Causality Rules

A light treatise on *dispersion relations and sum rules*

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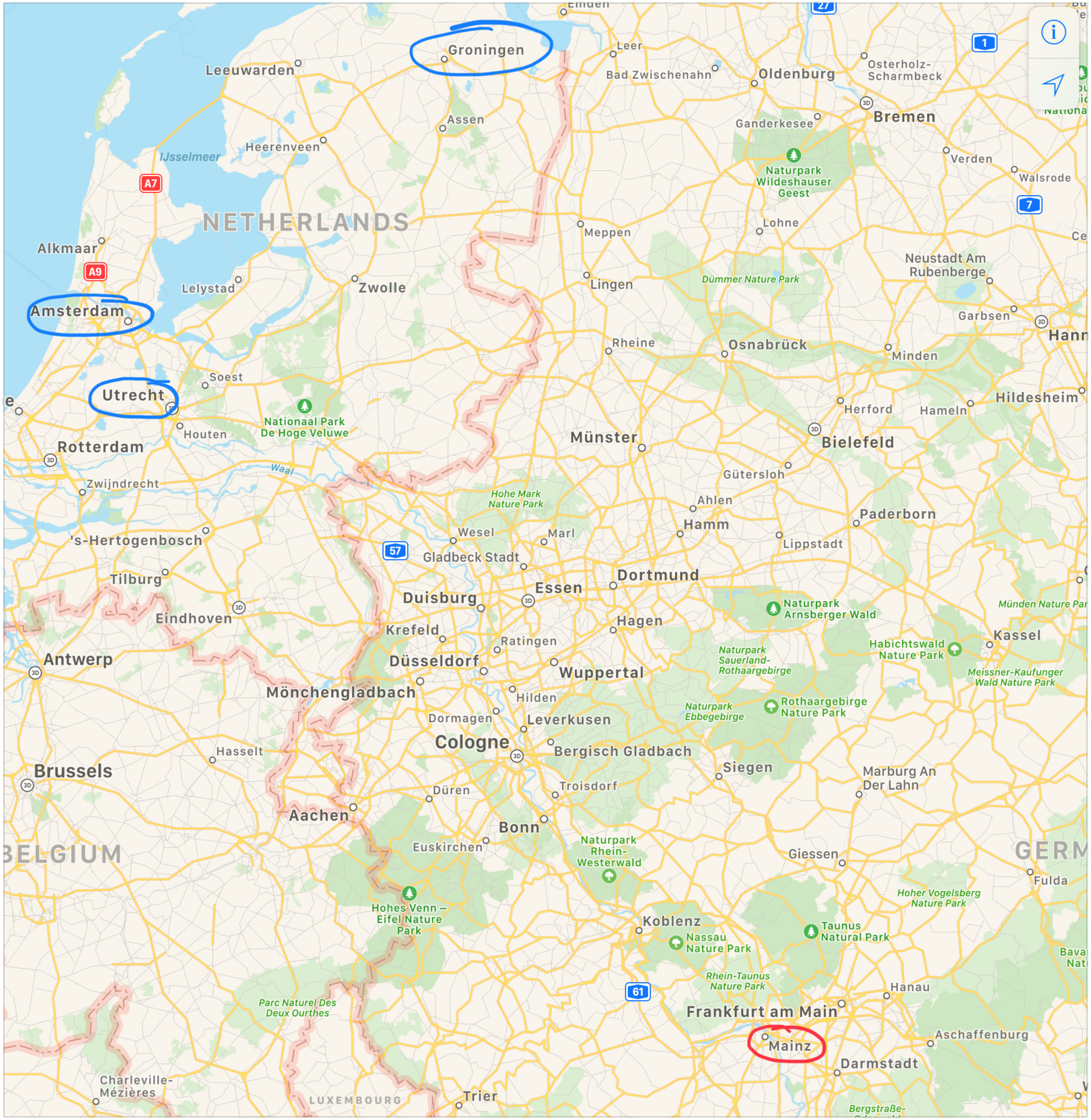
Education and Scientific Career

1989 – 1993	*Undergraduate Student,* Physics Department, Kiev University, Kiev, Ukraine
1993 – 1994	*NUFFIC Junior Fellow*, Kernfysisch Versneller Instituut, University of Groningen, Netherlands
1994 – 1998	*PhD researcher (OIO)*, Institute for Theoretical Physics, University of Utrecht, Netherlands
1998 – 1999	*Postdoctoral Researcher*, NIKHEF, Amsterdam, Netherlands
1999 – 2001	*Fellow of the Australian Research Council (ARC)*, Flinders University, Adelaide, Australia
2001 – 2003	*Postdoctoral Researcher*, Ohio University, Athens, Ohio, USA
2003 – 2006	*Research Assistant Professor*, College of William and Mary, Williamsburg, USA jointly with Thomas Jefferson Laboratory (JLab), Newport News, USA
2006 – 2008	*Assistant Professor (tenure track)*, European Centre for Theoretical Nuclear Physics and Related Areas (ECT*), Trento, Italy
2008 –	*Staff Scientist (tenured)*, Institute for Nuclear Physics, University of Mainz, Germany

Mainz U., Inst. Kernphys.	SENIOR	2008	
ECT, Trento	JUNIOR	2006	2008
William-Mary Coll.	PD	2003	2006
Jefferson Lab	PD	2003	2006
Ohio U.	PD	2001	2003
Flinders U.	PD	1999	2001
NIKHEF, Amsterdam	PD	1998	1999
Utrecht U.	PHD	1994	1998
Taras Shevchenko U.	UG	1989	1994



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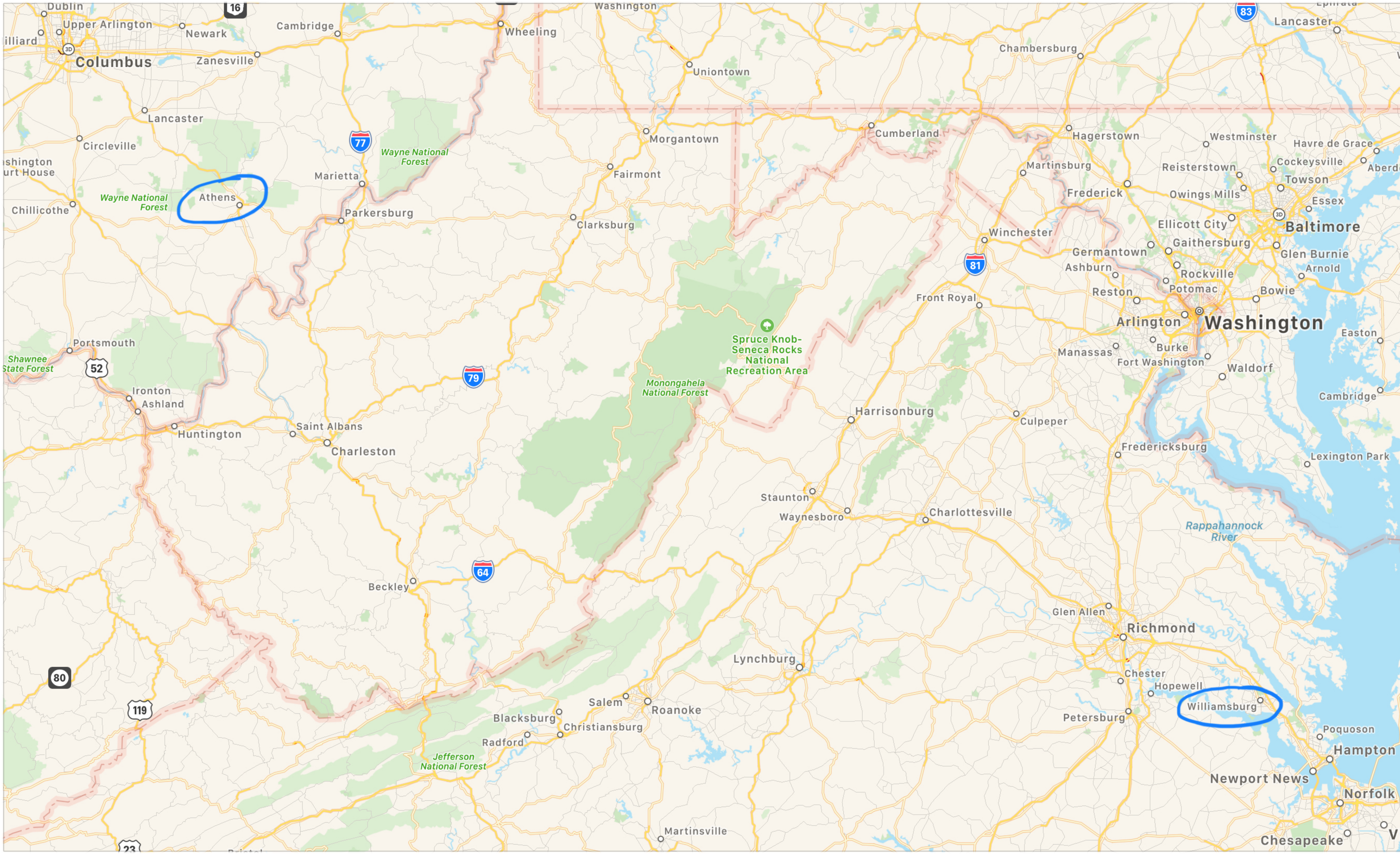

