Introduction to cosmology

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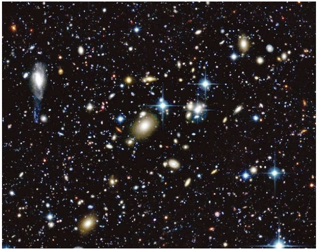


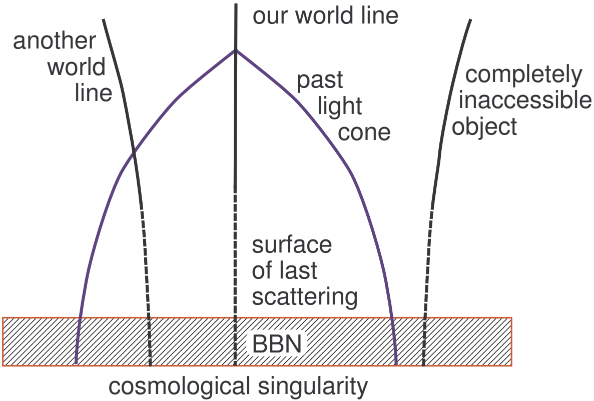
Outline

Cosmology

- Physical model of the structure and evolution of the universe
- Natural laboratory for testing physical theories

Sources of cosmological observations:

Astronomy "Geology" Local physics

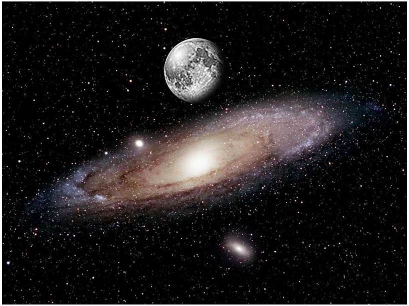


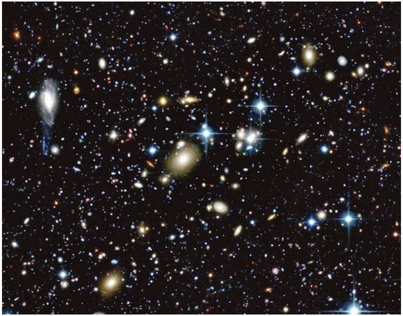
Horizons:

- Visible horizon
- Particle horizon
- Physical horizon

The world of galaxies

Dimension $\approx$ 20 kpc

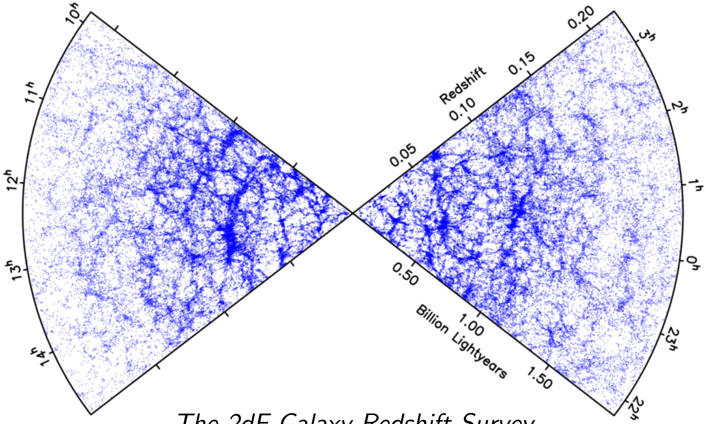




Characteristic intergalactic distance:

$$\text{Megaparsec (Mpc)} \approx 3.26 \times 10^6 \text{ light yr} \approx 3 \times 10^{24} \text{ cm}$$

The universe is homogeneous and isotropic on large spatial scales ($\gtrsim 100$ Mpc)



The 2dF Galaxy Redshift Survey

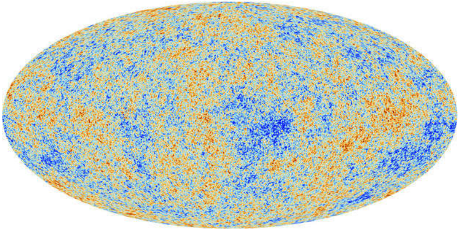
Temperature of cosmic microwave background (CMB) as a function of direction

Signifies isotropy of the early universe

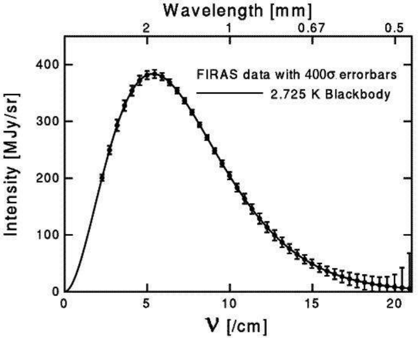
Everywhere isotropic universe is also homogeneous !

$$T = 2.725 \pm 0.002\ \text{K}$$

$$\frac{\Delta T}{T} \sim 10^{-5}$$

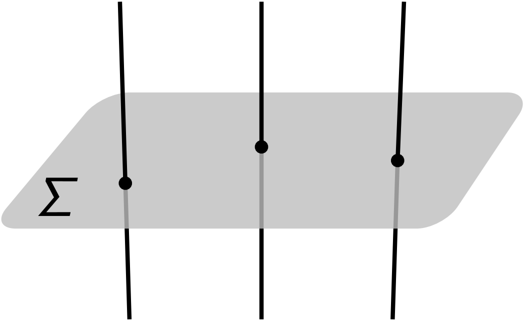


Planck spectrum of CMB



Fundamental (isotropic) observers

These are conventional observers for whom CMB is maximally isotropic



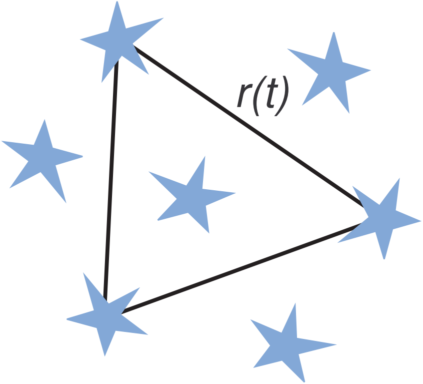
Residents of galaxies as "almost" isotropic observers ($v \sim 10^{-3} c$)

Copernican principle and cosmological principle

The universe is expanding
remaining homogeneous and isotropic !

The scale factor:

$$r(t) = a(t)\, r_0$$



Hubble law: $\dot{r} = Hr$

Hubble parameter: $H = \dfrac{\dot{a}}{a}$

Hubble constant:

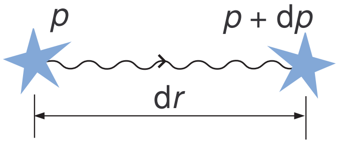
$$\boxed{H_0 \approx 70 \text{ km/s Mpc}}$$

Age of universe:

$$t_0 \sim H_0^{-1} \simeq 10^{10} \text{ yr}$$

Velocity $v = \dot{r}$ is measured by redshift of spectral lines in remote galaxies

Cosmological redshift



Performing Lorentz transformation with $v_{\rm rel} \ll c$, we have (*exercise*)

$$dp = -\frac{E v_{\rm rel}}{c^2} = -pHdt$$

Relative velocity:
$v_{\rm rel} = Hdr \ll c = 1$

$$\boxed{\frac{dp}{p} = -Hdt = -\frac{da}{a} \quad \Rightarrow \quad p \propto \frac{1}{a}}$$

For photons, $p = \hbar\omega/c$, which determines redshift z :

$$\boxed{1 + z \equiv \frac{\omega_{\rm em}}{\omega_{\rm obs}} = \frac{a_0}{a}}$$

Measures of cosmological time

The cosmological time can be "marked" in different ways:

- Physical time t by conventional clocks of isotropic observers
- Scale factor $a(t)$
- Cosmological redshift $z = a_0/a(t) - 1$
- Conformal time $\tau = c \int^t dt'/a(t')$

$$ds^2 = a^2(\tau) \left[d\tau^2 - \frac{d\boldsymbol{r}^2}{\left(1 + \kappa \boldsymbol{r}^2/4\right)^2} \right]$$

- Temperature T_γ of the photon gas (CMB as of today)

Expansion causes "cooling"
The universe was hot in the past !

Momentum: $p \propto \dfrac{1}{a}$

Number density: $n \propto \dfrac{1}{a^3}$

Temperature:

Mass–energy density:

- For nonrelativistic $(v \ll c)$ substance:

$$T \propto m \left\langle v^2 \right\rangle \propto \frac{1}{a^2} \propto (1+z)^2$$

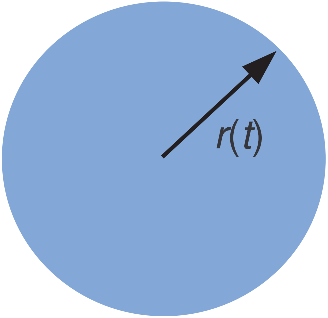
$$\rho \approx mn \propto \frac{1}{a^3} \propto (1+z)^3$$

- For relativistic $(v \sim c)$ substance:

$$T \propto \langle p \rangle \propto \frac{1}{a} \propto (1+z)$$

$$\rho \approx pn \propto \frac{1}{a^4} \propto (1+z)^4$$

Dynamics of cosmological expansion



Nonrelativistic "derivation":

$$\frac{\dot r^2}{2} - \frac{GM}{r} = \mathcal{E} = \text{const}$$

$$M = \frac{4\pi}{3}\rho r^3 \qquad \frac{\dot r}{r} = \frac{\dot a}{a} = H$$

The first Friedmann equation:

$$\boxed{H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho}$$

Energy conservation within a sphere:

$$\frac{d}{dt}\left(\rho r^3\right) + p\frac{d}{dt}\left(r^3\right) = 0$$

$$\Downarrow$$

The second Friedmann equation:

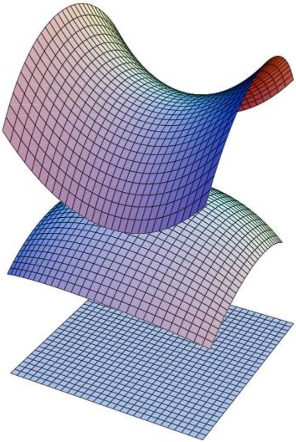
$$\boxed{\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right)}$$

p is pressure !

