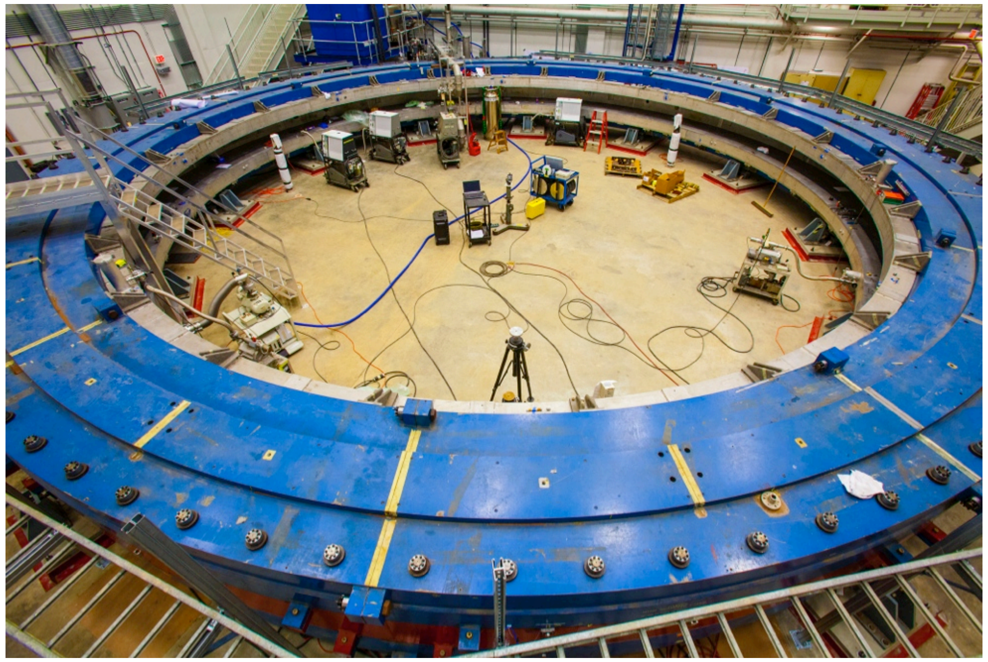
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

PRISMA

The anomalous magnetic moment of the muon



Marc Vanderhaeghen

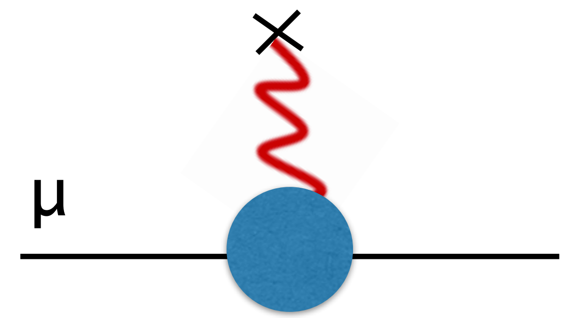
"The basic ideas and concepts behind the modern High-Energy Physics and Cosmology"

October 5 - 16, 2018, Truskavets, Ukraine

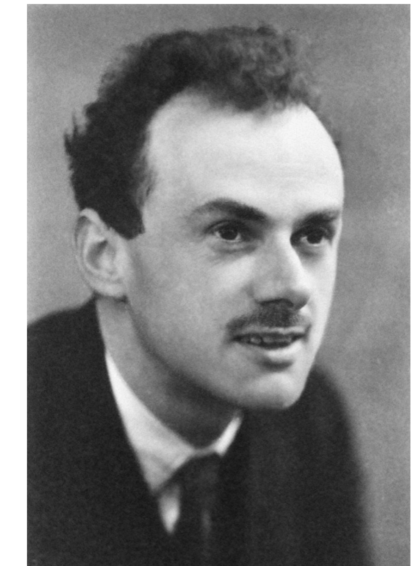
Motivation

- Magnetic moment of the muon

$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$



Classically g=1
Dirac equation g=2



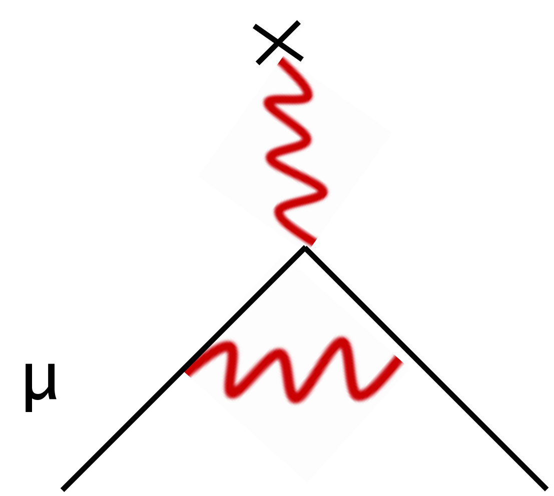
Dirac (1928)

- anomalous part

$$a_\mu = \frac{(g-2)_\mu}{2}$$

- first correction to LO result

$$a_\mu = \frac{\alpha}{2\pi} + \ldots$$



Schwinger(1947)

Standard model result for $(g-2)_\mu$

electroweak contributions: A **triumph** of perturbative QFT and computing

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had} = (\,11\,659\,182.8 \pm 4.9\,) \cdot 10^{-10}$$

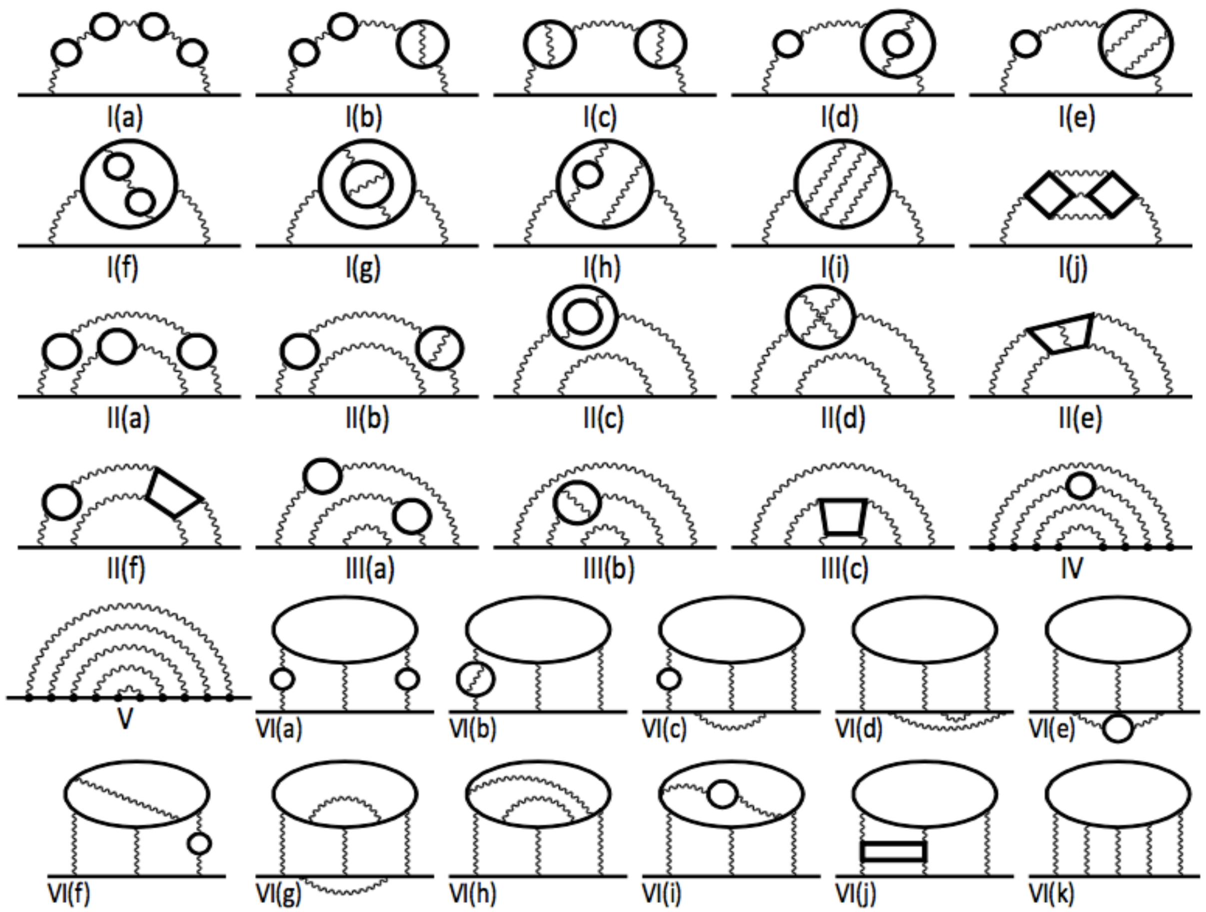
Czarnecki et al.

$(\,15.4 \pm \mathbf{0.2}\,) \cdot 10^{-10}$

Kinoshita et al. '12

$(\,11\,658\,471.808 \pm \mathbf{0.015}\,) \cdot 10^{-10}$

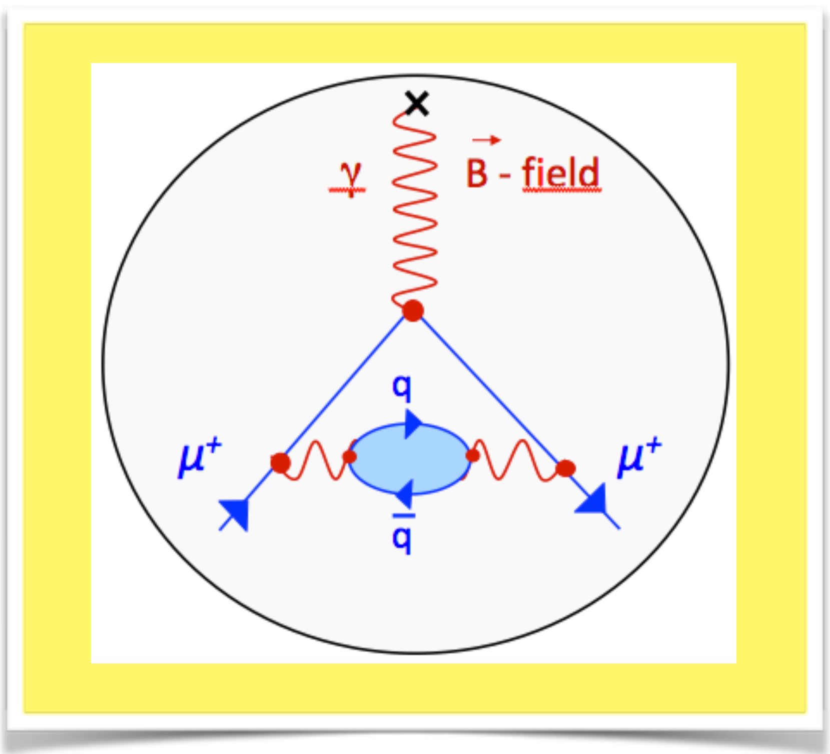
10th
12672
diagrams



Steinhauser et al. '14
First analytic calculation of part of the 8th order diagrams

Hadronic contributions to $(g-2)_\mu$

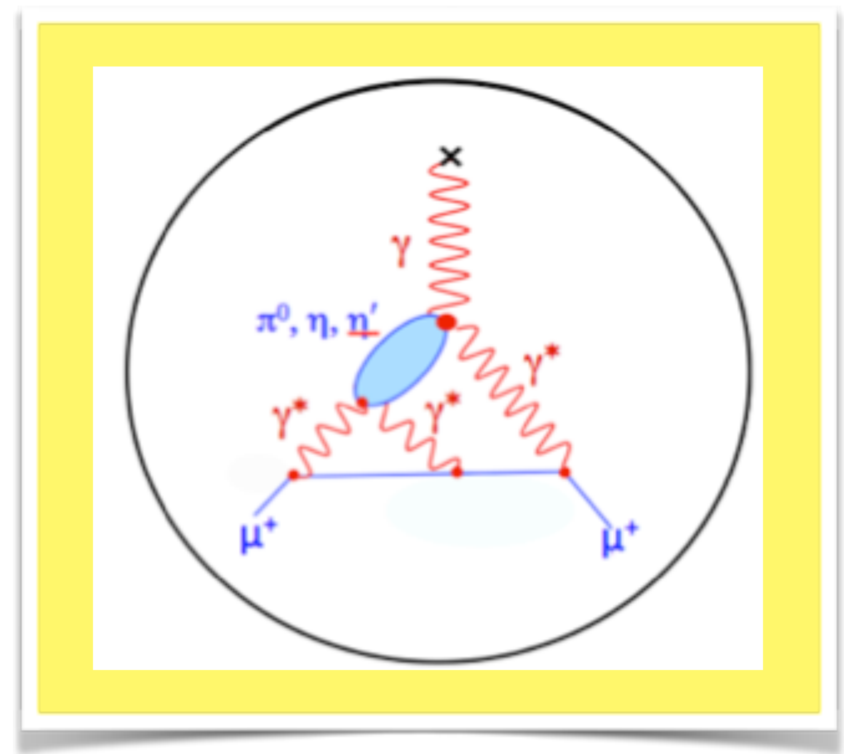
hadronic vacuum polarization (HVP)



$a_\mu^{\text{l.o. had, VP}} = (692.2 \pm 2.5) \times 10^{-10}$

Teubner et al. (2017)

hadronic light-by-light scattering (HLbL)



$a_\mu^{\text{had, LbL}} = (10.5 \pm 2.6) \times 10^{-10}$ (I)

$= (10.2 \pm 3.9) \times 10^{-10}$ (II)

(I) Prades, de Rafael, Vainshtein (2009)

(II) Jegerlehner, Nyffeler (2009)
Jegerlehner (2015)

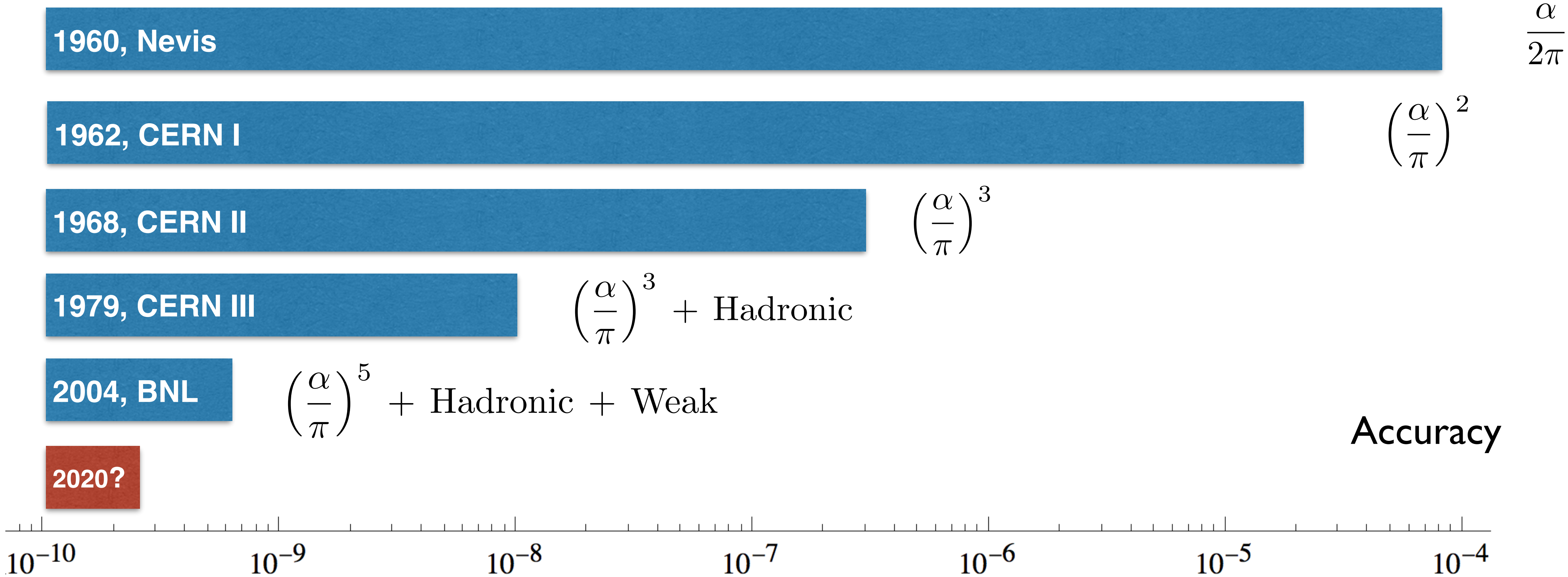
$(g\text{-}2)_\mu$: history of achieved accuracy / relevant corrections

- Magnetic moment of the muon

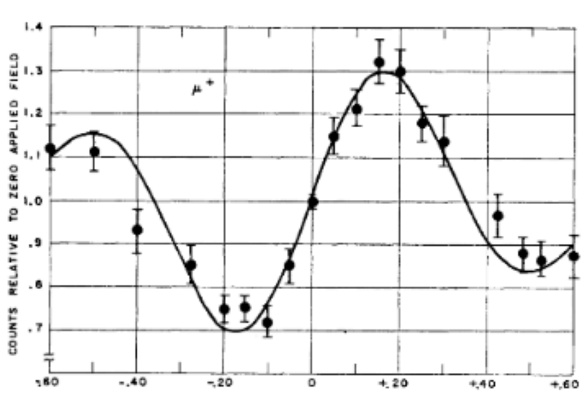
$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$

