Approximately L conserving seesaw models: Minimal Flavour Violation and Leptogenesis

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In collaboration with: B. Gavela, D. Hernandez and P. Hernandez, JHEP 09'
S. Blanchet and F.-X. Josse-Michaux, JCAP 10'

Following previous works with: A. Abada, C. Biggio, F. Bonnet and B. Gavela, JHEP 07', PRD 08'

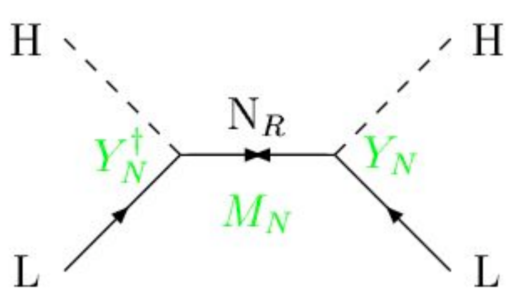
CERN, 16/09/2010

Neutrino mass origin?

appealing explanation: seesaw mechanism

3 basic seesaw models → leptogenesis

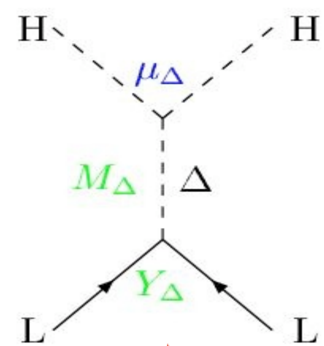
Right-handed singlet:
(type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

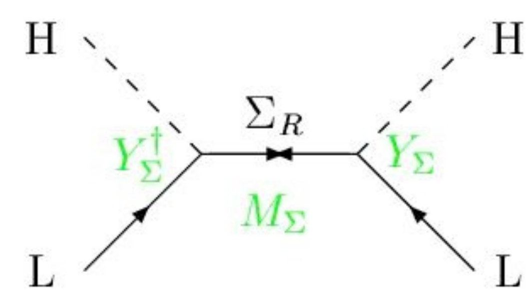
Scalar triplet:
(type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet:
(type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

requires experimental breakthrough on top of the ν mass matrix measurements

Lepton Flavour Violating processes ⟵ (L conserving)

⟶ $\mu \to e\gamma,\ \tau \to \mu\gamma,\ \tau \to e\gamma,\ \mu \to eee,\ \tau \to lll,\ \mu \to e$ atomic conversion, ...

⟶ expected small in seesaw models: dim-6 operator effect:

$$\mathcal{L}^{d=6} = Y_N^\dagger \frac{1}{M_N^2} Y_N\, (\bar{L}H)\,\partial\!\!\!/ (HL) \implies \Gamma(\mu \to e\gamma) \propto Y_N^4 \frac{m_\mu^5}{M_N^4}$$

$$m_\nu \sim Y_N \frac{1}{M_N} Y_N^T v^2$$

if $Y_N \sim 1, \quad m_\nu = 0.1\,\text{eV}$
e.g. requires $M_N \sim 10^{14}\,\text{GeV}$

if $M_N \sim 1$ TeV, $\quad m_\nu = 0.1\,\text{eV}$
e.g. requires $Y_N \sim 10^{-6}$

$$\Gamma(\mu \to e\gamma) \propto Y_N^4 \frac{m_\mu^5}{M_N^4} \text{ very suppressed!!}$$

⟶ but not necessarily: inverse seesaw models → m_ν violate L; → LFV conserve L

Approximately L conserving framework

assume a L conserving setup with not too large $M_N \sim 100\ \mathrm{GeV} - 100\ \mathrm{TeV}$
and large Yukawas $Y_N \sim 10^{-2} - 1$

$\Downarrow$

$$Br(\mu \to e\gamma) \sim 10^{-11} \sim \text{experimental upper limit}$$

$$m_\nu = 0$$ ← no L violation

assume L is broken by a small perturbation μ and/or Y'

$\Downarrow$

$$m_\nu \sim 0.1\,\mathrm{eV}$$ ← neutrino masses directly proport. to a small source of L violation μ and/or Y' rather than inversely proport. to a large mass M

Approximately L conserving type-I seesaw model

example with n N_1 and n N_2: $L_{N_1} = +1\,,\ L_{N_2} = -1$

$$
\begin{array}{c c}
 & \begin{array}{ccc} \nu_L & N_1 & N_2 \end{array} \\
\begin{array}{c} \nu_L \\ N_1 \\ N_2 \end{array} &
\left(\begin{array}{ccc}
0 & Y_N \frac{v}{\sqrt{2}} & 0 \\
Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\
0 & M_N & 0
\end{array} \right)
\end{array}
$$

