CERN, 17 September '10

Comments on TB vs Anarchy

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- Different models can accommodate the data on ν mixing

The main question is

- is TB mixing accidental or a hint?

Anarchy
Lopsided models
$U(1)_{FN}$,
••••••

discrete groups

Value of θ_{13} important for deciding

no supporting evidence from quarks

TB

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

A coincidence or a hint?

TB mixing agrees with data at ~ 1σ

At 1σ:

Schwetz et al '10

$\sin^2\theta_{12} = 1/3$: 0.30-0.34

$\sin^2\theta_{23} = 1/2$: 0.44-0.57

$\sin^2\theta_{13} = 0$: $< \sim 0.02$

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

LQC: Lepton Quark Complementarity

$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

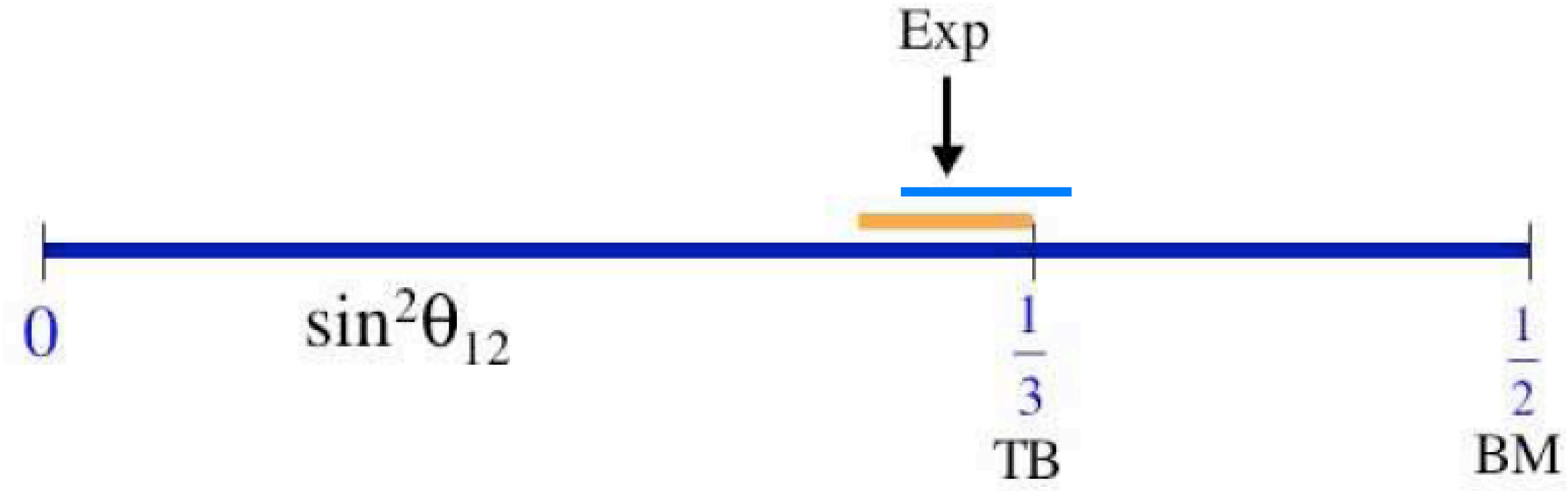
θ_C: Cabibbo angle

Raidal'04

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

A coincidence or a hint?



TB Mixing naturally leads to discrete flavour groups

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

This is a particular rotation matrix with specified fixed angles

A recent review: GA, F. Feruglio, ArXiv:1002.0211 (Review of Modern Physics, in press)