- anomalous part

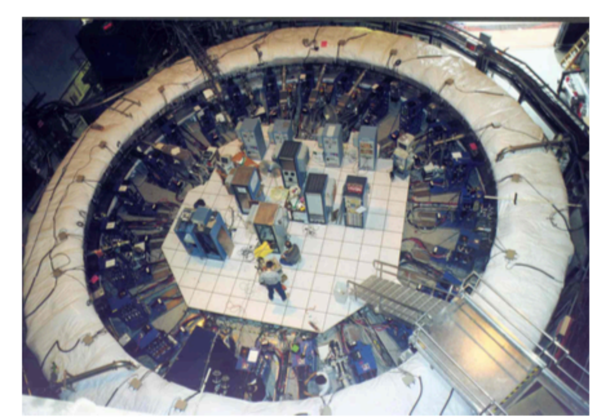
$$a_\mu = \frac{(g-2)_\mu}{2}$$



Nevis



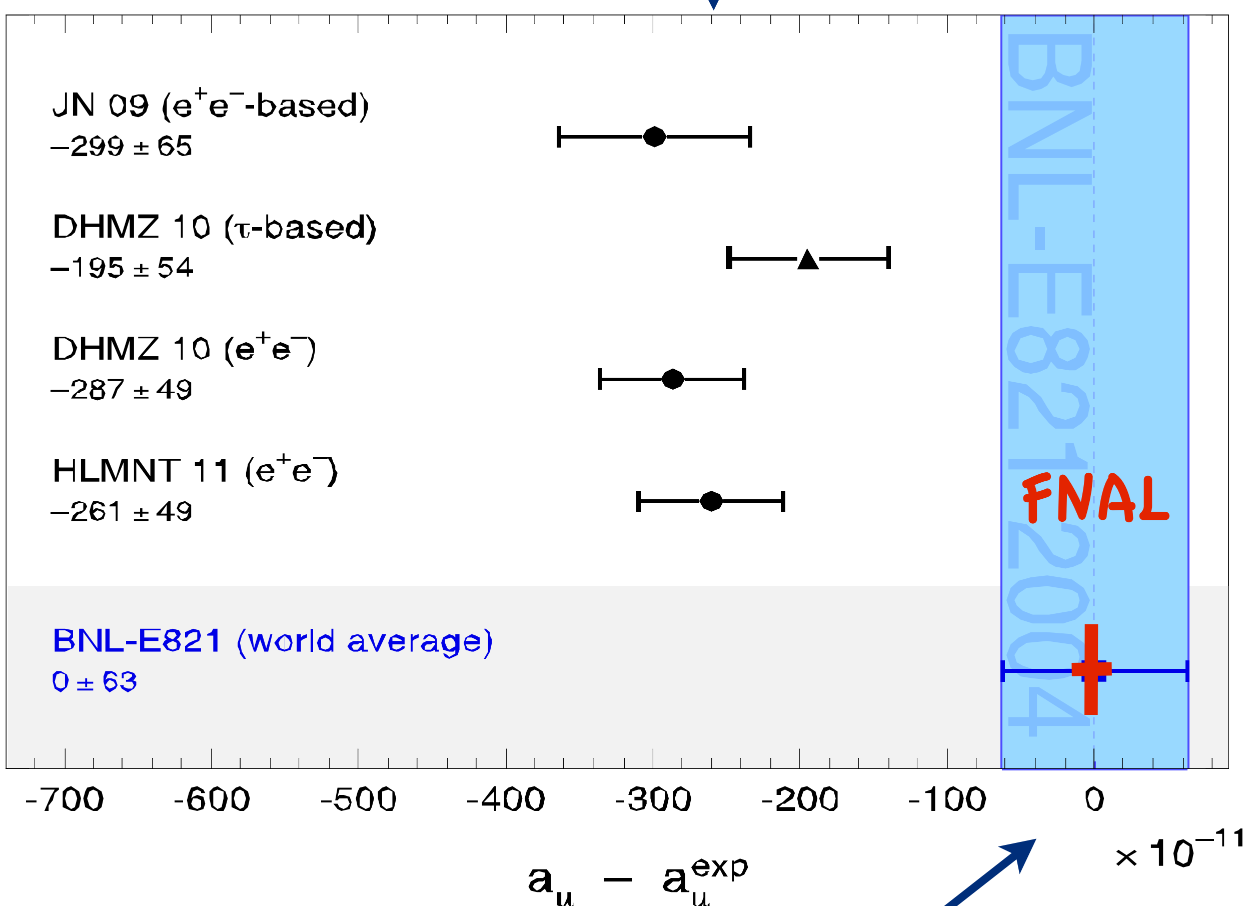
CERN I



Brookhaven

$(g-2)_\mu$: theory vs experiment

SM predictions for a_μ



BNL-E821 measurement of a_μ

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$

$a_\mu^{exp} - a_\mu^{SM} =$
$(28.1 \pm 3.6_{\ th} \pm 6.3_{\ exp}) \times 10^{-10}$

Teubner et al. (2017)

3 - 4 σ deviation from SM value !

Errors or new physics ?

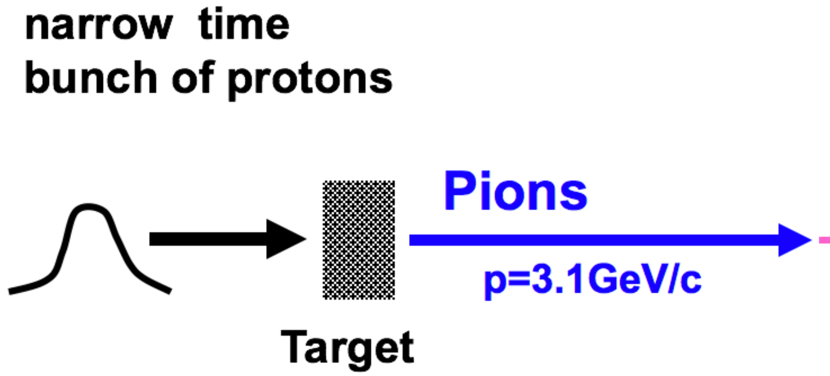
New FNAL, J-PARC experiments

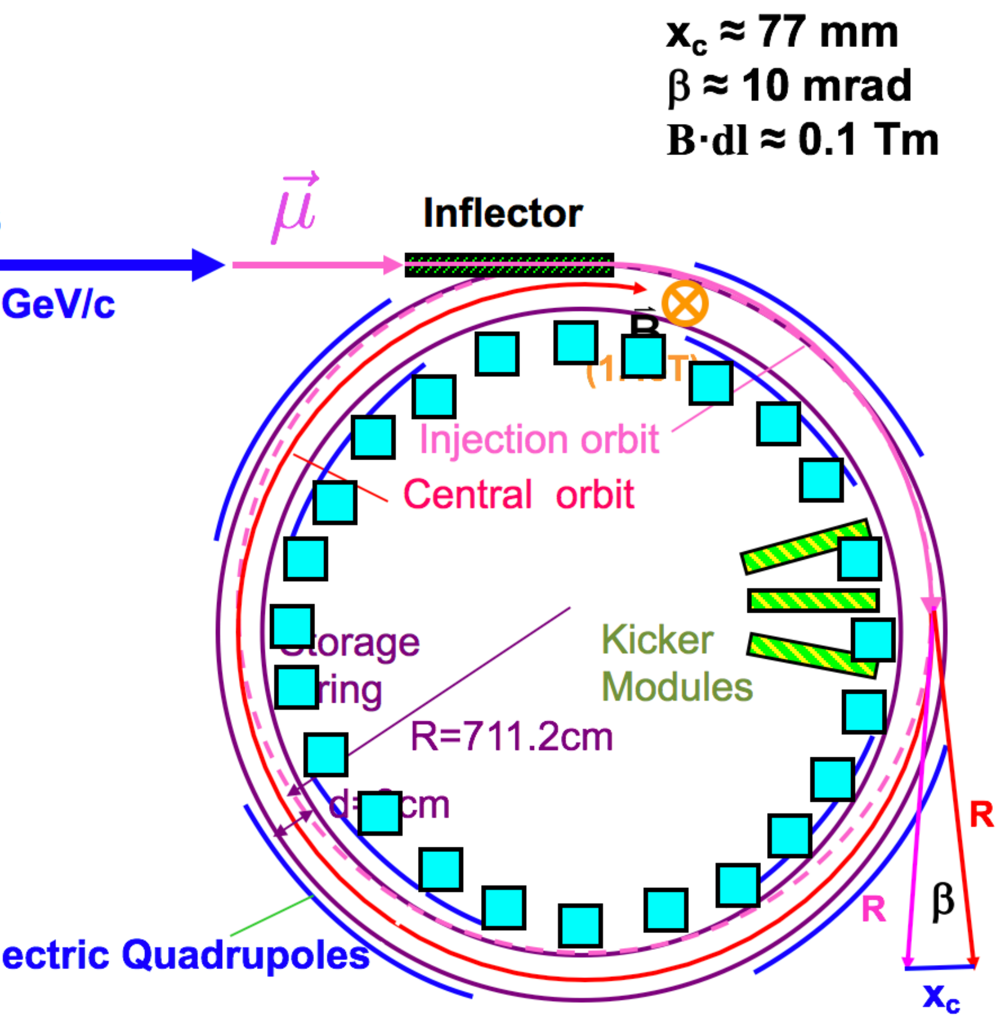
$\delta a_\mu^{FNAL} = 1.6 \times 10^{-10}$

factor 4 improvement in exp. error

-> Improve theory !

New $(g\text{-}2)_\mu$ experiment at Fermilab started!

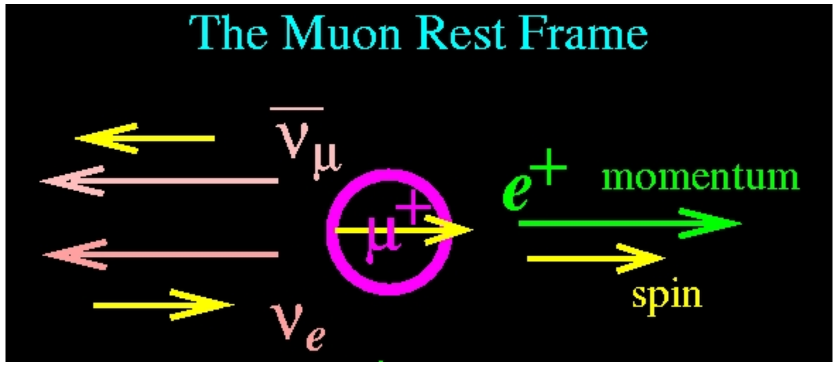




- Muon storage ring – weak focusing betatron
- Muon polarization
- Injection & kicking
- Focus with electric quads
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{Qe}{m} a_\mu \vec{B}$$

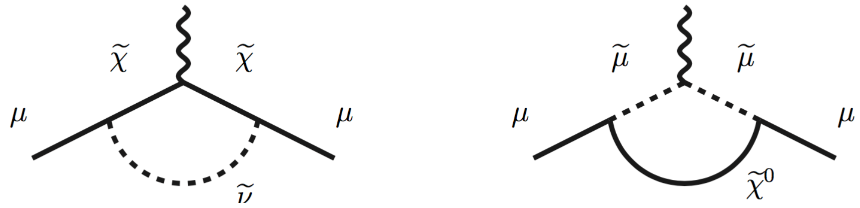
slide: Lee Roberts



Production run starting fall 2017,
BNL level result expected by end 2018,
Next data set 1/2 BNL error, final data: 1/4 BNL error

New physics in a_μ?

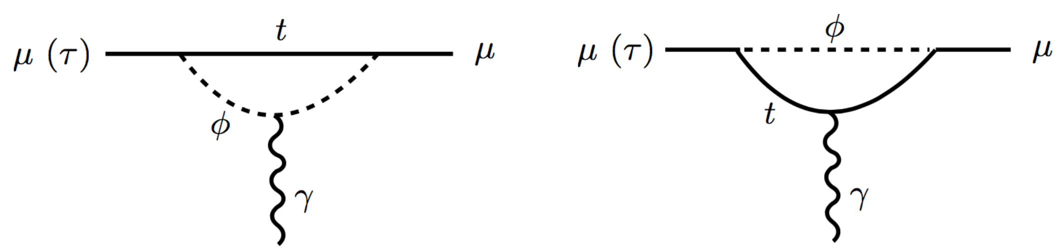
SUSY



Stockinger et al. (2017)

- Simplest case: $a_\mu^{\rm SUSY} \simeq sgn(\mu)\, 130 \times 10^{-11}\, \tan\beta \left(\frac{100\,{\rm GeV}}{\Lambda_{\rm SUSY}}\right)^2$
- Needs $\mu > 0$, low $\Lambda_{\rm SUSY}$, large coupling $\tan\beta$ to explain 281×10^{-11}
- Already excluded by LHC searches in simplest SUSY scenarios (like MSSM)
- However: SUSY could have large mass splittings (e.g. lighter sleptons), hadrophobic/leptophilic,…

1 TeV leptoquark



Bauer, Neubert (2016)

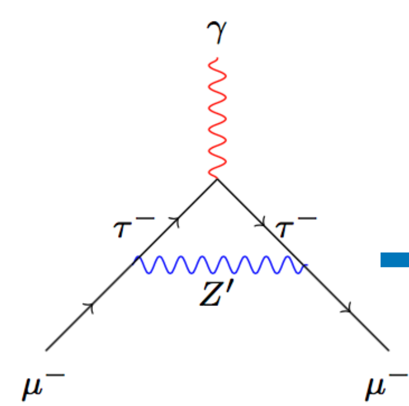
One new scalar could explain several anomalies seen by BaBar, Belle, and LHC in the flavor sector (e.g. violation of lepton universality in B -> K ll), explain $(g-2)_\mu$, and preserve LEP and LHC bounds

Light Z’

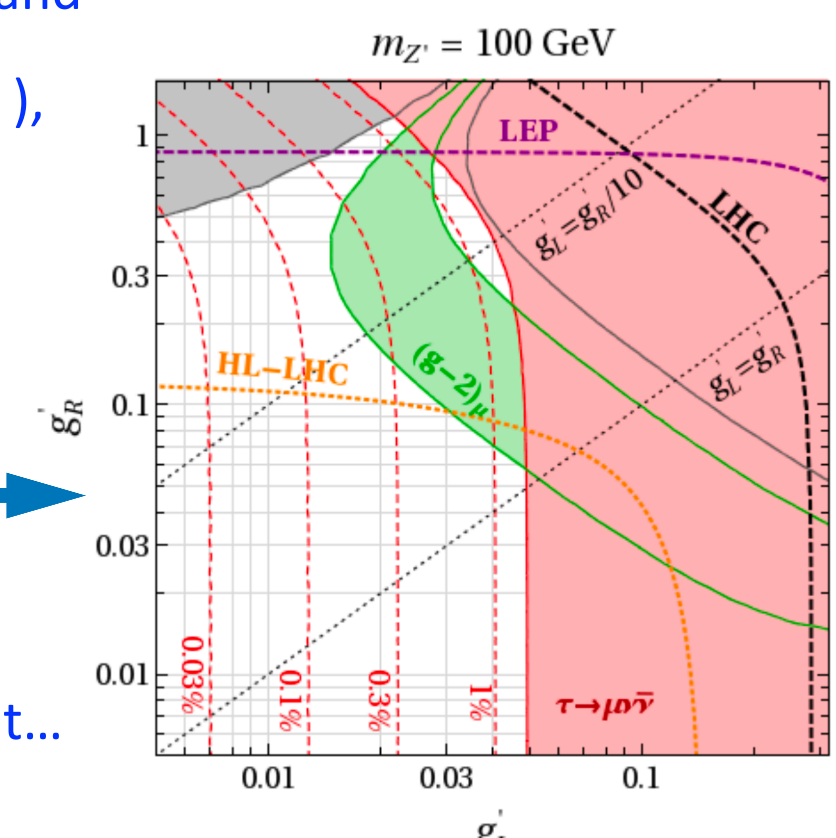
Altmannshofer et al. (2016)

With only flavor off-diagonal couplings to 2nd and 3rd generation of leptons: μ, ν_μ, τ, ν_τ

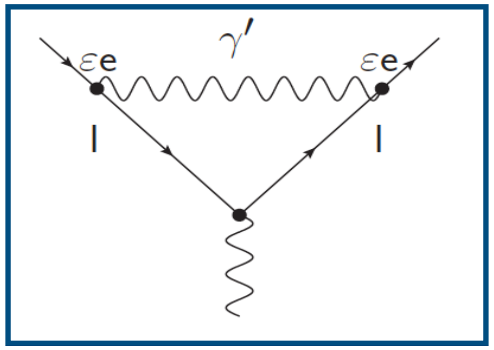
$$\mathcal{L}_{Z'} = g'_L(\bar\mu\gamma^\alpha P_L\tau + \bar\nu_\mu\gamma^\alpha P_L\nu_\tau)Z'_\alpha + g'_R(\bar\mu\gamma^\alpha P_R\tau)Z'_\alpha + {\rm H.c.}$$

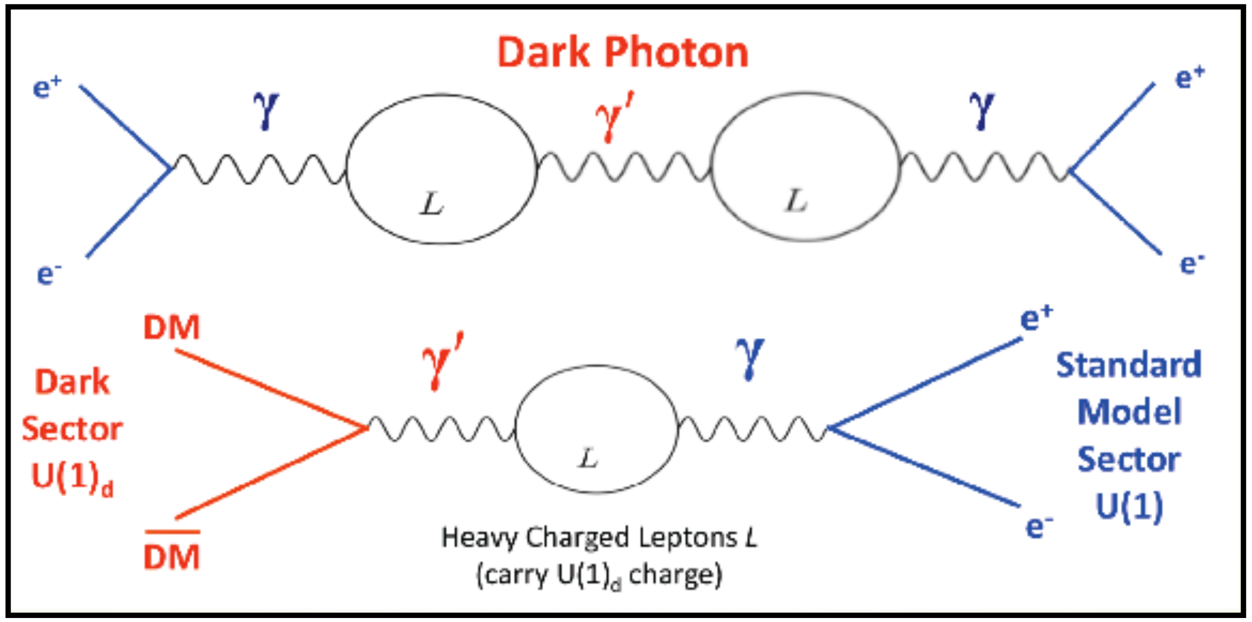


Z’ lighter than τ excluded, but…



Dark photons and $(g\text{-}2)_\mu$

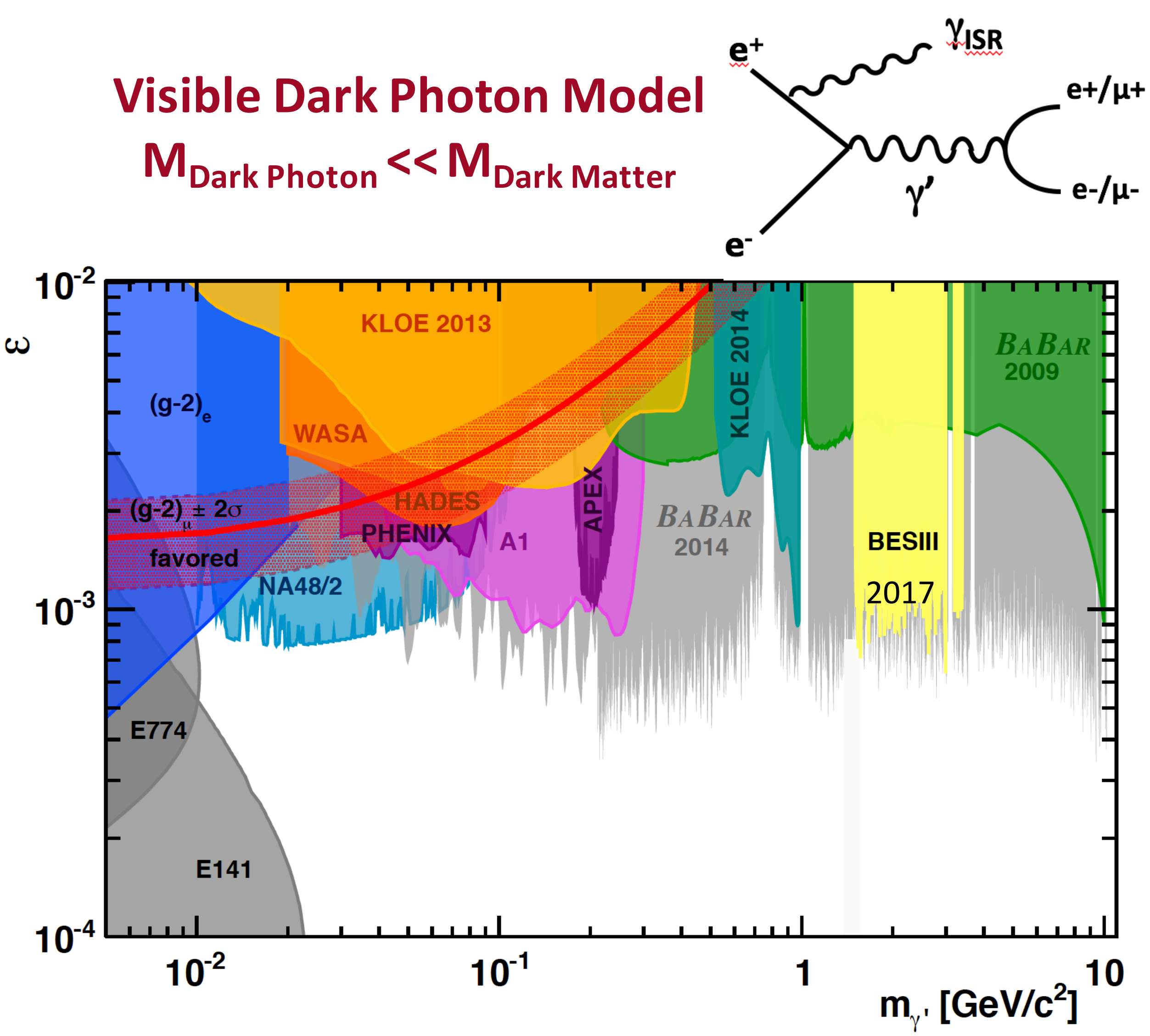




Holdom(1986),...