"inverse seesaw" as in
Mohapatra, Valle '86
Gonzalez-Garcia, Valle '89
Branco, Grimus, Lavoura '89
Kersten, Smirnov '07
Abada, Biggio, Bonnet, Gavela, T.H. '07

if Y_N is large, M_N not too high:

$$Br(\mu \to e\gamma) \sim 10^{-11} \sim \text{experimental upper limit}$$

$m_\nu = 0$ ← no L violation

Approximately L conserving type-I seesaw model

example with n N_1 and n N_2: $L_{N_1} = +1\,,\ L_{N_2} = -1$

$$\begin{array}{c} \\ \nu_L \\ N_1 \\ N_2 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_L & N_1 & N_2 \end{array} \\ \begin{pmatrix} 0 & Y_N \frac{v}{\sqrt{2}} & 0 \\ Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\ 0 & M_N & \mu \end{pmatrix} \end{array}$$

soft L breaking

"inverse seesaw" as in
Mohapatra, Valle '86
Gonzalez-Garcia, Valle '89
Branco, Grimus, Lavoura '89
Kersten, Smirnov '07
Abada, Biggio, Bonnet, Gavela, T.H. '07

if Y_N is large, M_N not too high:

$$Br(\mu \to e\gamma) \sim 10^{-11} \sim \text{experimental upper limit}$$

$$m_\nu = -Y_N^T \frac{\mu}{M_N^2} Y_N \, v^2 \ \sim \ 0.1\,\text{eV}$$

Approximately L conserving type-I seesaw model

example with n N_1 and n N_2: $L_{N_1} = +1\,,\ L_{N_2} = -1$

hard L breaking

$$
\begin{array}{c} \\ \nu_L \\ N_1 \\ N_2 \end{array}
\begin{array}{c} \begin{array}{ccc} \nu_L & N_1 & N_2 \end{array} \\
\begin{pmatrix} 0 & Y_N \frac{v}{\sqrt{2}} & Y_N' \frac{v}{\sqrt{2}} \\ Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\ 0 & M_N & 0 \end{pmatrix} \end{array}
$$

Branco, Grimus, Lavoura 89', ... ,
Kersten, Smirnov 07'; Blanchet, Asaka 08',
Abada, Biggio, Bonnet, Gavela, TH 07'
TH, Gavela, Hernandez, Hernandez 09',

if Y_N is large, M_N not too high:

$$Br(\mu \to e\gamma) \sim 10^{-11} \sim \text{experimental upper limit}$$

$$m_\nu = -({Y_N'}^T \frac{1}{M_N} Y_N + Y_N^T \frac{1}{M_N} Y_N')\, v^2 \ \sim\ 0.1\,\text{eV}$$

If we observe some lepton flavour viol. processes: seesaw?

would be the striking sign of new physics at a nearby scale and a very strong hint for the seesaw but not necessarily a proof at all: many models can lead to it and in general the dim. 6 coefficients are not known

But ways out do exist:

- In models where several processes are related to a single $c^{ij}_{d=6}$ coefficient independently of $c^{ij}_{d=5}$ ones in case their ratios are fixed:

type-III seesaw:

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 3.1 \cdot 10^{-4} \cdot R^{\mu\to e} \\
Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to e^- e^+ \mu^-) \\
Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee) = 2.1 \cdot 10^{-3} \cdot Br(\tau^- \to \mu^- \mu^+ e^-)
\end{aligned}
$$

proportional to the $c^{ij}_{d=6}$ coefficient:

$$
\begin{aligned}
|\epsilon_{e\mu}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\mu e} \lesssim 1.1 \cdot 10^{-4} \\
|\epsilon_{\mu\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau\mu} \lesssim 1.5 \cdot 10^{-2} \\
|\epsilon_{e\tau}| &= \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\tau e} \lesssim 2.4 \cdot 10^{-2}
\end{aligned}
$$

and similarly for type-I and type-II seesaw models

If we observe some lepton flavour viol. processes: seesaw?

- Minimal flavour violation: the flavour structure of the higher dimensional BSM induced operators can be determined from the flavour structure of the lowest dimension flavour structure

→ originally assumed in the quark sector to allow for new low scale physics without flavour changing problems

⇓

for leptons the context is $\neq$: we do have an evidence for new physics (neutrino masses) but the effect is so tiny that the new physics associated is not expected to bring any flavour changing problem ⇒ no need for MFV to avoid flavour violation but yet would lead to predictivity

→ all dim-6 induced processes could be predicted up to an overall normalization from the knowledge of $m_{\nu ij}$

Minimal Flavour Violation in lepton sector

Cirigliano, Grinstein, Isidori, Wise 05'

- 1) Large flavour violation with small L violation: a hierarchy between L-viol. scale Λ_{LN} and flavour-viol. scale Λ_F: $\Lambda_{LN} >> \Lambda_F$

- 2) The flavour structure of the dim-6 coefficients fixed by the dim-5 one

minimal setup: quadratically

$$c_{d=6} \propto c_{d=5}\, c_{d=5}$$

an explicit UV realization: type-II seesaw model

extended setup: linearly

$$c_{d=6} \propto c_{d=5}$$

an explicit UV realization: type-I seesaw model with 2 extra assumptions: $M_N \propto \mathbb{I}$ and no CP violation so that