The scale factor is an element of space-time metric:

$$ds^2 = dt^2 - a^2(t)\,\frac{d\boldsymbol{r}^2}{(1 + \kappa \boldsymbol{r}^2/4)^2}$$

$$\kappa = \pm 1/r_0^2$$



The parameter $0 < r_0 \leq \infty$ describes curvature of space

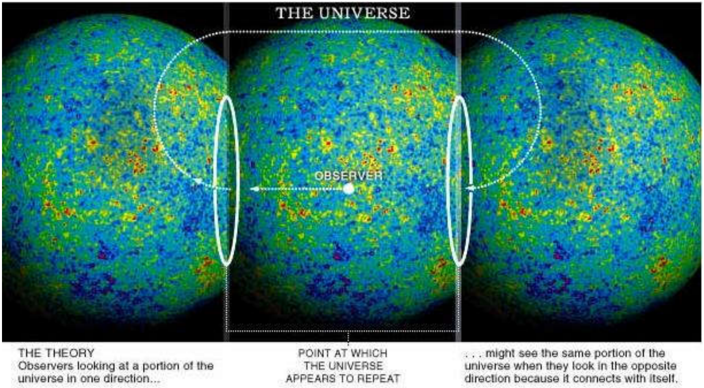
Current constraint:

$$a_0 r_0 \gtrsim \frac{1}{0.05\, H_0} \simeq 86 \text{ Gpc}$$

and this curvature is practically insignificant (the space is Euclidean)

The space of the universe: is it finite or infinite?

- In the case of positive curvature ($\kappa > 0$), the space is finite (local metric is that of three-sphere, topology may be different)
- In the case of non-positive curvature ($\kappa \leq 0$), the space can be infinite or finite depending on *topology* (picture by Max Tegmark)



Dynamics of cosmological expansion

Einstein equation (with two fundamental constants):

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda \, g_{\mu\nu}$$

Friedmann equations:

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

Currently accepted model is called ΛCDM ($\Lambda +$ Cold Dark Matter)

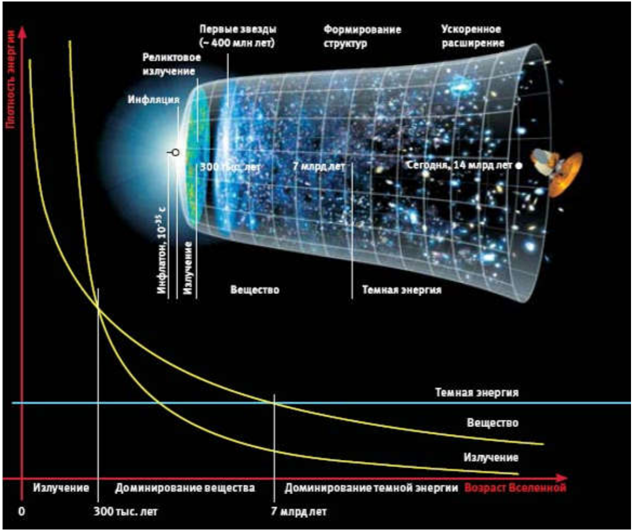
Main types of substance

- Matter (nonrelativistic): $p \ll \rho, \;\; \rho = mn \propto a^{-3}$
- Radiation (relativistic): $p = \rho/3, \;\; \rho = En \propto a^{-4}$
- "Vacuum": $p = -\rho = \text{const}$ (*equivalent to* Λ)

  This is the only law compatible with local Lorentz invariance of $T^{\mu}{}_{\nu} = (\rho + p) u^{\mu} u_{\nu} - p \delta^{\mu}{}_{\nu}$

In the currently standard cosmological model, the universe is dominated, in turn, by *radiation*, *matter*, and *dark energy* (Λ or something else)

Modern picture of evolution of the universe



The Ω parameters

The Friedmann equation can be recast in the form

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} + \frac{\Lambda}{3} = \frac{8\pi G}{3}\sum_i \rho_i$$

determines the critical density

$$\rho_c = \frac{8\pi G}{3H_0^2} \approx 10^{-29}\ \mathrm{g/cm^3}\,, \qquad \Omega_i = \frac{\rho_i}{\rho_c}$$

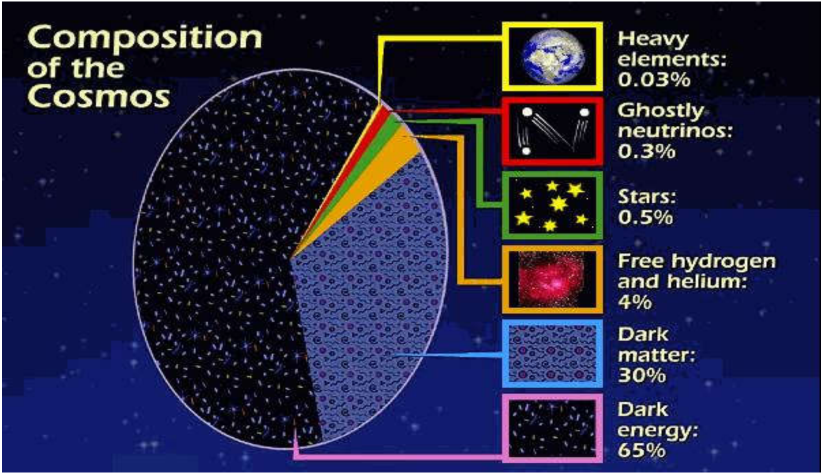
$$H^2(z) = H_0^2\left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\kappa(1+z)^2 + \Omega_\Lambda\right]$$

$$\Omega_r + \Omega_m + \Omega_\kappa + \Omega_\Lambda = 1$$

$$\Omega_\Lambda \approx 0.7\,, \quad \Omega_m \approx 0.3\,, \quad \Omega_r \approx 8.5 \times 10^{-5}\,, \quad \Omega_\kappa = 0.001 \pm 0.002$$

Matter–radiation equality: $z_{\mathrm{eq}} = \Omega_m/\Omega_r - 1 \approx 3400$

Current composition of the universe



Baryons and relic photons

- Mean number density of baryons today

$$n_b(t_0) = 2.7 \times 10^{-7}\,\mathrm{cm}^{-3}$$

- Mean concentration of relic photons is determined by the Planck spectrum with $T_\gamma = 2.73$ K :

$$n_\gamma(t_0) = \frac{2\,\zeta(3)}{\pi^2}\,T_\gamma^3 \approx 410\,\mathrm{cm}^{-3}$$

Thus (s is the mean entropy density):

$$\boxed{\frac{n_b}{s} \simeq \frac{n_b}{n_\gamma} \simeq 6 \times 10^{-10}}$$

What is the origin of this number?
This is one of open questions

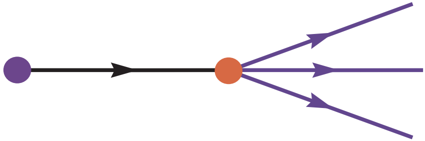
The early universe is called "hot" because of the smallness of this number

System of units

$$\boxed{\hbar = c = 1}$$

$$\text{energy} = \text{momentum} = \frac{1}{\text{length}} = \frac{1}{\text{time}}$$

Kinetics in the hot expanding universe



Mean free time of a particle

$$\tau = \frac{1}{\sigma n v}$$

σ – cross-section
n – number density of target particles
v – mean relative velocity

In the expanding universe, particles additionally recede from each other. The condition of local thermodynamical equilibrium becomes

$$\tau \ll t_{\mathrm{expansion}} = \frac{1}{H} \qquad \text{or} \qquad \Gamma \equiv \sigma n v \gg H$$

In the opposite case $\Gamma \ll H$, particles become free (*decoupling* or *freeze-out*)

Thermodynamics

Partition function for grand canonical ensemble

$$Z\left(T, V, \{\mu_i\}\right) = \sum_{\text{states}} e^{\left(\sum_i \mu_i Q_i - E\right)/T} = \sum_{\text{states}} e^{\left(\sum_A \mu_A N_A - E\right)/T}$$

μ_i is the chemical potential corresponding to a conserved charge Q_i

The last equality is obtained by using

$$\sum_i \mu_i Q_i = \sum_i \mu_i \sum_A Q_i^A N_A = \sum_A \mu_A N_A$$

with Q_i^A being the i^{th} charge of particle species A, and

$$\mu_A = \sum_i \mu_i Q_i^A$$

is their chemical potential. *Particles and antiparticles have all charges opposite, hence, opposite chemical potentials* $\Rightarrow \mu_\gamma = 0$

Chemical potentials

A simple consequence of the conservation of the total charges Q_i is that if there are reactions between particles in equilibrium

$$A_1 + \cdots + A_n \rightleftarrows B_1 + \cdots + B_m$$

then the corresponding chemical potentials are related by

$$\mu_{A_1} + \cdots + \mu_{A_n} = \mu_{B_1} + \cdots + \mu_{B_m}$$

Proof:

$$\sum_i \mu_i \times \quad Q_i^{A_1} + \cdots + Q_i^{A_n} = Q_i^{B_1} + \cdots + Q_i^{B_m}$$

Example: ionization/recombination of hydrogen: $p + e^- \leftrightarrow \mathrm{H} + \gamma$

$$\mu_p + \mu_{e^-} = \mu_{\mathrm{H}}$$

Example: lepton era

- The baryon number

$$B \equiv \frac{\Delta n_p + \Delta n_n + \Delta n_\Lambda}{s} \simeq 10^{-10}\text{–}10^{-9}\,,$$

where $\Delta n = n - \bar{n}$

- Electric charge

$$Q \equiv \frac{\Delta n_p - \Delta n_e - \Delta n_\mu - \Delta n_\tau - \Delta n_{\pi^-}}{s} = 0$$

- The lepton numbers

$$L_i \equiv \frac{\Delta n_i + \Delta n_{\nu_i}}{s} \quad \text{are likely to be} \sim B$$

since $B + b\sum_i L_i = 0$ for $T \gtrsim 100$ GeV

Nonrelativistic particles

Mean occupation numbers for particles in equilibrium

$$f_\epsilon = \frac{1}{\exp \frac{\epsilon - \mu}{T} \pm 1} \qquad \begin{cases} + & \text{Fermi–Dirac} \\ - & \text{Bose–Einstein} \end{cases}$$

Number density for nonrelativistic particles:

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{(\mu - m)/T} , \qquad \bar{n} = e^{-2\mu/T} n \, ,$$

where g is the number of internal degrees of freedom (spin, colour, etc)

