Thermodynamic-Mean-Field Approach and Equation of State of Strongly Interacting Matter

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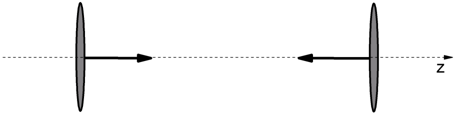
"The basic ideas and concepts behind the modern High-Energy Physics and Cosmology"
5-16 October 2018, Ukraine, Truskavets

Outline

- Introduction
- Thermodynamic mean field. The Grand Canonical Ensemble
- Single Particle Representation of Interacting Classical Gas
- Short-range repulsion by means of excluded volume
- Many-particle systems: Dynamical description
- Thermodynamic mean field: Applications I
- Thermodynamic mean field: Self-consistent equations
- Generalization of the van der Waals EOS to quantum statistics
- Properties of the nuclear matter at T = 0
- The hadron resonance gas model (HRG)
- The Hagedorn model
- Thermodynamic mean field: Applications II. Description of the lattice data

Fireball



- Relativistic energies, relativistic velocities ($v \approx 0.99c$)
- Lorentz contraction

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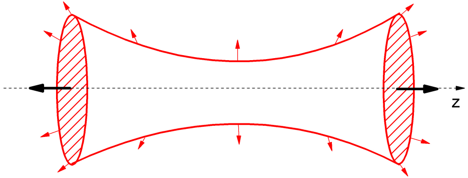
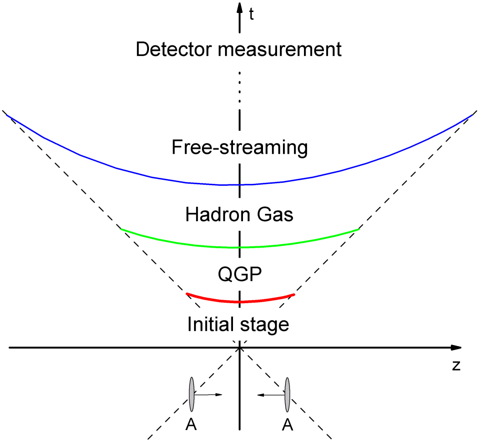
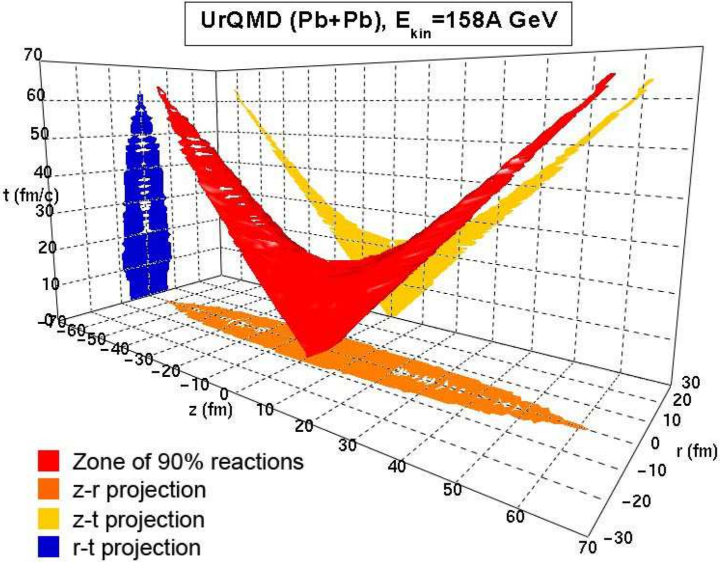


- High excited nonequilibrium system (energy density $\sim$GeV/fm^3)
- High multiplicity of secondary particles
- Short life-time of the system ($\sim$10–20 fm/c)

