Kaons, Lattice QCD and the Standard Model

High Performance Computing in
High Energy Physics

Central China Normal University

September 19, 2018

Norman H. Christ

Columbia University

RBC and UKQCD Collaborations

Outline

- Quick review of the standard model
- Lattice QCD: methods and status
- Four precision tests of the standard model:
  1) $K \to \pi\pi$ decay and direct ~~CP~~: ε'
  2) $K_L - K_S$ mass difference
  3) Long distance contribution to ε_K
  4) Long distance contribution to rare kaon decay: $K^+ \to \pi^+ \nu \bar{\nu}$

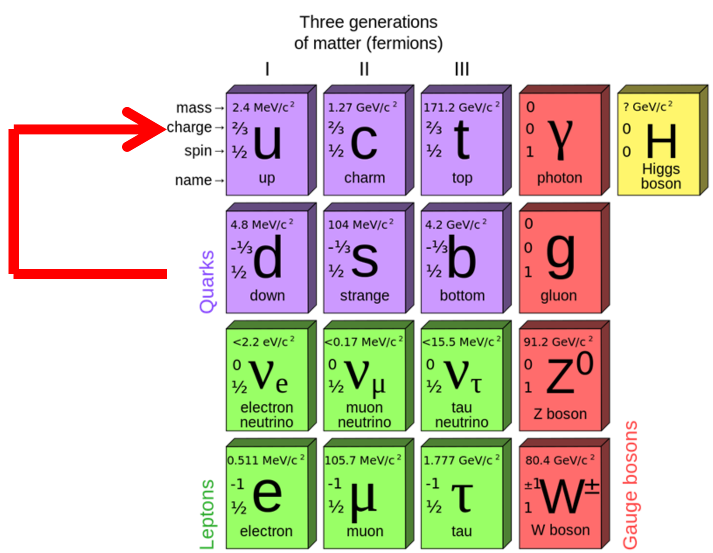
1[st]-order (item 1)

2[nd]-order (items 2–4)

Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$
$$\bar{\rho} = 0.131^{+0.026}_{-0.013}\,, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}\,.$$

Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CP violation!

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

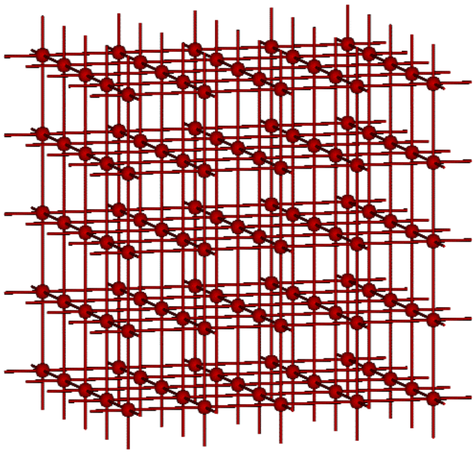
$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$

$$\bar{\rho} = 0.131^{+0.026}_{-0.013}\,, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}\,.$$

State-of-the-art Lattice QCD

Lattice QCD

- Introduce a space-time lattice.
- Evaluate the Euclidean Feynman path integral
  - Study $e^{-H_{QCD}\, t}$
  - Precise non-perturbative formulation
  - Capable of numerical evaluation



$$\sum_n \langle n| e^{-H(T-t)} \mathcal{O} e^{-Ht} |n\rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]} \det(D+m) \mathcal{O}[U](t)$$

- Evaluate using Monte Carlo importance sampling with hybrid, molecular dynamics/Langevin evolution.

Lattice QCD – 2018

- Physical quark masses (ChPT not needed)
- Chiral quarks (doubling problem solved)
- Large physical volumes: (6 -10 fm)^3
- Small lattice spacing: 1/a = 2.4 GeV
  - $(\Lambda_{\mathrm{QCD}}\, a)^2$ effects < 1% ☺
  - $(m_{\mathrm{charm}}\, a)^2$ effects ~ 5-15% ☹

QCD in Euclidean space

- Euclidean $e^{-H_{QCD}\,t}$ projects onto the ground state.
- Treat two-particle states using Luscher's finite-volume analysis
  - Finite-volume energy shifts determine scattering phase shifts.
  - Must work below multi-particle thresholds
  - Two-particle state of interest may not be the lowest energy state
- Extra problems for second-order weak calculations

Lattice QCD

$$\sum_n \langle n | e^{-H(T-t)} \mathcal{O} e^{-Ht} | n \rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]} \det(D+m) \mathcal{O}[U](t)$$

- Large computational challenge:
  - For a 64^3 x 128 lattice: Integrate over one billion variables
  - Spin-1/2 quarks are represented as 4-D states on the boundary of a 5-D volume.
  - Integrand contains the determinant of a (10 Billion) x (10 Billion) matrix

- Fast code running on 32K nodes of Mira sustains one Petaflops [10^{15} (adds + mults)/sec]
- Broad collaboration and substantial resources needed.

Support for Lattice QCD in the US

- Leadership class machines
  - Separate DOE and NSF installations
  - Allocated based on science promise
  - Approx 10% devoted to Lattice QCD
  - Computer centers are government-funded to solve critical science problems.
  - They are not supported by collecting money for computer cycles.
- USQCD is a federation of nearly all LQCD researchers in the US.
  - Plan, advocate and execute a national program
  - Exploit dedicated capacity resources at BNL, JLab and FNAL for smaller projects. ($2.5M/year from DOE.)

Mira, 2012, 10 Pflops

Summit, 2018, 200 Pflops

The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

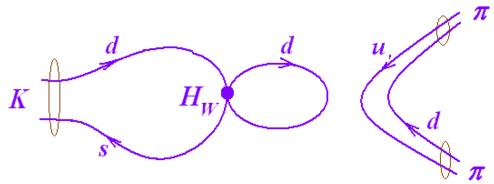
York University (Toronto)

Renwick Hudspith

Precision tests of the Standard Model

Direct CP violation $K \rightarrow \pi\pi$

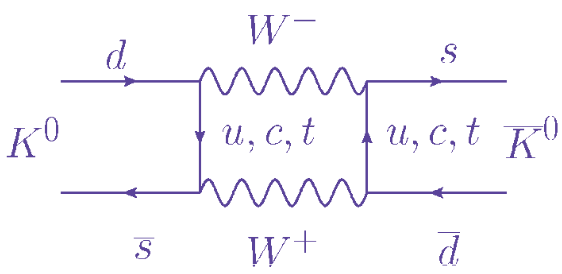
$|\varepsilon'| = 3.70(53) \times 10^{-6}$



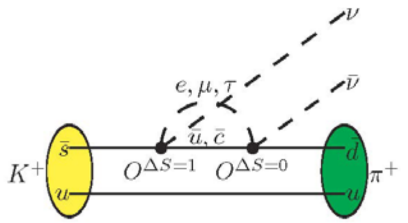
Indirect CP violation $K \rightarrow \pi\pi$