Ultrarelativistic particles

At $T \gg m, |\mu|$, we have

$$\rho_r(T) = \frac{\pi^2}{30} g T^4 , \qquad g = \sum_b g_b + \frac{7}{8} \sum_f g_f$$

$$H(t) = \left(\frac{8\pi G}{3} \rho_r \right)^{1/2} = 1.66 \sqrt{g(t) G}\, T^2 = \sqrt{G_*}\, T^2$$

To the leading order of $m/T \ll 1$ and $\mu/T \ll 1$, we have

$$\Delta n_b = \frac{g T^3}{3} \frac{\mu_b}{T} , \qquad \Delta n_f = \frac{g T^3}{6} \frac{\mu_f}{T}$$

Age of hot universe as a function of temperature:

$$t \simeq \frac{1}{2H} = \frac{1}{2\sqrt{G_*}\, T^2} \simeq \frac{2.4}{\sqrt{g}} \left(\frac{\mathrm{MeV}}{T} \right)^2 \ \mathrm{s}$$

Recombination of hydrogen

Thermal equilibrium is maintained due to the processes $p, e^- \rightleftarrows \mathrm{H}, \gamma$.
Hence, $\mu_p + \mu_{e^-} = \mu_{\mathrm{H}}$. The Saha equation:

$$\frac{n_p n_e}{n_{\mathrm{H}}} \approx \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-I_{\mathrm{H}}/T} , \qquad I_{\mathrm{H}} = m_p + m_e - m_{\mathrm{H}}$$

The hydrogen ionization energy $I_{\mathrm{H}} = 13.6\,\mathrm{eV} = 1.58 \times 10^5\,\mathrm{K}$

Use two conditions:

- Electroneutrality: $n_p = n_e$
- Mean number density of hydrogen nuclei is determined by $\eta = n_b/n_\gamma = 6 \cdot 10^{-10}$

One independent variable remains, e.g., $X_p = \dfrac{n_p}{n_p + n_{\mathrm{H}}}$

Taking, e.g., $X_p = 0.1$ and solving the Saha equation, we obtain recombination temperature $T_{\mathrm{rec}} \simeq 3400$ K, which corresponds to $z_{\mathrm{rec}} \simeq 1250$, $t_{\mathrm{rec}} = 4 \times 10^5$ yr

Decoupling of relic photons

Scattering rate of photons off free electrons is $\Gamma = \sigma_T n_e c$.
The Thomson cross-section

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 0.67 \times 10^{-24}\,\mathrm{cm}^2$$

From the Saha equation,

$$n_e \simeq n_{\mathrm{H}}^{1/2}\left(\frac{m_e T}{2\pi}\right)^{3/4} e^{-I_{\mathrm{H}}/2T}$$

Rate of the universe expansion (matter dominates)

$$H \simeq \left(\frac{8\pi G}{3}\rho_m\right)^{1/2} \simeq H_0 \Omega_m^{1/2}(1+z)^{3/2} = H_0 \Omega_m^{1/2}\left(\frac{T}{T_0}\right)^{3/2}$$

The condition $\Gamma \lesssim H$ implies $T_{\mathrm{dec}} = 0.26\,\mathrm{eV} = 3070\,\mathrm{K}$, or $z_{\mathrm{dec}} \simeq 1130$

Decoupling of neutrino

Typical processes:

$$\nu e^{\pm} \rightleftarrows \nu e^{\pm} , \qquad \nu\bar{\nu} \rightleftarrows e^{+}e^{-} , \qquad \nu\bar{\nu} \rightleftarrows \nu\bar{\nu}$$

Effective cross-section is $\sigma \simeq G_F^2 E^2$, where $G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant. Using $E \sim T$ and equating the rates, we obtain

$$\Gamma = \sigma n v \sim G_F^2 T^2 \times T^3 = H = \sqrt{G_*}\, T^2$$

At anticipated temperatures, the relativistic plasma consists of $e^{\pm}$, γ, ν_e, ν_μ, ν_τ, $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$, leading to $g = 39/4 = 9.75$.
Hence, the neutrino decoupling temperature

$$T_* \sim \frac{G_*^{1/6}}{G_F^{2/3}} \simeq 1.5\ \mathrm{MeV}$$

Today, we have $T_\nu = 1.95$ K (*Show this using the entropy conservation law* $sa^3 = \mathrm{const}$). Somewhat lower than $T_\gamma = 2.73$ K because annihilation of e^+e^- at $T \simeq 0.5$ MeV heat up the CMB

Neutron concentration freeze-out

The main processes are $n + \nu_e \rightleftarrows p + e^-$, $n + e^+ \rightleftarrows p + \bar{\nu}_e$
Characteristic energy parameters:

$$\Delta m \equiv m_n - n_p = 1.3 \text{ MeV}, \qquad m_e = 0.5 \text{ MeV}\,.$$

Free time of neutrons can be estimated as

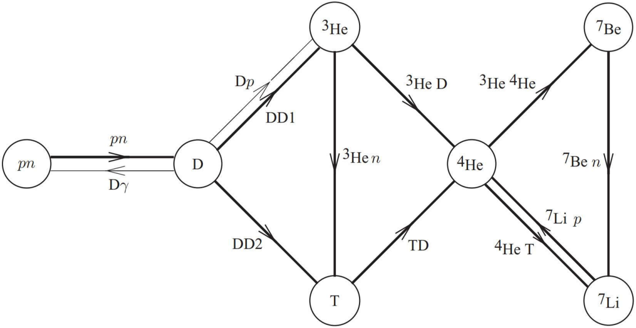
$$\Gamma \simeq G_F^2 T^5 \quad \left(\text{cf. with} \quad = \sqrt{G_*}\, T^2\right)$$

Accurate calculation using the condition $H \simeq \Gamma$ gives $T_n \approx 0.8$ MeV. We obtain neutron–proton relation at the time of freeze-out:

$$\frac{n_n}{n_p} = e^{-\frac{\Delta m}{T_n}} \approx \frac{1}{5}$$

Note: Fundamental constants and life in the universe

Big-Bang Nucleosynthesis (BBN)
out-of-equilibrium process !



From: Mukhanov, *Principles of Physical Cosmology* (2005)

$\Delta_{^4\mathrm{He}}/N = 7.75$ MeV, but the first step of nucleosynthesis is production of deuterium:

$$p + n \rightleftarrows \mathrm{D} + \gamma\,, \qquad \Delta_\mathrm{D} \equiv m_p + m_n - m_\mathrm{D} \simeq 2.23 \text{ MeV}$$

Because of large number of highly energetic photons and relatively small binding energy of D, nucleosynthesis is delayed till $T_\mathrm{NS} \approx 70$ keV. This phenomenon is called deuterium bottleneck.

Estimate of helium production

Age of the universe at the time of neutron decoupling:

$$t_n = \frac{1}{2\sqrt{G_*}\, T_n^2} \approx 1 \text{ s}\,, \qquad \frac{n_n}{n_p} = e^{-\Delta m / T_n} \approx \frac{1}{5}$$

Age of the universe at the time of commencement of nucleosynthesis

$$t_{\rm NS} = \frac{1}{2\sqrt{G_*}\, T_{\rm NS}^2} \approx 269 \text{ s}\,,$$

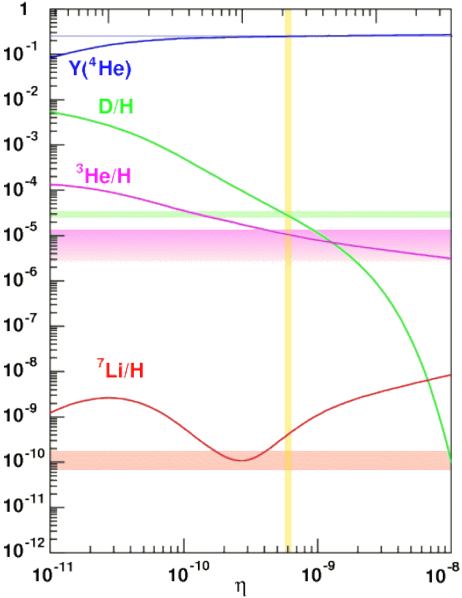
Lifetime of free neutrons is $\tau_n \approx 886$ s, whence

$$\left.\frac{n_n}{n_p}\right|_{T_{\rm NS}} \approx \frac{1}{5} \cdot e^{-t_{\rm NS}/\tau_n} \approx \frac{1}{7}$$

Mass fraction of helium:

$$Y_{^4{\rm He}} = \frac{M_{\rm He}}{M_b} = \frac{2}{\left.\frac{n_n}{n_p}\right|_{T_{\rm NS}} + 1} \approx 0.25$$

BBN and observations



$$\eta = \frac{n_b}{n_\gamma} \approx 6 \times 10^{-10}$$

Where does this number come from?

The smallness of this number is the reason why the universe is qualified as "hot"

$$n_\gamma(t_0) = \frac{2\,\zeta(3)}{\pi^2}\, T_\gamma^3 \approx 410\,\mathrm{cm}^{-3}$$

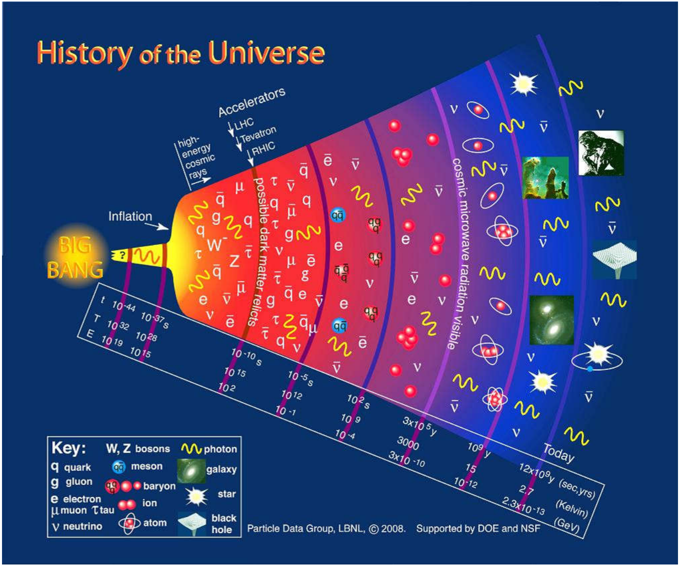
This determines the mean number density of baryons today:

$$n_b(t_0) = 2.7 \times 10^{-7}\,\mathrm{cm}^{-3}$$

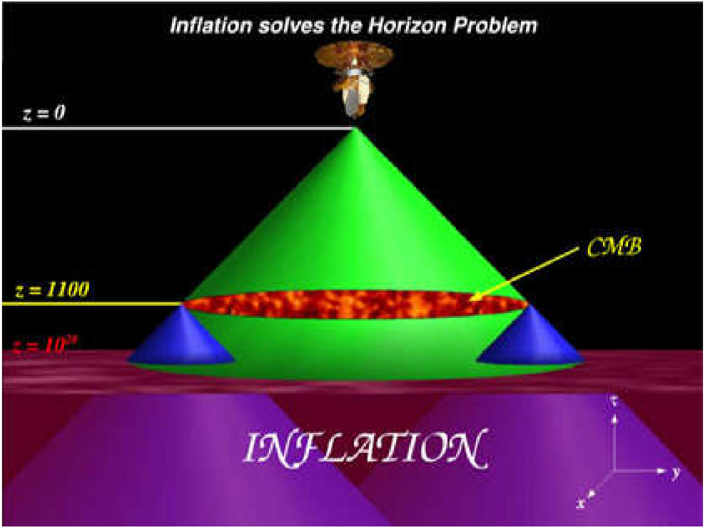
Thermodynamic history of the universe

- $T \sim 200$ GeV: symmetric electroweak phase, $g = 106.75$
- $T \sim 120$ GeV: electroweak symmetry breaking (cross-over), annihilation of $t\bar{t}$ quarks, $g = 96.25$
- $T < 80$ GeV: annihilation of $W^{\pm}$, Z^0, H^0, $g = 86.25$
- $T < 4$ GeV: $b\bar{b}$ annihilation, $g = 75.75$
- $T < 1$ GeV: $\tau^{-}\tau^{+}$ annihilation, $g = 72.25$
- $T \sim 150$ MeV: epoch of QCD; quarks and gluons are confined, forming baryons and mesons. Light hadrons (pions), leptons and photons give $g = 17.25$
- $T < 100$ MeV: annihilation of pions $\pi^{\pm}$, π^0 and muons $\mu^{\pm}$. Remaining particles $e^{\pm}$, γ and ν give $g_{\epsilon} = 10.75$
- $T \sim 1$ MeV: decoupling of neutrinos
- $T < 500$ keV: annihilation of $e^{+}e^{-}$, $g = 3.36$

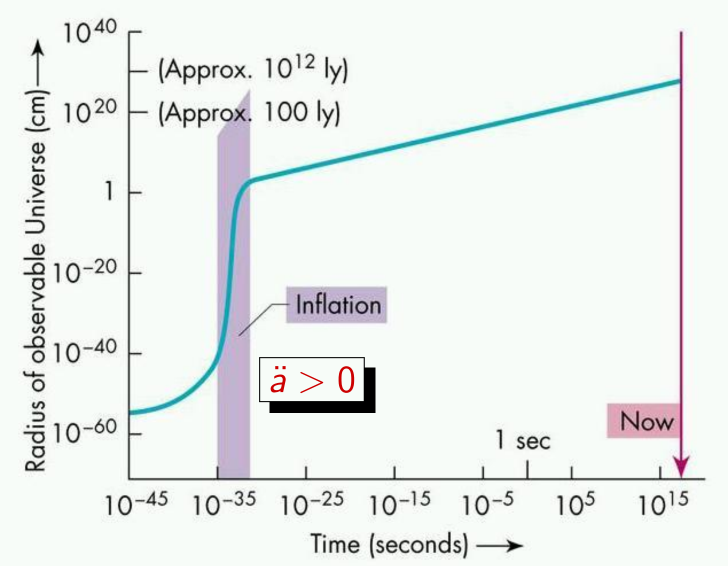
Thermodynamic history of the universe



Problem of initial conditions
and the problem of singularity



Cosmological inflation
beyond the physical horizon !



$\ddot{a} > 0$

Brief history

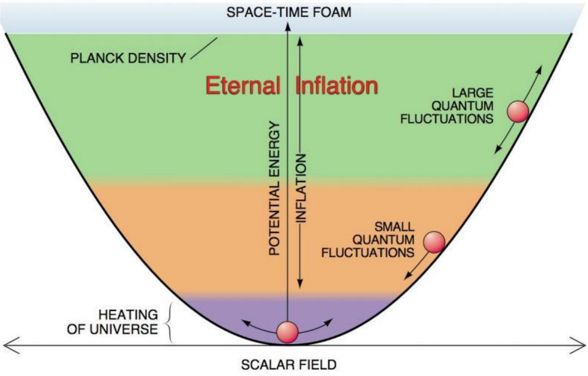
- Gliner (1965) — vacuum-like state before the hot phase
- Bugrij & Trushevsky (1975) — first-order phase transition with supercooling in nuclear matter
- Starobinsky (1979) — gravity with R^2 correction
- Guth (1981) — GUT with first-order phase transition and supercooling (proposed the term "inflation")
- Linde (1982, 1983) — field theory with generic initial conditions ("chaotic inflation")

Kavli prize in astrophysics 2014

The simplest model of inflation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$V(\phi) = \frac{m^2}{2}\phi^2$$



Regime of “slow rolling”

$$\left|\dot{\phi}/\phi\right| \ll H \equiv \dot{a}/a$$

$$\dot{\phi} \approx -\frac{V'(\phi)}{3H}$$

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \approx \frac{8\pi G}{3} V(\phi)$$

$$|\dot{H}| \ll H^2$$

quasi-exponential expansion

The Starobinsky model (1979)

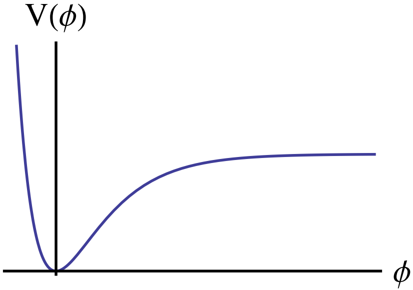
$$S[g] = -\frac{M^2}{3} \int \sqrt{-g}\, d^4x \left(R - \frac{R^2}{6m^2} \right)$$
$$= -\frac{M^2}{3} \int \sqrt{-g}\, d^4x \left[e^{\phi/M} R + \frac{3m^2}{2} \left(e^{\phi/M} - 1 \right)^2 \right]$$

Conformal transformation:

$$g_{\mu\nu} \to e^{-\phi/M} g_{\mu\nu}$$

$$S[g,\phi] = -\frac{M^2}{3} \int \sqrt{-g}\, d^4x\, R$$
$$+ \int \sqrt{-g}\, d^4x \left[\frac{1}{2} (\partial\phi)^2 - \frac{m^2 M^2}{2} \left(1 - e^{-\phi/M} \right)^2 \right]$$

$$M^2 = \frac{3}{16\pi G}$$



Inflationary origin of primordial perturbations

The inflaton field and the metric are perturbed by quantum uncertainties:

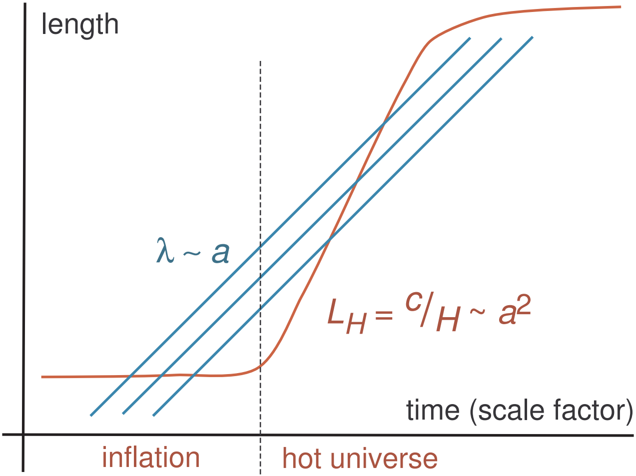
$$ds^2 = a^2(\eta)\Big[(1+2\Phi)d\eta^2 - (1-2\Psi)d\boldsymbol{x}^2 + h_{ij}dx^i dx^j\Big]$$

- Φ, Ψ — scalar type, accompanied by energy density perturbations $\delta \equiv \delta\rho/\rho$, $\boldsymbol{v} = \nabla v$

  $\Phi = \Psi$ in the model with single inflaton

- h_{ij} — tensor type (gravitational waves), transverse traceless field

Inflationary origin of primordial perturbations
Mukhanov & Chibisov (1981)



How do quantum uncertainties become classical ?

Power spectra of primordial perturbations

Scalar field (inflaton) with potential $V(\phi)$

$$\langle \Phi(\boldsymbol{x}) \Phi(\boldsymbol{y}) \rangle = \int \mathcal{P}_\Phi(k) e^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}-\boldsymbol{y})} \frac{d^3\boldsymbol{k}}{4\pi k^3} \qquad \mathcal{P}_\Phi(k) = \frac{\hbar H_k^4}{4\pi^2 \dot{\phi}_k^2} = \frac{128\pi\hbar G^3 \left[V(\phi_k)\right]^3}{3\left[V'(\phi_k)\right]^2}$$

$$\left\langle h_{ij}(\boldsymbol{x}) h^{ij}(\boldsymbol{y}) \right\rangle = \int \mathcal{P}_h(k) e^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}-\boldsymbol{y})} \frac{d^3\boldsymbol{k}}{4\pi k^3} \qquad \mathcal{P}_h(k) = \frac{16}{\pi} \hbar G H_k^2 = \frac{128}{3} \hbar G^2 V(\phi_k)$$

Conventional parameterization:

$$\mathcal{P}_\Phi(k) = A_S \left(\frac{k}{k_*}\right)^{n_S(k)-1} \qquad \mathcal{P}_h(k) = A_T \left(\frac{k}{k_*}\right)^{n_T(k)}$$

$$\left.\begin{aligned} r(k) \equiv \frac{\mathcal{P}_h(k)}{\mathcal{P}_\Phi(k)} = \frac{1}{\pi G}\left[\frac{V'(\phi_k)}{V(\phi_k)}\right]^2 &\approx 0.2 \\ n_S(k) - 1 &\approx -0.04 \\ n_T(k) &\approx -0.03 \end{aligned}\right] \quad \text{in simplest models}$$

Main predictions of inflationary scenario

1. Flatness (or Euclidean property) of space
   ($\Omega = 1$ with high precision)

$$H^2 = -\frac{\kappa}{a^2} + \frac{8\pi G}{3}\rho\,, \qquad |\Omega_\kappa| = \frac{|\kappa|}{a_0^2 H_0^2} \lesssim 10^{-5}$$

2. Adiabatic initial density perturbations with almost scale-invariant spectrum and Gaussian statistics
   [Mukhanov & Chibisov (1981), ...]