Predictions on the ν spectrum

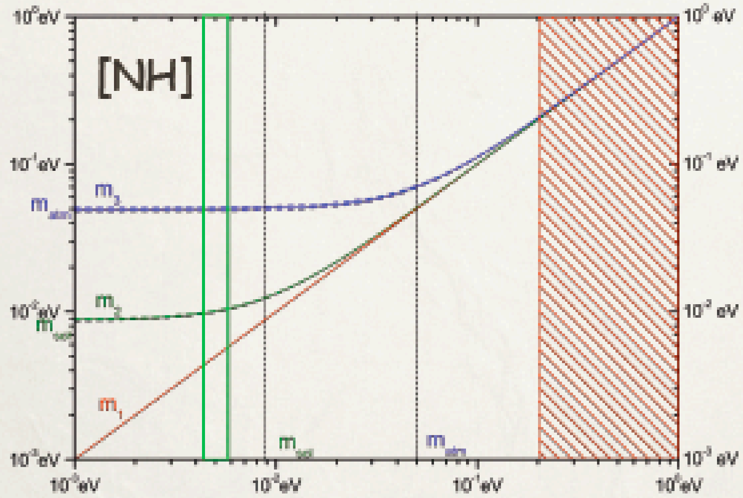
An example based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SUSY + SEE-SAW]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
at the LO neutrino mass spectrum depends on two complex parameters
there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

Feruglio, ICHEP'10

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

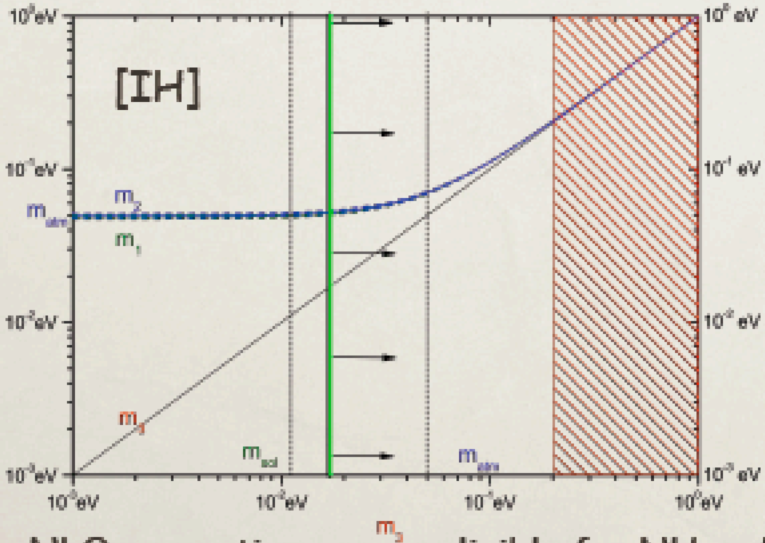
both normal [NH] and inverted [IH] hierarchy are allowed



in the NH case the sum rule completely determines the spectrum

$$m_1 \approx 0.005\ eV \quad m_2 \approx 0.01\ eV \quad m_3 \approx 0.05\ eV$$

$$|m_{ee}| \approx 0.007\ eV$$



in the IH case the sum rule provides a lower bound on m_3

$$m_3 \geq 0.017\ eV$$

$$|m_{ee}| \geq 0.017\ eV$$

NLO corrections are negligible for NH and for IH close to the lower bound

SUSY-SU(5) GUT with A4 and TB

GA, Feruglio, Hagedorn 0802.0090

A satisfactory ~ realistic model

Key ingredients:

- SUSY

  In general SUSY is crucial for hierarchy, coupling unification and p decay
  Specifically it makes simpler to implement the required alignment

- GUT's in 5 dimensions

  In general GUT's in ED are most natural and effective
  Here also contribute to produce fermion hierarchies

- Extended flavour symmetry: $A4 \times U(1) \times Z_3 \times U(1)_R$

  $U(1)_R$ is a standard ingredient of SUSY GUT's in ED

  Hall-Nomura'01

ED effects contribute to the fermion mass hierarchies

A bulk field is related to its zero mode by: $B = \frac{1}{\sqrt{\pi R}} B^0 + \ldots$

This produces a suppression parameter for couplings with bulk fields $s \equiv \frac{1}{\sqrt{\pi R \Lambda}} < 1$