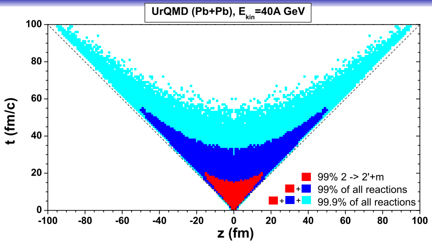
Stages of Fireball Evolution

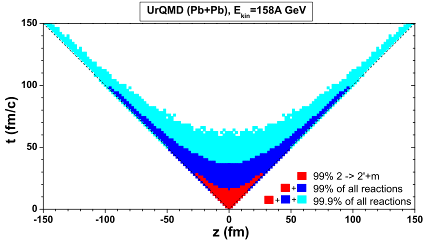


Reaction Zone in (t, r, z) coordinates, $r = \sqrt{x^2 + y^2}$



Reaction Zone in (z, t) coordinates for SPS conditions

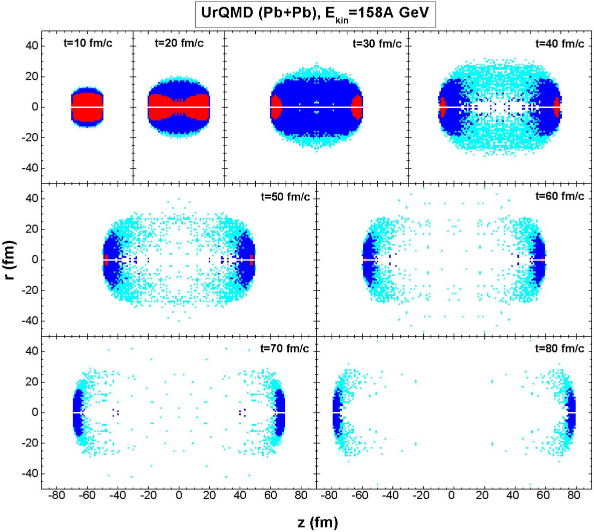




D. Anchishkin, A. Muskeyev, S. Yezhov, Phys. Rev. C81, 031902 (2010); arXiv:1004.0431 [nucl-th].

D. Anchishkin, V. Vovchenko, S. Yezhov, Int.J.Mod.Phys. E22, 1350042 (2013); arXiv:1302.6190 [nucl-th].

Cross sections of the reaction zone in coordinates (r, z) for different time moments



Thermodynamic mean-field

Free energy

$$F = E - T\,S\,, \qquad F = \mu\,N - p\,V$$

$$\mu = \frac{\partial F}{\partial N}\,, \qquad S = -\frac{\partial F}{\partial T}$$

$$\frac{F(N,V,T)}{V} = \phi(n,T)$$

Density of the free energy

$$\phi(n,T) = \varepsilon(n,T) - T\,s(n,T)\,, \qquad \phi(n,T) = n\,\mu(n,T) - p(n,T)$$

$$\mu = \frac{\partial \phi}{\partial n}\,, \qquad s = -\frac{\partial \phi}{\partial T}$$

where $\varepsilon(n,T)$ is the energy density, $p(n,T)$ is the pressure.
We adopt the system of units $k_B = c = \hbar = 1$.

Thermodynamic mean-field

The system of interacting particles → a sum of free and interacting contributions:

$$\phi(n,T) \;=\; \phi_0(n,T) \,+\, \phi_{\rm int}(n,T)$$

$$\mu \;=\; \mu_0 + \left(\frac{\partial \phi_{\rm int}}{\partial n}\right)_T ~, \quad \text{where} \quad \mu_0 \;\equiv\; \left(\frac{\partial \phi_0}{\partial n}\right)_T$$

$$p(n,T) \;=\; n\,\mu(n,T) - \phi(n,T) \quad \rightarrow \quad p \;=\; p_0(n,T) + n\left(\frac{\partial \phi_{\rm int}}{\partial n}\right)_T - \phi_{\rm int}~,$$

where

$$p_0(n,T) \;=\; n\,\mu_0(n,T) \,-\, \phi_0(n,T)$$

p_0 in the Grand Canonical Ensemble:

$$p_0(T,\mu_0) \;=\; \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{m^2+\boldsymbol{k}^2}}\,f_0(\boldsymbol{k};T,\mu_0)$$