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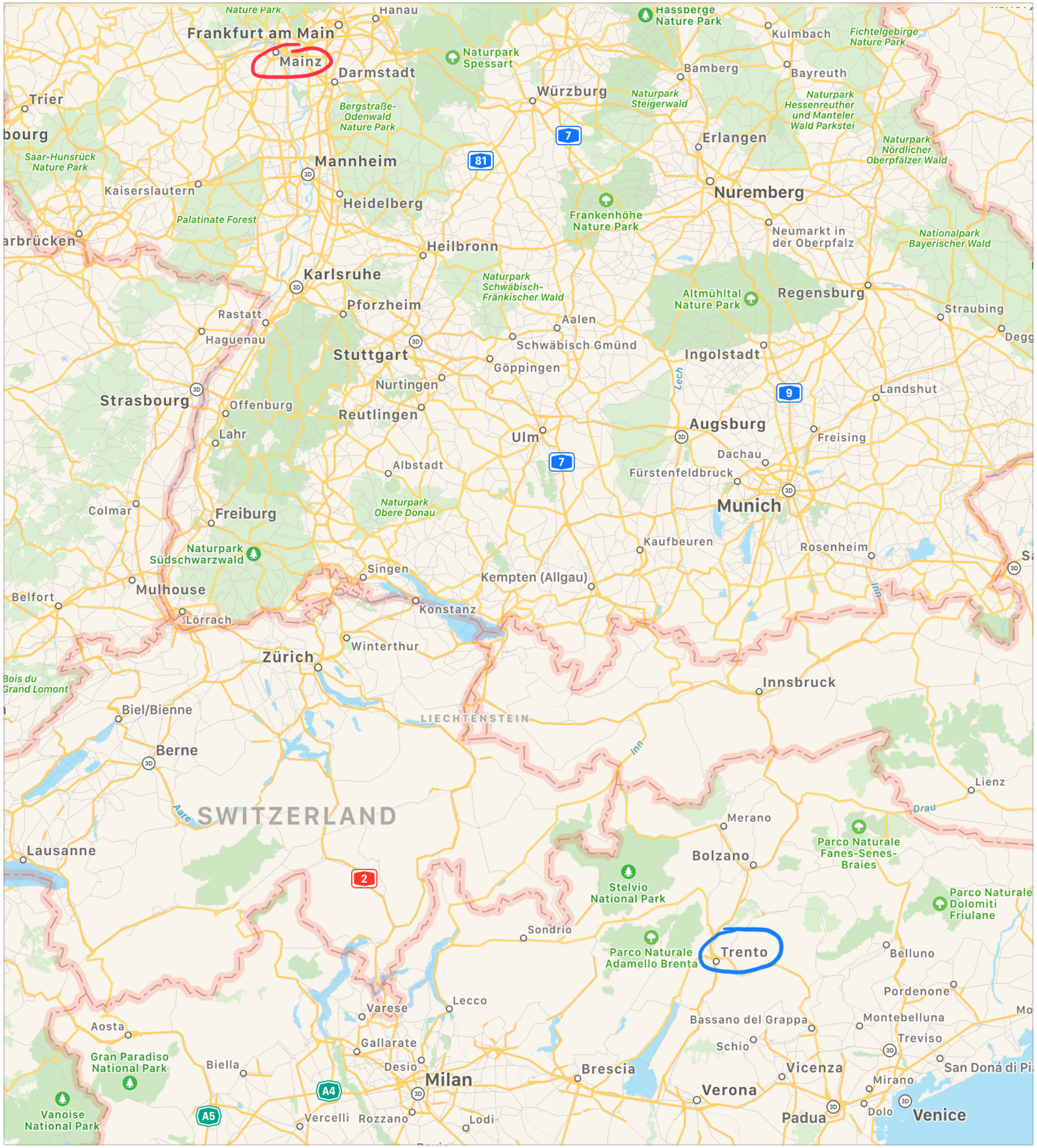
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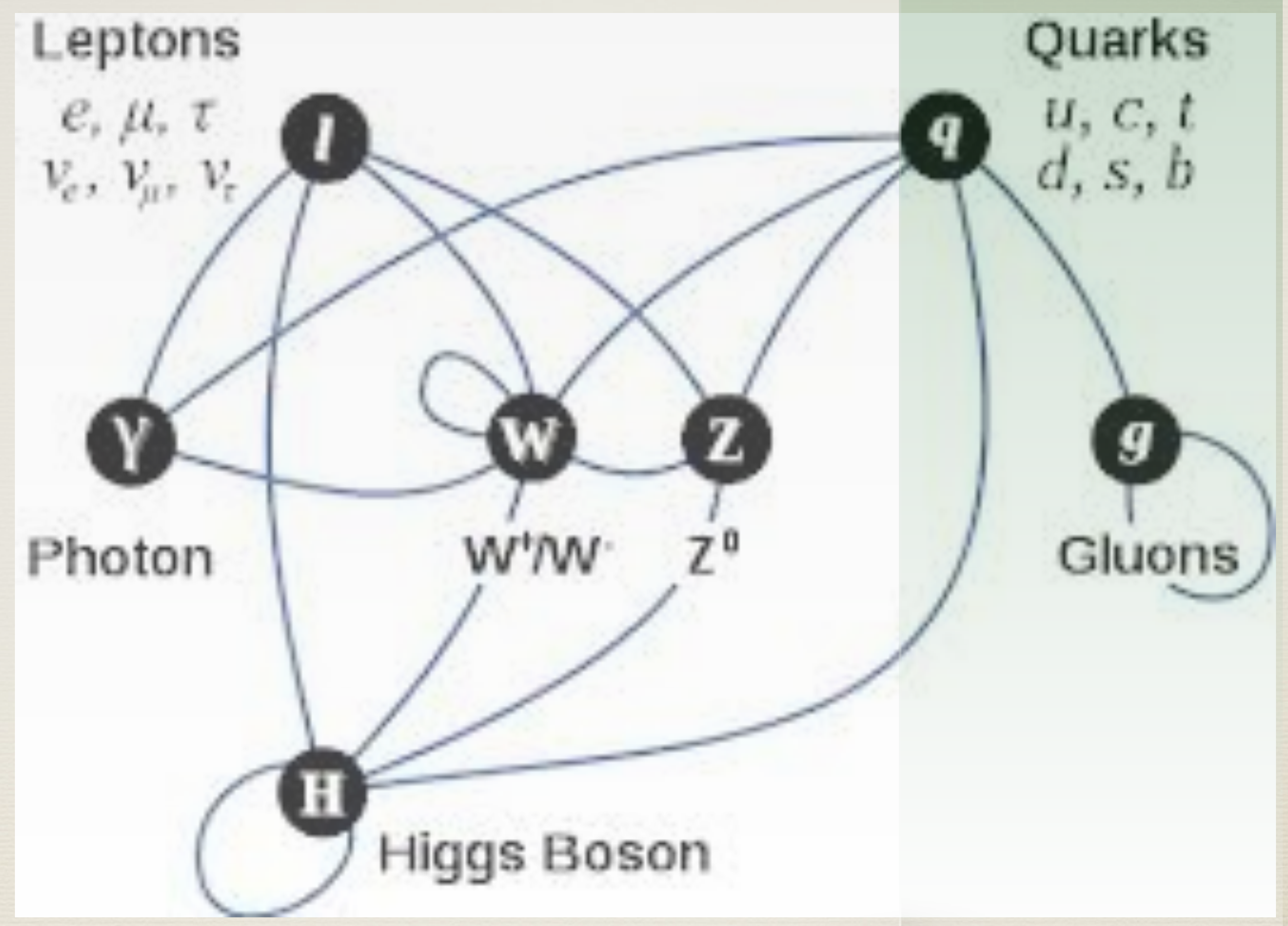
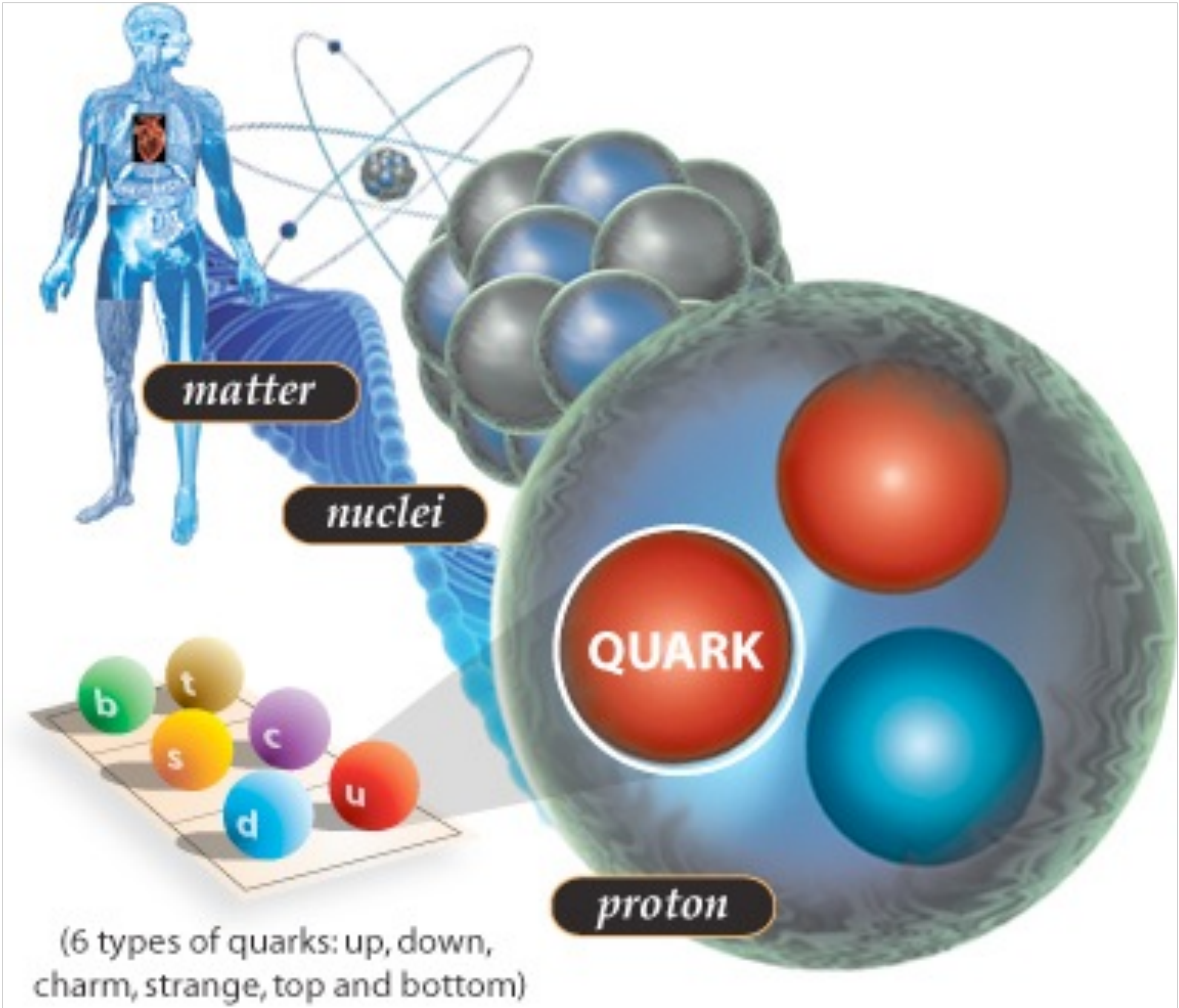
Lecture 1

