State of the art closed orbit feedback system

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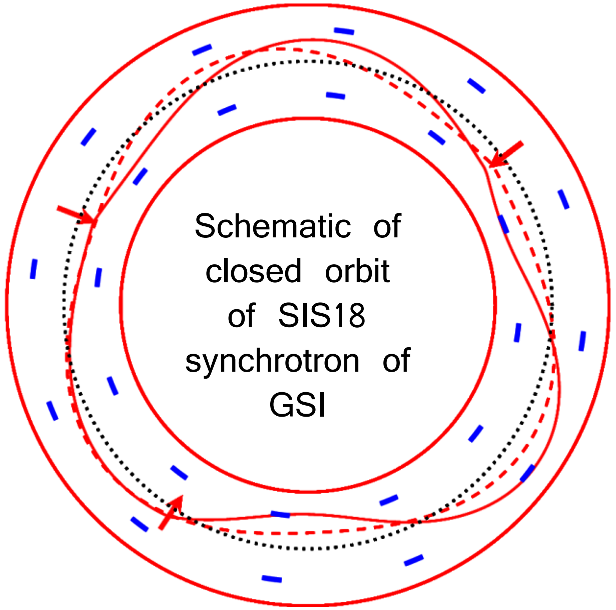
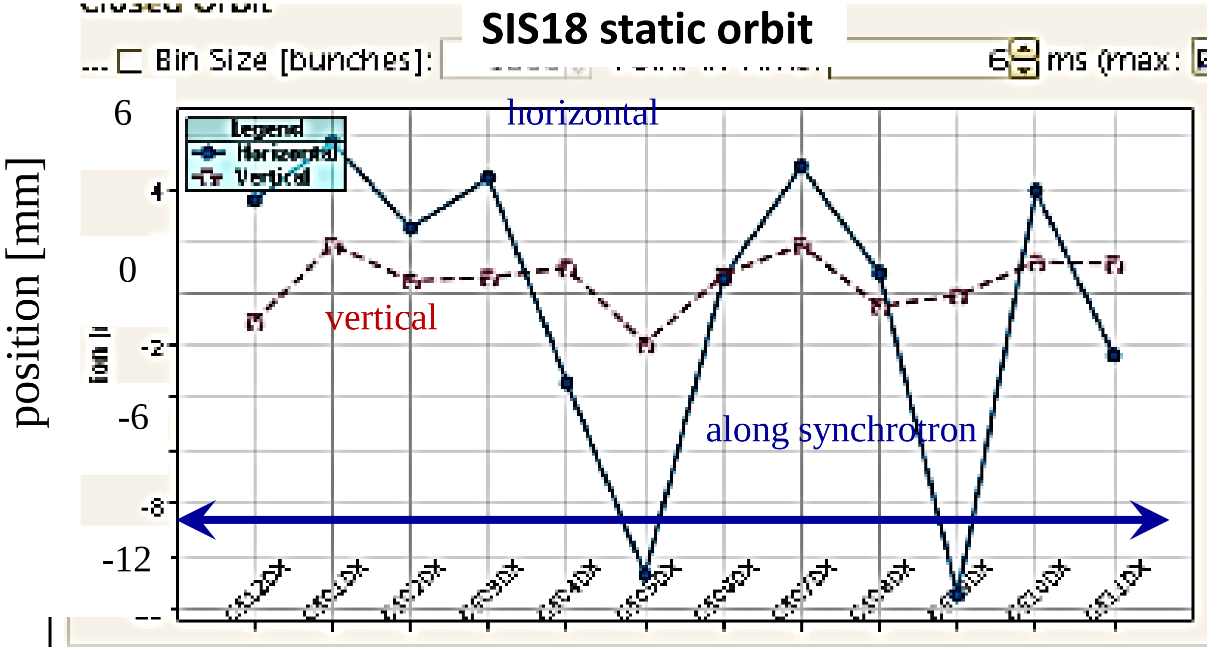
Outline

- Perturbed closed orbit : (examples)
- Brief comparison for Hadron Machines and Light Sources : motivation
- Components and key design players : orbit correction methods
- Symmetry exploitation
- Controller types
- Spatial model mismatch
- Closed orbit feedback system at SIS18 : results of first test
- Summary

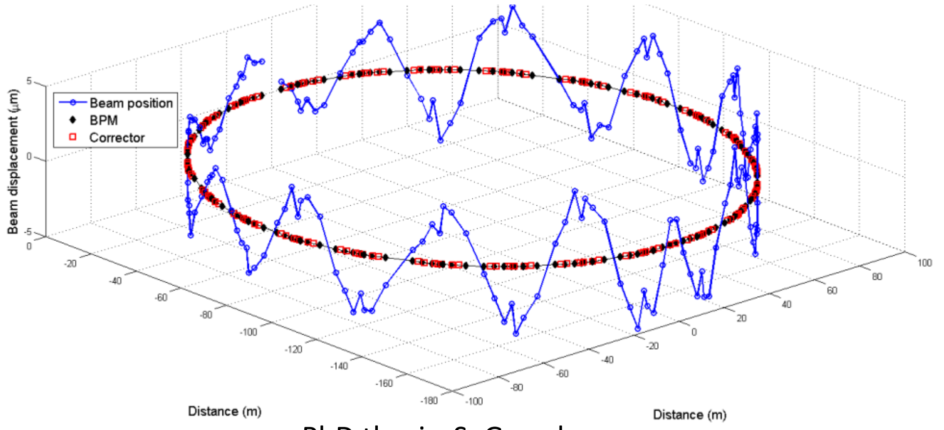
Closed orbit perturbation (examples)



SIS18 static orbit

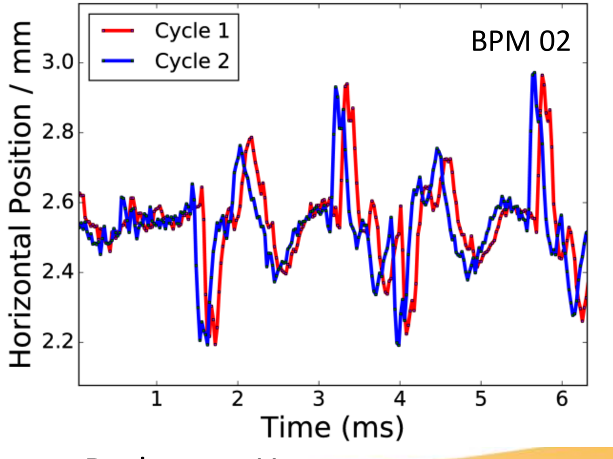


Diamond light source static orbit



PhD thesis, S. Gayadeen

SIS18 orbit movement



Closed orbit correction is the integral part of almost every synchrotron operation.

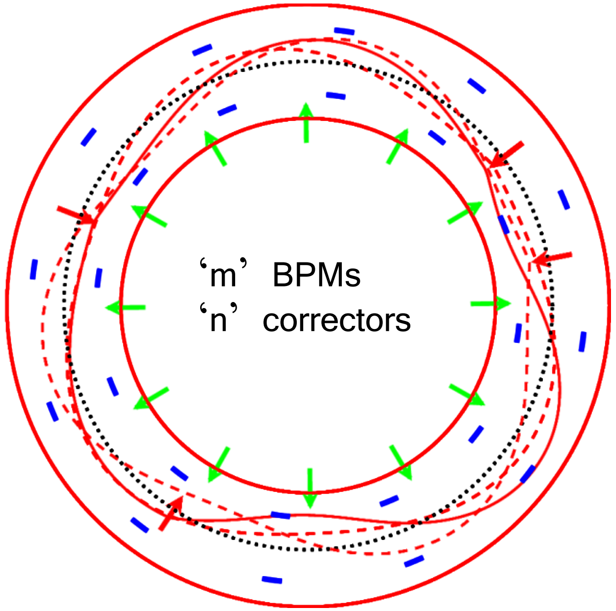
ARIES workshop on Next Generation Beam Position Acquisition and Feedback Systems

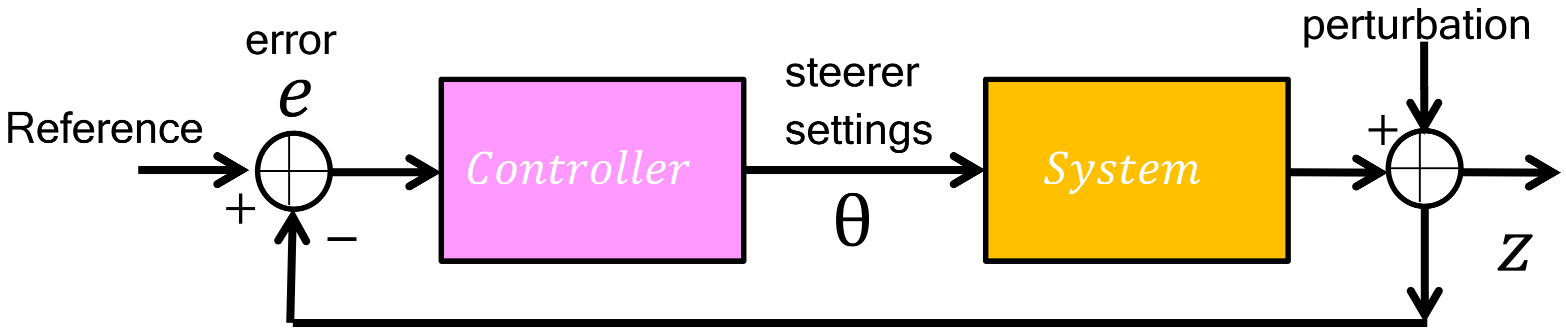
- A unique platform for the interaction of COFB community from light sources and hadron synchrotrons.
- Demands, achievements and challenges were discussed and compared for both kind of synchrotrons.
- A possibility of transfer of knowledge from light sources to hadron machines for fast orbit feedback systems.

