Study of Long-distance Processes Using LQCD: from Flavor Physics to Nuclear Physics

Xu Feng (冯旭)

北京大學
PEKING UNIVERSITY

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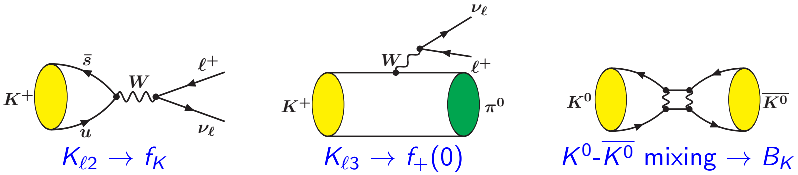
Congratulations to Heng-Tong and pepole at CCNU on the newly-built NSC^3 center!

Role of lattice QCD in flavor physics

Lattice QCD is powerful for observables such as



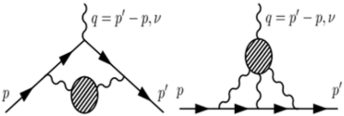
Flavor Lattice Averaging Group reported f_K/f_π, $f_+(0)$ and B_K

	N_f	FLAG average	Frac. Err.
f_K/f_π	$2+1+1$	1.1933(29)	0.25%
$f_+(0)$	$2+1+1$	0.9706(27)	0.28%
$\hat{B}_K$	$2+1$	0.7625(97)	1.27%

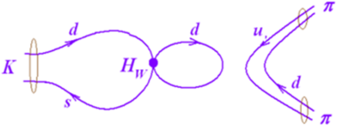
lattice QCD calculations play important role in precision flavor physics

Search for New Physics in the high-intensity frontiers [see Norman's talk]

$g_\mu - 2 = 0.00116592089(63)$

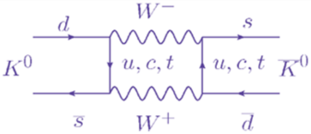


Direct CP violation $K \to \pi\pi$
$|\epsilon'| = 3.70(53) \times 10^{-6}$

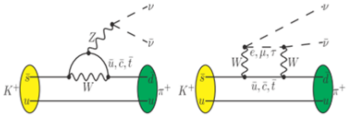


Indirect CP violation $K \to \pi\pi$
$\epsilon = 0.002228(11)$

$m_{K_L} - m_{K_S} = 3.2(1.0) \times 10^{-12}$ MeV

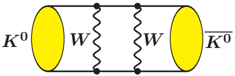


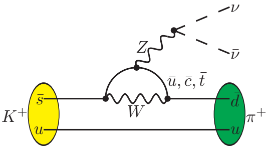
$K^+ \to \pi^+ \nu\bar{\nu}$: BR=$1.73^{+1.15}_{-1.05} \times 10^{-10}$



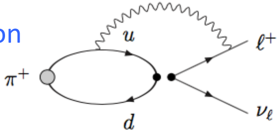
LD processes and non-local matrix elements $\langle f | O_1 \, O_2 | i \rangle$

2^{nd} order electroweak interaction

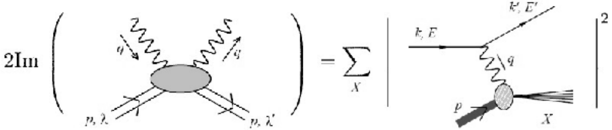




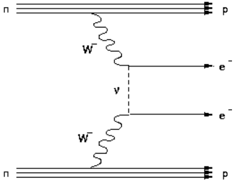
Electromagnetic correction



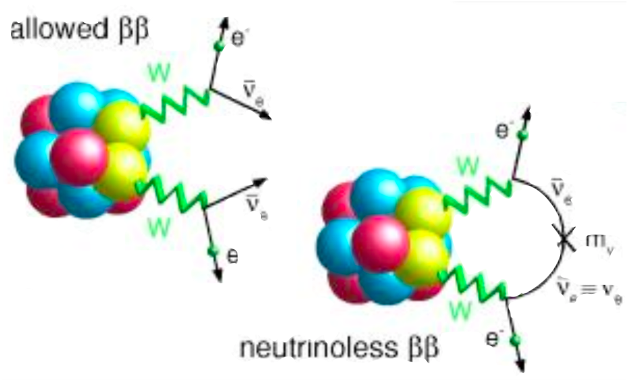
Inclusive decay and deep inelastic scattering



Neutrinoless double beta decay



From flavor physics to nuclear physics —— use double β decay as an example



Neutrinos: Dirac or Majorana?