Λ: UV cutoff

- In bulk: N=2 SUSY Yang-Mills fields + $H_5, H_5^{bar} + T_1, T_2, T_1', T_2'$
(doubling of bulk fermions to obtain chiral massless states at y=0)
also crucial to avoid too strict mass relations for 1,2 families:
(b-τ unification only for 3rd family)

- All other fields on brane at y=0 (in particular N, F, T_3)

$$m_u = \begin{pmatrix} s^2t^5t'' + s^2t^2t''^4 & s^2t^4 + s^2tt''^3 & stt''^2 \\ s^2t^4 + s^2tt''^3 & s^2t''^2 & st'' \\ stt''^2 & st'' & 1 \end{pmatrix} sv_u^0 \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

Note: all m of rank 1 in LO: only m_{33} ~o(1)!

dots=0 in 1st approx

$$m_d = \begin{pmatrix} st^3 + st''^3 & \dots & \dots \\ st^2t'' & st & \dots \\ stt''^2 & st'' & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

$$m_e = \begin{pmatrix} st^3 + st''^3 & st^2t'' & stt''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T s v_d^0 \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

with

A4 breaking — $U(1)_{FN}$ breaking

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (v_T, 0, 0) \quad , \quad \frac{\langle \varphi_S \rangle}{\Lambda} = (v_S, v_S, v_S) \quad , \quad \frac{\langle \xi \rangle}{\Lambda} = u \qquad \frac{\langle \theta \rangle}{\Lambda} = t \quad , \quad \frac{\langle \theta'' \rangle}{\Lambda} = t''$$

$s \sim t \sim t'' \sim \lambda \sim 0.22$ $\quad$ $v_T \sim \lambda^2 \sim m_b/m_t$ $\quad$ $v_S,\ u \sim \lambda^2$

Finally:

By taking $s \sim t \sim t'' \sim \lambda \sim 0.22$ $\quad v_T \sim \lambda^2 \sim m_b/m_t$ $\quad v_S, u \sim \lambda^2$

a good description of all quark and lepton masses is obtained. As for all U(1) models only $o(\lambda^p)$ predictions can be given (modulo o(1) coeff.s)

TB mixing for neutrinos is reproduced in first approximation

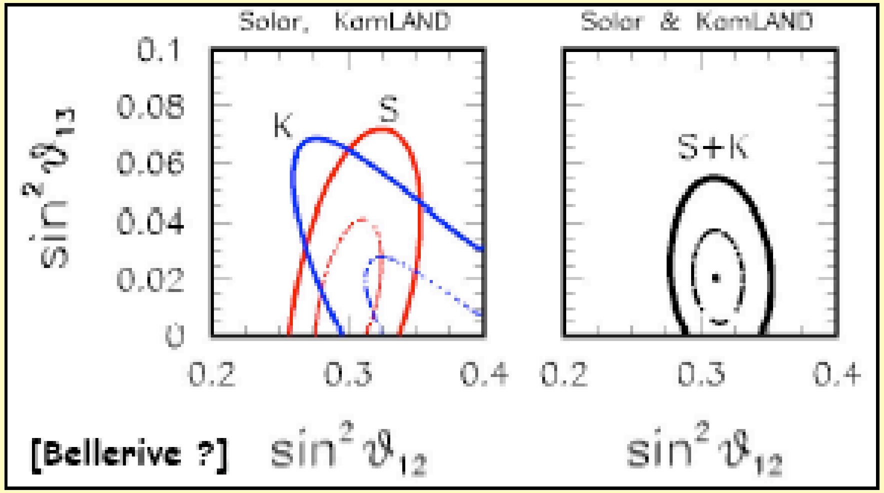
Quark hierarchies force corrections to TB mixing to be $o(\lambda^2)$ (in particular we predict $\theta_{13} \sim o(\lambda^2)$, accessible at T2K).