Thermodynamic mean-field

Definitions

$$U(n,T) \equiv \frac{\partial \phi_{\rm int}(n,T)}{\partial n}\,, \quad P^{\rm ex}(n,T) \equiv n\,\frac{\partial \phi_{\rm int}(n,T)}{\partial n} - \phi_{\rm int}(n,T)$$

The condition of thermodynamic self-consistency

$$n\,\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\rm ex}(n,T)}{\partial n}$$

Transition to the Grand Canonical Ensemble

$$\begin{aligned} p \; &= \; p_0(T,\mu_0) + P^{\rm ex}(n,T) \quad \rightarrow \\ \rightarrow \quad p(T,\mu) \; &= \; p_0[T,\mu_0(n,T)] + P^{\rm ex}[n(T,\mu),T] \\ \text{with} \quad \mu_0(T,\mu) \; &= \; \mu - U[n(T,\mu),T] \end{aligned}$$

Thermodynamic mean-field. The Grand Canonical Ensemble

Pressure

$$p(T,\mu) = \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\mathbf{k}^2}{\sqrt{m^2+\mathbf{k}^2}}\,f(\mathbf{k};T,\mu) + P^{\text{ex}}(n,T)$$

$$f(\mathbf{k};T,\mu) = \left\{\exp\left[\frac{\sqrt{m^2+\mathbf{k}^2}+U(n,T)-\mu}{T}\right]+a\right\}^{-1}$$

with $a=+1$ for fermions, $a=-1$ for bosons and $a=0$ for the Boltzmann statistics.

Particle density. We use: $n\,\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\text{ex}}(n,T)}{\partial n}$

$$n(T,\mu) = \frac{\partial p(T,\mu)}{\partial \mu} \quad \rightarrow \quad n = g\int \frac{d^3k}{(2\pi)^3}\,f(\mathbf{k};T,\mu)$$

Entropy density and energy density

$$s = \frac{\partial p(T,\mu)}{\partial T}\,, \qquad \varepsilon + p = T\,s + \mu\,n$$

Single Particle Representation of Interacting Classical Gas

Partition function

$$\Sigma(T,\mu,V) = \sum_{N=0}^{\infty} V^N g^N \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_N}{(2\pi)^3} \exp\left[-\frac{1}{T}\left(\sum_{j=1}^{N} \varepsilon_0(\boldsymbol{k}_j) - \mu N\right)\right]$$
$$\times \frac{1}{V^N N!} \int d^3r_1 \cdots d^3r_N \, \exp\left[-\frac{U_{\rm int}^N}{T}\right],$$

where $\varepsilon_0(\boldsymbol{k})$ is the free dispersion law; $\varepsilon_0(\boldsymbol{k}) = \boldsymbol{k}^2/2m$ in the nonrelativistic case, $\varepsilon_0(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2 + m^2}$ in the relativistic one,

$$U_{\rm int}^N = \sum_{i<j}^{N} u(|\mathbf{r}_i - \mathbf{r}_j|)$$

Thermodynamic mean-field. Classical gas with two-particle interaction

The Mayer's theorems

$$\ln \Sigma = V \left[n + \sum_{i=2}^{\infty} B_i(T)\, n^i \right] ,$$

$$\ln z_0 = \ln(n) + \sum_{i=2}^{\infty} \frac{i}{i-1} B_i(T)\, n^{i-1} ,$$

where

$$z_0 = g \int \frac{d^3k}{(2\pi)^3} \exp\left(- \frac{\sqrt{k^2+m^2} - \mu}{T} \right) ,$$

$$B_2(T) = \frac{1}{2} \int d^3r \left[1 - \exp\left(- \frac{u(r)}{T} \right) \right]$$

Definitions:

$$P_{\rm cl}(n,T) \equiv T \sum_{i=2}^{\infty} B_i(T)\, n^i , \qquad U_{\rm cl}(n,T) \equiv T \sum_{i=2}^{\infty} \frac{i}{i-1} B_i(T)\, n^{i-1}$$

