Properties of Nuclear Matter

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1. Equation of state (EoS) of nuclear matter: from Walecka model to thermodynamically self-consistent models (1.5 hours).

2. Induced surface tension EoS for nuclear and hadronic matter and quantum virial coefficients (1.5 hours).

3. Statistical multifragmentation model (SMM) of atomic nuclei, its exact analytical solution and nuclear liquid-gas phase transition (1 hour).

4. Critical exponents of classical and statistical EoS (1 hour).

Truskavets, October 11, 2018

Why Nuclear Matter?

1. Since it is sufficiently simple object and can be studied at low and intermediate energies of nuclear reactions

2. Nuclear Matter has a liquid-gas phase transition and hence it can be used as a realistic test site to verify the ideas on phase transition signals in finite systems

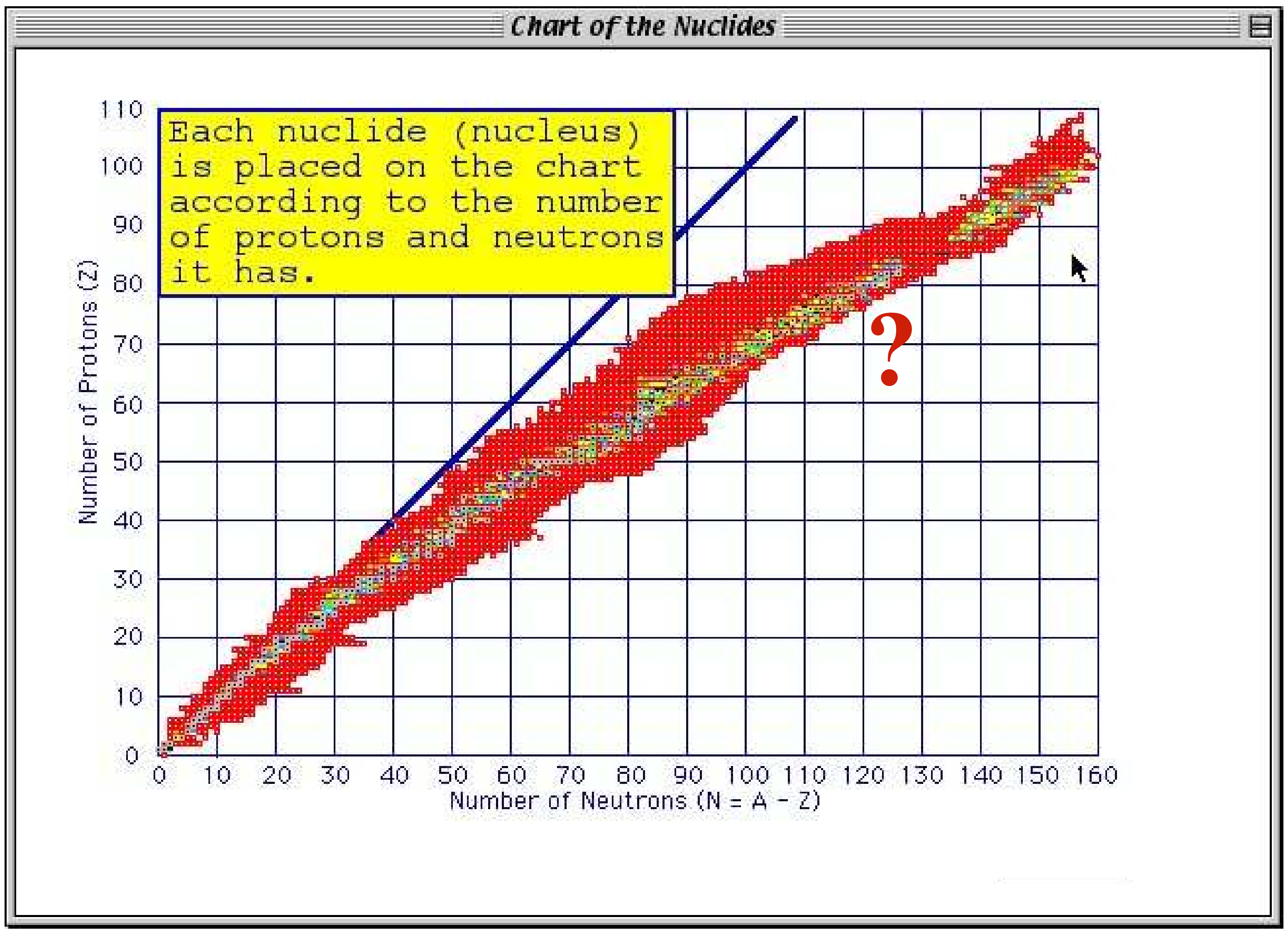
3. Still it is located at the frontier:

   superheavy elements;
   vacuum e.-m. instability against e+e- production, if total electric charge in reaction > 137;
   reactions with radioactive nuclei;
   neutron stars equation of state (EoS)

4. It has plenty of various applications in our life!

N-Z diagrams of the atomic nuclei

superheavy
elements



~ 400
stable isotops

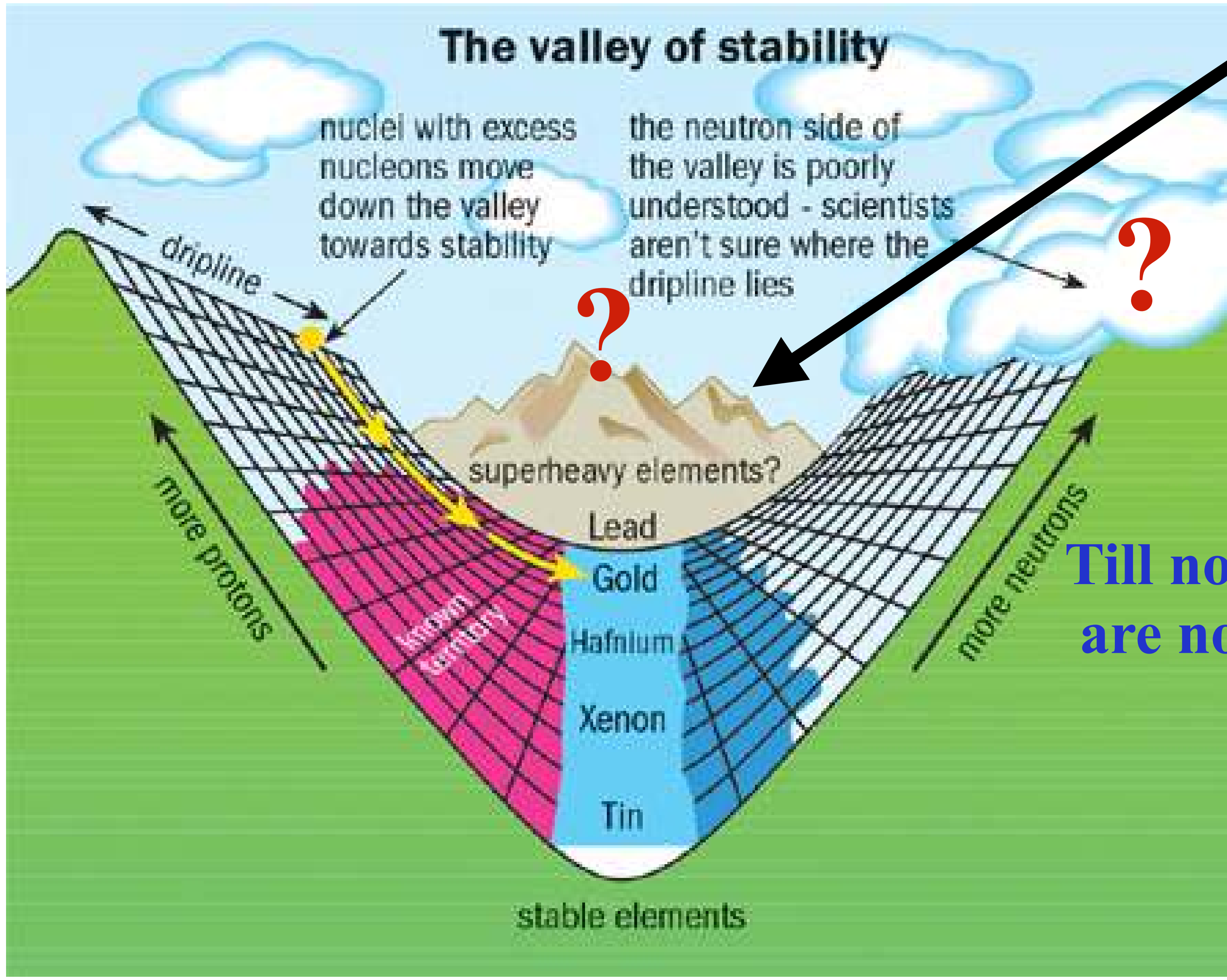
The valley of stability

$$Z = \frac{A}{1.98 + 0.015 A^{2/3}}$$

NM Frontiers

Till now we do not know whether stability island exists there!