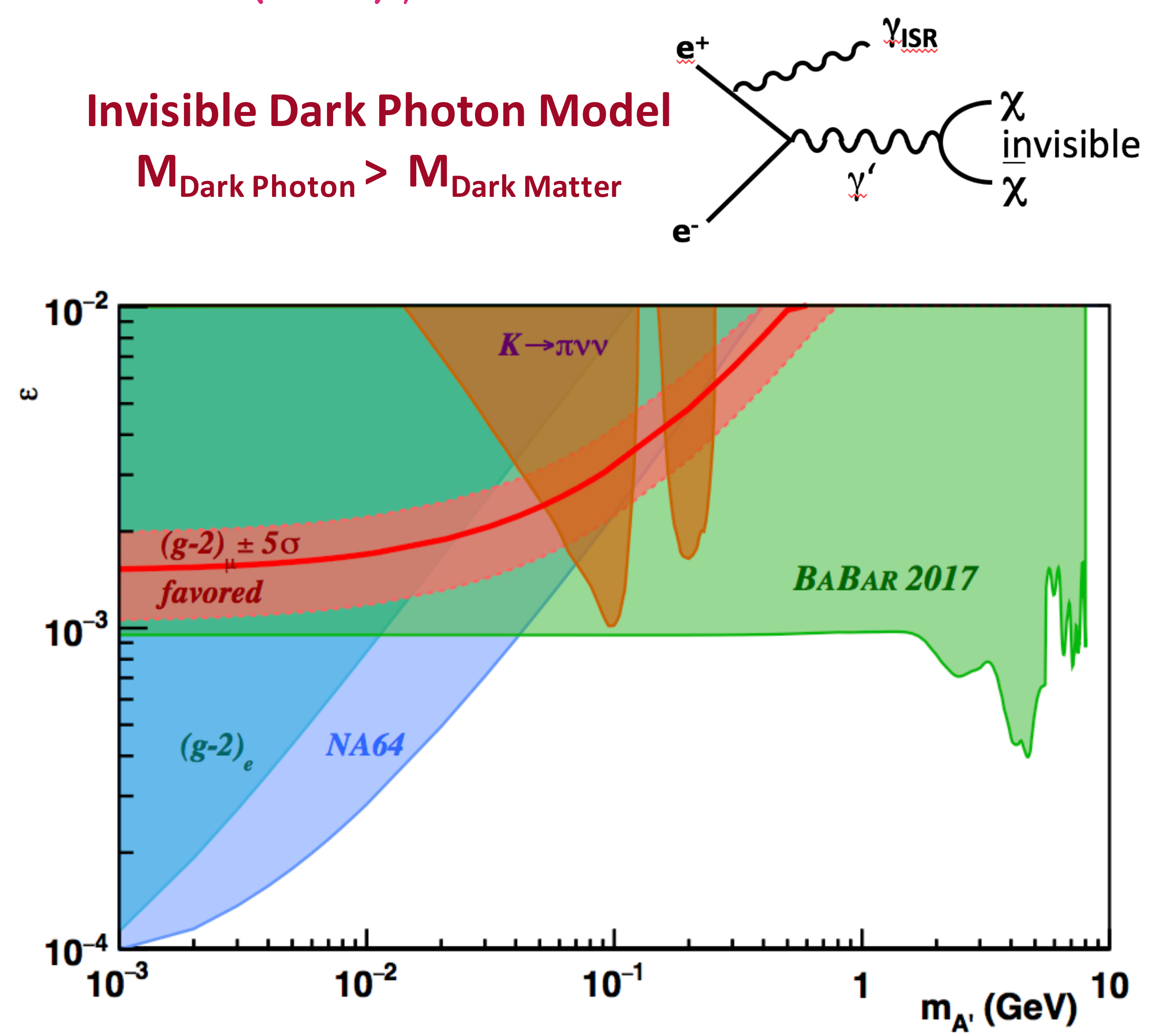
Visible Dark Photon Model

$M_{\text{Dark Photon}} << M_{\text{Dark Matter}}$



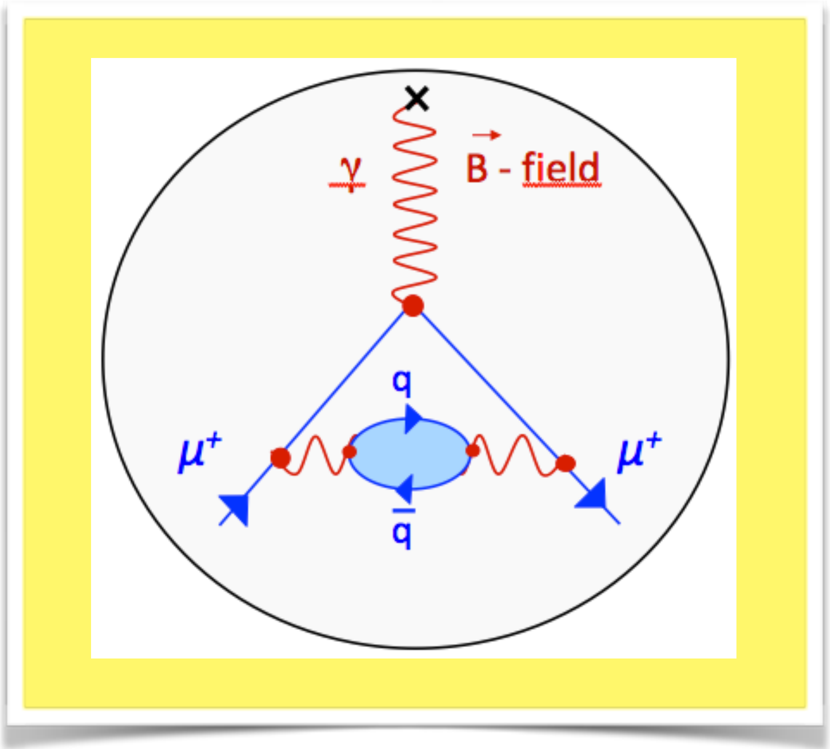
Invisible Dark Photon Model

$M_{\text{Dark Photon}} > M_{\text{Dark Matter}}$



Hadronic contributions to $(g\text{-}2)_\mu$

hadronic vacuum polarization (HVP)



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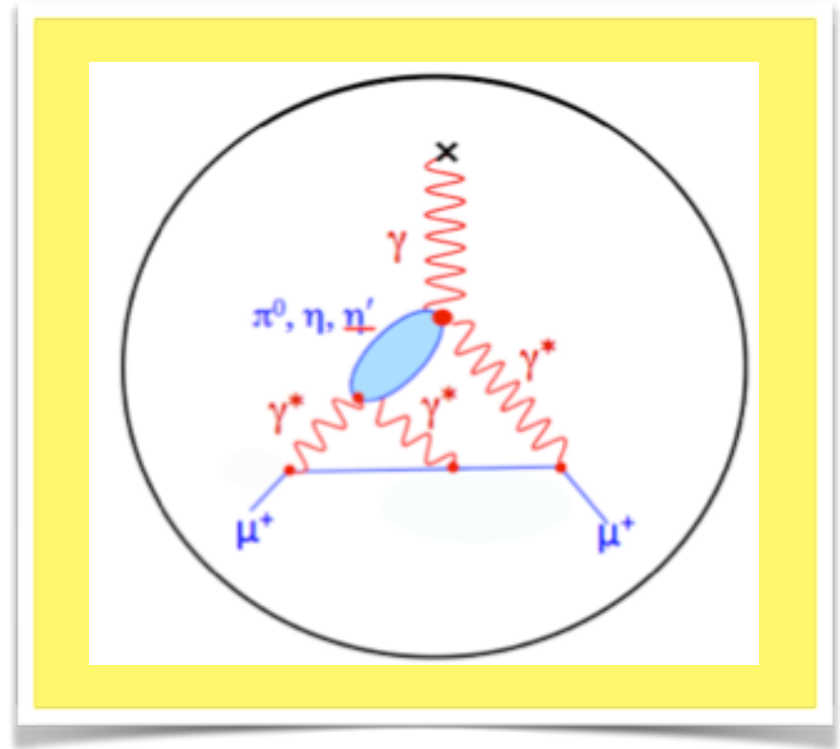
Teubner et al. (2017)

hadronic light-by-light scattering (HLbL)



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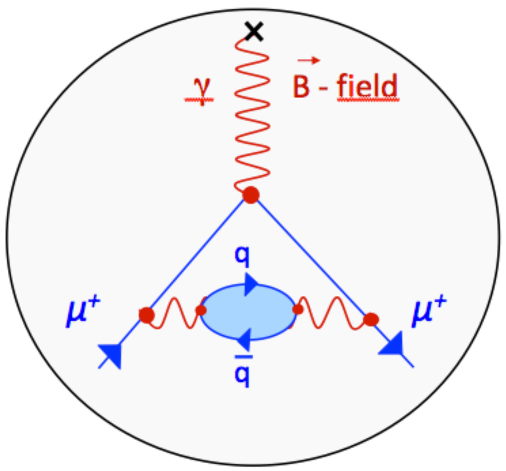
New FNAL and J-Parc $(g\text{-}2)_\mu$ expt. : $\delta a_\mu^{exp} = 1.6 \times 10^{-10}$

HVP determined by cross section measurements of e^+e^- -> hadrons

measurements of meson transition form factors required as input to reduce uncertainty

HVP corrections to $(g\text{-}2)_\mu$

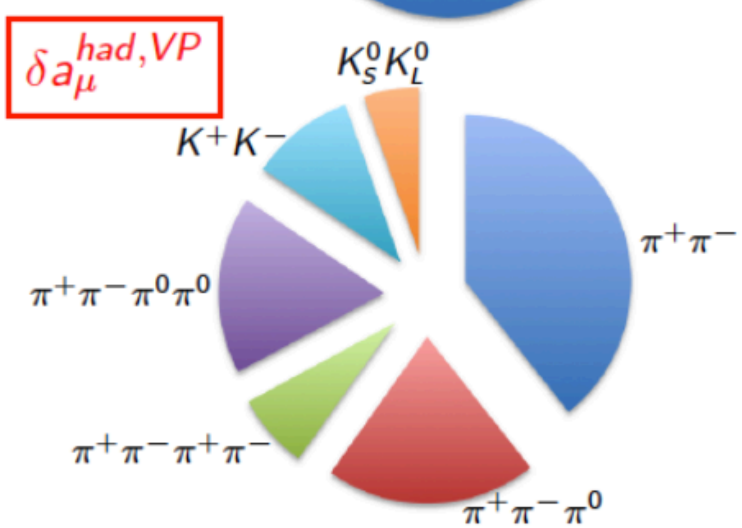


Optical theorem and analyticity allow to relate HVP contribution to $(g\text{-}2)_\mu$ with $\sigma_{had} = \sigma(e^+e^- \rightarrow \text{hadrons})$

$$a_\mu^{had,VP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

known Kernel function

Hadronic cross section



Future improvement of a_μ^{had} ?

1st priority:
Clarify situation regarding π+π− (KLOE vs. BABAR puzzle)

2nd priority:
Measure 3π, 4π channels

Ongoing ISR analyses
BESIII, BEPC-II collider

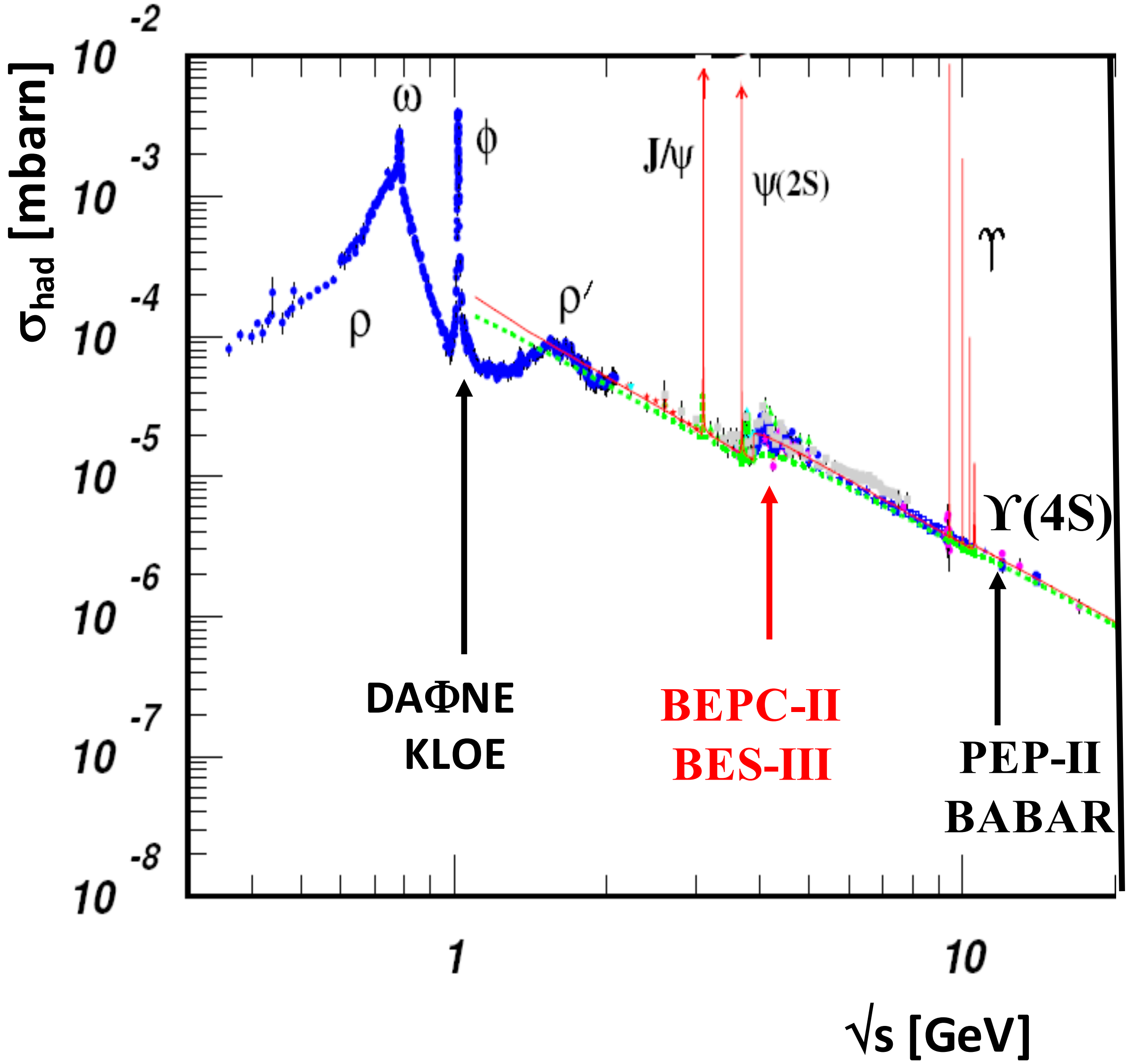
3rd priority:
KK and higher multiplicities

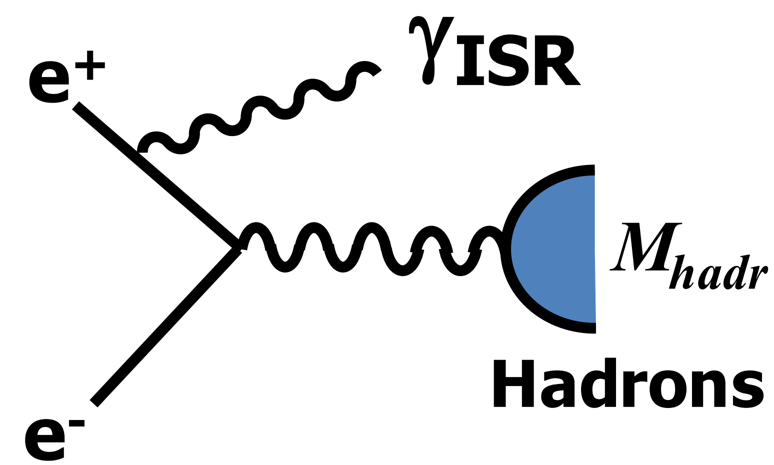
σ_{had}: Energy range up to 3 GeV essential !

aim: reduction of current error by factor of 2

HVP corrections to $(g-2)_\mu$

Approach for measuring hadronic cross section at modern particle factories with fixed c.m. energy $\sqrt{s}$: **Initial State Radiation (ISR)**



