High performance computing applied to static and dynamic phenomena in heavy-ion collisions

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Wayne State University

Workshop on HPC in HEP, CCNU, Sept 19-21, 2018

Outline

Intro to heavy-ion collisions and jet quenching

The many scales of jet quenching

The range of measurables and observables

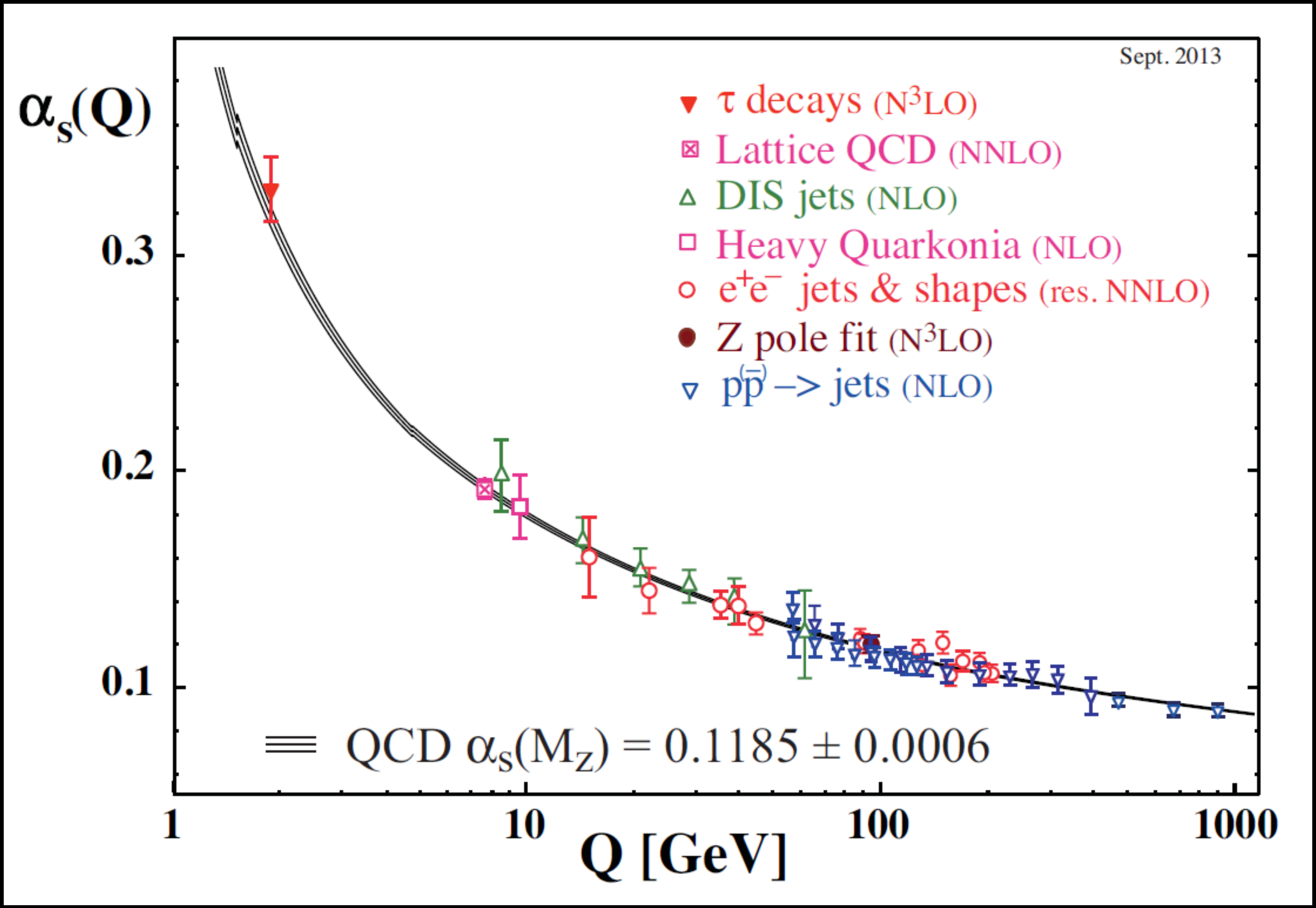
The JETSCAPE framework

Transport coefficients from first principles

Lattice calculations,

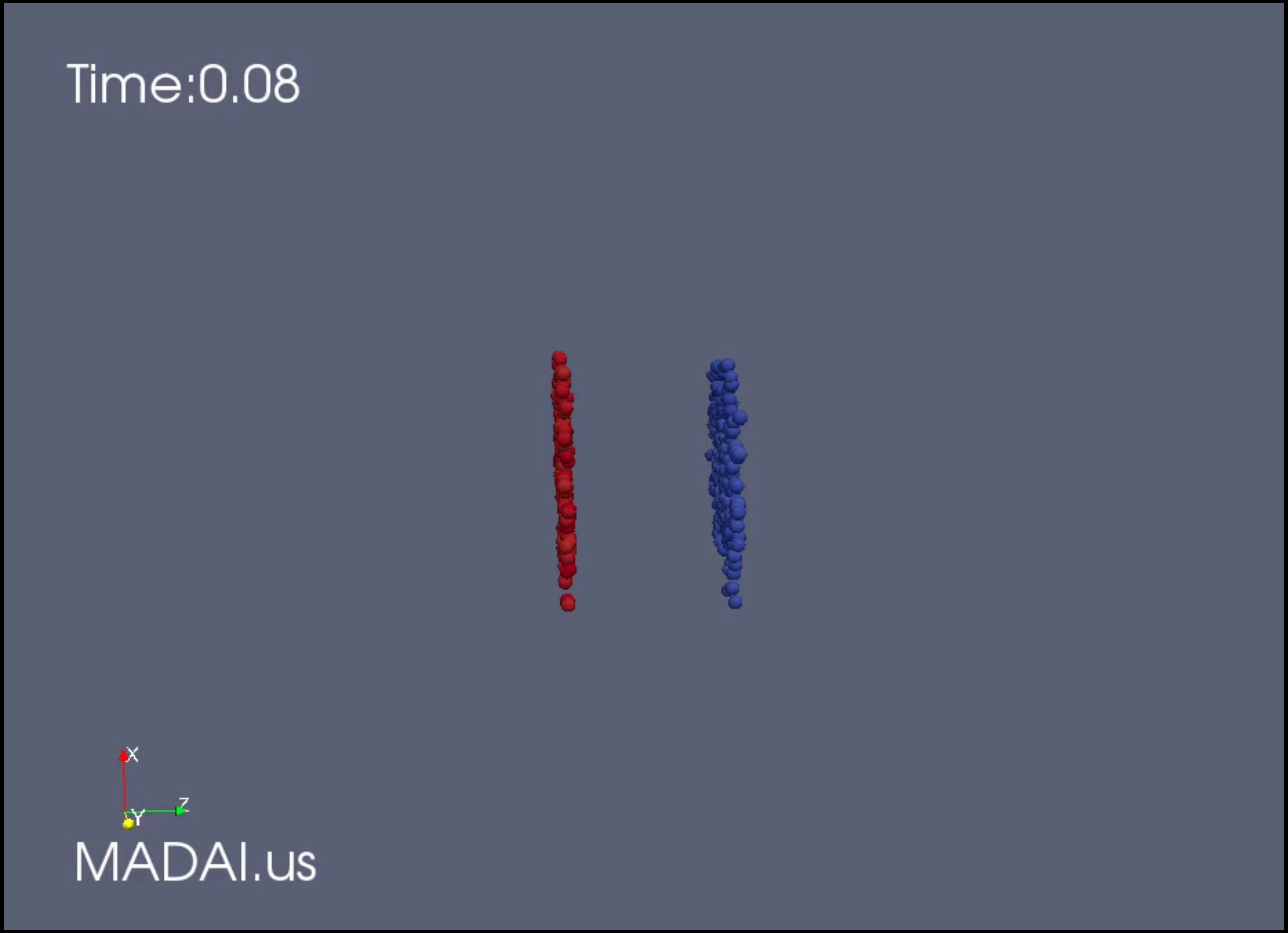
Outlook

QCD is all about scale!



Heavy-ion collisions span the entire curve above

A near established paradigm, on the soft side



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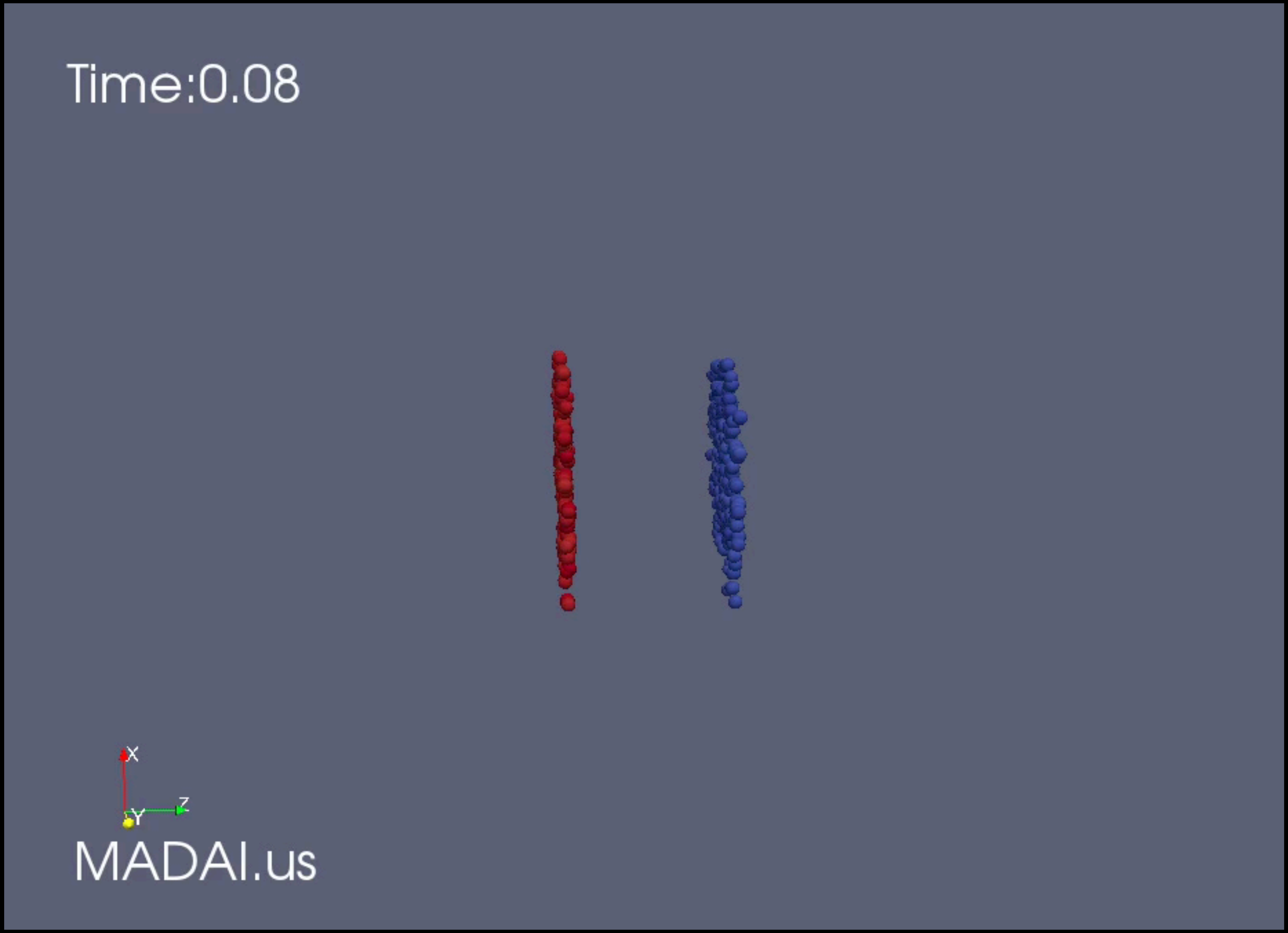
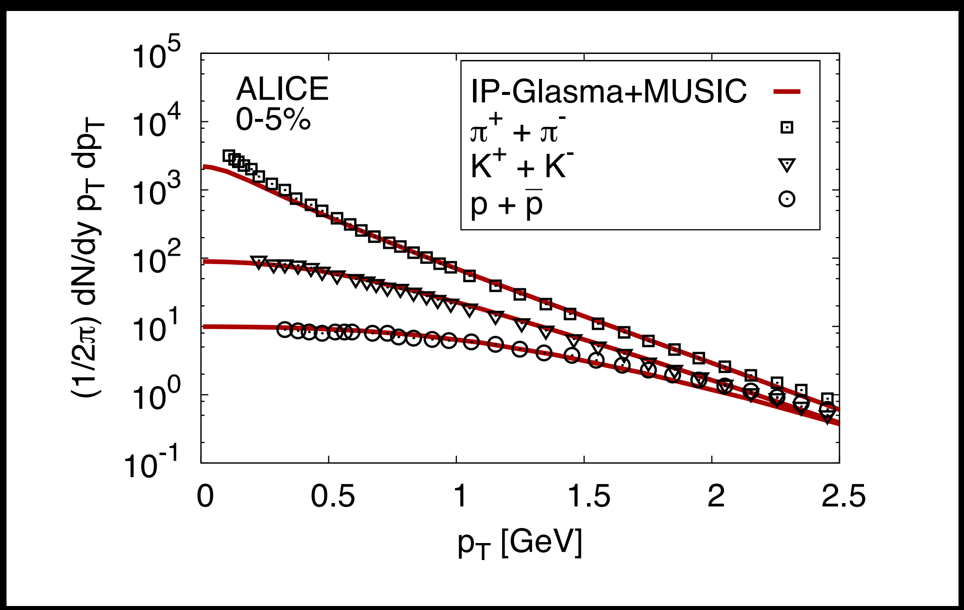
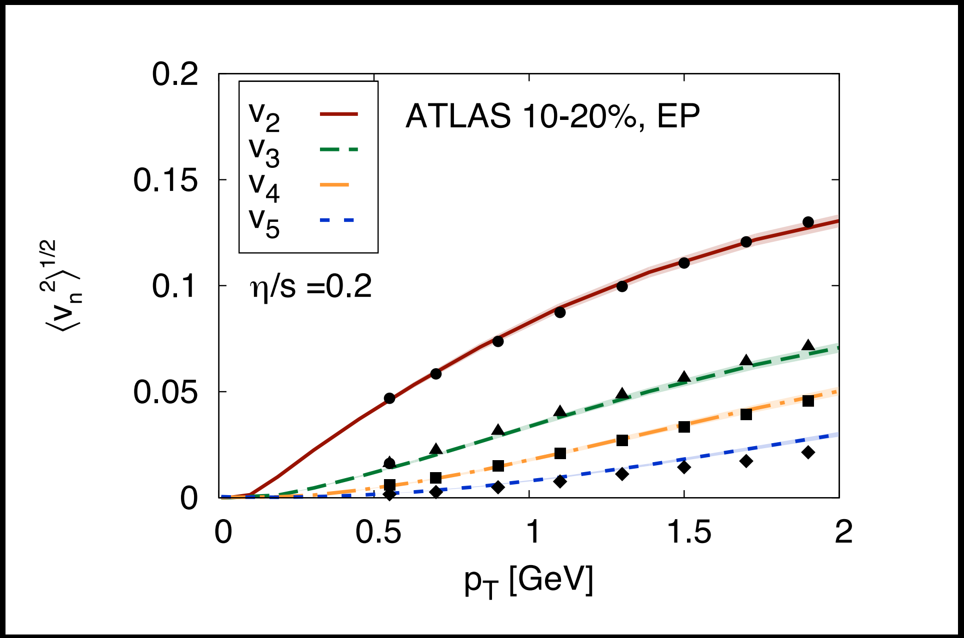
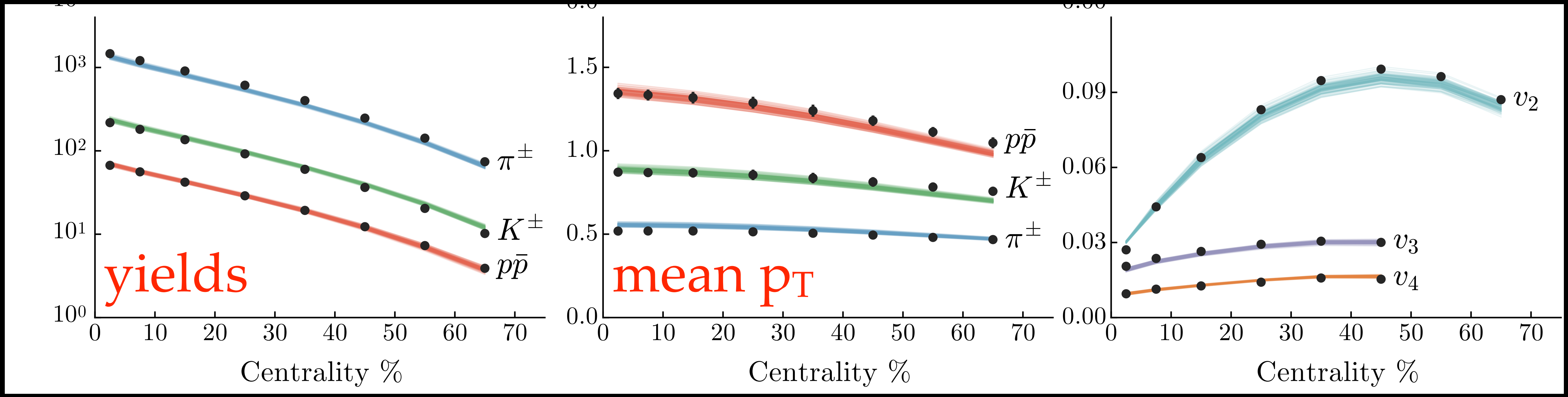


Well established results $p_T < 2$GeV

Excellent theoretical predictive power over "soft" spectrum
both centrality and p_T dependence