Standard Model

Electroweak QCD

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QUARK
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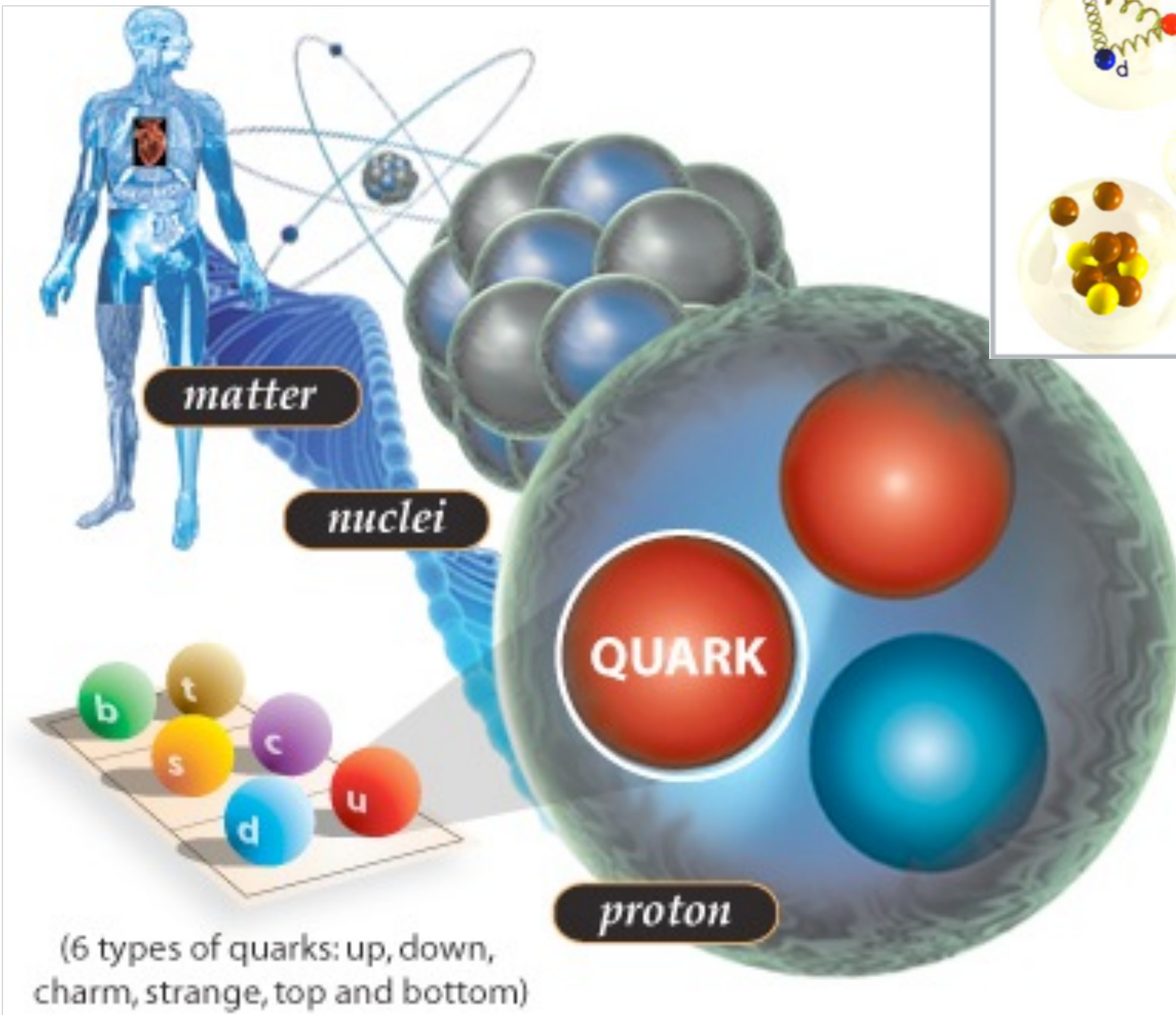
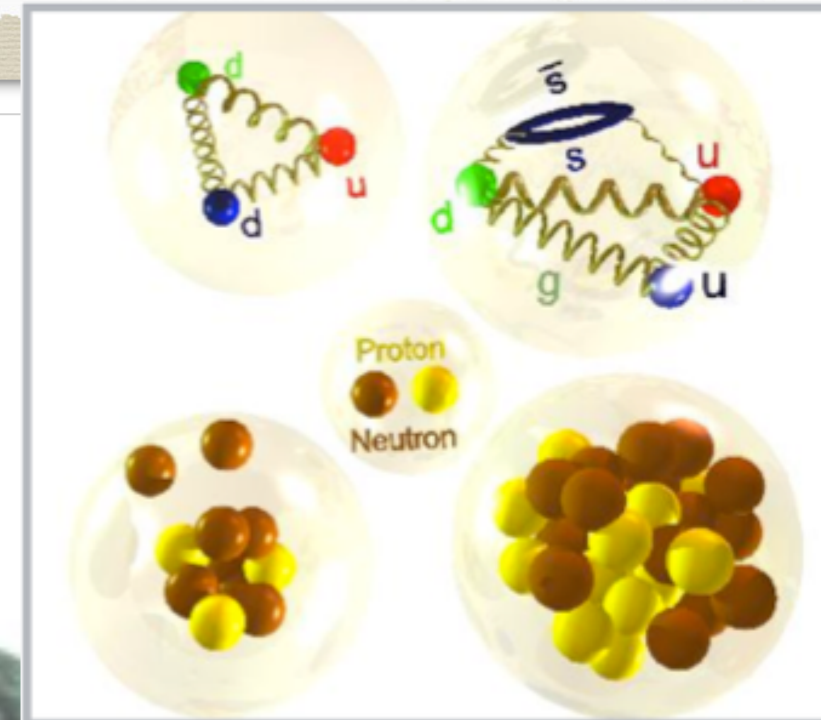
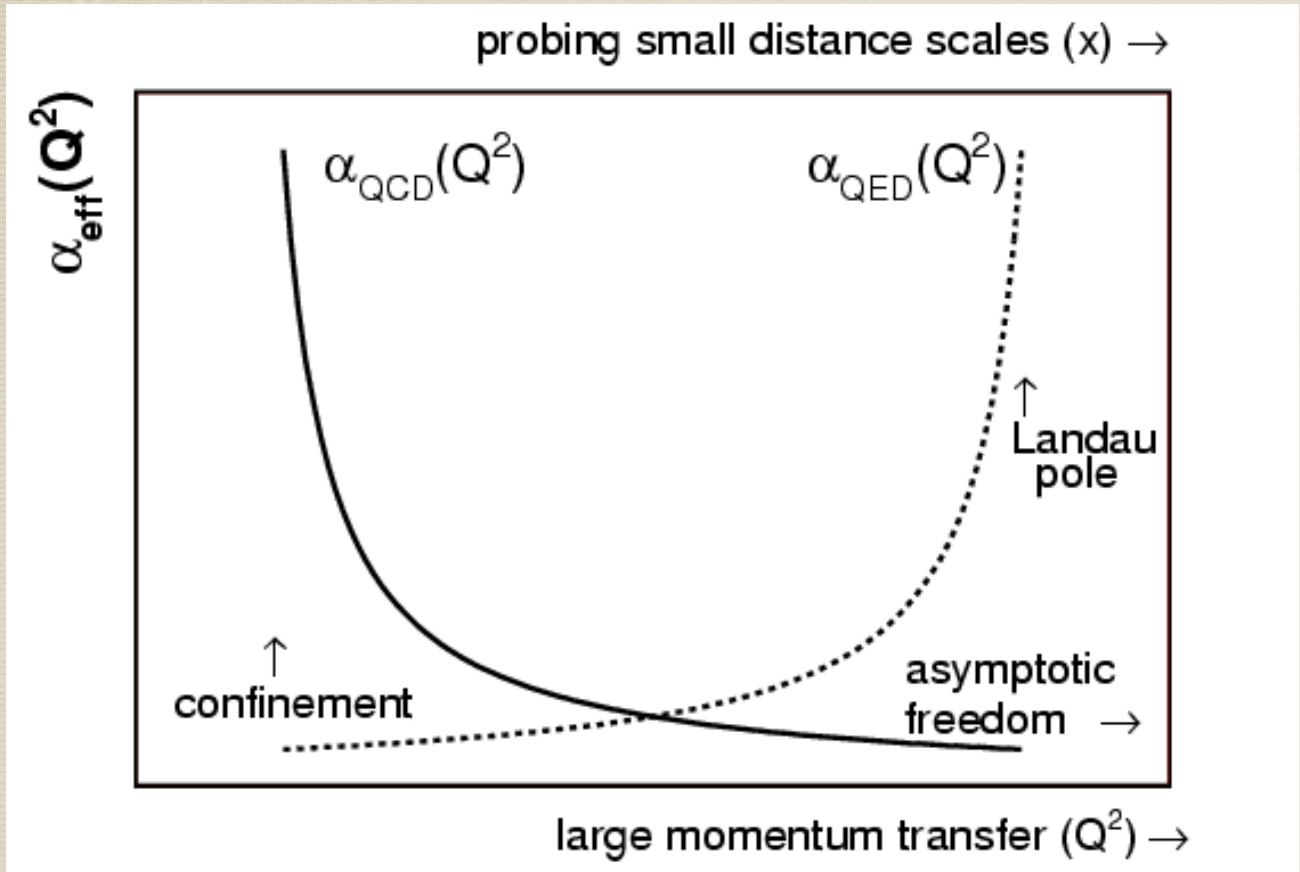
c
s
u
d
proton
(6 types of quarks: up, down,
charm, strange, top and bottom)



(6 types of quarks: up, down, charm, strange, top and bottom)