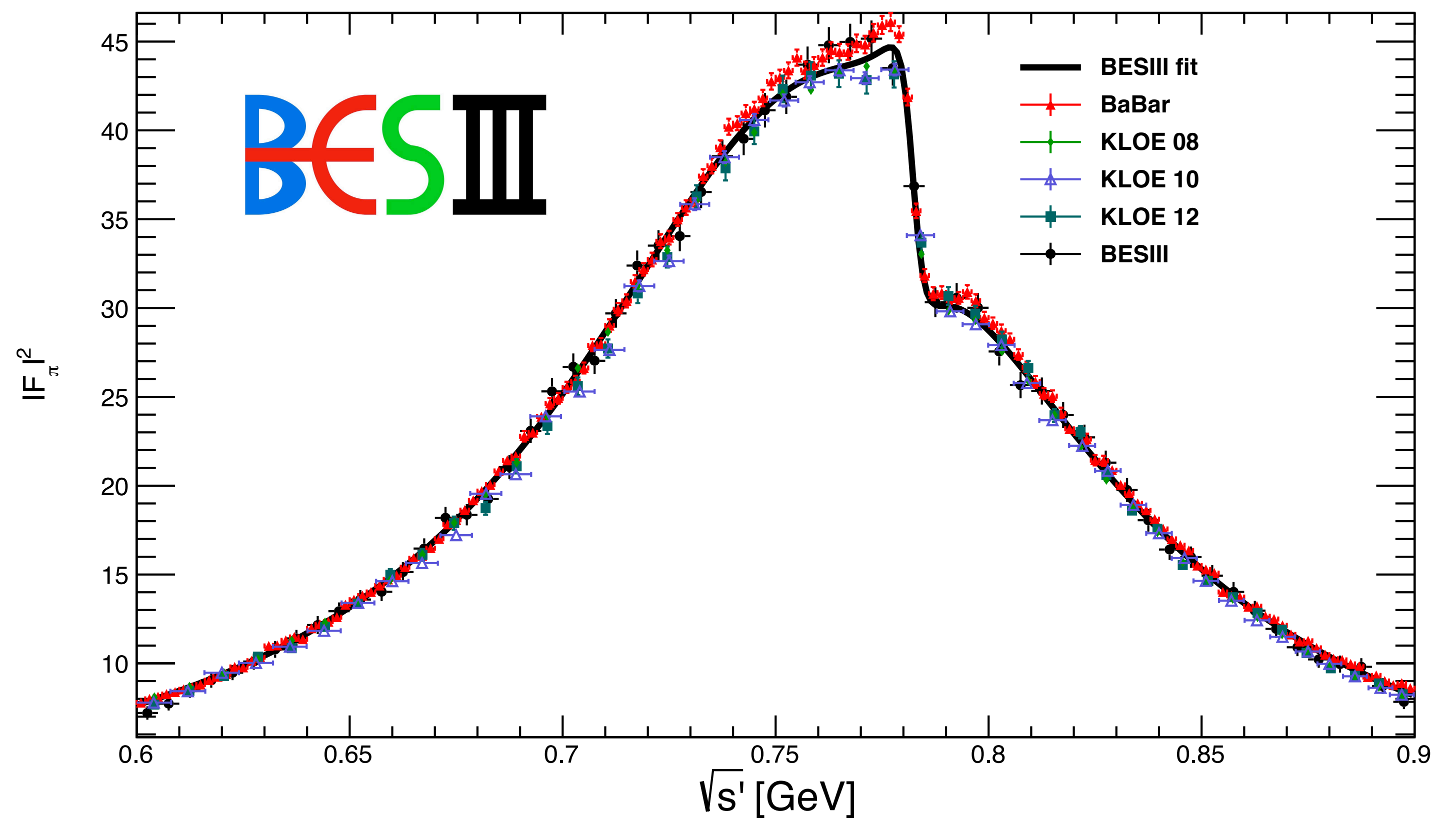


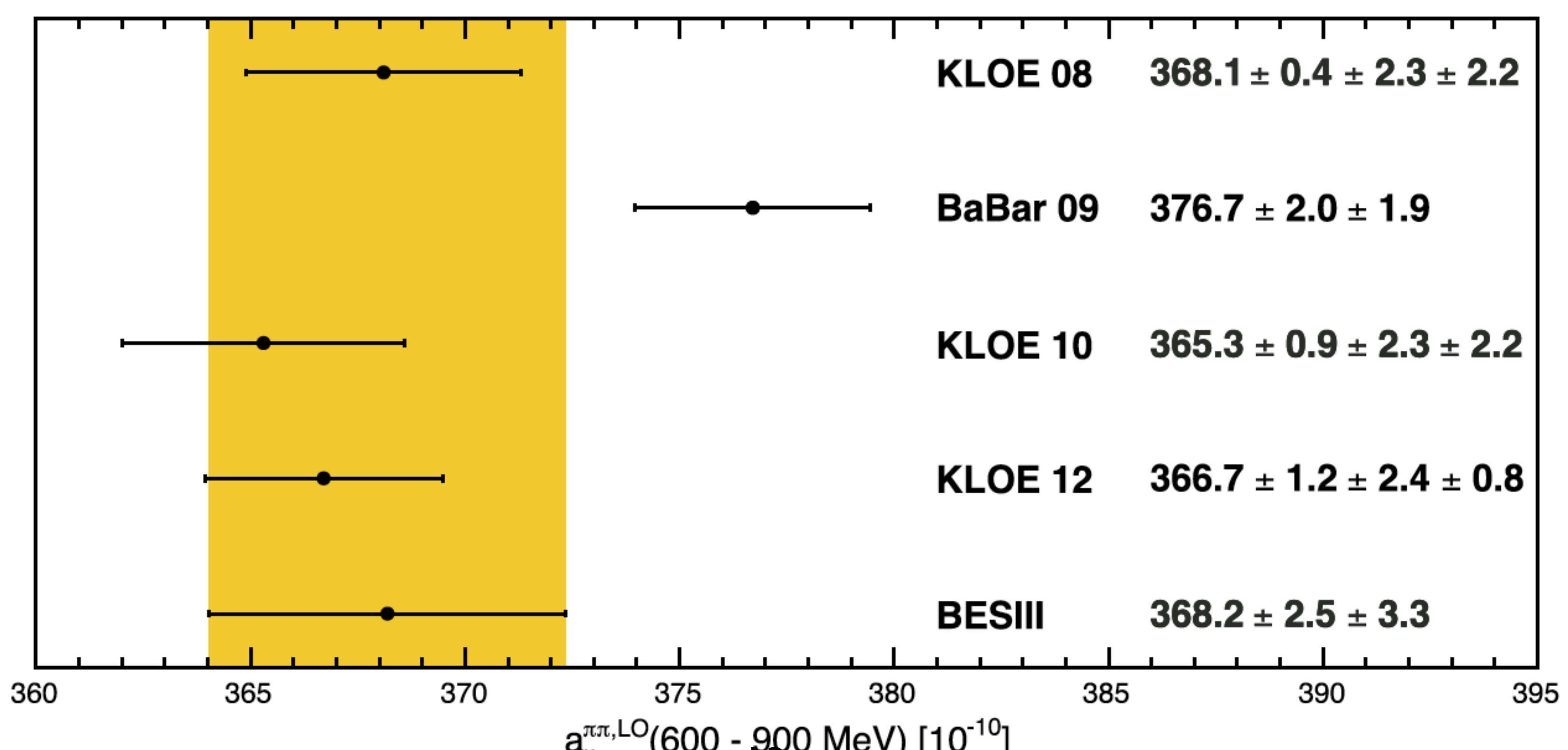
ISR method allows access to mass range M_{hadr} < 3 GeV at BES-III

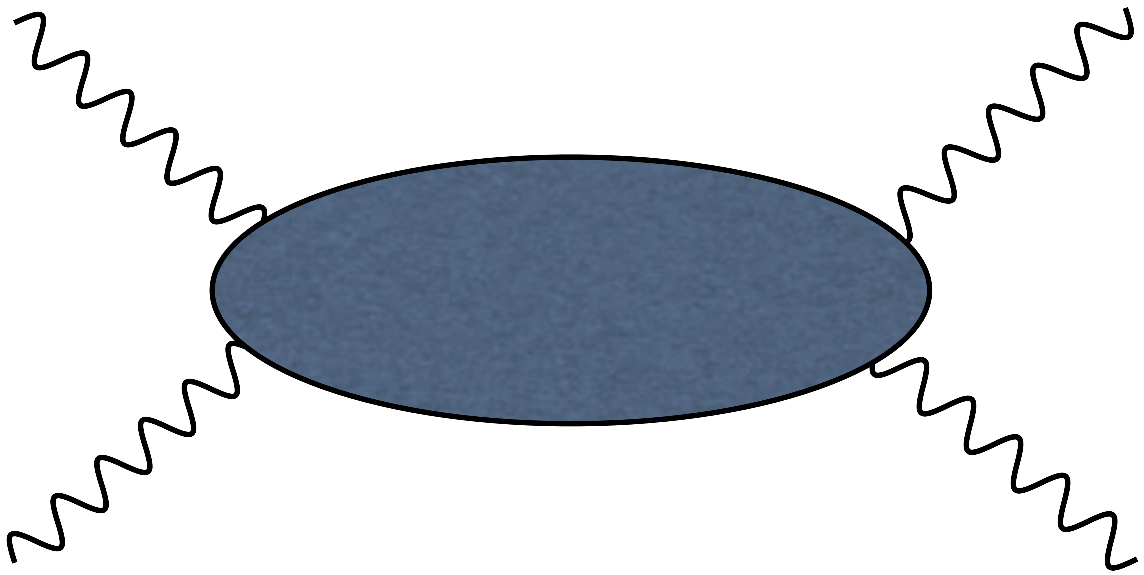
ISR channel $e^+e^- \to \pi^+\pi^-$: new BESIII results

- Tagged ISR analysis
- Systematic accuracy: 0.9%
- Some deviations seen from BaBar

Analysis by Mainz BESIII group
Denig et al.

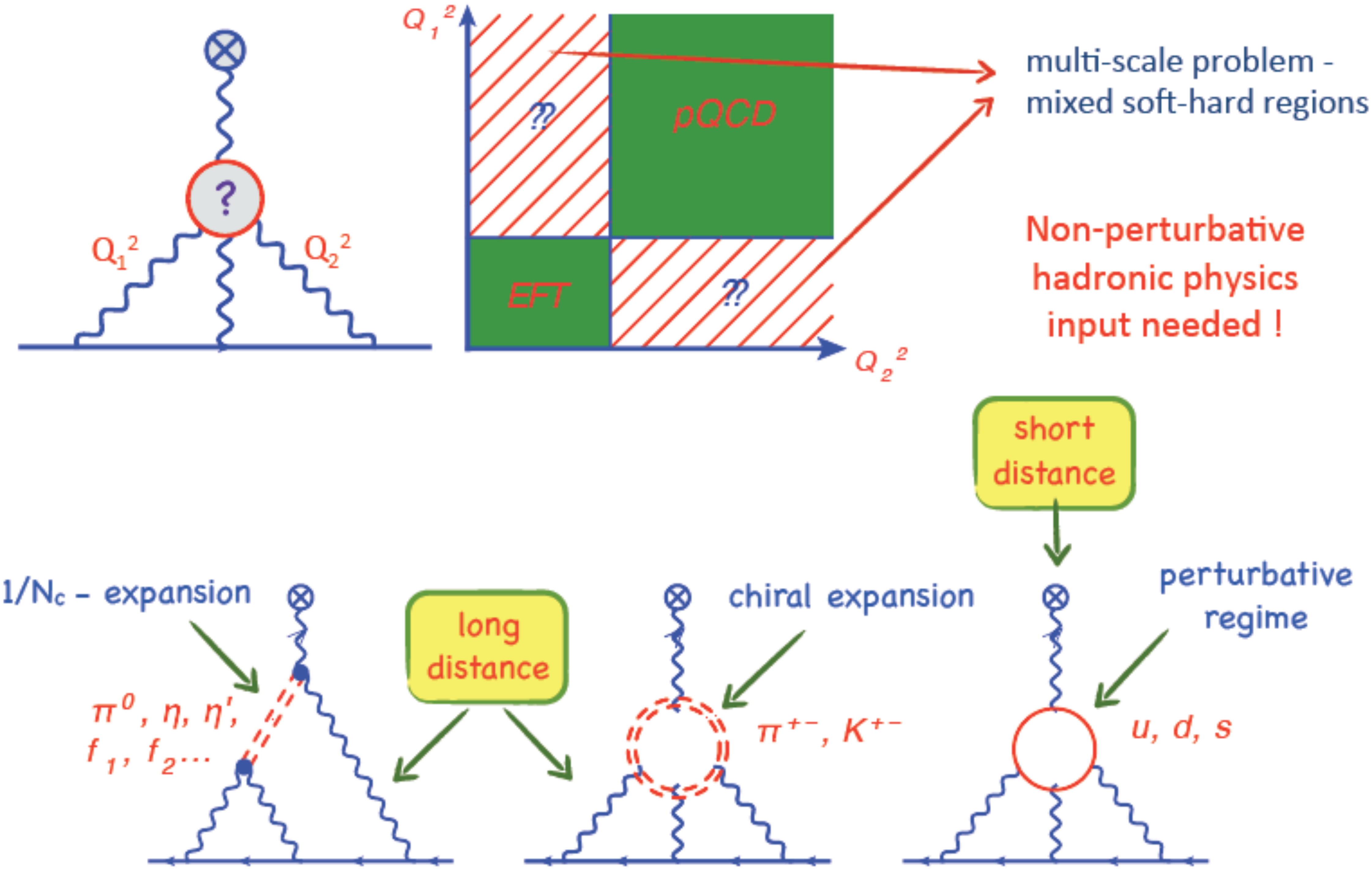




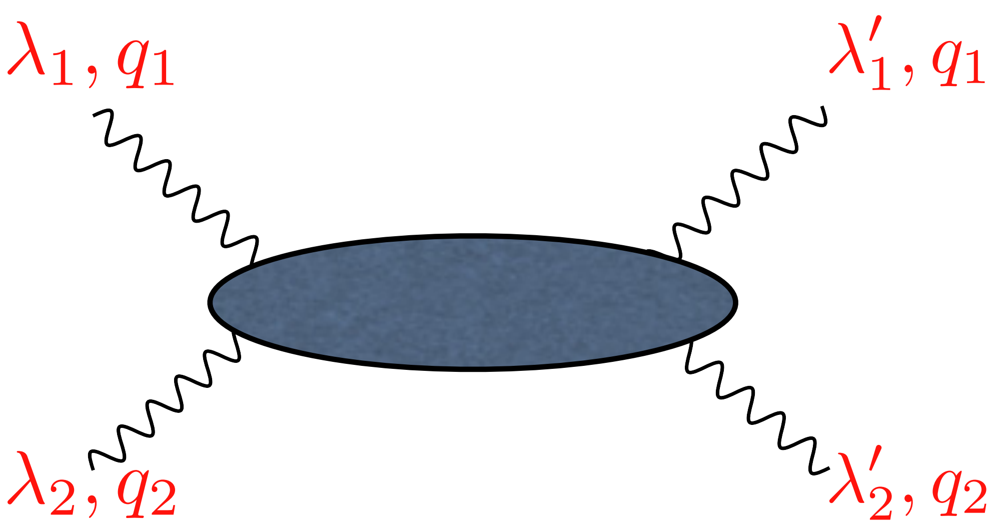


what is known about hadronic LbL scattering ?

hadronic LbL corrections to $(g\text{-}2)_\mu$: relevant contributions



Theory: sum rules for LbL scattering (I)



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda_1', q_1) + \gamma^*(\lambda_2', q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \qquad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes: $M_{\lambda_1'\lambda_2',\lambda_1\lambda_2}(\nu, Q_1^2, Q_2^2)$ $\qquad \lambda = 0, \pm 1$

discrete symmetries: 81 ➡ 8 independent amplitudes:

$$P: \quad M_{\lambda_1'\lambda_2',\lambda_1\lambda_2} = M_{-\lambda_1'-\lambda_2',-\lambda_1-\lambda_2}$$

$$T: \quad M_{\lambda_1'\lambda_2',\lambda_1\lambda_2} = M_{\lambda_1\lambda_2,\lambda_1'\lambda_2'}$$

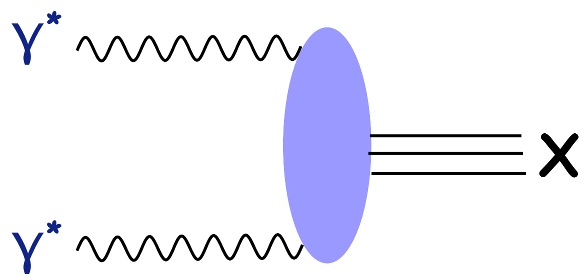
$M_{++,++},\ M_{+-,+-},\ M_{++,--},$ **T**

$M_{00,00},\ M_{+0,+0},\ M_{0+,0+},\ M_{++,00},\ M_{0+,-0}$ **T and L**

sum rules for LbL scattering (II)