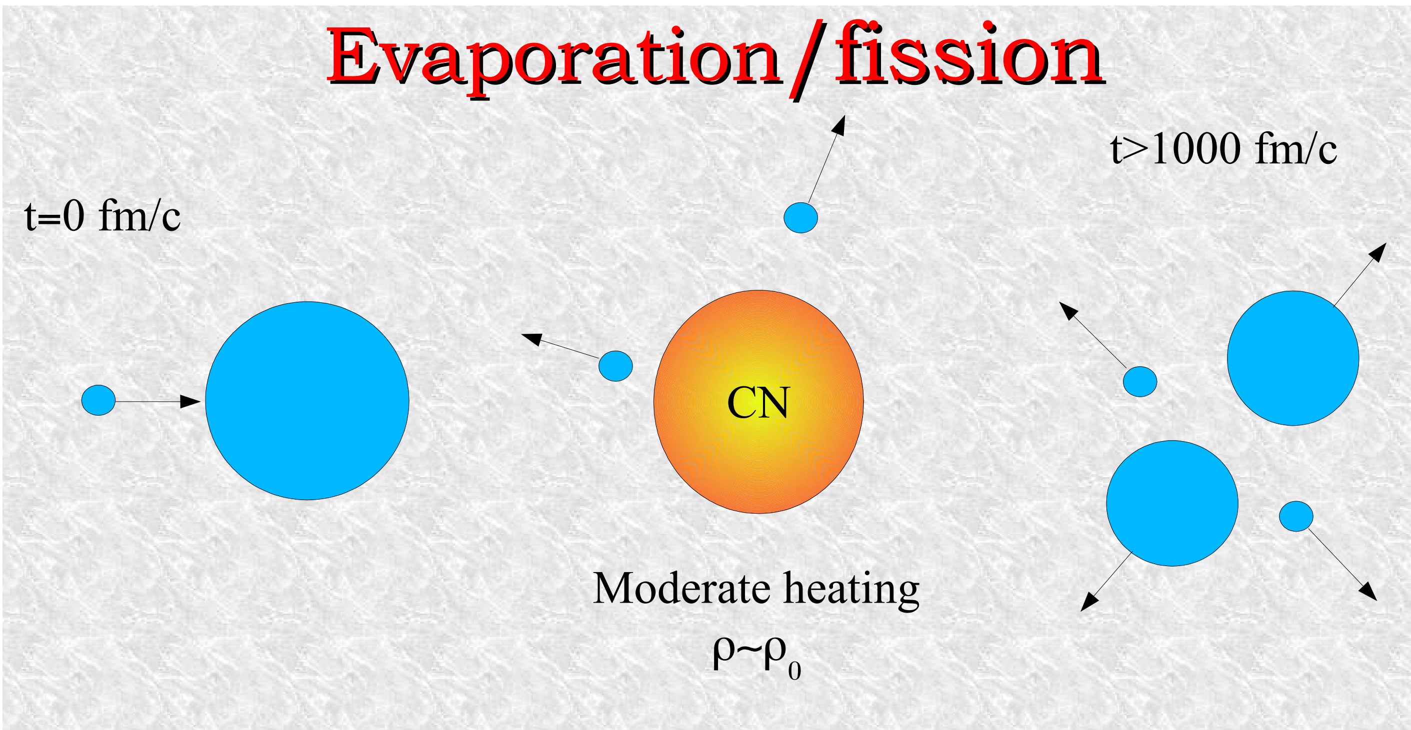
β^+ decay



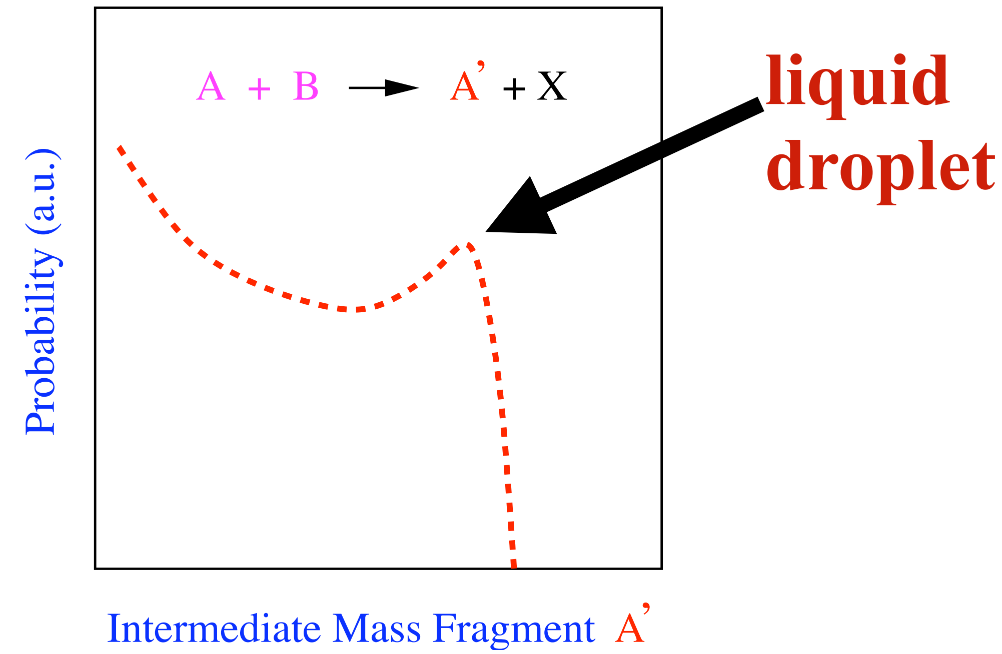
β^- decay

Till now these reactions are not well understood!

Nuclear Liquid = Compound Nucleus



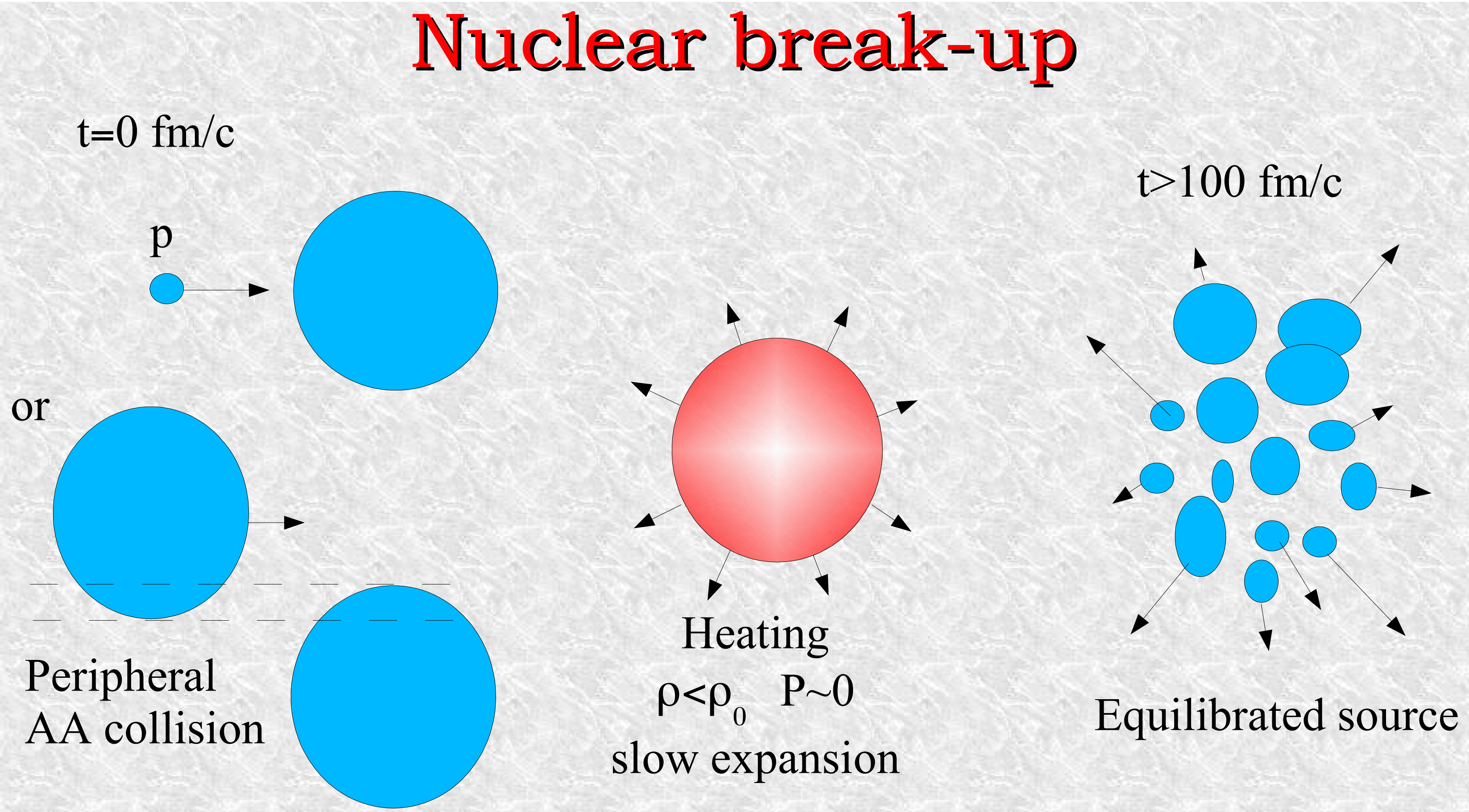
mass distribution of fragments



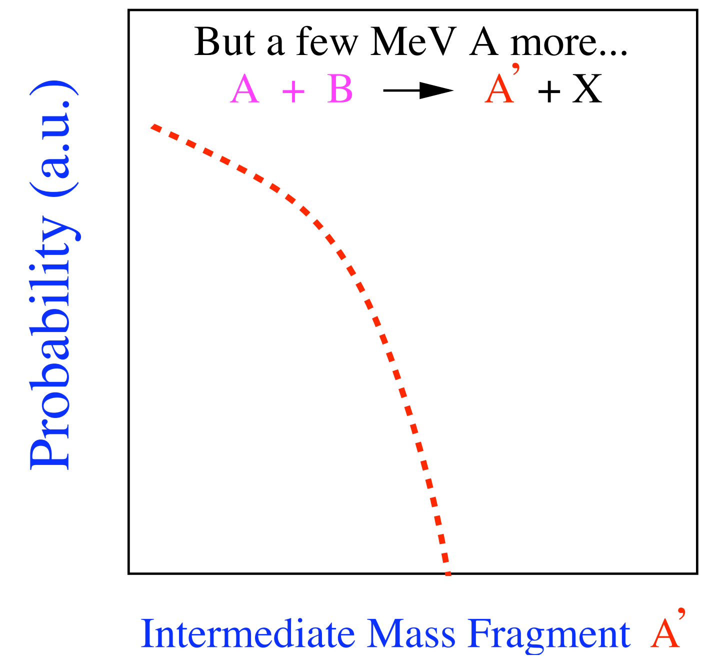
ρ_0 is normal nuclear density

Compound Nucleus (CN) is an equilibrated hot nucleus whose excitation energy is distributed over many microscopic d.o.f. (introduced by Niels Bohr in 1936-39)
Sequential evaporation model—Weiskopf 1937,
Statistical fission model—Bohr-Wheeler 1939, Frenkel 1939

Nuclear Multifragmentation = No Liquid!



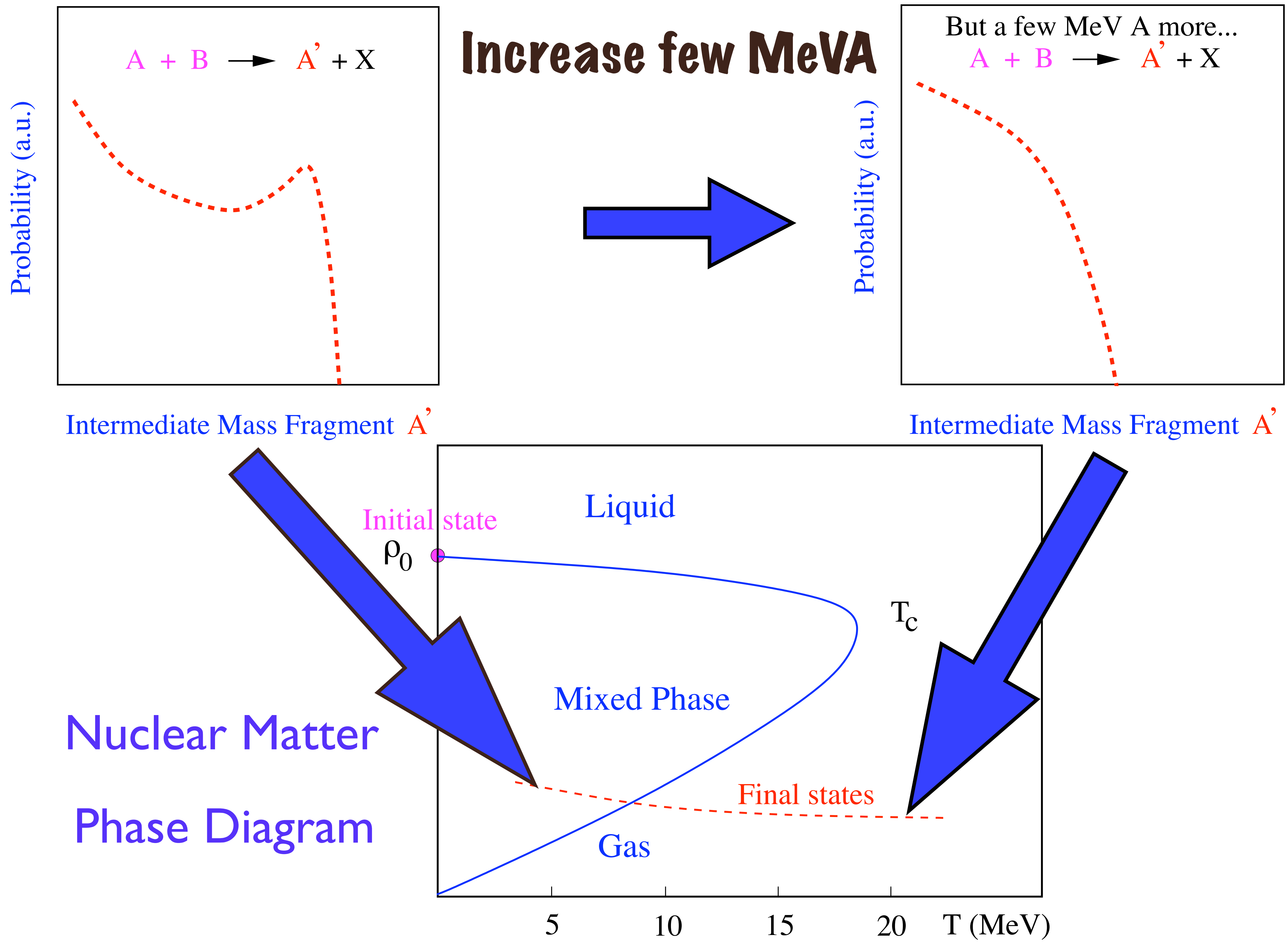
different mass distribution of fragments



Power-law fragment mass distribution around critical point, $Y(A) \sim A^{-\tau}$
Can be well understood within an equilibrium statistical approach

(Randrup&Koonin, D.H.E. Gross et al, Bondorf-Mishustin-Botvina, Hahn&Stoecker,...)