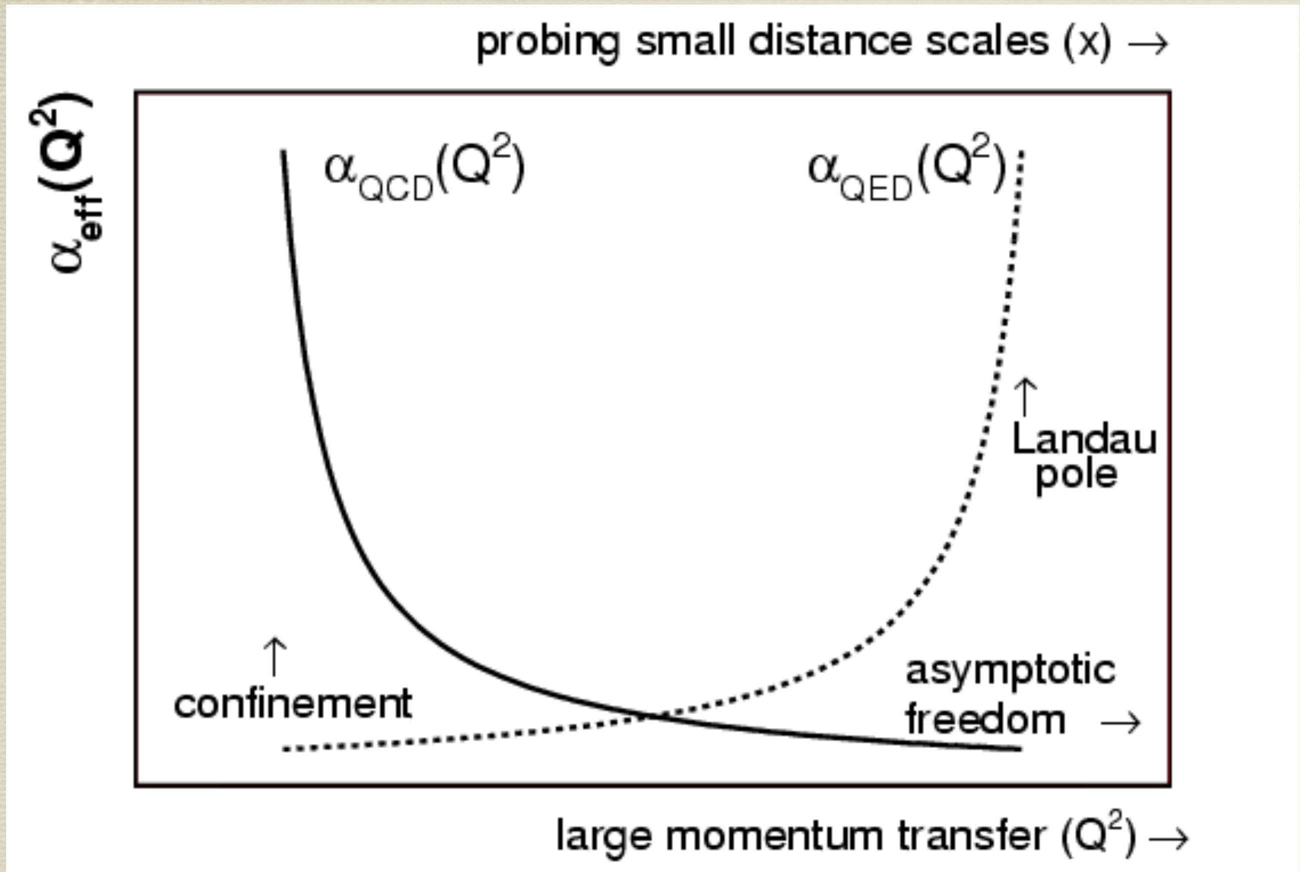


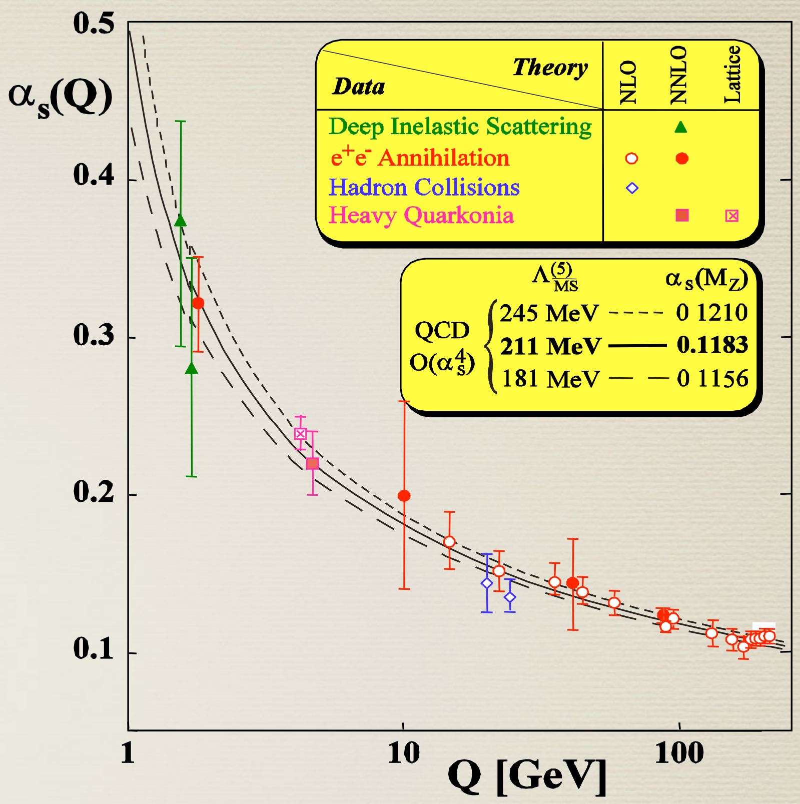
QCD coupling



For $Q^2 \to \infty, \alpha_s \to 0$: **asymptotic freedom**

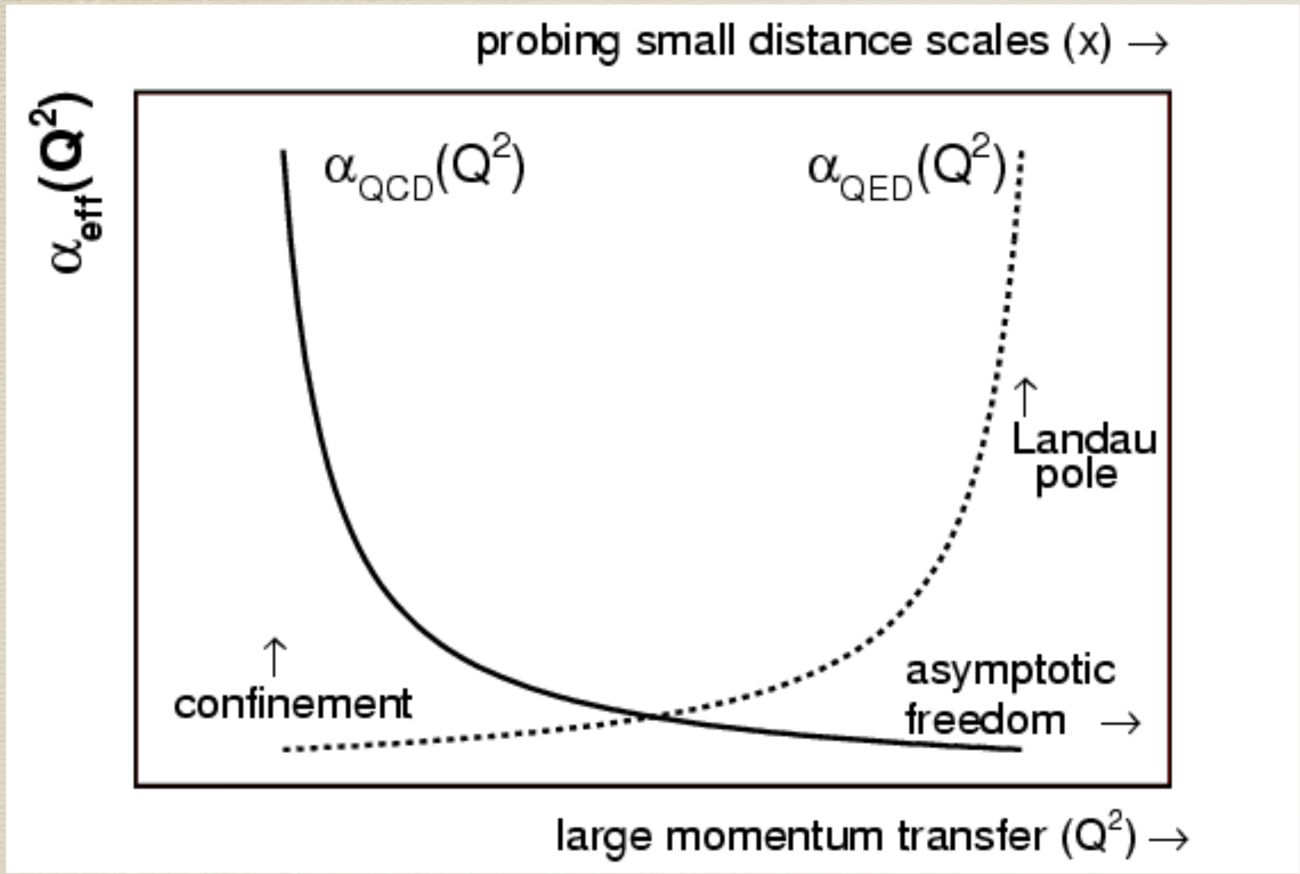
QCD coupling

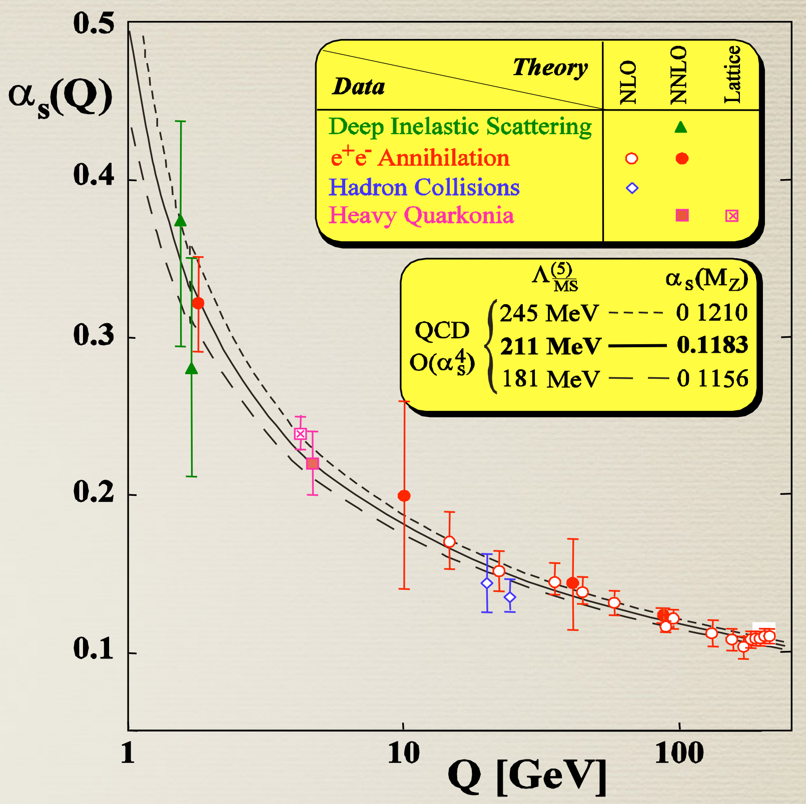




For $Q^2 \to \infty,\ \alpha_s \to 0$: **asymptotic freedom**

QCD coupling



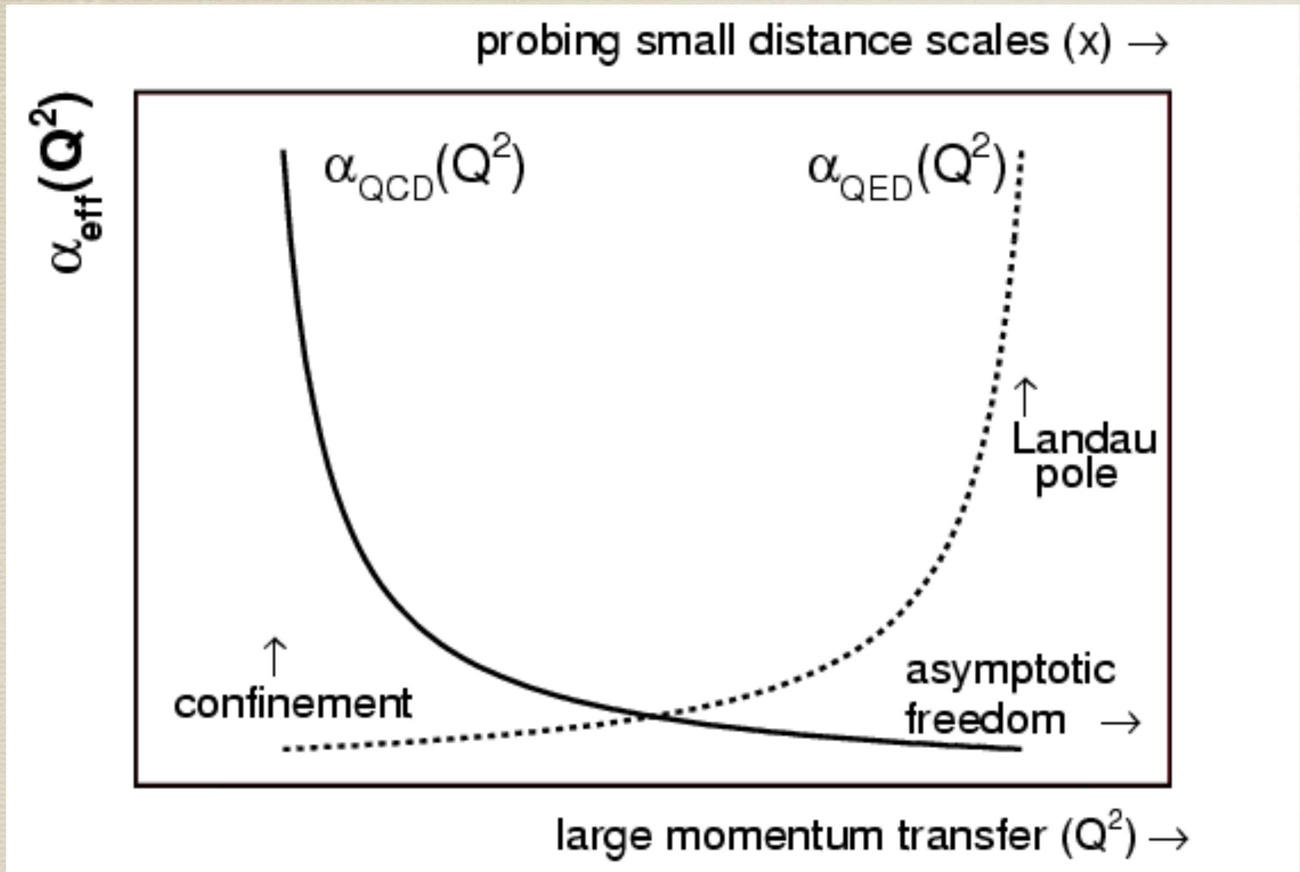


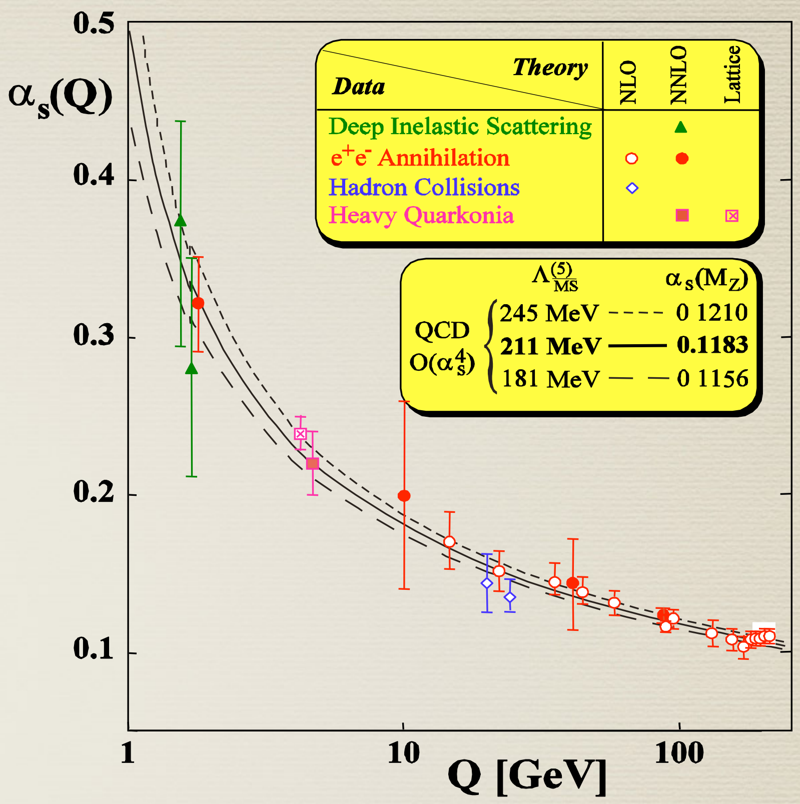
For $Q^2 \to \infty,\ \alpha_s \to 0$: **asymptotic freedom**

For $Q \sim \Lambda_{QCD}$ non-perturbative phenomena:

color confinement,
spontaneous chiral symmetry breaking,
generation of nucleon mass, ...

QCD coupling





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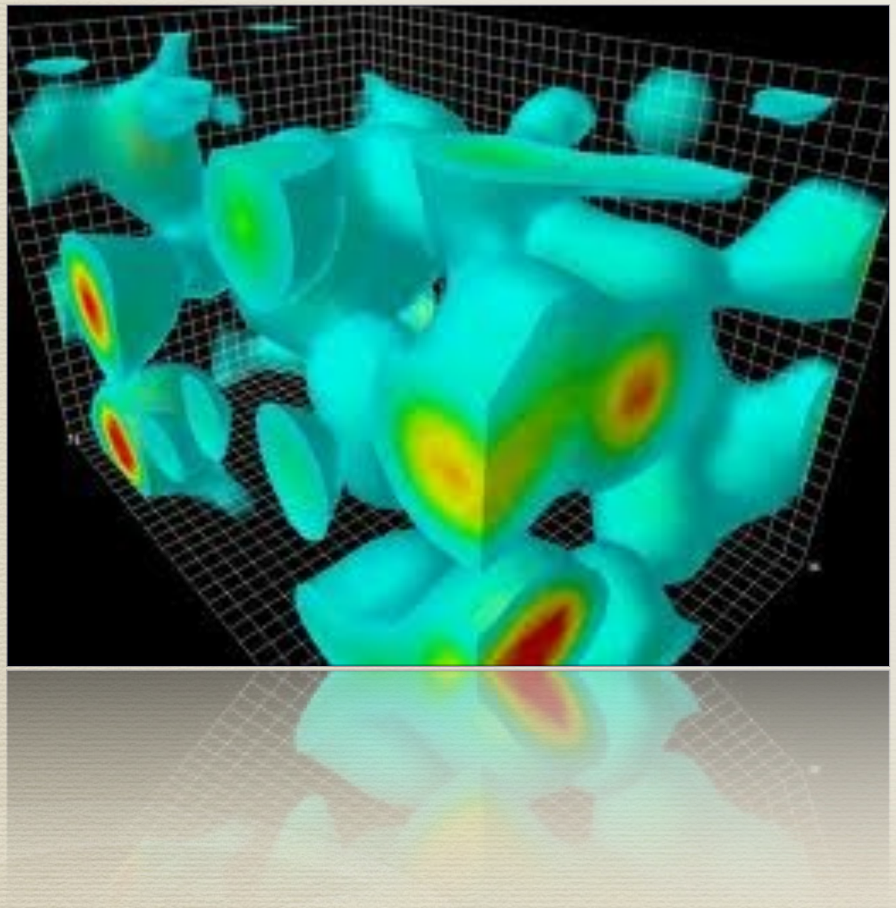
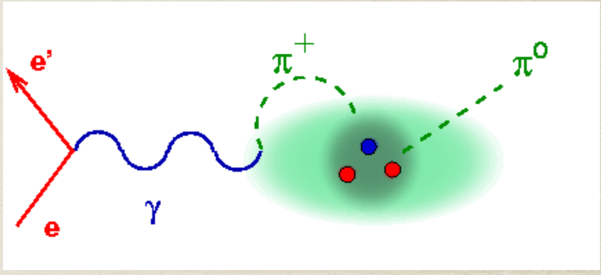
QFTs of low-energy QCD