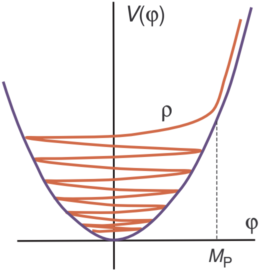
$$\mathcal{P}_\Phi(k) = A_S \left(\frac{k}{k_*}\right)^{n_S(k)-1}, \quad n_S(k) \approx 0.96$$

3. Relic gravitational waves
   [Grishchuk (1975), Starobinsky (1979), ...]

$$\mathcal{P}_h(k) = A_T \left(\frac{k}{k_*}\right)^{n_T(k)}, \quad n_T(k) \approx -0.03$$

(Re)Heating the universe after inflation

Phase trajectory of the scalar field $\varphi(t)$

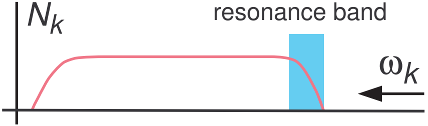


Model interaction:

$$L_{\rm int} = -f\varphi\overline{\psi}\psi - \eta\varphi\chi^2$$

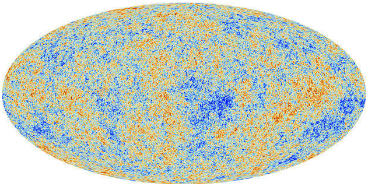
Rates of inflaton decay (Born approximation):

$$\Gamma_\psi \simeq \frac{f^2}{4\pi}\,\omega\,, \qquad \Gamma_\chi \simeq \frac{\eta^2}{4\pi\omega}$$



For strong coupling, the leading effect is parametric resonance [J. Traschen & R. Brandenberger (1990), Yu.S., J. Traschen & R. Brandenberger (1994), L. Kofman, A. Linde & A. Starobinsky (1994)].
For weak coupling, Born approximation works because of universe expansion [I. Rudenok, Yu.S., S. Vilchinskii (2014)]

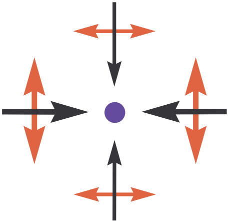
CMB temperature anisotropy and polarization



Temperature anisotropy: $\Delta T(\boldsymbol{n})$

Scattering cross-section of photon on electron

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_{\rm T}}{8\pi} \left|\boldsymbol{\epsilon}' \cdot \boldsymbol{\epsilon}\right|^2$$



Polarization is caused by the quadrupole anisotropy of the incident flow of last scattered photons

CMB polarization tensor

$$\mathcal{P}_{ab} = \frac{\langle E_a E_b^* \rangle}{\langle E_c E_c^* \rangle} - \frac{1}{2} g_{ab} = \frac{1}{2} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

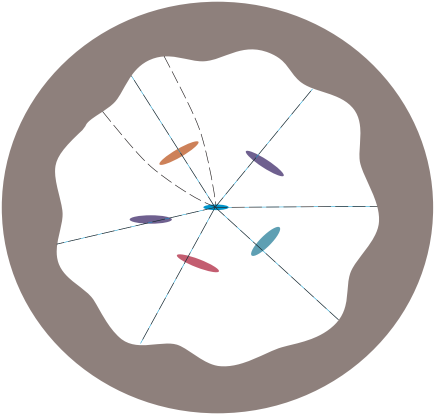
E and B polarization modes: $E(\boldsymbol{n}) \equiv \nabla^a \nabla^b \mathcal{P}_{ab}$, $B(\boldsymbol{n}) \equiv \nabla^a \nabla^c \mathcal{P}_a{}^b \epsilon_{cb}$

Correlation functions and spectra C_ℓ

Integer ℓ corresponds to angular scale $\theta \sim \pi/\ell$

$$
\begin{aligned}
\langle \Delta T(\boldsymbol{n}_1) \Delta T(\boldsymbol{n}_2) \rangle &= \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell^{TT} P_\ell(\cos\theta) \\
\langle \Delta T(\boldsymbol{n}_1) E(\boldsymbol{n}_2) \rangle &= \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell^{TE} P_\ell(\cos\theta) \\
\left\langle E(\boldsymbol{n}_1) E(\boldsymbol{n}_2) \right\rangle &= \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell^{EE} P_\ell(\cos\theta) \\
\left\langle B(\boldsymbol{n}_1) B(\boldsymbol{n}_2) \right\rangle &= \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell^{BB} P_\ell(\cos\theta)
\end{aligned}
$$

CMB on the way to Earth



The last-scattering surface is 'thick'

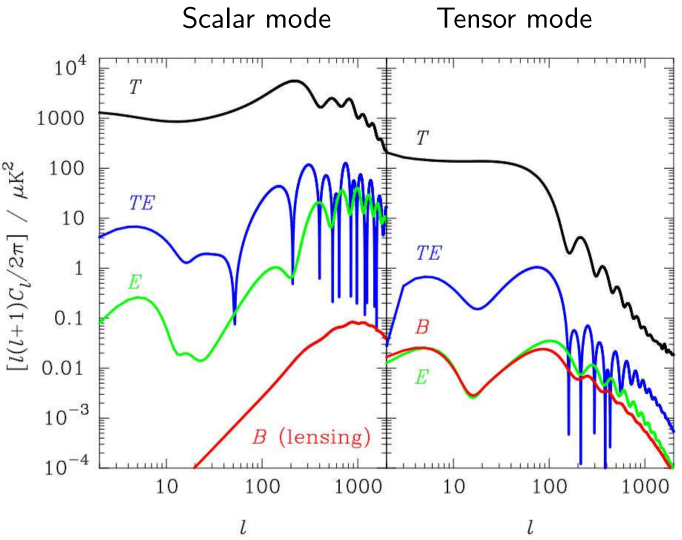
$$z_{\rm rec} \simeq 1100 \qquad \Delta z_{\rm rec} \approx 300$$

$$T_{\rm rec} \simeq 3000\,\mathrm{K} \qquad T_0 = 2.725\,\mathrm{K}$$

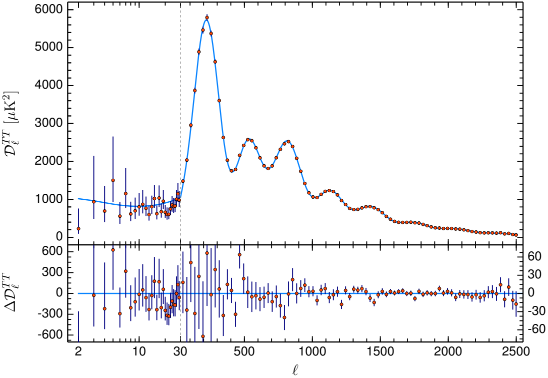
CMB is additionally lensed by the LSS

Its spectrum is also distorted by rescattering on hot gas in clusters (Zunyaev–Zeldovich effect)

Contribution of scalar and tensor modes to the CMB temperature anisotropy and polarization



Temperature anisotropy



$$\mathcal{P}_\Phi(k) = A_S \left(\frac{k}{k_*} \right)^{n_S(k)-1}$$

Planck (2018):

$$n_S = 0.965 \pm 0.004$$

In 2013 it was first established that $n_S < 1$, as predicted by a simple class of inflationary theories

Position of the peaks depends on the cosmological parameters, including the spatial curvature (Ω_κ) and amount of dark matter (Ω_c) $\longrightarrow$ gives the most accurate estimate for these parameters

CMB polarization and primordial gravitational waves

A joint analysis of BICEP2/KEK and Planck (2017) gives an upper limit

$$\boxed{r < 0.04}$$

Inflationary models are 'filtered'

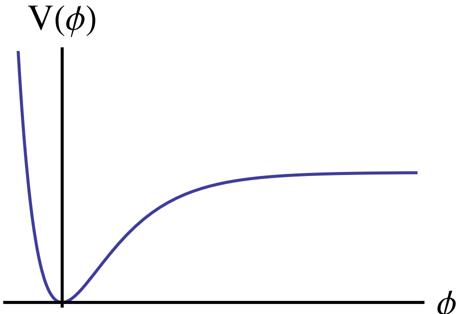
Best-fit models based on 'plateau' potential:

- Starobinsky model (1979)

$$\mathcal{L}_{\rm grav} \propto R + \alpha R^2$$

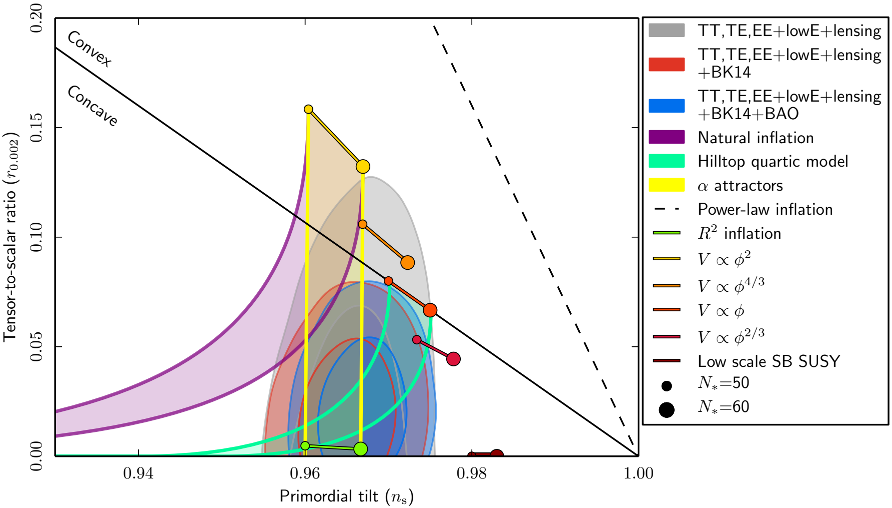
- Higgs inflation (Bezrukov & Shaposhnikov, 2007)

$$\mathcal{L}_{\rm grav} \propto \left(M_{\rm P}^2 + \xi h^2 \right) R$$



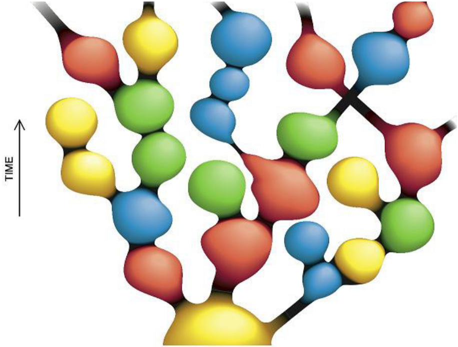
These models predict $r \approx 0.004$

Planck constraints on inflationary models (2018)



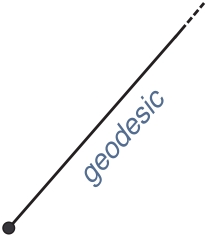
Eternal inflation

Multiverse



Did it have a beginning?