Unitarity: link to $\gamma^* \gamma^* \to$ X cross sections

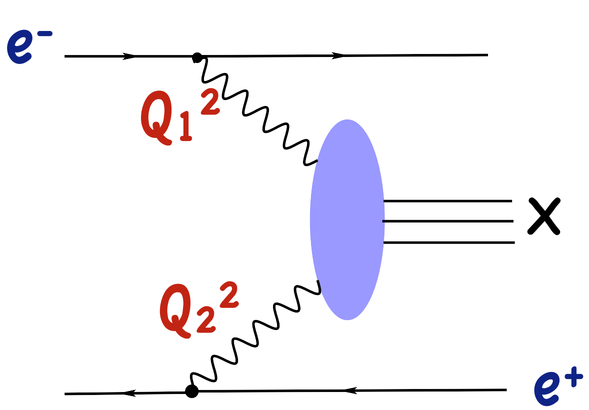
$$W_{\lambda_1'\lambda_2',\lambda_1\lambda_2} \equiv \mathrm{Im}\, M_{\lambda_1'\lambda_2',\lambda_1\lambda_2}$$



$$\begin{aligned}
W_{++,++} + W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0 + \sigma_2) = 2\sqrt{X}\,(\sigma_\parallel + \sigma_\perp) \equiv 4\sqrt{X}\,\sigma_{TT},\\
W_{++,++} - W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0 - \sigma_2) \equiv 4\sqrt{X}\,\tau^a_{TT},\\
W_{++,--} &\equiv 2\sqrt{X}\,(\sigma_\parallel - \sigma_\perp) \equiv 2\sqrt{X}\,\tau_{TT},\\
W_{00,00} &\equiv 2\sqrt{X}\,\sigma_{LL},\\
W_{+0,+0} &\equiv 2\sqrt{X}\,\sigma_{TL},\\
W_{0+,0+} &\equiv 2\sqrt{X}\,\sigma_{LT},\\
W_{++,00} + W_{0+,-0} &\equiv 4\sqrt{X}\,\tau_{TL},\\
W_{++,00} - W_{0+,-0} &\equiv 4\sqrt{X}\,\tau^a_{TL}.
\end{aligned}$$

$$X \equiv \nu^2 - Q_1^2 Q_2^2$$

Experiment: $e^- e^+ \to e^- e^+$ X cross sections

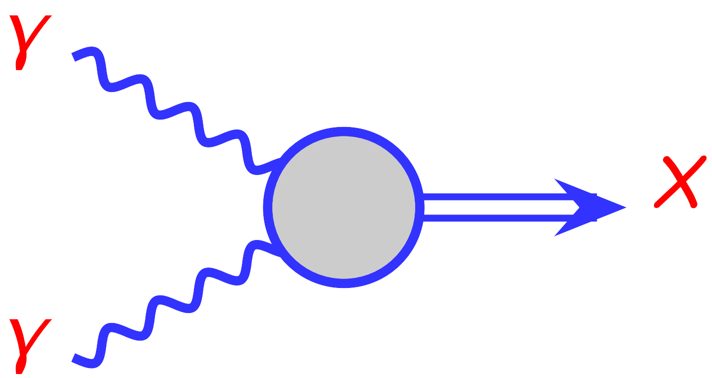


$$\begin{aligned}
d\sigma = {} & \frac{\alpha^2}{16\pi^4\, Q_1^2\, Q_2^2}\, \frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1^{\,\prime}}{E_1'} \cdot \frac{d^3\vec{p}_2^{\,\prime}}{E_2'}\\
& \times \Big\{ 4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT}\\
& + 2\,\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\left(\cos 2\tilde{\phi}\right)\tau_{TT} + 8\left[\frac{\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)}{\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\tau_{TL}\\
& + h_1 h_2\, 4\left[\left(\rho_1^{00}+1\right)\left(\rho_2^{00}+1\right)\right]^{1/2}\tau^a_{TT} + h_1 h_2\, 8\left[\left(\rho_1^{++}-1\right)\left(\rho_2^{++}-1\right)\right]^{1/2}\left(\cos\tilde{\phi}\right)\tau^a_{TL}\Big\}
\end{aligned}$$

lepton beam polarization

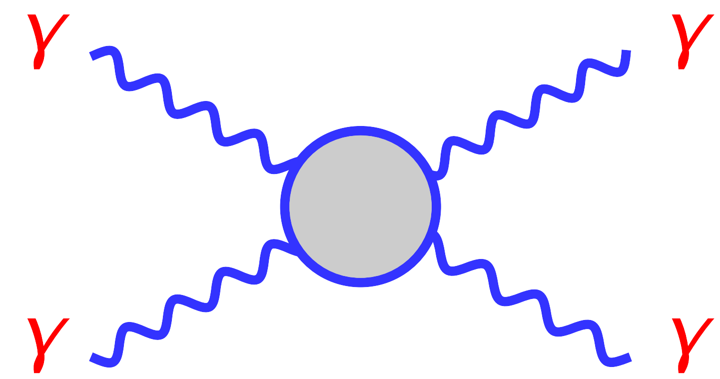
ρ's, ϕ : kinematical quantities

sum rules for LbL scattering (III)



micro-causality

unitarity → dispersion theory ← LETs



imaginary part of the amplitude - photon-photon fusion into leptons and hadrons:

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}\left[\sigma_2(s) - \sigma_0(s)\right]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}\left[\sigma_{tot}(s)\right]$$

real part of the amplitude - low-energy structure of the elastic LbL scattering:

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

Sum rules

$$\int_{s_0}^{\infty} \frac{ds}{s}\left[\sigma_2(s) - \sigma_0(s)\right] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi}\int_{s_0}^{\infty} \frac{ds}{s^2}\left[\sigma_\parallel(s) \pm \sigma_\perp(s)\right]$$

sum rules for LbL scattering: 3 superconvergence relations

➡ helicity difference sum rule   for $Q^2 = 0$: GDH sum rule   Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

the I=0 channel

meson contributions to helicity SR for $Q_1^2 = 0$ (in nb)

State	m (MeV)	$\Gamma_{\gamma\gamma}$(keV)		SR_1 $(Q^2=0)$
η	547.862±0.017	0.516±0.020		-193±7
η'	957±0.06	4.35±0.25		-304±17
$f_2(1270)$	1275.5±0.8	2.93±0.40	(Λ=2)	434±60
			(Λ=0)	≈0
$f_2(1565)$	1562±13	0.70±0.14		56±11
.....				
sum				-7±64

lowest few meson states saturate sum rules

➡ sum rules involving L photons

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a\right]_{Q_2^2=0}$$

Pascalutsa, Vdh (2010)

Pascalutsa, Pauk, Vdh (2012, 2014)

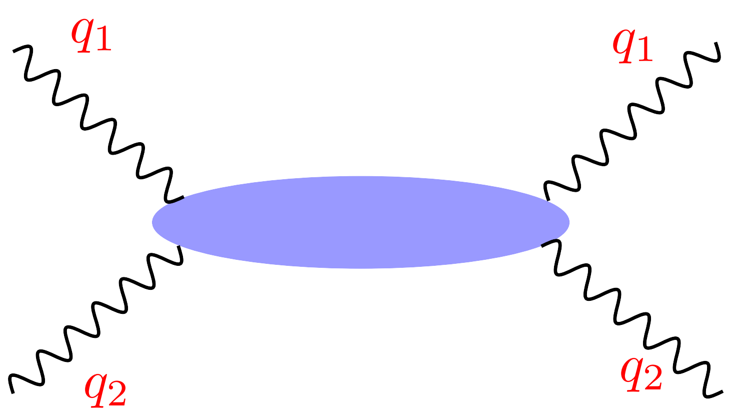
lattice calculation of forward $\gamma^*\gamma^*$ scattering

Green, Gryniuk, von Hippel, Meyer, Pascalutsa (2015), Gerardin et al.(2018)

- Euclidean correlator for LbL scattering

$$\Pi^E_{\mu_1\mu_2\mu_3\mu_4}(P_4; P_1, P_2) \equiv \int d^4X_1\, d^4X_2\, d^4X_4 e^{-i\sum_a P_a\cdot X_a} \langle J_{\mu_1}(X_1) J_{\mu_2}(X_2) J_{\mu_3}(0) J_{\mu_4}(X_4)\rangle_{\mathrm{E}}$$

- forward amplitude for two transverse (T) γ^*



$$q_1^2 = -Q_1^2, \quad q_2^2 = -Q_2^2, \quad \nu = q_1 \cdot q_2$$

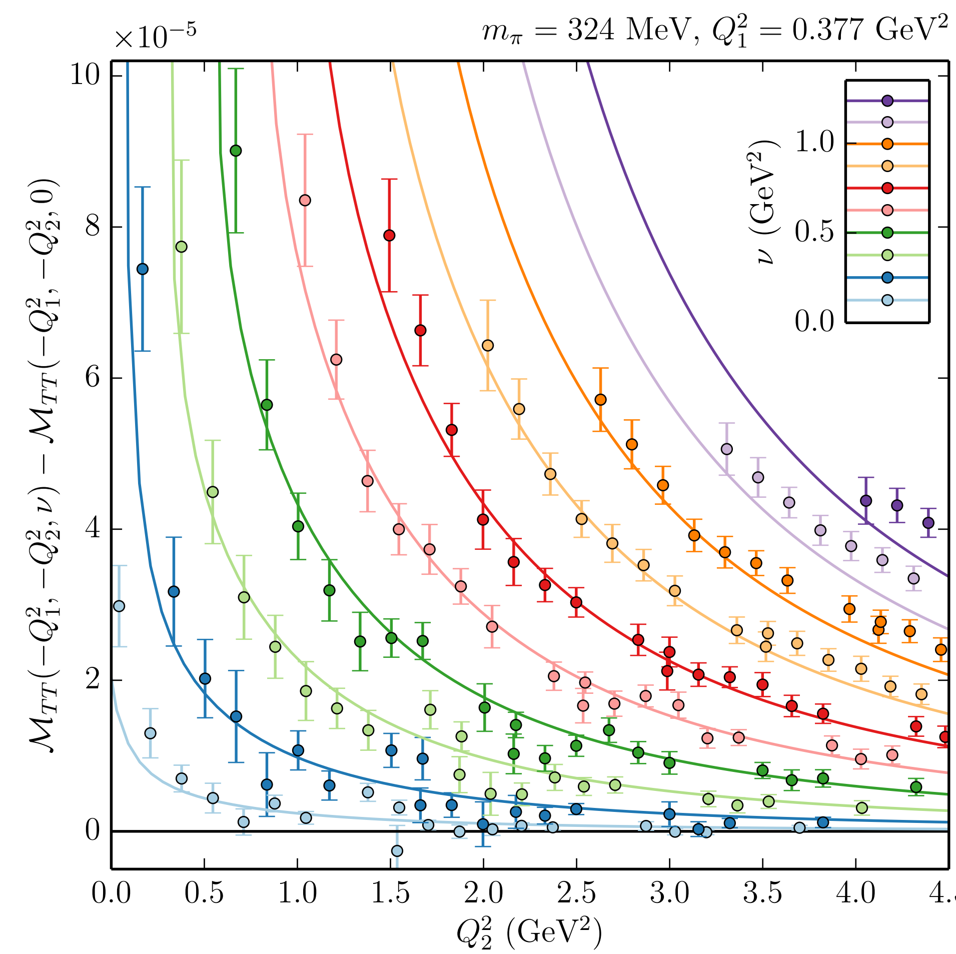
$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, -Q_1\cdot Q_2) = \frac{e^4}{4} R^E_{\mu_1\mu_3} R^E_{\mu_2\mu_4} \Pi^E_{\mu_1\mu_3\mu_4\mu_2}(-Q_2; -Q_1, Q_1)$$

R^E : transverse projectors

- comparison with dispersive sum rule evaluation

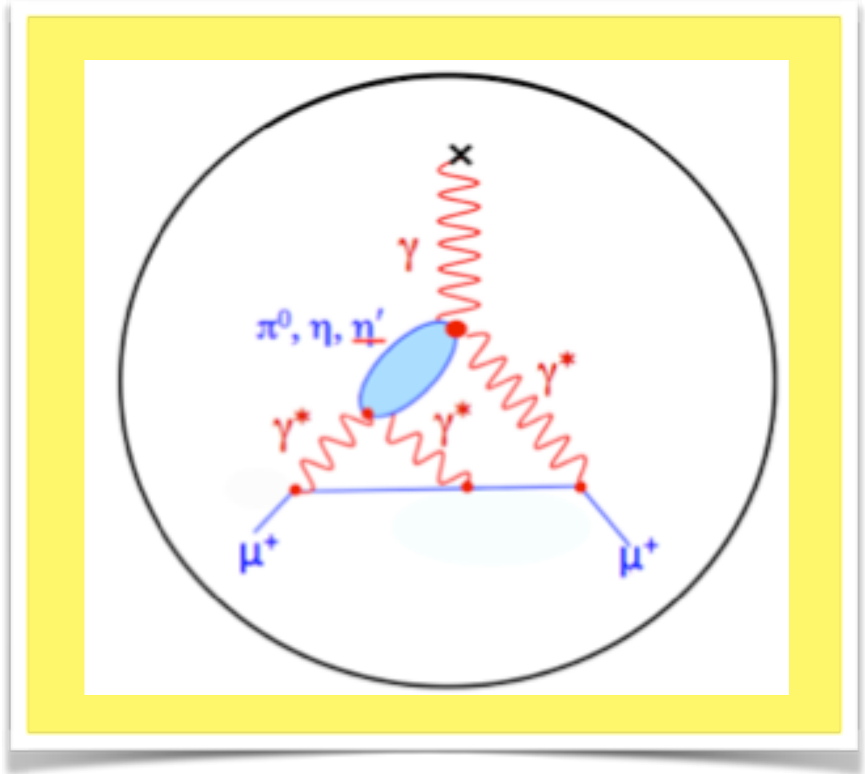
$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, \nu) - \mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, 0)$$
$$= \frac{2\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\sqrt{\nu'^2 - Q_1^2 Q_2^2}}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} (\sigma_0 + \sigma_2)(\nu')$$

2-flavor QCD, quark connected contribution →



promising consistency between lattice and dispersive estimates

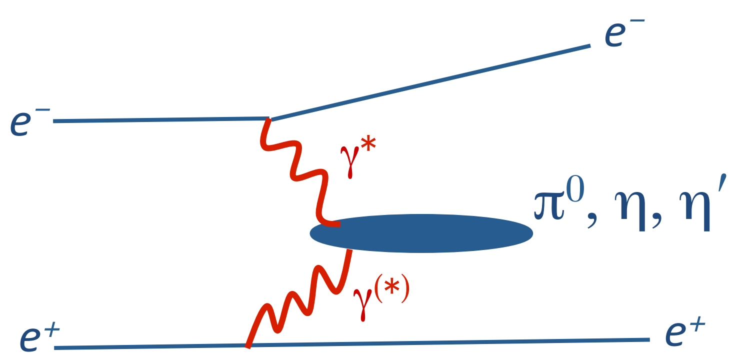
next steps: disconnected, lattice evaluation of a_μ from Π^E



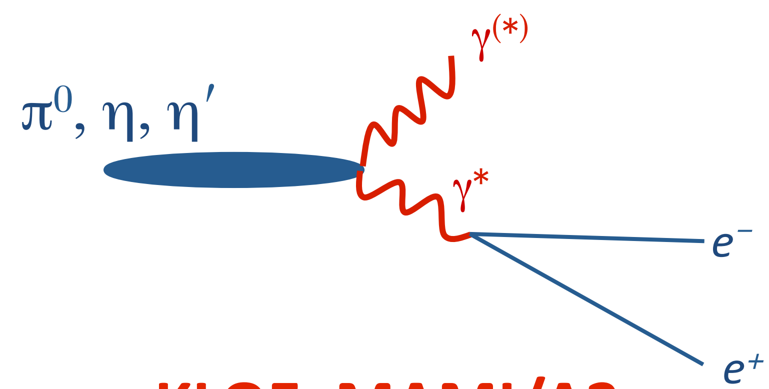
how to estimate the HLbL contribution to a_μ ?

hadronic LbL corrections to $(g\text{-}2)_\mu$

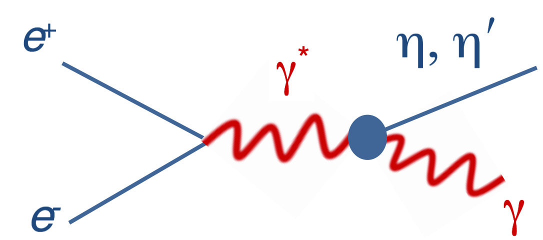
➡ **experimental input:** meson transition FFs, $\gamma^*\,\gamma^*$ -> multi-meson states, meson Dalitz decays



CLEO, BaBar, Belle, BESIII, ...



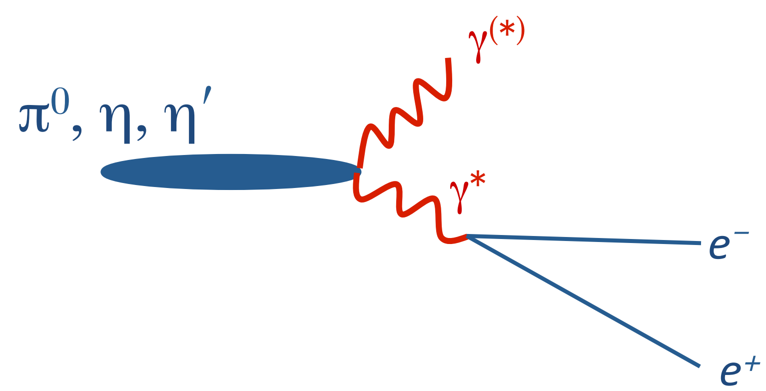
KLOE, MAMI/A2, BESIII, ...

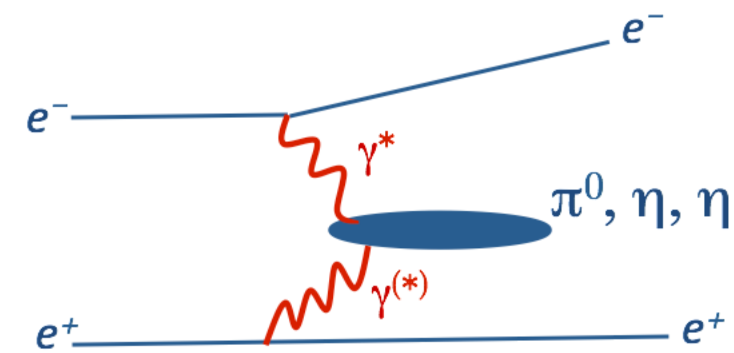


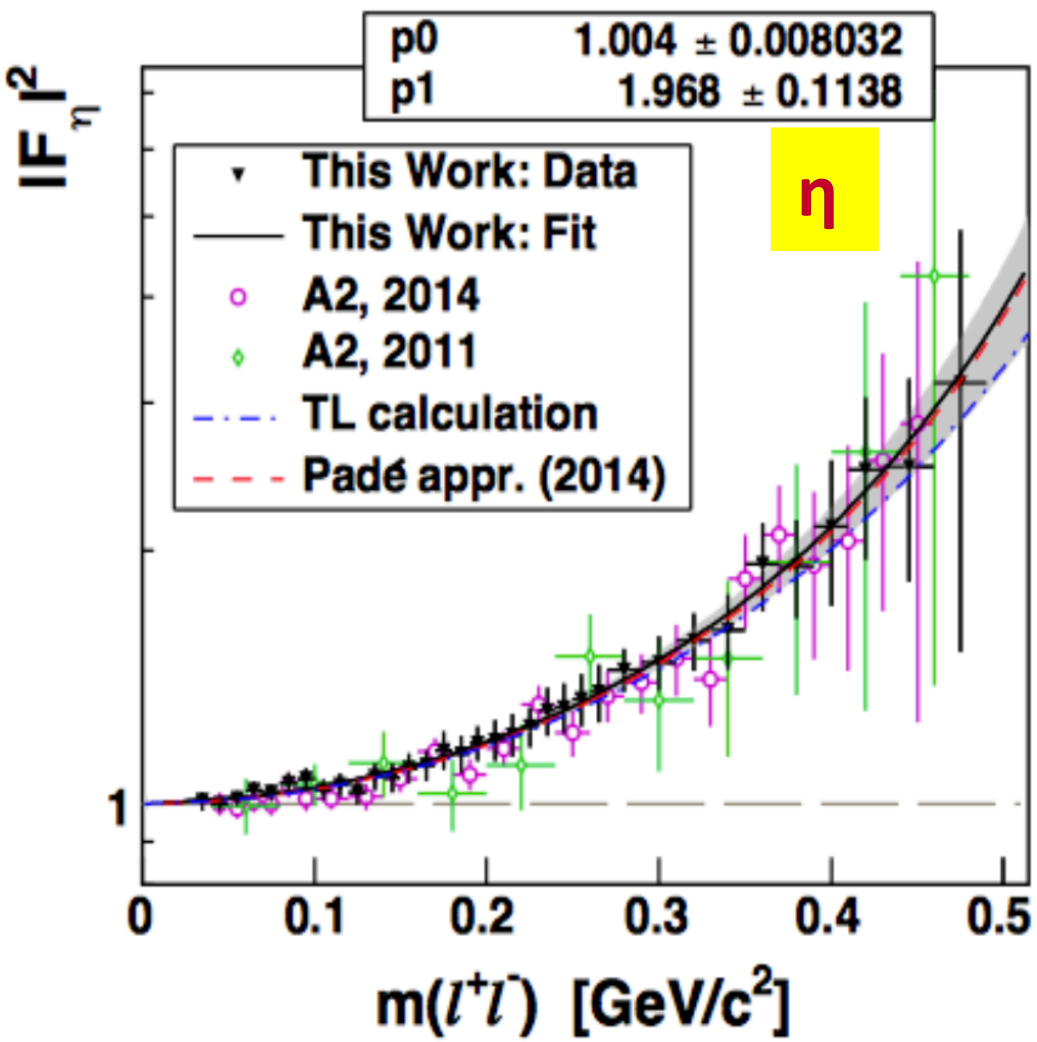
SND, CMD-2, BESIII, ...