$|\varepsilon| = 0.002228\ (11)$

$m_{K_L} - m_{K_S} = 3.19(41)(96) \times 10^{-12}$ MeV



$K^+ \rightarrow \pi^+ \nu \overline{\nu}$, BR=$1.73\ ^{+1.15}_{-1.05} \times 10^{-10}$



$K \to \pi\,\pi$ Decay

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi\pi$ state.
- Resulting mixing described by Wigner-Weisskopf

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and $\overline{K^0}$

$$|K_S\rangle = \frac{K_+ + \bar{\epsilon}K_-}{\sqrt{1+|\bar{\epsilon}|^2}}$$

$$|K_L\rangle = \frac{K_- + \bar{\epsilon}K_+}{\sqrt{1+|\bar{\epsilon}|^2}}$$

$$\bar{\epsilon} = \frac{i}{2}\left\{ \frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}} \right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon'$$

- Where: $\epsilon = \overline{\epsilon} + i \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

$K \rightarrow \pi\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

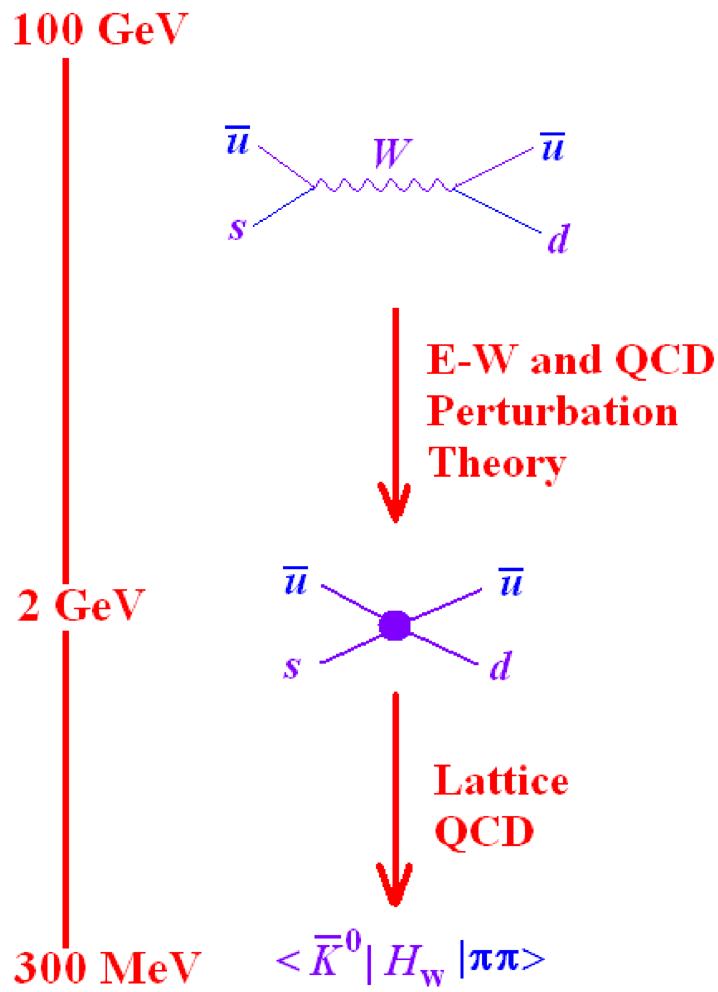
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

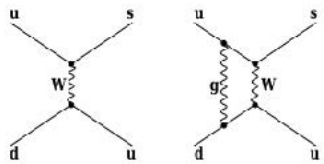
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



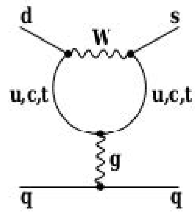
Local four quark operators

- **Current-current operators**



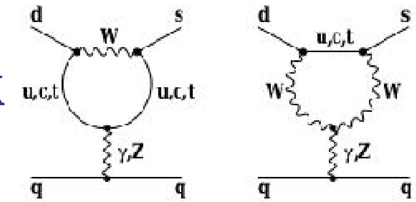
$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

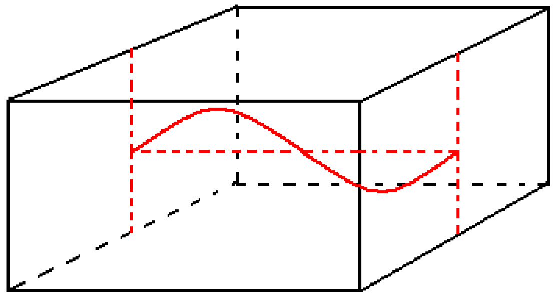
- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Lattice calculation of $<\pi\pi|H_W|K>$

- The operator product $\overline{d}(x)s(x)$ easily creates a kaon.
- Use finite-volume energy quantization (Lellouch-Luscher) and adjust L so that n^{th} excited state obeys: $E_{\pi\pi}^{(n)} = M_K$



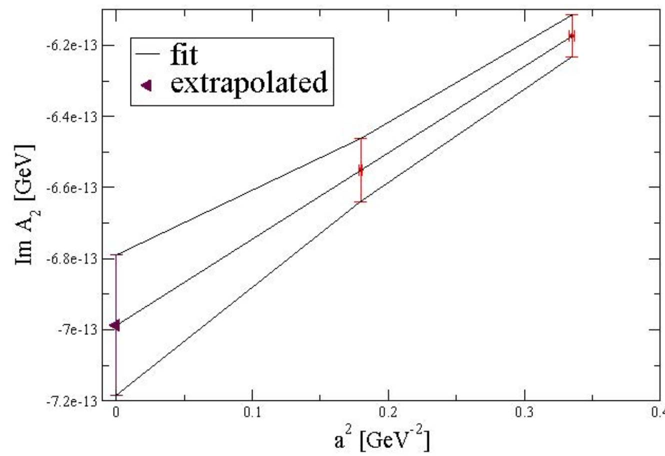
$$\langle \pi^+\pi^- | H_W | K^0 \rangle \quad \propto \quad \langle \overline{d}u(t_{\pi_1})\overline{u}d(t_{\pi_2})\ H_W(t_{\mathrm{op}})\ \overline{d}u(t_K) \rangle$$

- Use boundary conditions on the quarks: $E_{\pi\pi}^{(\mathrm{gnd})} = M_K$
- For $(\pi\pi)_{I=2}$ make d anti-periodic
- For $(\pi\pi)_{I=0}$ use G-parity boundary conditions

Calculation of A_2

ΔI = 3/2 – Continuum Results

(M. Lightman, E. Goode T. Janowski)