Inflationary space-time is geodesically incomplete in the past
Borde, Guth, Vilenkin (2001)



Dark energy

- About 7 billion years ago, the universe proceeded to accelerated expansion:
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) + \frac{\Lambda}{3}$$
It is inflation, but on a much lower energy scale

- In frames of homogeneous cosmology based on GR, one needs either dark energy – a form of energy with $\rho + 3p < 0$, or the cosmological constant Λ

- Alternative explanation: effect of inhomogeneities of the universe on relatively small scales (clusters of galaxies)
Buchert, Ellis, Wiltshire, … Criticized by Green & Wald

First evidence for dark energy

2011 Nobel Prize in physics

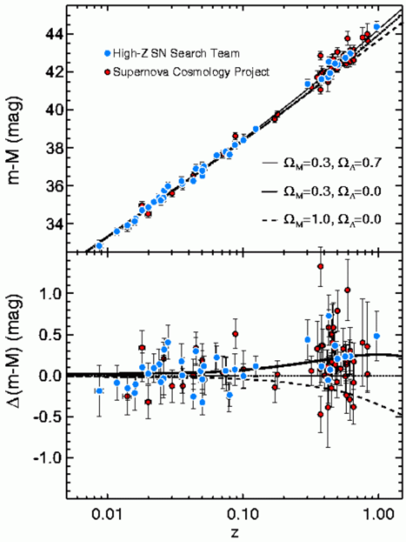
Luminosity distance d_L to supernovae type Ia

$$F = \frac{L}{4\pi d_L^2}$$

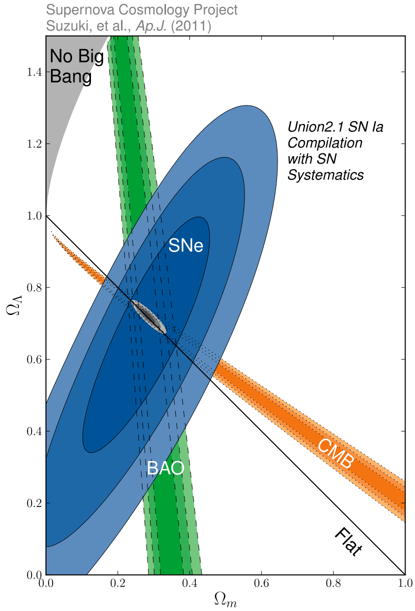
For a spatially flat universe:

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_\Lambda\right]$$



Combined constraints



Dark energy:

- Hampers the development of large-scale structure
- Affects the dynamics of galaxy clusters
- Affects the picture of CMB anisotropy and distribution of galaxies

Theoretical issues

- Scales (in units $\hbar = c = 1$):

$$\Lambda \simeq (2.8 \text{ Gpc})^{-2} \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G} \sim \left(2.5 \times 10^{-3}\,\text{eV}\right)^4$$

Comparable to the neutrino mass-squared difference

$$\Delta m^2_{\rm sol} = \left(8 \times 10^{-3}\,\text{eV}\right)^2$$

- Coincidence:

$$\rho_\Lambda \sim \rho_{\rm m} \quad \text{today}$$

Perhaps, dark energy is a dynamical substance (described, e.g., by a scalar field) $\longrightarrow$ it is evolving

Supersymmetry and naturalness

Vacuum energy density ($\rho = T^0{}_0$) and pressure ($p = -T^i{}_i$) per one degree of freedom:

$$\rho_b = -\rho_f = \int_0^{\Lambda_c} \frac{4\pi k^2 dk}{(2\pi)^3} \cdot \frac{1}{2}\sqrt{k^2 + m^2}$$
$$= \frac{1}{16\pi^2}\left(\Lambda_c^4 + \Lambda_c^2 m^2 - \frac{1}{2} m^4 \ln \frac{\Lambda_c}{m}\right) + \frac{m^4}{128\pi^2}(1 - 4\ln 2) + o\left(\frac{m}{\Lambda_c}\right)$$

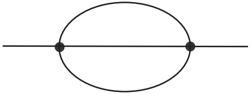
$$p_b = -p_f = \int_0^{\Lambda_c} \frac{4\pi k^2 dk}{(2\pi)^3} \cdot \frac{k^2}{6\sqrt{k^2 + m^2}}$$
$$= \frac{1}{16\pi^2}\left(\frac{1}{3}\Lambda_c^4 - \frac{1}{3}\Lambda_c^2 m^2 + \frac{1}{2} m^4 \ln \frac{\Lambda_c}{m}\right) - \frac{m^4}{128\pi^2}\left(\frac{7}{3} - 4\ln 2\right) + o\left(\frac{m}{\Lambda_c}\right)$$

Canceling divergences:

$$N_b = N_f\,, \qquad \sum m_b^2 = \sum m_f^2\,, \qquad \sum m_b^4 = \sum m_f^4$$

$$\rho_v = \frac{1}{32\pi^2}\left(\sum m_b^4 \ln m_b - \sum m_f^4 \ln m_f\right) \sim m_{\rm SUSY}^4 \gtrsim \text{TeV}^4$$

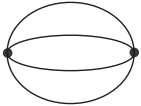
Relation to ultraviolet divergences



Quadratically divergent graph contributes to

$$m^2 = m_0^2 + \Lambda_c^2 f(m_0, \Lambda_c)$$

leading to the "mass hierarchy problem"



Divergent vacuum graphs contribute to the cosmological constant $\Lambda_{\rm GR} \sim \Lambda_c^4 / M_{\rm P}^2$

However, there exist mathematical frameworks in which all such graphs correspond to *finite* expressions. In particular, the vacuum graphs are exactly zero! Dos this mean that the problem of "vacuum energy" is artificial?

Equation-of-state parameter: $w = p_{\rm DE}/\rho_{\rm DE}$

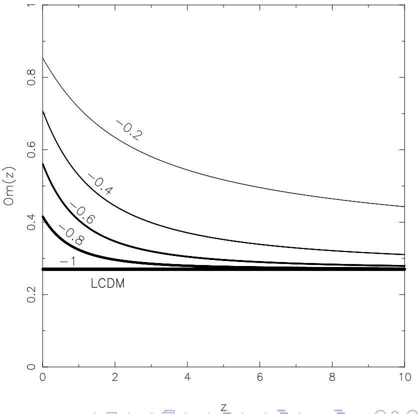
$$\rho_{\rm DE} \propto \exp\left[-3\int (1+w)\frac{da}{a}\right] = \exp\left[3\int (1+w)\frac{dz}{1+z}\right]$$

For a spatially flat universe:

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_{\rm DE}(z)\right]$$

‘Om’ diagnostics for dark-energy evolution [Sahni, Shafieloo & Starobinsky (2008)] :

$$\mathrm{Om}(z) = \frac{h^2(z)-1}{(1+z)^3-1}\,, \qquad h(z) = \frac{H(z)}{H_0}$$



Examples of theoretical models

Quintessence

Just a scalar field with potential:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$
$$p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$
$$w = \frac{p}{\rho} > -1$$

Extra dimensions

$$S = M_5^3 \int_{\mathrm{bulk}} (\mathcal{R} - \Lambda_5) + M_4^2 \int_{\mathrm{brane}} (R - \Lambda_4 + L_{\mathrm{matter}})$$

Effective dark energy is evolving and has $w < -1$ [Sahni & Shtanov (2002)]

Problem of structure formation

- For baryonic component,

$$\delta\rho_b/\rho_b \sim \delta T/T \sim 10^{-5}$$

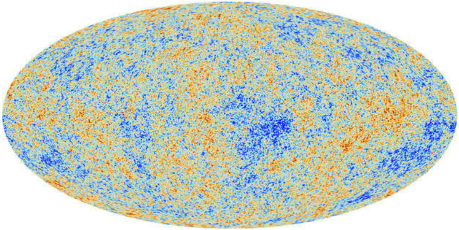
at $z_{\rm rec} \simeq 1100$

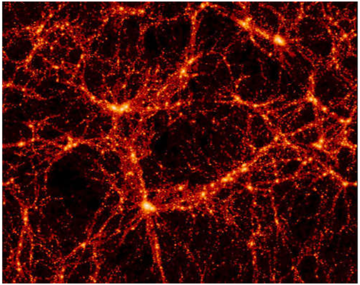
- Density perturbations grow as

$$\delta\rho/\rho \propto a \propto (1+z)^{-1}$$

- Today we would have $\delta\rho_b/\rho_b \sim 10^{-2}$ — surely insufficient for formation of observable structure

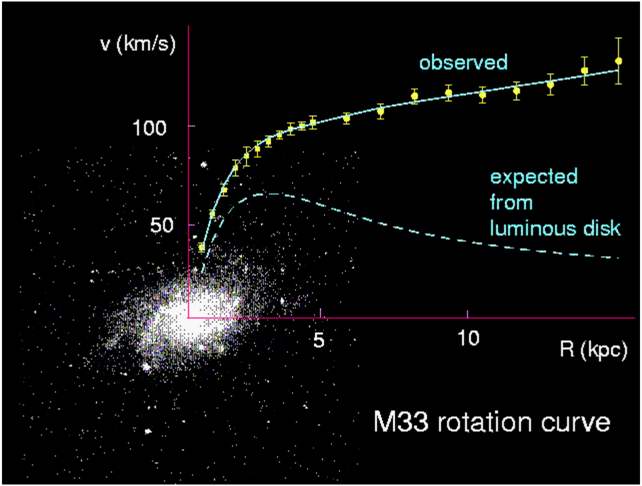
- Way out: dark matter with $\delta\rho/\rho \gg \delta\rho_b/\rho_b$ at $z_{\rm rec} \simeq 1100$