1. Lattice QCD



2. Chiral effective-field theory (ChEFT)

[Weinberg (1979), Gasser & Leutwyler (1984, 85)]



3. Dispersive Methods (these lectures)

General constraints:

causality,

unitarity,

symmetries,

low-energy theorems

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General constraints:

causality,
unitarity,
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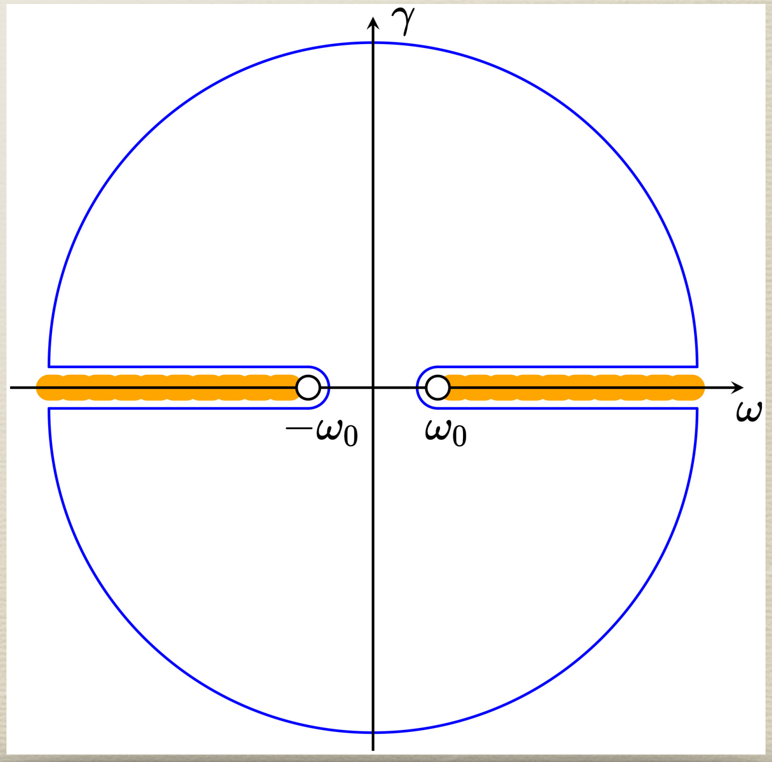
low-energy theorems

$$\boxed{f(w) = \frac{1}{2\pi i} \oint_C \frac{f(z)\,dz}{z - w}}$$

for any interior pt. w of C



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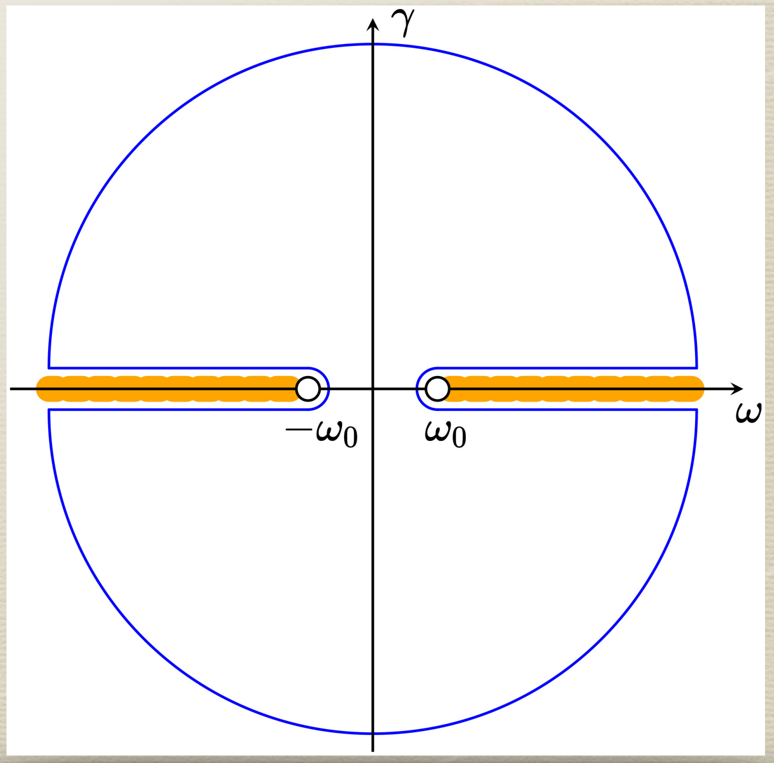
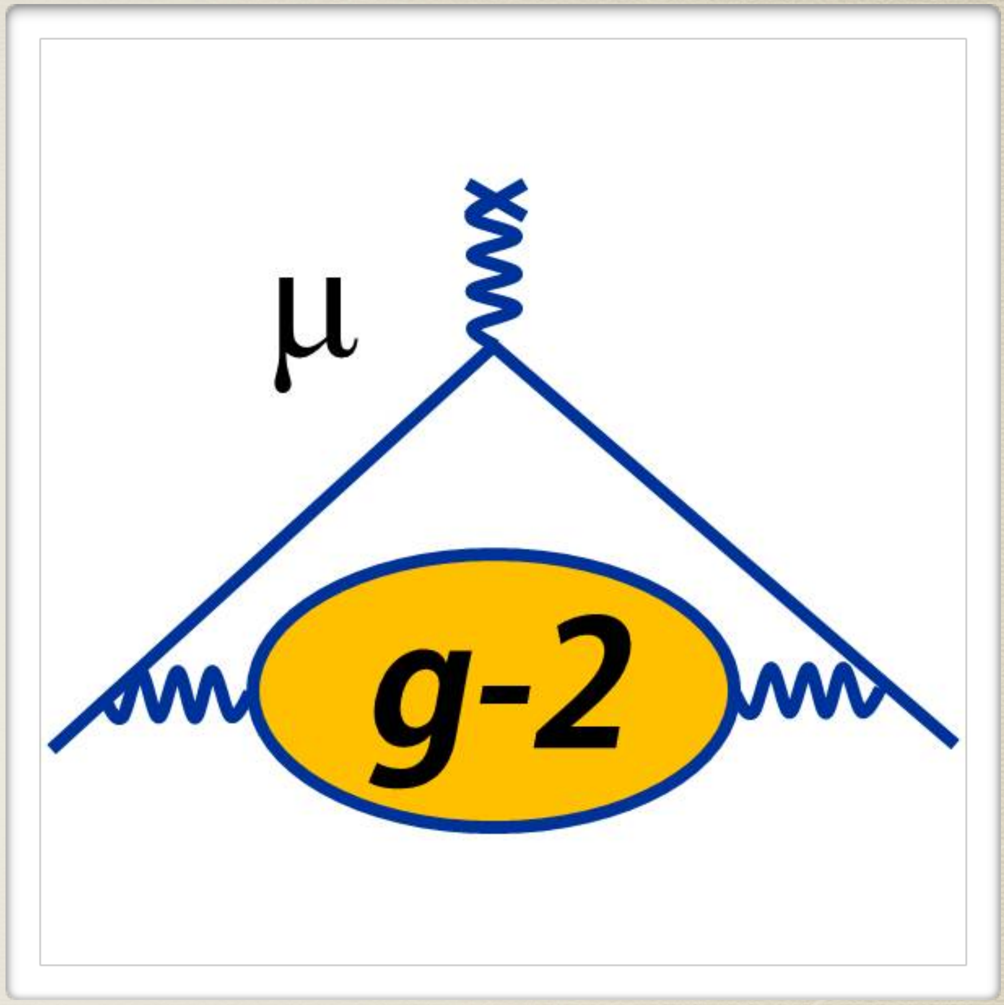
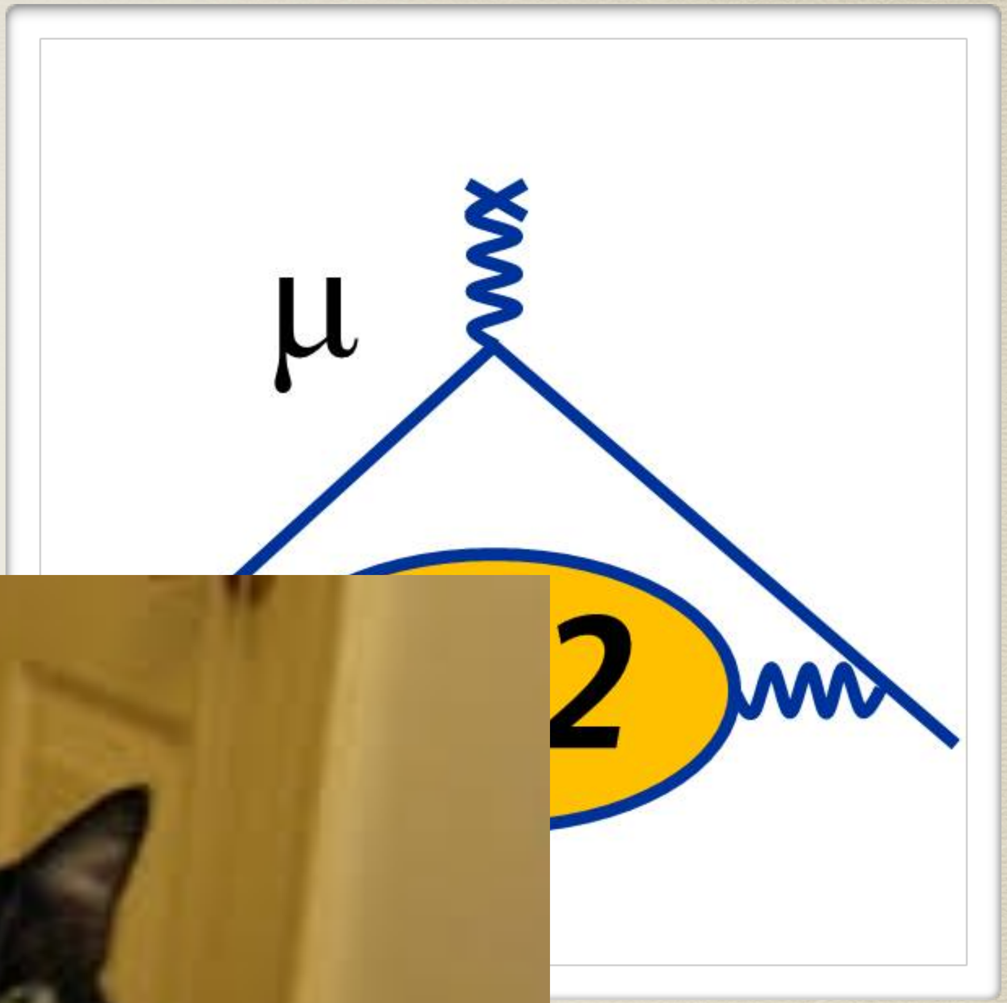