- Use two large ensembles to remove a^2 error (m_p=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV
- Continuum results:
  - Re(A_2) = 1.50(0.04_{stat}) $(0.14)_{syst}$×10^{-8} GeV
  - Im(A_2) = - 6.99$(0.20)_{stat}$ $(0.84)_{syst}$×10^{-13} GeV
- Experiment: Re(A_2) = 1.479(4) 10^{-8} GeV
- $E_{\pi\pi}$ → δ_2 = −11.6(2.5)(1.2)^o
- [Phys.Rev. **D91**, 074502 (2015)]

Calculation of A_0 and ε'

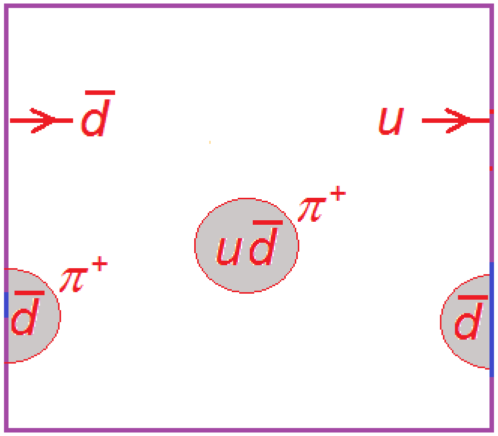
Overview of 2015 calculation

(Chris Kelly and Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - $1/a$ = 1.3784(68) GeV, L = 4.53 fm.
  - G-parity boundary condition in 3 directions
  - 216 configurations separated by 4 time units
- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV

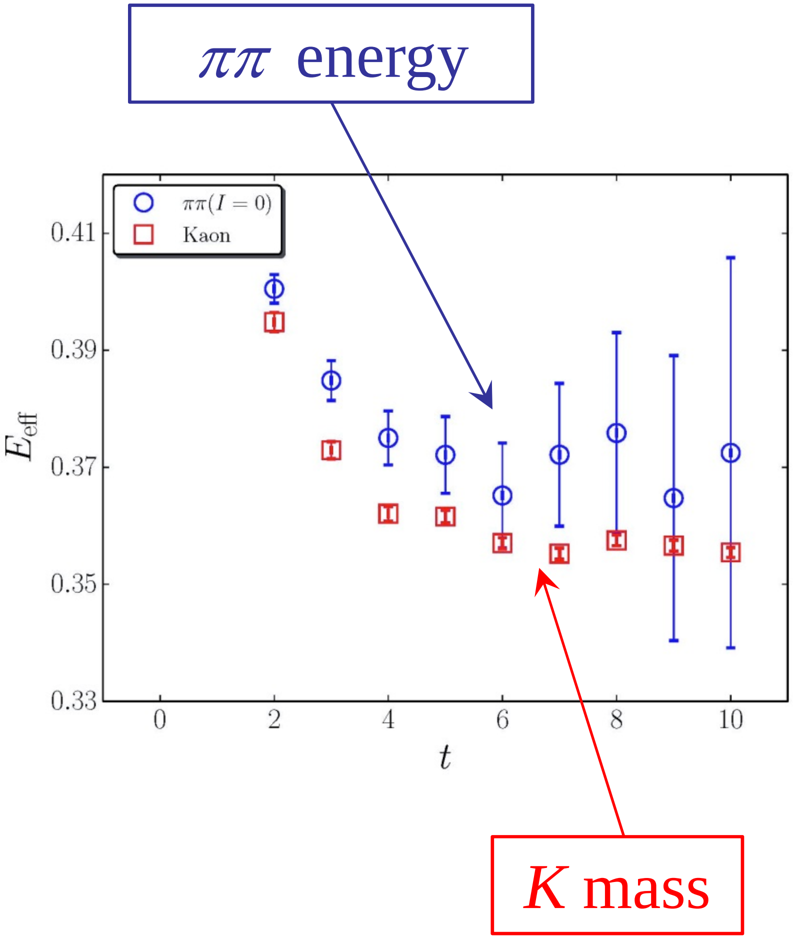
$I = 0$ $K \rightarrow \pi\pi$ with $E_{\pi\pi} = M_K$
(Chris Kelly & Daiqian Zhang)

- Use **G-parity** BC to obtain $p_\pi = 205$ MeV (Changhoan Kim, hep-lat/0210003)
  - $G = C e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$
  - Gauge fields obey C BC
  - Extra $I = 1/2$, s' quark adds $e^{-m_K L}$ error
  - Must take non-local square root of s-s' determinant.
  - Tests: f_K and B_K correct within errors.