Start from Dirac fermion

- Lagrangian density for a classical Dirac field

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}(i\gamma^0\partial_t + i\vec{\gamma}\cdot\vec{\nabla} - m)\psi$$

- Using Weyl representation of γ, one can write ψ as

$$\psi = \begin{pmatrix}\phi \\ \chi\end{pmatrix}, \quad \psi_L = \begin{pmatrix}0 \\ \chi\end{pmatrix}, \quad \psi_R = \begin{pmatrix}\phi \\ 0\end{pmatrix}$$

- Lagrangian becomes

$$\mathcal{L}_{\mathrm{Dirac}} = \chi^\dagger \underbrace{i(\partial_t - \vec{\sigma}\cdot\vec{\nabla})}_{O_-}\chi + \phi^\dagger \underbrace{i(\partial_t + \vec{\sigma}\cdot\vec{\nabla})}_{O_+}\phi - m\underbrace{(\chi^\dagger\phi + \phi^\dagger\chi)}_{\text{Dirac mass}}$$

- Lorentz transform for the fermionic fields $\psi(x) \to \psi'(x) = \Lambda_{\frac{1}{2}}\psi(\Lambda^{-1}x)$

$$\Lambda_{\frac{1}{2}} = \exp\left(-\frac{i}{2}\omega_{\mu\nu}S^{\mu\nu}\right), \quad \omega_{ij} = \epsilon_{ijk}\theta_k, \quad \omega_{0i} = \beta_i, \quad S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$$

Under infinitesimal rotations $\vec{\theta}$ and boosts $\vec{\beta}$, χ and ϕ transform differently

$$\chi'(x) = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\chi(\Lambda^{-1}x)$$

$$\phi'(x) = (1 - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\phi(\Lambda^{-1}x)$$

Construct Majorana fermion

- Realizing that $\tilde{\phi} = i\sigma_2\phi^*$ behaves like a left-handed spinor, one can define

$$\text{Left hand: } \eta = \frac{\chi + \tilde{\phi}}{\sqrt{2}}, \quad \text{Right hand: } \xi = -i\frac{\tilde{\chi} + \phi}{\sqrt{2}}$$

- Define the charge-conjugate field

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \psi^c = C\bar{\psi}^T = i\gamma^2\psi^* = \begin{pmatrix} -\tilde{\chi} \\ \tilde{\phi} \end{pmatrix}$$

One can construct self-charge-conjugate (Majorana) field N_1 and N_2

$$N_1 = \frac{\psi + \psi^c}{\sqrt{2}}, \quad N_2 = -i\frac{\psi - \psi^c}{\sqrt{2}}$$

N_1 is purely related to η and N_2 is related to ξ

$$N_1 = N_{1L} + N_{1L}^c = \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \begin{pmatrix} -\tilde{\eta} \\ 0 \end{pmatrix}, \quad N_2 = N_{2L} + N_{2L}^c = \begin{pmatrix} 0 \\ \tilde{\xi} \end{pmatrix} + \begin{pmatrix} \xi \\ 0 \end{pmatrix} = (N_{2R})$$

- Lagrangian density for Dirac field can also be expressed as

$$\mathcal{L}_{\rm Dirac} = \mathcal{L}_{\rm Majorana}(N_1) + \mathcal{L}_{\rm Majorana}(N_2)$$

$$\begin{aligned}
\mathcal{L}_{\rm Majorana}(N_1) &= \frac{1}{2}\overline{N_1}(i\gamma^\mu\partial_\mu - m)N_1 \\
&= \overline{N_{1L}}(i\gamma^\mu\partial_\mu)N_{1L} - \frac{1}{2}m\left(\overline{N_{1L}}(N_{1L})^c + \text{h.c.}\right) \\
&= \underbrace{\eta^\dagger O_-\eta}_{\text{Left handed}} \quad -\frac{1}{2}m\underbrace{\left(\eta^\dagger(-\tilde{\eta}) + \text{h.c.}\right)}_{\text{Majorana mass}}
\end{aligned}$$

  - Dirac fermion consists of a pair of mass degenerate Majorana fermion
  - Majorana fermion also satisfies Dirac equation

- Under global phase transformation

$$\text{Dirac: } \chi \to e^{i\alpha}\chi, \quad \phi \to e^{i\alpha}\phi \quad \Rightarrow \quad \text{lepton number conservation}$$
$$\text{Majorana: } \eta \to e^{i\alpha}\eta, \quad \tilde{\eta} \to e^{-i\alpha}\tilde{\eta} \quad \Rightarrow \quad \text{lepton number violation}$$