Nuclear Multifragmentation as a Phase Transition



Our Major Aims

1. **Study the nuclear liquid-gas PT using different approaches**

2. **Become familiar with mean-field approximation and statistical models of cluster type etc**

Outline of lecture I

1. Historical introduction

2. Formal definition of grand canonical ensemble (GCE) and L. van Hove axioms of statistical mechanics

3. Properties of heavy nuclei and nuclear matter

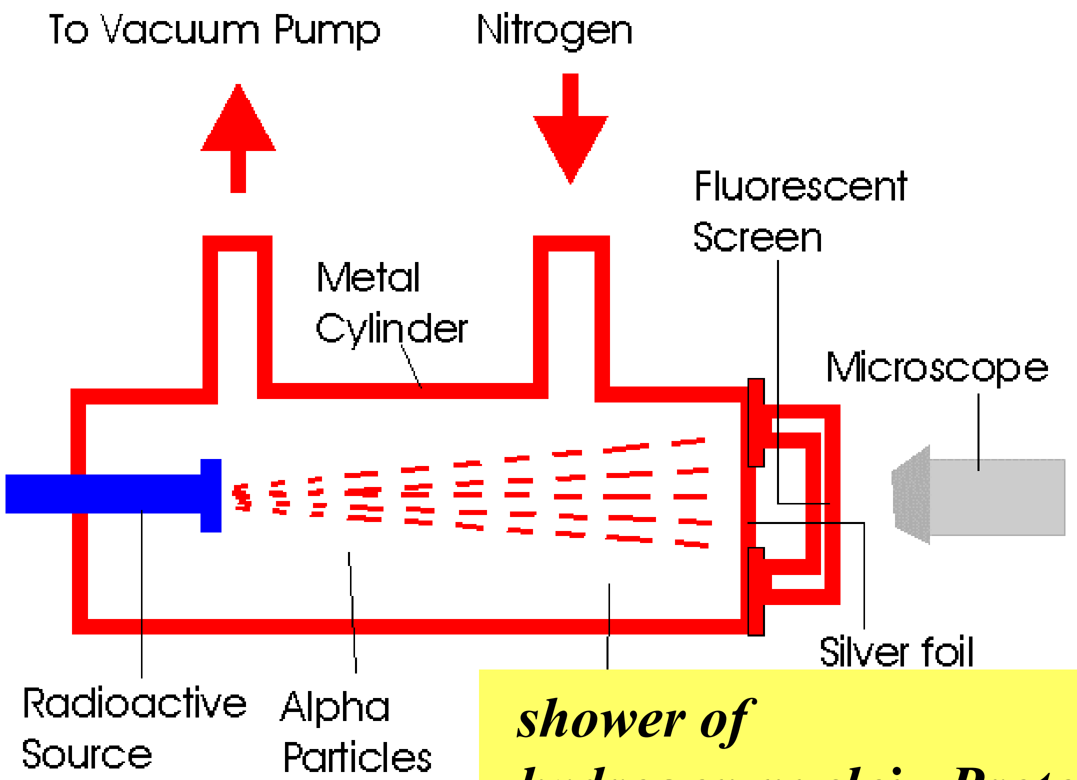
4. Walecka model and nuclear matter EoS

5. Phenomenological generalization of Walecka model

6. Summary

Strong Interaction Discovery

Discovery of the proton (Rutherford 1918)



shower of hydrogen nuclei - Protons

atomic number of hydrogen is 1

$$\alpha + {}^{14}_{7}N \rightarrow p + {}^{17}_{8}O$$

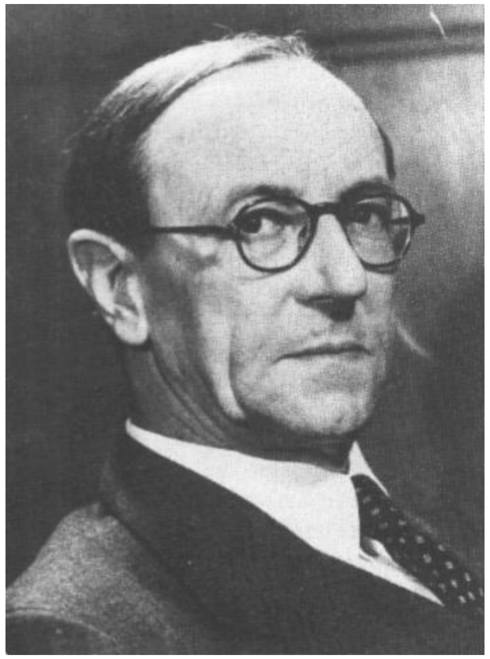
$$M_p \simeq 2000 M_e, \quad e_p = -e_e$$

Conclusion: **proton** is a ***constituent*** of a nucleus

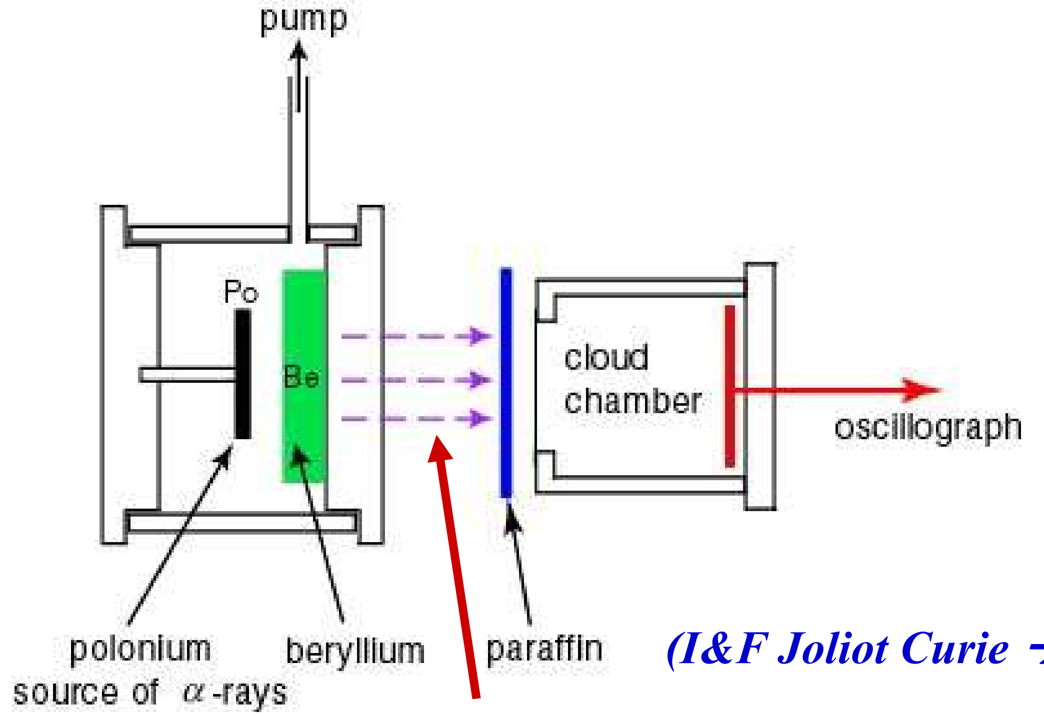
First nuclear reaction performed in a laboratory!

Strong Interaction Discovery

Discovery of the neutron (Chadwick 1932)



James Chadwick
(1891-1974)



(I&F Joliot Curie → p flux)

unknown high penetrating radiation (neutral particle)

$$x + p \to x' + p'$$

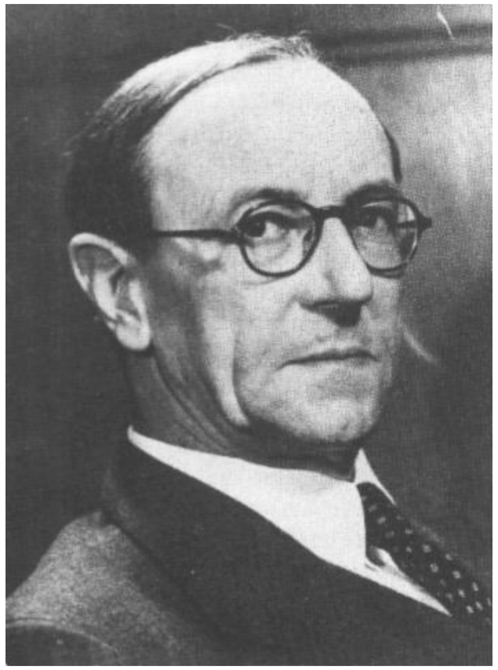
$$p_{p'} \simeq \frac{2 p_x M_p \cos\theta}{E_x + M_p}$$

→ $20 \cos\theta$ MeV/c ($x = \gamma$) with $M_x = 0$

→ $280 \cos\theta$ MeV/c ($x = n$) with $M_x \simeq M_p$

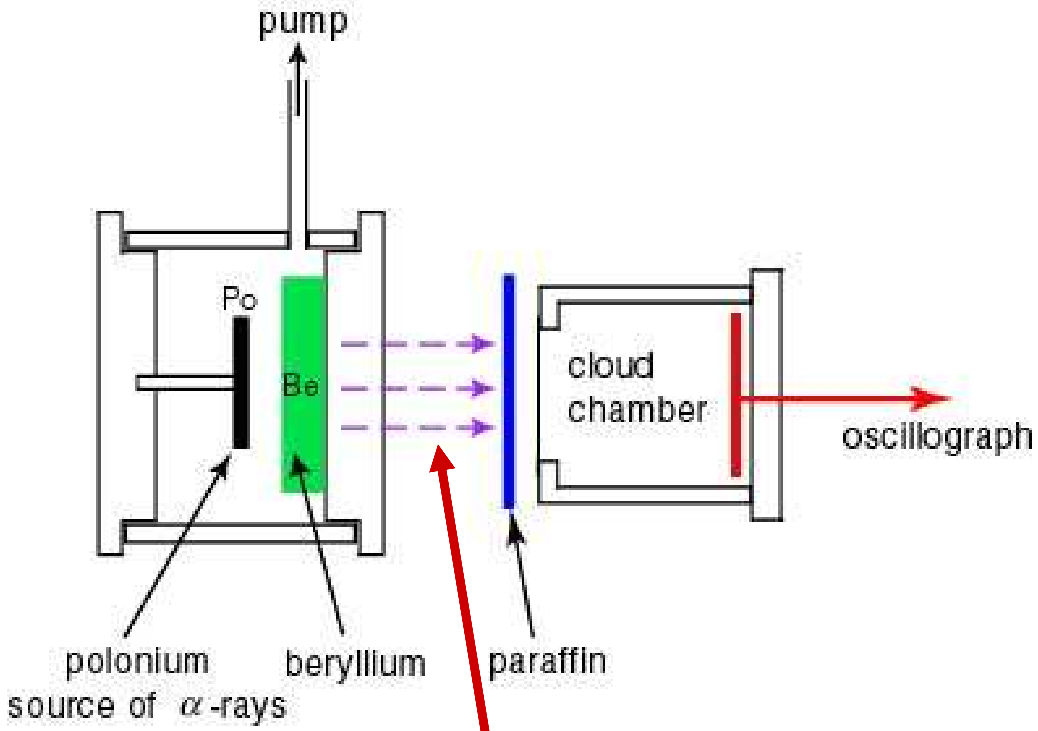
Strong Interaction Discovery

Discovery of the neutron (Chadwick 1932)



James Chadwick
(1891-1974)

The Nobel Prize
in Physics 1935



unknown neutral particle = ***neutron***

$$\alpha + {}^{9}_{4}Be \rightarrow n + {}^{12}_{6}C$$

$$M_n \simeq M_p$$

neutron is a ***constituent*** of a nucleus

Strong Interaction Discovery

Hideki Yukawa
(1907-1981)

Nobel Prize 1949

Prediction of pion (π)

In 1935 H. Yukawa introduced interaction between nucleons (proton and neutron) in nucleus. ***"Nucleons (protons and neutrons) are held together by force stronger than electrostatic repulsion of protons"***

$$U = U_o \frac{\exp(-m_\pi r/hc)}{r}$$

In fact, Yukawa predicted a *new particle* –quanta of strong interaction ***meson,*** with mass

$$m_\pi \simeq \frac{\hbar c}{R} \simeq \frac{200\text{MeV}\cdot\text{fm}}{1\div 2(\text{fm})} = 100 \div 200 \text{ MeV}$$

Discovery of muons came first…

Discovery of muon: J. Street and E. Stevenson 1937

They found (in cloud chamber) penetrating cosmic ray tracks with unit charge but mass in between electron and proton (Yukawa particle?).