A moderate fine tuning is needed to fix λ_C and r (nominally of $o(\lambda^2)$ and 1 respectively)

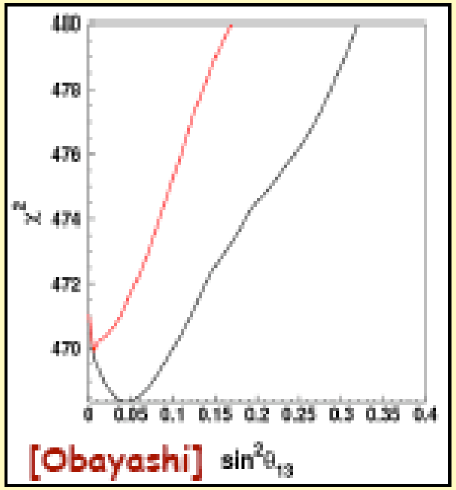
Normal or inverse hierarchy are possible, degenerate ν's are excluded

Hints of $\theta_{13}>0$? [Fogli, EL, Marrone, Palazzo, Rotunno.] **Current status:**

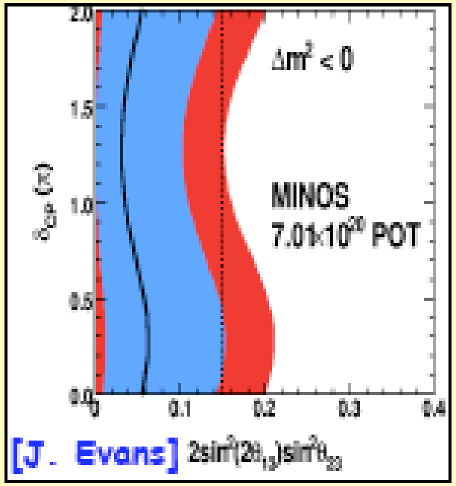
Solar & KamLAND: ~1.5σ



SK atmos.: ~1.5σ



MINOS: ~0.7σ



Overall significance close to ~2σ. Intriguing, but still weak.

Lisi, ICHEP'10

In A4 we typically expect $\theta_{13} \sim o(\lambda_C^2)$

Note: $\lambda_C/3\mathrm{sqrt}(2) \sim 0.05 \sim o(\lambda_C^2)$

King......

If we assume that TB mixing is accidental then an "improved anarchy" is a good alternative

This is a SU(5) GUT with $U(1)_{FN}$ charges

SU(5)xU(1)

G.A., Feruglio, Masina'02

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim \bar{5}\ 10$
$m_{\nu D} \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)

all charges positive

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)