  - Electric charge conservation forces charged fermion to be Dirac type
  - Neutrino can be Dirac, Majorana or the mixed type

Light-neutrino exchange and $0\nu 2\beta$ decay

Light-neutrino exchange in $0\nu 2\beta$ decay

Minimal extension of SM – exchange of three light Majorana neutrinos

- Effective Lagrangian for β decay

$$\mathcal{L}_{eff} = 2\sqrt{2} G_F V_{ud} (\overline{u}_L \gamma_\mu d_L)(\overline{e}_L \gamma_\mu \nu_{eL})$$

- Effective Hamlitonian for 2β decay

$$\mathcal{H}_{eff}^{2\beta} = \frac{1}{2!} \int d^4x \, \mathcal{L}_{eff}(x) \mathcal{L}_{eff}(0)$$

- Neutrino flavor eigenstate mixes with three mass eigenstates

$$\overline{e}_L \gamma_\mu \nu_{eL} \quad \rightarrow \quad \sum_k \overline{e}_L \gamma_\mu U_{ek} \nu_{kL}$$

U_{ek} is the mixing matrix element.

- These neutrinos are very light

Long-distance contribution dominated

Light-neutrino exchange in $0\nu 2\beta$ decay

Assume that $0\nu2\beta$ is mediated by exchange of light Majorana neutrinos

$$
\begin{aligned}
& \sum_k \overline{e}_L(x)\gamma_\mu U_{ek}\nu_{kL}\underbrace{(x)\,\overline{e}_L(0)\gamma_\nu U_{ek}\nu_{kL}}(0) \\
= \quad & -\sum_k \overline{e}_L(x)\gamma_\mu U_{ek}\nu_{kL}\underbrace{(x)\,\overline{\nu^c_k}_L}(0)\gamma_\nu U_{ek} e^c_L(0) \\
= \quad & -\sum_k \overline{e}_L(x)\gamma_\mu U_{ek} P_L \left(\int \frac{d^4q}{(2\pi)^4} \frac{-i\not{q} + m_k}{q^2 + m_k^2} e^{iqx} \right) P_L \gamma_\nu U_{ek} e^c_L(0) \\
\approx \quad & -m_{\beta\beta} \int \frac{d^4q}{(2\pi)^4} \frac{e^{iqx}}{q^2} \overline{e}_L(x)\gamma_\mu\gamma_\nu e^c_L(0)
\end{aligned}
$$

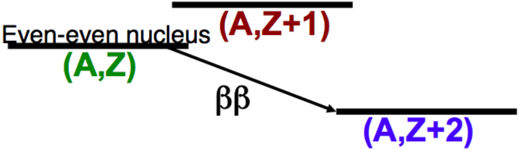
In the last step, $\not{q}$ vanishes and m_k enters into the effective mass $m_{\beta\beta}$

$$m_{\beta\beta} = \sum_k m_k U_{ek}^2$$

$0\nu2\beta$ decay amplitude is proportional to the absolute neutrino mass

Double beta decay

Double beta decay exists in nature



$2\nu\beta\beta$ decay is the rarest SM process that has been measured

- Small mass difference ΔM between initial and final state
  $\Rightarrow$ suppressed by phase space
- 2^{nd}-order EW interaction $\Rightarrow$ suppressed by $\frac{\Delta M^2}{M_W^2}$

$2\nu\beta\beta$ has been detected in total of 10 nuclei: $^{48}\mathrm{Ca}$, $^{76}\mathrm{Ge}$, $\cdots$ $^{238}\mathrm{U}$

- For all decays, half-life time: $10^{18} - 10^{21}$ yr (Age of universe: 1.38×10^{10} yr)

$0\nu\beta\beta$ decay

- The easiest way to determine whehter ν is a Majorana fermion
- Give the information on the absolute mass scale of ν
- Provide the evidence of lepton number violation

It can be measured by using tons of materials such as TeO_2

CANDLES	Ca-48	60 CaF_2 crystals in liq. scint	6 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	100 kg	
COBRA	Cd-116, Te-130	CdZnTe detectors	10 kg	R&D
CUROICINO	Te-130	TeO_2 Bolometer	11 kg	Operating
CUORE	Te-130	TeO_2 Bolometer	206 kg	Construction
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D
EXO200	Xe-136	Xe TPC	200 kg	Construction
EXO	Xe-136	Xe TPC	1-10t	R&D
GEM	Ge-76	Ge diodes in LN	1 t	
GERDA	Ge-76	Seg. and UnSeg. Ge in	35-40 kg	Construction