Comparison for Hadron Machines and Light Sources : motivation

Parameter	Light sources	SIS18 (example of Hadron machines)
Stability criteria (vertical plane)	Less than 1 µm (10% of beam size ~ 10 µm)	Less than 1 mm
Bandwidth	~ 250 Hz	Up to 900 Hz
On-ramp orbit correction	Not needed	Required (also at LHC)
Sources	Mechanical vibrations (*water cooling pumps*) /power supply ripples	Power supply ripples / Cycle to cycle hysteresis
Reaction time	Fractions of seconds	< 1 ms
Lattice	Fixed lattice settings	Lattice settings changes
Flexibility of operations	Electron beams, Fixed energies Almost fixed intensities	Protons to heavy ions, Variable beam intensities, Variable beam energies
BPM failure/malfucntion	Less probability(?)	More probability due to high radiation
Beta beating	Lattice model more understood	Variable optics

Simple closed orbit feedback system





Sensors : Beam Position Monitor (BPM)

Actuators: Dipole magnets (other than main bending magnets) along with their power supplies

System : The closed orbit to be controlled + BPMs+ steerers+ P.S.+ vac. chamber

Controller : The control logic and the hardware for controller action

$$z_c(s) = \sum_{i=1}^{N} \theta_i \frac{\sqrt{\beta(s_i)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{si}| - \pi Q_y)$$

$$[\mathbf{Z}]_{m\times 1} = [\mathbf{R}]_{m\times n}[\Theta]_{n\times 1}$$

R is called the orbit response matrix

$$g(s) = e^{-e\tau} \frac{s}{s + \omega_c}$$

Latency **bandwidth**

Controller

$$C = K_p + \frac{K_i}{s}$$

Steerer settings calculation (orbit correction methods)

$$\Theta = \mathbf{R}^{-1}\mathrm{Z}$$

Spatial model, **R**

Latency, τ

Orbit correction methods

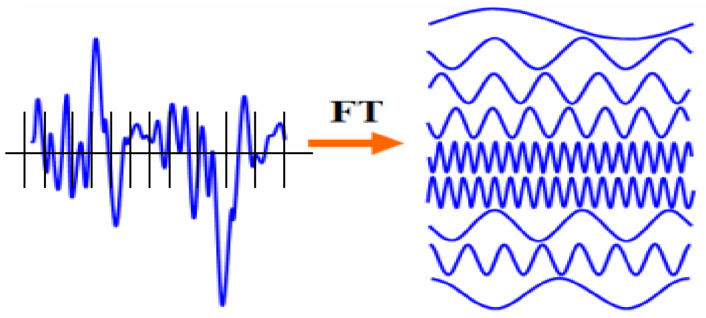
System bandwith, ω_c

Controller gains

Controller hardware, e.g. FPGA

Orbit correction methods : Harmonic analysis

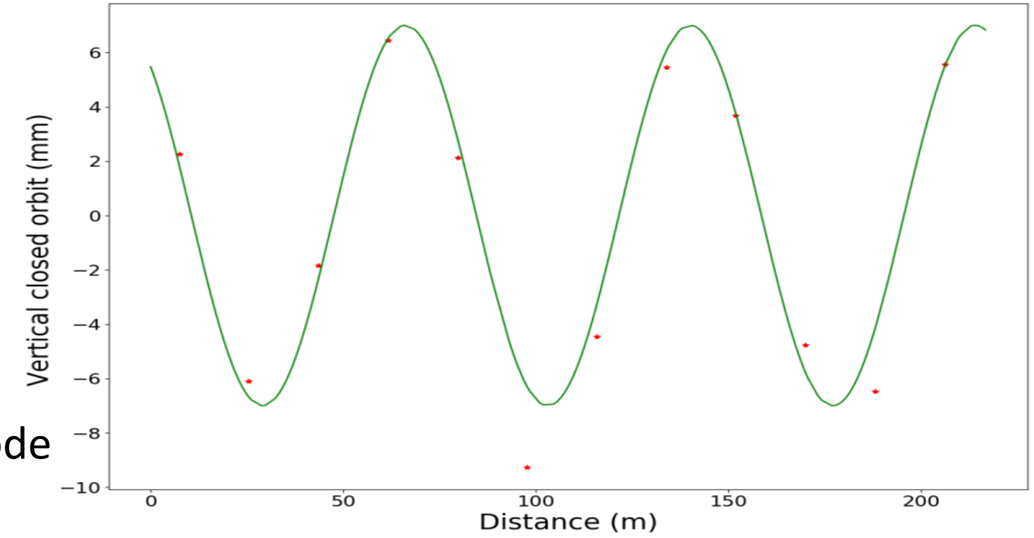
$$y_c(s) = \theta \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin(\pi Q_y)} \cos(|\mu(s) - \mu_{s0}| - \pi Q_y)$$



Perturbed orbit can be Fourier expanded

Modes to be removed (corrected) are selected before-hand and measured orbit is fitted over corresponding mode e.g. modes around tune frequency.

$$y_i = \sum_{k=1}^{n} (a_k \cos k\varphi + b_k \sin k\varphi)$$



Corrector strengths are proportional to the Fourier coefficients

Mode switching is possible because of separate channels for each mode

Fitting for each mode is mathematically complicated procedure

Reference: L.H.Yu et al."Real time harmonic closed orbit correction", Nucl. Instr. Meth. A, vol. 284, pp. 268–285, 1989

Orbit correction methods : Singular value decomposition (SVD)

$\Theta = \mathbf{R}^{-1} \mathrm{Z}$

$\mathbf{R} = \mathbf{USV^T}$

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}^T$$

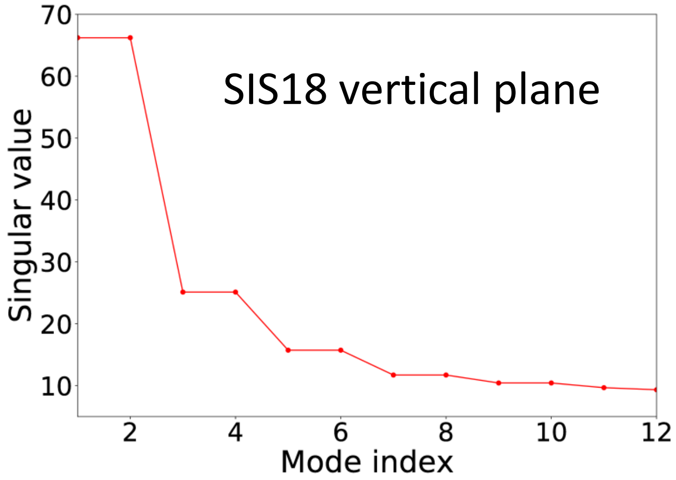
s_i are called singular values arranged as

$$s_1 > s_2 > s_3 \ldots . s_n$$

U and V are orthogonal matrices such that

$$U^{-1} = U^T \text{ and } V^{-1} = V^T$$

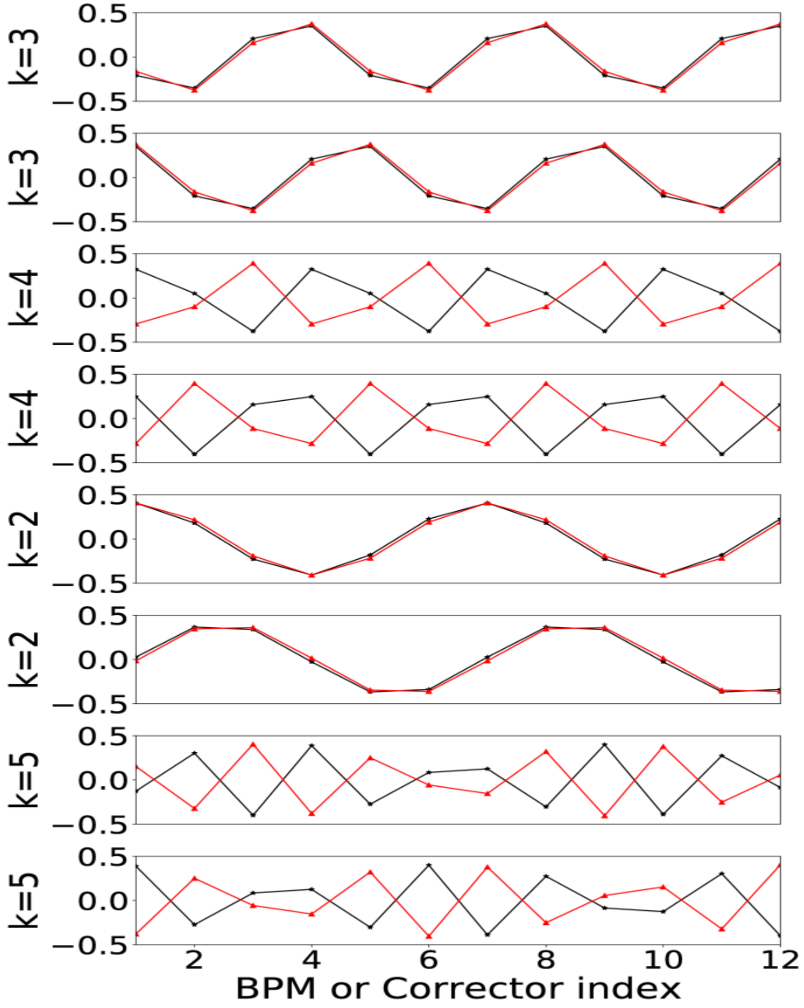
where the columns of U and V are the eigenvectors of $\mathbf{RR^T}$ and $\mathbf{R^TR}$



Which helps to find inverse R^{-1} (if R is invertible) as

Pseudo-inverse

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix}^{-1} = \begin{bmatrix} V_{11} & \cdots & V_{1m} \\ \vdots & \ddots & \vdots \\ V_{m1} & \cdots & V_{mm} \end{bmatrix} \begin{bmatrix} 1/s_1 & \cdots & 0 \\ \vdots & 1/s_2 & \vdots \\ 0 & \cdots & 1/s_n \end{bmatrix} \begin{bmatrix} U_{11} & \cdots & U_{1n} \\ \vdots & \ddots & \vdots \\ U_{n1} & \cdots & U_{nn} \end{bmatrix}^T$$



Columns of U: Black (BMP space)

Columns of V: Red (Steerer space)

Reference: William H. Press, Numerical recipes; The art of scientific computing (2007) Cambridge university press

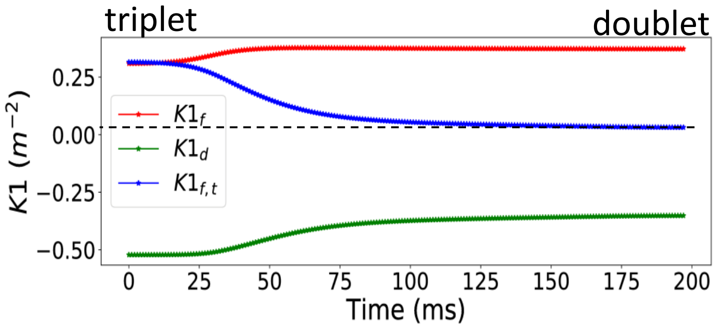
Let us come back to real world!