Classical gas with two-particle interaction. Thermodynamic consistency

Thermodynamic consistency:

$$\Rightarrow \qquad n\,\frac{\partial U_{\rm cl}(n,T)}{\partial n} = \frac{\partial P_{\rm cl}(n,T)}{\partial n}$$

Thermodynamic quantities

$$p(T,\mu) = \frac{T}{V}\ln\Sigma = T\,n(T,\mu) + P_{\rm cl}(n,T)$$

$$n(T,\mu) = g\int\frac{d^3k}{(2\pi)^3}\,\exp\left[-\frac{\sqrt{k^2+m^2}-\mu+U_{\rm cl}(n,T)}{T}\right]$$

D.V. Anchishkin, Sov. Phys. JETP 75, 195 (1992) [Zh. Eksp. Teor. Fiz. 102, 369 (1992)].

D. Anchishkin, E. Suhonen, Generalization of mean-field models to account for effects of excluded-volume, Nucl. Phys. A586, 734 (1995).

Many-particle systems: Dynamical description

The state of N-particle system $\Psi(t, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_N)$ is determined by the Schrödinger equation (we adopt $\hbar = 1$ and $c = 1$)

$$i\hbar \frac{\partial \Psi(t, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_N)}{\partial t} = H\Psi(t, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) \Rightarrow \Psi(t, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) = e^{-iH(t-t_0)}\Psi_0(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N),$$

where $H = H_0 + H_{\text{int}}$ is the total Hamiltonian of the system which consists of N identical particles ($U(\boldsymbol{r})$ is an external field)

$$H_0 = \sum_{i=1}^{N} \left[\frac{-\hbar^2}{2m} \nabla_i^2 + U(\boldsymbol{r}_i) \right], \qquad H_{\text{int}} = \frac{1}{2} \sum_{i,j=1\,(i \neq j)}^{N} W(|\boldsymbol{r}_i - \boldsymbol{r}_j|).$$

We define field operators at initial time moment $t = t_0$:

$$\psi(\boldsymbol{r}) = \frac{1}{V} \sum_{\boldsymbol{k}} \mathbf{a}(\boldsymbol{k})\, \varphi_{\boldsymbol{k}}(\boldsymbol{r}), \qquad \psi^+(\boldsymbol{r}) = \frac{1}{V} \sum_{\boldsymbol{k}} \mathbf{a}^+(\boldsymbol{k})\, \varphi^*_{\boldsymbol{k}}(\boldsymbol{r})$$

where for the single-particle Hamiltonian $\hat{h}_0 = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2$ it is valid

$$\hat{h}_0 \varphi_{\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon(\boldsymbol{k}) \varphi_{\boldsymbol{k}}(\boldsymbol{r})$$

Many-particle system: Occupation number representation. Homogeneous system

Hamiltonian at $t = t_0$:

$$H \, = \, H_0 \, + \, H_{\rm int} \, ,$$

where

$$H_0 \, = \int d^3r \, \psi^+(\boldsymbol{r}) \left(-\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \right) \psi(\boldsymbol{r}) \, ,$$

$$H_{\rm int} \, = \, \frac{1}{2} \int d^3r_1 \, d^3r_2 \, \psi^+(\boldsymbol{r}_1) \, \psi^+(\boldsymbol{r}_2) \, W(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \, \psi(\boldsymbol{r}_2) \, \psi(\boldsymbol{r}_1)$$

For homogeneous system one can accept the Hartree approximation:

$$\psi^+(\boldsymbol{r}_1) \, \psi(\boldsymbol{r}_1) \quad \rightarrow \quad \langle \psi^+(\boldsymbol{r}_1) \, \psi(\boldsymbol{r}_1) \rangle \, = \, n \, ,$$

or

$$\psi^+(\boldsymbol{r}_2) \, \psi(\boldsymbol{r}_2) \quad \rightarrow \quad \langle \psi^+(\boldsymbol{r}_2) \, \psi(\boldsymbol{r}_2) \rangle \, = \, n \, .$$

$$H \;\; \approx \;\; \int d^3r \, \psi^+(\boldsymbol{r}) \left[-\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 + 2n \int d^3r_2 \, W(|\boldsymbol{r} - \boldsymbol{r}_2|) \right] \psi(\boldsymbol{r}) \, - \\ - \int d^3r \, \psi^+(\boldsymbol{r}) \, n \int d^3r_2 \, W(|\boldsymbol{r} - \boldsymbol{r}_2|) \, \psi(\boldsymbol{r}) \, .$$