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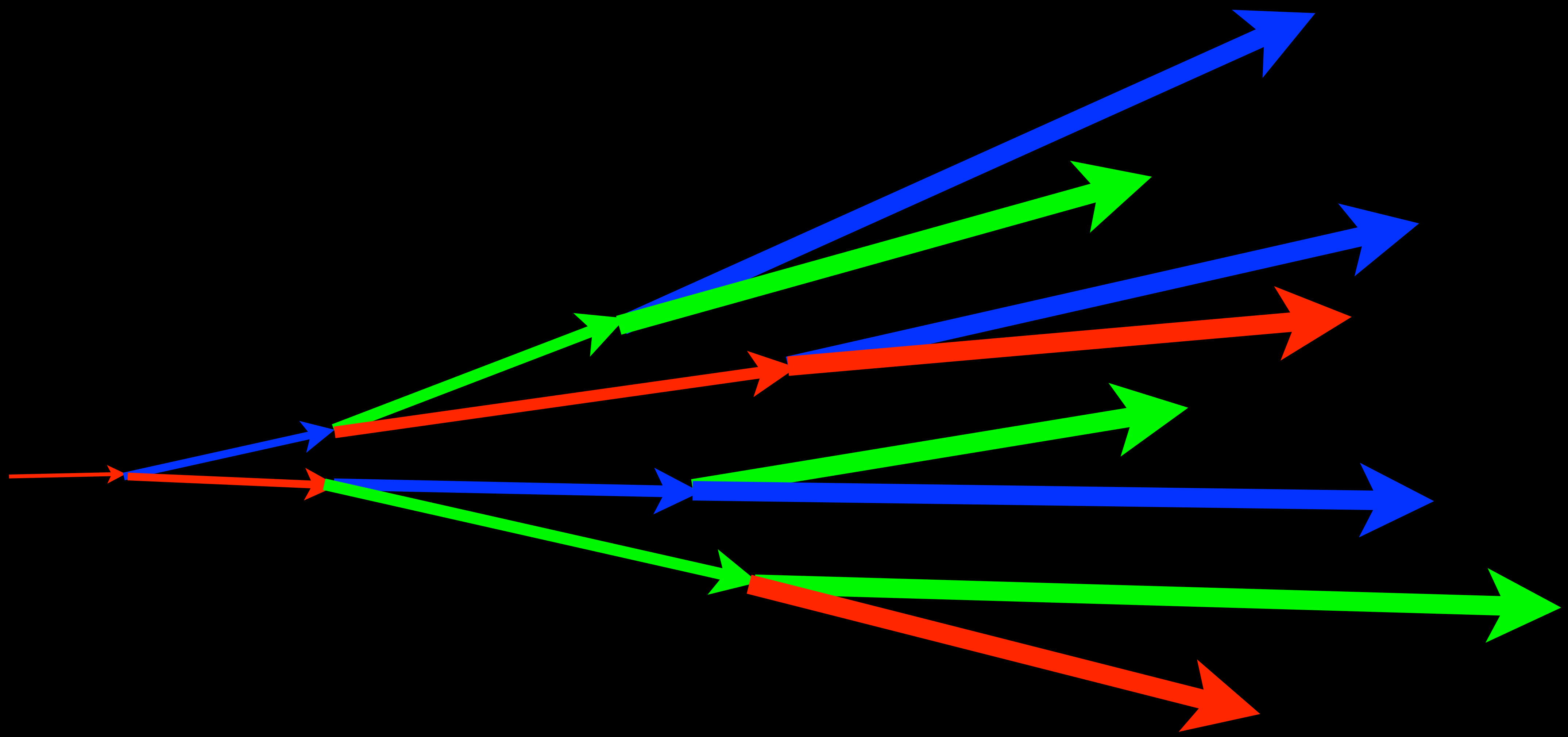

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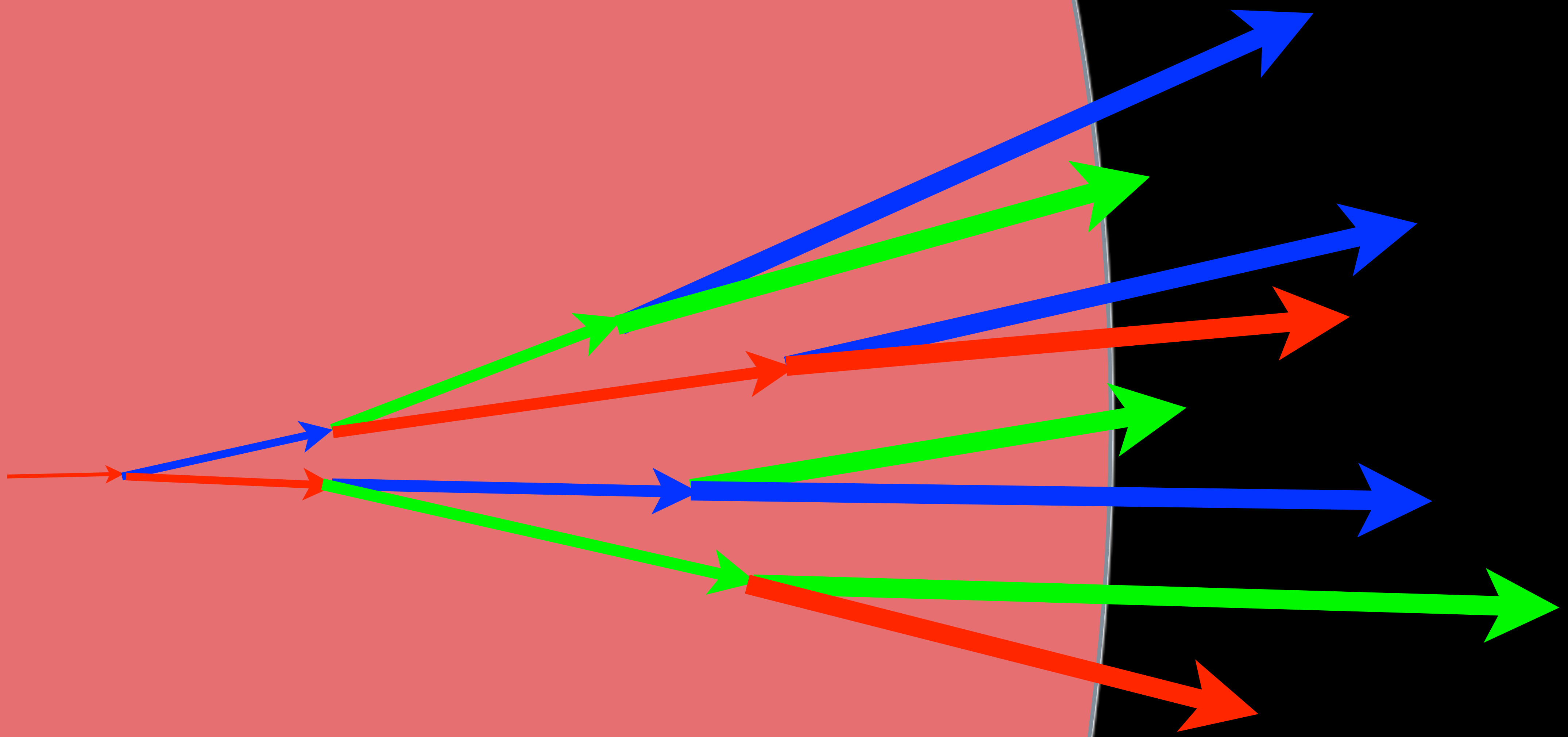

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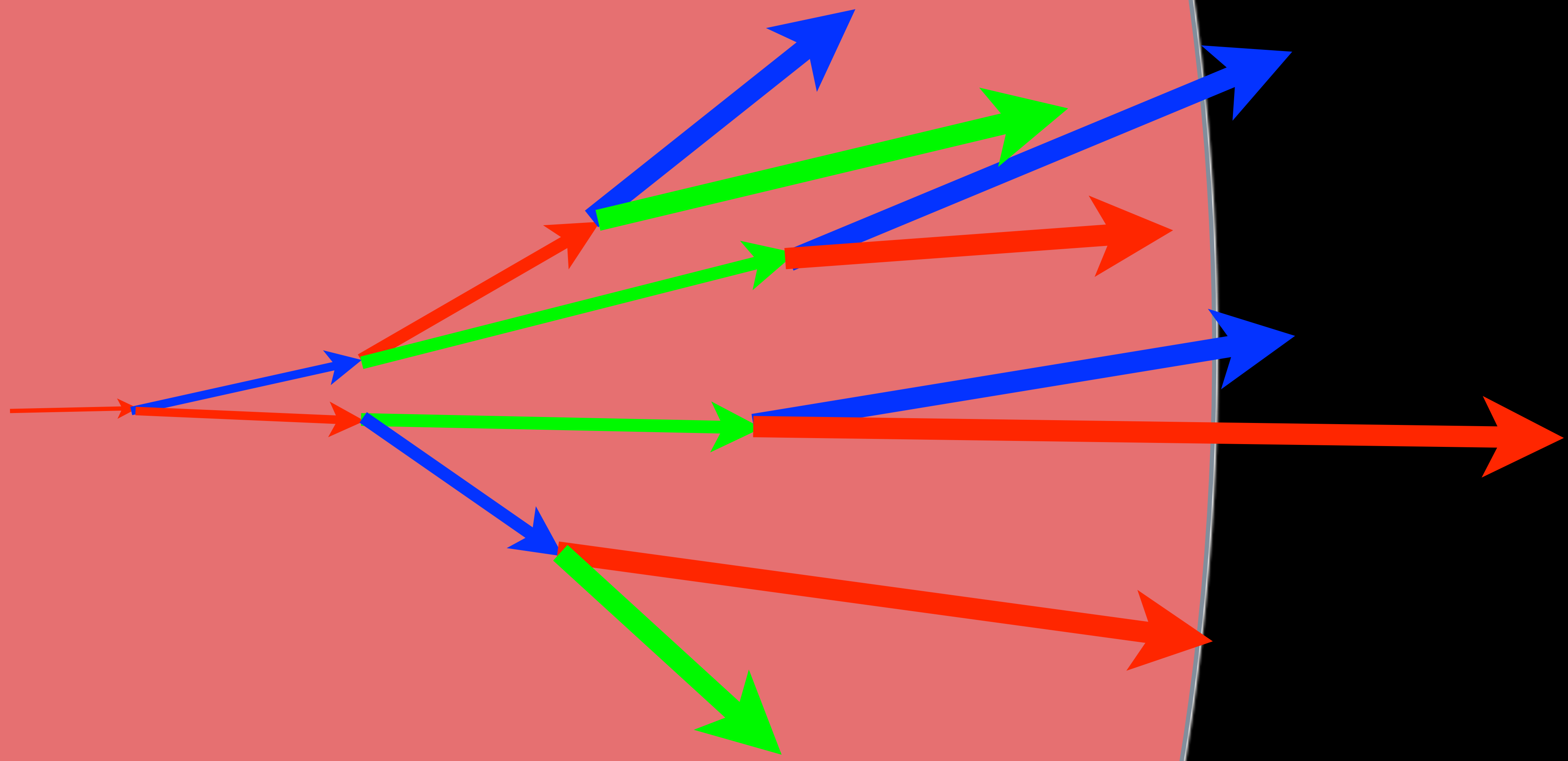

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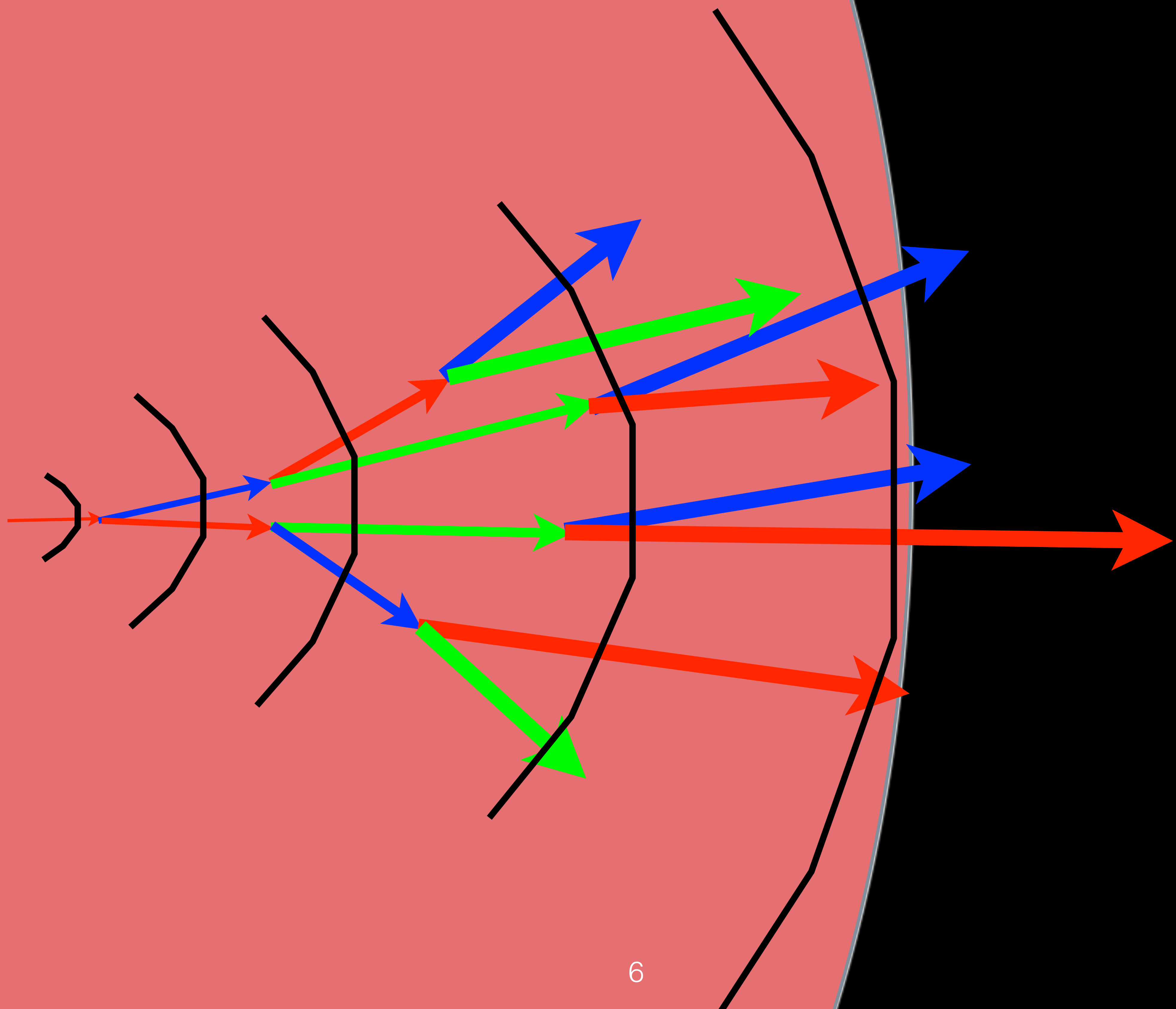

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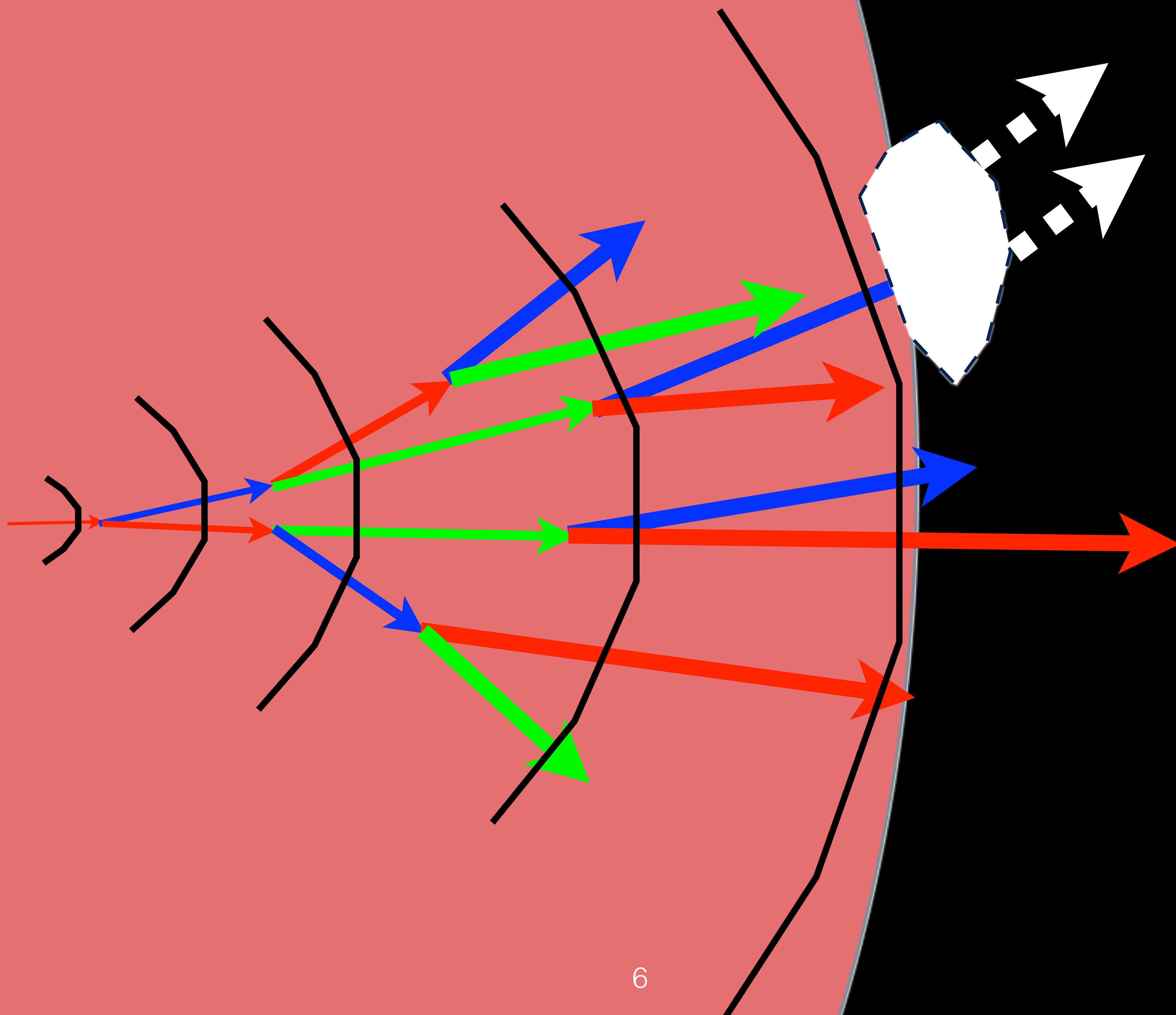

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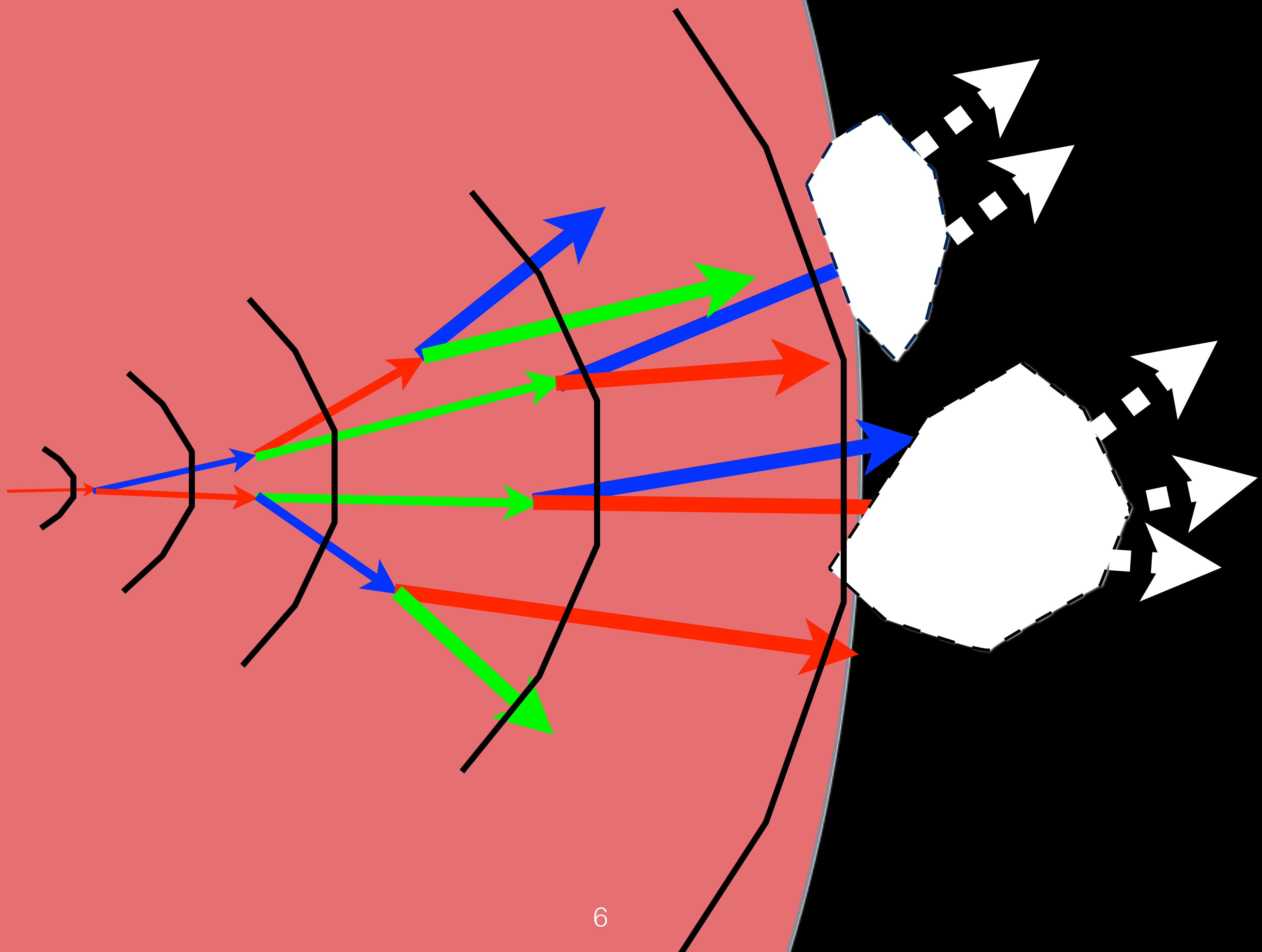