Evidence of dark matter

Galactic rotation curves

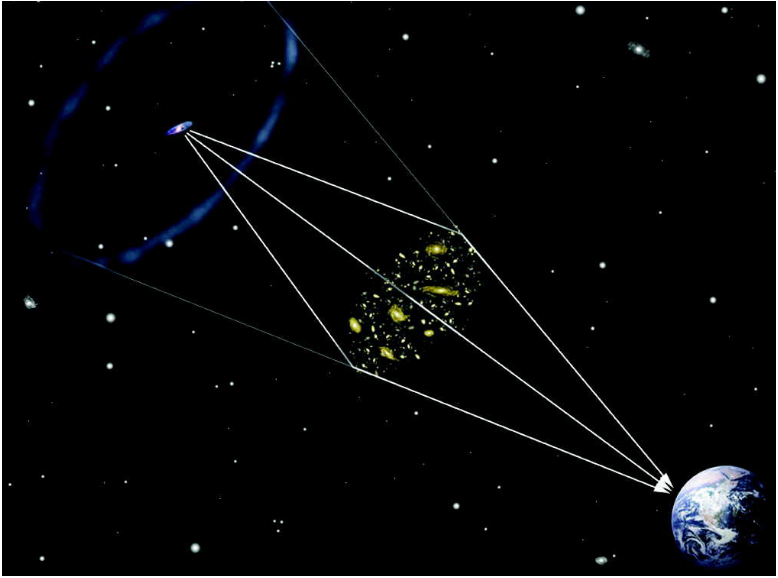


$$\frac{v^2}{r} = \frac{GM(r)}{r^2}$$

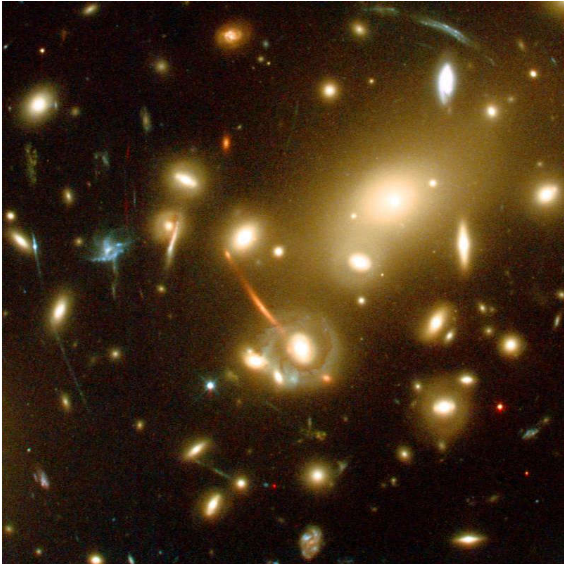
$$v^2 = \frac{GM(r)}{r}$$

Total mass in a galaxy is 6 to 7 times its "visible" mass

Gravitational lens

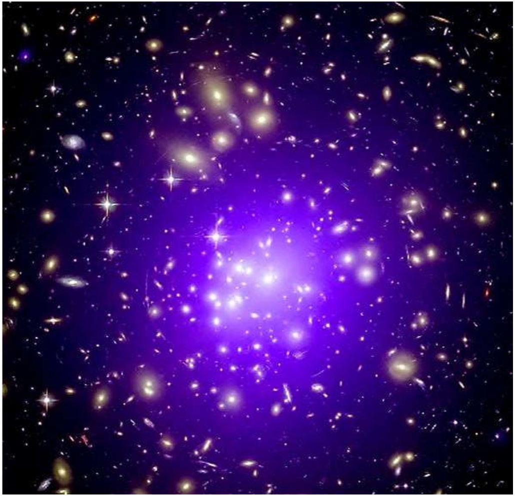


Galaxy cluster as a gravitational lens



Dark-matter halo in a galaxy cluster

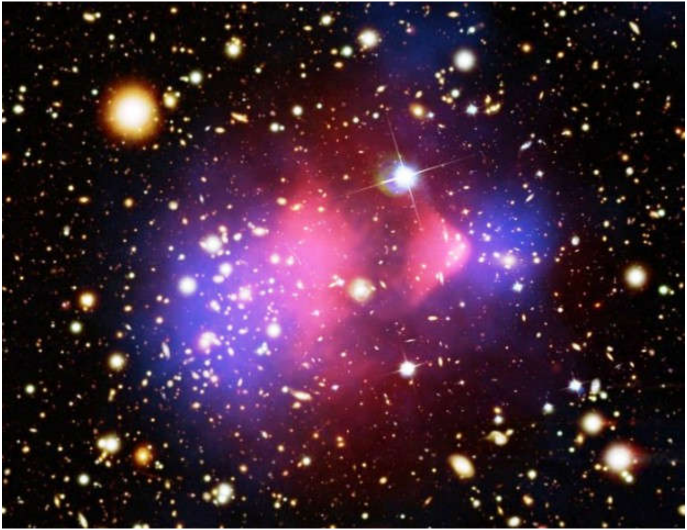
by observation of hot intergalactic gas
and gravitational lensing



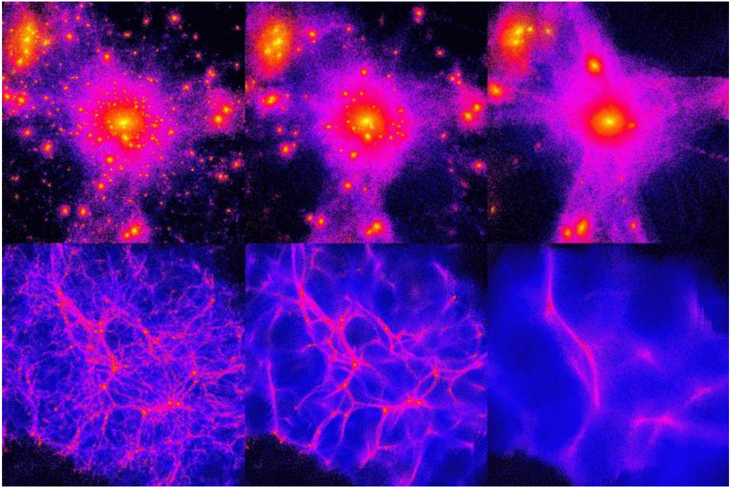
Total mass of a cluster exceeds by order of magnitude its “visible” mass (in gas and stars)

Bullet cluster

at a distance of 3.8 million light years



"Cold" "Warm" "Hot"



Active neutrino cannot compose (all of) dark matter

- For each neutrino specie, $T_\nu \simeq 1.95$ K and $n_\nu \simeq 56$ cm^{-3}. Thus, $\sum_i m_{\nu_i} \simeq 11$ eV is required, which contradicts experiments on β-decay, which give $m_{\nu_e} < 2$ eV and neutrino oscillations $\sqrt{\Delta m^2_{\mathrm{atm}}} = 5 \times 10^{-2}\,\mathrm{eV}$, $\sqrt{\Delta m^2_{\mathrm{sol}}} = 8 \times 10^{-3}\,\mathrm{eV}$

- Phase density is limited for neutrinos being fermions $\Rightarrow$ for galaxies and clusters, $m_{\mathrm{DM}} \gtrsim 20$ eV and 0.4 keV, respectively

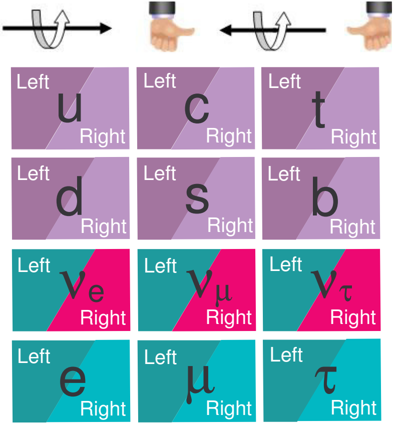
- "Free-streaming": for $m_\nu \simeq 3$ eV, the current velocities $v_\nu \simeq 3500$ km/s $\Rightarrow$ most of the observed gravitationally bound objects would not be able to form

Standard Model requires extension

Possible nature of dark matter

- Dark objects made of usual matter, such as planets, comets or faded stars, can comprise only an insignificant part of undetected matter, which, in particular, follows from theory and observations of CMB and BBN.
- Primordial black holes (formed in some models)
- Weakly or superweakly interacting particles yet to be discovered (sterile neutrinos, superpartners, ...)
- Scalar fields (remnants of the inflaton, axions, ...)
- Perhaps, the laws of gravity are modified on large scales?
  A satisfactory theory of this sort is still missing.

Right-handed (sterile) neutrino
as a natural extension of SM



Potentially describes:

- Neutrino masses and oscillations
- Origin of baryon asymmetry

  νMSM model:
  (Asaka & Shaposhnikov, 2005)
- Dark matter

$$L = L_{SM} + \sum_n \left(\bar{N}_n i \gamma^\mu \partial_\mu N_n - \frac{M_n}{2} \bar{N}_n^C N_n \right) - \sum_{\alpha n} f_{\alpha n} \bar{L}_\alpha N_n \varphi^C + \mathrm{h.c.}$$

Detection of dark matter

- By signals of decay, e.g.,

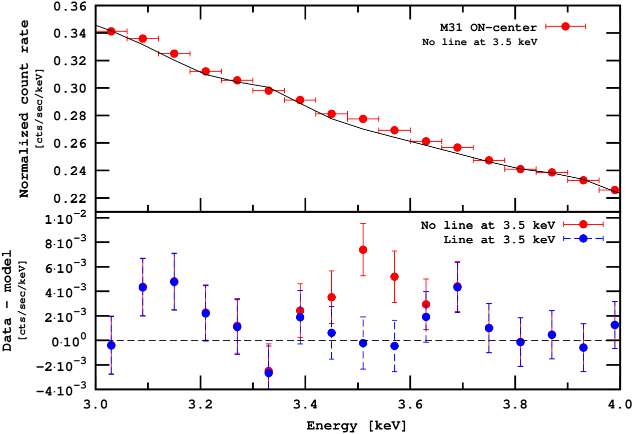
$$N \to \nu + \gamma$$

  The photon in the rest frame of N has definite energy (approximately equal to $m_N c^2/2$, which can be manifest as a radiation line from dark-matter halos