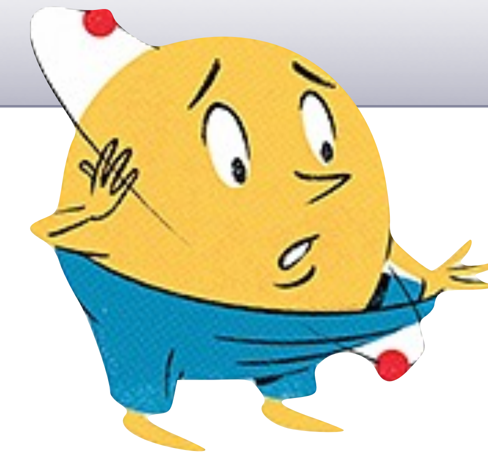
Timely applications

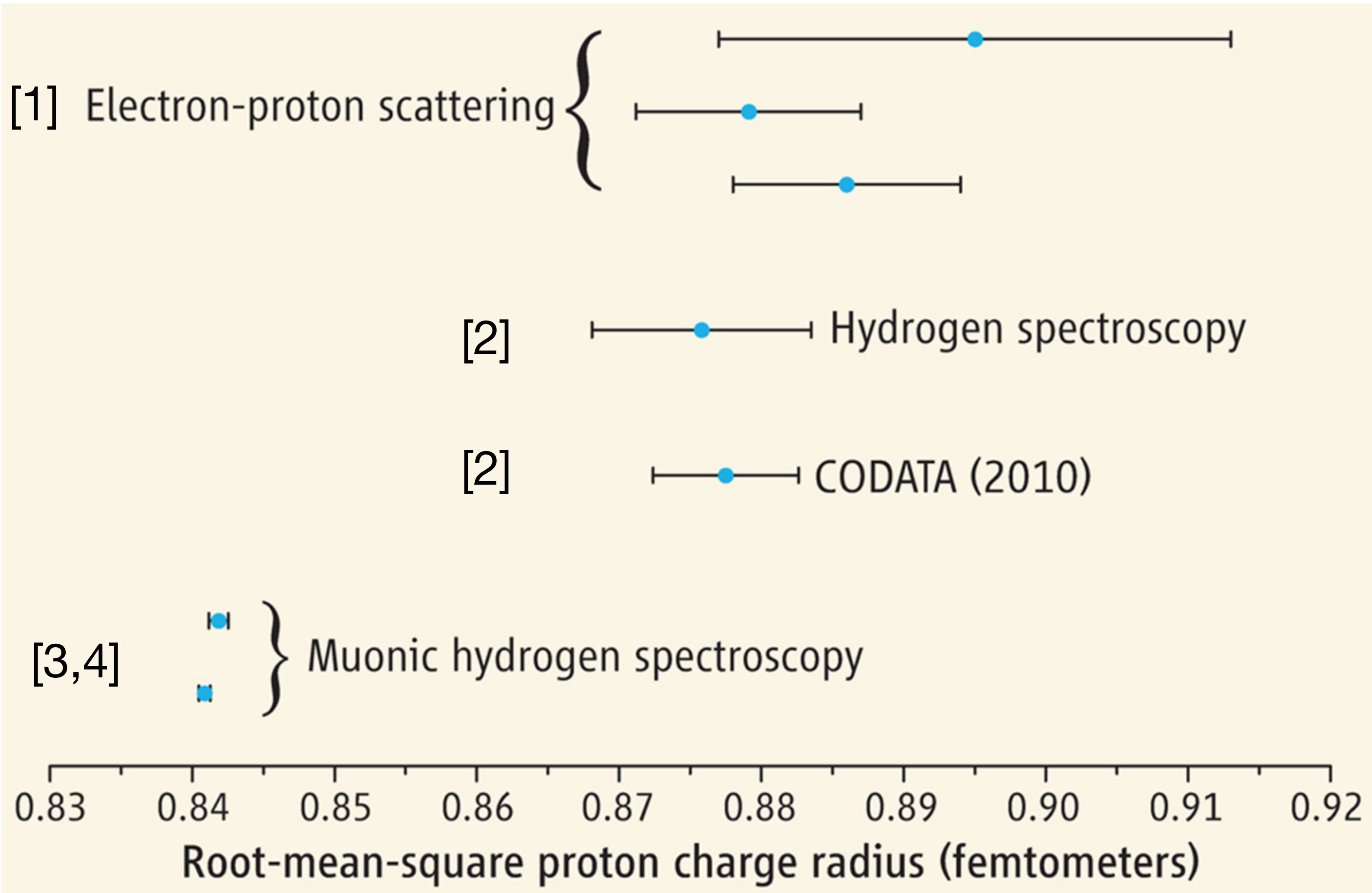


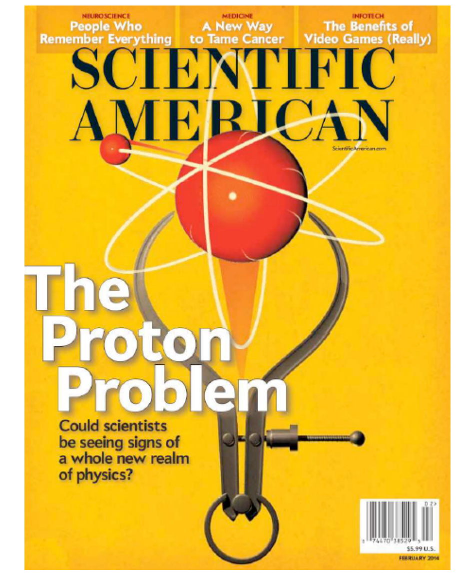
Timely applications



Proton radius puzzle





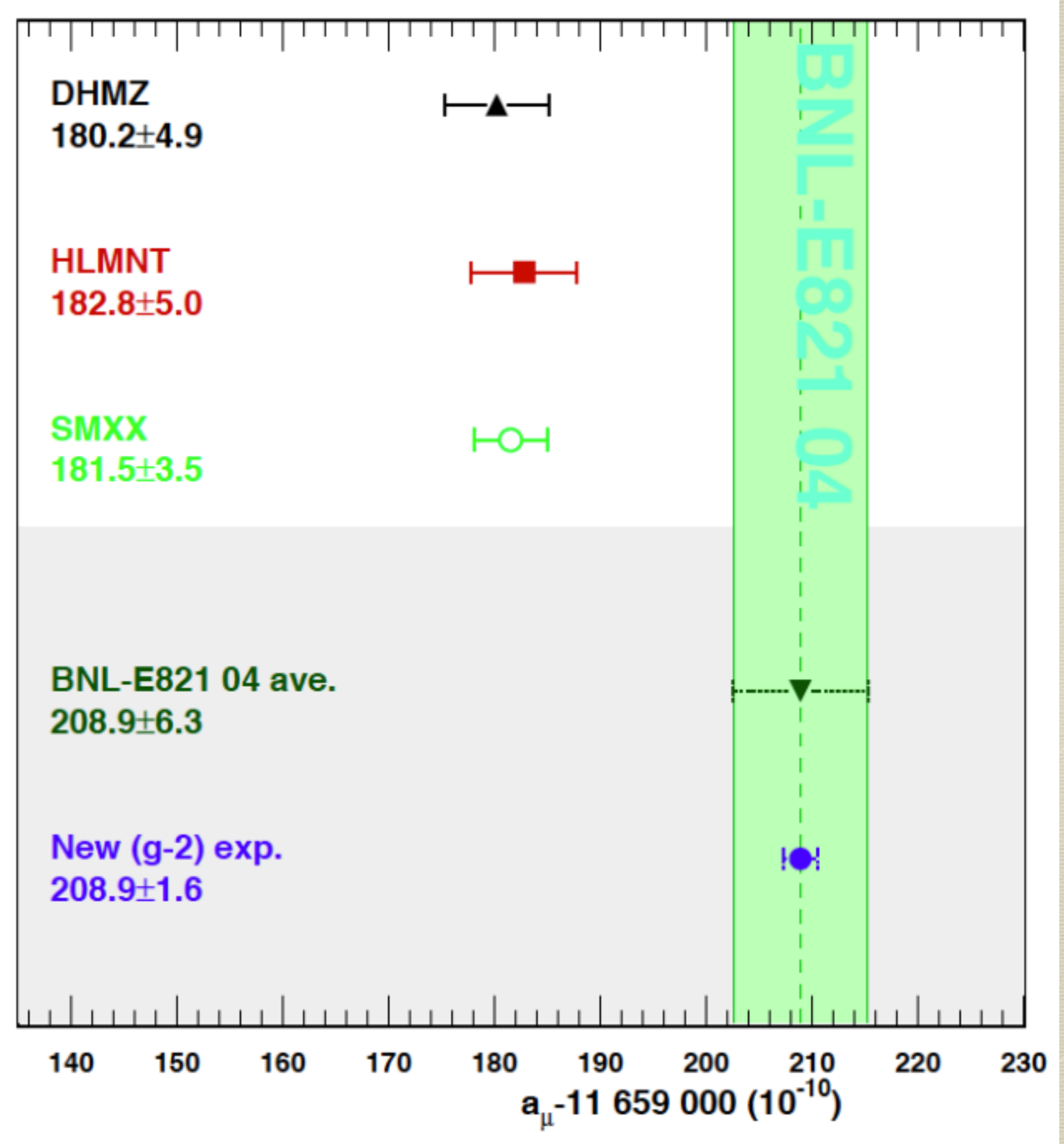


[1] J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).
[2] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).
[3] R. Pohl, A. Antognini *et al.*, Nature **466**, 213 (2010).
[4] A. Antognini *et al.*, Science **339**, 417 (2013).

7σ discrepancy

$$[R_E^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}] \longleftrightarrow [R_E^{\mathrm{CODATA\ 2010}} = 0.8775(51)\ \mathrm{fm}]$$

Muon anomaly

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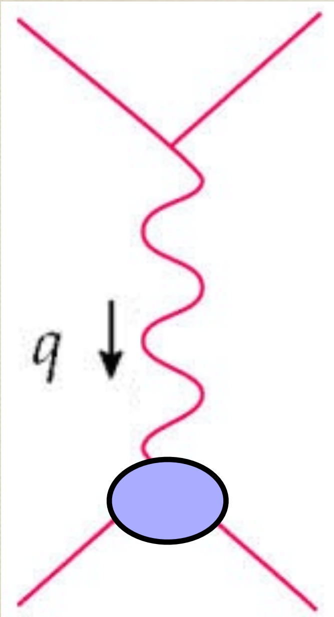
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Proton structure in hydrogen—finite-size effect

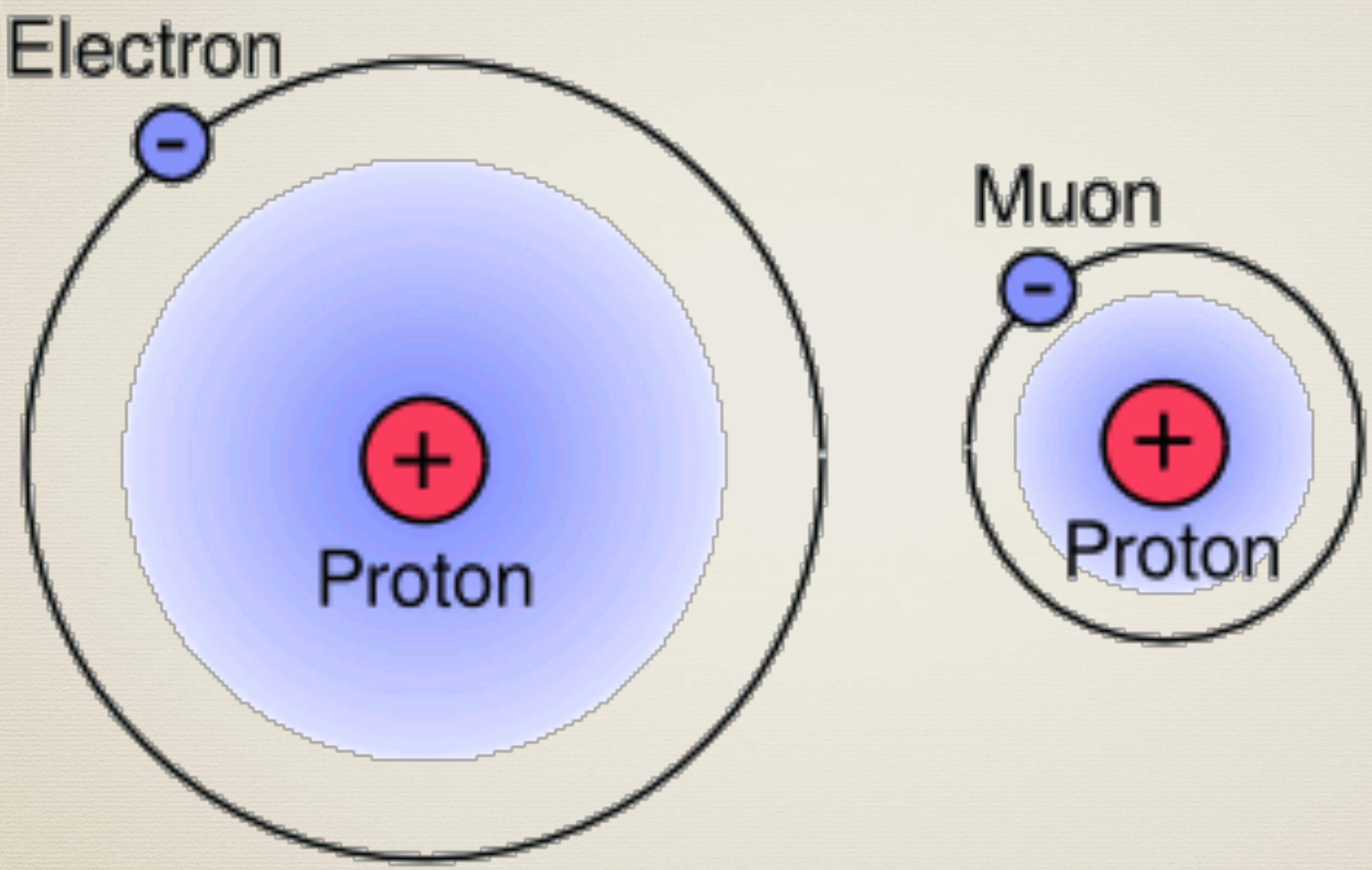


$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}\left[G_E(-\vec{q}^{\,2}) - 1\right] = \frac{2}{3}\pi\alpha\, r_E^2 + O(\vec{q}^{\,2})$$