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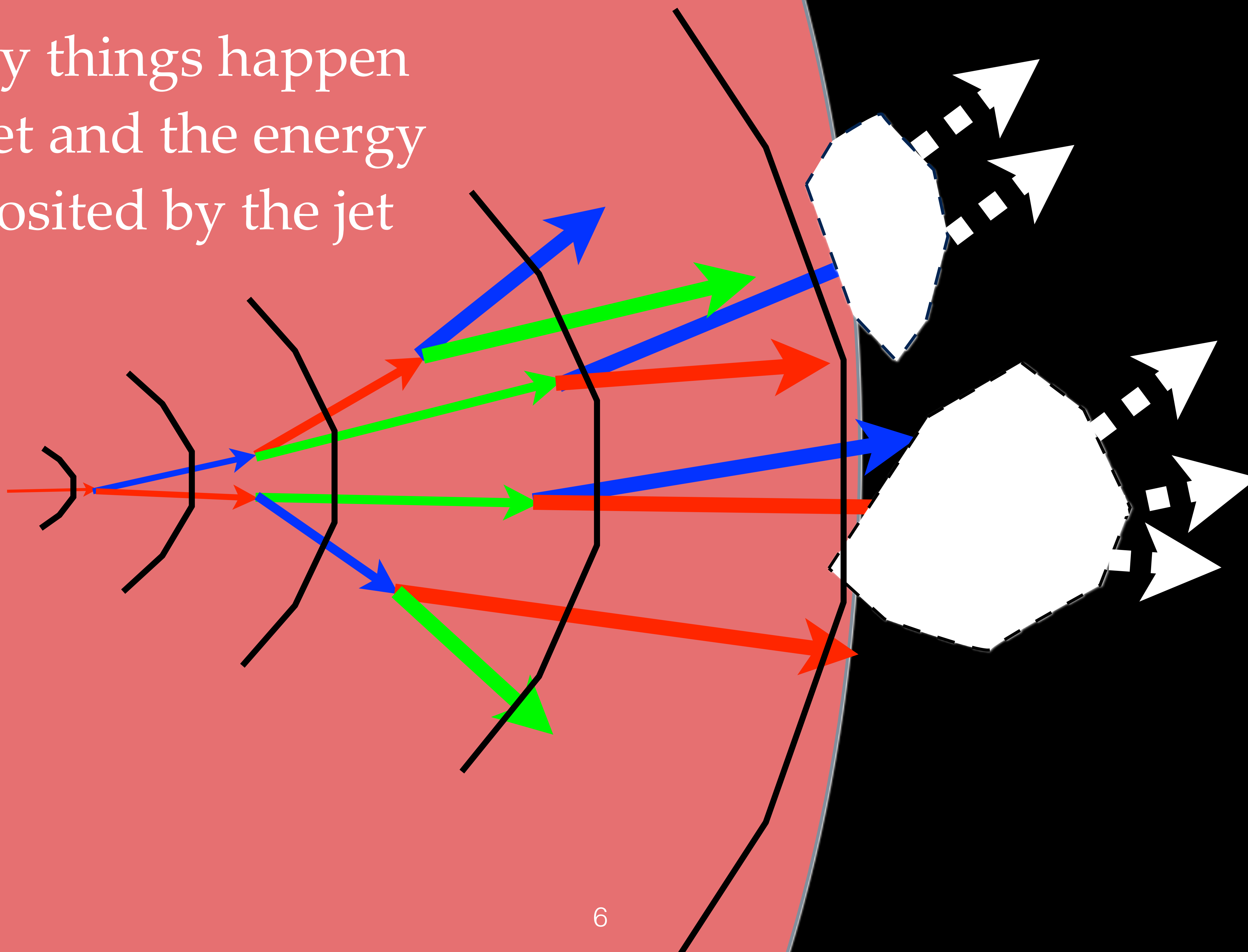

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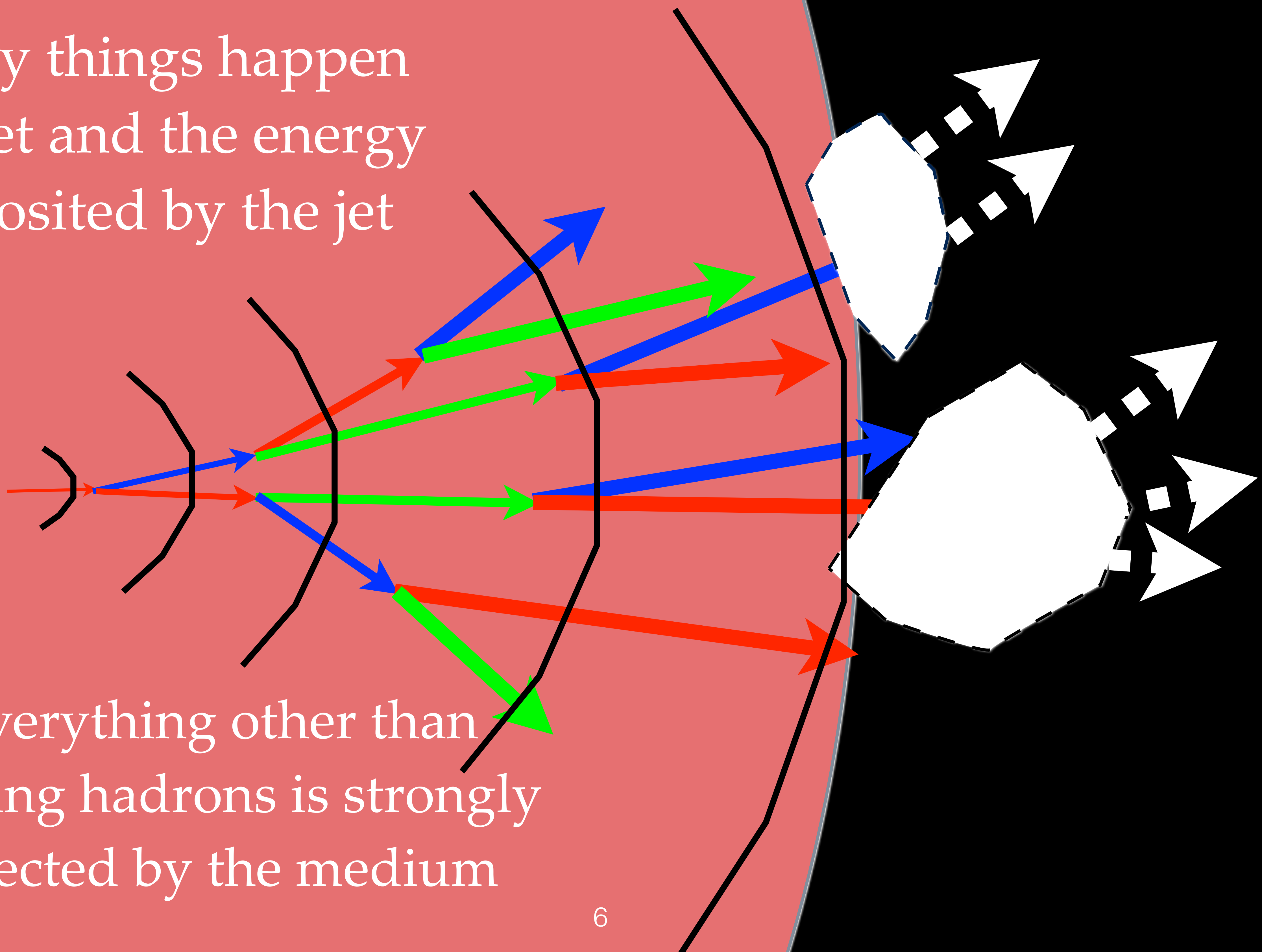
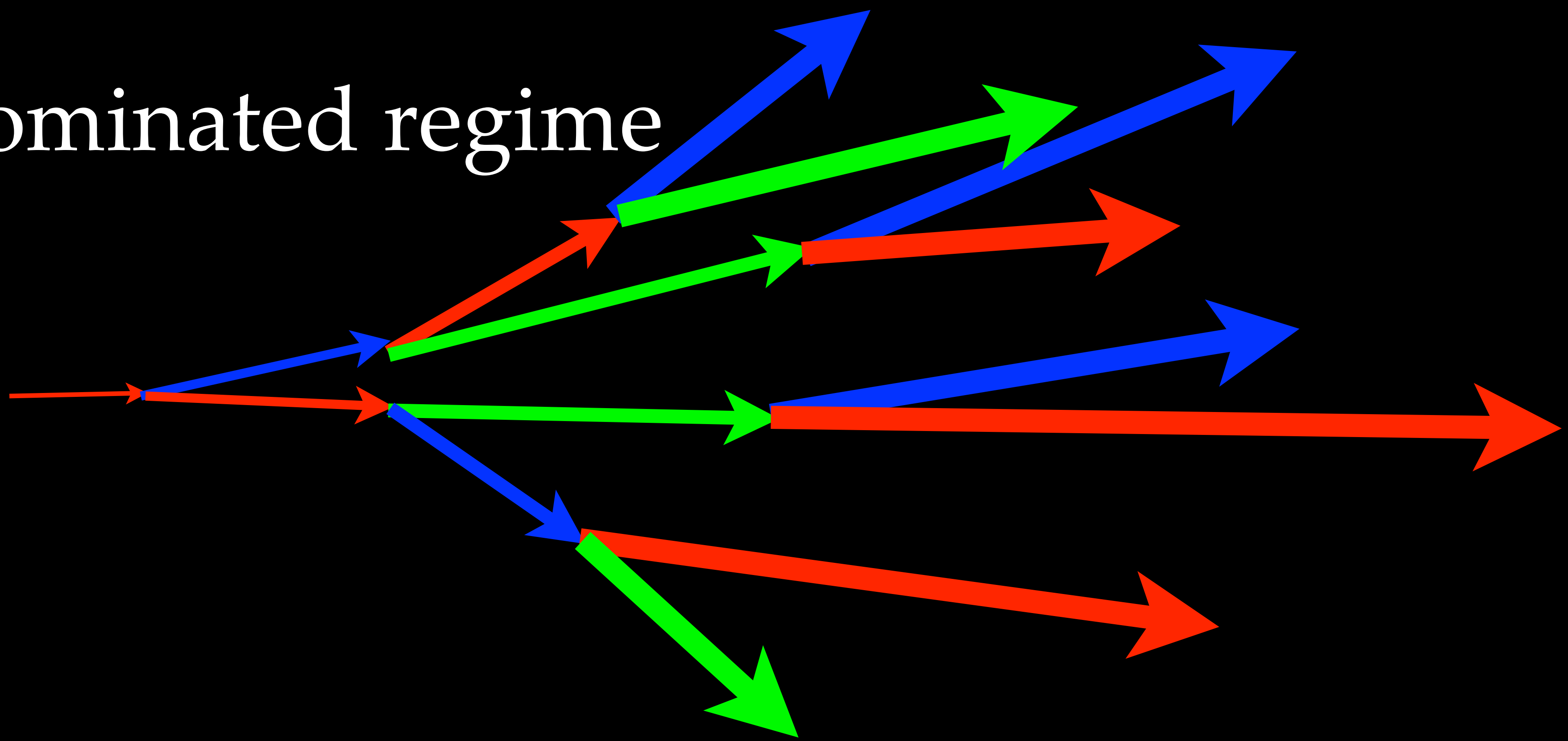


Everything other than leading hadrons is strongly affected by the medium

High energy and high virtuality part of shower

- Radiation dominated regime



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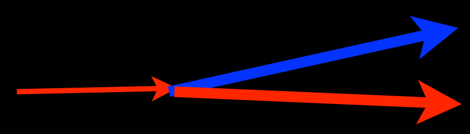

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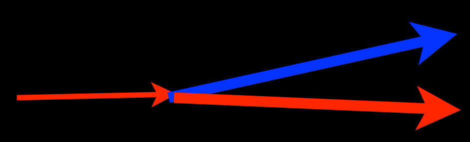

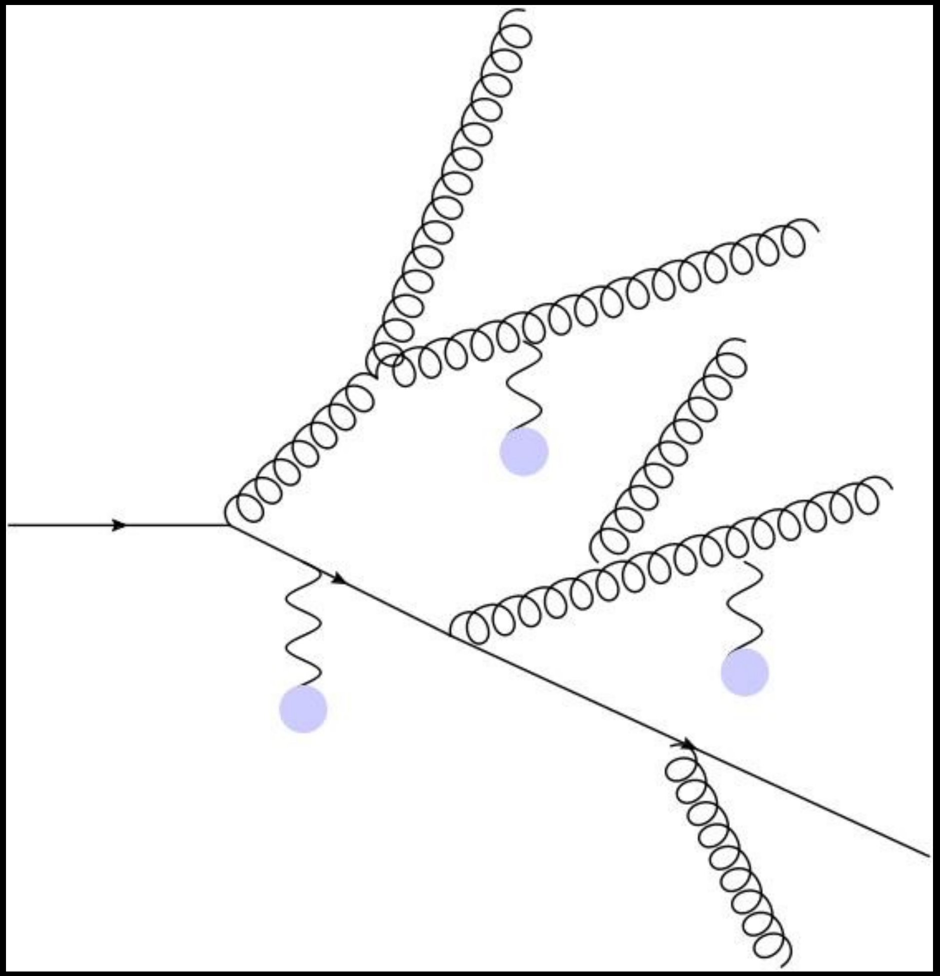

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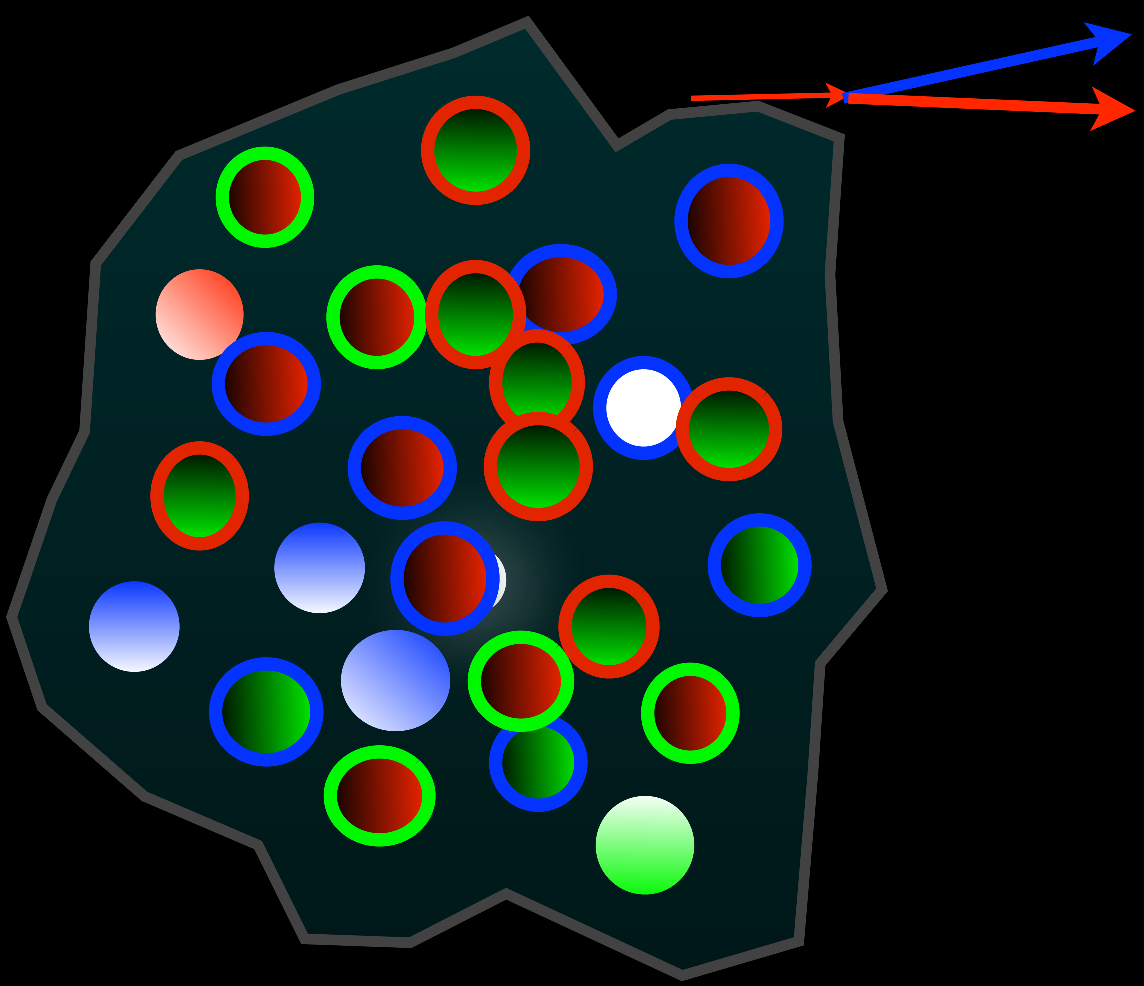

