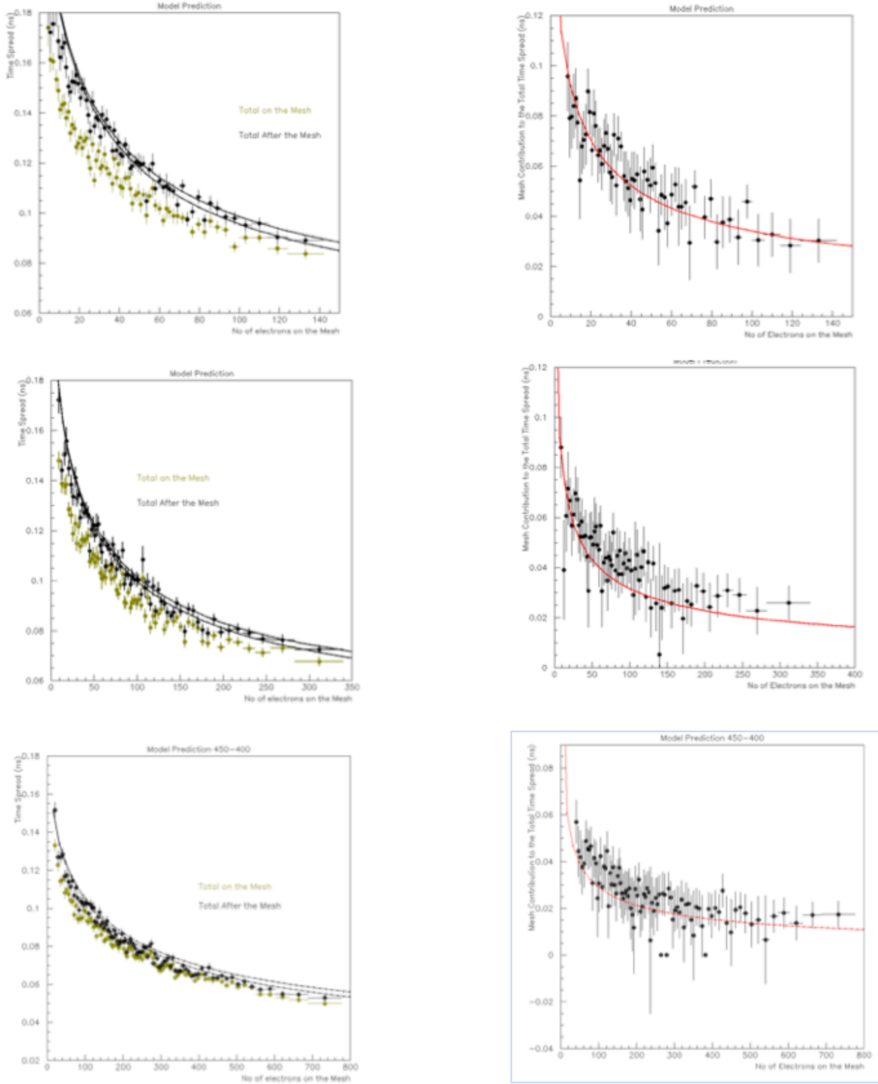


Figure 12: *The points represent GARFIELD++ simulation results. The left plots show the spread of the total time on the mesh (golden points) and after the mesh (black points) versus the number of pre-amplification electrons. The right plots show the mesh contribution (that is the square root of the difference between the variance of the total time after and on the mesh) versus the electron multiplicity. The solid lines represent the respective model predictions. The double lines in the left-row plots indicate a 100% uncertainty in the RMS of the time-gain variable. The voltage settings considered in these comparisons were 450 V at the anode and 325 V (top row), 350 V (middle row) and 400 V (bottom row) and the Penning Transfer Rate was 50%.*

$$V\left[T_m(L)\right] = \left\langle V_m(L,N)\right\rangle_N = \frac{\theta+1}{2\theta}\exp\left[-a_{eff}L\right]\cdot\left[\sigma_0^2\cdot L\left(\frac{1}{tr}-1\right)+\frac{\delta^2}{tr}\right]+V\left[T_{tot}(L)\right]$$

•Due to the correlation terms, the variance of the total-time after the mesh is not proportional to the variance of the total time on the mesh.