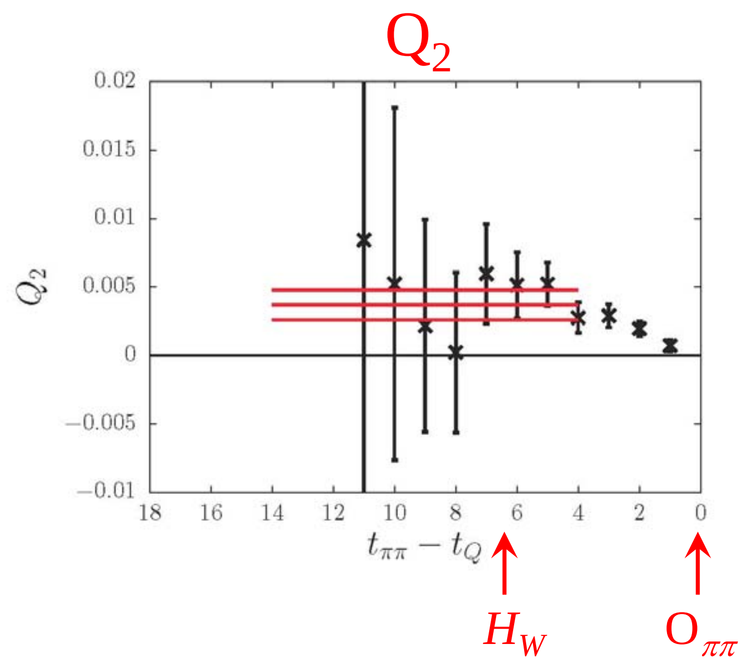
I = 0, $\pi\pi - \pi\pi$ correlator

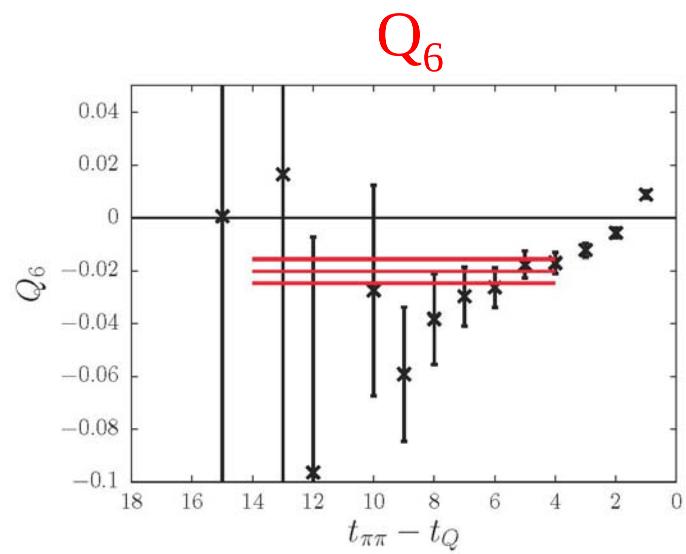
- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, I = 0, $\pi\pi$ state:
  $E_{\pi\pi}$ = 498(11) MeV
- Implies I = 0 $\pi\pi$ phase shift:
  δ_0 = 23.8(4.9)(2.2)^o
- Dispersion theory result:
  δ_0 = ~35^o [G. Colangelo, *et al.*]



$I = 0$ $K \rightarrow \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations $t_Q - t_K \geq 6$ and $t_{\pi\pi} - t_K = 10, 12, 14, 16$ and 18.
- Fit correlators with $t_{\pi\pi} - t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi} - t_Q \geq 3$ or 5





Systematic errors

Description	Error
Operator renormalization	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch-Luscher factor	11%
Finite volume	7%
Parametric errors	5%
Excited states	5%
Unphysical kinematics	3%
Total	27%

Results

Determine the complex ΔI=1/2 amplitude A_0

$\mathrm{Re}(A_0) = (4.66 \pm 1.00_{stat} \pm 1.26_{sys}) \times 10^{-7}$ GeV

Expt: $(3.3201 \pm 0.0018) \times 10^{-7}$ GeV

$\mathrm{Im}(A_0) = (-1.90 \pm 1.23_{stat} \pm 1.08_{sys}) \times 10^{-11}$ GeV

Calculate $\mathrm{Re}(\varepsilon'/\varepsilon)$:

$\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$

Expt.: $(16.6 \pm 2.3) \times 10^{-4}$

2.1 σ difference

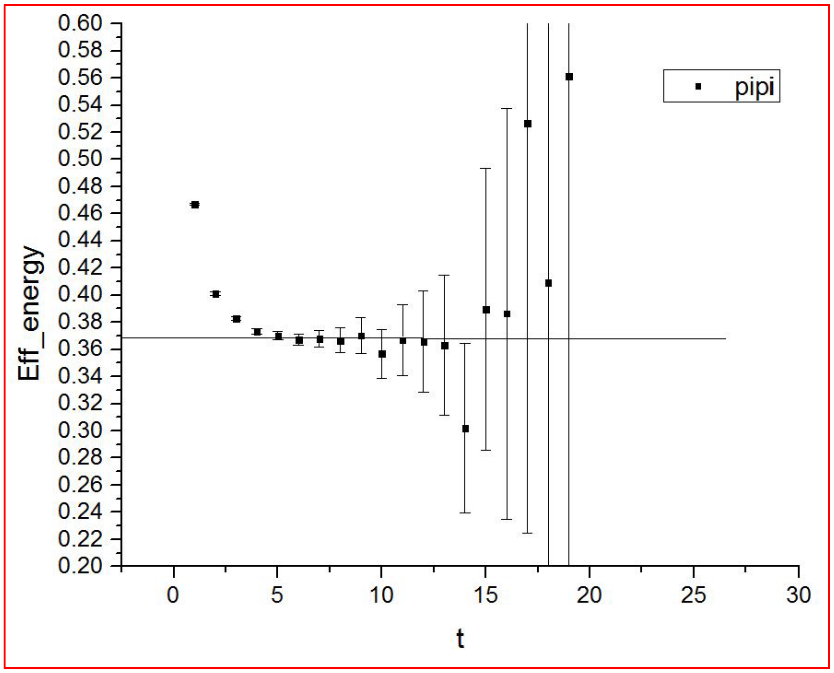
[Phys. Rev. Lett. 115 (2015) 212001]

Extend and improve calculation
(Chris Kelly and Tianle Wang)

- ✓- Increase statistics: 216 → 1400 configs.
  - Reduce statistical errors
  - Allow in depth study of systematic errors
- ✓- Study operators neglected in our NPR implementation
- ✓- Use step-scaling to allow perturbative matching at a higher energy
- Use an expanded set of $\pi\pi$ operators
- Use X-space NPR to cross charm threshold (Masaaki Tomii).

Adding more statistics

- Increasing statistics: 216 → 1400 configs.
  - $\pi\pi - \pi\pi$ correlator well-described by a single $\pi\pi$ state
  - δ_0 = 23.8(4.9)(2.2)° → 19.1(2.5)(1.2)° ?? χ^2 / DoF = 1.6



Adding more $\pi\pi$ operators

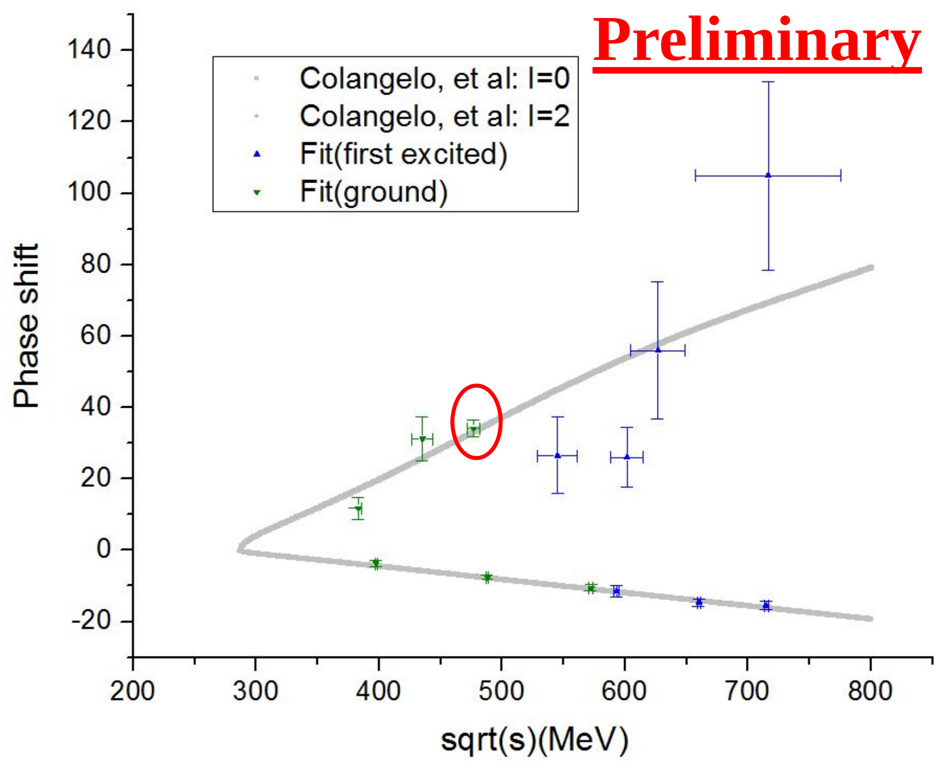
- Adding a second σ-like ($\bar{u}u+\bar{d}d$) operator reveals a second state!
- If only one state, 2 x 2 correlator matrix will have determinant 0. For $t_f - t_i = 5$:

$$\det \begin{pmatrix} \langle \pi\pi(t_f)\pi\pi(t_i)\rangle & \langle \pi\pi(t_f)\sigma(t_i)\rangle \\ \langle \sigma(t_f)\pi\pi(t_i)\rangle & \langle \sigma(t_f)\sigma(t_i)\rangle \end{pmatrix} = 0.439(50)$$

	E0(MeV)	δ	amplitude
sim-fit(5-10)	483.1(1.4)(2.7)	30.9(1.5)(3.0)	11.86(11)(12)
GEVP(6,3)	475.6(2.6)(2.7)	32.8(1.2)(3.0)	11.52(15)(12)
Dispersion	474.6	35.0	

I=0 $\pi\pi$ Scattering

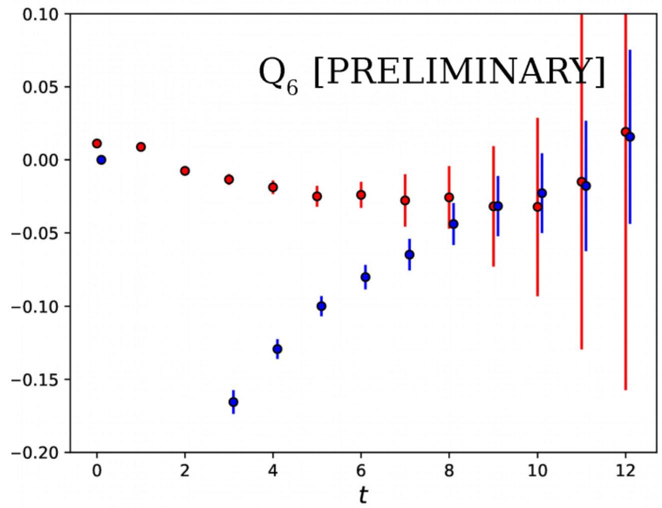
- Combine moving pions to create states with $p_{cm} \neq 0$, giving more values for E_{cm} :



Extend and improve calculation

(Chris Kelly and Tianle Wang)

- Extend $\pi\pi \rightarrow \pi\pi$ and $K \rightarrow \pi\pi$ calculation:
  - Use three operators: $(\pi\pi)_{\text{rel=235}}$, σ, $(\pi\pi)_{\text{rel=449}}$
  - Currently 134 configurations
- Improve the calculation of re(A_0) and ε'
- Expect results by the end of the year.

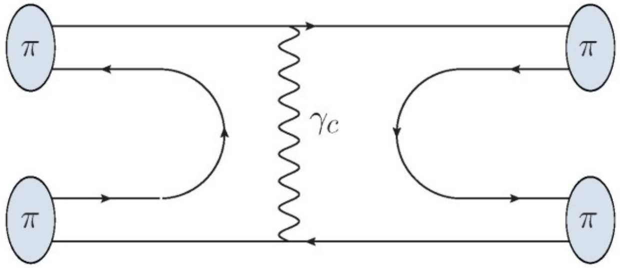


- $K \rightarrow \pi\pi$
- $K \rightarrow \sigma$

Add E&M corrections

(Xu Feng)

- Avoid QED_L, instead use:
  - Use $$V_T(r) = \begin{cases} \frac{e^2}{r} & r \le R_T \\ 0 & r > R_T \end{cases}$$
  - Choose $R_{strong} < R_T < L/2$



- Standard two-channel, finite-volume quantization can be employed.
- Missing long-distance effects, including $\eta \ln(2kr)$ term, cancel in the ratios η_{+-} , η_{00} and ε'

$$\eta_{+-} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_S\rangle} \qquad \eta_{00} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_S\rangle}$$

$$\epsilon' = \frac{1}{3}(\eta_{+-} - \eta_{00}) = \frac{\sin 2\theta}{\sin 2\theta^{\gamma}} \frac{i e^{i(\delta_2^{\gamma} - \delta_0^{\gamma})}}{\sqrt{2}} \frac{\text{Re}A_2^{\gamma}}{\text{Re}A_0^{\gamma}} \left(\frac{\text{Im}A_2^{\gamma}}{\text{Re}A_2^{\gamma}} - \frac{\text{Im}A_0^{\gamma}}{\text{Re}A_0^{\gamma}} \right)$$