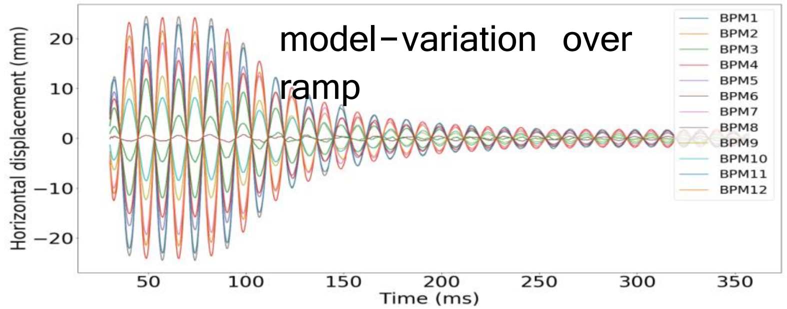
$$\mathbf{R}_{mn} = \frac{\sqrt{\beta_m \beta_n}}{2\sin(\pi Q_y)} \cos(|\mu_m - \mu_n| - \pi Q_y)$$

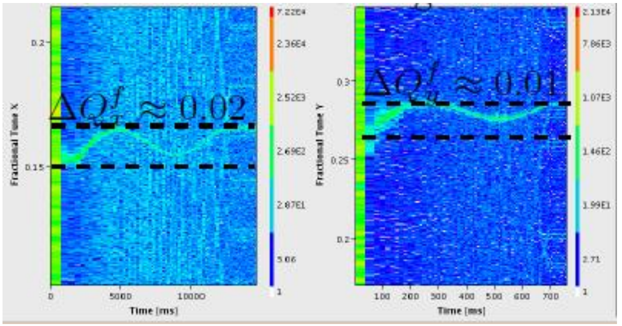
beta function

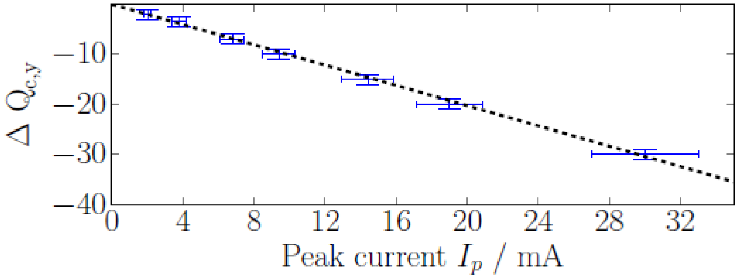
tune

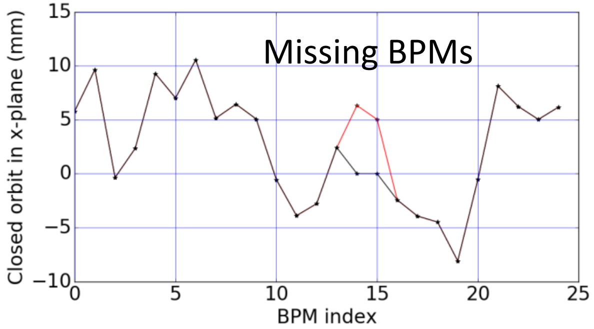
phase advance



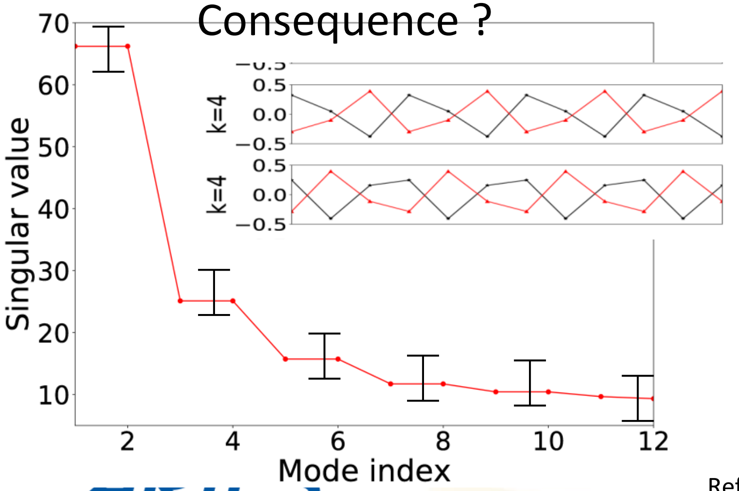




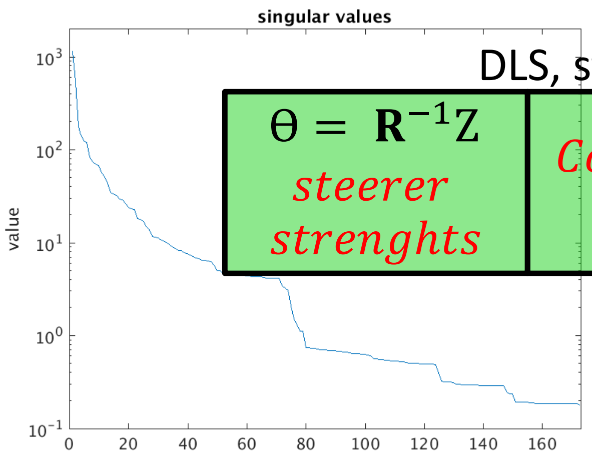




Consequence ?

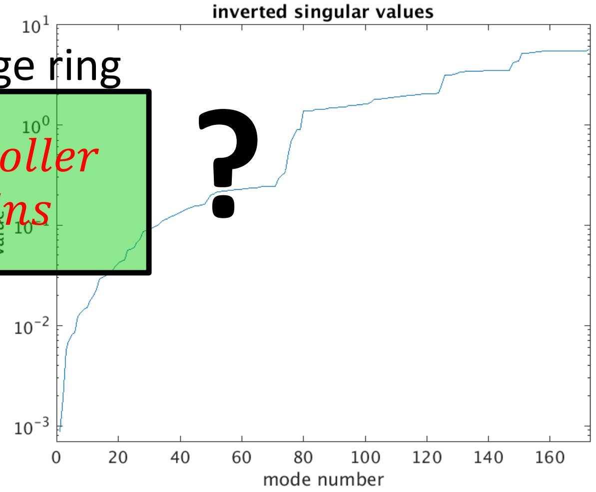


DLS, storage ring



$\theta = \mathbf{R}^{-1}Z$ steerer strenghts	Controller gains

?



Reference: R. Singh, PhD Thesis, TU Darmstadt 2014.

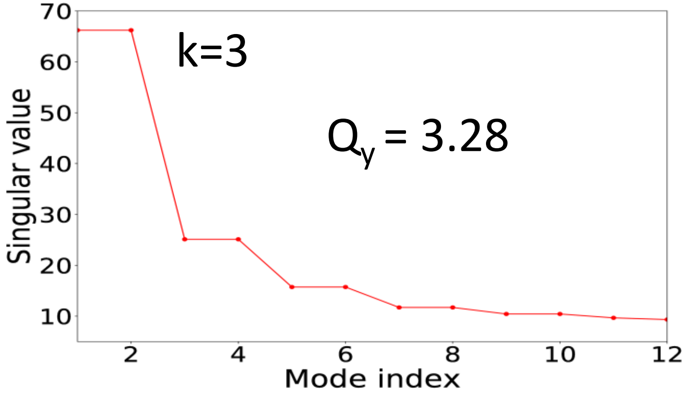
SVD in case of spatial model uncertainties

- **U** and **V** are interconnected through a phase relation
- Uncertainty modeling is required in all three matrices

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}^T$$

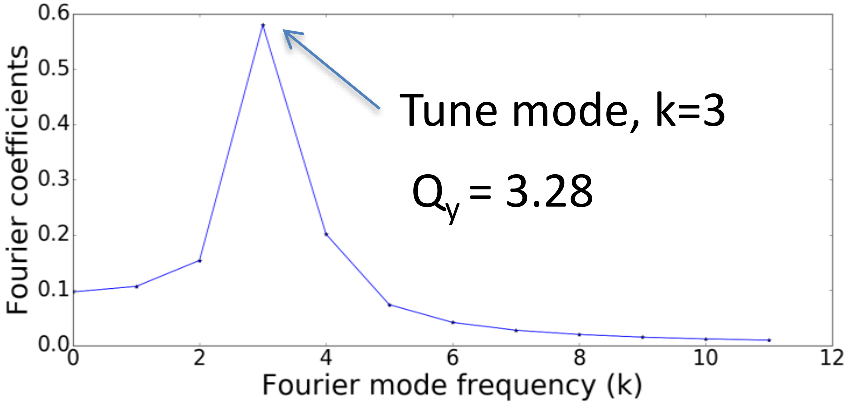
$$(\mathbf{I} + \Delta_\mathbf{R})\mathbf{R} = (\mathbf{I} + \Delta_\mathbf{R})\mathbf{U}\,(\mathbf{I} + \Delta_\mathbf{R})\mathbf{S}\,(\mathbf{I} + \Delta_\mathbf{R})\mathbf{V}$$

- Over the ramp, updating of all three matrices required
- Loss of physical interpretation of modes (SVD is a numerical technique)



Fourier coefficients of harmonic analysis have been proposed for uncertainty modeling

$$\sigma_f = \frac{1}{2\pi}\frac{2Q}{Q^2 - f^2}$$



Reference: S. Gayadeen, “Synchrotron Electron Beam Control”, Ph.D. thesis, St. Hugh’s College, University of Oxford, UK, 2014.

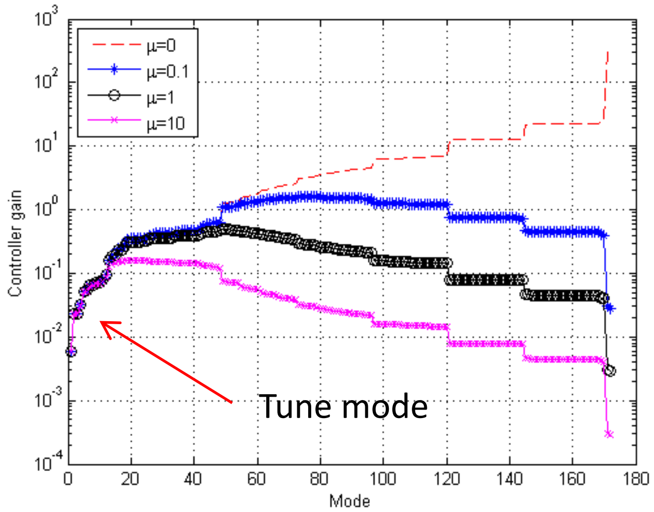
Tikhonov regularization (idea implemented at DLS)

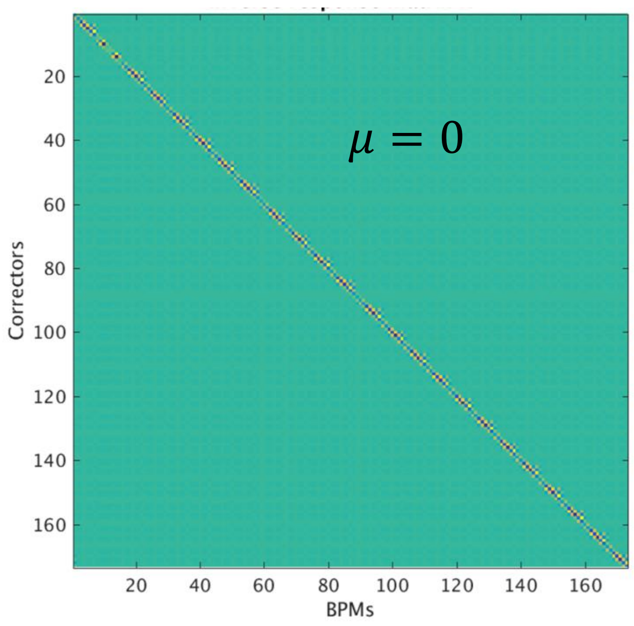
- “Calming down” higher modes, but not eliminating