$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm \right| \delta V^{(1\gamma)} \left| nlm \right\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, r_E^2\, \frac{\alpha^3 m_r^3}{\pi n^3} + O(\alpha^5)$$

wave function at origin

Normal vs. muonic hydrogen



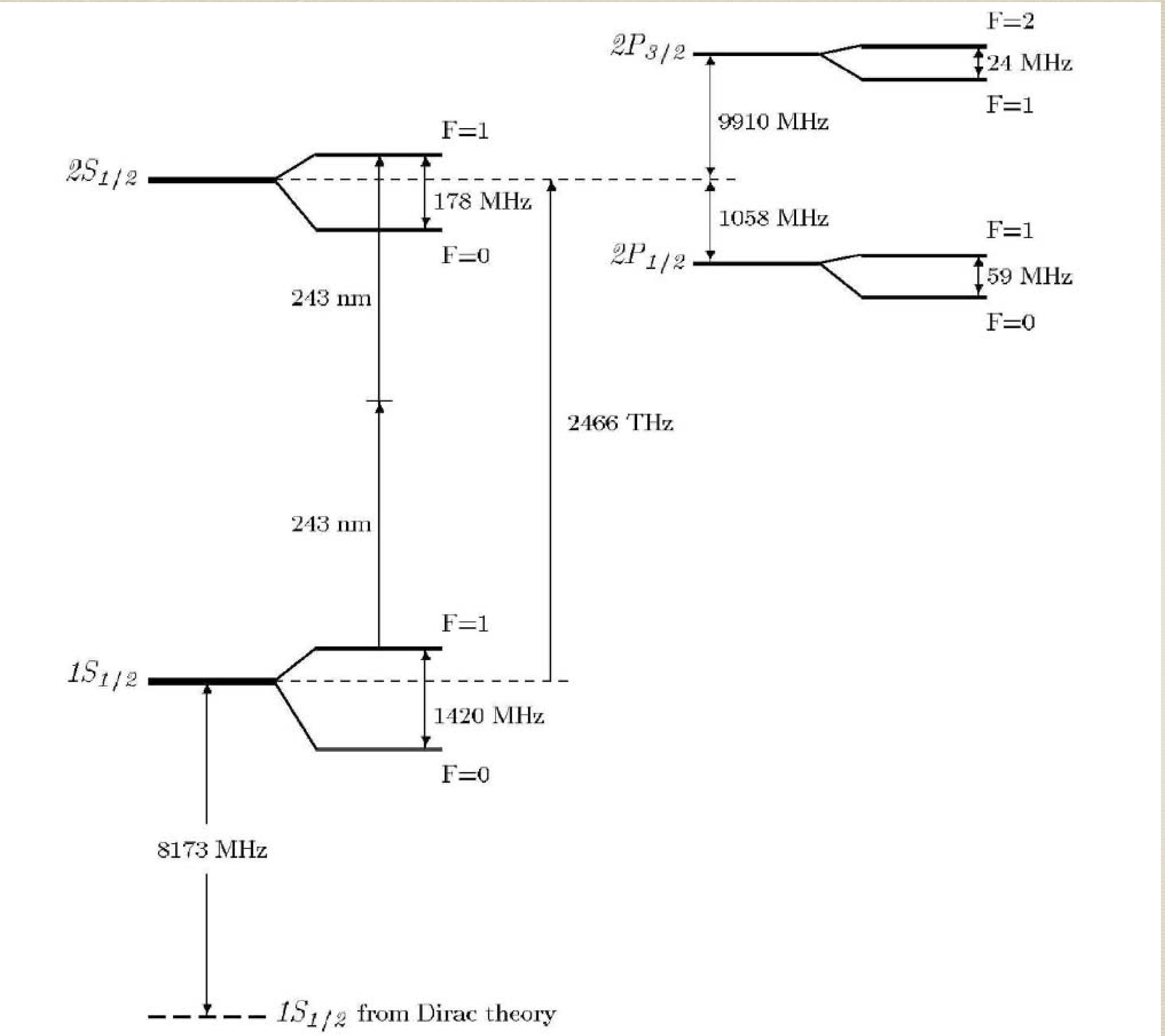
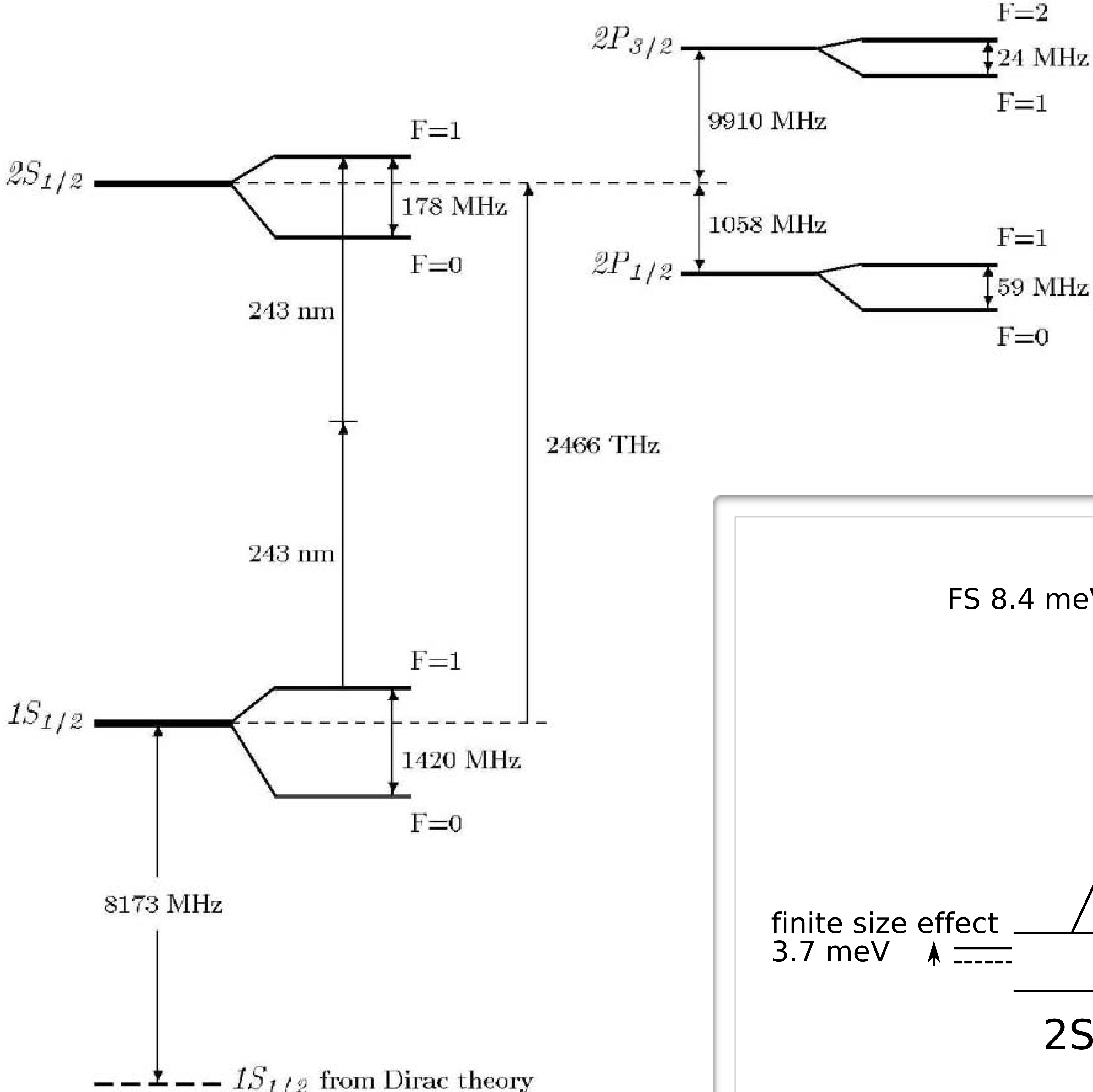


Fig. 1.1. Hydrogen energy levels

1 neV $\simeq$ 1.5 MHz

$1\ \text{neV} \simeq 1.5\ \text{MHz}$



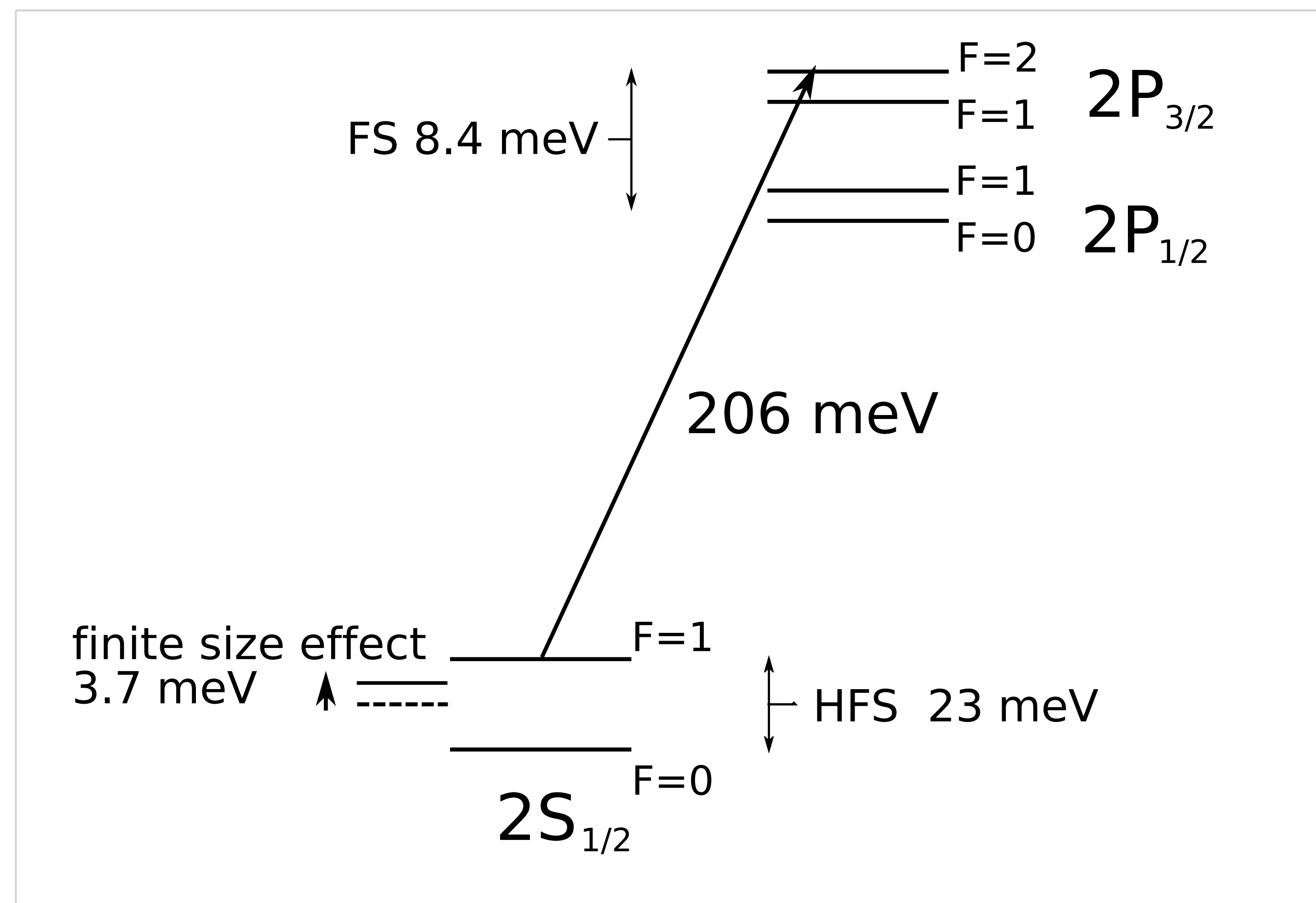
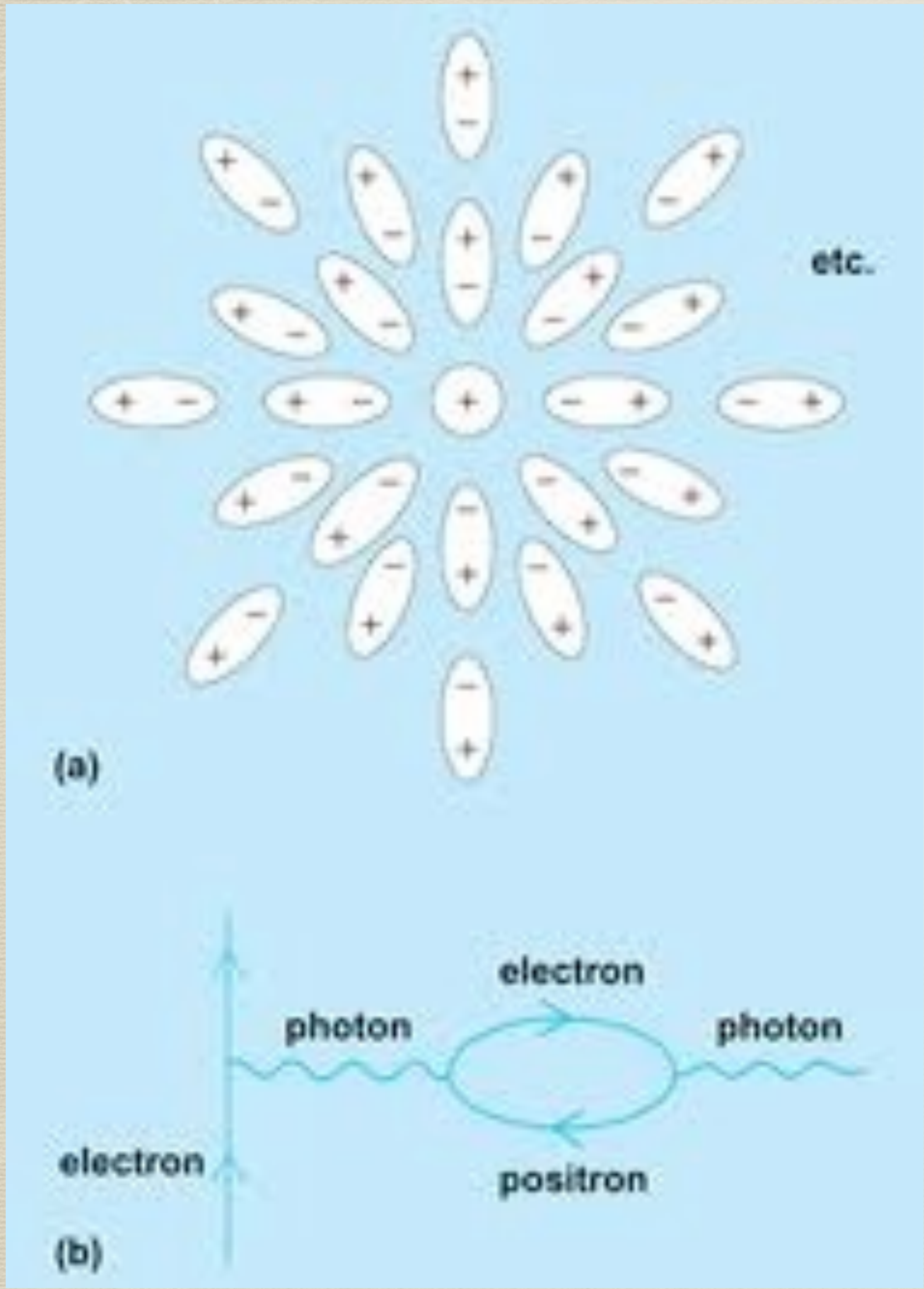