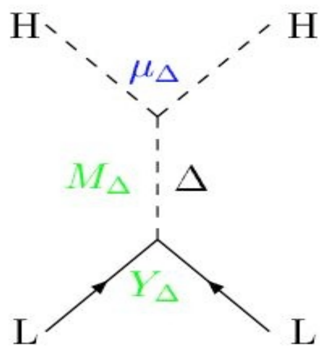
$$c_{d=6} = Y_N^\dagger \frac{1}{M_N^2} Y_N = \frac{1}{M_N^2} Y_N^T Y_N = \frac{1}{M_N} c_{d=5}$$

A seesaw model automatically of the MFV type: the type-II model

see e.g. TH, Gavela, Hernandez, Hernandez 09'

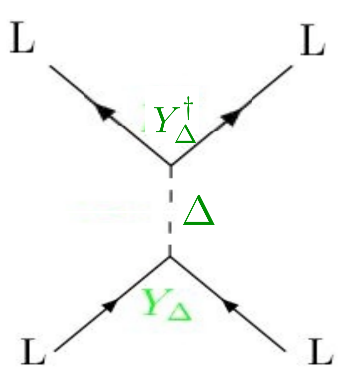
of minimal setup type:

- 1) Neutrino masses:



$$\Rightarrow \; c^{d=5}_{ij} = -2\, Y_{\Delta ij} \frac{\mu_\Delta}{M^2_\Delta} \quad \leftarrow \quad = -\frac{m_{\nu ij}}{v^2}$$

- 2) Flavour changing L conserving processes:



$$\Rightarrow \; c^{d=6}_{ijkl} = -\frac{1}{M^2_\Delta} Y^\dagger_{\Delta ij} Y_{\Delta kl} \propto c^{d=5\dagger}_{ij} c^{d=5}_{kl}$$

and there is effectively a separation of scale: $\Lambda_F \sim M_\Delta \longleftrightarrow \Lambda_{LN} \sim M^2_\Delta / \mu_\Delta$

MFV in type-I model? The simplest realization

B. Gavela, TH, D. Hernandez and P. Hernandez, JHEP 09'

→ There exists a particularly minimal and predictive MFV type-I seesaw model!

A model with 2 right-handed neutrinos: $L_{N_1} = +1\,,\ L_{N_2} = -1$

$$\begin{array}{c|ccc} & \nu_L & N_1 & N_2 \\ \hline \nu_L & 0 & Y_N \frac{v}{\sqrt{2}} & 0 \\ N_1 & Y_N \frac{v}{\sqrt{2}} & 0 & M_N \\ N_2 & 0 & M_N & 0 \end{array} \Longrightarrow \quad c_{d=6} = Y_N^\dagger \frac{1}{M_N^2} Y_N$$

$$\begin{array}{c|ccc} & \nu_L & N_1 & N_2 \\ \hline \nu_L & 0 & Y_N \frac{v}{\sqrt{2}} & Y_N' \frac{v}{\sqrt{2}} \\ N_1 & Y_N \frac{v}{\sqrt{2}} & \mu_1 & M_N \\ N_2 & Y_N' \frac{v}{\sqrt{2}} & M_N & \mu_2 \end{array}$$

hard L breaking (Y_N')

soft L breaking (μ_2)

$$\Longrightarrow \quad \begin{aligned} c_{d=5} &= -\frac{m_\nu}{v^2} \\ &= Y_N^T \frac{\mu_2}{M_N^2} Y_N \\ &\quad + ({Y_N'}^T \frac{1}{M_N} Y_N + Y_N^T \frac{1}{M_N} Y_N') \end{aligned}$$

→ separation of scales: $\Lambda_F \sim M_N \longleftrightarrow \Lambda_{LN} \sim M_N^2/\mu_2\,,\ M_N/Y'$

The simplest MFV type-I model

counting of parameters in the pure hard case:

$$\begin{array}{llll} M_N & \rightarrow 1 \text{ real} + 1 \text{ phase} & \rightarrow \ 1 \text{ real} & \leftarrow 1 \text{ normalizat.} \\ Y_N & \rightarrow 3 \text{ real} + 3 \text{ phases} & \rightarrow \ 3 \text{ real} & \leftarrow 1 \text{ normalizat.} + 2 \text{ flavour param.} \\ Y'_N & \rightarrow 3 \text{ real} + 3 \text{ phases} & \rightarrow \ 3 \text{ real} + 2 \text{ phases} & \leftarrow 1 \text{ normalizat.} + 2 \text{ flavour param.} + 2 \text{ phases} \end{array}$$

rephasing $N_1, \ N_2$ and the 3 L_i

to be compared with the $m_{\nu ij}$ matrix from 2 N's:

$$\begin{array}{lll} m_{\nu_i} & \rightarrow 2 \text{ real } \nu \text{ masses} & \leftarrow 1 \text{ normalizat.} + 1 \text{ flavour param.} \\ \theta_{ij} & \rightarrow 3 \text{ real mixing angles} & \leftarrow 3 \text{ flavour param.} \\ \delta\,,\alpha_1 & \rightarrow 1 \text{ CKM} + 1 \text{ Majorana phase} & \leftarrow 2 \text{ phases} \end{array}$$

⇒ the full flavour structure of the model can be reconstructed from $m_{\nu ij}$!