not all charges positive

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

With suitable charge assignments all relevant patterns can be obtained

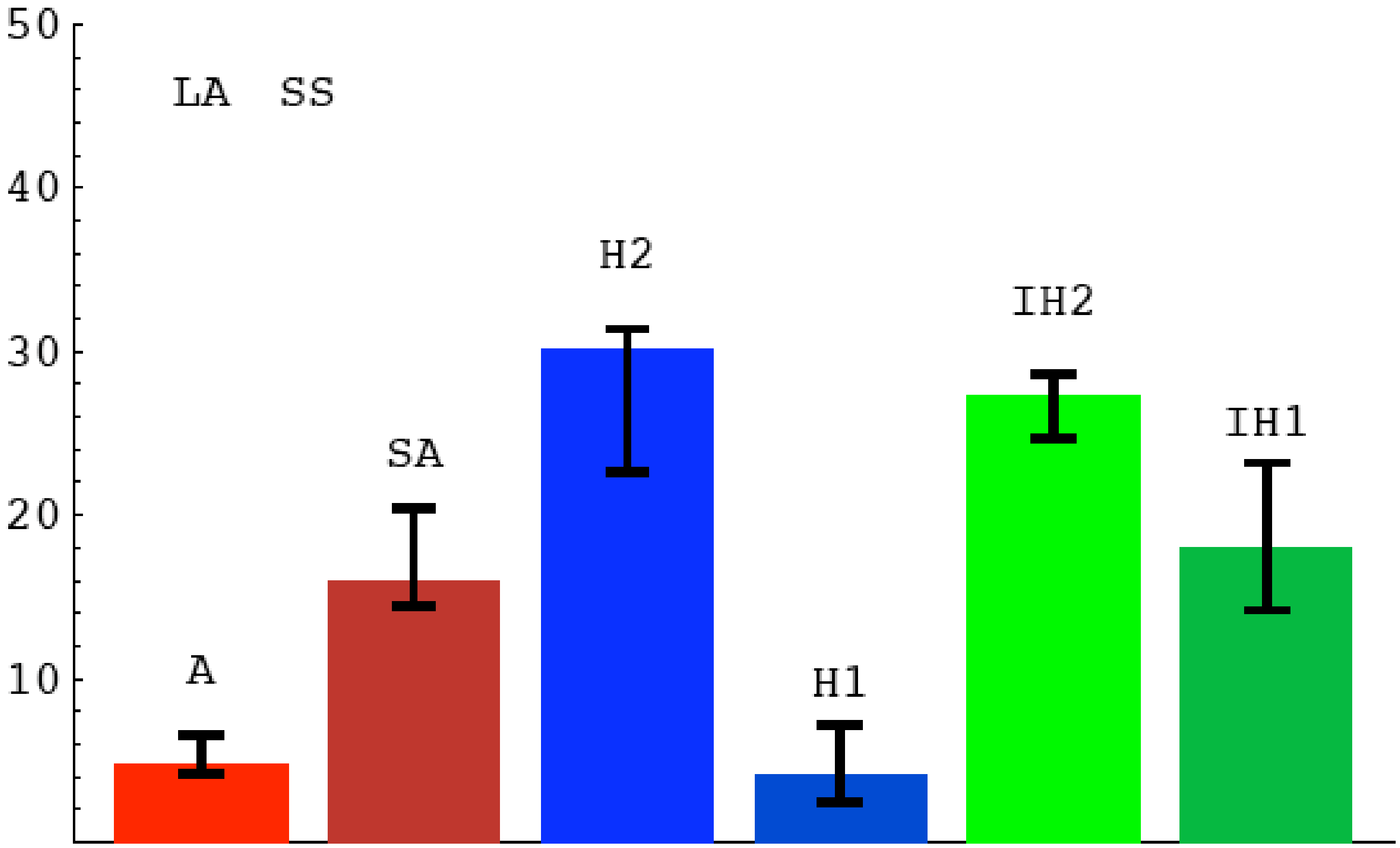


Figure 3: Relative success rates for the LA solution, with see-saw. The sum of the rates has been normalized to 100. The results correspond to the default choice $\mathcal{I} = [0.5, 2]$, and to the following values of $\lambda = \lambda'$: 0.2, 0.3, 0.35, 0.5, 0.15, 0.2 for the models A_{SS}, SA_{SS}, $H_{(SS,II)}$, $H_{(SS,I)}$, $IH_{(SS,II)}$ and $IH_{(SS,I)}$, respectively. The error

Example: Normal Hierarchy

G.A., Feruglio, Masina'02

1st fam. → 2nd → 3rd

q(10): (5, 3, 0)
$q(\bar{5})$: (2, 0, 0)
q(1): (1,-1, 0)

Note: not all charges positive --> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. O(1) omitted, only orders of magnitude predicted with $\lambda \sim \lambda'$