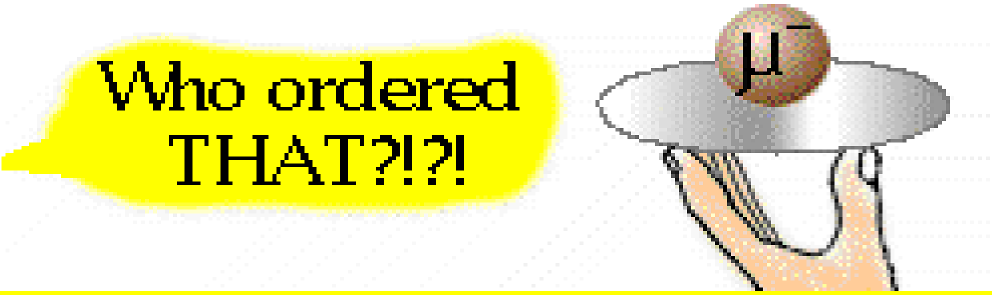
Muons were proven not to have any nuclear interactions and to be just heavier versions of electrons.

H. Bethe and R. Marshak suggested that the muon might be the decay product of the particle needed in the Yukawa theory, so the search of Yukawa particle was continued.

Later was known that μ meson decays to electron and two invisible neutrinos via weak interactions (β decay): $\mu \rightarrow 2\nu + e$.

Particle	*Electric charge (x 1.6 10^{-19} C)*	*Mass (GeV=x 1.86 10^{-27} kg)*
e	−1	0.0005
μ	−1	0.106
p	+1	0.938
n	0	0.940
γ	0	0

elementary particles by 1938



Why does the Nature need muons?

Discovery of pion

Cecil F. Powell
(1903-1969)
Nobel Prize 1950

Discovery of pion (1947)

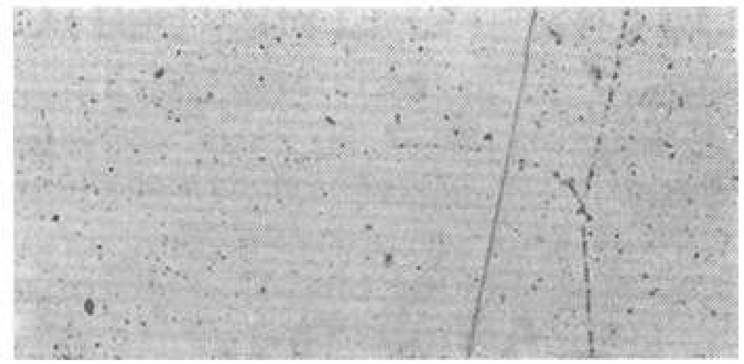


Fig. 1 *a*. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK OF MESON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSION OBJECTIVE. × 500)

details on the next slide

- Detected in cosmic rays captured in photographic emulsion.
- Unlike muons, they do interact with nuclei.
- Charged pions eventually decay to muons:
  $\pi \rightarrow \mu + \nu$.

$\tau_{\pi^\pm} \simeq 2.6 \times 10^{-8}\text{s}$ $\tau_{\pi^0} \simeq 8.4 \times 10^{-17}\text{s}$

Discovery of pion

First Pion

Nuclear capture of pion

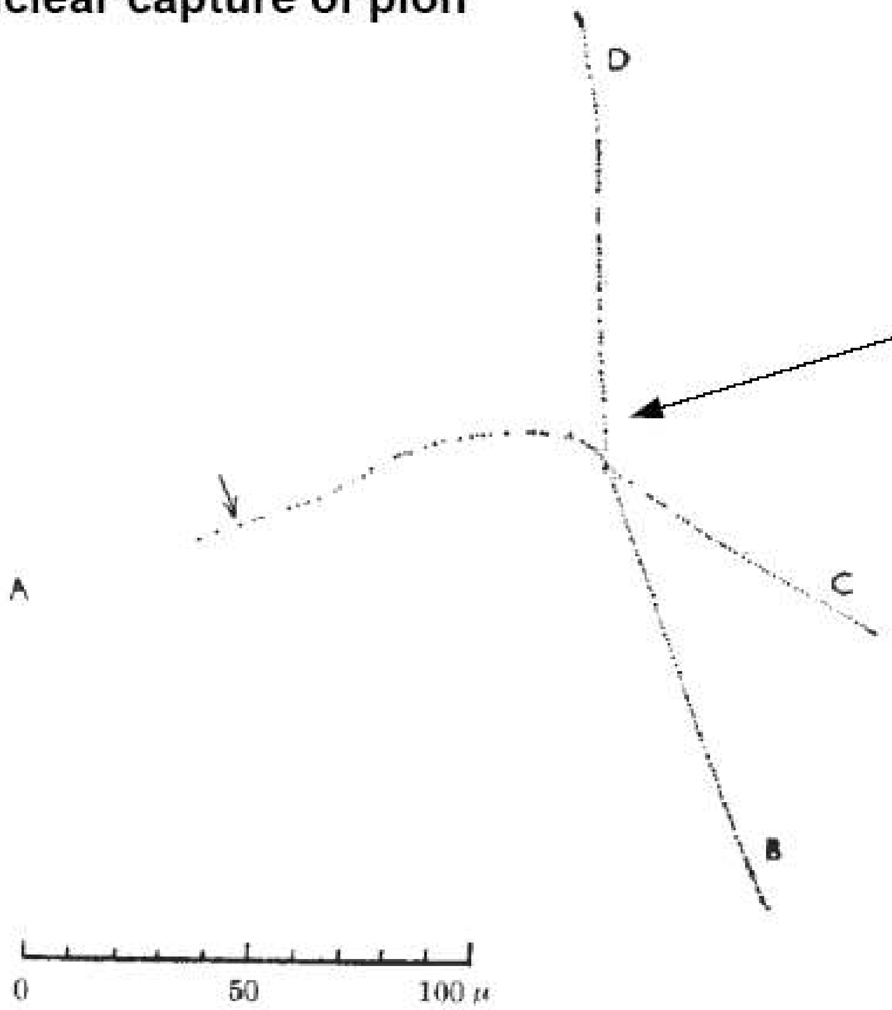
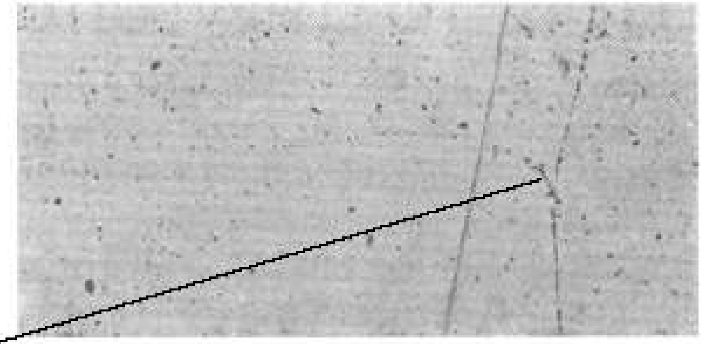


Fig. 1 b. TRACE OF COMPLETE STAR ON SCREEN OF PROJECTION MICROSCOPE, SHOWING PROJECTION OF THE TRACKS IN THE PLANE OF THE EMULSION. TRACK A CANNOT BE TRACED WITH CERTAINTY BEYOND THE ARROW

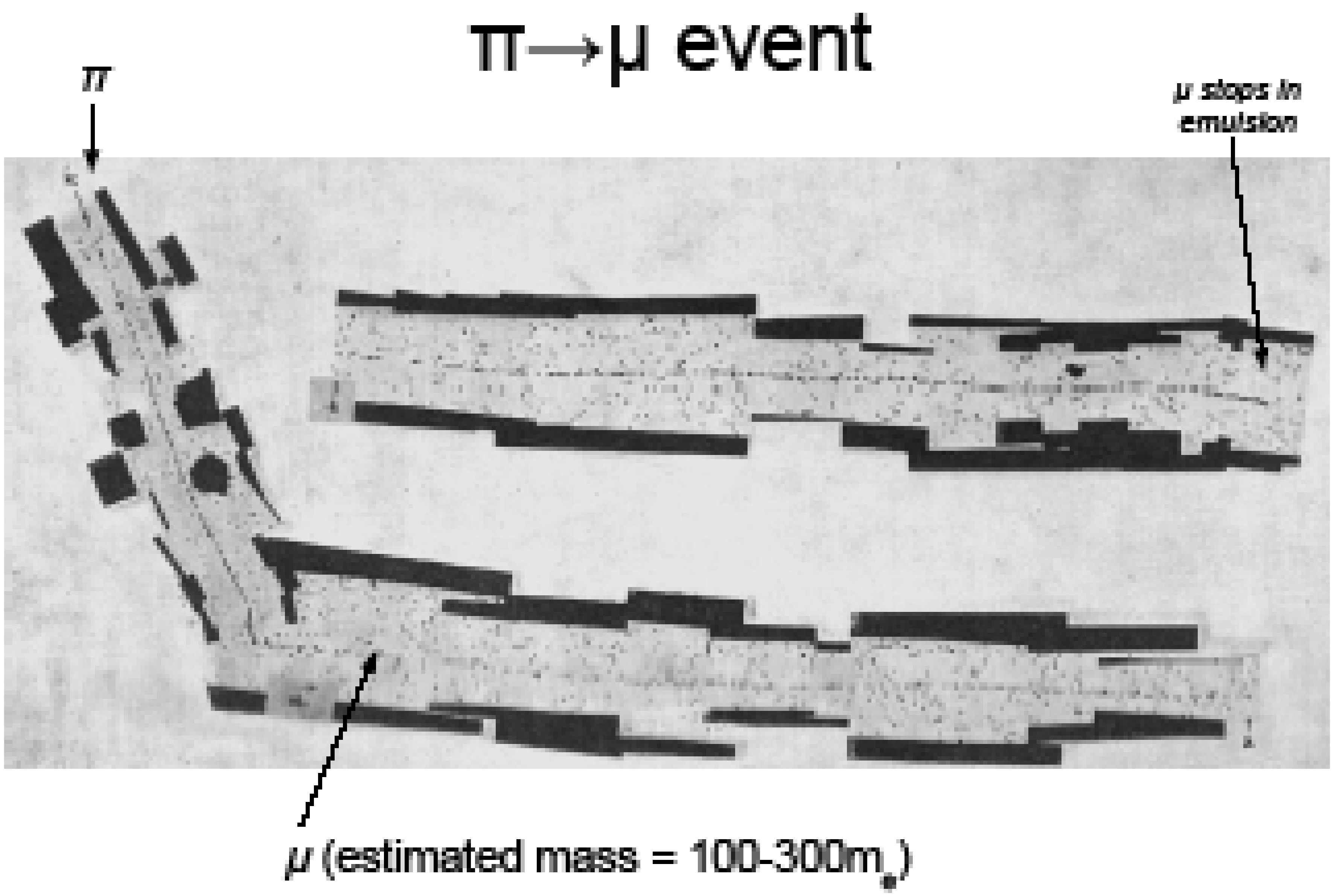


g. 1 a. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK ESON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSI OBJECTIVE. × 500)

- A is the new meson
- B,D,C are likely protons
- Track C goes into the page

Why A is a new meson:
electron: range too large
proton: scattering too large
muon: frequent nuclear interaction