➡ **theory deveolpments:**
- sum rules, dispersion relations
- lattice QCD
- Dyson-Schwinger
- phenomenology, modeling

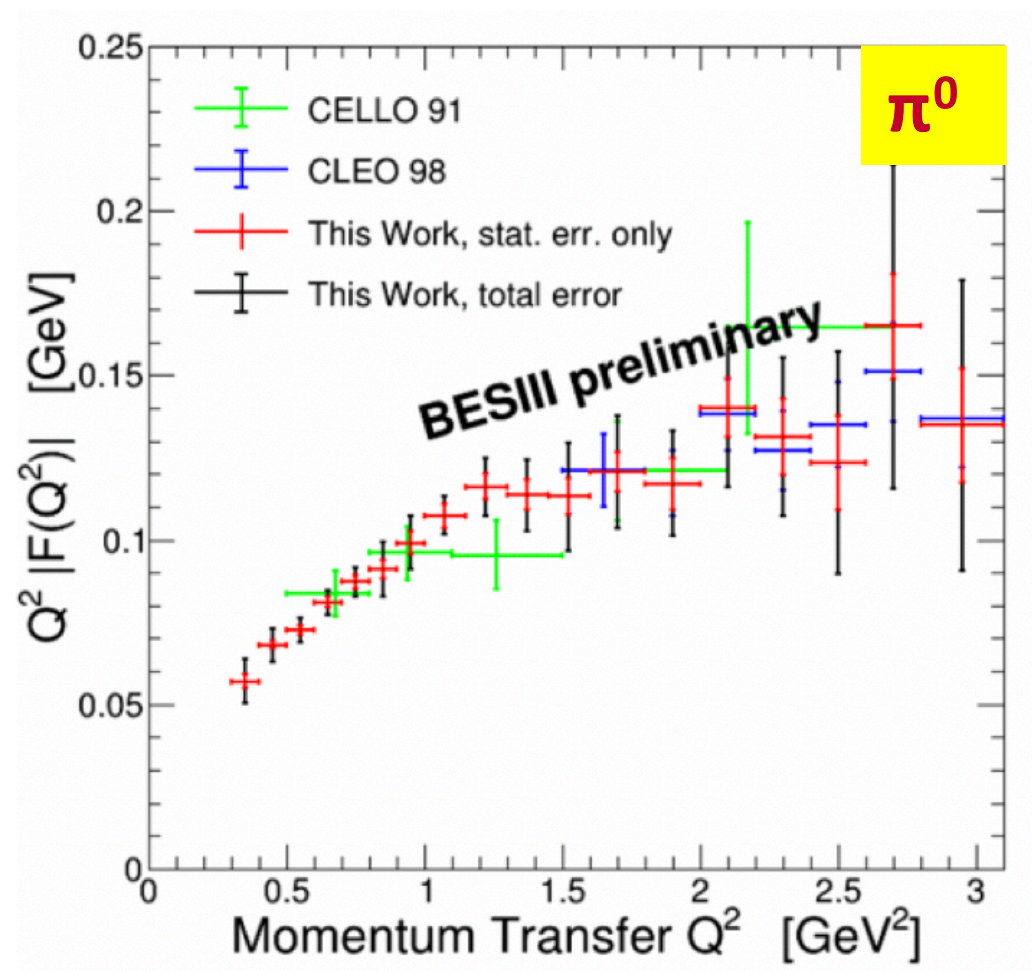
$\gamma^*\gamma^* \to$ M processes: meson transition form factors (TFFs)







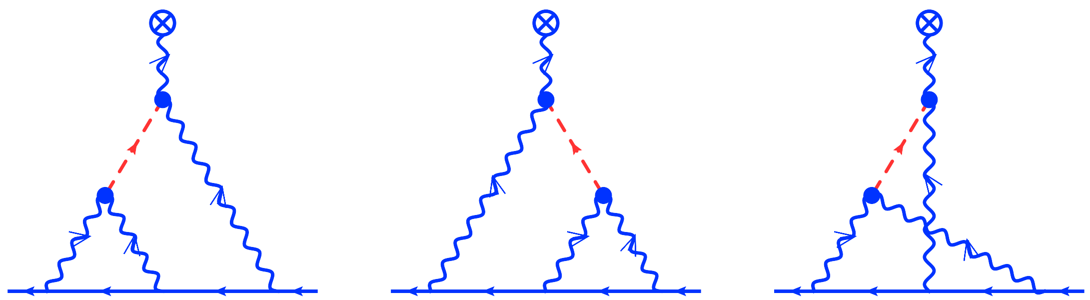
MAMI/A2 data



BESIII data set: significantly extends data set below $Q^2 < 1.5$ GeV^2: input to $(g-2)_\mu$

Redmer et al.

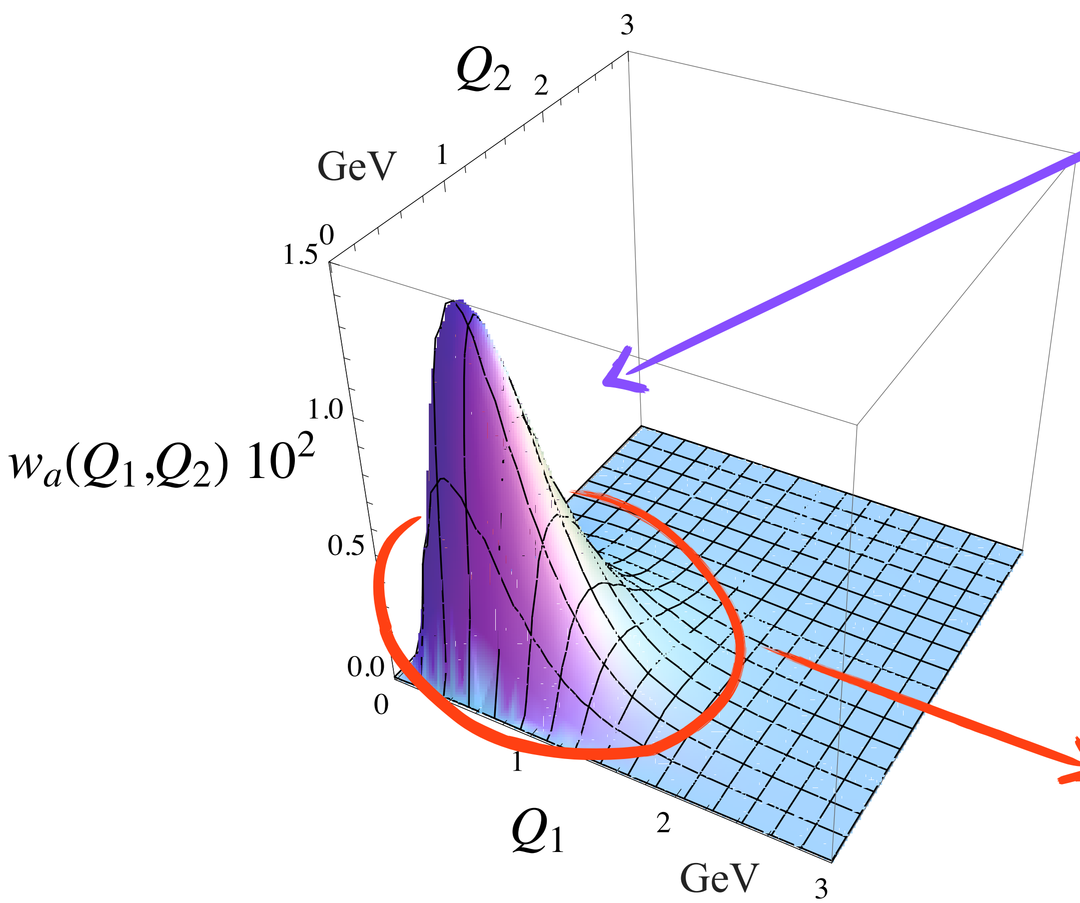
single meson contributions to a_μ



for π^0 : Knecht, Nyffeler (2002)

extended in many works

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int \mathrm{d}Q_1 \int \mathrm{d}Q_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2) \right]$$

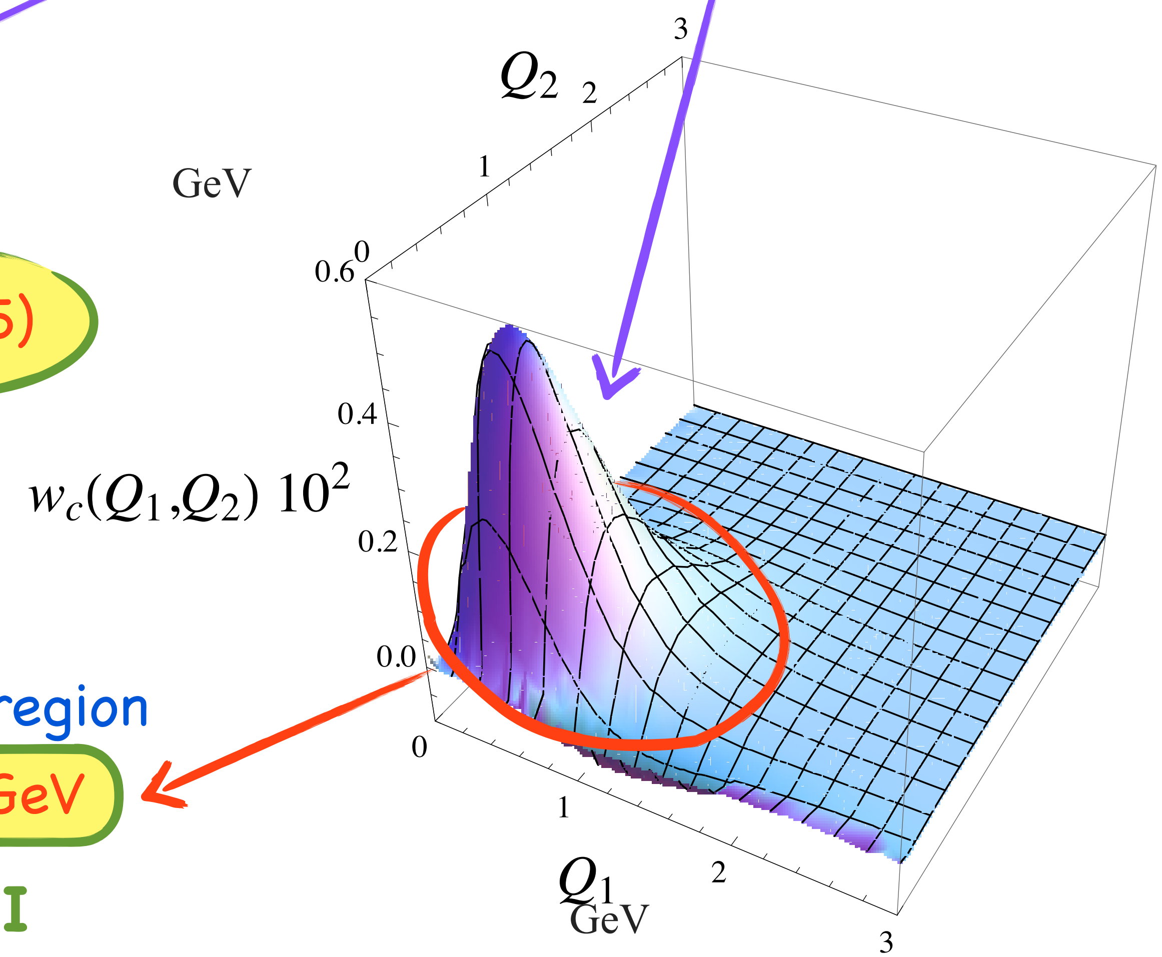


f1(1285)

dominating region

$Q_1 \sim Q_2 \sim 1$ GeV

BES III



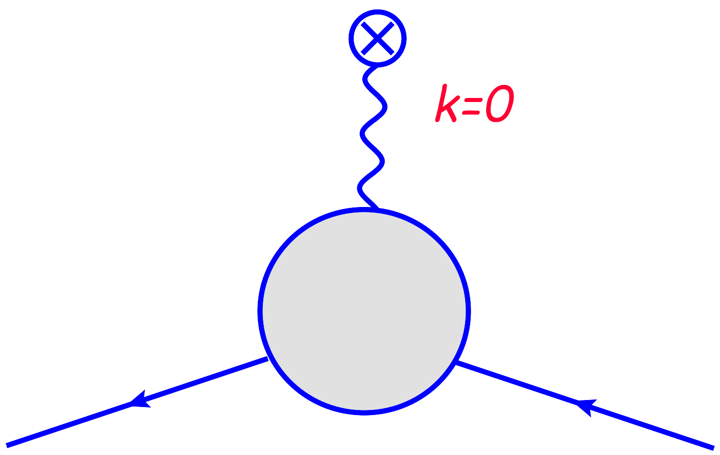
Pauk, Vdh (2013)

dispersive analysis for a_μ (I)

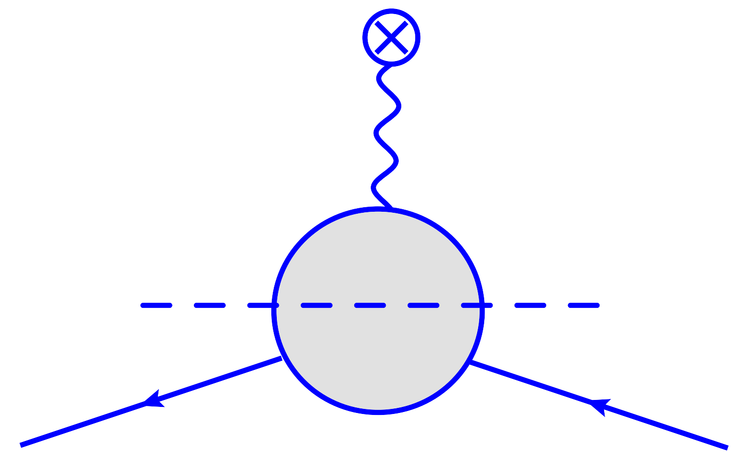
➡ **dispersion formalism** directly for a_μ Pauk, Vdh (2014)

$$a_\mu = \lim_{k\to 0} F_2(k^2, (p+k)^2, p^2)$$

$$F_2(0) = \frac{1}{2\pi i}\int \frac{dk^2}{k^2}\, \mathrm{Abs}\, F_2(k^2)$$



$$a_\mu = F_2(0)$$



weighting functions (entire)

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p, p', q_1, k-q_1-q_2, q_2)$$

analytic structure ⟶

$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)}{q_1^2 q_2^2 (k-q_1-q_2)^2 \left[(p+q_1)^2 - m^2\right]\left[(p+k-q_2)^2 - m^2\right]}$$

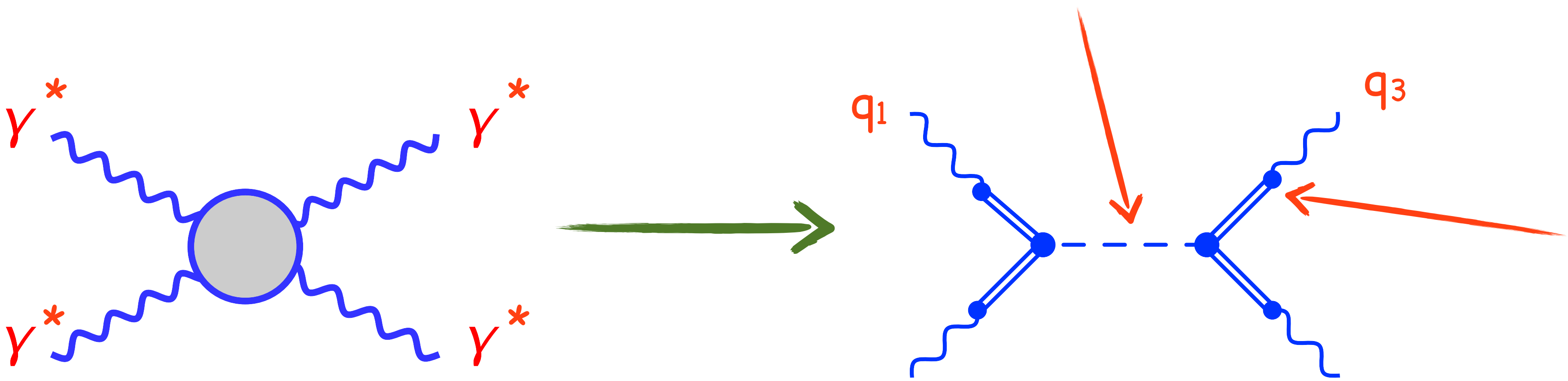
$$\Pi_{\lambda_1\lambda_2\lambda_3\lambda_4}(q_1, q_2, q_3) = \epsilon^\mu(q_1, \lambda_1)\epsilon^\nu(q_2, \lambda_2)\epsilon^\lambda(q_3, \lambda_3)\epsilon^\rho(q_4, \lambda_4)\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3)$$

dispersive analysis for a_μ (II)