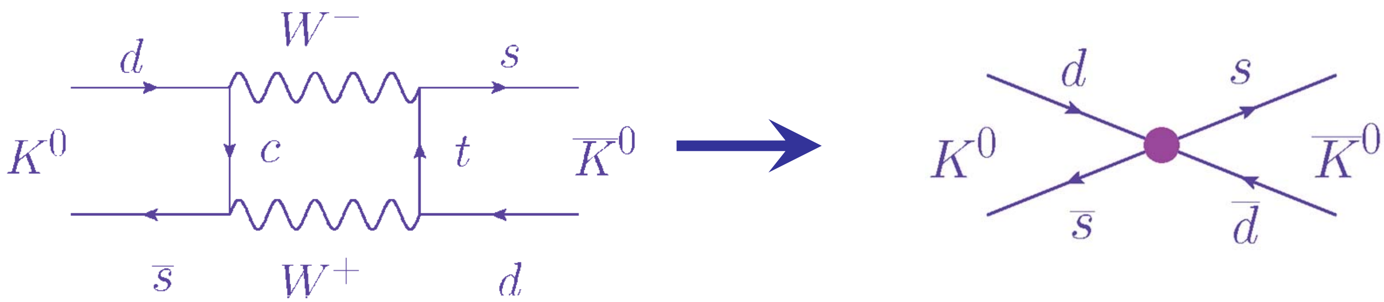
$K^0 - \overline{K^0}$ mixing

ΔM_K & ε_K

$K^0 - \overline{K^0}$ Mixing

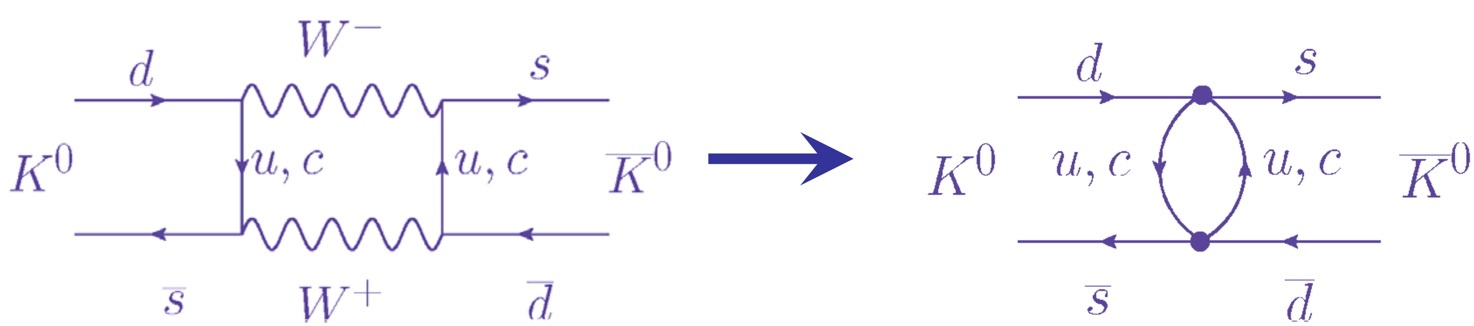
- CP violating: $p \sim m_t$

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re}M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$



- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



$K^0 - \overline{K^0}$ Mixing

- ΔS=1 weak decay allows K^0 and $\overline{K^0}$ to decay to the same $\pi - \pi$ state
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where

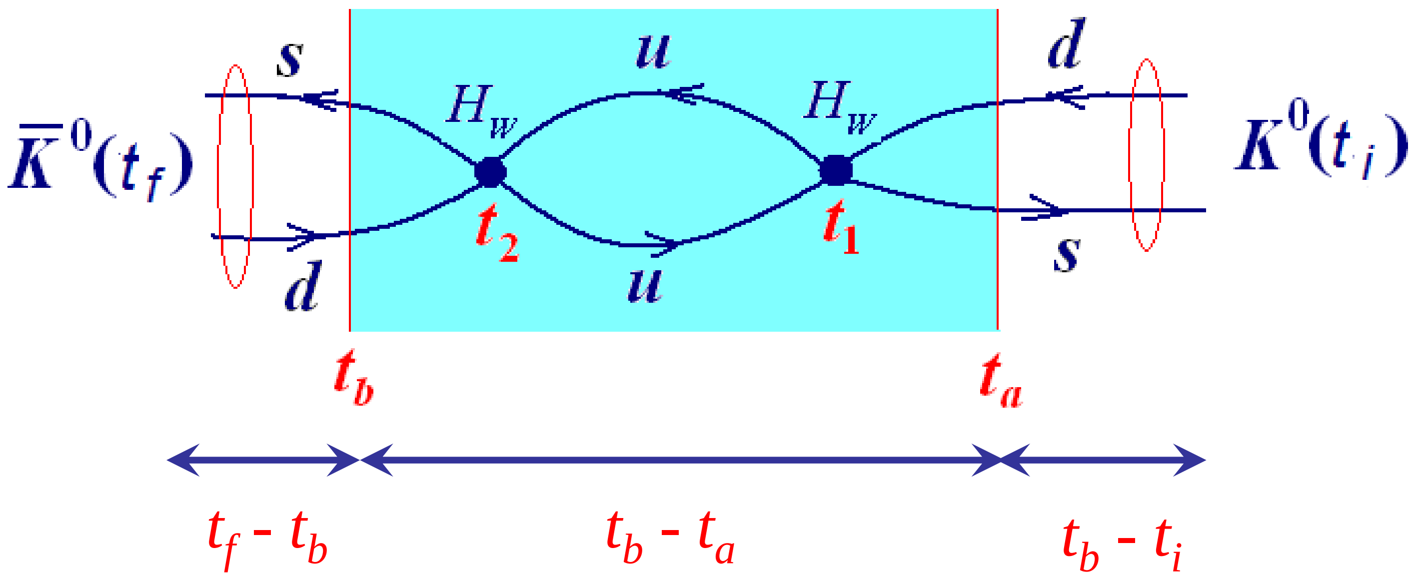
$$\Gamma_{ij} = 2\pi \sum_\alpha \int_{2m_\pi}^{\infty} dE \langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle \delta(E - m_K)$$

$$M_{ij} = \sum_\alpha \mathcal{P} \int_{2m_\pi}^{\infty} dE \frac{\langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle}{m_K - E}$$

Lattice Version

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\dagger}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{M_K - E_n} \left(\underset{1.}{-(t_b - t_a)} \underset{2.}{- \frac{1}{M_K - E_n}} \underset{3.}{+ \frac{e^{(M_K-E_n)(t_b-t_a)}}{M_K - E_n}} \right)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential: states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2\sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{M_K - E_n} - 2\frac{d(\phi+\delta_0)}{dk}\Big|_{m_K} |\langle n_0 | H_W | K^0\rangle|^2 \cot(\phi+\delta_0)\Big|_{M_K}$$

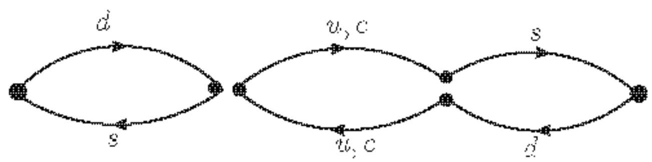
(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, arXiv:1504.01170)

$K_L - K_S$ mass difference

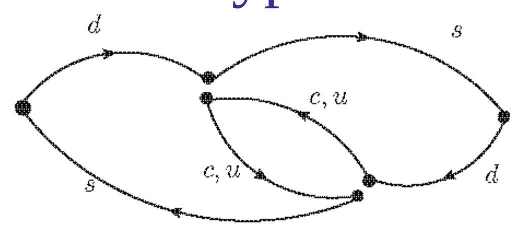
$K_L - K_S$ mass difference

- M_{K_L}-M_{K_S} = 3.483(6) x 10^{-12} MeV: sensitive to 1000 TeV scale physics.
- Perturbative result integrates out charm and shows poor convergence (Brod and Gorbahn).
- Finite when charm quark is included (GIM).