Discovery of pion



Observed by Powell, Oct. 1947

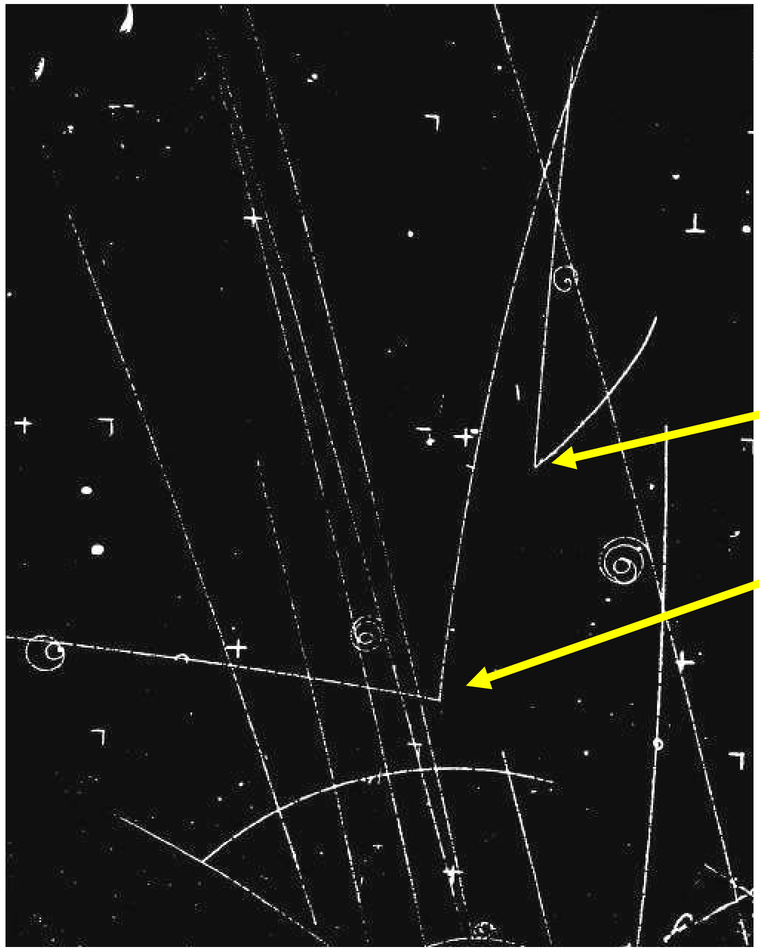
After the discovery of pion…

Elementary particles (discovered) at end of 40th

Particle	*Electric charge (x 1.6 10^{-19} C)*	*Mass (GeV=x 1.86 10^{-27} kg)*
e	-1	0.0005
μ	-1	0.106
γ	0	0
p	+1	0.938
n	0	0.940
π	+1,−1,(0)	0.14

After the discovery of pion more hadrons were found!

Strange particles 1947



$$\pi^- + p \rightarrow \Lambda + K^0$$

$$\Lambda \rightarrow p + \pi^-$$

$$K^0 \rightarrow \pi^+ + \pi^-$$

"V-particles"

$$M_K \simeq 498 \text{ MeV}$$

$$M_\Lambda \simeq 1116 \text{ MeV}$$

since them come in pairs with V-like tracks

After the discovery of pion more hadrons and hyper nuclei were found!

Why they are "strange"

1. They are produced in pairs

$$\pi^- + p \to \Lambda + K^0$$

2 The probability of a production is much greater, than probability of their decay

$$\Lambda \to p + \pi^-$$

$$K^0 \to \pi^+ + \pi^-$$

$\tau_{\text{prod}} \sim 10^{-23}$ s $\quad \tau_{\text{dec}} \sim 10^{-10}$ s

Murray Gell-Mann and **Kazuhiro Nishijima** introduced new quantum number: "***Strangeness***"

$$S_\Lambda = -1$$

$$S_{K^0} = +1$$

… and concluded that "Strangeness is conserved in strong interactions (production) and violated in weak interactions (decay)

$\Sigma^\pm, \Sigma^0, \Xi^-, \Xi^0, \Omega, \ldots \qquad {}_\Lambda A, {}_{\Lambda\Lambda}A$

hypernuclei!

NB: Hagedorn Spectrum Follows from

Stat.Bootstrap Model,
S.Frautschi, 1971

Hadrons are built from hadrons

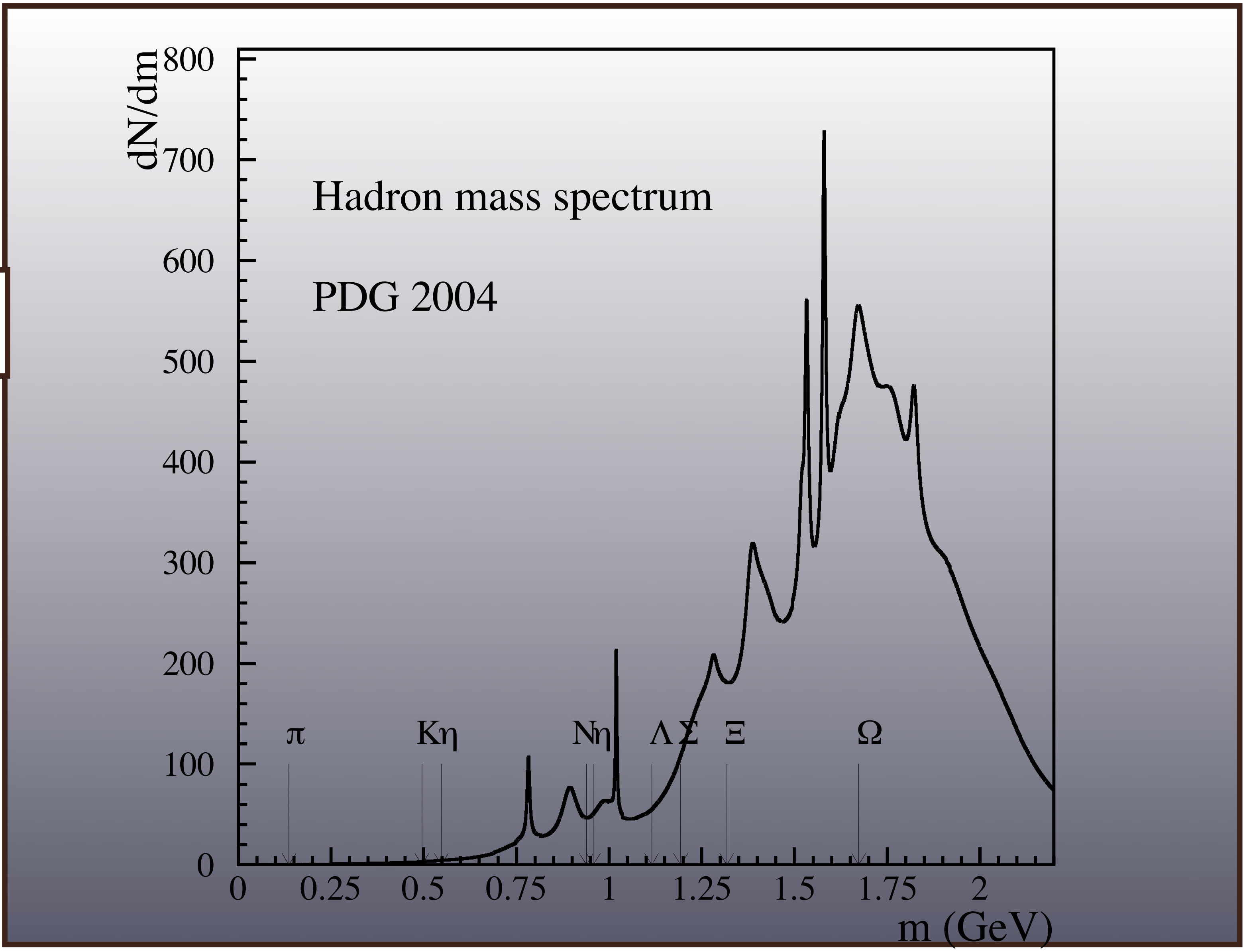
Veneziano Model,
K.Huang,S.Weinberg,
1970

Used in string models

M.I.T. Bag Model,
J.Kapusta, 1981

Hadrons are quark-gluon bags

Large Nc limit of 3+1
QCD
T. Cohen, 2009



- But experimentally it is not seen...

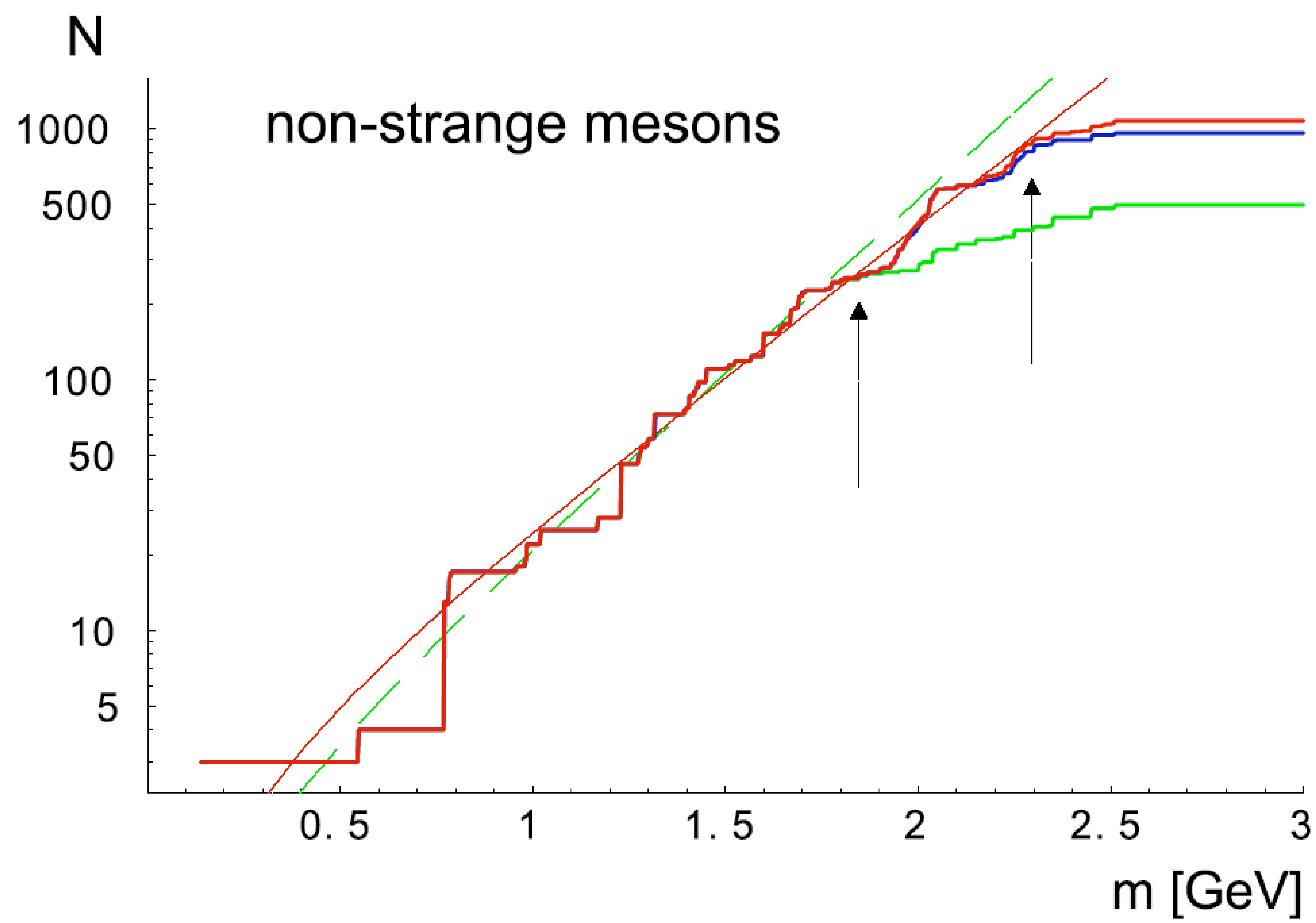
...Hagedorn Spectrum is seen, but not where it is supposed to be!