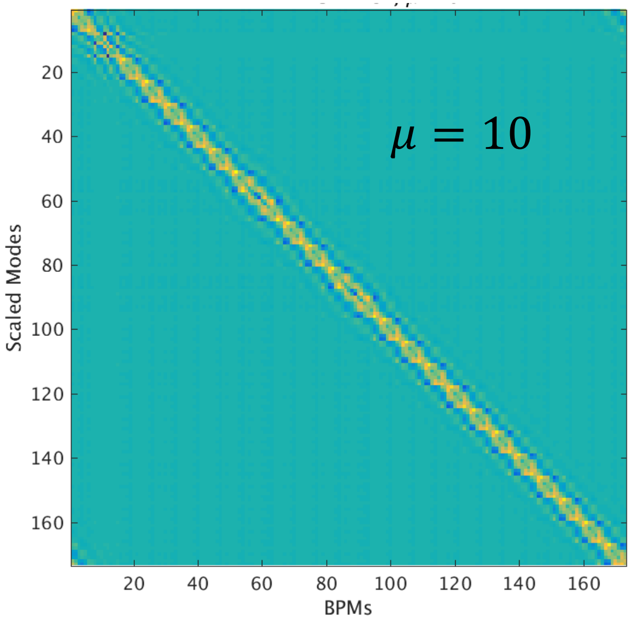
- Replaces inversion of singular values with

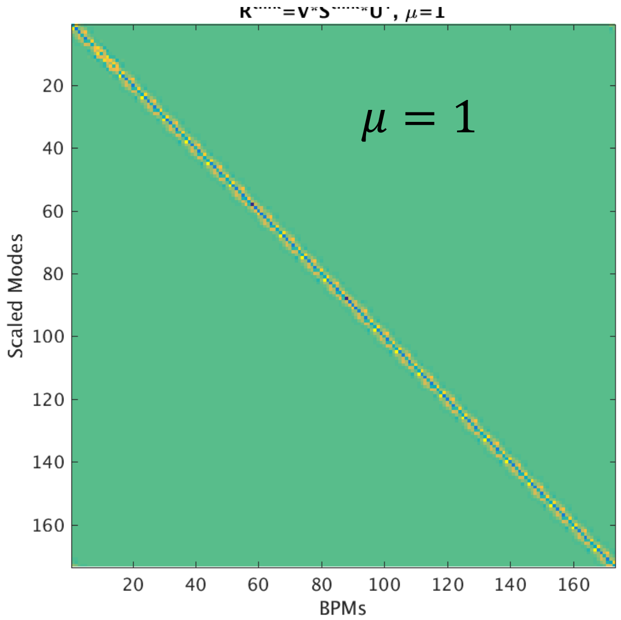
$$\tilde{S}_n = \frac{s_n}{s_n^2 + \mu^2}$$

$$\tilde{R} = V\tilde{\Sigma}U^T$$









How many ?

After multiple applications, the result will still be a fully corrected orbit

Symmetry exploitation in SIS 18 vertical ORM

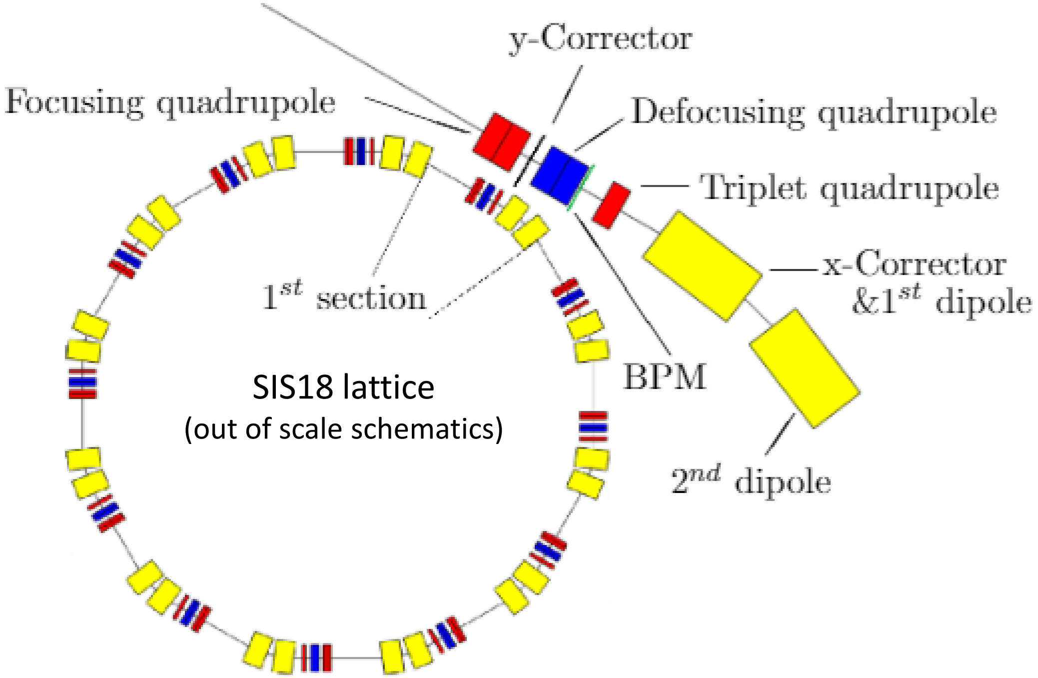
$$\beta_{bpm1} = \beta_{bpm2} = \beta_{bpm3} \cdots \cdots = \beta_{bpm12}$$

$$\beta_{corr1} = \beta_{corr2} = \beta_{corr3} \cdots \cdots = \beta_{corr12}$$

$$\Delta\mu_{bpm} = constant$$

$$\Delta\mu_{corr} = constant$$

$$R = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & \cdots & R_n \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$



Each row is cyclic shift of previous row

All diagonal elements are identical

Such a square matrix is called **Circulant Matrix**

Reference: Philips J.Davis, Circulant matrices, (1994), Chelsea

Diagonalization of a Circulant matrix

$$R = \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & \cdots & R_n \\ R_n & R_1 & R_2 & R_3 & \cdots & R_{n-1} \\ R_{n-1} & R_n & R_1 & R_2 & \cdots & R_{n-2} \\ R_{n-2} & R_{n-1} & R_n & R_1 & \cdots & R_{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ R_2 & R_3 & R_4 & R_5 & \cdots & R_1 \end{bmatrix}$$

$$\sigma_k = \sigma_{rk} + j\,\sigma_{ik} = \sum_{i}^{n-1} R_n\, e^{-j2\pi ki/n}$$

$$\mathrm{R} = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$

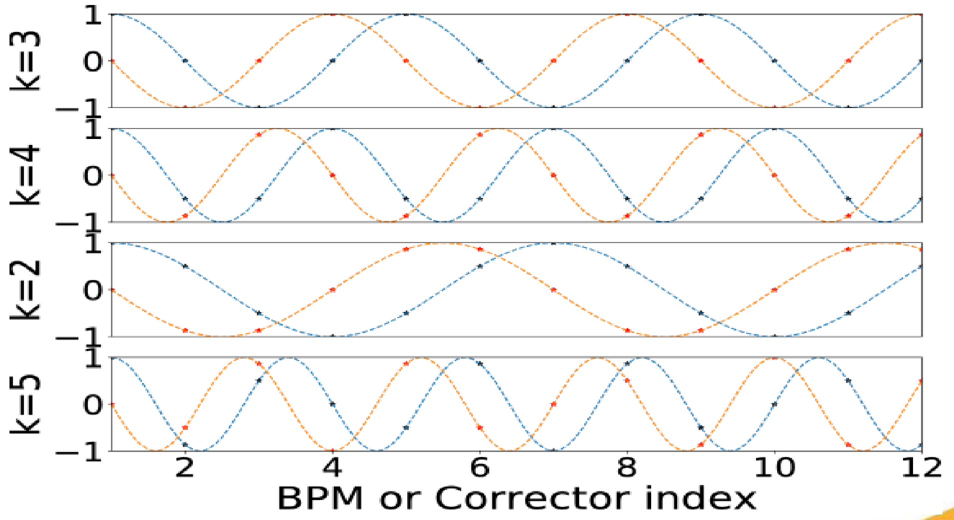
$$F_k = F_{kc} + jF_{ks} \qquad F_{ks} = \sin\left(\frac{2\pi km}{n} + \varphi_k\right)$$

Inverse is straightforward

$$R^{-1} = F^* H^{-1} F$$

$$H^{-1} = \mathrm{diag}\left(\frac{1}{\sigma_k}\right),\ k=1\ldots n$$

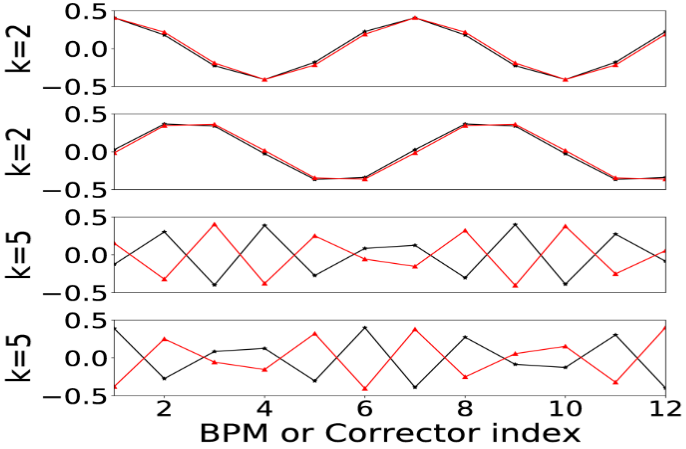
Standard Fourier matrix



Equivalence of DFT and SVD

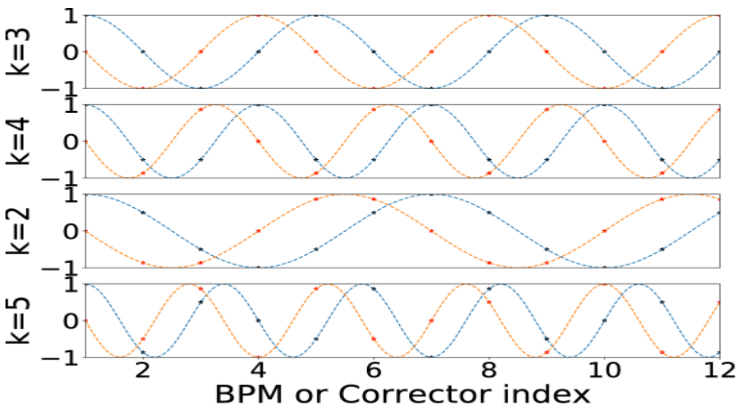
DFT:

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} F_{11} & \cdots & F_{1m} \\ \vdots & \ddots & \vdots \\ F_{m1} & \cdots & F_{mm} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_2 & \vdots \\ 0 & \cdots & \sigma_n \end{bmatrix} \begin{bmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{bmatrix}$$



$$\varphi_{dk} = \text{phase}(\sigma_k)$$

$$s_k = |\sigma_k| = \sqrt{{\sigma_{rk}}^2 + {\sigma_{ik}}^2}$$



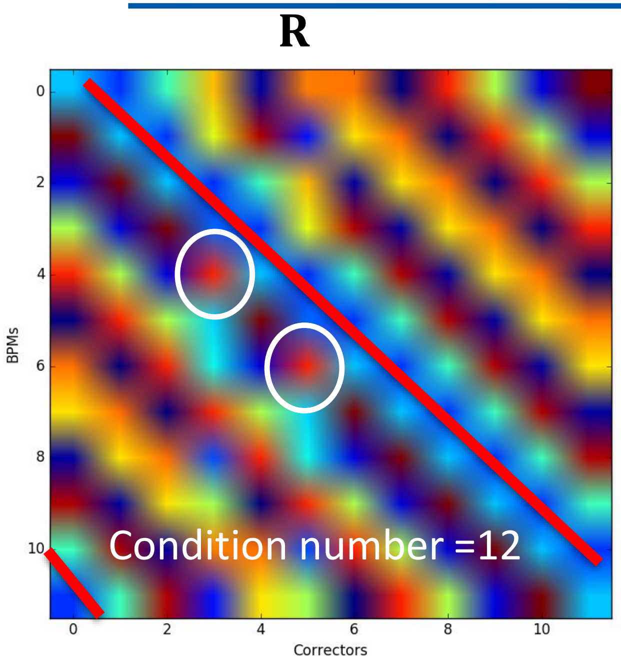
SVD:

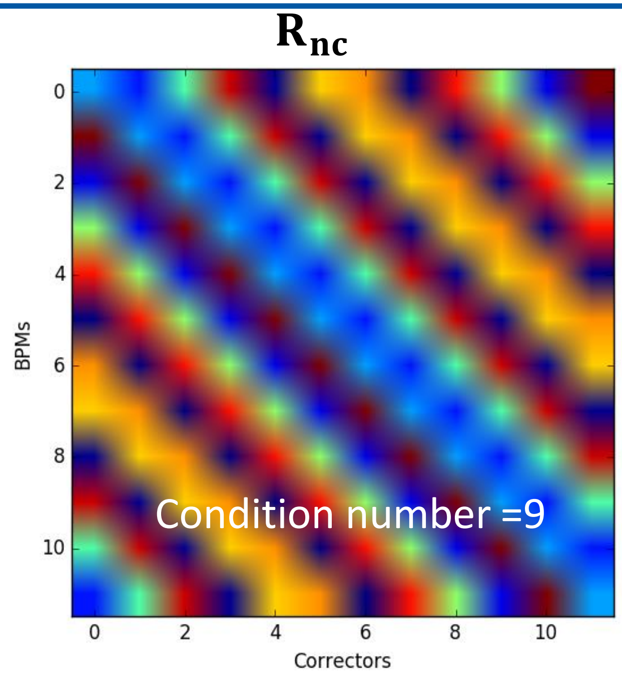
$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} = \begin{bmatrix} U_{11} & \cdots & U_{1m} \\ \vdots & \ddots & \vdots \\ U_{m1} & \cdots & U_{mm} \end{bmatrix} \begin{bmatrix} s_1 & \cdots & 0 \\ \vdots & s_2 & \vdots \\ 0 & \cdots & s_n \end{bmatrix} \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{n1} & \cdots & V_{nn} \end{bmatrix}$$

Why to do SVD when Circulant symmetry exits?

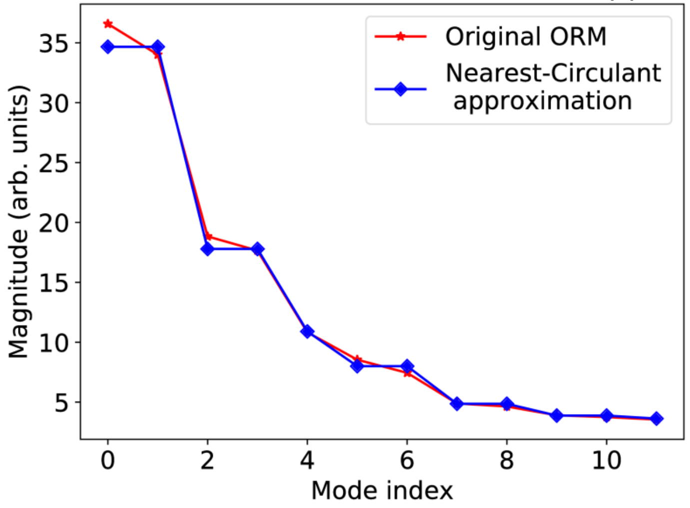
Reference: Herbert Karner et al. Spectral decomposition of real Circulant matrices, Linear Algebra and its Applications, Volume 367, 2003

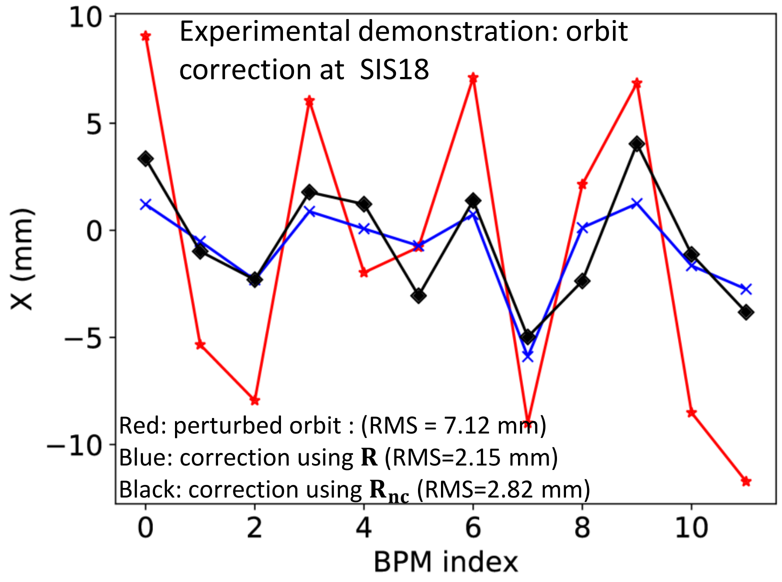
Nearest-Circulant symmetry : SIS18 x-plane





Singular values of **R** and its nearest-Circulant approximation





Frobenius product

$$c_k = \frac{1}{n} < \mathbf{R}, \pi^k >$$

$$\mathbf{R}_{nc} = circ(c_0, c_1, c_2, .., c_{11})$$

$$c_1 = \frac{1}{n}\left(R_{1,2} + R_{2,3} + \cdots R_{12,1}\right)$$

$$\pi^1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ . & . & . & . & . \\ . & . & . & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Ignoring the dispersion induced orbit shift at SIS18

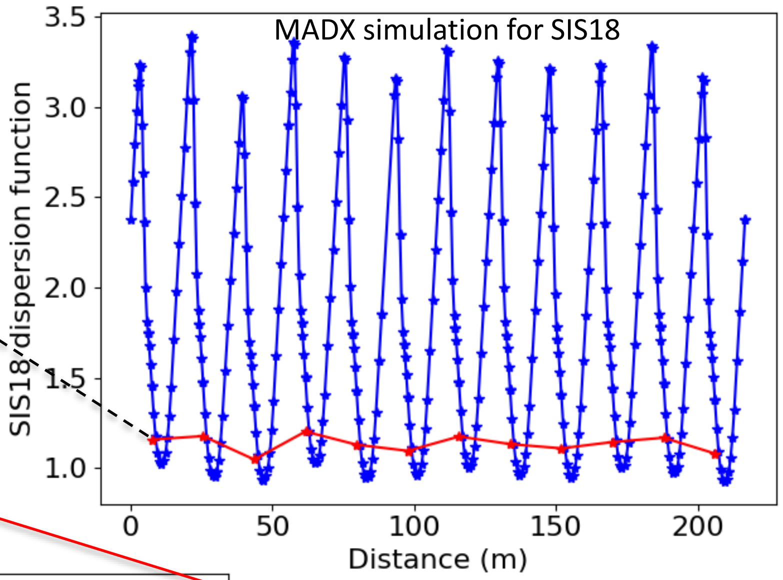
Mismatch between RF frequency and the dipole field

$$\Delta x_D(s) = D(s)\frac{\Delta p}{p}$$

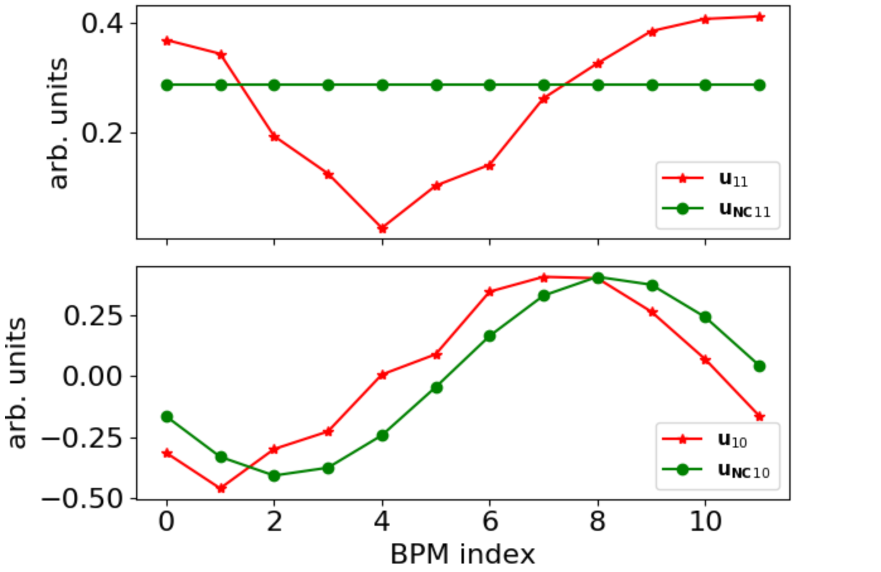
$$\Delta x = \Delta x_{co} + \Delta x_D(s)$$

an attempt to correct it can saturate the corrector magnets.