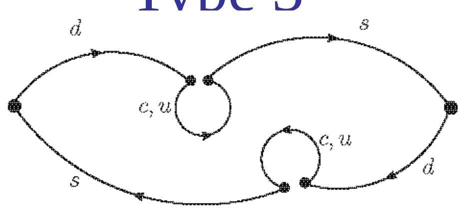
Type 1



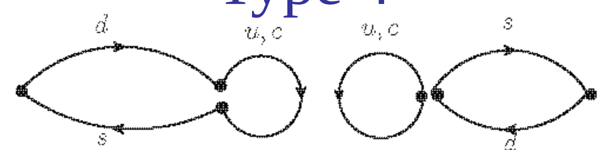
Type 2



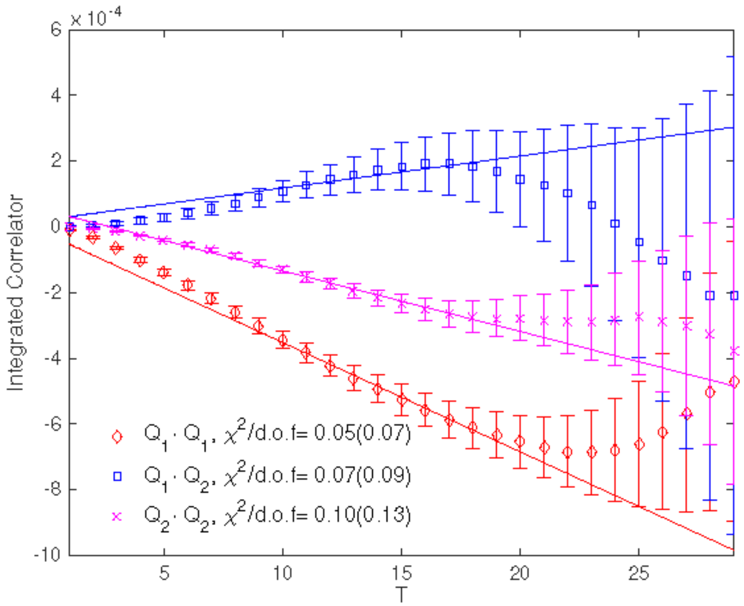
Type 3



Type 4



ΔM_K Preliminary Results
(Ziyuan Bai & Bigeng Wang)



	ΔM_K x 10^{+12} MeV
ΔM_K	5.8(1.7)
Δ_{FV}	0.27(18)
Expt.	3.483(6)

- Physical quark masses
- 57 → 151 configurations

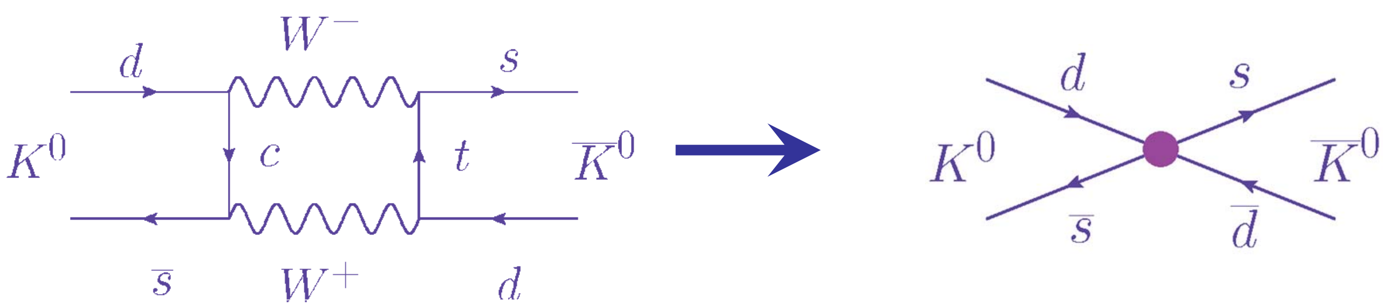
- $m_c^{\overline{MS}}$(2 GeV) ~ 1.2 MeV, M_π = 138 MeV
- 64^3x128, 1/a=2.36 GeV
- Uncorrelated fit: 10 ≤ T ≤ 20
- FV correction ~5%
- a^2 errors 5-10%

Long distance part of ε_K

$K^0 - \overline{K}^0$ mixing: Indirect CP Violation

- CP violating: $p \sim m_t$

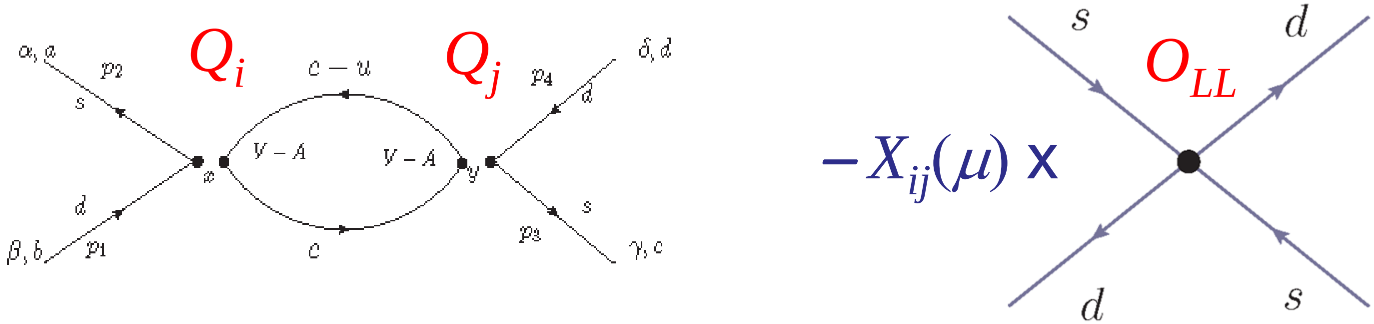
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$



- Where $|\varepsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$
  – Short distance prediction [W.Lee, *et al*. 1710.06614]:
  $|\varepsilon_K| = 1.58 \pm 0.16$ (V_{cb} dominant error)
  – Long distance estimate [Buras, *et al*. 1002.3612] :
  results in 6% reduction

New ΔS = 2 counter term

(Ziyuan Bai)



- Subtract $X_{ij}(\mu)$ $(\bar{s}\gamma^\nu(1-\gamma^5)d)$ $(\bar{s}\gamma^\nu(1-\gamma^5)d)$ to make off-shell Greens function vanish at $p_i^2 = \mu_{RI}{}^2$
- Define infrared-safe Rome-Southampton normalization for bi-local operator.