⇒ the full flavour structure of dim-6 effects can be reconstructed!

this remains true in the full hard + soft case too

Predictions

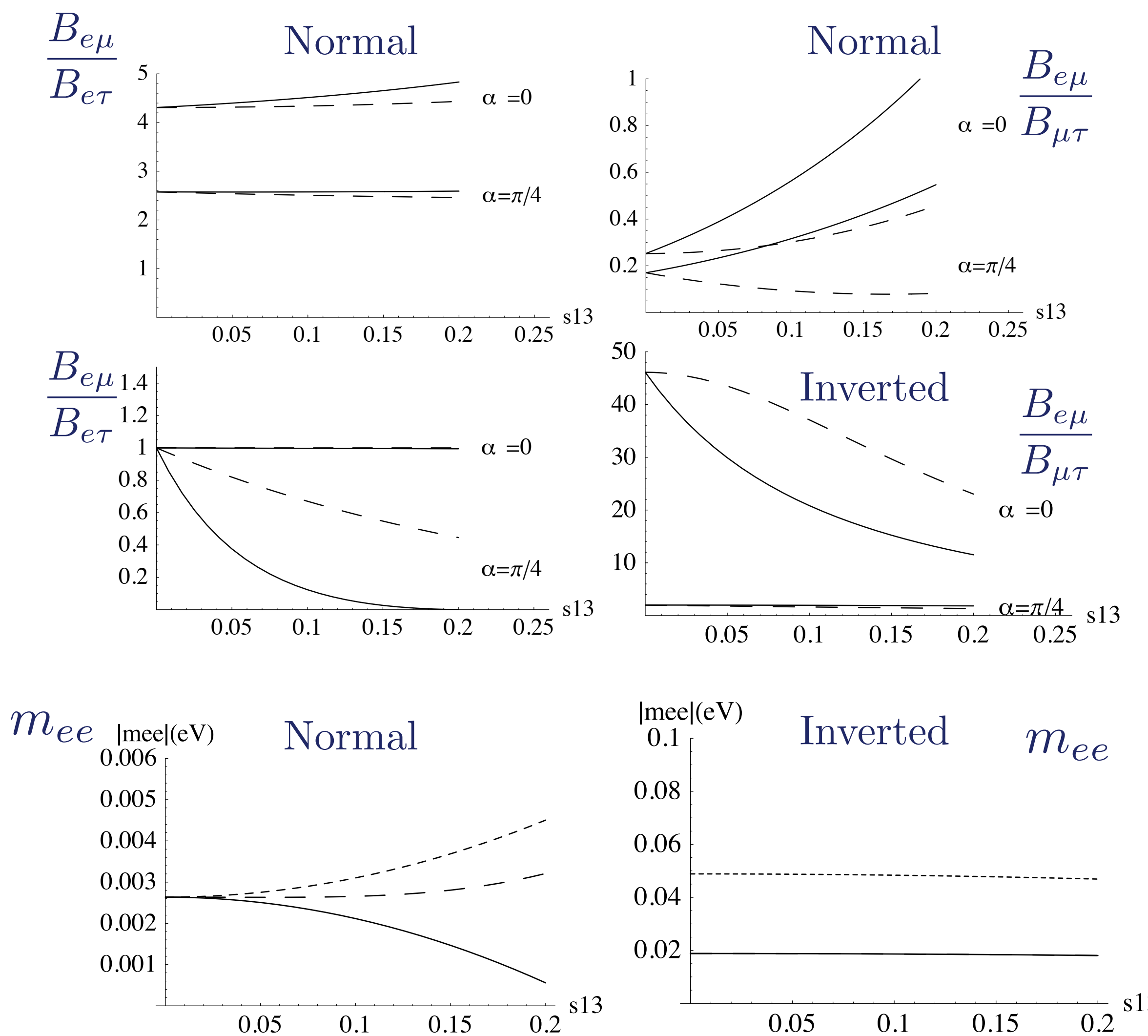
B. Gavela, TH, D. Hernandez and P. Hernandez, JHEP 09'

in terms of the 3 unknown parameters of $m_{\nu ij}$: $\theta_{13},\ \delta,\ \alpha$

$$B_{e\mu} \equiv \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)}$$

$$B_{e\tau} \equiv \frac{\Gamma(\tau \to e\gamma)}{\Gamma(\tau \to e\nu_\tau\bar{\nu}_e)}$$

$$B_{\mu\tau} \equiv \frac{\Gamma(\tau \to \mu\gamma)}{\Gamma(\tau \to \mu\nu_\tau\bar{\nu}_\mu)}$$



Leptogenesis in approximately L conserving seesaw models???