•The mesh adds to the total time variance a term almost proportional to Lexp[-a_{eff}L]. As the drift voltage increases, because the electron multiplication factor, a_{eff}, increases, the above term decreases for all values of L.

• Because the above term is an decreasing functions of L, in connection to the fact that the average avalanche length is an increasing function of the drift field, it is expected that the observed influence of the mesh to the resolution decreases at higher drift voltages.

Extend the Model to predict the p.d.f. followed by the above microscopic variables (instead of just mean values and variances) and compare with GARFIELD++ simulations

The model is complemented with the extra assumption that the transmission times (photoelectron time, avalanche time, total time on and after the mesh), which correspond to a certain avalanche length, follow* an Inverse Gaussian distribution (Wald) function.

$$f(x;\mu,\lambda)=\left(\frac{\lambda}{2\pi x^3}\right)^{1/2}\exp\left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right]$$

with the parameter μ to be the mean value and the shape parameter λ to be related with the variance of the distribution as $V[x]=\mu^3/\lambda$.

Then using the expressions, derived in this work, for the mean values and variances as functions of L and convoluting with the Wald distributions

$$f_p\left(T_p;\mu_p(L),\lambda_p(L)\right)=\left(\frac{\lambda_p(L)}{2\pi\cdot T_p^3}\right)^{1/2}\cdot\exp\left[\frac{-\lambda_p(L)\cdot\left(T_p-\mu_p(L)\right)^2}{2\mu_p^2(L)\cdot T_p}\right]$$

$$f\left(T;\mu(L),\lambda(L)\right)=\left(\frac{\lambda(L)}{2\pi\cdot T^3}\right)^{1/2}\cdot\exp\left[\frac{-\lambda(L)\cdot\left(T-\mu(L)\right)^2}{2\mu^2(L)\cdot T}\right]$$

$$f_{tot}\left(T_{tot};\mu_{tot}(L),\lambda_{tot}(L)\right)=\left(\frac{\lambda_{tot}(L)}{2\pi\cdot T_{tot}^3}\right)^{1/2}\cdot\exp\left[\frac{-\lambda_{tot}(L)\cdot\left(T_{tot}-\mu_{tot}(L)\right)^2}{2\mu_{tot}^2(L)\cdot T_{tot}}\right]$$

$$f_m\left(T_m;\mu_m(L),\lambda_m(L)\right)=\left(\frac{\lambda_m(L)}{2\pi\cdot T_m^3}\right)^{1/2}\cdot\exp\left[\frac{-\lambda_m(L)\cdot\left(T_m-\mu_m(L)\right)^2}{2\mu_m^2(L)\cdot T_m}\right]$$

In general, the convolution of two Wald distributions is not also a Wald distribution [10]. Consequently, even if the photoelectron and avalanche transmission times are described by Wald distributions, it is not necessarily true that the total-times are distributed according to the same function. However (see K. Kordas talk) it was found that the distributions of the total-times, on and after the mesh, are very well approximated by Wald functions.

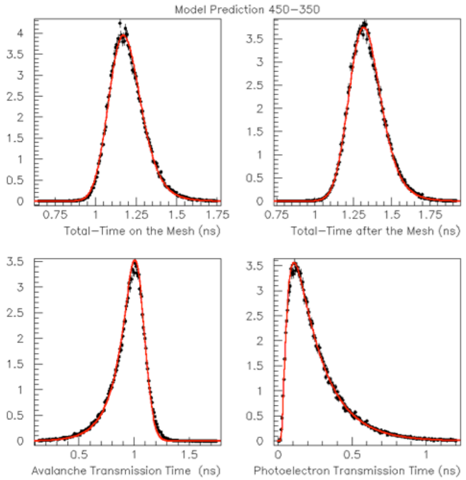


Figure 14: *Transmission time distributions for all events at 350 V and 450 V drift and anode voltage respectively and 50% Penning Transfer Rate: (top-left) Total time on the mesh, (top-right) total-time after the mesh, (bottom-left) avalanche transmission time and (bottom-right) photoelectron transmission time. The points are results of GARFIELD++ simulations whilst the red lines represent the respective model predictions, as it is described in the text.*