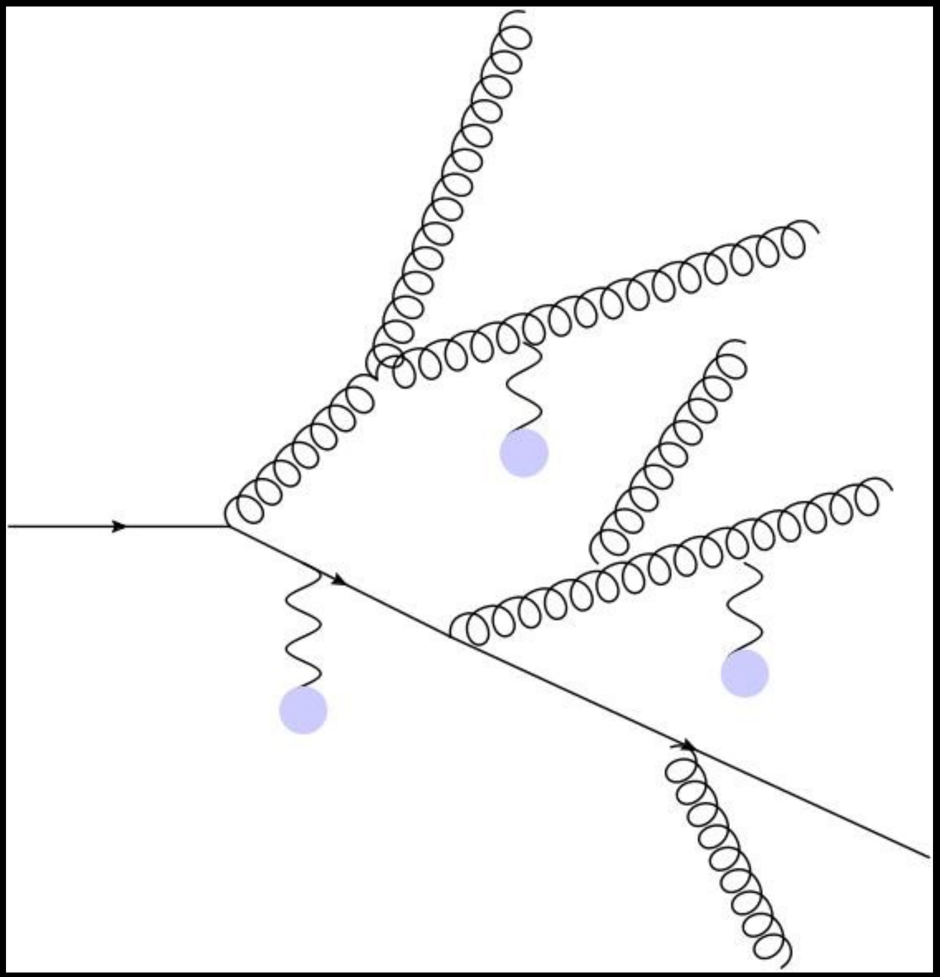

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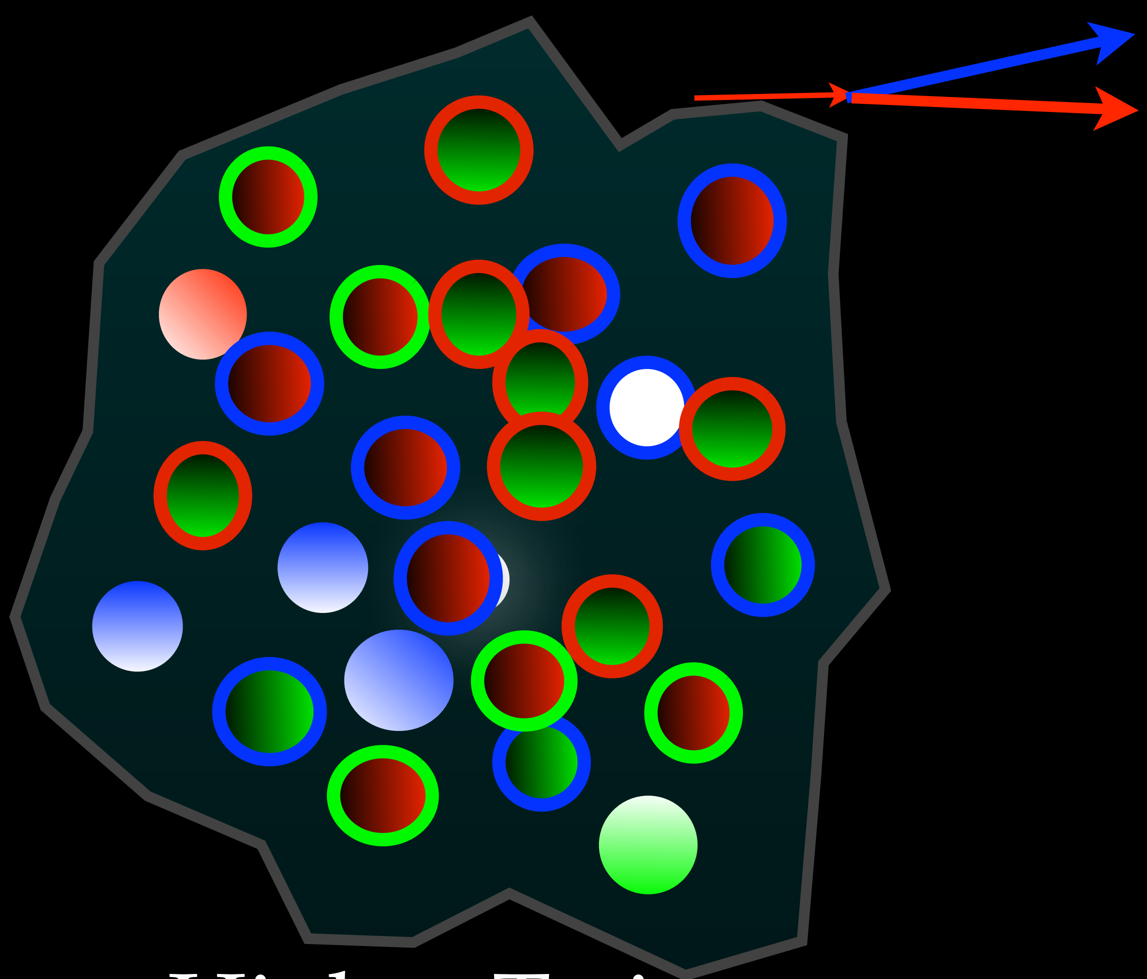
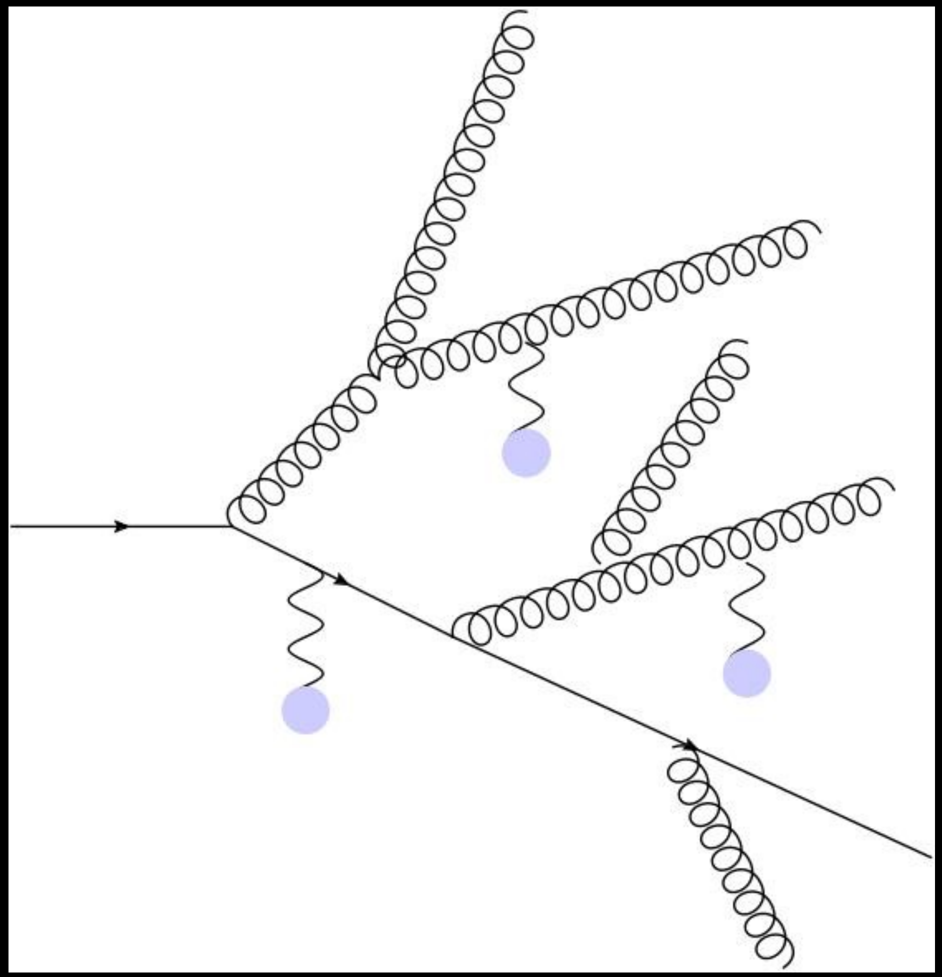


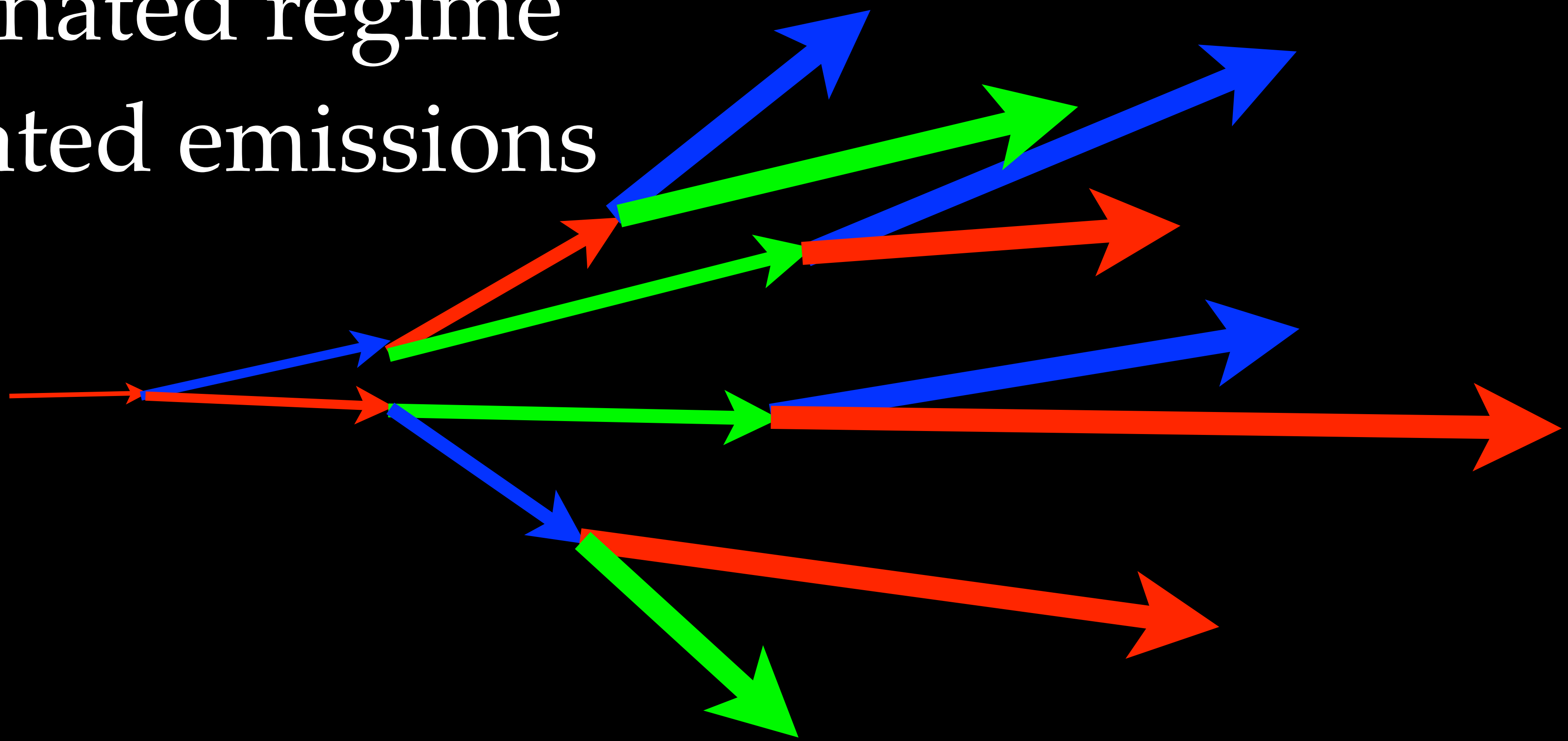


Theory: Higher Twist (X. Guo X.-N. Wang)

MC: MATTER, YaJEM, Qin F P approach

Low virtuality, high energy part

Scattering dominated regime
Few, time separated emissions



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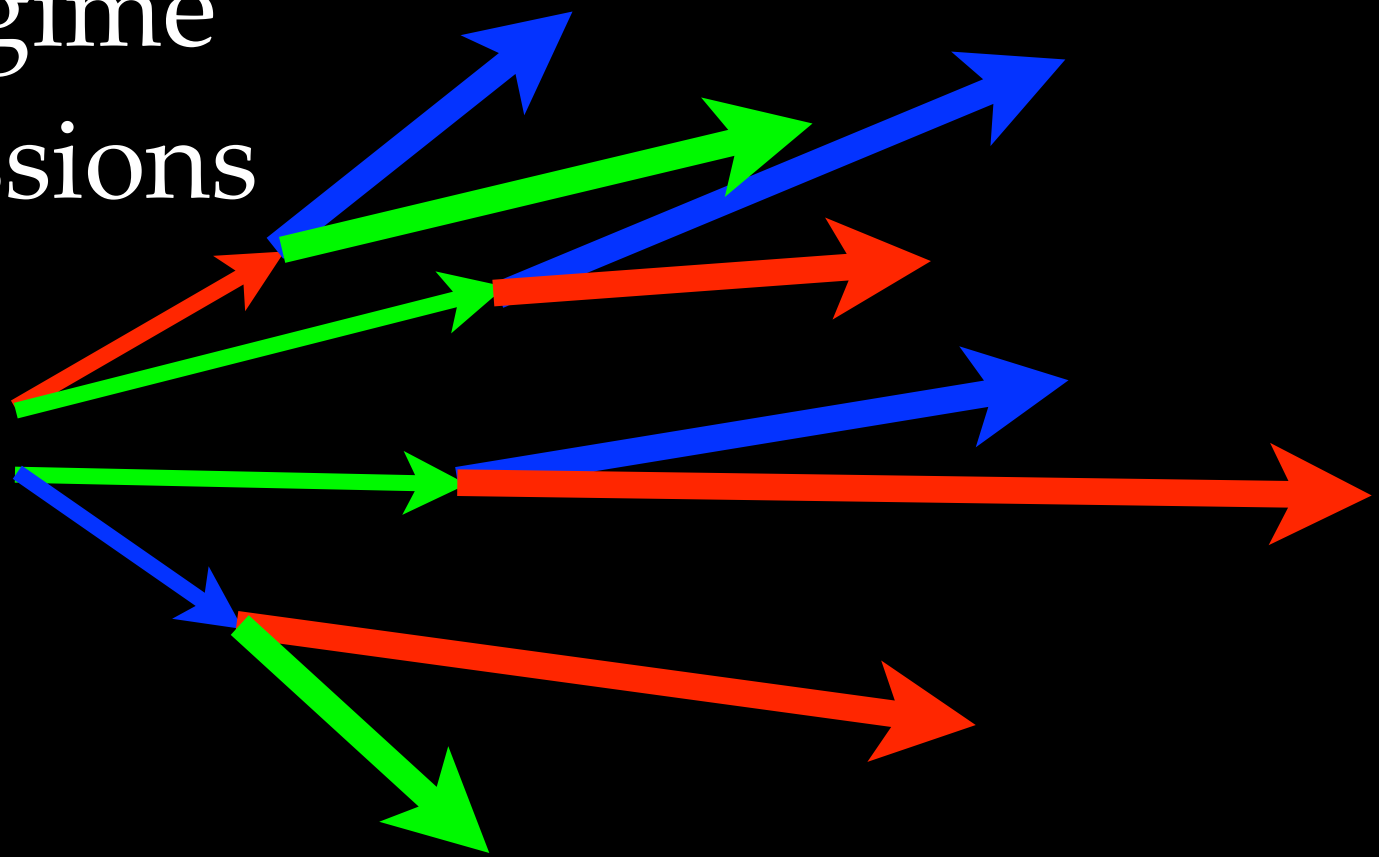

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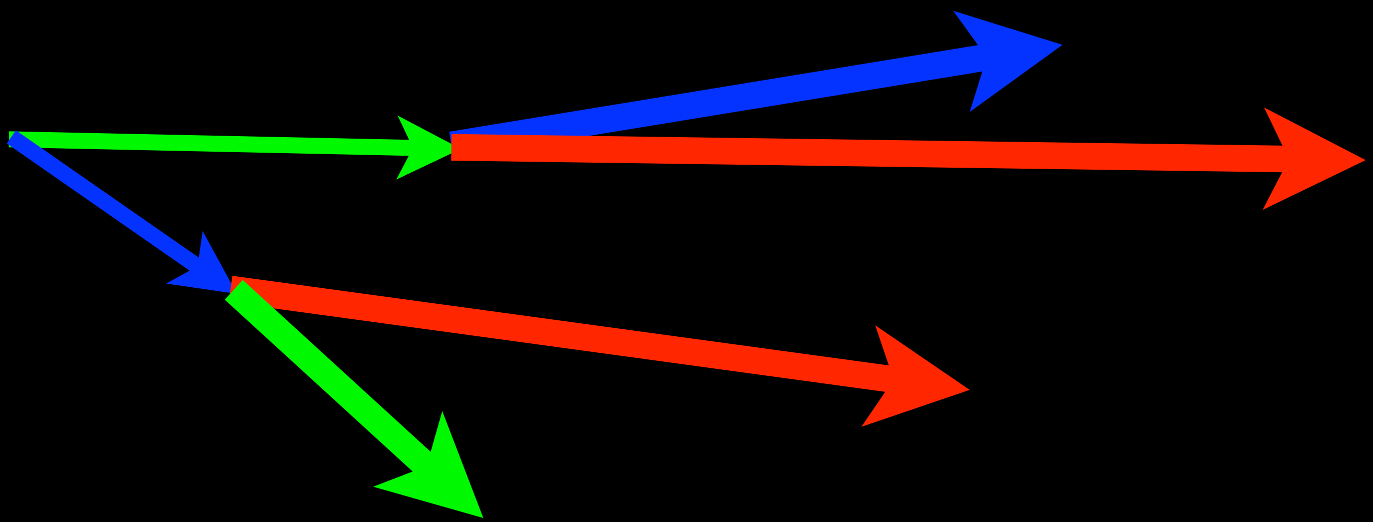