Homogeneous system: Statistics

Next step

$$\psi^+(\mathbf{r})\,\psi(\mathbf{r}) \quad \to \quad \langle \psi^+(\mathbf{r})\,\psi(\mathbf{r})\rangle = n$$

$$H \approx \int d^3r\,\psi^+(\mathbf{r})\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + 2\,a\,n\right)\psi(\mathbf{r}) - a\,n^2\,V\,,$$

where

$$H_{\mathrm{int}} \approx -a\,n^2\,V\,, \qquad \text{and} \quad a \equiv \int d^3r\,W(|\mathbf{r}|)\,.$$

$$H = \int \frac{d^3k}{(2\pi)^3}\left(\sqrt{m^2+\mathbf{k}^2} + 2\,a\,n\right) a^+_{\mathbf{k}} a_{\mathbf{k}} - a\,n^2\,V\,,$$

The thermodynamic potential Ω is obtained with making use of the statistical operator:

$$\rho = \exp\left(-\beta H\right), \qquad \text{where} \quad \beta = 1/T\,,$$

$$\Omega = -\frac{1}{\beta}\,\ln\left(\mathrm{Tr}\,e^{-\beta H}\right)\,.$$

Homogeneous system: Statistics

$$\Omega(\beta) = -\frac{g}{\eta\beta}\sum_{\boldsymbol{k}} \ln\left\{1 + \eta \exp\left[-\beta\left(\sqrt{\boldsymbol{k}^2 + m^2} + 2\,a\,n\right)\right]\right\} - a\,n^2\,V\,,$$

where $\eta = 1$ for fermions, $\eta = -1$ for bosons and $\eta \to 0$ in case of the Boltzmann approximation.
The pressure is related to the thermodynamic potential by $p = -\Omega/V$ and is given by

$$p(\beta) = \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{\boldsymbol{k}^2 + m^2}}\,f(k,\beta) + a\,n^2$$

with distribution function

$$f(\boldsymbol{k},\beta) = \left\{\exp\left[\beta\left(\sqrt{\boldsymbol{k}^2 + m^2} + 2an\right)\right] + \eta\right\}^{-1}.$$

So, one indicates that

$$U(n) = 2\,a\,n \qquad \text{and} \qquad P^{\text{ex}} = a\,n^2\,.$$

$$n\,\frac{\partial U(n)}{\partial n} = \frac{\partial P^{\text{ex}}(n)}{\partial n}$$

Thermodynamic mean-field. Applications I

$$p = nT + P^{\text{ex}}(n,T), \qquad n\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\text{ex}}(n,T)}{\partial n}$$

$$U(n,T) = U(n=0,T) + \int_0^n \frac{1}{n'}\frac{\partial P^{\text{ex}}(n',T)}{\partial n'}\,dn'$$

1. Virial expansion for hard-sphere potential, $v_0 = 4\,\frac{4\pi}{3}\,r^3$

$$P^{\text{ex}}(n,T) = T\,v_0 n^2 \qquad \rightarrow \qquad U(n,T) = 2\,T\,v_0 n$$