➡ **proof of principle:** pole contributions

π^0 - pole at $(q_i+q_j)^2 = M^2$

analytical structure of LbL amplitude

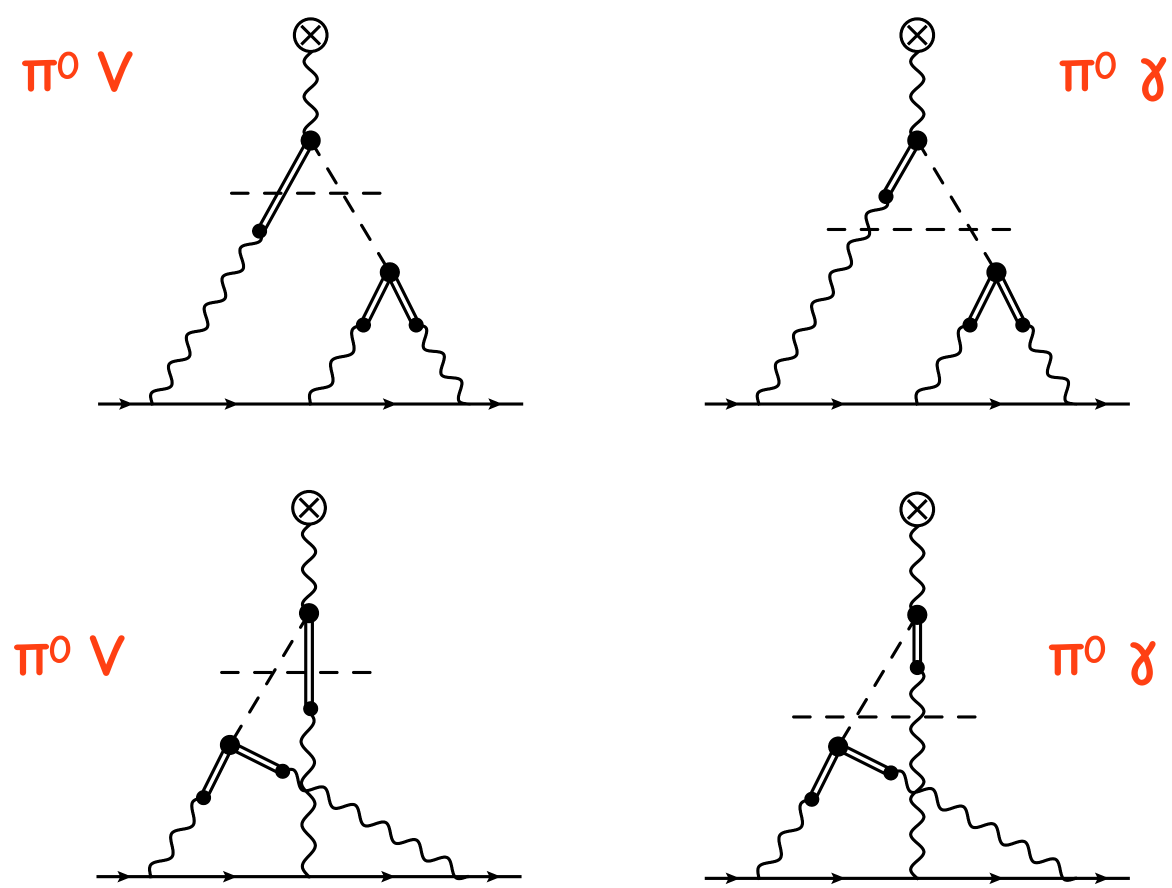


vector - poles at $q_i^2 = \Lambda^2$

➡ **2-particle cuts**

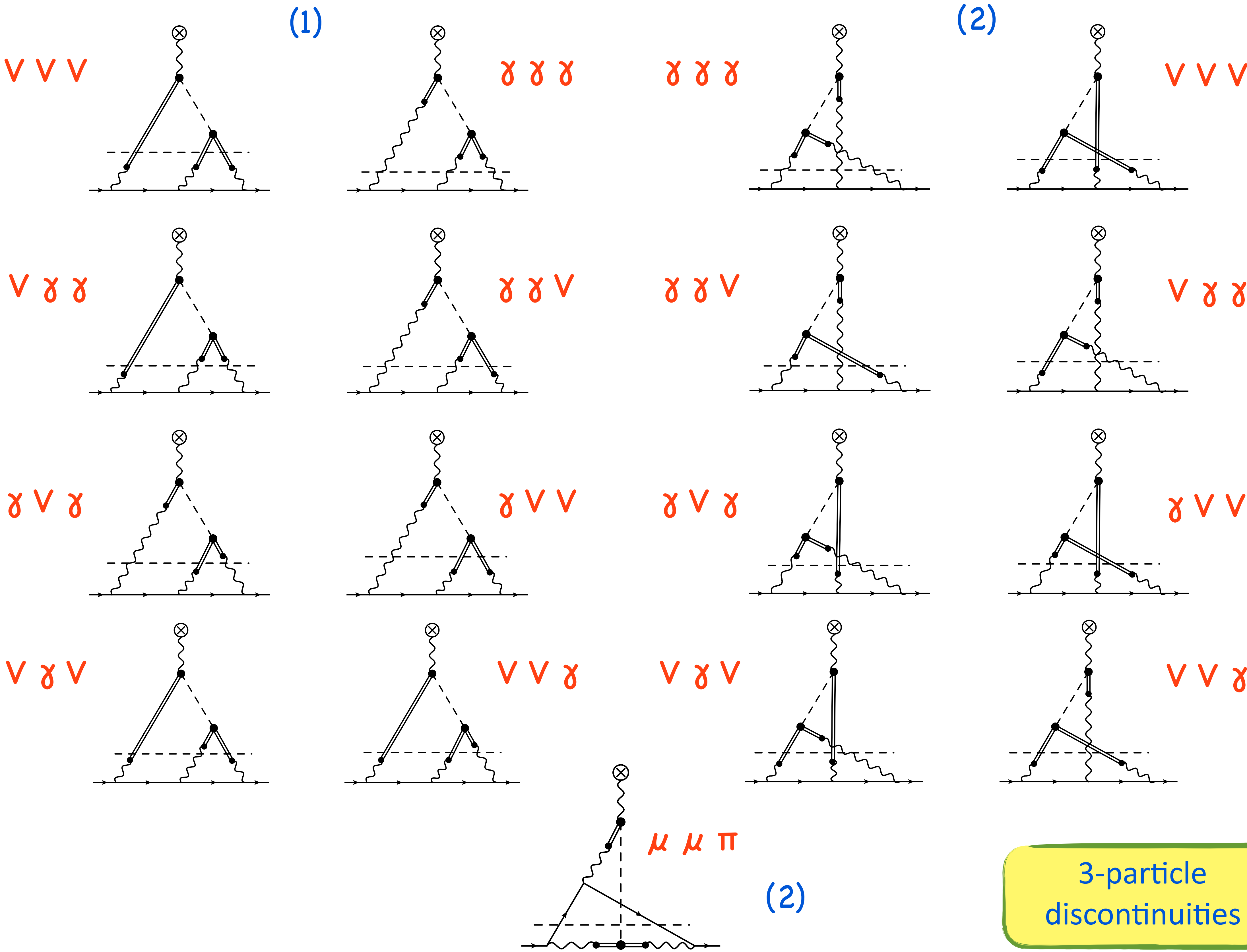
(1)

(2)



2-particle discontinuities

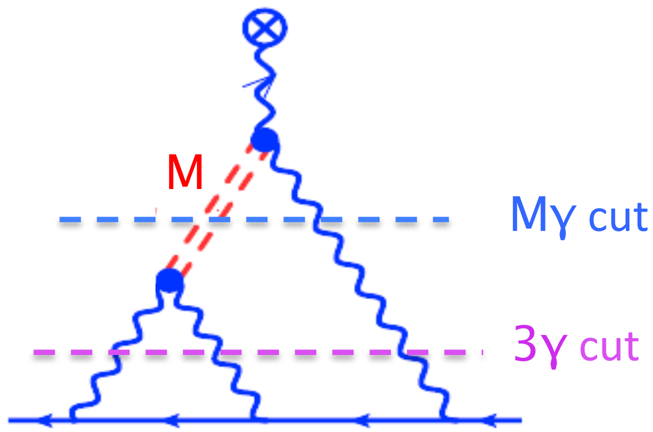
dispersive analysis for a_μ (III)



3-particle discontinuities

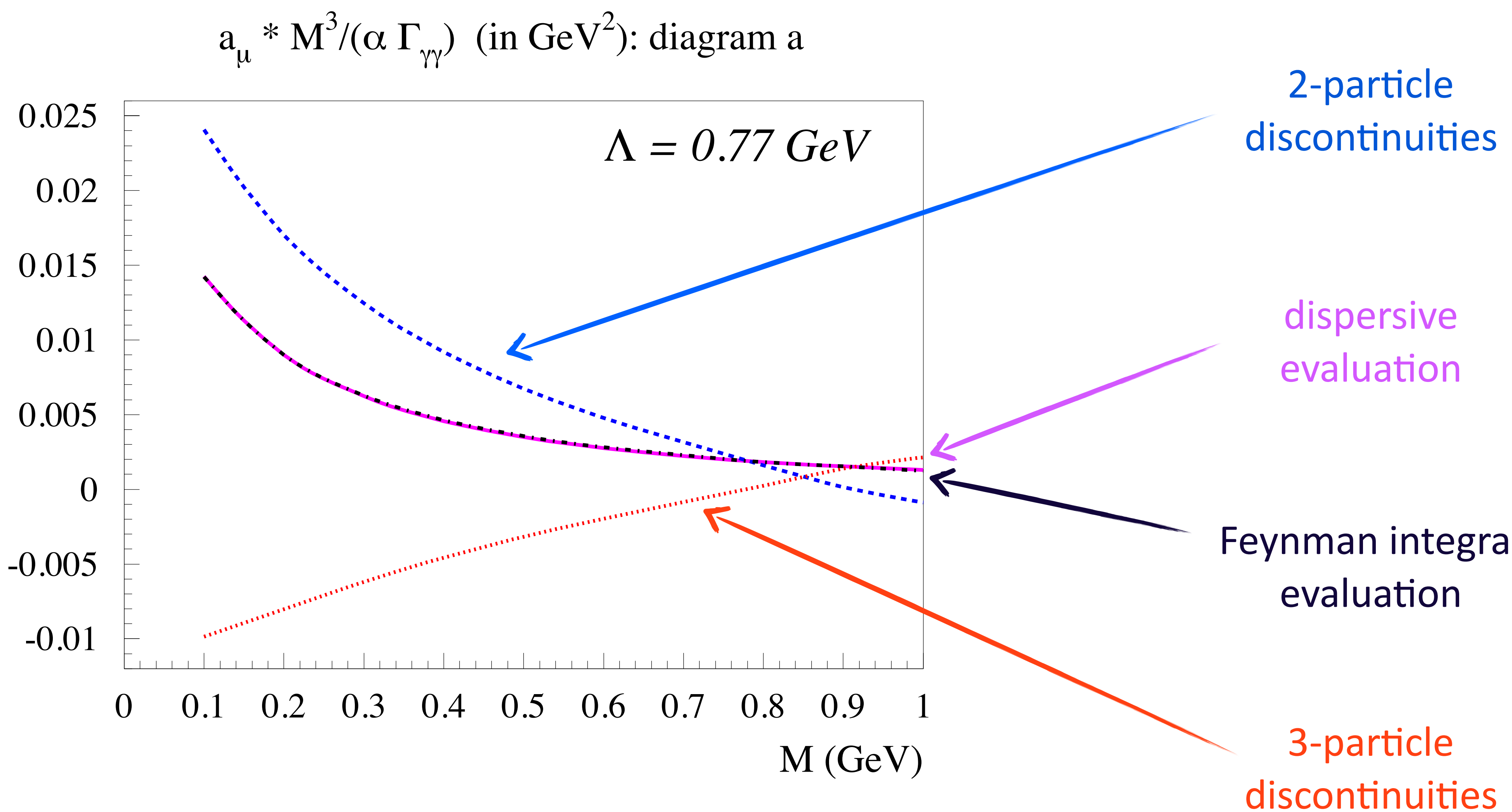
dispersive analysis for a_μ (IV)

reconstruction of a_μ from dispersion integral: **proof of principle**



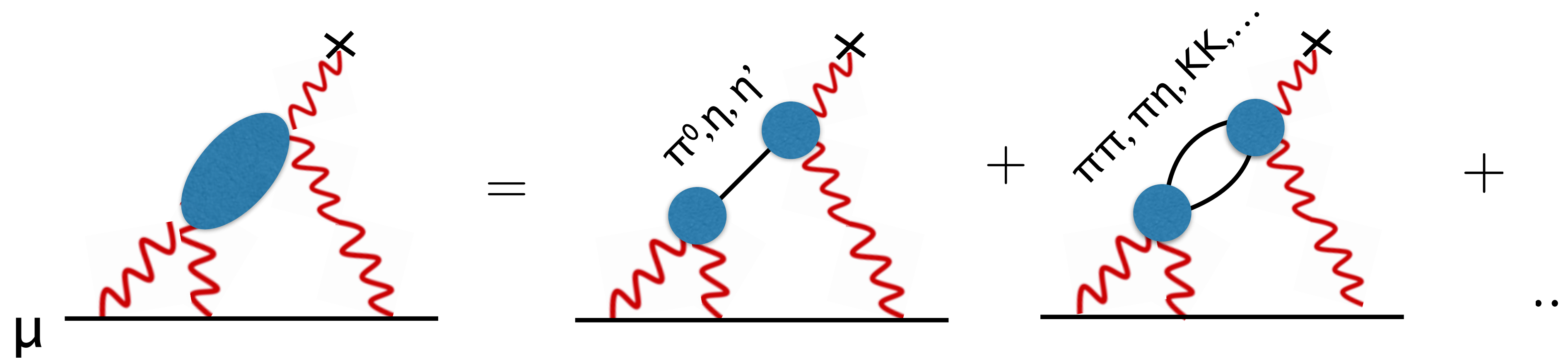
$$F_2^{(i)}(0) = \frac{1}{2\pi i}\int_{M^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(2)} F_2^{(i)}(t) + \frac{1}{2\pi i}\int_{0}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(3)} F_2^{(i)}(t)$$

Pauk, Vdh (2014)



exact agreement between direct 2-loop and dispersive calculation found

HLbL to a_μ : present status and outlook



Total HLbL [a_μ in units 10^{-10}]

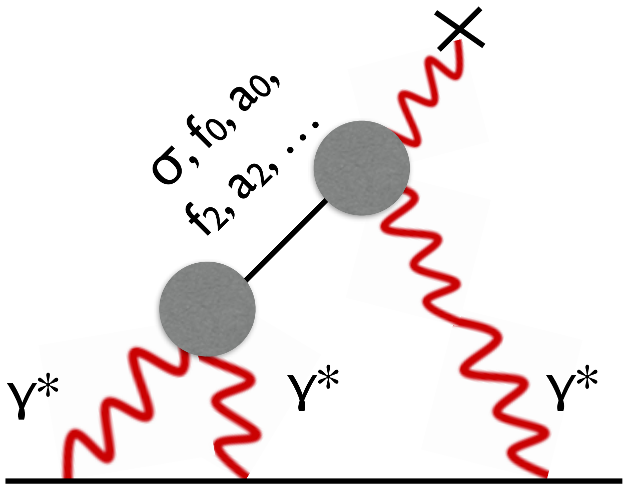
Authors	π^0, η, η'	$\pi\pi$, KK	scalars	axial vectors	quark loops	Total
BPaP(96)	8.5(1.3)	-1.9(1.3)	-0.68(0.20)	0.25(0.10)	2.1(03)	8.3(3.2)
HKS(96)	8.3(0.6)	-0.5(0.8)	—	0.17(0.17)	1.0(1.1)	9.0(1.5)
KnN(02)	8.3(1.2)	—	—	—	—	8.0(4.0)
MV(04)	11.4(1.0)	—	—	2.2(0.5)	—	13.6(2.5)
PdRV(09)	11.4(1.3)	-1.9(1.9)	-0.7(0.7)	1.5(1.0)	0.23	**10.5(2.6)**
N/JN(09)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	2.2(0.5)	2.1(0.3)	11.6(3.9)
J(15)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	**0.75(0.27)**	2.1(0.3)	**10.2(3.9)**

B=Bjnens, Pa=Pallante, P=Prades, H=Hayakawa, K=Kinoshita, S=Sanda, Kn=Knecht, N=Nyffeler, M=Melnikov, V=Vainshtein, dR=de Rafael, J=Jegerlehner

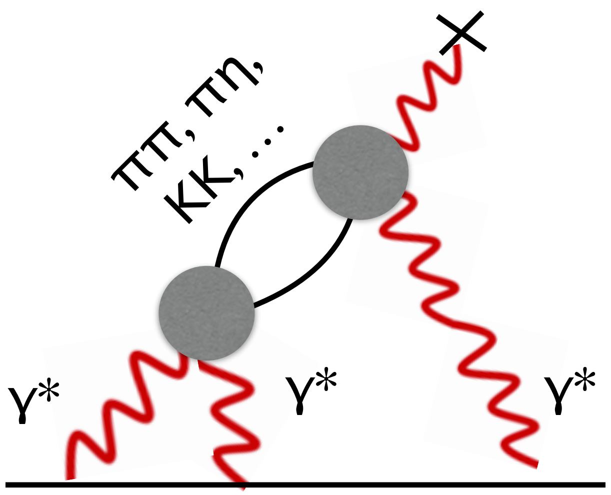
Improvements: include multi-meson channels in a data-driven / dispersive approach

Dispersive formalism for HLbL scattering

Important contributions beyond **pseudo-scalar** poles



dispersive analysis for
ππ, πη, … loops



3 dispersive formalisms have been proposed:

1) Bern group: two-loop integral with full HLbL tensor

Colangelo, Hoferichter, Procura, Stoffer (2014,2015, 2017)

$a_\mu^{\pi\text{-box}} = (-\,1.59 \pm 0.02) \times 10^{-10}$ $\quad$ $a_\mu^{\text{s-wave}\,\pi\pi} = (-\,0.8 \pm 0.1) \times 10^{-10}$

2) Mainz group: DR for Pauli FF of muon (involves space- and timelike data)

Pauk, Vdh (2014)