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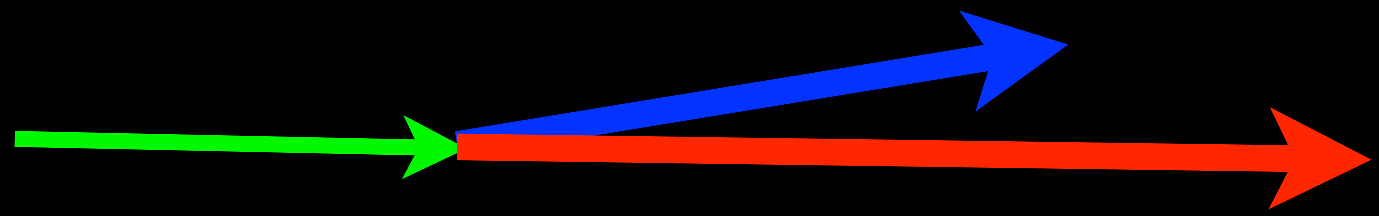

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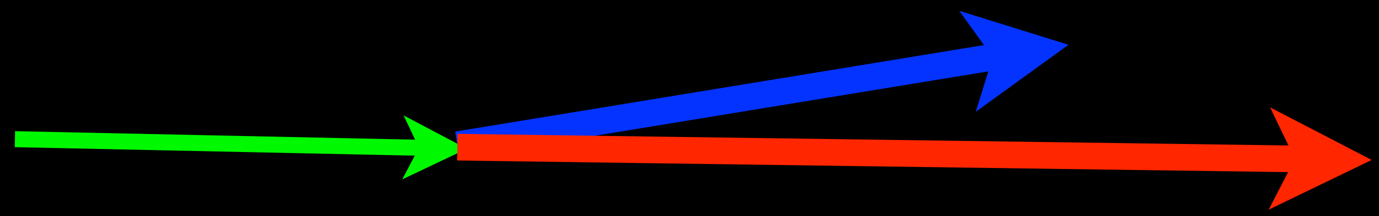

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MC: MARTINI, JEWEL,LBT

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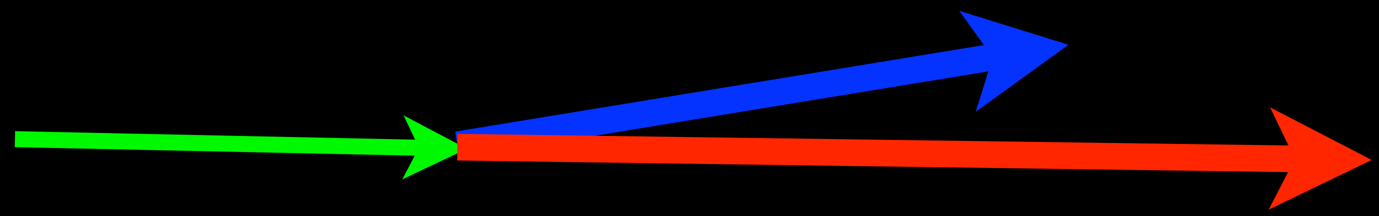
$Q^2 = q\,\tau$

τ: *lifetime of a parton*



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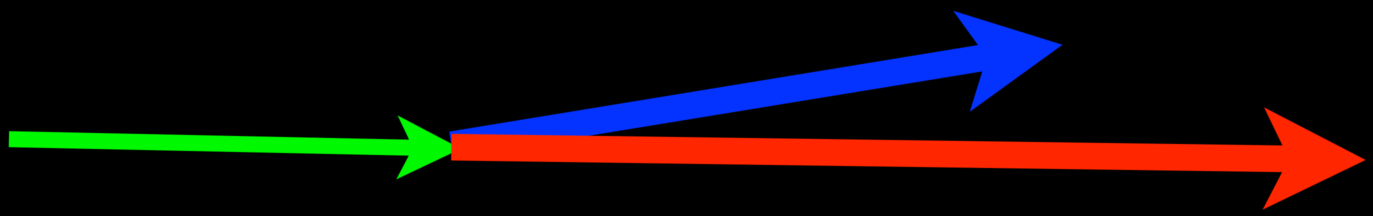

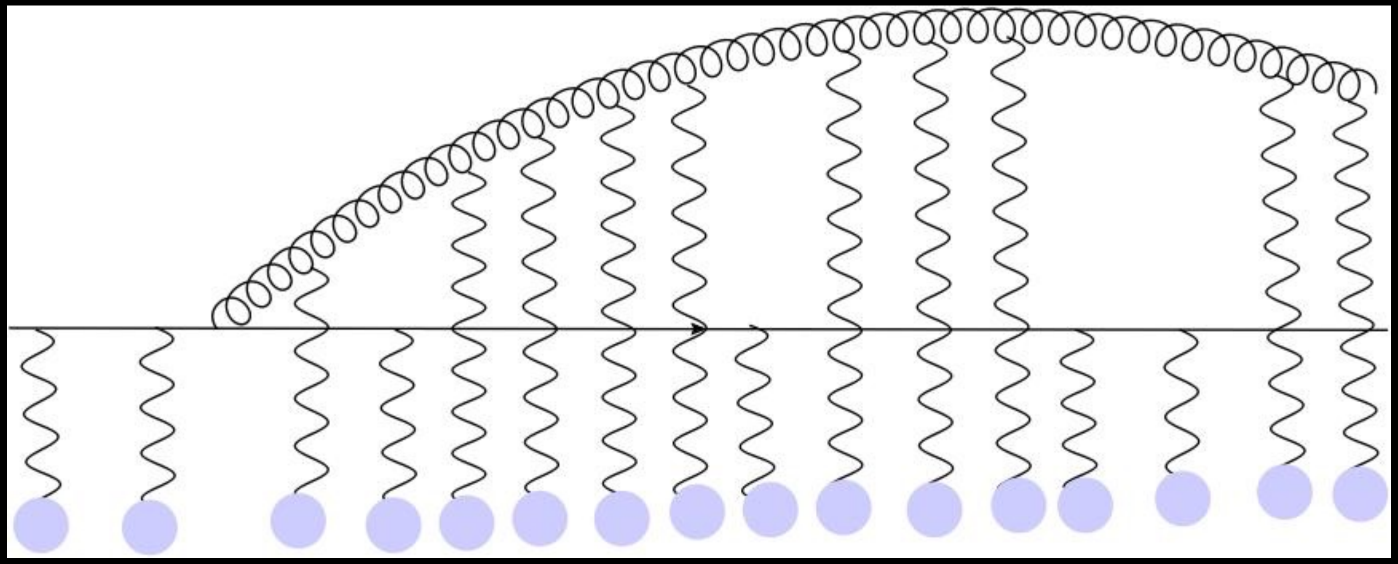

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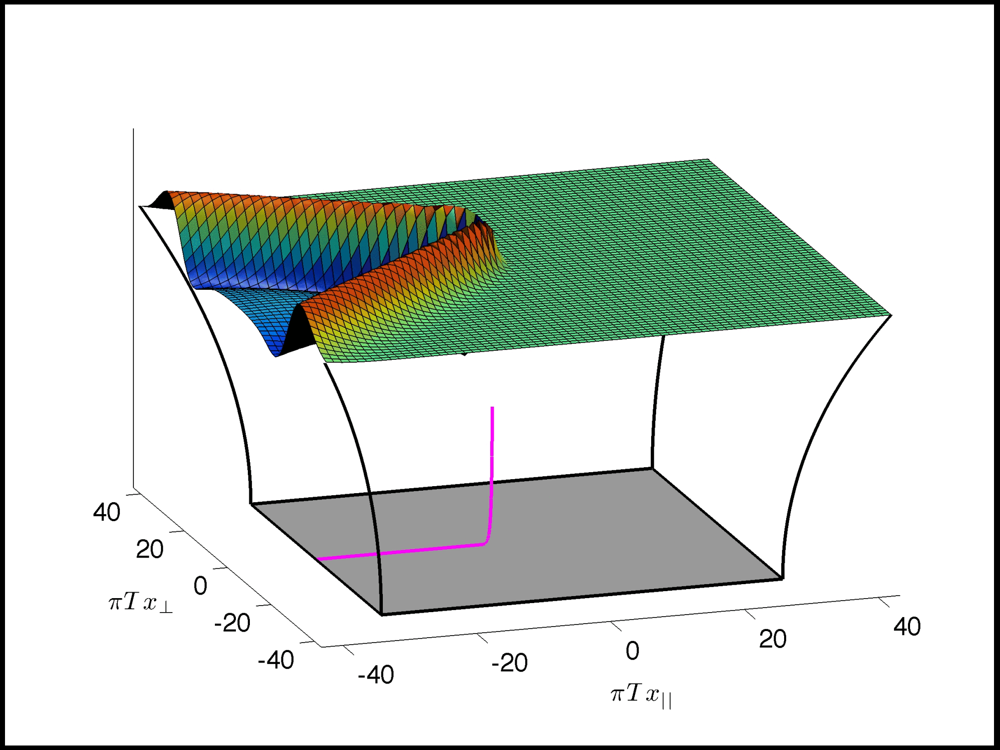

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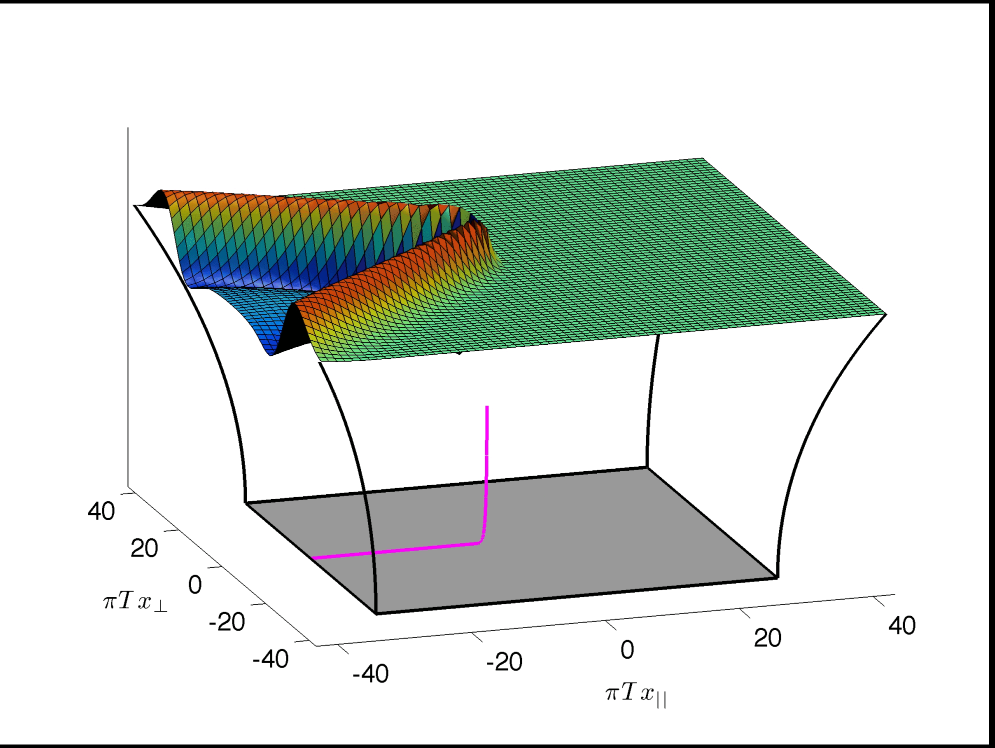
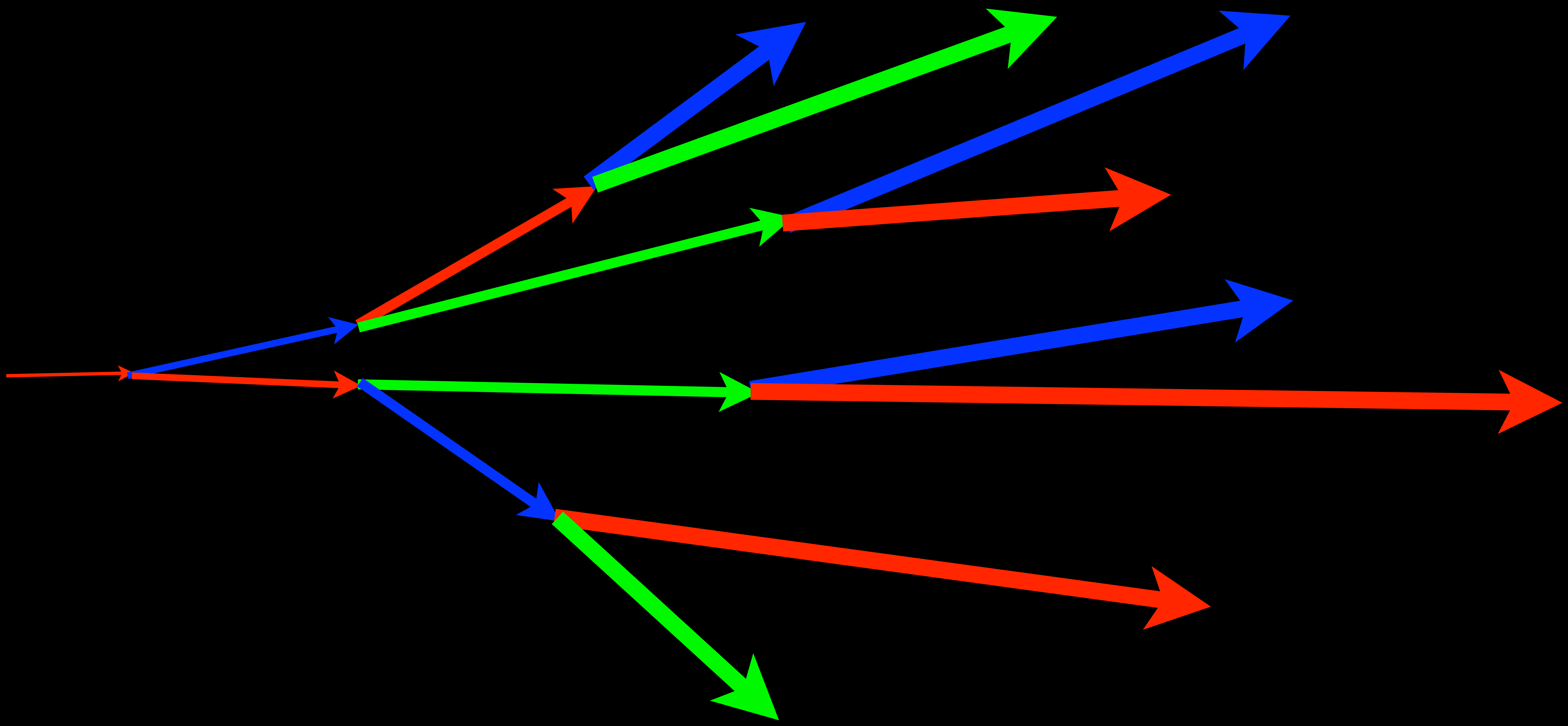


P. Chesler, W. Horowitz J. Casalderrey-Solana, G. Milhano, D. Pablos, K. Rajagopal

Grand picture (leading hadrons)



In a static brick

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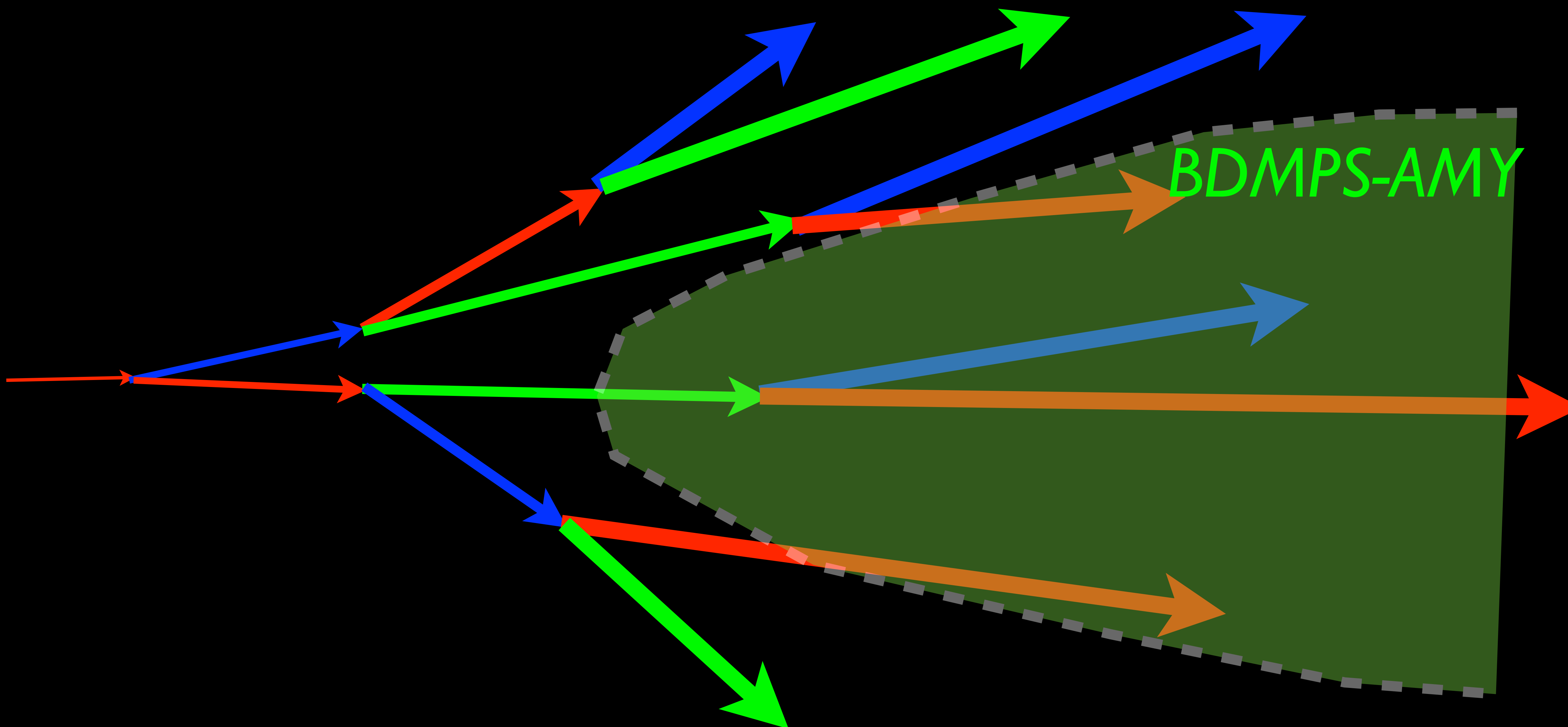