S. Blanchet, TH and F.-X. Josse-Michaux, JCAP 10'

→ at first sight very difficult:

- leptogenesis at low scale: $M_N \sim TeV$
- large Y_N ⇒ $\Gamma_N >>> H|_{T=M_N}$ ⇒ the N are in deep thermal equilibrium
- L broken by a small perturbation ⇒ we would expect suppression of CP-asym.

→ at second sight: leptogenesis appears to be generically
successful in these models

Apparent contradiction in approximate L models

$M_N \sim \mathrm{TeV}$

to saturate $Br(\mu \to e\gamma)_{Exp} < 1.2 \cdot 10^{-11}$ we need $Y_N \sim 10^{-(1-2)}$

$\Downarrow$

$$\frac{\Gamma_N^{\mathrm{TOT}}}{H(T = M_N)} \sim 10^8$$

$\Downarrow$

$N_{1,2}$ are in deep thermal equilibrium

$\Downarrow$

huge η suppression expected: $\eta \sim 10^{-10}$

$\Downarrow$

forget about successful leptogen.

this is what we get from usual Boltzmann equations

Apparent contradiction in approximate L models

$Y_L = (n_l - n_{\bar{l}})/s$

$z \equiv \frac{M_N}{T}$

"Usual" Boltzmann equations

$$\frac{s}{z}\frac{dY_L}{dz} = \varepsilon_N\left(\frac{Y_N}{Y_N^{EQ}} - 1\right)\cdot\frac{\gamma_D}{H(T=M_N)} - \frac{Y_L}{2Y_l^{EQ}}\cdot\frac{\gamma_D}{H(T=M_N)} - 2\,\frac{Y_L}{Y_l^{EQ}}\cdot\frac{\gamma_{\Delta L=2}^{off-shell}}{H(T=M_N)}$$

each decay produces a $\Delta L = \varepsilon_N$

each inverse decay produces a $\Delta L = -\varepsilon_N$

if more l than $\bar{l}$: more $lH \to N$ inverse decays than $\bar{l}H^* \to N$

if more l than $\bar{l}$: more $l\,H \to N \to \bar{l}\,H^*$ processes than $\bar{l}\,H^* \to N \to l\,H$

$$= \varepsilon_N\left(\frac{Y_N}{Y_N^{EQ}} - 1\right)\cdot\frac{\gamma_D}{H(T=M_N)} - 2\,\frac{Y_L}{Y_l^{EQ}}\cdot\frac{\gamma_{\Delta L=2}^{full}}{H(T=M_N)}$$

$$\simeq \varepsilon_N\left(\frac{Y_N}{Y_N^{EQ}} - 1\right)\cdot\frac{\gamma_D}{H(T=M_N)} - \frac{Y_L}{2Y_l^{EQ}}\cdot\frac{\gamma_D}{H(T=M_N)}$$

$$\frac{\gamma_D}{H(T=M_N)} \equiv \underbrace{\frac{\Gamma_N^{TOT}}{H(T=M_N))}}_{\sim 10^8}\frac{K_1(z)}{K_2(z)}\,n_N^{EQ}(z)$$

⇓ huge washout

⇒ main condition to avoid an efficiency suppression: $\Gamma_N^{TOT} < H(T=M_N)$

Apparent contradiction in approximate L models

→ but with $Y_N \sim 10^{-(1-2)}$ suppose $Y'_N = \mu_1 = \mu_2 = 0$

↓ $\frac{\Gamma_N^{\rm TOT}}{H(T = M_N)} \sim 10^8$

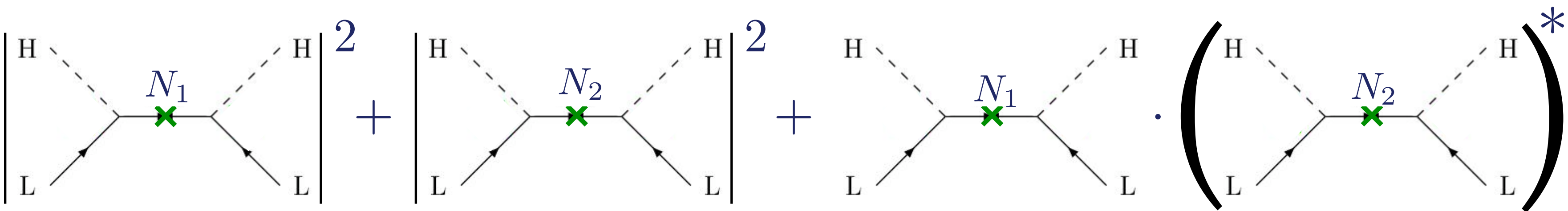
↓ L is conserved!

⇒ how comes a decay could washout a L asymmetry if there is no L violation in the model??