Consider the integral of experimental density of hadronic states:

$$N_{\exp}(m) = \sum_i g_i \Theta(m - m_i),$$

- It is exponential for 1 GeV< m< 2.5 GeV!
- What is above masses of 2.5 GeV ?

Can QCD models explain this?



K. A.Bugaev, V. K. Petrov and G. M. Zinovjev,

Europhys. Lett. 85, (2009) 22002; and PRC 79 (2009)

Strong Interaction

Yukava interaction is not a fundamental one

The fundamental interaction of strongly interacting particles is due to colored gluons, the source of color interaction are quarks

Yukava interaction is not sufficient to construct nuclei, we need strong repulsion at very short range and moderate range attraction

Necessary Apparatus

Microcanonical Ensemble (MCE) of N Boltzmann particles

$$Z_{mc}(E,N,V) = \frac{1}{N!}\int \prod_{k=1}^{N} \frac{d^3x_k d^3p_k}{(2\pi\hbar)^3}\delta\left(E - \sum_j \sqrt{m^2+p_k^2} - \underbrace{\sum_{l,j} U(x_l,x_j)}_{potential\ \ energy}\right)$$

EXACTLY conserves energy E and number of particles N (or charge)

x_k and p_k are, respectively, coordinate and momentum of particle k

Canonical Ensemble (CE) of N Boltzmann particles

$$Z_{ce}(T,N,V) \;=\; \int_0^{\infty} d\,E\,e^{-\frac{E}{T}}\,Z_{mc}(E,N,V) =$$

$$= \frac{1}{N!}\int \prod_{k=1}^{N} \frac{d^3x_k d^3p_k}{(2\pi\hbar)^3}\exp\left[-\frac{\sum_j \sqrt{m^2+p_k^2} - \sum_{l,j} U(x_l,x_j)}{T}\right]$$

conserves E on average, but number of particles N (or charge) EXACTLY

Grand Canonical Ensemble

Grandcanonical Ensemble (GCE) of Boltzmann particles

$$Z_{gc}(T,\mu,V) = \sum_{N=0}^{\infty} e^{\frac{\mu N}{T}} Z_{ce}(E,N,V) =$$

$$= \sum_{N=0}^{\infty} \frac{e^{\frac{\mu N}{T}}}{N!} \int \prod_{k=1}^{N} \frac{d^3x_k d^3p_k}{(2\pi\hbar)^3} \exp\left[-\frac{\sum_j \sqrt{m^2+p_k^2} - \sum_{l,j} U(x_l,x_j)}{T}\right]$$

conserves E and N on average only!

Pressure is defined as

$$\text{thermal} \quad p = T \lim_{V\to\infty} \frac{\ln\left[Z_{ce}(T,N,V)\right]}{V}$$

$$\text{mechanical} \quad p = T \frac{\partial \ln\left[Z_{ce}(T,N,V)\right]}{\partial V}$$

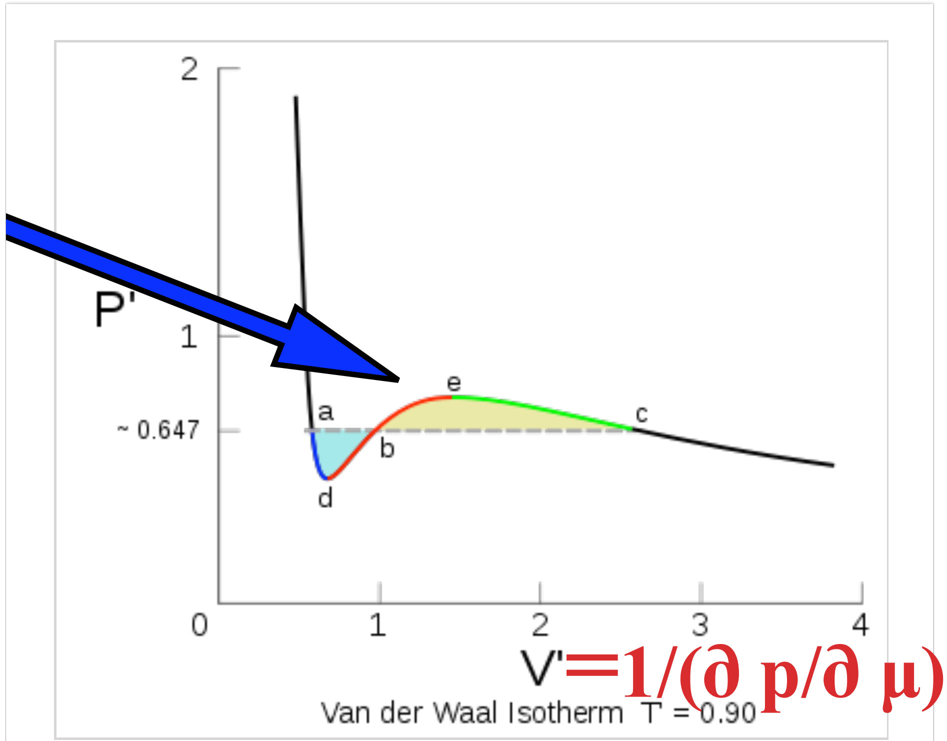
Apparently, in thermodynamic limit they should coincide

L. van Hove Axioms of Statistical Mechanics

A1: in the Grand Canonical Ensemble the pressure $p \geq 0$ can be only the function of V, T and μ

A2: in thermodynamic limit along the curve T=const the pressure p can be only a monotonically decreasing function of inverse density $1/(\partial p/\partial \mu)$.

Exception is a phase transition region, where p=const for T=const



Maxwell's rule eliminates the oscillating behavior of the isotherm in the phase transition zone by defining it as a certain isobar in that zone.

A3: kinks of pressure p can exist in thermodynamic limit only! For finite V they are transformed into regular behavior of isotherms.

Hence the Van der Waals EoS does not considers phase transitions correctly!

Thermodynamics in Grand Canonical Ensemble

Other thermodynamic quantities can be found from identities:

I Law $$p + \epsilon = Ts + \mu n$$

II Law $$s = \frac{\partial p}{\partial T}; \quad n = \frac{\partial p}{\partial \mu}$$

III Law $$s \to 0, \text{ if } T \to 0$$

ϵ is energy density (E/V), s is entropy density, n is baryonic charge density

Nuclear Matter Properties

Bethe—Weizsaecker formula for binding energy of nucleus of Z protons and (A-Z) neutrons

$$E_W = (m_N + W_0)A + a_2 A^{\frac{2}{3}} + a_3 \frac{(A/2 - Z)^2}{A} + a_4 \frac{Z^2}{A^{\frac{1}{3}}}$$ **+ corrections due to shell effects**

Binding energy of nucleons at $T = 0$ $W_0 =$-16 MeV

Surface energy of spherical fragments $a_2 = 16 \div 18.5$ MeV

Symmetry energy $a_3 = 100$ MeV

Coulomb energy $a_4 = 0.72$ MeV **small!**

Imagine a matter with 50% of protons and 50% of neutrons, but protons have no electric charge = symmetric nuclear matter

Nuclear Matter Properties

1. Normal nuclear density $n_0 \simeq 0.16$ fm^{-3} is density at the center of heavy nuclei

2. At temperature $T = 0$ and normal nuclear density the system pressure p is zero.

   $p = 0$ is mechanical stability condition

3. Binding energy/nucleon at $T = 0$ and $n = n_0$ is $W_0 = -16$ MeV (see prev. slide)

4. Incompressibility constant of normal nuclear matter is

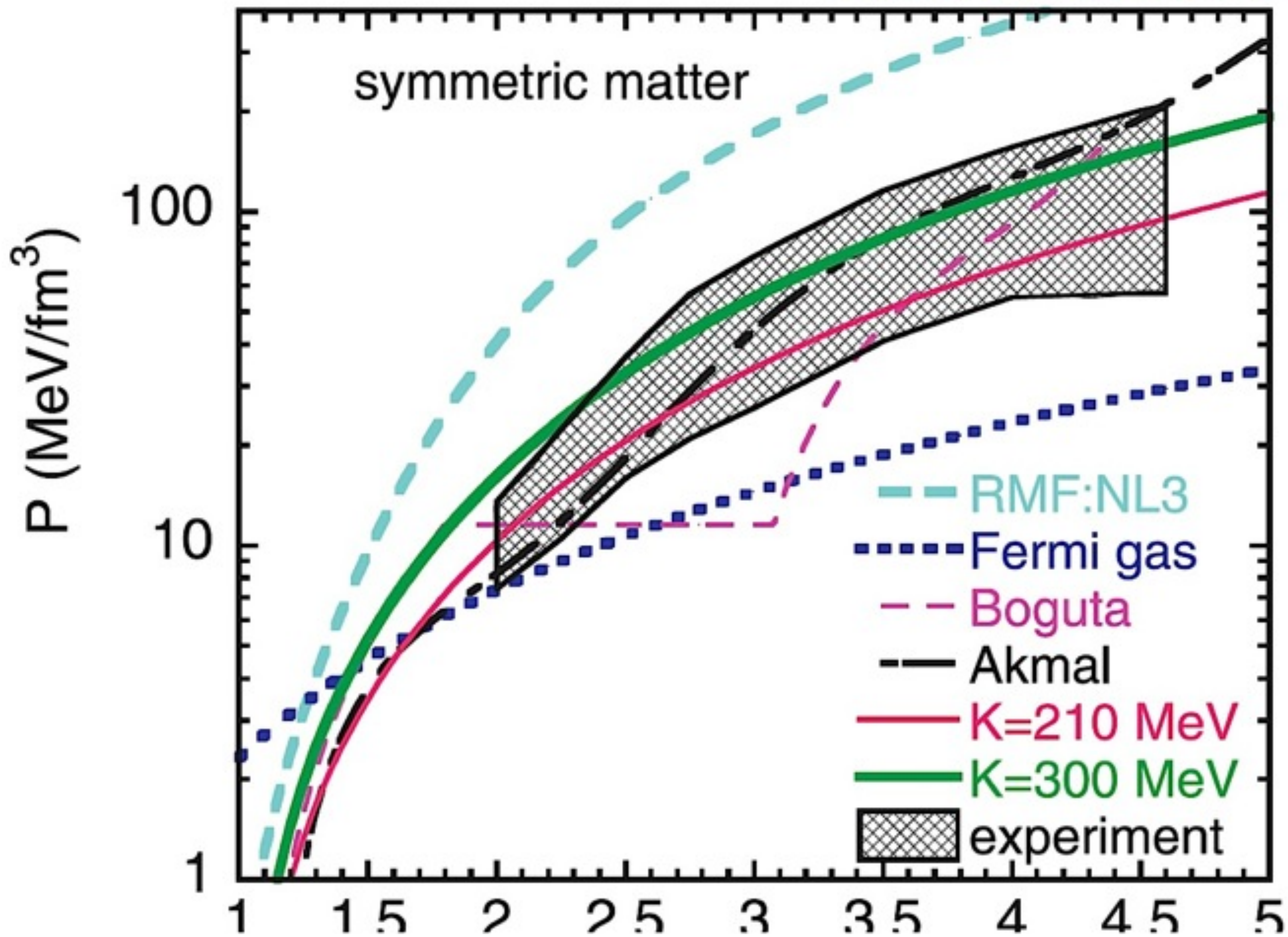
   $$K_0 \equiv 9 \frac{\partial p}{\partial n}\Big|_{T=0,\ n=n_0} \in [200; 315] \text{ MeV}$$

5. Proton flow constraint ($p(n)$ dependence at high n values)

6. Hard-core radius of nucleons $R_n \in [0.3; 0.35]$ fm (see later)

Proton flow constraint

P. Danielewicz et al.
Science 298, 1592 (2002)



$$n/n_0$$

As you can see from these examples, some EoS do not obey this constraint!

Nuclear Matter Properties at T=0

EoS at $T = 0$:

$$p(n) = \mu n - \epsilon \equiv n\left[\mu - (m + W(n))\right]$$

ϵ is energy density, $W(n)$ is binding energy per nucleon.