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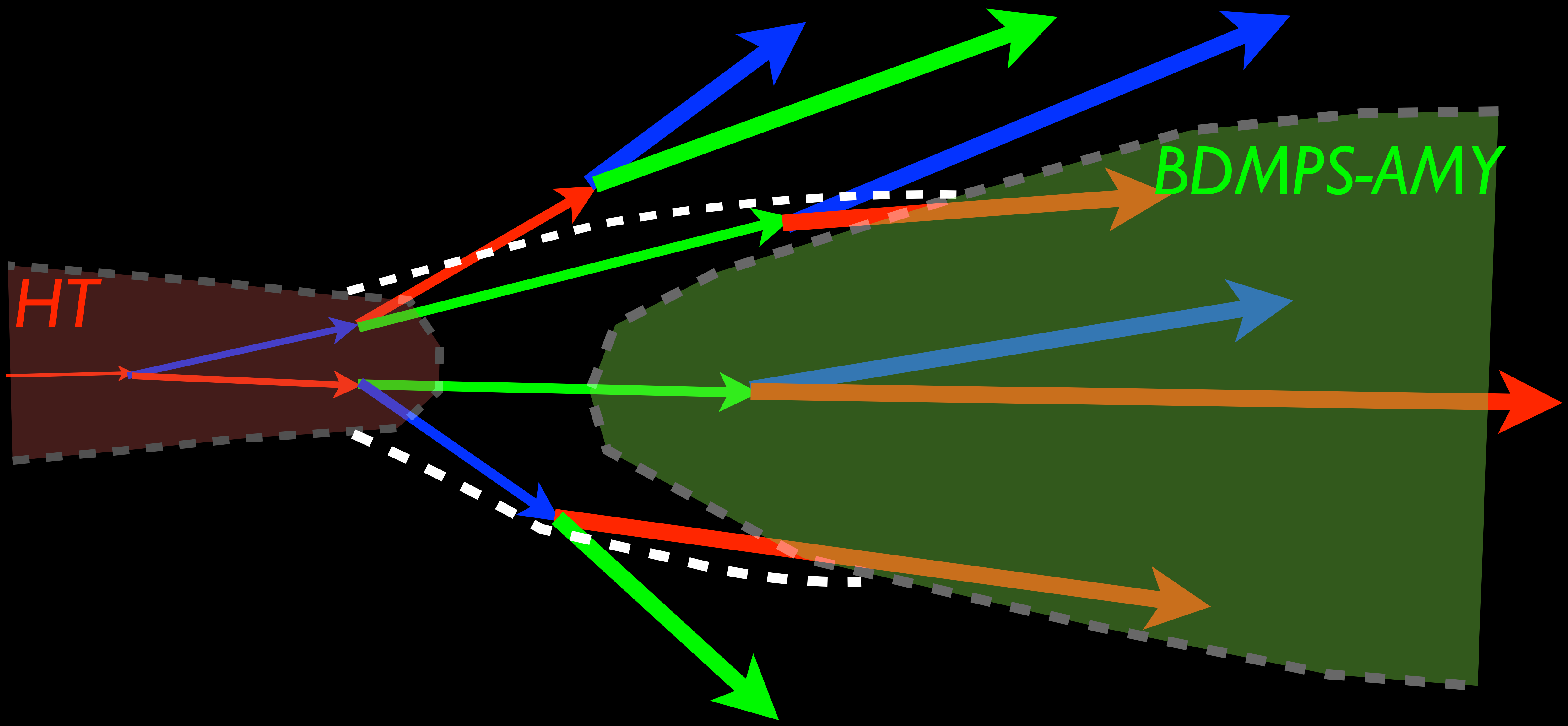

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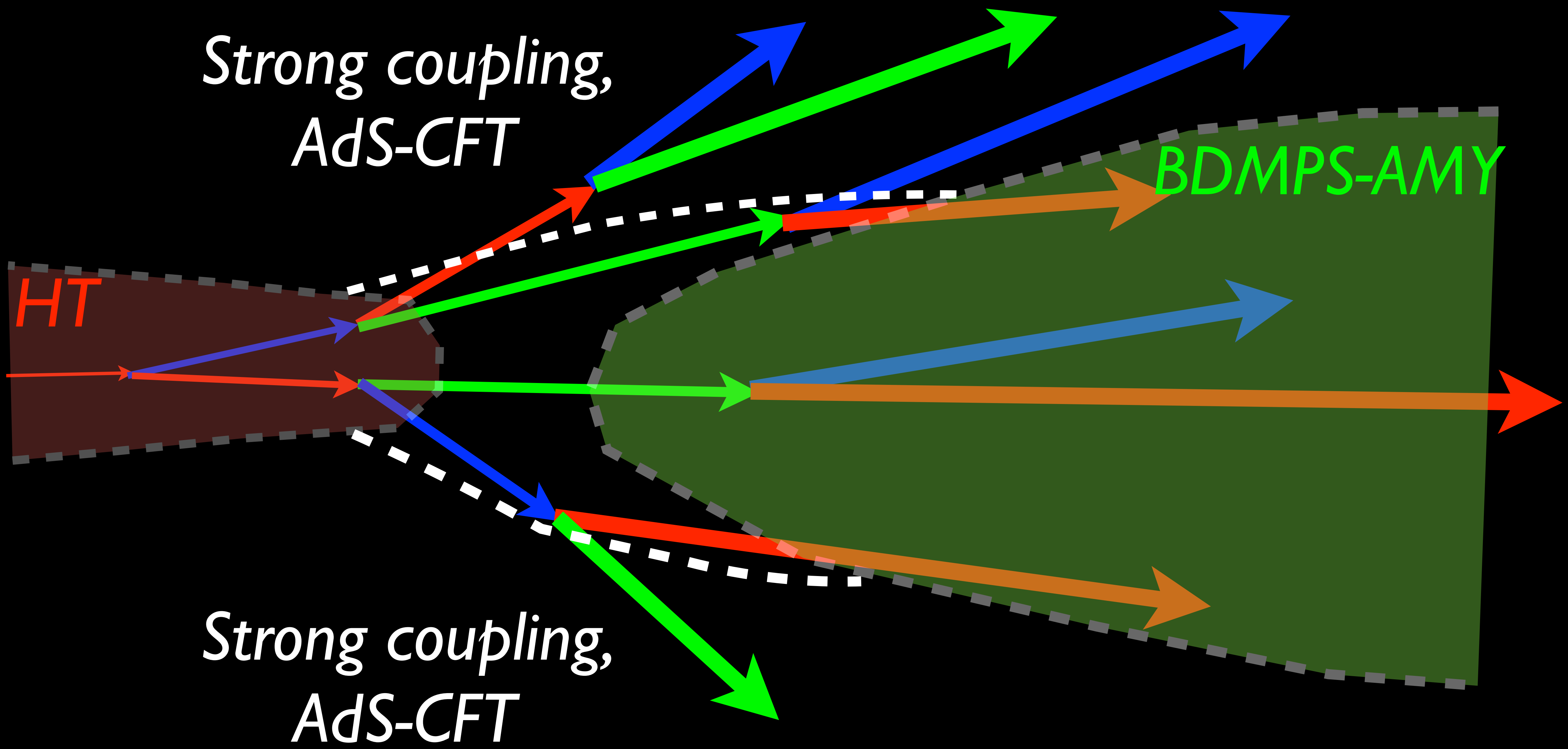

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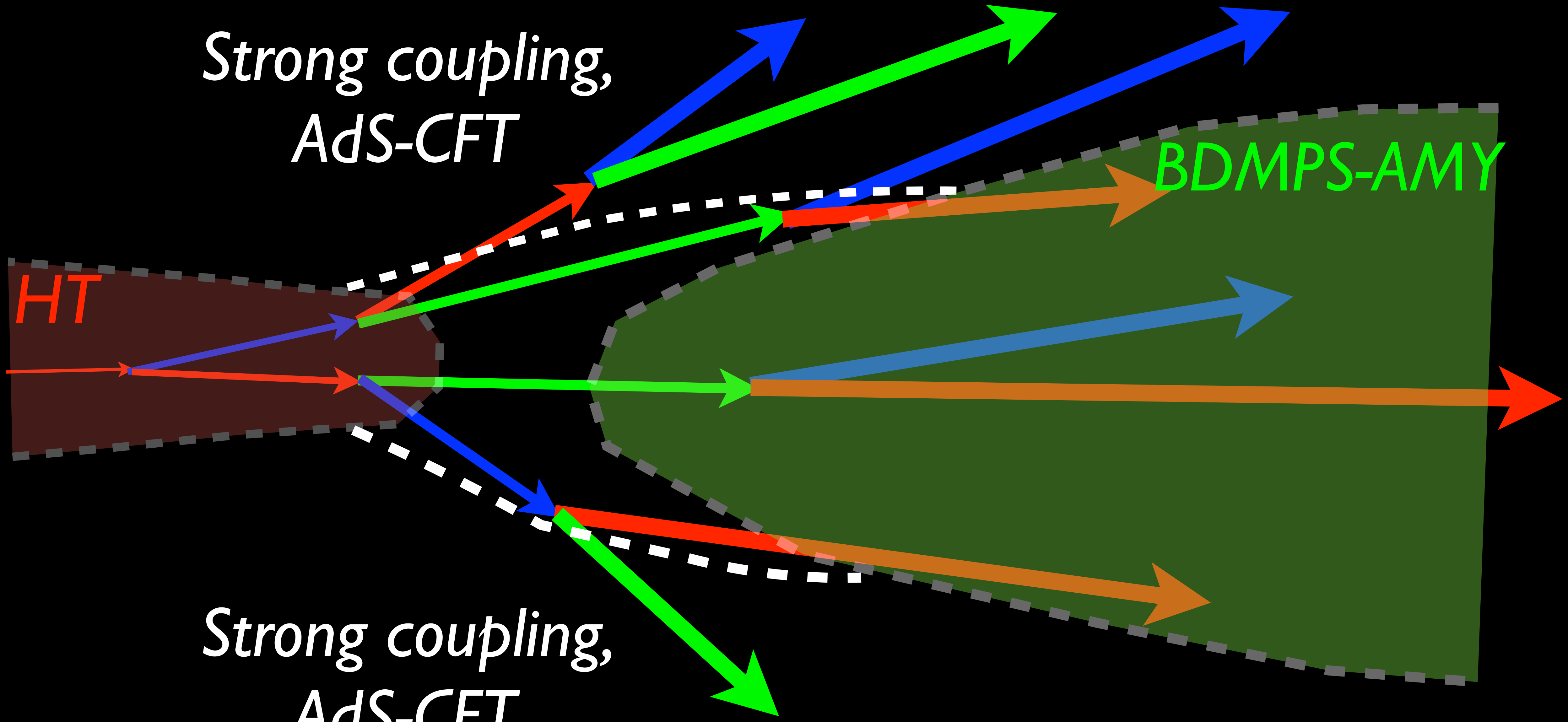

In an expanding QGP

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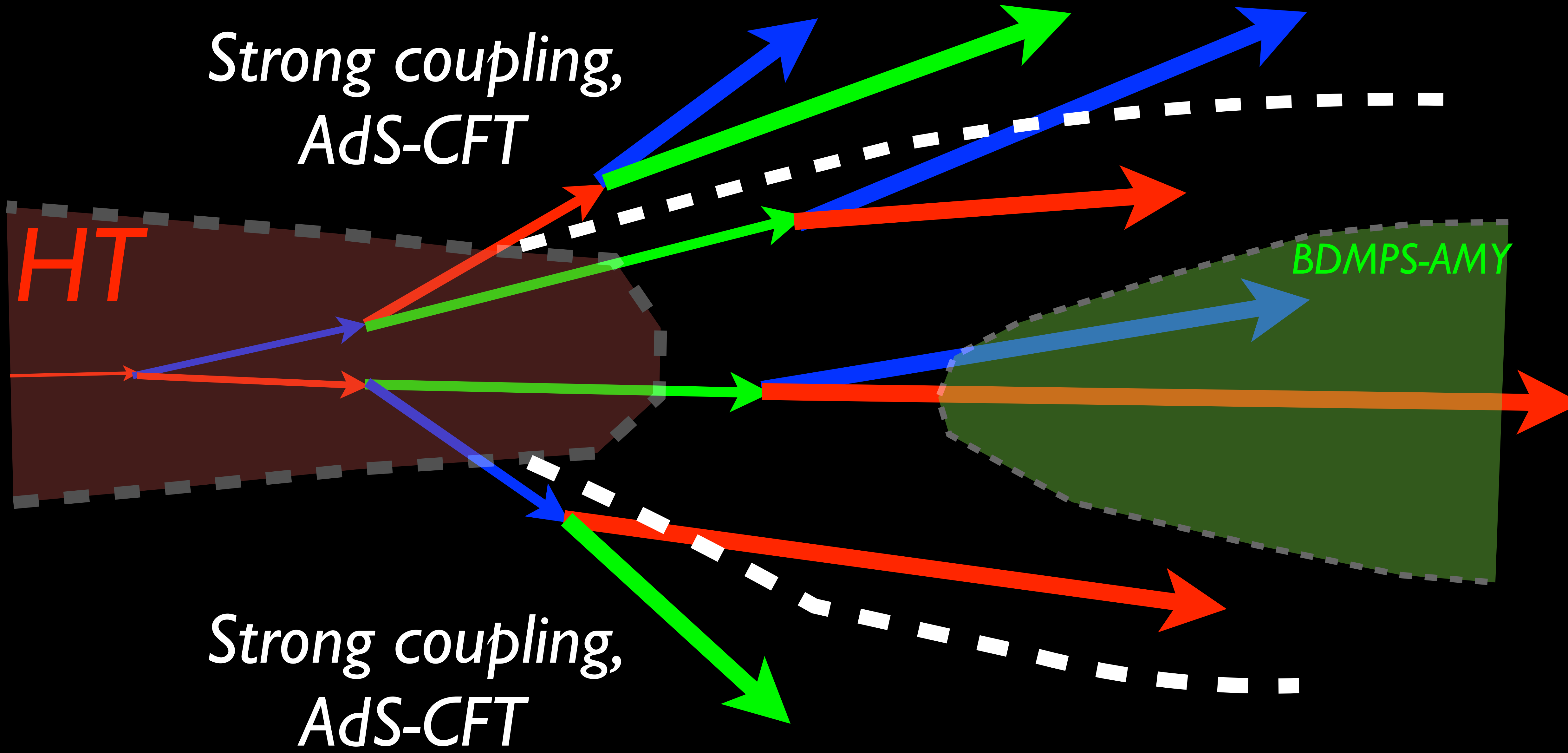
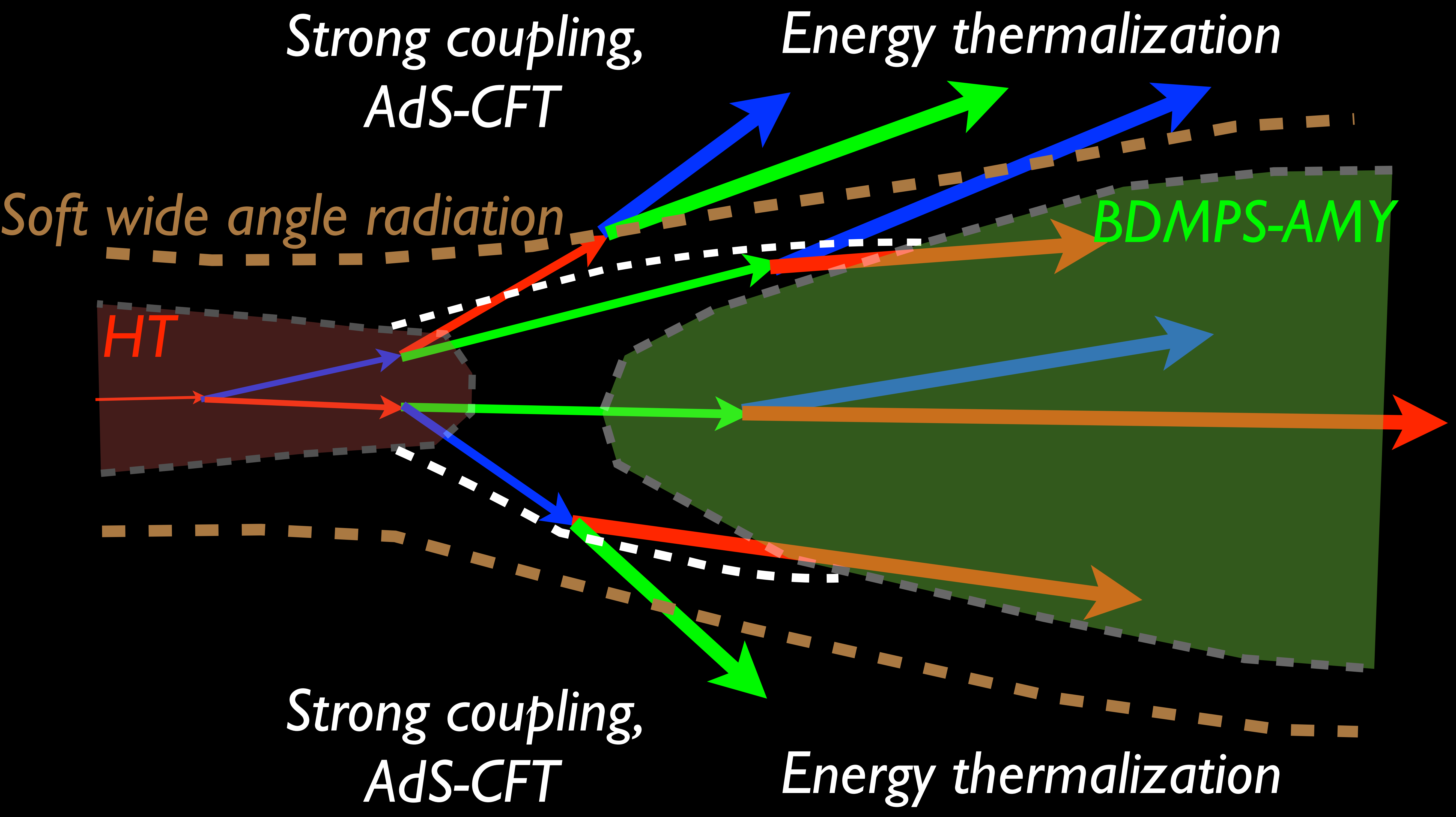