2. The van der Waals equation of state (the Boltzmann statistics)

$$p = \frac{Tn}{1-bn} - an^2 \quad \rightarrow \quad P^{\text{ex}} = p - nT = Tn\frac{bn}{1-bn} - an^2$$

$$\rightarrow \quad U(n,T) = T\frac{bn}{1-bn} - T\ln(1-bn) - 2an$$

Thermodynamic mean-field. Applications I

3. The Carnahan-Starling equation of state (the Boltzmann statistics)

$$P^{\rm ex}_{\rm CS}(n,T) \;=\; Tn\,\frac{v_0 n \;-\; (v_0 n)^2/8}{(1 \;-\; v_0 n/4)^3}\,,$$

$$U_{\rm CS}(n,T) \;=\; -3T\left[1 - \frac{1 - v_0 n/12}{(1 - v_0 n/4)^3}\right]\,.$$

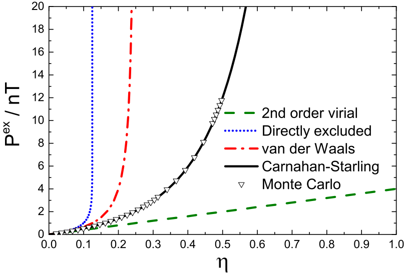


Рис.: The dependence of the quantity $P^{\rm ex}/nT$ on the packing fraction $\eta = v_0 n/4$.

V. Vovchenko, Equations of state for real gases on the nuclear scale, Phys. Rev. C96, 015206 (2017).

Thermodynamic mean field: Excluded volume $n\,\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\rm ex}(n,T)}{\partial n}$

$$U(n,T) \;=\; T\widetilde{U}(n) \qquad \Leftrightarrow \qquad P^{\rm ex}(n,T) \;=\; T\widetilde{P}^{\rm ex}(n)$$

$$n(T,\mu) \;=\; g\int \frac{d^3k}{(2\pi)^3}\,\exp\left[\frac{-\sqrt{m^2+\boldsymbol{k}^2} - T\widetilde{U}(n) + \mu}{T}\right],$$

$$\varepsilon(T,\mu) \;=\; g\int \frac{d^3k}{(2\pi)^3}\,\sqrt{m^2+\boldsymbol{k}^2}\,\exp\left[\frac{-\sqrt{m^2+\boldsymbol{k}^2} - T\widetilde{U}(n) + \mu}{T}\right],$$

For the Boltzmann statistics:

$$N(T,\mu) \;=\; e^{-\widetilde{U}(n)}V\,n^{\rm id}(T,\mu)\,, \qquad E(T,\mu) = e^{-\widetilde{U}(n)}V\,\varepsilon^{\rm id}(T,\mu)\,.$$

$$N(T,\mu) = \widetilde{V}\,n^{\rm id}(T,\mu)\,, \quad E(T,\mu) = \widetilde{V}\varepsilon^{\rm id}(T,\mu)\,.$$

$$V \to \widetilde{V} \;=\; V\,e^{-\widetilde{U}(n)}\,, \qquad V\,e^{-\widetilde{U}(n)} \;\equiv\; V - N\widetilde{v}_0 \quad \to \quad \widetilde{v}_0 = \frac{V}{N}\left(1 - e^{-\widetilde{U}(n)}\right)$$

Thermodynamic mean field: Self-consistent equations

The van der Waals equation of state (the Boltzmann statistics). Self-consistent equation for n

$$(i) \quad n \,=\, (1-b\,n)\,\exp\left[-\,\frac{b\,n}{1\,-\,b\,n}\,+\,\frac{2an}{T}\right]\,n^{\rm id}(T,\mu)$$

$$n^{\rm id}(T,\mu)\,=\,g\,e^{\mu/T}\,\frac{1}{2\pi^2}\,m^2\,T\,{\rm K}_2(m/T)$$

$$(ii) \quad n\,=\,\frac{n^{\rm id}(T,\mu^*)}{1\,+\,b\,n^{\rm id}(T,\mu^*)} \quad \text{with} \quad \mu^*=\mu-T\,\frac{b\,n}{1-bn}+2an$$