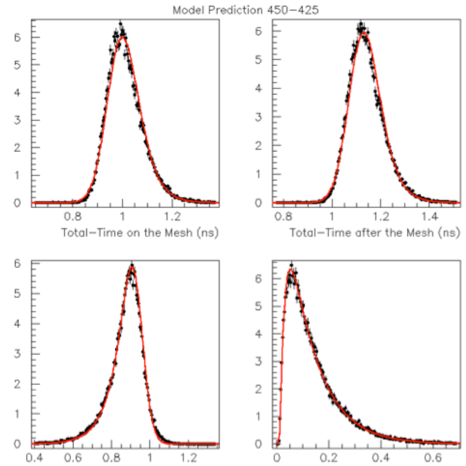
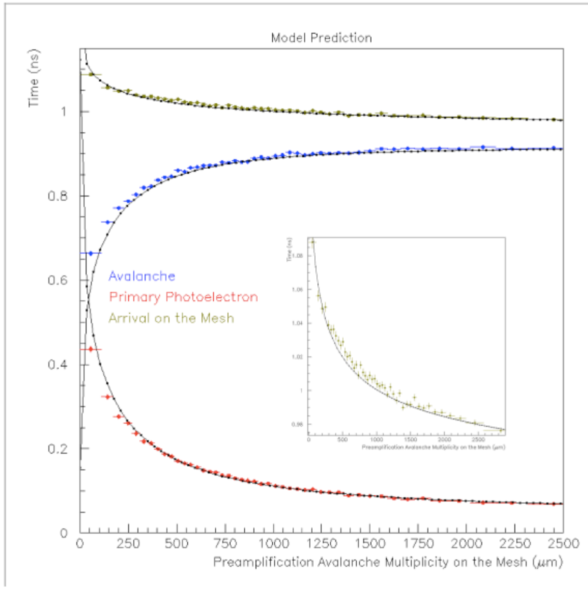
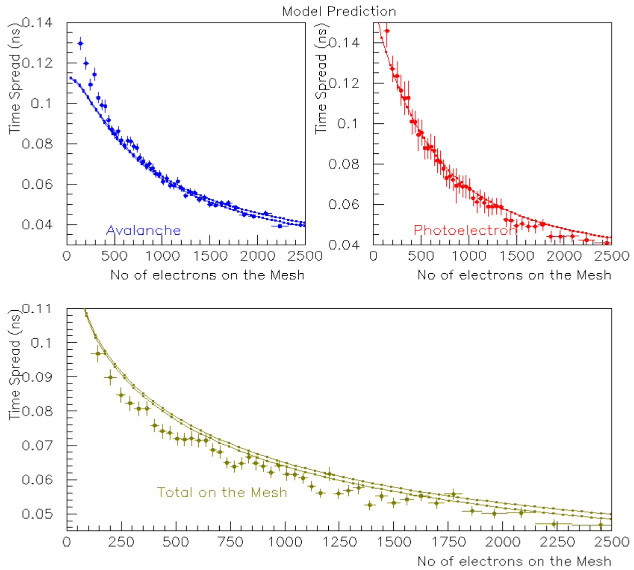


Figure 1: *Transmission time distributions for all events at 425 V and 450 V drift and anode voltage respectively and 50% Penning Transfer Rate: (top-left) Total time on the mesh, (top-right) total-time after the mesh, (bottom-left) avalanche transmission time and (bottom-right) photoelectron transmission time. The points are results of GARFIELD++ simulations whilst the red lines represent the respective model predictions, as it is described in the text.*

Concluding Remarks -I





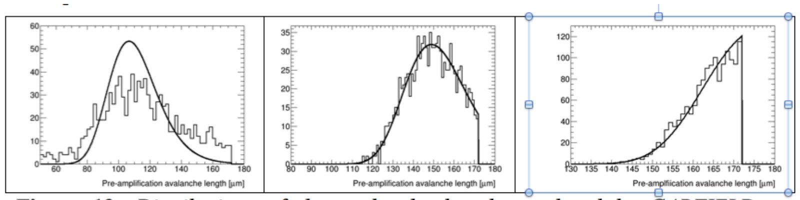


Figure 13: *Distributions of the avalanche length, produced by GARFIELD++ simulations (assuming: 50% Penning Transfer Rate, 425V drift and 450 V anode voltages) in case that the multiplicity of pre-amplification electrons is less than 120 (left plot), between 400 and 440 (center plot) and 1230 and 1300 (right plot). In the above distributions, only simulated events that the avalanche started further than 10 μm from the photocathode (D-L>10 μm) have been used. The solid lines represent the related prediction of the distribution function G(L|N) defined with eq. (16).*

•The deviations at low electron multiplicities result from the inadequacy of the employed p.d.f.' s to approximate accurately the avalanche statistical properties at its very beginning (small avalanche length, low electron multiplicity).