Fig. 1.1. Hydrogen energy levels

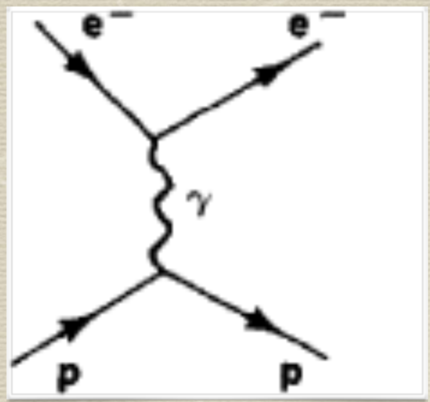
Vacuum polarization



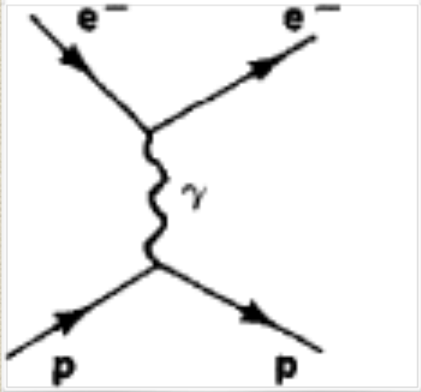
$$\Pi^{\mu\nu}(q) = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$

$$q_\mu \Pi^{\mu\nu}(q) = 0 = q_\nu \Pi^{\mu\nu}(q)$$

$$\Pi^{\mu\nu}(q) = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$



$$V_C(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} = \frac{e_1 e_2}{4\pi r} = -\frac{\alpha}{r}$$



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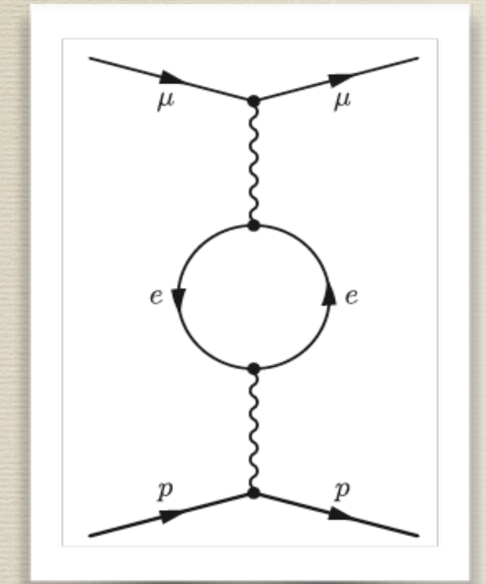

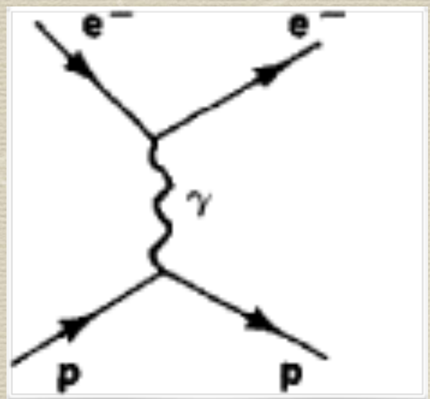

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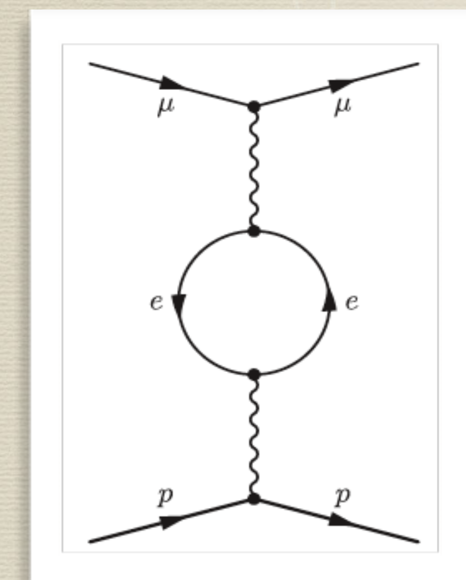
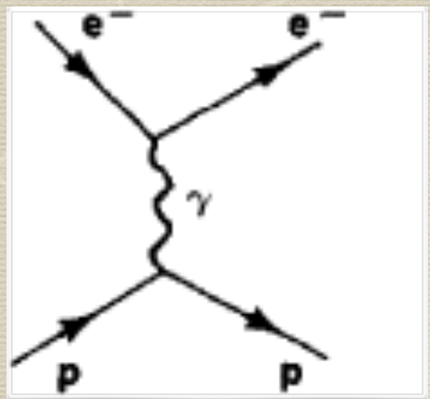


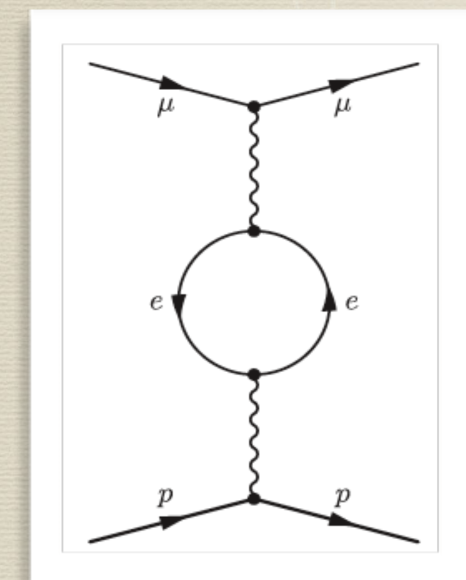
$$\delta V_C^{(\mathrm{V.P.})}(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} \Pi(-\vec{q}^2) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t+\vec{q}^2}$$

$$= \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} \frac{e_1 e_2}{4\pi r} - \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} \frac{e_1 e_2}{4\pi r} e^{-r\sqrt{t}}$$

$$= V_C^{(renorm.)} + \frac{\alpha}{r} \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} e^{-r\sqrt{t}}$$



$$V_C(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} = \frac{e_1 e_2}{4\pi r} = -\frac{\alpha}{r}$$



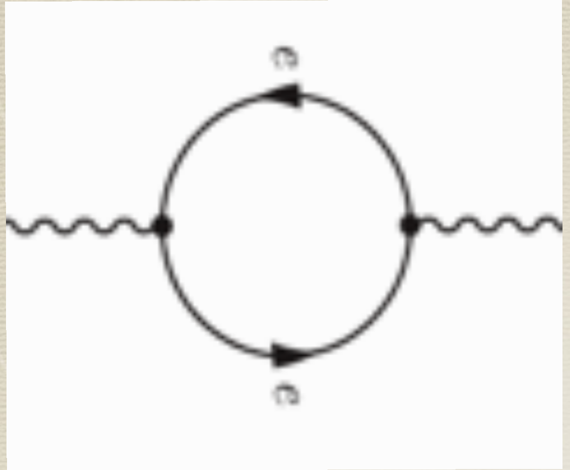
$$\delta V_C^{(\mathrm{V.P.})}(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} \Pi(-\vec{q}^2) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{e_1 e_2}{\vec{q}^2} \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t + \vec{q}^2}$$

$$= \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} \frac{e_1 e_2}{4\pi r} - \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} \frac{e_1 e_2}{4\pi r} e^{-r\sqrt{t}}$$

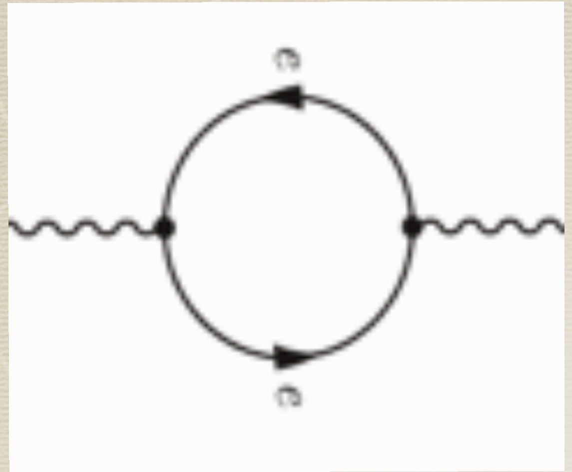
$$= V_C^{(renorm.)} + \frac{\alpha}{r} \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} e^{-r\sqrt{t}}$$

Modified Coulomb potential:

$$\tilde{V}_C(r) = -\frac{\alpha}{r} \left[1 - \frac{1}{\pi} \int_0^\infty dt \, \frac{\mathrm{Im}\,\Pi(t)}{t} e^{-r\sqrt{t}} \right]$$



$$\text{QED :} \quad \mathrm{Im}\,\Pi(t) = -\frac{\alpha}{3}\sqrt{1-\frac{4m_\ell^2}{t}}\left(1+\frac{2m_\ell^2}{t}\right) + O(\alpha^2)$$



$$\text{QED}: \quad \mathrm{Im}\,\Pi(t) = -\frac{\alpha}{3}\sqrt{1-\frac{4m_\ell^2}{t}}\left(1+\frac{2m_\ell^2}{t}\right) + O(\alpha^2)$$

Exercise:

from $$\Pi^{\mu\nu}(q) \quad = \quad ie^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{tr}\left[\gamma^\mu \frac{\not{k} + m}{k^2 - m^2} \gamma^\nu \frac{\not{q} + \not{k} + m}{(q+k)^2 - m^2}\right]$$

show

1) $$\Pi^{\mu\nu}(q) = (q^2 \eta^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$

2) $$\Pi(q^2) = \frac{q^2}{\pi}\int_{4m^2}^{\infty} dt \frac{\mathrm{Im}\,\Pi(t)}{t(t-q^2)}$$