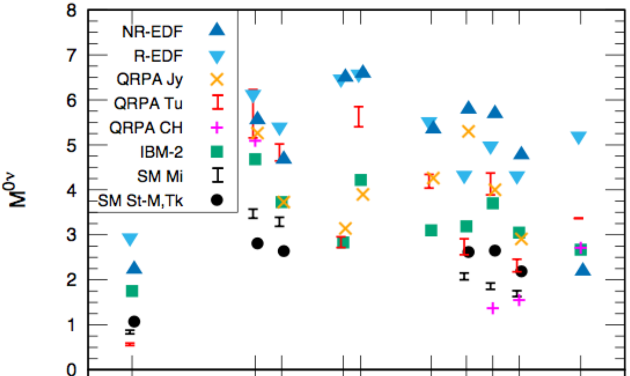
- 4 Exp. (Majorana, EXO, CUORE, GERDA) reached $T^{0\nu}_{1/2} > 10^{25}$ year
- 1 Exp. (KamLAND-Zen) exceeded the level of 1×10^{26} year

Double β decay: generic difficulties

At present, lattice QCD mainly targets on light nuclei
– because of two exponential difficulties

- For nucleus A: $\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2m_\pi)t\right] \quad \Rightarrow \quad$ a sign problem!
- Complexity: number of Wick contractions $= N_u!N_d!N_s!$
  - e.g. $^4\mathrm{He} \Rightarrow$ naively 5×10^5 contractions!

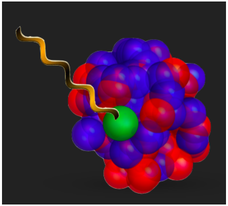
For nuclear matrix element, various models yield O(100%) discrepancies



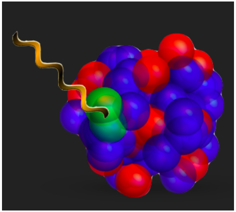
Single β decay of nuclei

Coupling of currents to nuclei in nuclear EFT [Detmold, talk at Lat18]

- One body coupling dominates



- Two nucleon contributions are subleading but non-negligible



A promising way to provide few-body inputs to ab initio many-body calculations

Double β decay of nuclei

Begin with the effective Lagrangian $\mathcal{L}_{\mathrm{eff}}$ for the single β decay

$$\mathcal{L}_{\mathrm{eff}} = 2\sqrt{2} G_F V_{ud} (\bar{u}_L \gamma_\mu d_L)(\bar{e}_L \gamma_\mu \nu_{eL})$$

Contributions are identified into three regions in EFT

- Hard region: $\Lambda \gg 1$ GeV

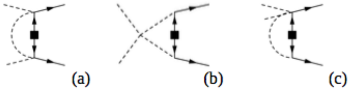
$$\int d^4x\, e^{i\Lambda x} \mathcal{L}_{\mathrm{eff}}(x) \mathcal{L}_{\mathrm{eff}}(0) \sim 8 G_F^2 V_{ud}^2 \frac{m_{\beta\beta}}{\Lambda^2} (\bar{u}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu d_L) \bar{e}_L e_L^c.$$

  In lattice QCD, a hard cutoff is introduced by $1/a \Rightarrow O(a^2)$ effects

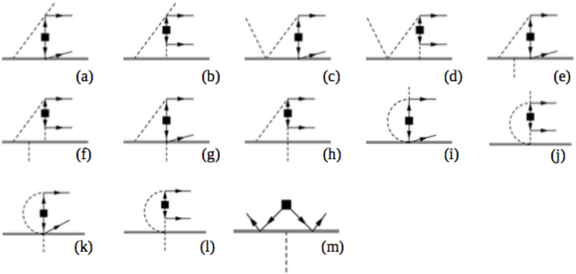
- Soft region: $O(100$ MeV$)$ - $O(1$ GeV$)$
  - Few-body decay dominates
  - Nuclear potential mediated by pions: $\pi\pi \to ee$, $\pi n \to pee$, $nn \to ppee$, $\cdots$
- Ultrasoft or radiative region: $\Lambda \ll 100$ MeV
  - Neutrinos feel the complete nucleus instead of just the nucleons