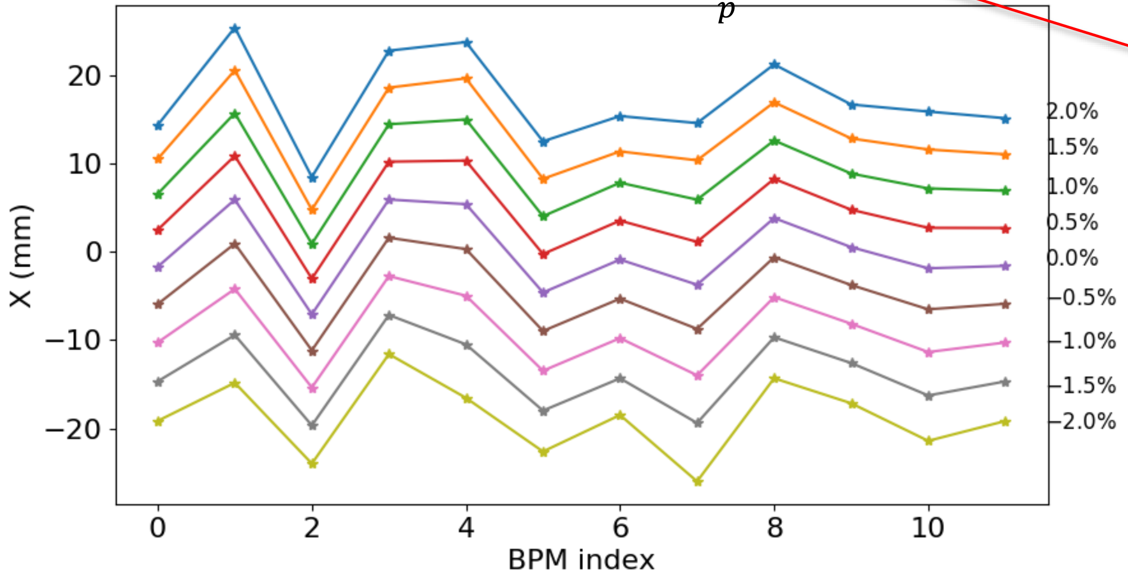
$$\boldsymbol{\theta} = \mathbf{V}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{U}\,\Delta \boldsymbol{x}$$

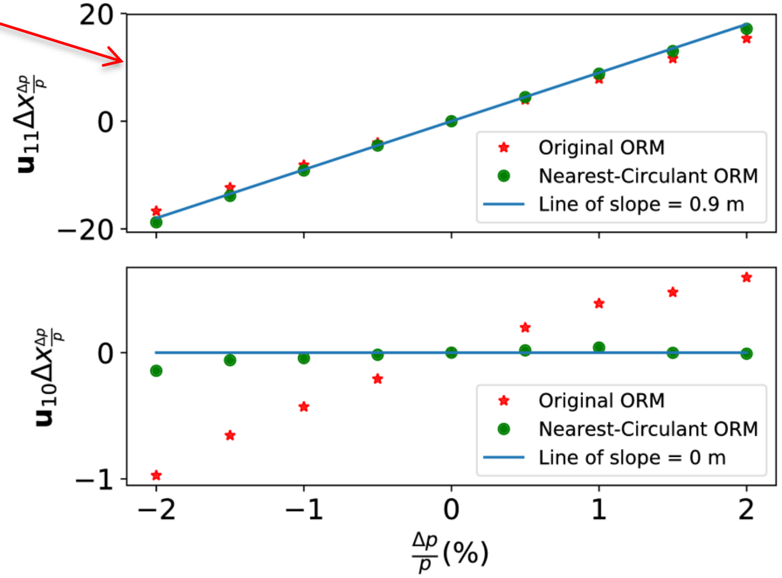


Last two modes of original ORM and its Circulant approximation



Measured closed orbit for various values of $\frac{\Delta p}{p}$

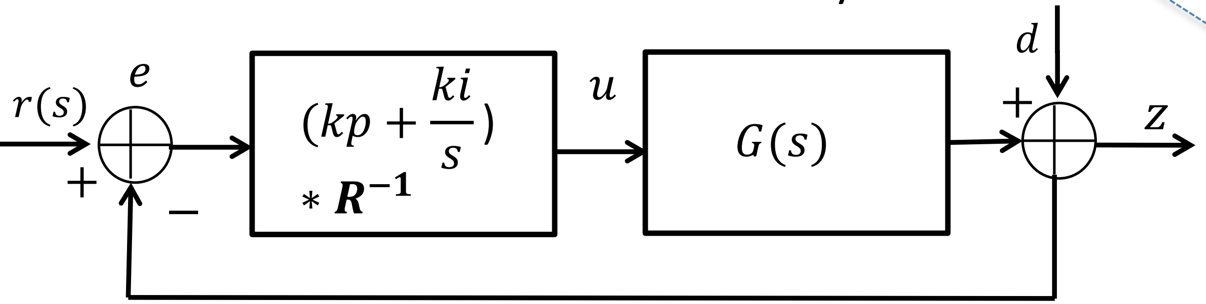




Coupling of dispersion to last two modes of original ORM and its nearest-Circulant approximation. Dispersion couples to one mode only for Circulant ORM.

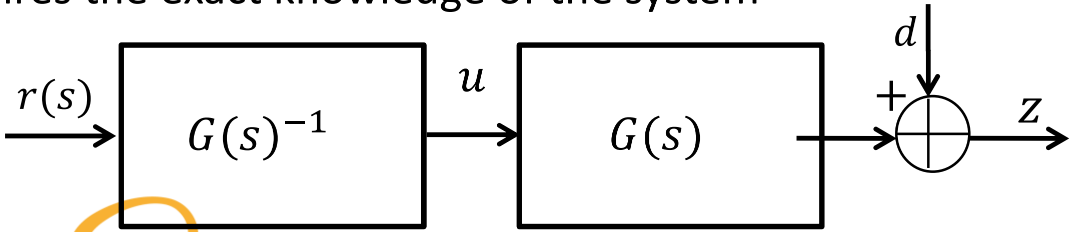
Controller types used in COFB : temporal domain

- Proportional Integral (PI) controller (Simple controller)
- Can be implemented with least knowledge of the system
- Tuning of gains according to the application (trial and error)
- Easy to implement on hardware
- Widely used at synchrotrons.
- A stable controller can destabilize the system



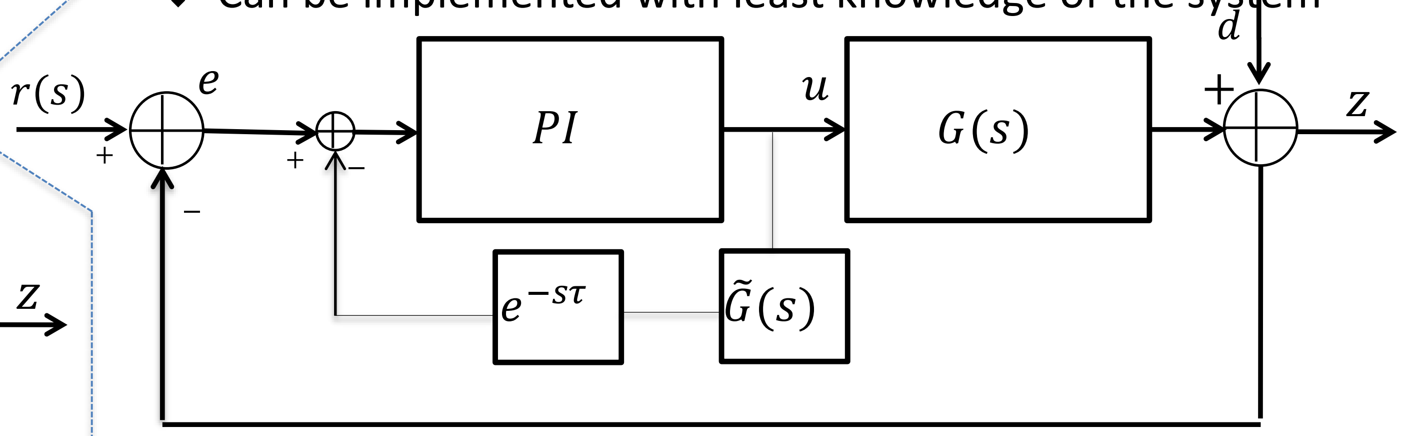
Inverse controller (ideal controller)

Requires the exact knowledge of the system



Real systems have delays and model discrepancies.

- Smith predictor with classical (PI) controller
- Tuning the controller assuming no delay
- Can be implemented with least knowledge of the system

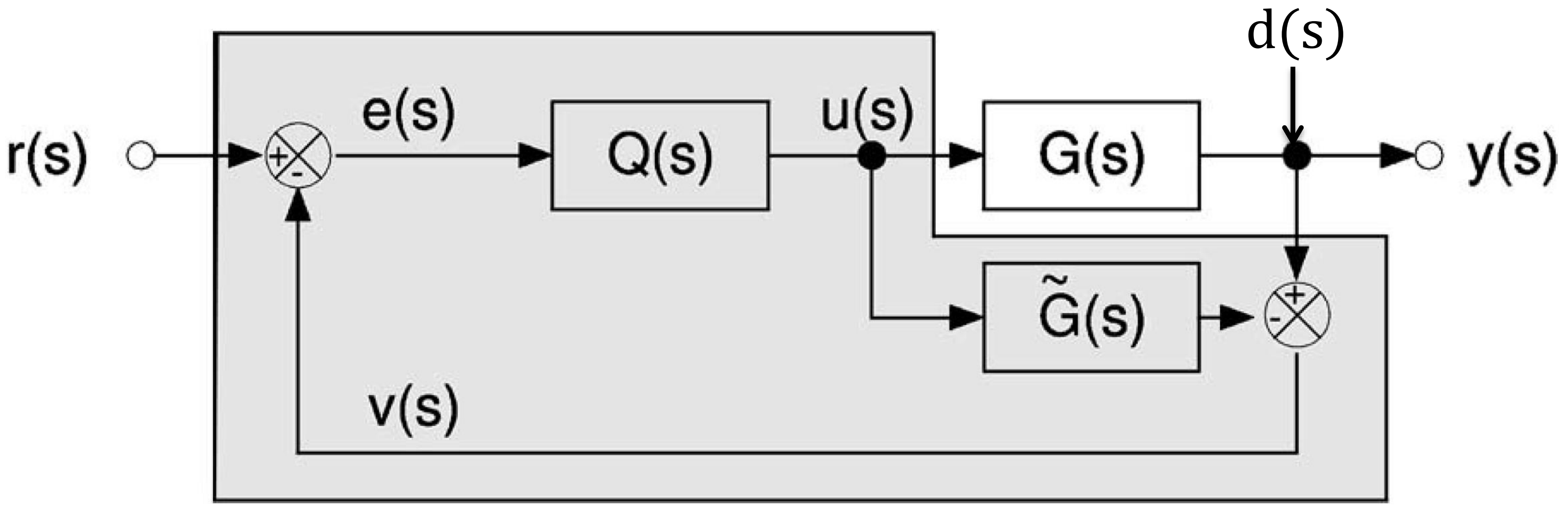


- Model is included later in order to compensate pure delays

Latest implementation at Diamond Light Source is a blend of classical smith predictor and inverse controller Internal model controller (IMC)

Internal model controller

- System identification provides the predictions of plant behaviour : Transfer function of the model, monitor and actuator dynamics and latency.