The van der Waals equation of state (the Boltzmann statistics). Self-consistent equation for p

$$p\,=\,p_{\rm id}(T,\mu^*)\,-\,a\,n^2$$

$$\text{with} \quad \mu^*\,=\,\mu\,-\,b\,p\,-\,b\,a\,n^2\,+\,2a\,n\,,$$

$$n\,=\,\frac{n^{\rm id}(T,\mu^*)}{1\,+\,b\,n^{\rm id}(T,\mu^*)}$$

$$\text{If} \quad a=0 \quad \Rightarrow \quad p\,=\,p_{\rm id}(T,\mu^*) \quad \text{with} \quad \mu^*\,=\,\mu\,-\,bp$$

The Boltzmann statistics

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner,
Excluded volume effect for the nuclear matter equation of state,
Z. Phys. C - Particles and Fields 51, 485-489 (1991)

Excluded volume: Self-consistent equation for pressure

$$Z^{\mathrm{excl}}(T,\mu,V) = \sum_{N=0}^{\infty} e^{\mu N/T} Z(T,N,V-v_0N)\,\theta(V-v_0N)$$

$$p^{\mathrm{excl}}(T,\mu) = p(T,\mu^*)\,, \qquad \text{where} \qquad \mu^* = \mu - v_0\,p^{\mathrm{excl}}(T,\mu)$$

Excluded volume in the ideal gas

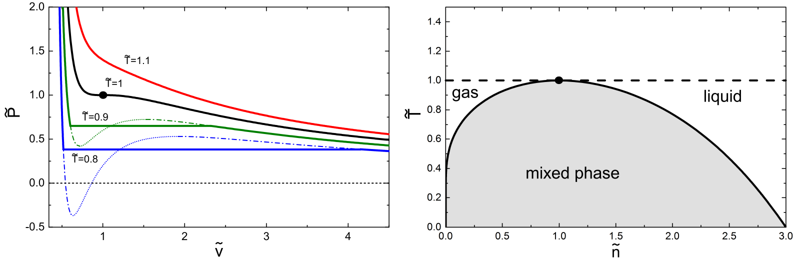
$$p_{\mathrm{id}}^{\mathrm{excl}}(T,\mu) = p_{\mathrm{id}}(T,\mu^*)\,, \qquad \text{where} \qquad \mu^* = \mu - v_0\,p_{\mathrm{id}}^{\mathrm{excl}}(T,\mu)$$

Phase Diagram for the VdW Gas

$$p(T, n) = \frac{nT}{1 - bn} - an^2, \qquad n = \frac{N}{V},$$

$$T_C = \frac{8a}{27b}, \qquad n_C = \frac{1}{3b}, \qquad p_C = \frac{a}{27b^2}, \qquad \Rightarrow \qquad \tilde{p} = \frac{8\tilde{T}\tilde{n}}{3 - \tilde{n}} - 3\tilde{n}^2,$$

where $\tilde{n} = n/n_C$, $\tilde{p} = p/p_C$, and $\tilde{T} = T/T_C$.



(a): The dimensionless form of the VDW isotherms for pressure, $\tilde{p}$, versus the volume per particle $\tilde{v} = \tilde{n}^{-1}$. The dashed-dotted lines present the metastable parts of the VDW isotherms at $\tilde{T} < 1$, whereas the dotted lines correspond to unstable parts. The full circle on the $\tilde{T} = 1$ isotherm corresponds to the critical point.

(b): The phase diagram for the VDW equation of state on the $(\tilde{n}, \tilde{T})$ plane. The phase coexistence region resulting from the Maxwell construction is depicted by grey shaded area.

Generalization of the van der Waals EOS to quantum statistics

The van der Waals equation of state (quantum statistics). Self-consistent equation for p

$$p = p^{\mathrm{id}}(T,\mu^*) - a\,n^2$$

$$\text{with}\quad \mu^* = \mu - b\,p - b\,a\,n^2 + 2a\,n\,, \qquad n = \frac{n^{\mathrm{id}}(T,\mu^*)}{1+b\,n^{\mathrm{id}}(T,\mu^*)}$$