Loop diagrams in EFT

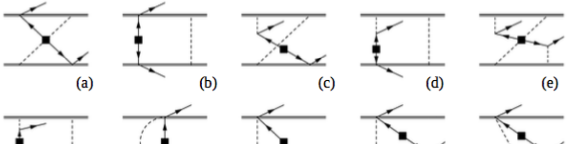
- $\pi\pi \to ee$



- $\pi n \to pee$

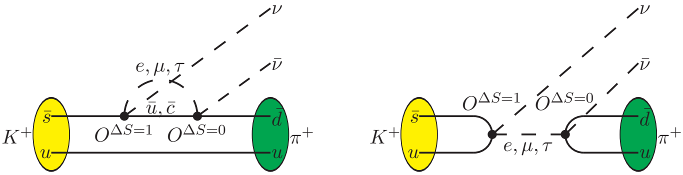


- $nn \to ppee$

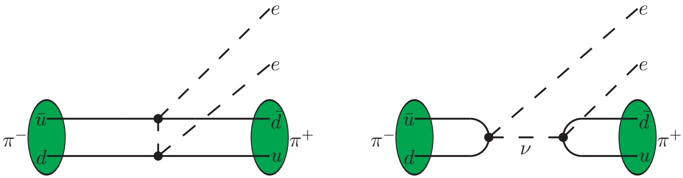


Rare kaon decay:

N. Christ et. al PRD93 (2016), 114517
Z. Bai et. al, PRL118 (2017), 252001
Z. Bai et. al, 1806.11520, to be published



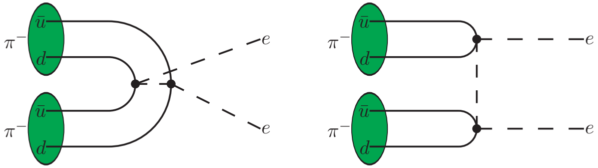
Double beta decay:



Our setup

$\pi\pi \to ee$

- Two pions stay at rest
- Two electrons carry spatial momentum $\vec{p}_1 = -\vec{p}_2$, $\vec{p}_{1,2} = E_{\pi\pi}/2$



- Decay goes through $\pi\pi \to e_1\bar{\nu}n \to e_1e_2$ or $\pi\pi \to e_2\bar{\nu}n \to e_1e_2$
- Decay amplitude is simplified as

$$\mathcal{A} = -T_{\mathrm{lept}} \sum_n \int \frac{d^3\vec{p}_n}{(2\pi)^3} \sum_{i=1,2} \frac{\langle 0|J_{\mu L}|n\rangle\langle n|J_{\mu L}|\pi\pi\rangle}{2E_{\nu,i}E_n(E_n + E_{\nu,i} + E_i - E_{\pi\pi})}$$

with $T_{\mathrm{lept}} = 4G_F^2 V_{ud}^2 m_{\beta\beta}\bar{u}_L(p_1)u_L^c(p_2)$.

Implementation of neutrino propagator

Scalar propagator $S_0(x,y)e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}$ with $\vec{k}=\vec{p}_{1,2}$ can be implemented

$$S_0(x,y)e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} = \int \frac{d^4q}{(2\pi)^4}\frac{e^{iq(x-y)}}{q_t^2+(\vec{q}+\vec{k})^2}$$
$$\Rightarrow \frac{1}{VT}\sum_{\vec{q},q_t}\frac{e^{iq(x-y)}}{\widehat{q}_t^{\,2}+\sum_i \widehat{q_i+k_i}^{\,2}}$$

- $\widehat{q}_i = 2\sin(q_i/2)$ are the lattice discretized momenta

Zero mode ($\vec{q}=0$) of the propagator is computed exactly

$$\frac{1}{VT}\sum_{q_t}\frac{e^{iq_t(t_x-t_y)}}{\widehat{q}_t^{\,2}+\sum_i \widehat{k}_i^{\,2}}$$

Non-zero modes ($\vec{q}\neq\vec{0}$) of the propagator can be constructed as

$$\frac{1}{N_r}\sum_{r=1}^{N_r}\phi_r(x)\phi_r^*(y), \quad \phi_r(x)=\frac{1}{\sqrt{VT}}\sum_{\vec{q}\neq\vec{0},q_t}\frac{\xi_r(q)e^{iqx}}{\sqrt{\widehat{q}_t^{\,2}+\sum_i \widehat{q_i+k_i}^{\,2}}}.$$