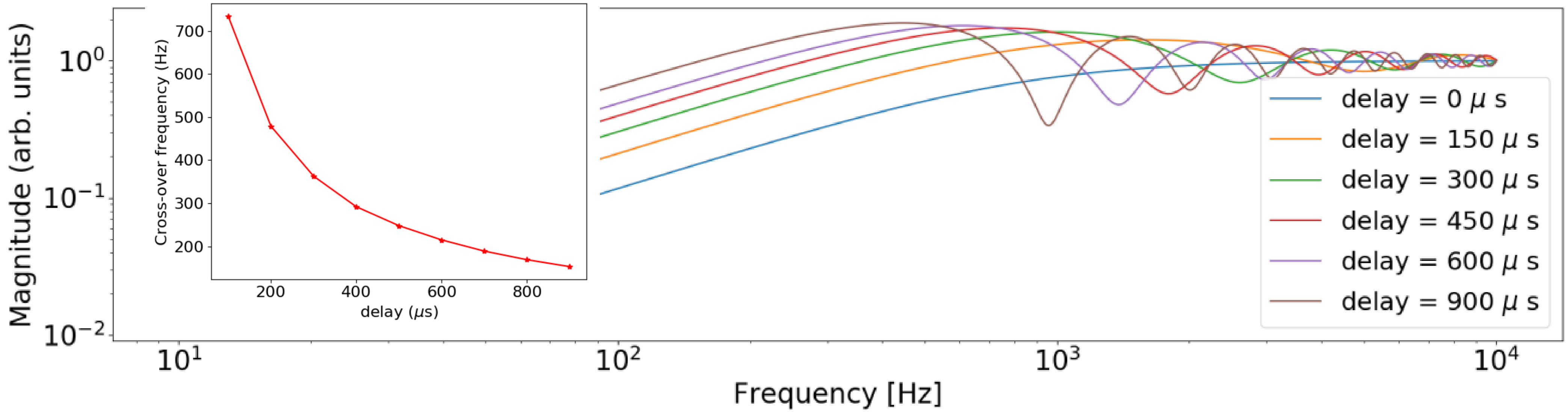


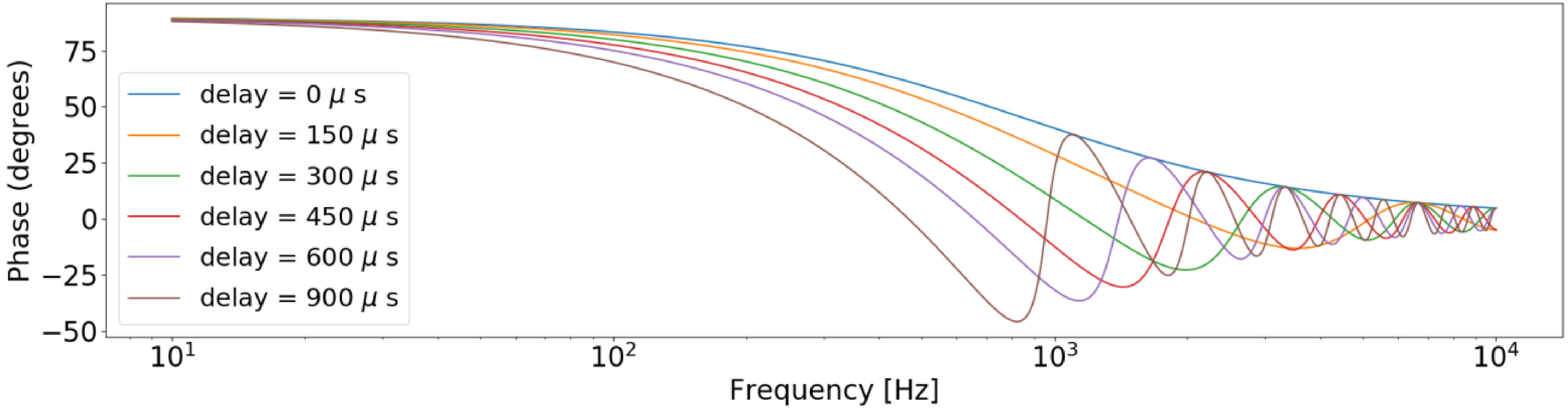
$$Q(s) \cong \tilde{G}(s)^{-1}$$

$$\tilde{G}(s) = e^{-e\tau} \frac{s}{s+\omega_c}$$

- This system is one way to explicitly address the significant latency as found in digital systems
- If G(s) is stable, a stable Q(s) will ensure internal stability in time domain
- Solution for a simple 1-pole low pass model is straight forward and feature three parameters: **latency, bandwidth and gain**
- Works for non-linear systems if their difference can be approximated by linear

Sensitivity to temporal delay (disturbance to output and $\omega_c = 850Hz$)





Characterizing the effect of spatial model mismatch

$\Theta = \mathbf{R}'^{-1}\Delta z_0$ → Corrector settings

$r_1 = \Delta z_0 - \mathbf{R}\Theta$ → First iteration residual

$r_1 = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})\Delta z_0$

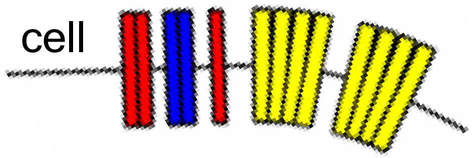
Multiple applications

$r_n = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})^n\Delta z_0$ → n^{th} iteration residual（n costs the bandwidth）

Focusing magnets

defocusing magnets

SIS18 cell



triplet focusing magnet

Bending magnets

$\mathbf{S} = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})$ → Spatial sensitivity function

$\mathbf{S} = \mathbf{P}\mathbf{\Lambda_S}\mathbf{P}^{-1}$

$r_n = (\mathbf{P}\mathbf{\Lambda_S}\mathbf{P}^{-1})^n\Delta z_0$

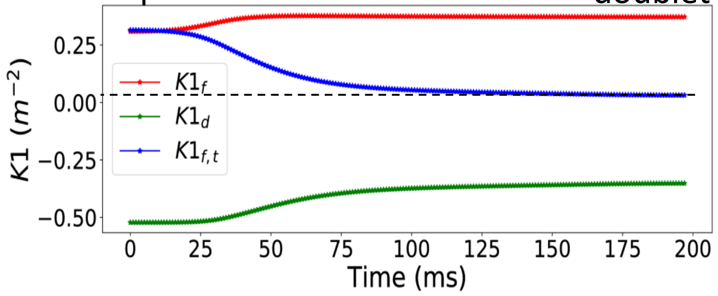
$r_n = \mathbf{P}(\mathbf{\Lambda_S})^n\mathbf{P}^{-1}\Delta z_0$

$\lambda_k \approx (1 - \lambda_{R,k}\lambda^{-1}{}_{R',k})$

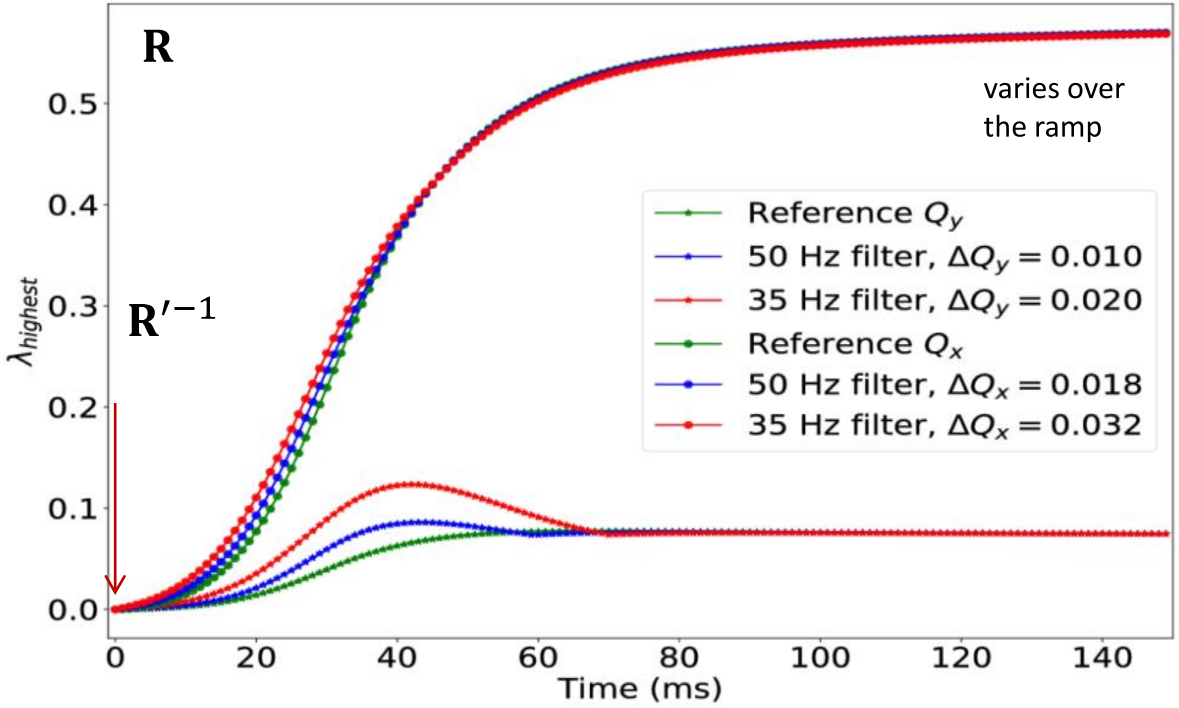
$\lambda_k < 1$	Correctability
$\lambda_k = 1$	No correction
$\lambda_k > 1$	Instability