From the stability condition $p = 0$ at $T = 0$ and $n = n_0$ $\Rightarrow$

$$\mu_0 \equiv \mu(T = 0, n = n_0) = m + W(n = n_0) = 923 \text{ MeV}$$

Nuclear Matter Properties at T=0

EoS at $T = 0$: $$p(n) = \mu n - \epsilon \equiv n\,[\mu - (m + W(n))]$$

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$$\mu_0 \equiv \mu(T = 0, n = n_0) = m + W(n = n_0) = 923 \text{ MeV}$$

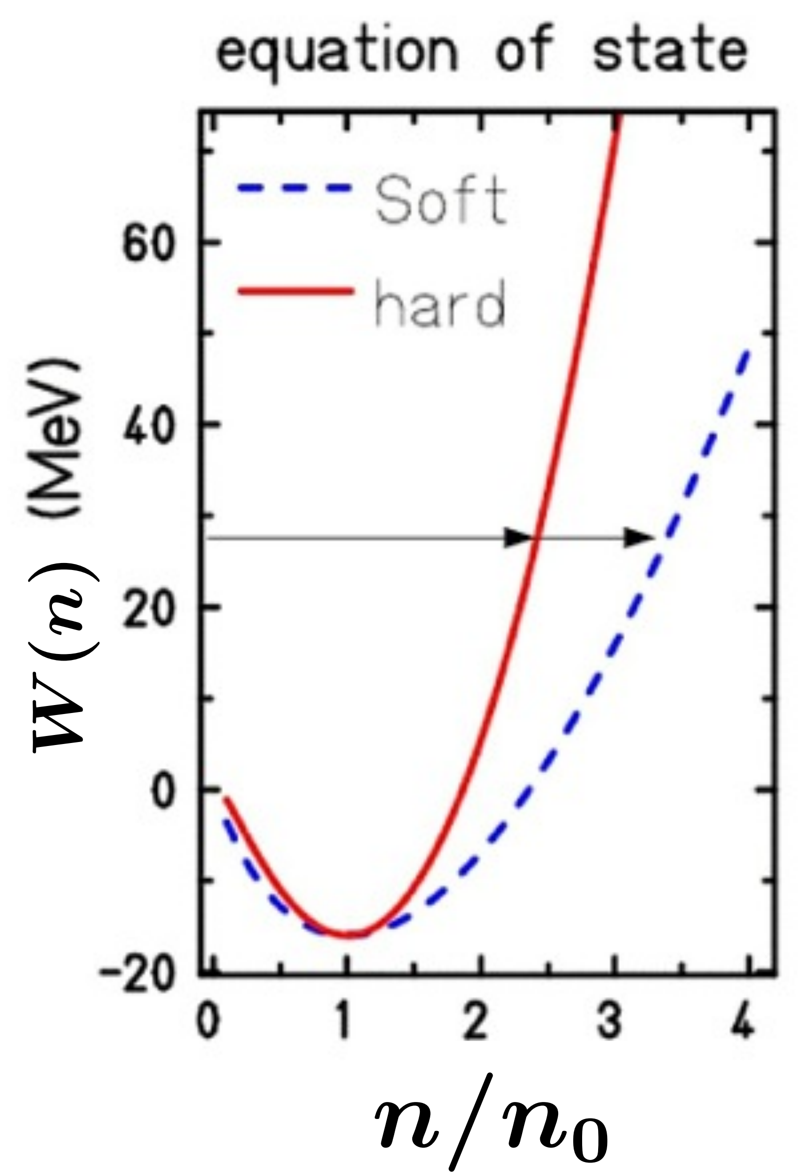
$$\underbrace{\frac{\partial p}{\partial n}}_{\displaystyle \| \atop \displaystyle \frac{\partial p}{\partial \mu}\frac{\partial \mu}{\partial n} = n\frac{\partial \mu}{\partial n}} = [\mu - (m + W(n))] + n\frac{\partial \mu}{\partial n} - n\frac{dW(n)}{dn}$$

$$\Rightarrow \quad \frac{dW(n)}{dn} = 0, \quad \text{for } n = n_0,$$

since $[\mu - (m + W(n))] = 0$ for $n = n_0$

At normal nuclear matter W(n) has an extremum!

Nuclear Matter Properties at T=0



From expression for $\frac{\partial p}{\partial n}$ find μ and get

$$\mu = m + W(n) + n\frac{dW(n)}{dn}\,,\ \text{differentiating it} \quad \Rightarrow$$

$$\frac{\partial \mu}{\partial n} = 2\frac{dW(n)}{dn} + n\frac{d^2W(n)}{dn^2}\,, \quad \Rightarrow$$

$$\boxed{\frac{\partial \mu}{\partial n} = n_0 \frac{d^2W(n=n_0)}{dn^2}} \quad \text{for } n = n_0$$

$$K_0 \equiv 9\,\frac{\partial p}{\partial n} = 9\,n_0 \frac{\partial \mu}{\partial n}\Big|_{T=0,\ n=n_0} \quad \Rightarrow \quad \boxed{K_0 \equiv 9\,n_0^2\,\frac{d^2W(n)}{dn^2}\Big|_{T=0,\ n=n_0}}$$

At normal nuclear matter W(n) has a minimum!

Nuclear Matter within Walecka (σ-ω) model

J. D. Walecka, Annals Phys. 83, (1974) 491

Also Jonson&Teller, Duerr contributed:

constituents are nucleons and static σ-ω mesons

Lagrangian is a Lorentz scalar, then interaction one is (x = t, x, y, z)

$$\mathcal{L}_{\text{int}} = g_\sigma\, \sigma(x)\, \overline{\psi}(x)\psi(x) - g_\omega\, \omega_\mu(x)\, \overline{\psi}(x)\gamma^\mu \psi(x)\,,$$

scalar **vector**

Full Lagrangian is

$$\mathcal{L} = \overline{\psi}\Big[i\gamma_\mu(\partial^\mu + i g_\omega \omega^\mu) - (m - g_\sigma \sigma)\Big]\psi$$
$$+ \tfrac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2} m_\omega^2 \omega_\mu \omega^\mu\,.$$

Euler-Lagrangian equations of motion are

Coupling constants are g_σ and g_ω

σ meson $$(\Box + m_\sigma{}^2)\sigma(x) = g_\sigma\, \overline{\psi}(x)\psi(x)\,,$$

ω meson $$(\Box + m_\omega^2)\omega_\mu(x) - \partial_\mu \partial^\nu \omega_\nu(x) = g_\omega\, \overline{\psi}(x)\gamma_\mu \psi(x)$$

nucleons $$\Big[\gamma_\mu\big(i\partial^\mu - g_\omega \omega^\mu(x)\big) - \big(m - g_\sigma \sigma(x)\big)\Big]\psi(x) = 0.$$

For point-like nucleons => Yukawa potential!

Coupled system is still complicated!

m is nucleon mass, m_σ is σ-meson mass, m_ω is ω-meson mass

Mean-field approximation to Walecka (σ-ω) model

We are interested in a static uniform and isotropic matter being in the ground state

=> all spatial points and all directions are equivalent!

=> σ = <σ> averaged value,
ω_1 = ω_2 = ω_3 =0,
ω_0 = < ω_0> averaged value

$$m_\sigma^2 \langle \sigma \rangle = g_\sigma \langle \bar{\psi} \psi \rangle ,$$
$$m_\omega^2 \langle \omega_0 \rangle = g_\omega \langle \psi^\dagger \psi \rangle ,$$

=> for nucleons

$$\left[\gamma_\mu (i \partial^\mu - g_\omega \omega_\mu \delta^{0\mu}) - (m - g_\sigma \sigma) \right] \psi(x) = 0 ,$$

There is no explicit x-dependence in Euler-Lagrange equation!

Effective nucleon mass is $m^*(\sigma) = m - g_\sigma \sigma$

$$\psi(x) = \psi(k) e^{-ik\cdot x}$$

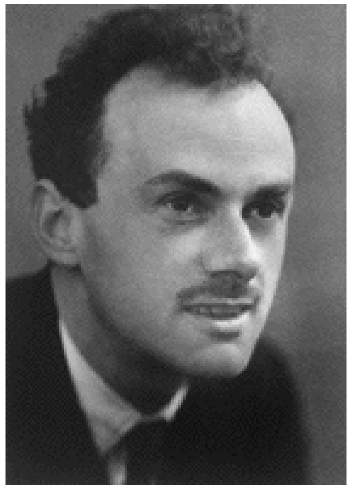
With nucleon 4-momentum

Formal solution for nucleons

$$k \cdot x \equiv k_\mu x^\mu = k_0 t - \boldsymbol{k} \cdot \boldsymbol{r} .$$

$$K^\mu = k^\mu - g_\omega \omega_\mu \delta^{0\mu}$$

Walecka (σ-ω) model: equation for nucleons



Paul Dirac
(1902-1984)

Dirac equation:

$$(\not{K} - m^*)\psi(K) = 0.$$ **where** $$\not{K} = \gamma_\mu K^\mu$$

Effective nucleon mass is

$$m^*(\sigma) = m - g_\sigma \sigma$$

With nucleon 4-momentum is

$$K^\mu = k^\mu - g_\omega \omega_\mu \delta^{0\mu}$$

Using properties of γ-matrices one can find the eigenvalues of Dirac operator

$$(\not{K} + m^*)(\not{K} - m^*) = \not{K}\not{K} - m^{*2} = \gamma_\mu K^\mu \gamma_\nu K^\nu - m^{*2}$$
$$= K^\mu K^\nu \frac{\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu}{2} - m^{*2} = K_\mu K^\mu - m^{*2}.$$

=> $$(K_\mu K^\mu - m^{*2})\psi(K) = 0.$$

The standard procedure gives the eigen values for energy of

$$\text{for nucleons } E^+ = \sqrt{k^2 + (m^*)^2} + g_\omega \omega_\mu \, ,$$

$$\text{for antinucleons } E^- = \sqrt{k^2 + (m^*)^2} - g_\omega \omega_\mu \, .$$

Walecka (σ-ω) model: pressure

pressure

$$p(T,\mu) = \frac{\gamma_N}{3}\int \frac{d^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2 + \boldsymbol{m}^{*2}}}(f_+ + f_-) + \frac{\boldsymbol{m}_\omega^2}{2}\boldsymbol{\omega}_0^2 + \frac{\boldsymbol{m}_\sigma^2}{2}\boldsymbol{\sigma}^2$$

mean-field contributions

ideal gas with m* mass

Degeneracy factor $\gamma_N = (2S_N + 1)(2I_N + 1) = 4$,

nucleon spin $S_N = \frac{1}{2}$ and isospin $I_N = \frac{1}{2}$

distribution function

$$f_\pm \equiv \left[\exp\left(\frac{\sqrt{k^2 + \boldsymbol{m}^{*2}} \mp \mu \pm \boldsymbol{g}_\omega\boldsymbol{\omega}_0}{T}\right) + 1\right]^{-1}$$

μ baryonic chemical potential

Formally, pressure is known, but how to find the values of σ and ω_0 fields?

Walecka (σ-ω) model: pressure

pressure

$$p(T,\mu) = \frac{\gamma_N}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{k^2}{\sqrt{k^2 + \boldsymbol{m}^{*2}}}\,(f_+ + f_-) + \frac{\boldsymbol{m_\omega^2}}{\boldsymbol{2}}\boldsymbol{\omega_0^2} + \frac{\boldsymbol{m_\sigma^2}}{\boldsymbol{2}}\boldsymbol{\sigma^2}$$

mean-field contributions

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μ baryonic chemical potential

Formally, **pressure is known, but how to find the values of σ and ω_0 fields?**

Thermodynamics requires extremum values of corresponding potential for a given set of variables (ensemble)!

For the GC ensemble one has to require a maximum of pressure!

Walecka (σ-ω) model: maximum of pressure

$$\frac{\delta p}{\delta \omega_0} = 0 \quad \Rightarrow \quad g_\omega \gamma_N \int \frac{dk^3}{(2\pi)^3} [f_+ - f_-] - m_\omega^2 \omega_0 = 0$$

instead of field σ it is more convenient to use m* =>

$$\frac{\delta p}{\delta \phi} = 0$$

$$\left(\frac{\delta \; p(T,\mu)}{\delta \quad m^*} \right)_{T,\; \mu} \equiv \frac{m_\sigma^2}{g_\sigma^2}(m - m^*) - \; \gamma_N \int \frac{d^3k}{(2\pi)^3} \underbrace{\frac{m^*}{\sqrt{k^2 \; + m^{*\,2}}}}_{\text{scalar density}} (f_+ + f_-) \;=\; 0\,.$$

These conditions provide maximum of pressure and allow one to recover the standard thermodynamics identities!