Here the stochastic sources $\xi_r(q)$ satisfy

$$\lim_{N_r\to\infty}\frac{1}{N_r}\sum \xi_r(q)\xi_r^*(q') = \delta_{q,q'}.$$

Gauge configuraiton ensembles

m_π [MeV]	a^{-1} [GeV]	$L^3 \times T$	N_{conf}	N_r
420	1.73	$16^3 \times 32$	200	32
140	1.01	$24^3 \times 64$	60	64

- Generate Coulomb-gauge-fixed wall-source propagators at all T time slices
- At $m_\pi = 140$ MeV, use all mode average to reduce the computational cost
- Average the temporal PBC and APBC condtion to make $T \to 2T$

Construct the correlation function

$$C(t_x, t_y, t_{\pi\pi}) = \frac{1}{2!}\langle e_1 e_2|\mathcal{L}_{eff}(t_x)\,\mathcal{L}_{eff}(t_y)\,\phi_{\pi\pi}(t_{\pi\pi})|0\rangle$$

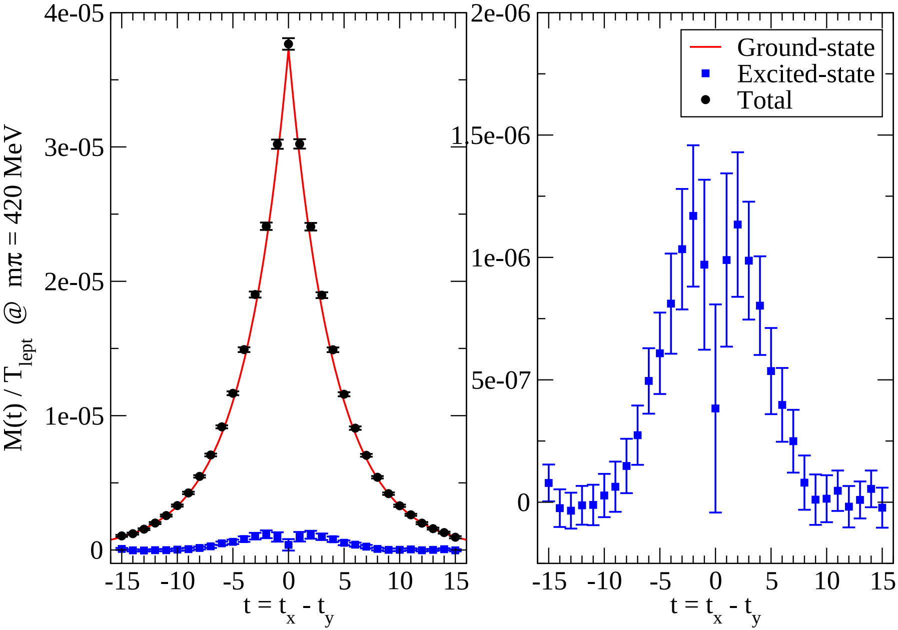
Define the unintegrated amplitude $\mathcal{M}(t)$ with $t = t_x - t_y$:

$$\mathcal{M}(t) = C(t_x, t_y, t_{\pi\pi}) / \left(V\frac{N_{\pi\pi}}{2E_{\pi\pi}} e^{E_{\pi\pi} t_{\pi\pi}} \right)$$

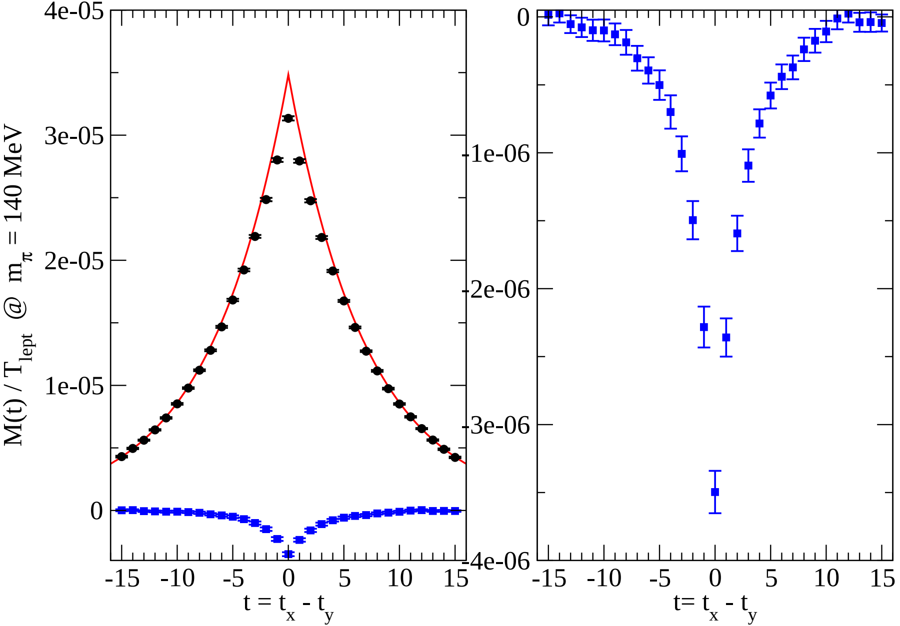
At large $|t|$, $\mathcal{M}(t)$ is saturated by ground intermediate state - $e\bar{\nu}\pi$