Pressure of the ideal quantum gas

$$p^{\mathrm{id}}(T,\mu) = \frac{g}{3}\int\frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{m^2+\boldsymbol{k}^2}}\,f_0(\boldsymbol{k};T,\mu)$$

$$f_0(\boldsymbol{k};T,\mu) = \left\{\exp\left[\frac{\sqrt{m^2+\boldsymbol{k}^2}-\mu}{T}\right]+a\right\}^{-1}$$

Quantum statistics, nuclear-matter properties at $T = 0$

Parameters a and b are fixed to reproduce the properties of nuclear matter in its ground state at $T = 0$:

$$p = 0\,, \qquad \frac{\varepsilon}{n} = m + E_B \cong 922\,\mathrm{MeV}\,, \qquad n = n_0 \cong 0.16\,\mathrm{fm}^{-3}\,.$$

Here $E_B \cong -16$ MeV is the binding energy per nucleon.

$$\Rightarrow \qquad a \cong 329\,\mathrm{MeV\,fm}^3\,, \qquad b \cong 3.42\,\mathrm{fm}^3\,.$$

Parameter b of the proper particle volume can be expressed in terms of hard-core radius r as $b = 16\pi r^3/3$. This gives $r \cong 0.59$ fm for the hard-core nucleon radius.

Quantum statistics, properties at $T = 0$

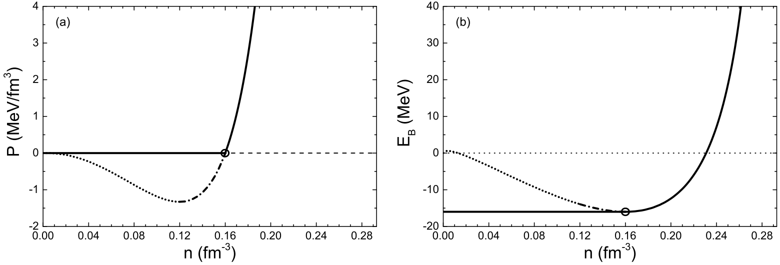


Рис.: Dependence of pressure p (a) and binding energy E_B (b) on the nucleon density n at $T = 0$. The VDW parameters are $a \cong 329$ MeV·fm^3 and $b \cong 3.42$ fm^3 ($r \cong 0.59$ fm). The open circle corresponds to the ground state of nuclear matter. The dash-dotted line corresponds to the metastable part of VDW isotherm, whereas the dotted line corresponds to the unstable part.

Phase diagram at $T \neq 0$

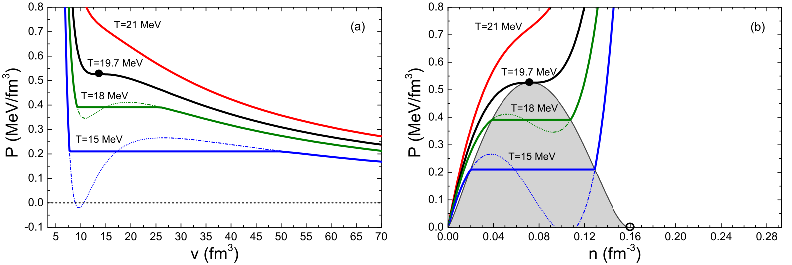


Рис.: Pressure isotherms in (a) (p, v) and (b) (p, n) coordinates, calculated in the quantum van der Waals equation of state with parameters $a \cong 329$ MeV·fm^3 and $b \cong 3.42$ fm^3 ($r \cong 0.59$ fm). The dashed-dotted lines present the metastable parts of the VDW isotherms at $T < T_C$, whereas the dotted lines correspond to unstable parts. The full circle on the $T = T_C$ isotherm corresponds to the critical point, while the open circle at $T = 0$ in (b) shows the ground state of nuclear matter. Shaded grey area in (b) depicts the mixed phase region obtained from the Maxwell construction of equal areas for $p(v)$ isotherms in (a).

The classical VDW equation with Boltzmann statistics, would give

$$T_c = \frac{8a}{27b} \cong 28.5\,\mathrm{MeV}\,, \qquad n_c = \frac{1}{3b} \cong 0.10\,\mathrm{fm}^{-3}$$