Proof of principle calculation for pion pole has been demonstrated

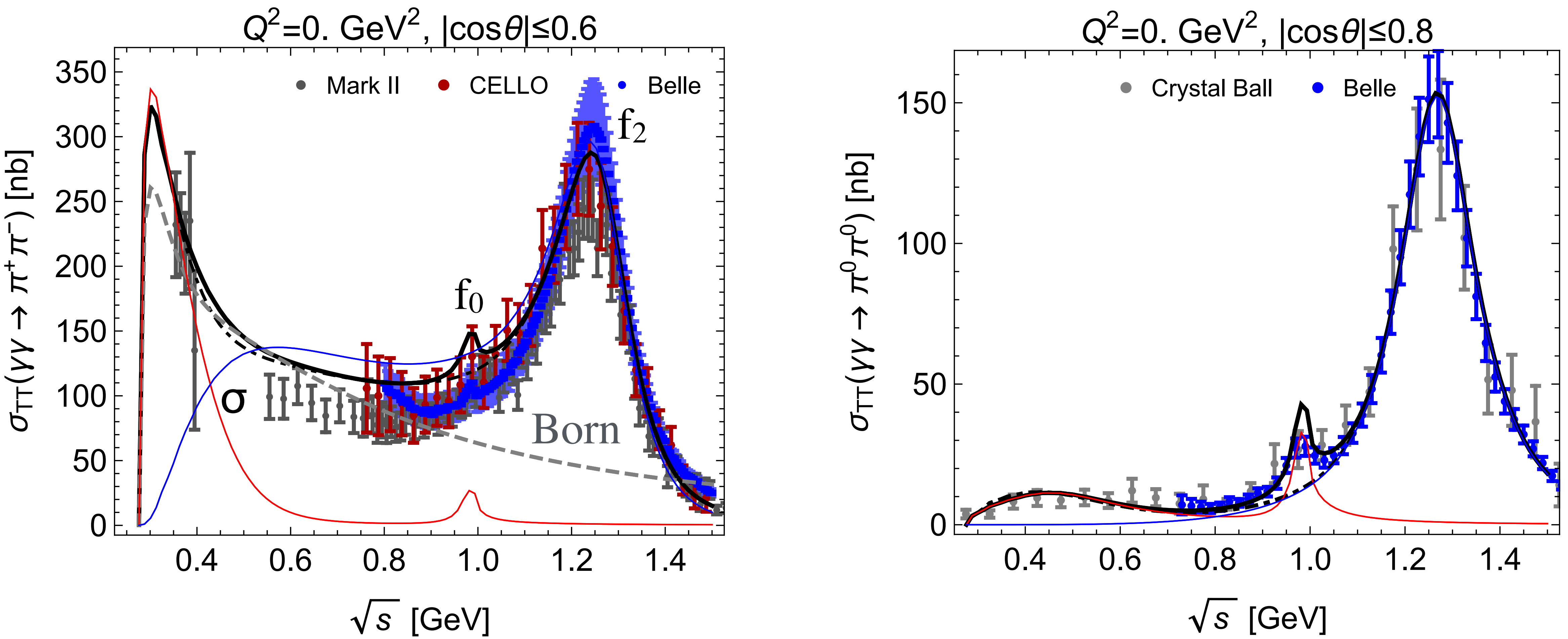
3) Schwinger sum rule: Hagelstein, Pascalutsa: PRL 120, 072002 (2018)

multi-meson production in $\gamma\gamma$ collisions

$\gamma\gamma \to \pi^+\pi^-$

$Q^2 = 0$ GeV^2

$\gamma\gamma \to \pi^0\pi^0$



Coupled-channel dispersive treatment of $f_0(980)$ is **crucial**

$f_2(1270)$ described dispersively through Omnes function

Danilkin, Vdh

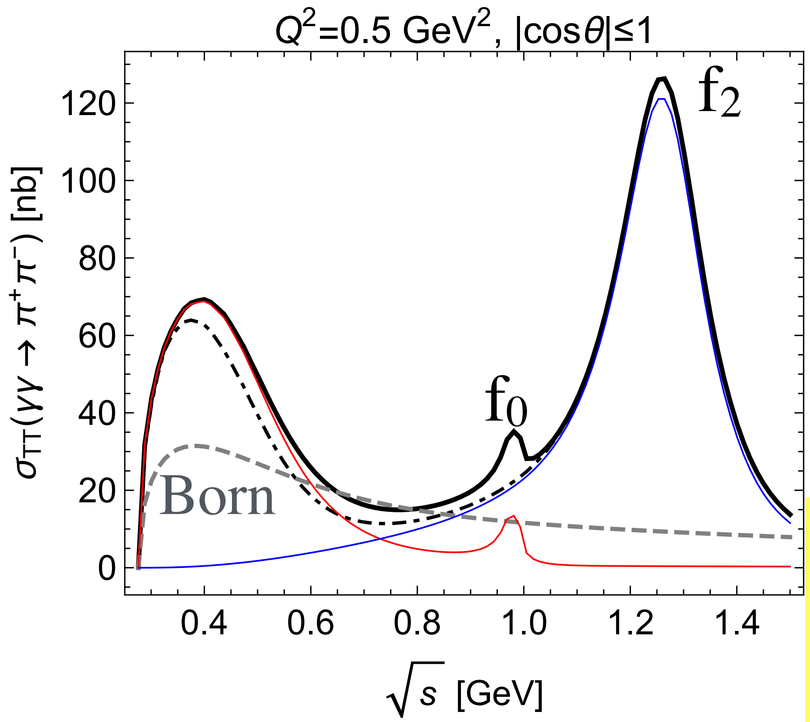
arXiv:1810.03669[hep-ph]

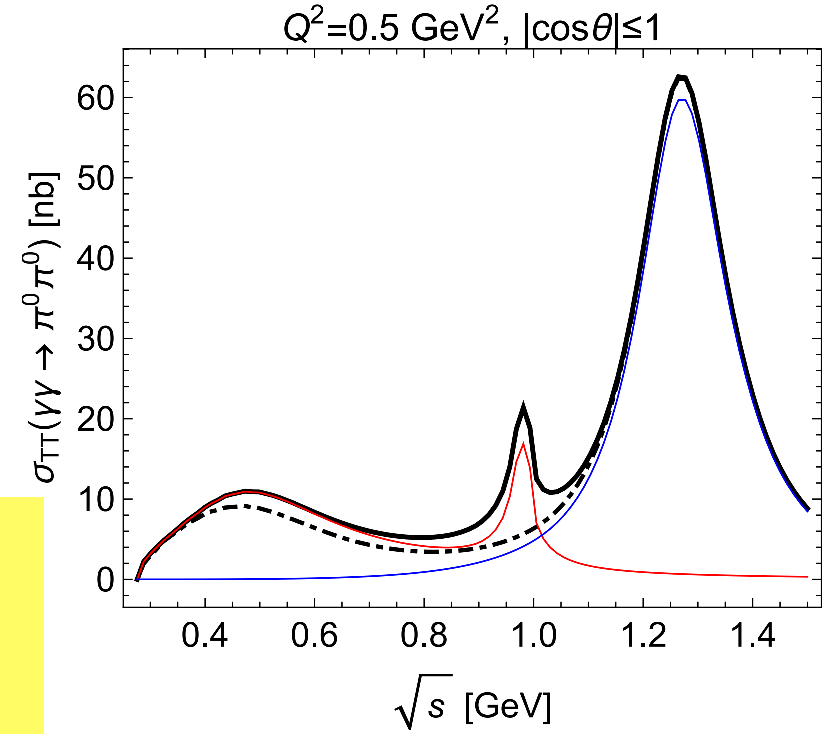
multi-meson production in $\gamma^*\gamma$ collisions

$\gamma^*\gamma \to \pi^+\pi^-$

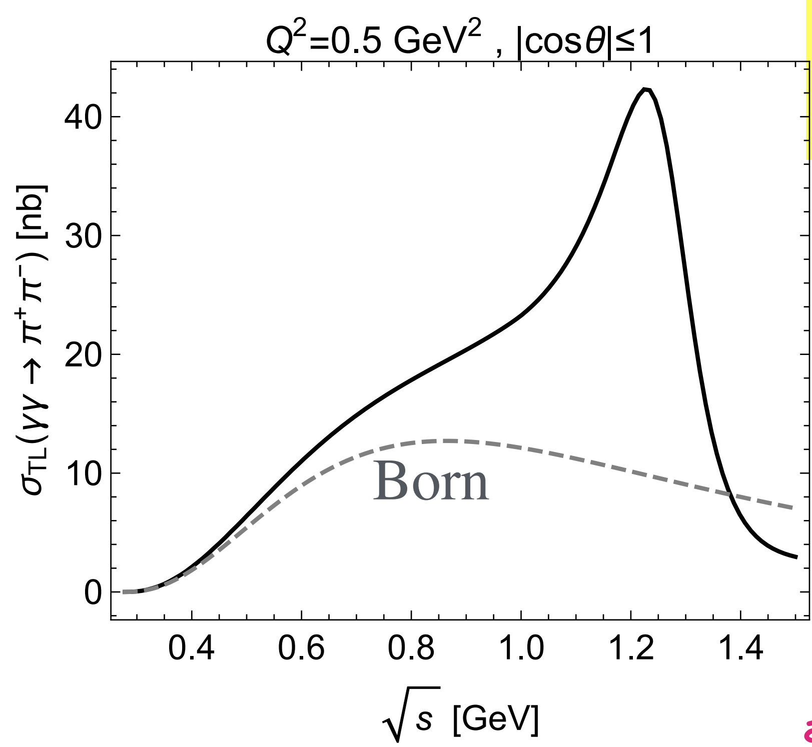
$Q^2 = 0.5$ GeV^2

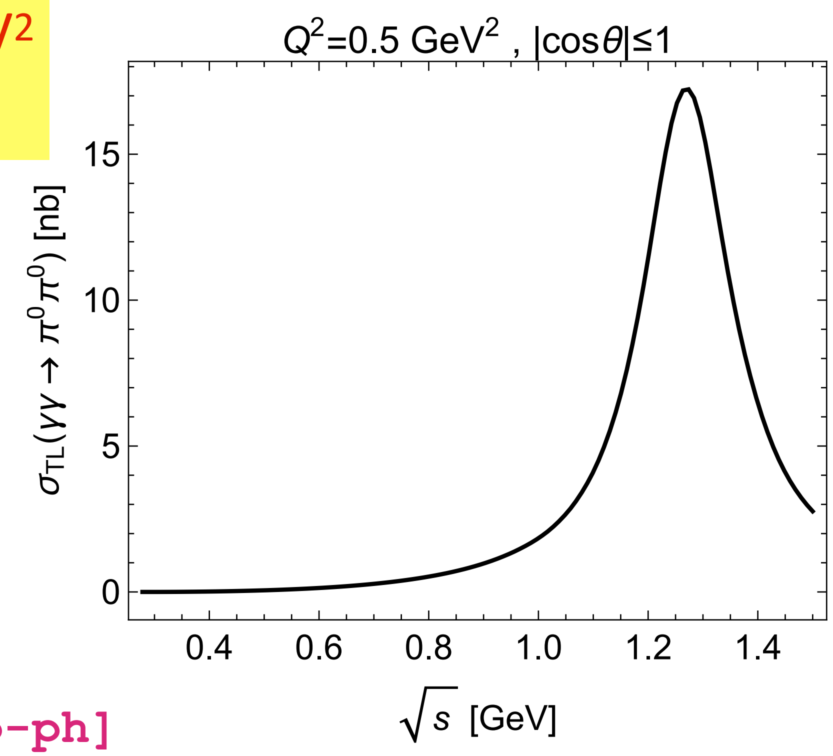
$\gamma^*\gamma \to \pi^0\pi^0$





Single tagged BES-III data for $\pi^+\pi^-$, $\pi^0\pi^0$ in range 0.2 GeV$^2 < Q^2 < 4$ GeV^2 under analysis



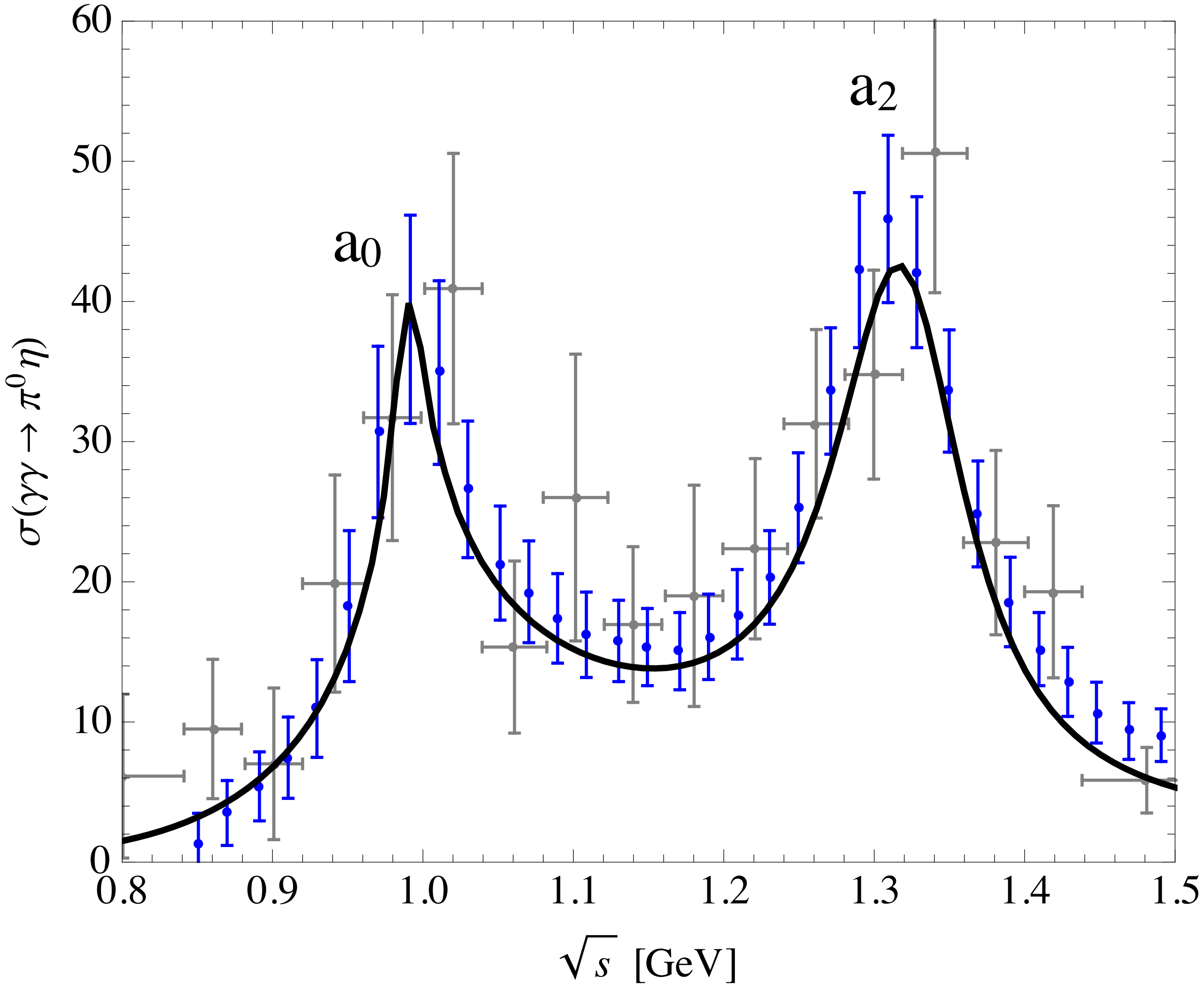


Danilkin, Vdh

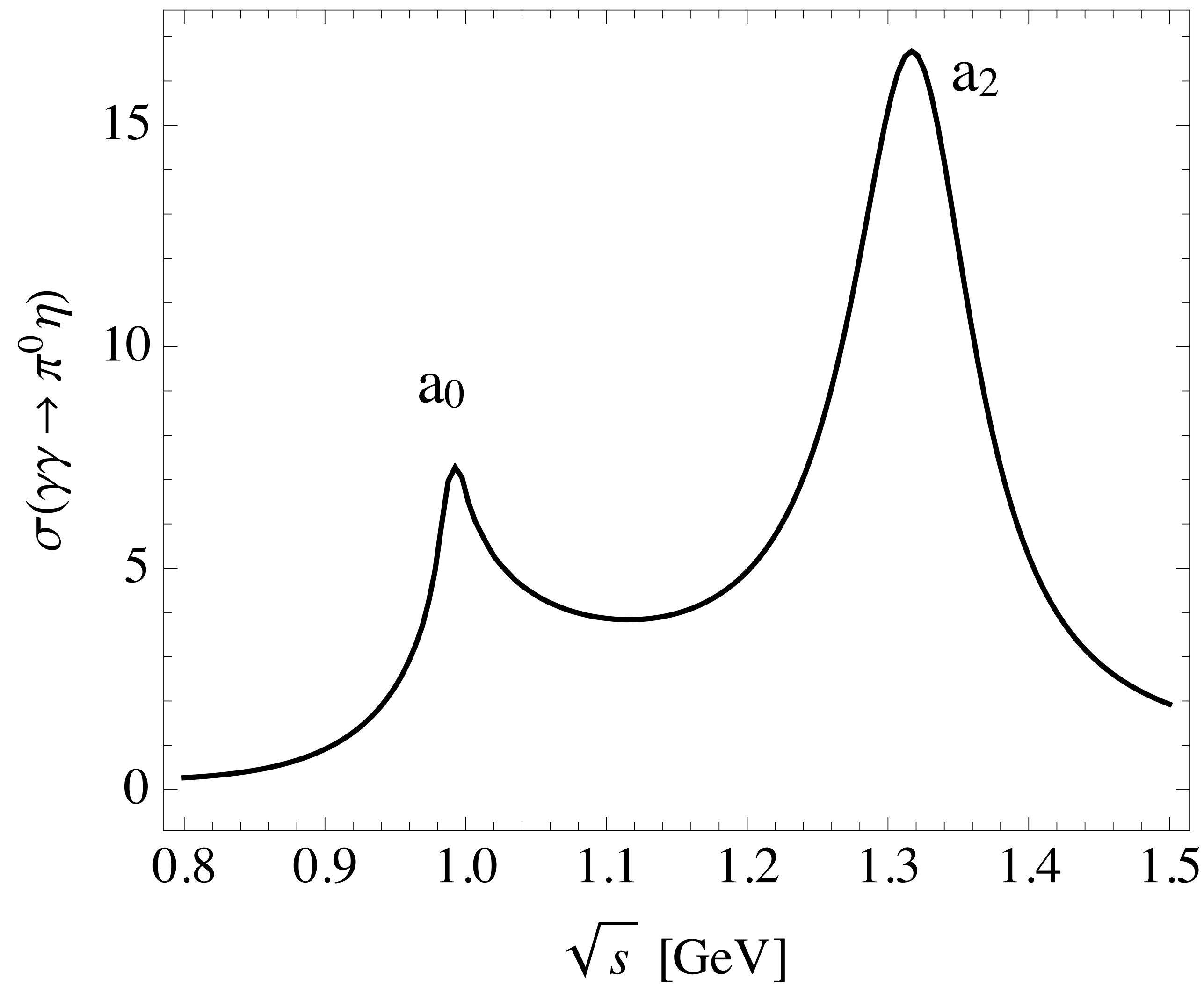
arXiv:1810.03669[hep-ph]

$\gamma^*\gamma \to \pi^0\eta$

$Q^2 = 0\ \text{GeV}^2$



$Q^2 = 0.5\ \text{GeV}^2$



Coupled-channel dispersive treatment for $a_0(980)$

$a_2(1320)$ as Breit Wigner resonance, TFF taken from Belle data

Danilkin, Deineka, Vdh (2017)

Summary and outlook

- **new a_μ Fermilab and J-Parc experiments ongoing:**
  aim: factor 4 improvement in experimental value

- **complementary experimental program (BESIII, Belle II) ongoing as input for the hadronic contributions to the HVP and HLbL contributions to a_μ**
  aim: factor 2 improvement for HVP

- **new dispersion relation frameworks for HLbL to a_μ :**
  -> require close collaboration with experiment (spacelike, timelike, meson decays)
  aim: data driven approach also in HLbL

- **dedicated lattice QCD effort for HVP and HLbL to pin down hadronic contributions**

- **Theory goal: realistic error estimate on a_μ / reduce to 2 x 10^{-10} (20 % of HLbL) to match accuracy of forthcoming experiments -> Muon (g-2) Theory Initiative**

Aim of concerted effort is to allow for a conclusive statement on the present 4σ deviation in a_μ between experiment and SM prediction !