$$\mathcal{M}(t) \xrightarrow{|t|\gg 0} -T_{\mathrm{lept}}\frac{1}{V}\frac{2\langle 0|J_{\mu L}|\pi\rangle_{\mathrm{V}}\langle\pi|J_{\mu L}|\pi\pi\rangle_{\mathrm{V}}}{(2m_\pi)(2E_\nu)}e^{-m_\pi|t|}$$

$\pi\pi \to ee$ decay amplitude @ $m_\pi = 420$ MeV



$\pi\pi \to ee$ decay amplitude @ $m_\pi = 140$ MeV



Double integration

N. Christ et. al, PRD88 (2013) 014508

Integrate both t_x and t_y over a fixed window $[t_a, t_b]$ with $t_a \gg t_{\pi\pi}$

$$\mathcal{M} = \sum_{t_x=t_a}^{t_b} \sum_{t_y=t_a}^{t_b} C(t_x, t_y, t_{\pi\pi}) / \left(V \frac{N_{\pi\pi}}{2E_{\pi\pi}} e^{E_{\pi\pi} t_{\pi\pi}} \right)$$

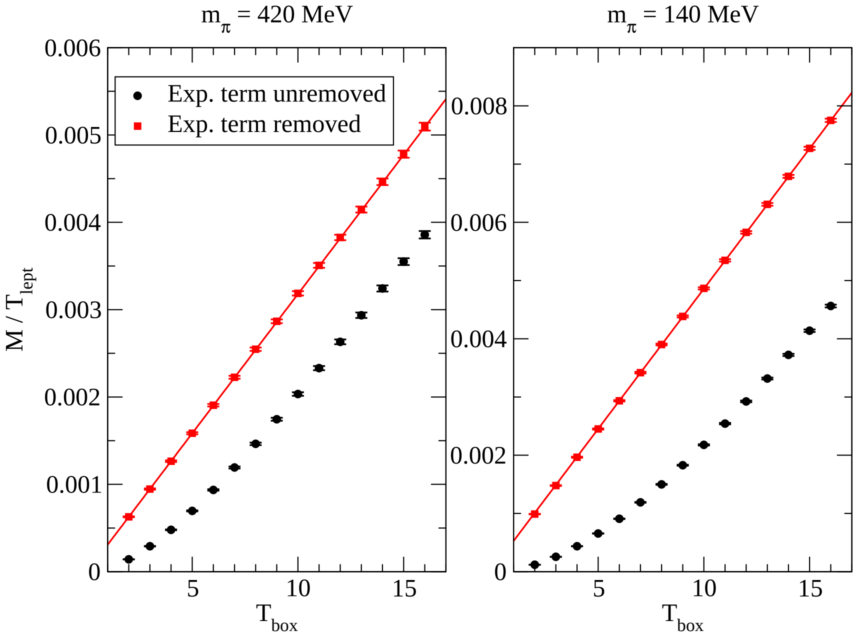
$$= -T_{\mathrm{lept}} \sum_n \frac{1}{V} \sum_{\vec{p}_n} \sum_{i=1,2} \frac{\langle 0|J_{\mu L}|n\rangle \langle n|J_{\mu L}|\pi\pi\rangle}{2E_n E_{\nu,i}(E_n + E_{\nu,i} + E_i - E_{\pi\pi})}$$

$$\times \left(T_{\mathrm{box}} + \frac{e^{-(E_n+E_{\nu,i}+E_i-E_{\pi\pi})T_{\mathrm{box}}} - 1}{E_n + E_{\nu,i} + E_i - E_{\pi\pi}} \right)$$

with $T_{\mathrm{box}} = t_b - t_a + 1$ the time extent of the integration window

- T_{box} is sufficiently large $\Rightarrow$ exp. term vanishes as $E_n + E_{\nu,i} + E_i > E_{\pi\pi}$
- The coefficient of the term proportional to T_{box} provides $\mathcal{A}(\pi\pi \to ee)$