In an expanding QGP

Energy deposition-thermalization



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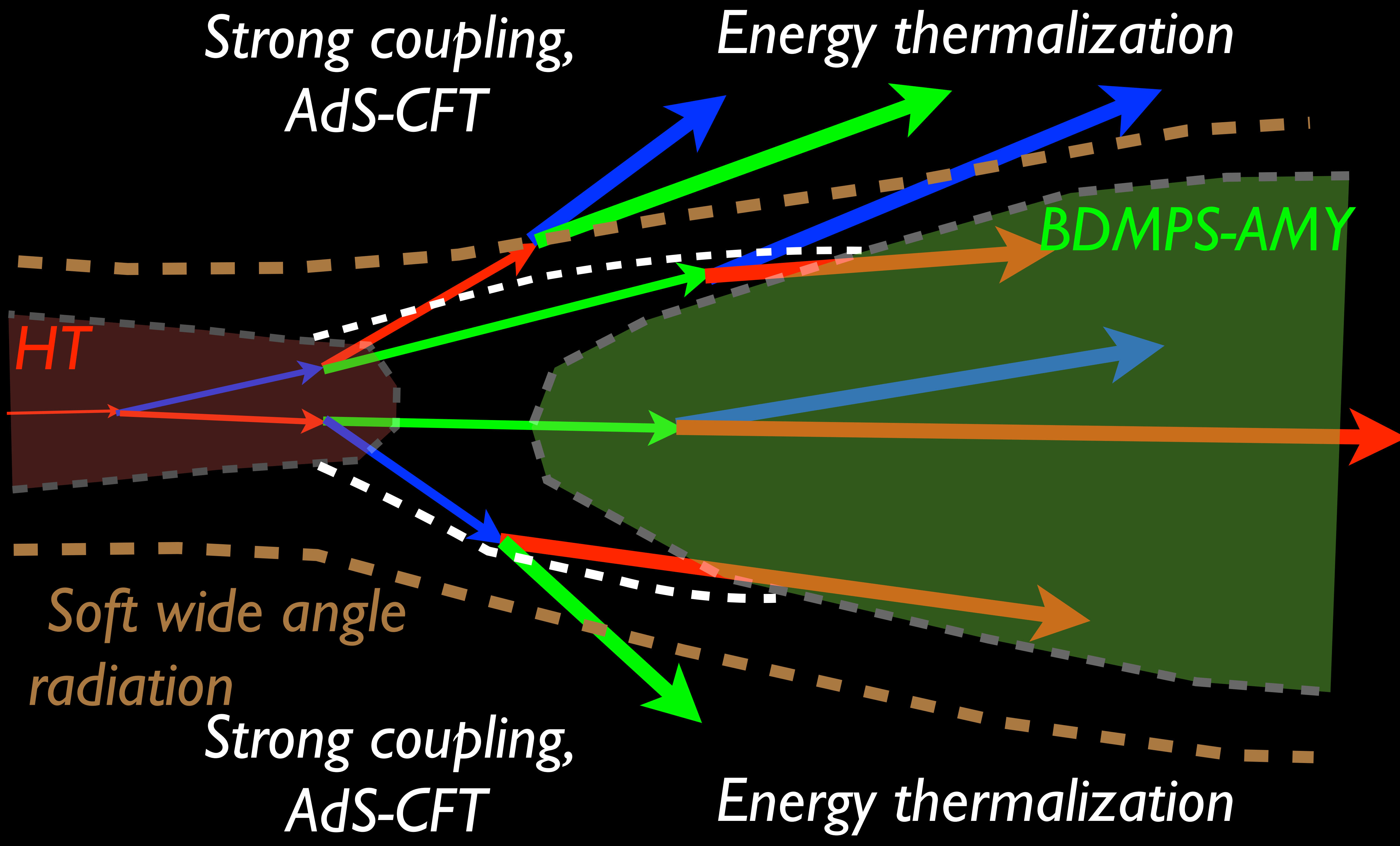

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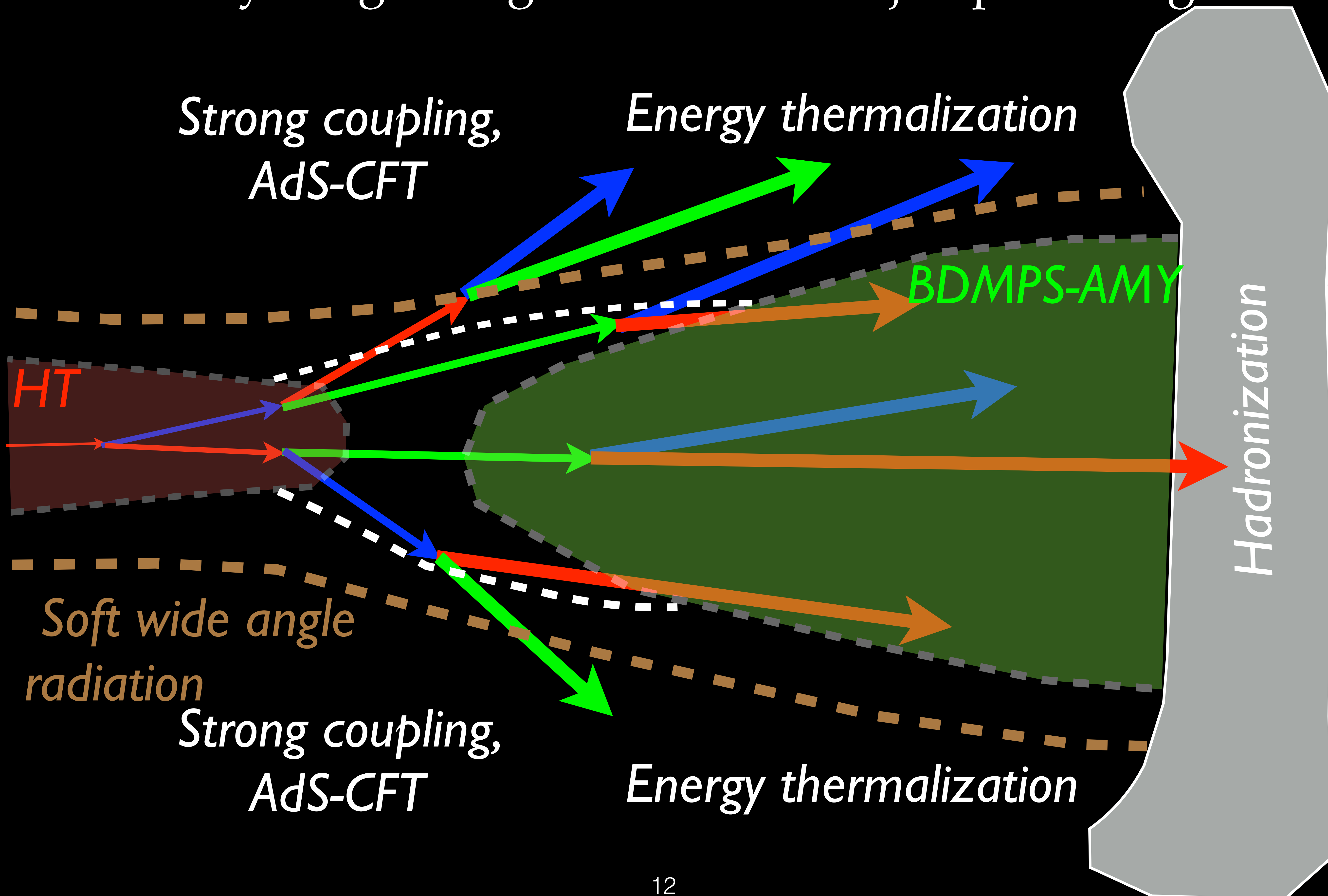
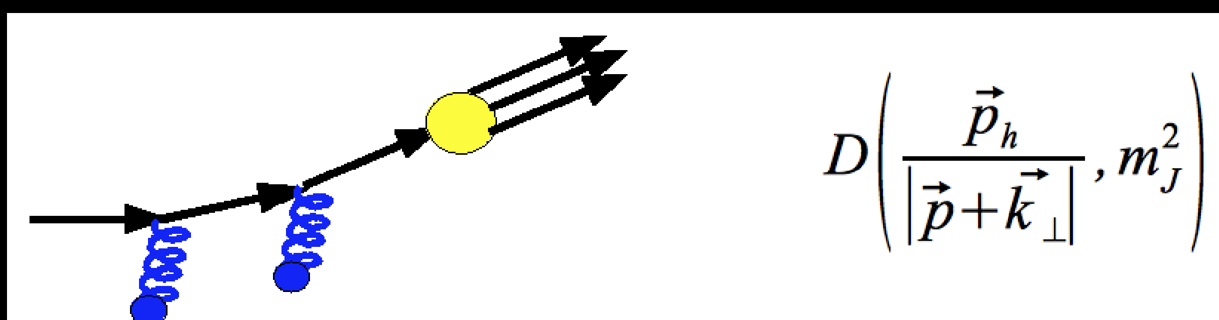


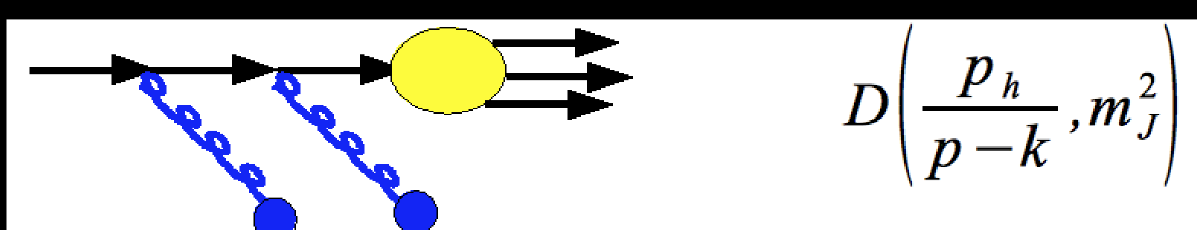
Transport coefficients partons in a dense medium

$$p_z^2 \simeq E^2 - p_\perp^2 \qquad\qquad p^+ \simeq p_\perp^2/2p^-$$



$$\hat{q} = \frac{\langle p_\perp^2 \rangle_L}{L}$$

Transverse momentu
diffusion rate



$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

Elastic energy
loss rate
also diffusion
rate e_2

By definition, describe how the medium modifies the jet parton!

In general, 2 kinds of transport coefficients

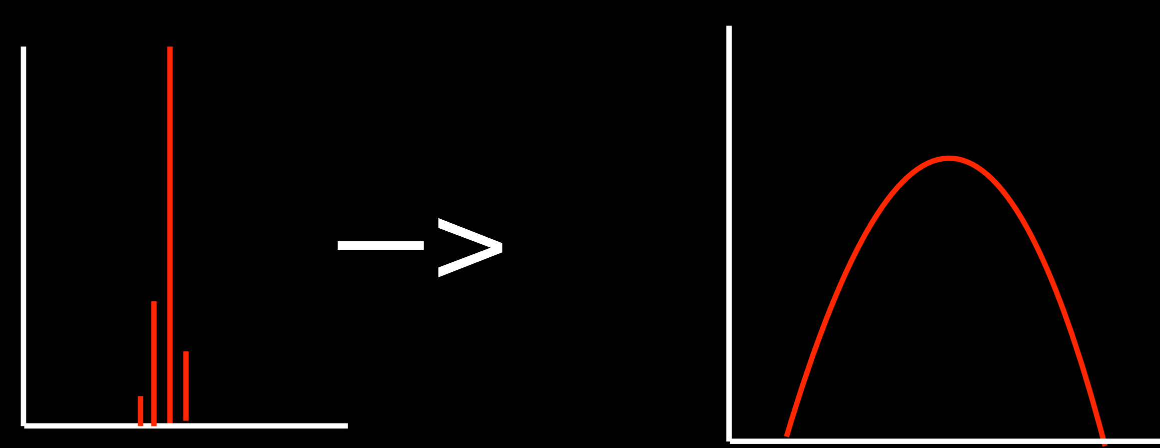
Type 1: which quantify how the medium changes the jet

$$\hat{q}(E, Q^2) \qquad \hat{q}_4(E, Q^2) = \frac{\langle p_T^4 \rangle - \langle p_T^2 \rangle^2}{L} \ldots$$

$$\hat{e}(E, Q^2) \qquad \hat{e}_2(E, Q^2) = \frac{\langle \delta E^2 \rangle}{L} \qquad \hat{e}_4(E, Q^2) = \frac{\langle \delta E^4 \rangle - \langle \delta E^2 \rangle^2}{L} \ldots$$

Type 2: which quantify the space-time structure of the deposited energy momentum at the hydro scale

$$\delta T^{\mu\nu}$$



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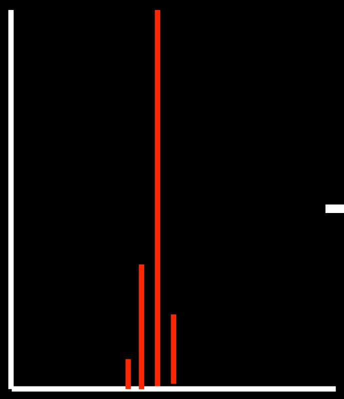

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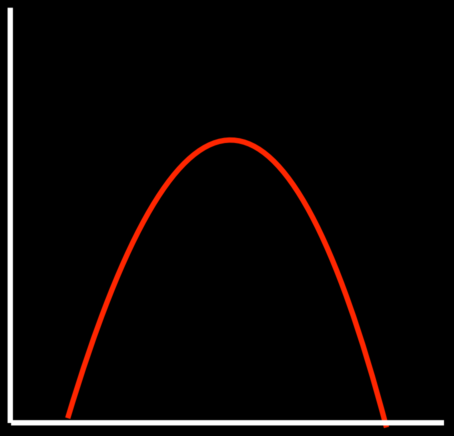

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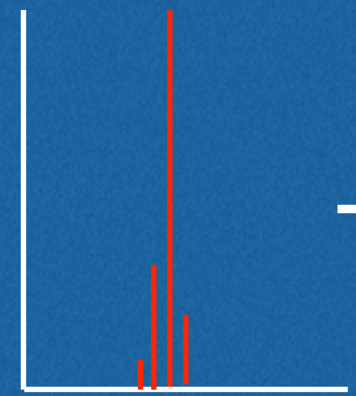

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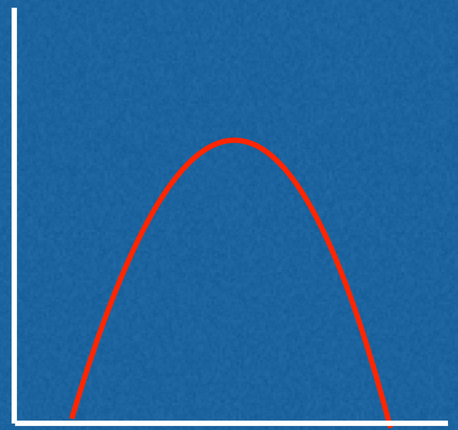

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        2. Dihadron, I_{AA}, γ-Hadron

(clear dependence on q, but also require fragmentation functions)

    2. Weaker dependence on hard σ :
        1. Near side I_{AA} ! *(badly surface biased)*
2. Observables that depend on type 1 and some type 2
    1. Strong dependence on hard σ :
        1. Jet R_{AA}, high p_T v_2!
        2. DiJets (X_J), γ-Jet

(reduce dependence on type 2 by increasing E, lose sensitivity, reduce R, requires resummation)

    2. Weaker dependence on hard σ :
        1. z_g
        2. Jet Mass, Jet shape

3. Observables that depend strongly on type 2

Jet medium correlations

Observables: more type 2, more MC

1. Observables that only depend on type 1
   1. Strong dependence on hard σ :
      1. Hadron R_{AA}, high p_T v_2!
      2. Dihadron, I_{AA}, γ-Hadron

(clear dependence on q, but also require fragmentation functions)

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   2. Weaker dependence on hard σ :
      1. z_g
      2. Jet Mass, Jet shape

3. Observables that depend strongly on type 2

   Jet medium correlations

What is the goal of this enterprise?

- We focus on about 5 jet coefficients, and 15 soft coefficients
- All of them are non-perturbative
- Determine these unambiguously from detailed phenomenology
- Have an extendable phenomenological framework
- Calculate them (if possible) from first principles
- Deeper understanding of the structure of the QGP.

Need a Monte-Carlo event generator based approach

Need to have a framework

- That can modularly incorporate a variety of theoretical approaches
- Which can allow you to model medium response, and entire range of transport coefficients
- Can address all observables simultaneously

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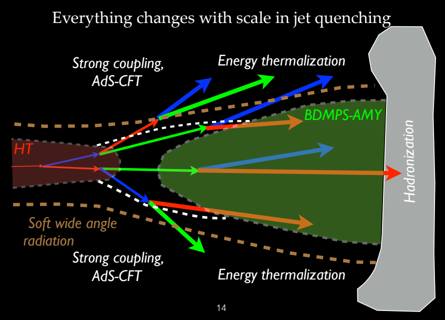

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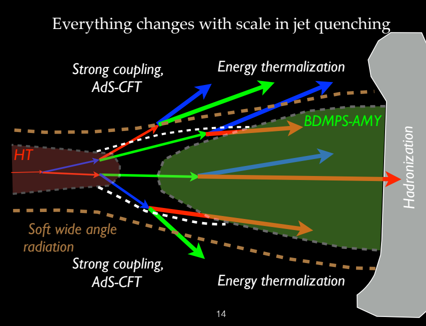

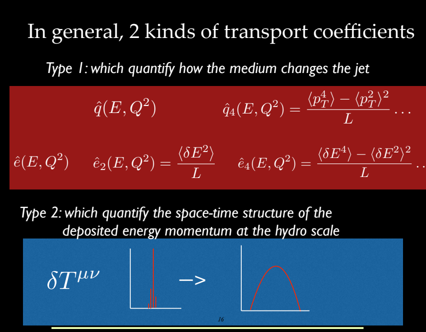

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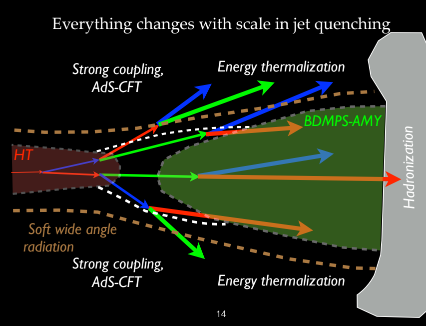

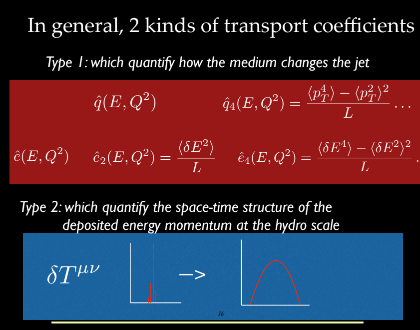

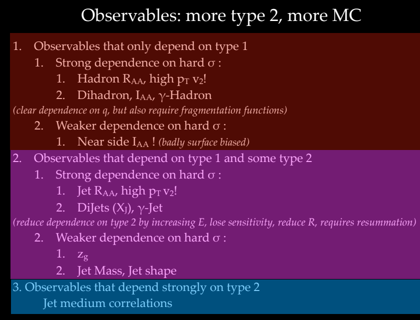

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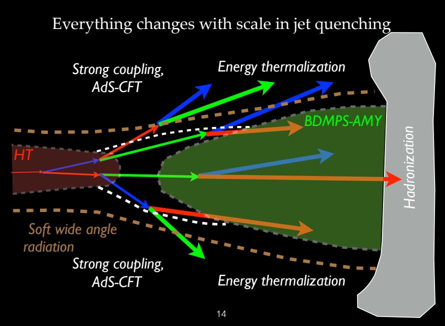
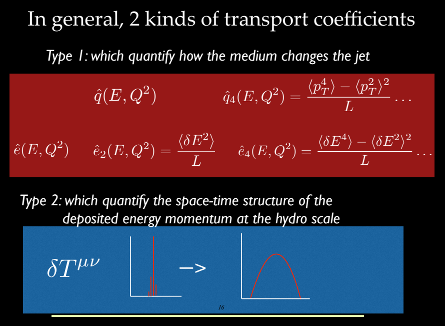
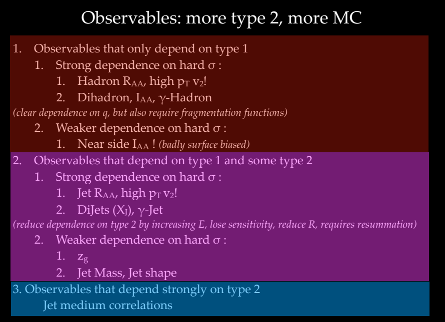