$\bar{5}_i 1_j$ → $m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}$,

$1_i 1_j$ → $M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$

see-saw $m_\nu \sim m_{\nu D}^T M_{RR}^{-1} m_{\nu D}$

$$m_\nu \sim v_u^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

$\mathbf{H}_{(SS,II)}$

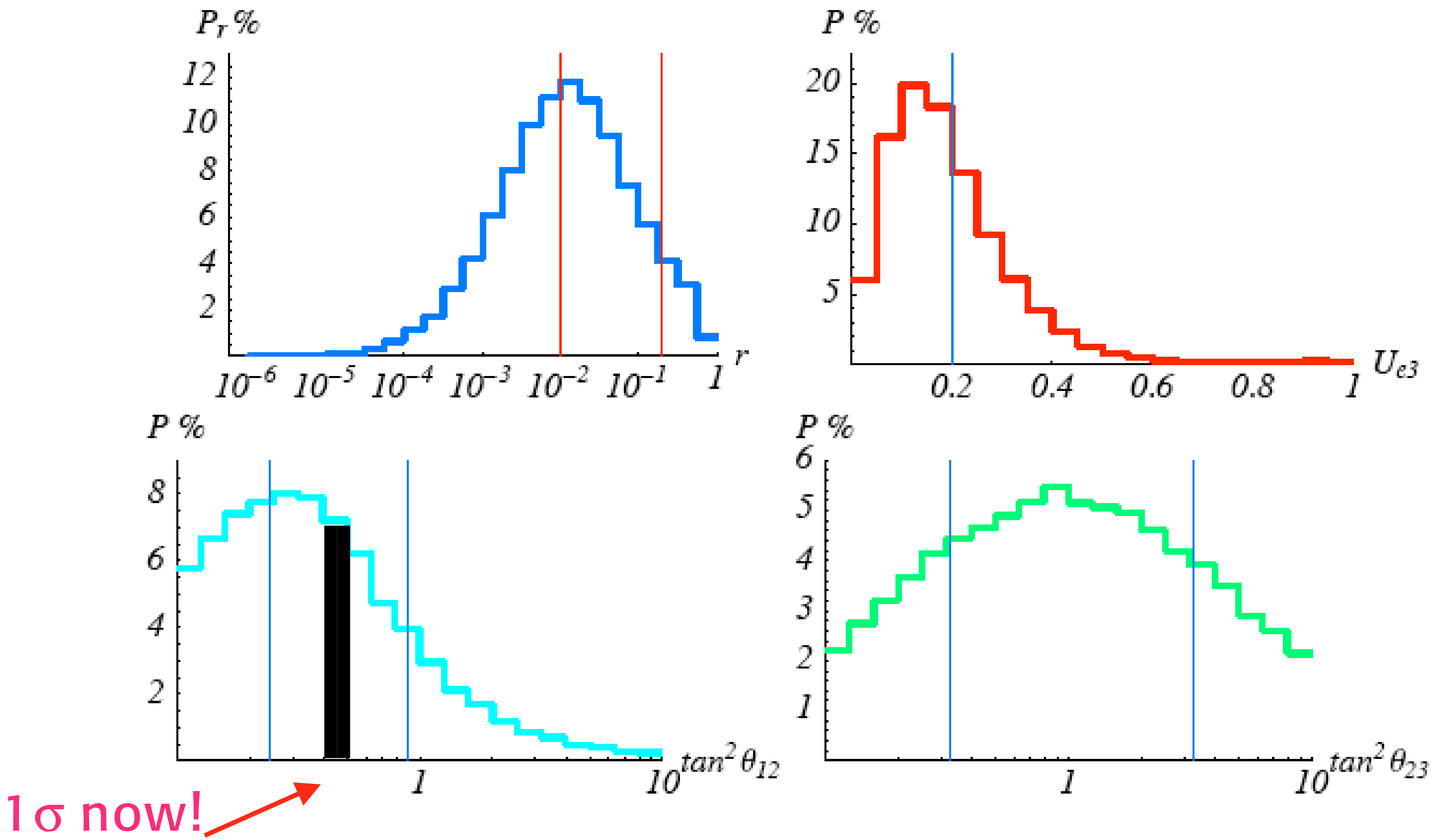


Figure 8: Distributions for $H_{(SS,II)}$, $\mathcal{I} = [0.5, 2]$, $\lambda = \lambda' = 0.35$, obtained with 50000 points $\mathcal{P}$.

- Different models can accommodate the data on ν mixing

The main question is

- is TB mixing accidental or a hint?

Anarchy
Lopsided models
$U(1)_{FN}$,
••••••

discrete groups

Value of θ_{13} important
for deciding

no supporting
evidence from
quarks