Recall 1-st L. van Hove axiom!

$$s = \frac{\partial p}{\partial T}; \quad n = \frac{\partial p}{\partial \mu}$$

from 1-st Eq. above $\Rightarrow \quad \omega_0 = \frac{g_\omega}{m_\omega^2} n$

$$\varepsilon(T,\mu) \equiv T \left(\frac{\partial \; p}{\partial \; T} \right)_\mu + \mu \left(\frac{\partial \; p}{\partial \; \mu} \right)_T - p = \gamma_N \int \frac{dk^3}{(2\pi)^3} \left[\left(\sqrt{k^2 + (m^*)^2} + g_\omega \omega_0 \right) f_+ + \left(\sqrt{k^2 + (m^*)^2} - g_\omega \omega_0 \right) f_- \right] -$$

$$- \frac{m_\omega^2}{2} \omega_0^2 - \frac{m_\sigma^2}{2} \phi^2 = \gamma_N \int \frac{dk^3}{(2\pi)^3} \sqrt{k^2 + (m^*)^2} \, [f_+ + f_-] + \frac{m_\omega^2}{2} \omega_0^2 - \frac{m_\sigma^2}{2} \phi^2$$

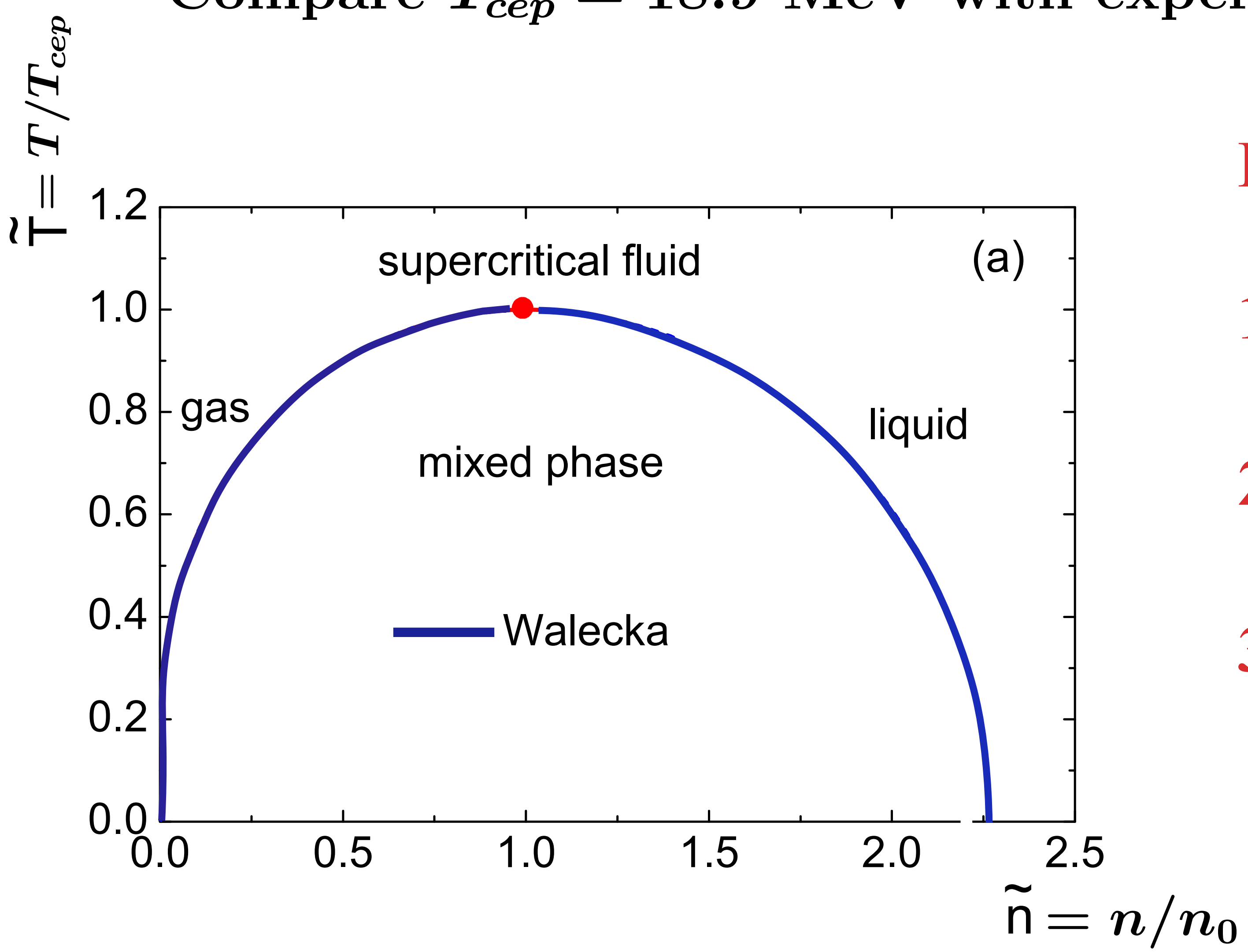
Walecka (σ-ω) model: normal nuclear matter

Having 2 parameters $C_v = \frac{g_\omega^2}{m_\omega^2} \simeq 285.9\ \text{GeV}^{-2},\quad C_s = \frac{g_\sigma^2}{m_\sigma^2} \simeq 377.6\ \text{GeV}^{-2}$

one can describe the properties of nuclear matter:
p=0 at T=0 and n=n_0 and W(n_0) = -16 MeV

Provides a reasonable value for the critical endpoint temperature

Compare $T_{cep} = 18.9$ MeV with experimental value $T_{cep} = 17 \pm 1$ MeV



But there are three problems:

1. K_0 = 553 MeV is too huge!

2. cannot reproduce flow constraint!

3. m*/m =0.55 is too small!

should be m*/m =[0.6; 0.8]

How can we improve this model?

Improving Walecka (σ-ω) model

1. One can add more meson fields, add nonlinear interaction for σ field
=> relativistic mean-field approach

=> difficulties with the flow constraint even having 10-15 parameters!

2. One can add a phenomenological repulsion a la Van der Waals
to weaken the vector meson repulsion

D. H. Rischke, M. I. Gorenstein, H. Stoecker and W. Greiner, Z. Phys. C 51, (1991) 485

=> same difficulties remain up to huge value for nucleon hard-core radius
$R_N = 0.7$ fm!

3. One can add another phenomenological attraction which depends on
baryonic (vector, not a scalar!) density

M. I. Gorenstein, D. H. Rischke, H. Stoecker, W. Greiner and K. A.Bugaev, J. Phys. G 19, (1993) 69

However, the problem is how to recover the 1-st L. Van Hove axiom?

In Walecka model this occurred automatically, since the rule
to calculate pressure from Lagrangian is known!

Thermodynamically Self-consistent EoS for Nuclear Matter

generalized pressure

$$p(T,\mu) = \frac{\gamma_N}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{k^2}{\sqrt{k^2 + M^{*2}}}\,(f_+ + f_-) + \boldsymbol{n\,U(n) - \int_0^n d\rho\,U(\rho) + P(M^*)}$$

ideal gas with M* mass

mean-field contributions

generalized distribution function

$$f_\pm \equiv \left[\exp\left(\frac{\sqrt{k^2 + M^{*2}} \mp \mu \pm U(n)}{T}\right) + 1\right]^{-1}$$

μ baryonic chemical potential

n-dependent interaction pressure

$$\boldsymbol{P_{int}(n) = \int_0^n d\rho\,U(\rho) - n\,U(n)}$$

Should obey a self-consistency condition

$$\boldsymbol{\frac{dP_{int}(n)}{dn} = n\frac{dU(n)}{dn}}$$

Maximum pressure with respect to effective mass

$$\left(\frac{\delta\,p(T,\mu)}{\delta\ M^*}\right)_{T,\ \mu} \equiv \frac{d\ P(M^*)}{d\ M^*} - \gamma_N\int \frac{d^3k}{(2\pi)^3}\,\frac{M^*}{\sqrt{k^2 + M^{*2}}}\,(f_+ + f_-) = 0\,.$$

Thermodynamically Self-consistent EoS for Nuclear Matter

Similarly to Walecka model, the self-consistency condition provides fulfillment of thermodynamic identities

$$\mathbf{n}(T,\mu) \equiv \left(\frac{\partial\, p}{\partial\, \mu}\right)_T = \gamma_N \int \frac{d^3k}{(2\pi)^3} \left(f_+ - f_-\right)$$

baryonic charge density

$$\boldsymbol{\varepsilon}(T,\mu) \equiv T\left(\frac{\partial\, p}{\partial\, T}\right)_\mu + \mu \left(\frac{\partial\, p}{\partial\, \mu}\right)_T - \boldsymbol{p}$$

energy density

$$= \gamma_N \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M^{*2}} \left(f_+ - f_-\right) + \boxed{\int_0^n d\rho\, U(\rho) - P(M^*)}$$

Home work: derive Eq. for ε from expression for pressure; use thermodynamic identities for ideal gas with chemical potential ν=μ-U(n)

Requirements to Mean-field Potentials

In contrast to Walecka model, our functions P_int(n) and P(M*) are not restricted by some Lagrangian!

But we have to pay for this freedom and have to formulate some general conditions on these functions:

$$U(-n) = -U(n)$$

odd function of baryonic charge density

$$\text{for } n \to \infty \;\Rightarrow\; U(n) \to n^a,\; a \leq 1$$

to obey causality condition, i.e. speed of sound < speed of light

$$\text{for } n \to 0 \;\Rightarrow\; U(n) \to n^b,\; 0 \leq b$$

i.e. interaction must vanish at vanishing density

$$P(M^*) = \sum_{k \geq 2} a_k \left(M - M^*\right)^k \quad \text{with} \quad a_2 < 0$$

higher powers than one can get from Lagrangians

Simplest Realization of the Model

$$P(M^*) \;=\; -\,\frac{1}{2}\,C_s^2\,(M - M^*)^2 \quad , \quad \boldsymbol{U(n) = C_v^2 n - C_d^2 n^{\frac{1}{3}}}$$

Compared to Walecka model there is **additional attraction and one additional parameter**

This attraction is generated by a peculiar Lagrangian $\frac{3}{4}\,C_d^2\,\left(\bar{\psi}\,\gamma^\mu\,\psi\,\bar{\psi}\,\gamma_\mu\,\psi\right)^{\frac{2}{3}}$

$$\varepsilon(T,\mu) = \gamma_N \int \frac{d^3k}{(2\pi)^3}\,\sqrt{k^2 + M^{*2}}\,(f_+ - f_-) \;+\; \boldsymbol{\frac{1}{2}C_v^2 n^2 - \frac{3}{4}C_d^2 n^{\frac{4}{3}} - \frac{1}{2}C_s^2(M - M^*)^2}$$

These parameters are normalized on properties of nuclear matter

M^*/M	C_v^2 (GeV^{-2})	C_s^2 (GeV^{-2})	C_d^2	K_o (MeV)
0.543	285.90	377.56	0	553
0.600	257.40	326.40	0.124	380
0.635	238.08	296.05	0.183	300
0.688	206.79	251.14	0.254	210
0.720	186.94	244.52	0.288	170

allowed range of values (left brace: rows 0.600–0.720)

allowed range of values (right brace: rows 0.600–0.688)

Model with K_0 =220-300 MeV obeys the proton flow constraint

Summary

1. We discussed the necessary apparatus to describe the nuclear matter EoS

2. The properties of normal nuclear matter are used to normalize the phenomenological EoS

3. The Walecka model is presented and its mean-field approximation is applied to normal nuclear matter

4. A phenomenological generalization of Walecka model is discussed and the self-consistency condition is obtained