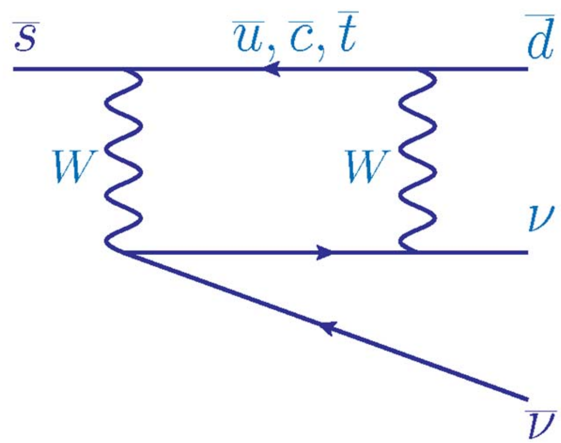
Exploratory calculation

- Use 24^3 x 64, 1/*a* = 1.73 GeV ensemble
- m_π = 329 MeV, m_K = 575 MeV, m_c = 941 MeV (0.363/*a*)
- Average over 64 separate, time-translated measurements on 200 configurations.
- Study 1.4 GeV $\leq \mu \leq$ 2.6 GeV
- Find $\Delta\varepsilon_K^{LD}$ = 0.108(76) x 10^{-3} a 5% correction.

Rare Kaon Decays

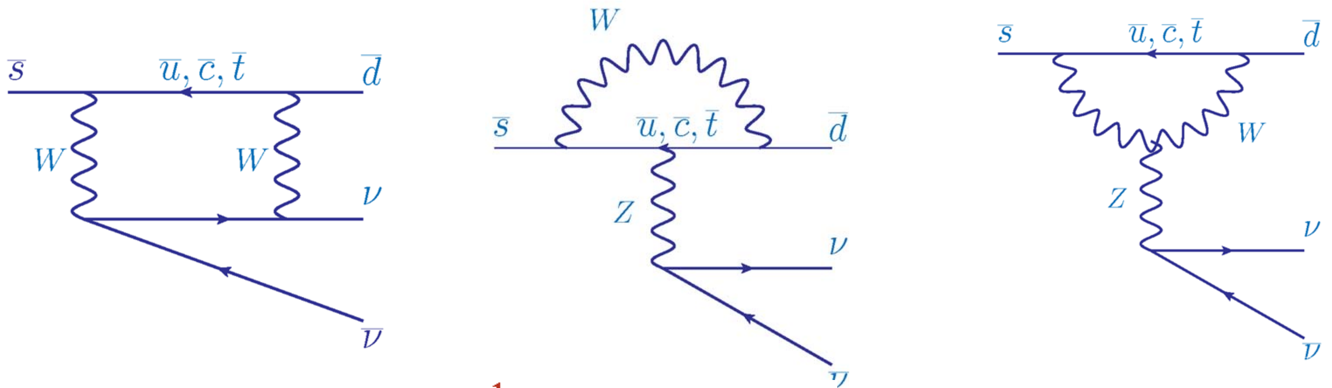
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$
(Xu Feng)



- Flavor changing neutral current
  - Allowed in the Standard Model only in second order
  - Short distance dominated
- Target of NA62 at CERN
  - 100 events in 2-3 years
  - Test Standard Model prediction at 10% level
  - Use lattice for long distance part: 5% effect ?

$K^+ \to \pi^+ \nu \bar{\nu}$ in the Standard Model



- Factors of $\frac{1}{M_W^4}$ or $\frac{1}{M_W^2 M_Z^2}$ force the largest contribution to come from short distance

Pert. Th.
- Top quark contribution largest.
- GIM implies charm-up $\sim \frac{m_c^2 - m_u^2}{M_W^4} \ln(M_W^2/m_c^2)$

Lattice
- Long distance part $\sim \frac{m_c^2}{M_W^4}$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$: Effect of bilocal operator

Bilocal Local

$$\mathcal{A}(K^+ \to \pi^+ \nu\overline{\nu}) = \langle \pi^+ \nu\overline{\nu} |\, T\left\{ \int d^4x \mathcal{H}'_{\text{eff}}(x) \mathcal{H}'_{\text{eff}}(0) \right\} + O_0(0)\ | K^+ \rangle$$

- Standard continuum treatment
  - Replace bilocal term with (perturbative coefficient) x (local operator)
- Lattice treatment: Evaluate $H_{eff}(x)$ $H_{eff}(0)$ product
  - Revolve logarithmic divergence as $x \rightarrow 0$
  - Deal with intermediate states with $E \leq M_K$
  - Exploit methods from M_{K_L}-M_{K_S} calculation

Exploratory Lattice Calculation
(Xu Feng)

- 16^3 x 32, RBC-UKQCD ensemble
  - 2+1 flavor DWF, $1/a$ = 1.73 GeV
  - M_π = 420 MeV, M_K = 540 MeV,
  - $m_c(2\text{ GeV})^{\overline{MS}}$ = 863 GeV
- Calculate all diagrams
- 800 configurations
- Phys.Rev.Lett. 118 (2017) 252001

Compare lattice and perturbative:

- Decay rate is short distance dominated:

$$\mathrm{Br} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\underbrace{\frac{\mathrm{Im}\lambda_t}{\lambda^4}X(x_t)}_{0.270\ \mathrm{x}1.481}\right)^2 + \left(\underbrace{\frac{\mathrm{Re}\lambda_c}{\lambda}P_c}_{-0.974\ \mathrm{x}\ 0.365} + \underbrace{\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)}_{-0.533\ \mathrm{x}1.481}\right)^2\right]$$

- Result for P_c :
  - Perturbation theory [Buras, et al.,1503.02693]: $P_c = 0.365(12)$
  - LD correction [Isidori, et al., hep-ph/0503107]: $\delta P_{cu} = 0.04(2)$

    (estimate of non-perturbative and $(\Lambda_{QCD}/m_c)^2$ effects)
  - Exploratory lattice result:
    - lattice evaluation of bilocal matrix element minus PT estimate)

$$P_c(\mu_{\overline{\mathrm{MS}}}) - P^{\mathrm{PT}}(\mu_{\overline{\mathrm{MS}}}) = 0.0040\,(\pm 13)_{\mathrm{stat}}\,(\pm 32)_{\mathrm{scale}}\,(-45)_{\mathrm{FV}}$$

    - small because of unexpected 4x cancellation.

Outlook

- Lattice QCD is now capable of 1st-principles calculation of:
  - $K \rightarrow \pi\pi$, ΔI = 3/2 and 1/2, ε'/ε.
  - $M_{K_L} - M_{K_S}$ and long distance contribution to ε.
  - Long distance parts of $K \rightarrow \pi \bar{l} l$, $K \rightarrow \pi \bar{\nu} \nu$.
- Physical quark mass calculations underway:
  - $M_{K_L} - M_{K_S}$ (Mira/ANL)
  - $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ (Mira/ANL), $K \rightarrow \pi \bar{l} l$ (Tesseract/UK)
- Study feasibility of computing $K_L \rightarrow \mu^+\mu^-$: provides a new 10% test for SM rare decay.
- New CORAL computer (Summit at ORNL) can perform $a^2 \rightarrow 0$ limit with charm.