• The model predictions of both the mean value and the variance of the avalanche time utilize the function G(L|N). Recall that this conditional pdf describes the distribution of the length of an avalanche given that the avalanche electron multiplicity is N.

•Apparently, the model prediction for G(L|N) approximates poorly the GARFIELD++ distributions for N ~80 but successfully describes the detailed-simulation results for higher values of electron multiplicity. Therefore, the model predictions, for low avalanche electron multiplicities, are suffering from the poor success of G(L|N) to describe the GARFIELD++ results at low electron multiplicities.

However, PICOSEC data are collected with non-zero experimental, amplitude thresholds corresponding to e-peak charge greater than 3-4 pC, which translates (for 425V drift and 450 V anode voltages, and 50% Penning Transfer Rate) to 400-500 pre-amplification electrons on the mesh. At this experimentally observed region of pre-amplification electron multiplicities, the model predictions are in an excellent agreement with the results of GARFIELD++ simulations.

Concluding Remarks -II

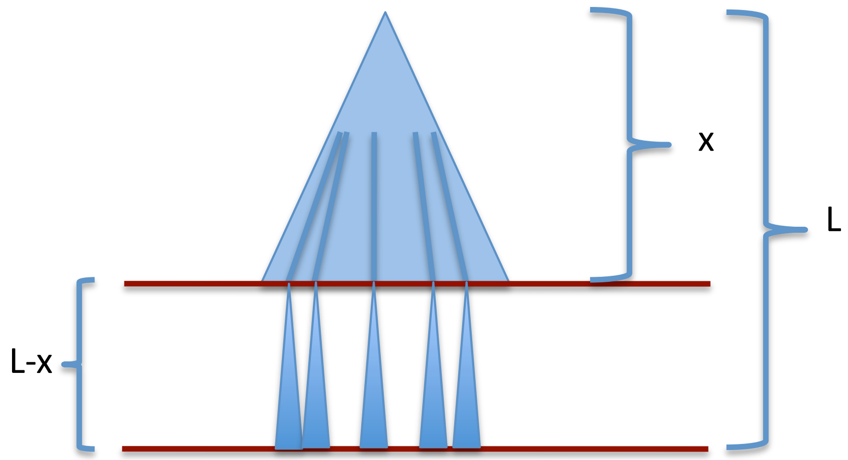
•The developed model is very successful in providing insights for the major microscopic mechanisms, which determine the timing characteristics of the detector, and in explaining coherently the unexpected behavior of microscopic quantities, predicted by GARFIELD++ simulations.

•Due to the very good agreement of the model predictions with the detailed GARFIELD++ simulation results, the formulae developed in this work can be used easily as a tool for fast predictions, provided that the values of the model input-parameters are known for the considered operational conditions.

•This necessity, obviously limits the application of the developed model as a stand-alone tool. However, having available sets of input parameter values for certain operational settings, it is possible to derive an empirical parameterization of the input parameters (see K. Kordas talk), which can be used to provide input to the model for the whole region of operational settings covered by the above parameterization.

•K. Paraschou has prepared a user friendly tool, in MATHEMATICA, which can be used get quantitative results with the model

On going work: Use the model to study timing effects due to lowe anode fields (I. Maniatis).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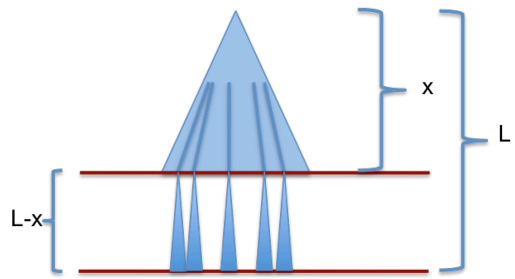
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