triplet

doublet



On- ramp model change in SIS18

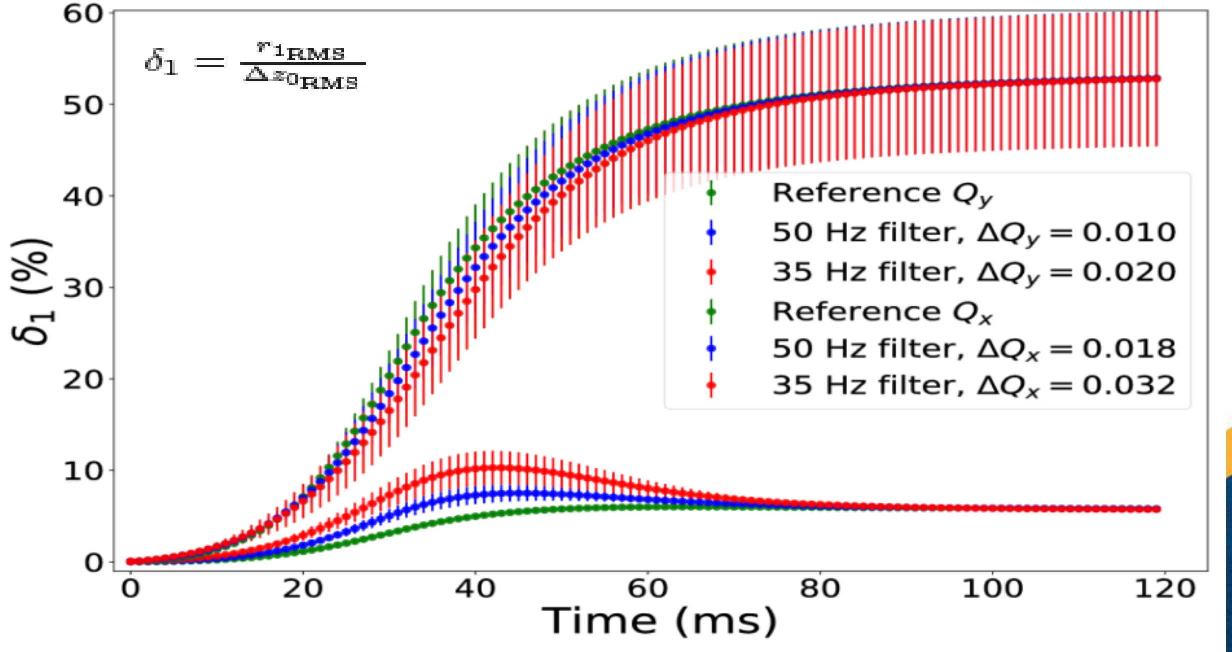


$$\mathbf{S} = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})$$

$$\mathbf{S} = \mathbf{P}\boldsymbol{\Lambda}_{\mathbf{S}}\mathbf{P}^{-1}$$

$$\lambda_{max} \ \ \textbf{of} \ \ \mathbf{S}$$

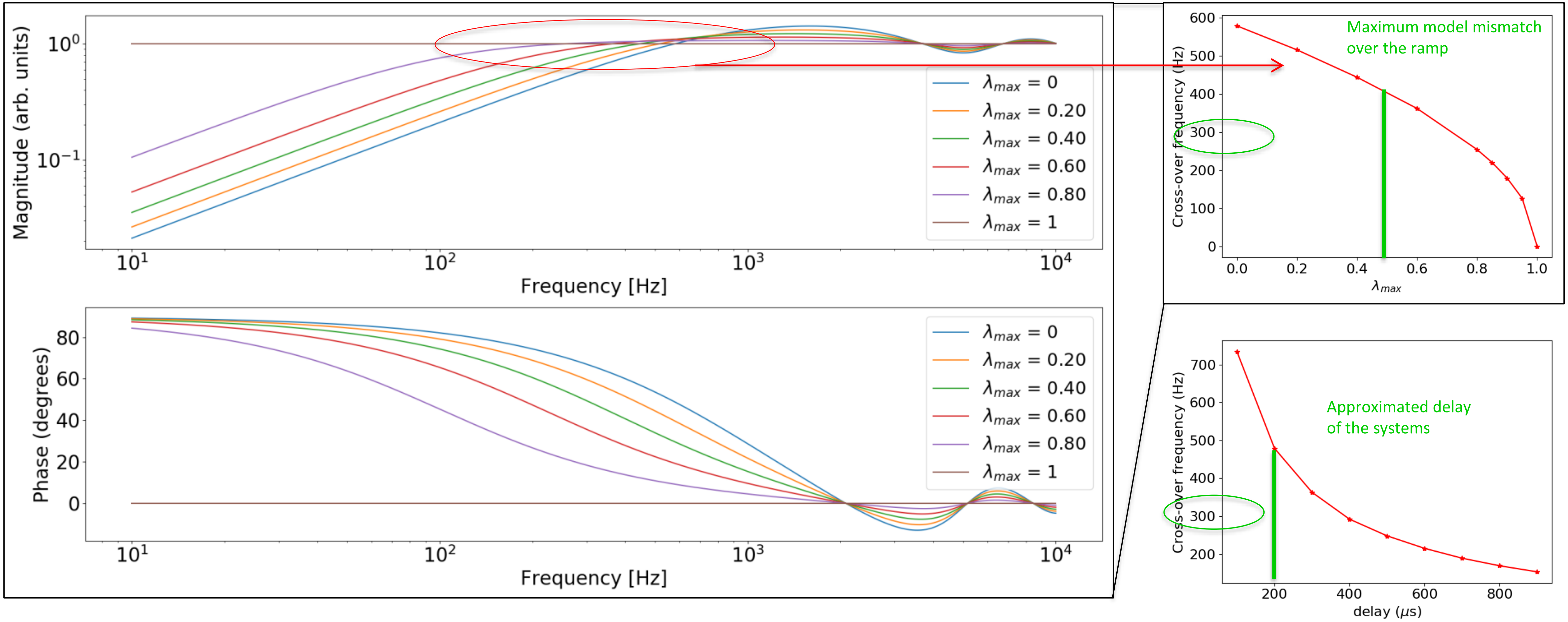
First iteration residual calculated in MADX



Conclusion:

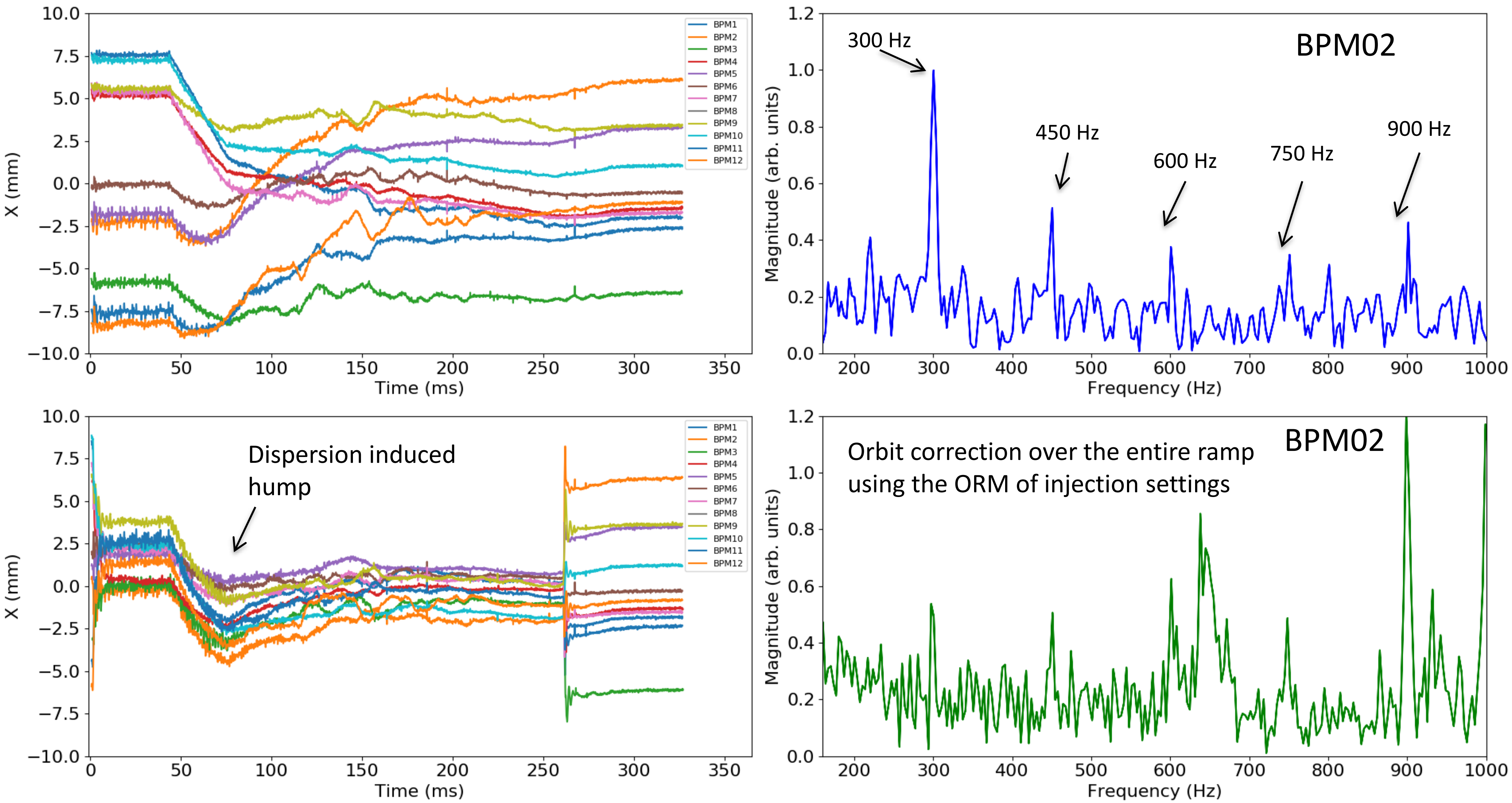
- In y-plane the effect of model mismatch can be ignored
- In x-plane, closed orbit will not be instable even for using ORM of injection settings at extraction energy.
- Only 2-3 ORMs might be needed over the entire ramp in x-plane

Sensitivity to spatial model mismatch at SIS18 (disturbance to output)



With the approximated delay of $\tau = 200\mu s$ and model mismatch up to $\lambda_{max} = 0.5$
Orbit correction up to 300 Hz was expected in SIS18 COFB

First test of COFB system for the on-ramp orbit correction



Summary

- Closed orbit feedback system has is an integral part of synchrotron operations.
- Light sources and hadron machines have similarities and differences in requirements.
- Harmonic analysis and Singular value decomposition are two methods of orbit correction. SVD being more popular.
- DFT based diagonalization and inversion of the ORM can replace SVD in case of Circulant symmetry and provides more physical interpretation of the mode-space.
- Odd position of BPMs and correctors can loss of Circulant symmetry that can be explored by the nearest-Circulant approximation.
- Internal model controller design is replacing the classical controllers in COFB design and covers the temporal features like latency, bandwidth and gain more efficiently than classical controller.
- The effect of spatial model mismatch is investigated particularly for the on-ramp orbit correction in SIS18 and is found to decrease the achievable bandwidth of the closed loop.
- The first results of the on-ramp orbit correction at SIS18 synchrotron are presented.

Thanks