Solution of the apparent contradiction

Blanchet, TH, Josse-Michaux 10‘

proper calculation of the $\Delta L = 2$ scattering contribution



$+h.c. = 0$

if L conser.

on-shell part:
inverse decay of N_1
$\neq 0$ even if L conser.

on-shell part:
inverse decay of N_2
$\neq 0$ even if L conser.

interference term: gives also a
resonance part always omitted
$\neq 0$ even if L conser.

especially if $M_{N_1} \sim M_{N_2}$

⇒ no washout if L is conserved even if the N are deeply in thermal equilibrium!!

⇒ In practice the washout turns out to be controlled by the $N_1 - N_2$ mass splitting:

$$\delta = \frac{M_2 - M_1}{\Gamma^{TOT}_{N_1}} << 1 \rightarrow \gamma^{on-shell}_{\Delta L=2} = \frac{\gamma_D}{4} \cdot \frac{2\delta^2}{1 + \delta\, \Gamma^{TOT}_{N_1}/M_{N_1} + \delta^2} \rightarrow$$ small washout

$$\delta = \frac{M_2 - M_1}{\Gamma^{TOT}_{N_1}} >> 1 \rightarrow \gamma^{on-shell}_{\Delta L=2} = \frac{\gamma_D}{4}$$ → huge washout

usual inverse decay term

Naturalness of small mass splitting in approximate L models

→ automatic in approximate L models!

$$\begin{array}{c} \\ \nu_L \\ N_1 \\ N_2 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_L & N_1 & N_2 \end{array} \\ \begin{pmatrix} 0 & Y_N \frac{v}{\sqrt{2}} & Y'_N \frac{v}{\sqrt{2}} \\ Y_N \frac{v}{\sqrt{2}} & \mu_1 & M_N \\ 0 & M_N & \mu_2 \end{pmatrix} \end{array} \Rightarrow \delta = \frac{M_2 - M_1}{\Gamma_{N_1}^{\rm TOT}} = \frac{\mu}{\Gamma_{N_1}^{\rm TOT}}$$

$$\mu_1 + \mu_2 e^{i\alpha} \equiv \mu e^{i\phi}$$

↓

$\propto$ small L violating perturbations

↓

protected by L symmetry

CP-asymmetry in approximate L models

Blanchet, TH, Josse-Michaux 10'

for $M_N \sim \text{TeV}$ a large asymmetry can be obtained only through resonance

⇓

the condition to have a resonance of the CP asymmetry is the same as to avoid washout: a small mass splitting

⇓

in the approximate L setup not only the numerator of the CP asymmetry is suppressed by the small L violating entries but also the denominator

$$\epsilon_{1\alpha} = \epsilon_{2\alpha} \simeq -\frac{|Y_\alpha|^2}{4\pi}\left(\sin\alpha\frac{\mu_1\mu_2}{2\mu M} + \frac{\sum_\beta \mathrm{Im}(Y_\beta Y_\beta'^{*} \mathrm{e}^{\mathrm{i}\phi})}{\sum_{\beta'}|Y_{\beta'}|^2}\right) f_{\rm self}$$

$$f_{\rm self} = \frac{a_2 - a_1}{(a_2-a_1)^2 + \left(\sqrt{a_2 c_2} - \sqrt{a_1 c_1}\right)^2} \overset{\delta \ll 1}{\simeq} \frac{1}{2\delta\sqrt{c}}$$