Integrated matrix element



Results

m_π [MeV]	$t_a - t_{\pi\pi}$	$\mathcal{A}^{(g)}$	$\mathcal{A}^{(e)}$	$\mathcal{A}^{(g)} + \mathcal{A}^{(e)}$
	6		0.055(13)	1.517(13)
420	7	1.462(10)	0.060(13)	1.522(13)
	8		0.052(14)	1.514(14)
	6		−0.0664(70)	1.8199(63)
140	7	1.8863(50)	−0.0660(73)	1.8203(62)
	8		−0.0665(70)	1.8199(60)

We obtain the result with sub-percent statisitcal errors:

$$\left.\frac{\mathcal{A}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}}\right|_{m_\pi=420\ \mathrm{MeV}} = 1.517(13),$$

$$\left.\frac{\mathcal{A}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}}\right|_{m_\pi=140\ \mathrm{MeV}} = 1.820(6).$$

At $m_\pi = 420$ and 140 MeV, $\mathcal{A}(\pi\pi \to ee)$ are 24% and 9% smaller than the leading-order ChPT predication

$$\frac{\mathcal{A}^{\mathrm{LO}}(\pi\pi \to ee)}{F_\pi^2\, T_{\mathrm{lept}}} = 2$$

Outlook

Today, LQCD is entering Exaflop generation

- Standard quantity: expect the precision significantly enhanced
- Non-standard quantity, such as LD processes: worthwhile for study

For flavor physics:

- lattice QCD provides useful low-energy QCD information
- plays important role in high-precision frontier

The techniques developed in flavor physics can be used in nuclear physics

- help to study the rare processes related to nuclear matter
- Can one day, nuclear physics become a new flavor physics?

Backup slides

Three types of power-law finite-volume effects

- Generic FV effects associated to long-distance processes $i \to n \to f$

  N. Christ et. al, PRD91 (2015), 114510

  - n is the multi-particle intermediate state
  - Energy of n is smaller than initial-state energy

- FV effects caused by $\pi\pi$-rescattering in the initial state

$$|\pi\pi\rangle_\infty = \left(2\pi \frac{E_{\pi\pi}}{k^3}\right)^{\frac{1}{2}} \left(q\frac{d\phi}{dq} + k\frac{d\delta}{dk}\right)^{\frac{1}{2}} |\pi\pi\rangle_{\mathrm{V}}$$

At threshold

$$\frac{2\pi}{k^3}\left(q\frac{d\phi}{dq} + k\frac{d\delta}{dk}\right) = V\left[1 + d_1\frac{a_{\pi\pi}}{L} + d_2\left(\frac{a_{\pi\pi}}{L}\right)^2 + d_3\left(\frac{a_{\pi\pi}}{L}\right)^3 - 2\pi\frac{a_{\pi\pi}^2 r_{\pi\pi}}{L^3} + O(L^{-4}\right.$$

- FV effects due to the long-range property of neutrino propagator
  - e.g FV effects relevant for the $e\bar{\nu}\pi$-intermediate state

Three types of power-law finite-volume effects

- FV effects relevant for the $e\bar{\nu}\pi$-intermediate state

$$\Delta_{\rm FV} = \left(\frac{1}{V}\sum_{\vec{p}} - \int \frac{d^3\vec{p}}{(2\pi)^3}\right) \frac{\langle 0|J_{\mu L}|\pi(\vec{p})\rangle\langle \pi(\vec{p})|J_{\mu L}|\pi\pi\rangle}{E_\nu E_\pi(E_\pi + E_\nu + E_e - E_{\pi\pi})}$$

  - Integrand is singular at $E_\nu = |\vec{p}_\nu| = |\vec{p} + \vec{p}_e|$
  - Define the regular part $f(\vec{p}) \equiv \frac{\langle 0|J_{\mu L}|\pi(\vec{p})\rangle\langle \pi(\vec{p})|J_{\mu L}|\pi\pi\rangle}{E_\pi(E_\pi + E_\nu + E_e - E_{\pi\pi})}$ and write

$$f(\vec{p}) = f(-\vec{p}_e) + [f(\vec{p}) - f(-\vec{p}_e)]$$

  - Finally we have

$$\Delta_{\rm FV} = f(-\vec{p}_e)\left(\frac{1}{V}\sum_{\vec{p}} - \int \frac{d^3\vec{p}}{(2\pi)^3}\right) \frac{1}{|\vec{p} + \vec{p}_e|}$$