- By signals of annihilation
- By direct detection in underground laboratories

Unidentified emission line at $E \approx 3.5$ keV

A. Boyarsky, O. Ruchayskiy, D. Iakubovskyi, J. Franse, Phys. Rev. Lett. **113**, 251301 (2014)

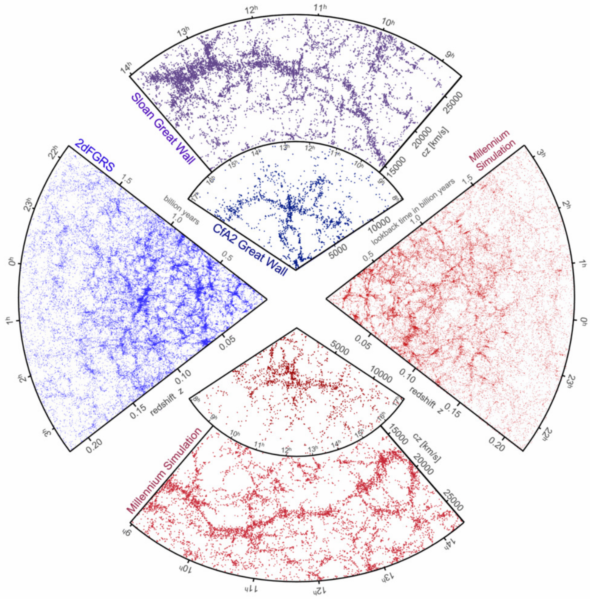


Can be the line of sterile neutrino of mass ≈ 7 keV

Status of the line currently debated (chemical origin, instrumental systematics etc)

E. Bulbul *et al*, Astrophysical Journal **789**, 13 (2014)

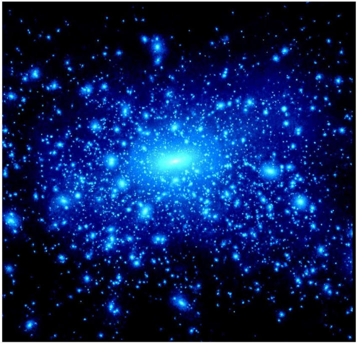
Observations and simulations

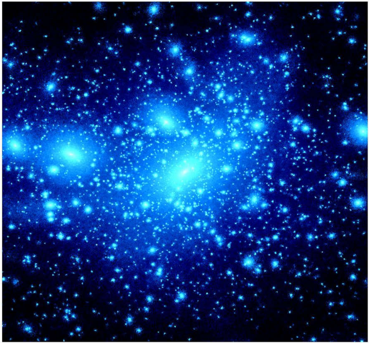


Springel, Frenk & White, Nature **440**, 1137–1144 (2006)

Problem on galactic scales

"Missing satellite" problem: they are abundant in computer simulations, and are scarcely visible in the neighborhoods of big galaxies (such as Milky Way or Andromeda)





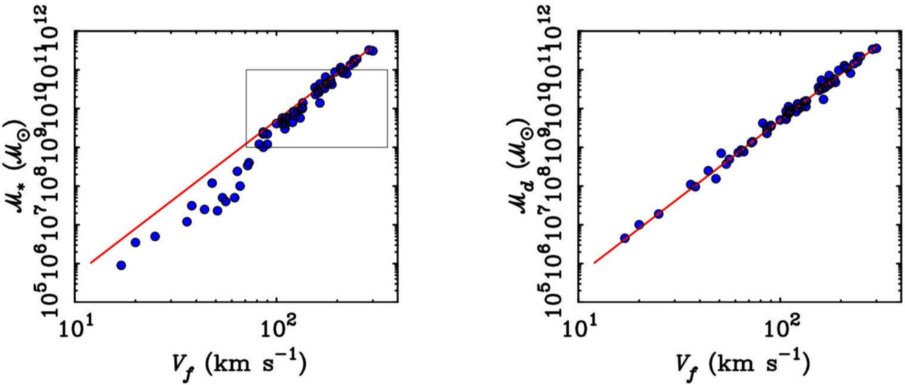
$M = 3 \times 10^{14}\, M_\odot$ $\qquad$ $M = 3 \times 10^{12}\, M_\odot$

Kravtsov, Advances in Astronomy **2010**, 281913 (2010)

Possible solutions: warm or light fermionic dark matter or peculiarities of galaxy formation

Regularities on galactic scales

Tully–Fisher relation



$$M_b = \mathcal{A} v_f^4 \,, \qquad \mathcal{A} = 50\, M_\odot\, \mathrm{km}^4/\mathrm{s}^4$$

What is the reason of this dependence with power 4 ?

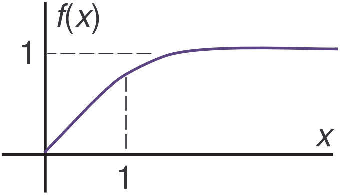
What determines the constant $\mathcal{A}$?

MOND

MOdified Newtonian Dynamics (Milgrom, 1983):

$$m\vec{g}\,\mu\left(\frac{g}{a_0}\right) = \vec{F}$$

For instance, $\quad \mu(x) = x/\sqrt{1+x^2}$



The theory describes:

- Galactic rotation curves (asymptotically "flat")
- Tully–Fisher law:

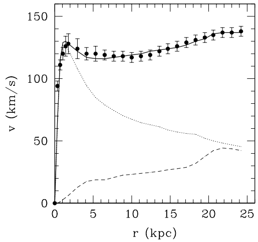
$$\frac{m}{a_0}\left(\frac{v^2}{r}\right)^2 = \frac{GmM}{r^2} \quad \Rightarrow \quad v^4 = a_0 GM$$

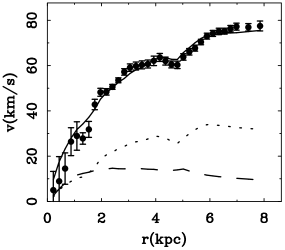
$$a_0 \simeq 1.2 \times 10^{-8}\,\mathrm{cm/s^2} \simeq cH_0/2\pi$$

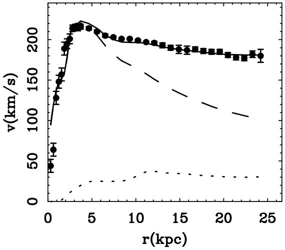
Is this relation a coincidence?

Galactic rotation curves in MOND

$\boxed{\text{The acceleration parameter } a_0 \text{ is universal}}$







However, to account for dark matter in clusters, twice as large value of a_0 is required

Open issues in the theory of dark matter

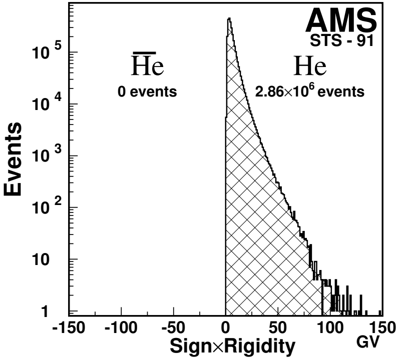
- The nature of dark-matter particles and their detection:
  - Scattering off the usual particles in laboratory
  - Observation of decay and/or annihilation
  - Discovery of a suitable particle at a collider
- Why does MOND fit observations so well? What determines the scale of the fitting parameter $a_0 = 1.2 \times 10^{-8}\ \mathrm{cm/s^2}$?

Origin of baryon asymmetry
beyond the physical horizon !

- Visible part of universe contains scarce amount of antimatter
- Should one explain this asymmetry?
  Yes, if one assumes inflation

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \approx 6 \times 10^{-10}$$

- Three Sakharov's conditions of successful baryogenesis:
  1. B not conserved
  2. C and CP broken
  3. Processes 1 and 2 are not in thermodynamic equilibrium



Inflation begs for baryogenesis

Inflation leaves zero baryonic charge in the universe $\rightarrow$ it is necessary to generate it

Possible scenarios:

- Baryogenesis via leptogenesis

  Keywords: sphalerons, $B - L$ conservation

- GUT baryogenesis

- MSSM and Affleck–Dine scenario

Standard Model (SM) of particle physics

- Quantum anomalies break B and L, preserving $B - L$:

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = \frac{3g^2}{16\pi^2}\left[\mathrm{tr}\left(\mathcal{F}^{\mu\nu}\widetilde{\mathcal{F}}_{\mu\nu}\right)_{\mathrm{SU}(2)_L} - \left(F^{\mu\nu}\widetilde{F}_{\mu\nu}\right)_{\mathrm{U}(1)_Y}\right]$$

- These processes are effective at $10^2\,\mathrm{GeV} \lesssim T \lesssim 10^{12}\,\mathrm{GeV}$ resulting in the equilibrium condition

$$B + bL = 0\,, \qquad b = \frac{28}{51} \quad \text{in the Standard Model}$$

Together with the condition $B - L = 0$, this implies $B = L = 0$

In SM, breaking of thermal equilibrium and non-conservation of B are insufficiently strong for successful baryogenesis

$\boxed{\textit{SM requires extension}}$

Baryogenesis via leptogenesis

$$L = L_{SM} + \sum_n \left(\bar{N}_n i\gamma^\mu \partial_\mu N_n - \frac{M_n}{2} \bar{N}_n^C N_n \right) - \sum_{\alpha n} f_{\alpha n} \bar{L}_\alpha N_n \varphi^C + \mathrm{h.c.}$$

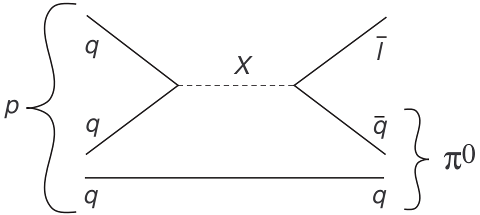
First, lepton asymmetry L is generated due to CP-breaking interactions:

$$\Gamma\left(N \to l\varphi\right) \neq \Gamma\left(N \to \bar{l}\bar{\varphi}\right)$$

After that

$$\left.\begin{array}{r} B - L = -L_i \\ B + bL = 0 \end{array}\right\} \quad \to \quad B = -\frac{b}{1+b} L_i\,, \qquad b = \frac{28}{51} \quad \text{(in SM)}$$

Baryogenesis in Great Unification theories



$$p \to e^+ \pi^0$$

$$\tau_p^{-1} \sim \frac{\alpha_{\rm GUT}^2}{M_{\rm GUT}^4} m_p^5$$

- These processes should violate B and $B - L$ (otherwise electroweak anomalies will destroy B)
- The bound on the proton lifetime $\tau_p \gtrsim 10^{32}$ yr gives $M_{\rm GUT} \gtrsim 10^{16}$ GeV
- New physics implied at $E \sim M_{\rm GUT}$

Summary

The ΛCDM model + inflation is a fairly successful model of the universe

Principal questions:

- Dark matter

  Many candidates
  How to explain correlation of DM and baryonic matter in galaxies (described by MOND)?

- Dark energy

  What determines its value?
  Does it evolve?

- Initial conditions

  Baryon asymmetry?
  Inflation?
  Beginning of the universe?