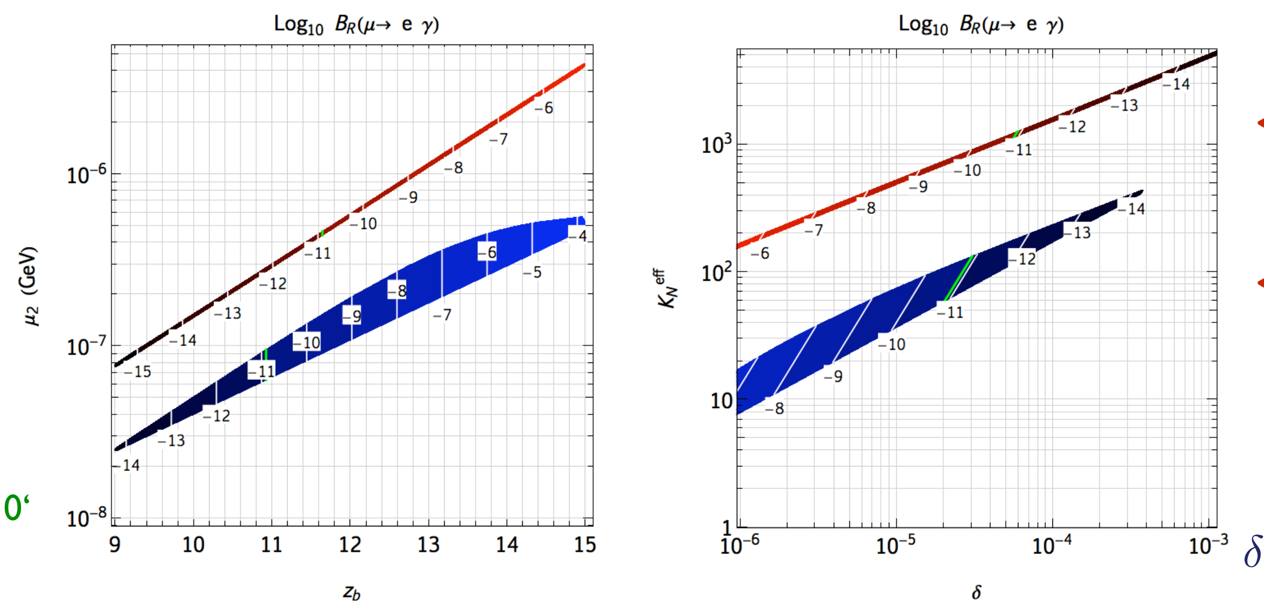
$a_i \equiv (M_{N_i}/M_{N_1})^2$

$c_i \equiv (\Gamma^{\rm TOT}_{N_i}/M_{N_1})^2$

⇒ despite that the CP-asymmetry is suppressed by the small L-violating entries one gets a large enough CP-asym if: $2\,\mathrm{Re}(Y_N Y_N')/|Y^2| << \delta \equiv \frac{M_2 - M_1}{\Gamma^{\rm TOT}_{N_1}} << 1$

Summing up: results on $\mu \to e\gamma$ imposing successful leptogenesis

- approximately L conserving seesaw models can lead to large flavour violation in agreement with small m_ν
- successful leptogenesis can be generically obtained:
  - a large washout of the L asymmetry can be avoided despite the N are in deep thermal equilibrium: requires a small enough mass splitting
  - a small mass splitting is a prediction of the model
  - a large enough CP-asymmetry is obtained through resonance from the same small mass splitting



normal hierarchy

inverted hierarchy

$M_N = 250\,\text{GeV}$

$\alpha = 0$

$\theta_{13} = 10^\circ$

Blanchet, TH, Josse-Michaux 10'

$\Rightarrow$ an observable $\mu \to e\gamma$ process is compatible with successful leptogenesis (without SUSY)

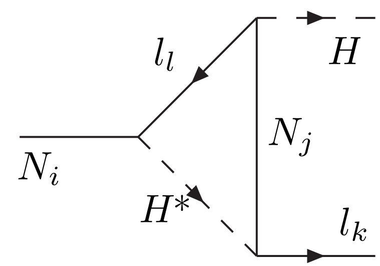
Backup

The 3 leptogenesis ingredients

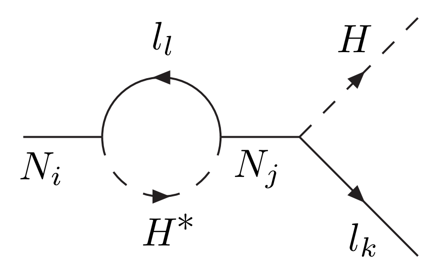
- 1) The CP-asymmetry: $\leftarrow$ averaged ΔL produced per N decay

$$\varepsilon_{N_i} = \sum_k \frac{\Gamma(N_i \to L_k H) - \Gamma(N_i \to \bar{L}_k H^*)}{\Gamma^{\rm TOT}_{N_i}}$$

$\Rightarrow$ CP-violation from 2 one-loop diagrams:



vertex diagram



self-energy diagram

$$\Rightarrow \quad \varepsilon_{N_i} = \frac{1}{8\pi}\sum_j \frac{\sum_{jl}\mathrm{Im}[Y_{Nik}Y^\dagger_{Nkj}Y_{Nil}Y^\dagger_{Nlj}]}{\sum_k |Y_{Nik}|^2}\frac{M_{Nj}}{M_{Ni}}\cdot\left[1-\left(1+\frac{M^2_{Nj}}{M^2_{N_i}}\right)\log\left(1+\frac{M^2_{Ni}}{M^2_{N_j}}\right) + \frac{M^2_{N_i}(M^2_{N_i}-M^2_{Nj})}{(M^2_{N_i}-M^2_{Nj})^2+\Gamma^2_{N_j}M^2_{N_i}}\right]$$

Vertex term: can be effective only for $M_N \gtrsim 10^8\,\mathrm{GeV}$

Self-energy term: can be effective for $M_N \sim 1\,\mathrm{TeV}$ if $M_{N_2} \sim M_{N_1}$ (resonance)

The 3 leptogenesis ingredients

- 2) The efficiency η: $\dfrac{n_L}{s} = \varepsilon_{N_i} \cdot \left(\dfrac{n_{N_i}}{s}\right)\Big|_{T>>M_{N_i}} \cdot \eta$

$\eta \sim 1 \quad \leftarrow \quad \text{out-of-equilibrium}$

$\eta << 1 \quad \leftarrow \quad \text{thermal equilibrium}$

can be obtained integrating the Boltzmann equations:

$Y_N = n_N/s$

$Y_L = (n_l - n_{\bar{l}})/s$

$z \equiv \dfrac{M_N}{T}$

$$\frac{s}{z}\frac{dY_N}{dz} = \left(1 - \frac{Y_N}{Y_N^{EQ}}\right) \cdot \frac{\gamma_D}{H(T=M_N)}$$

$$\frac{\gamma_D}{H(T=M_N)} \equiv \frac{\Gamma_N^{\rm TOT}}{H(T=M_N))}\frac{K_1(z)}{K_2(z)} n_N^{EQ}(z)$$

$$\frac{s}{z}\frac{dY_L}{dz} = \varepsilon_N \cdot \left(\frac{Y_N}{Y_N^{EQ}} - 1\right) \cdot \frac{\gamma_D}{H(T=M_N)} - 2\,\frac{Y_L}{Y_l^{EQ}} \cdot \frac{\gamma_{\Delta L=2}}{H(T=M_N)}$$

each decay produces a $\Delta L = \varepsilon_N$

each inverse decay produces a $\Delta L = -\varepsilon_N$

if more l than $\bar{l}$: more $l\,H \to N \to \bar{l}\,H^*$ processes than $\bar{l}\,H^* \to N \to l\,H$

⇒ main condition to avoid an efficiency suppression: $\Gamma_N^{\rm TOT} < H(T=M_N)$

The 3 leptogenesis ingredients

- 3) The L to B conversion from SM sphalerons:

→ above the EW scale B+L violating but B-L conserving SM sphalerons are in thermal equilibrium

⇒ put B+L to ~ 0 but conserving B-L:

$$\left.\begin{array}{rcl} (B+L)_{Fin} & \sim & 0 \\ (B-L)_{Fin} & = & (B-L)_{In} \\ B_{In} & = & 0 \end{array}\right\} \Rightarrow B_{Fin} \sim -L_{Fin} \sim -\frac{L_{In}}{2}$$

⇓

$$\frac{n_B}{s} \simeq -\frac{1}{2}\frac{n_L}{s} = -\frac{1}{2}\,\eta\,\varepsilon_{N_i}\,\frac{n_{N_i}}{s}\Big|_{T>>M_{N_i}}$$

↕

$$\frac{n_B}{s} = (8.82 \pm 0.23)\cdot 10^{-11}$$ WMAP

3 nus +3 N DFV case

$$(\nu_e, \nu_\mu, \nu_\tau, N_1, N_2, N_3)$$

$$\begin{pmatrix} 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & e & 0 & 0 \\ c & d & e & 0 & 0 & a \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & a & 0 & 0 \end{pmatrix}$$

Details on delta L=2 scattering calculation:

$$\gamma^{\rm on}_{\Delta L=2,\alpha} = \frac{\gamma^D_{N,\alpha}}{4}\cdot 2\left(1+2\delta\sqrt{c}-\frac{1+3\delta\sqrt{c}}{1+\delta\sqrt{c}+\delta^2}+\mathcal{O}(Y'^2,\mu^2)\right) = \frac{\gamma^D_{N,\alpha}}{4}\cdot\frac{2\,\delta^2}{1+\sqrt{c}\delta+\delta^2}$$

$$K^{\rm eff}_\alpha \equiv K_\alpha\cdot\frac{\delta^2}{1+\sqrt{c}\delta+\delta^2} \overset{\delta\ll 1}{\simeq} K_\alpha\cdot\delta^2$$

$$\mathcal{L} \;=\; {\rm i}\overline{N_i}\,\not\partial N_i - \left(h_{i\alpha}\,\overline{N_i}\tilde{\phi}^\dagger\,\ell_{L\alpha}+\frac{1}{2}M_i\overline{N_i}N_i^c + h.c.\right), \quad (i=1,2;\ \alpha=e,\mu,\tau)$$

$$M_{1,2}\simeq M\mp\frac{1}{2}\mu \qquad\qquad \mu_1+\mu_2{\rm e}^{{\rm i}\alpha}\equiv\mu{\rm e}^{{\rm i}\phi}$$

$$h_{1\alpha} \;\simeq\; \frac{\rm i}{\sqrt{2}}{\rm e}^{-{\rm i}(\phi-\lambda)/2}\left[\left(1+\frac{\mu_2^2-\mu_1^2}{4M\mu}\right){\rm e}^{{\rm i}\phi}Y_\alpha - Y'_\alpha\right] \qquad \lambda=\sin\alpha\frac{\mu_1\mu_2}{\mu M}$$

$$h_{2\alpha} \;\simeq\; \frac{1}{\sqrt{2}}{\rm e}^{-{\rm i}(\phi+\lambda)/2}\left[\left(1-\frac{\mu_2^2-\mu_1^2}{4M\mu}\right){\rm e}^{{\rm i}\phi}Y_\alpha + Y'_\alpha\right]$$