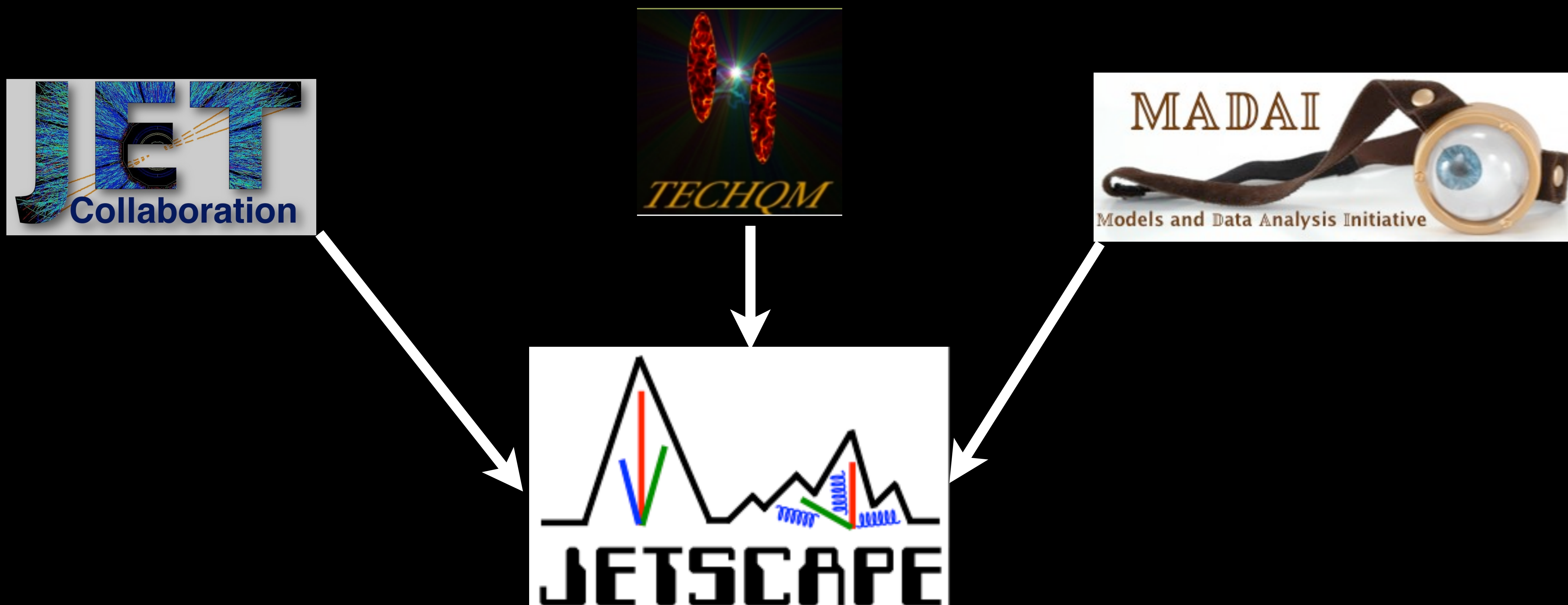


Such a framework now exists: JETSCAPE
https://github.com/JETSCAPE

JETSCAPE

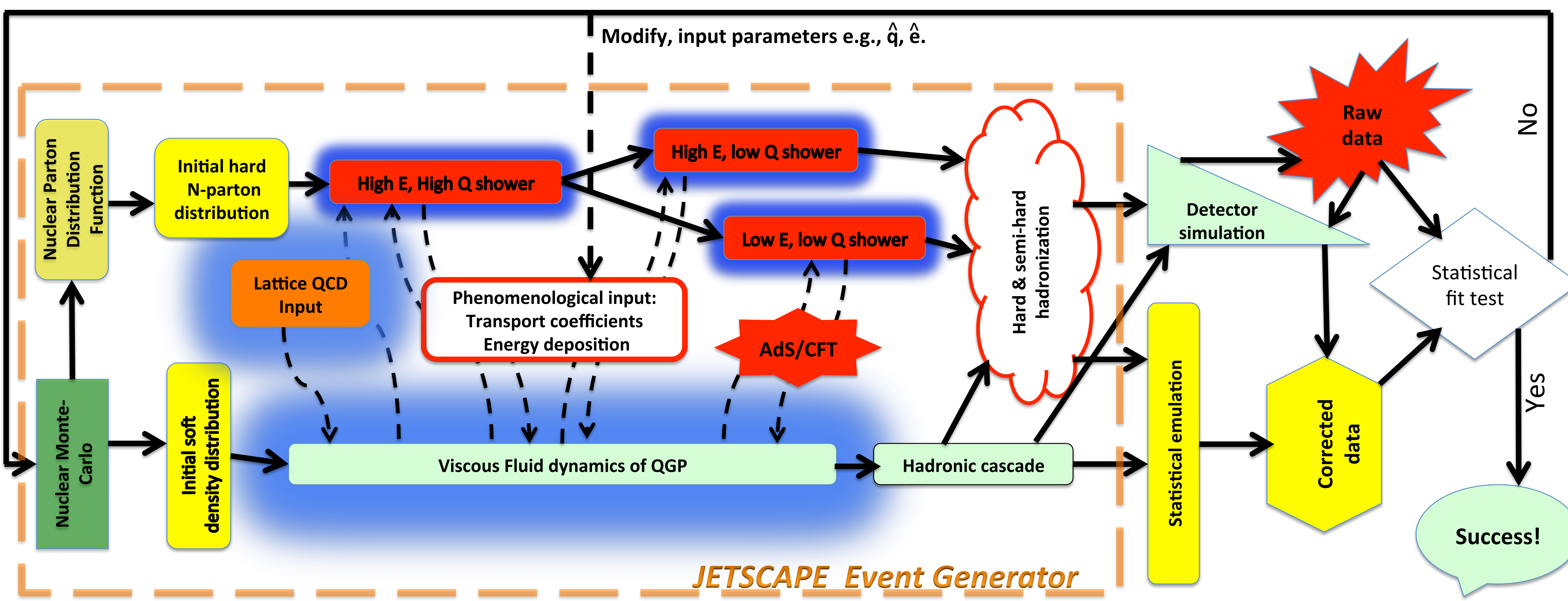
What is JETSCAPE?

Jet Energy-loss Tomography with a Statistically and Computationally Advanced Program Envelope.



10 Institutions, $4 M, 4 year NSF project

How would this work?



2 streams: one CPU stream and one GPU stream
Ideal for hybrid architecture.

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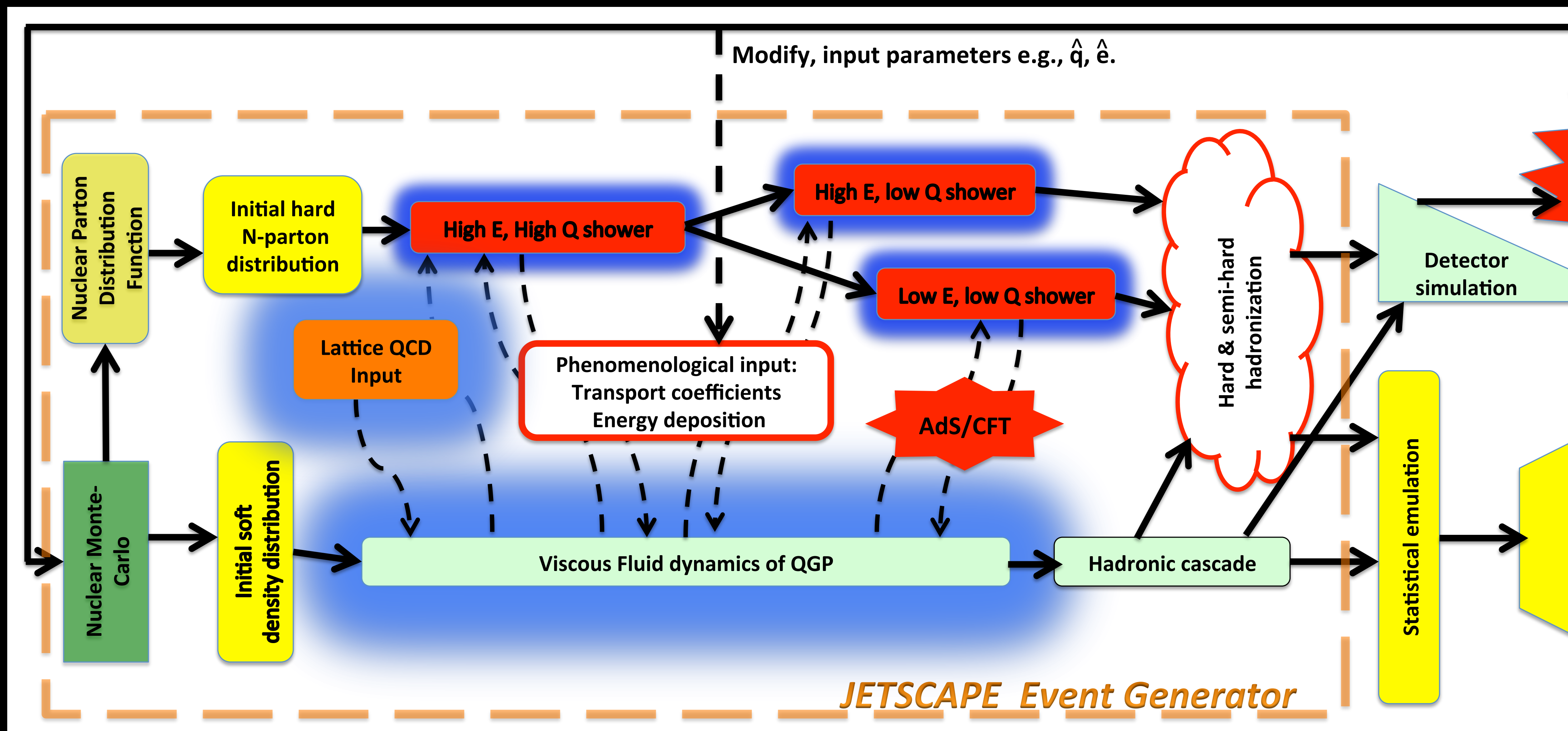

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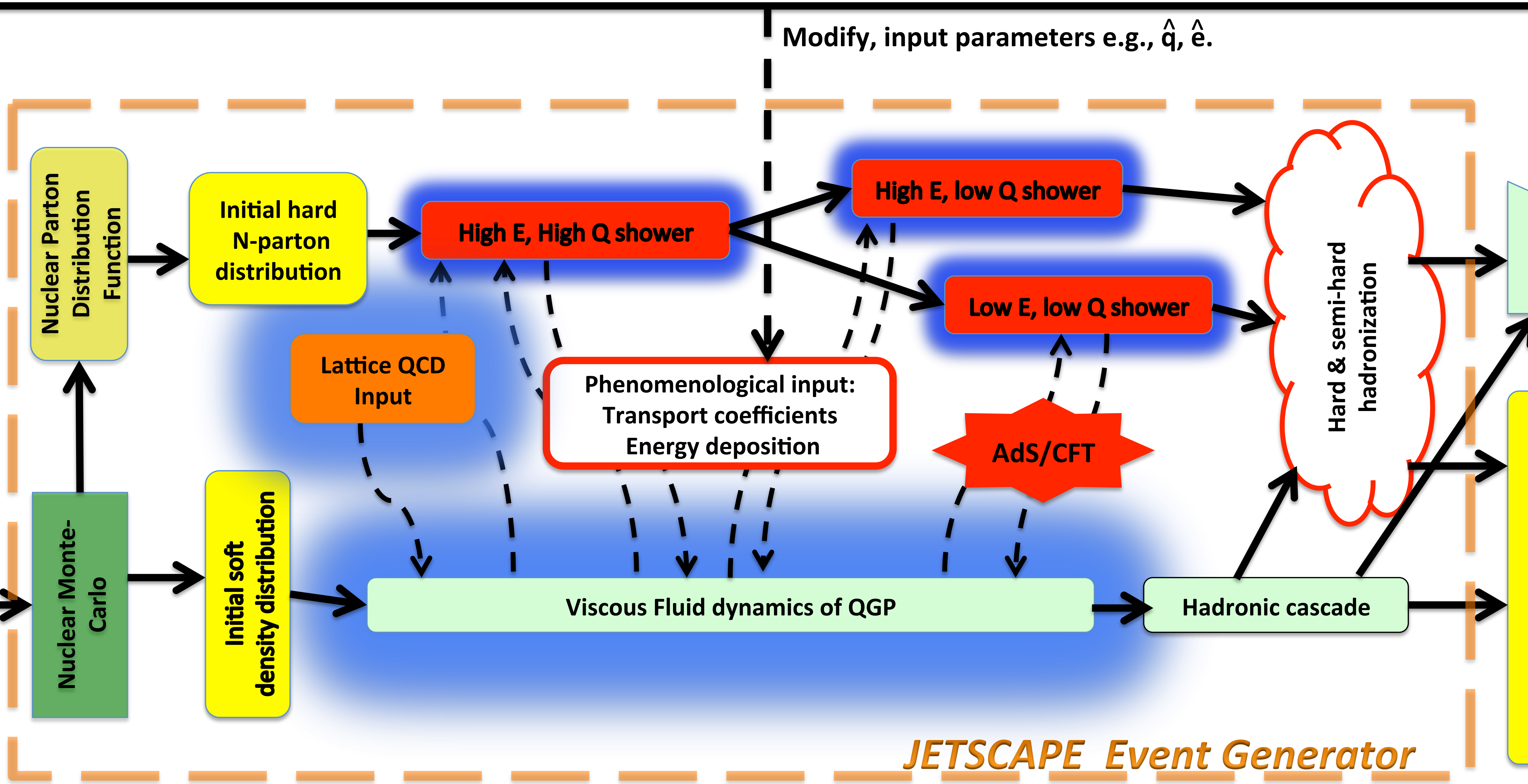


2 streams: one CPU stream and one GPU stream

Ideal for hybrid architecture.

Calibrating/tuning the generator

Need a Bayesian analysis to determine best value of 20 parameters

Each require 25 values that have to be sampled

Thus Number of sample points = 500.

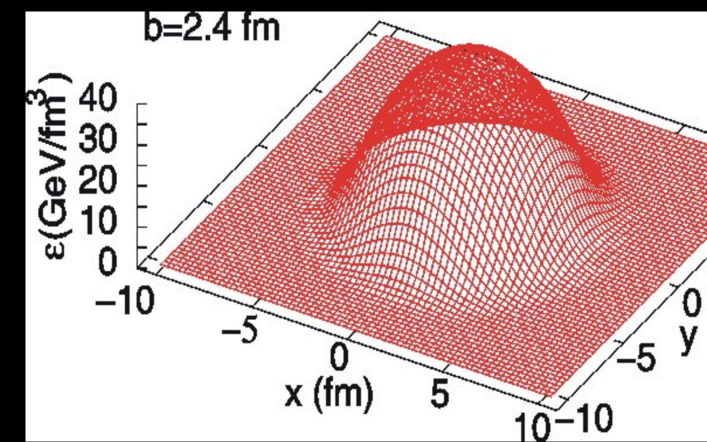
We need 500 X 3 energies X 5 Centralities X 400 events = 3,000,000 events

1 JETSCAPE event takes 1hr on default node { 8 core + 1P100 GPU}

After we do all of this, the parameters will be determined …

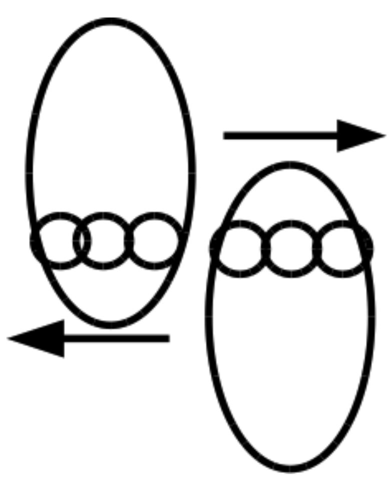
In all calculations presented bulk medium described by viscous fluid dynamics

Medium evolves hydro-dynamically as the jet moves through it
Fit the $\hat{q}$ for the initial T in the hydro in central coll.

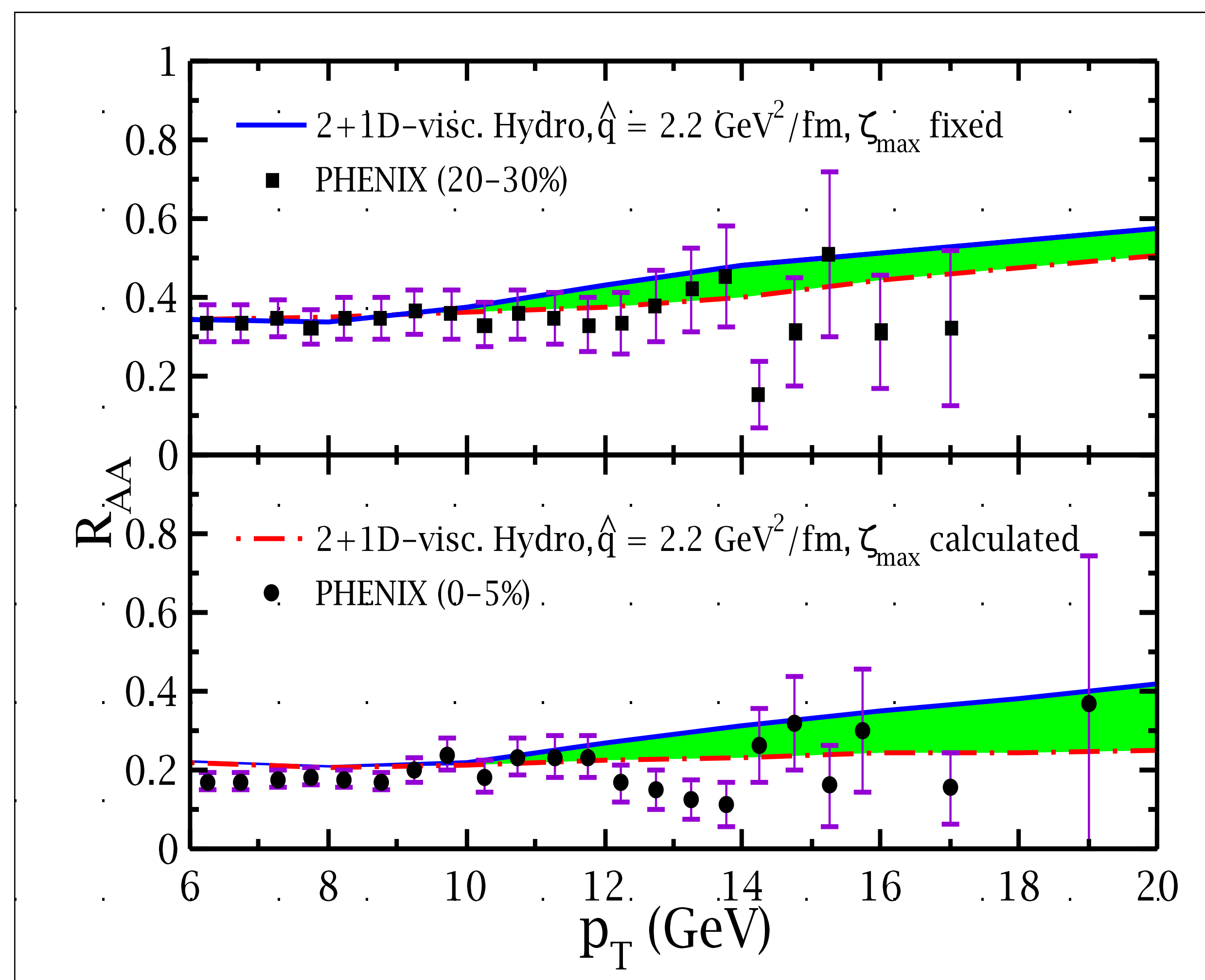


$$\hat{q}(\vec{r},t) = \hat{q}_0 \frac{s(\vec{r},t)}{s_0}$$

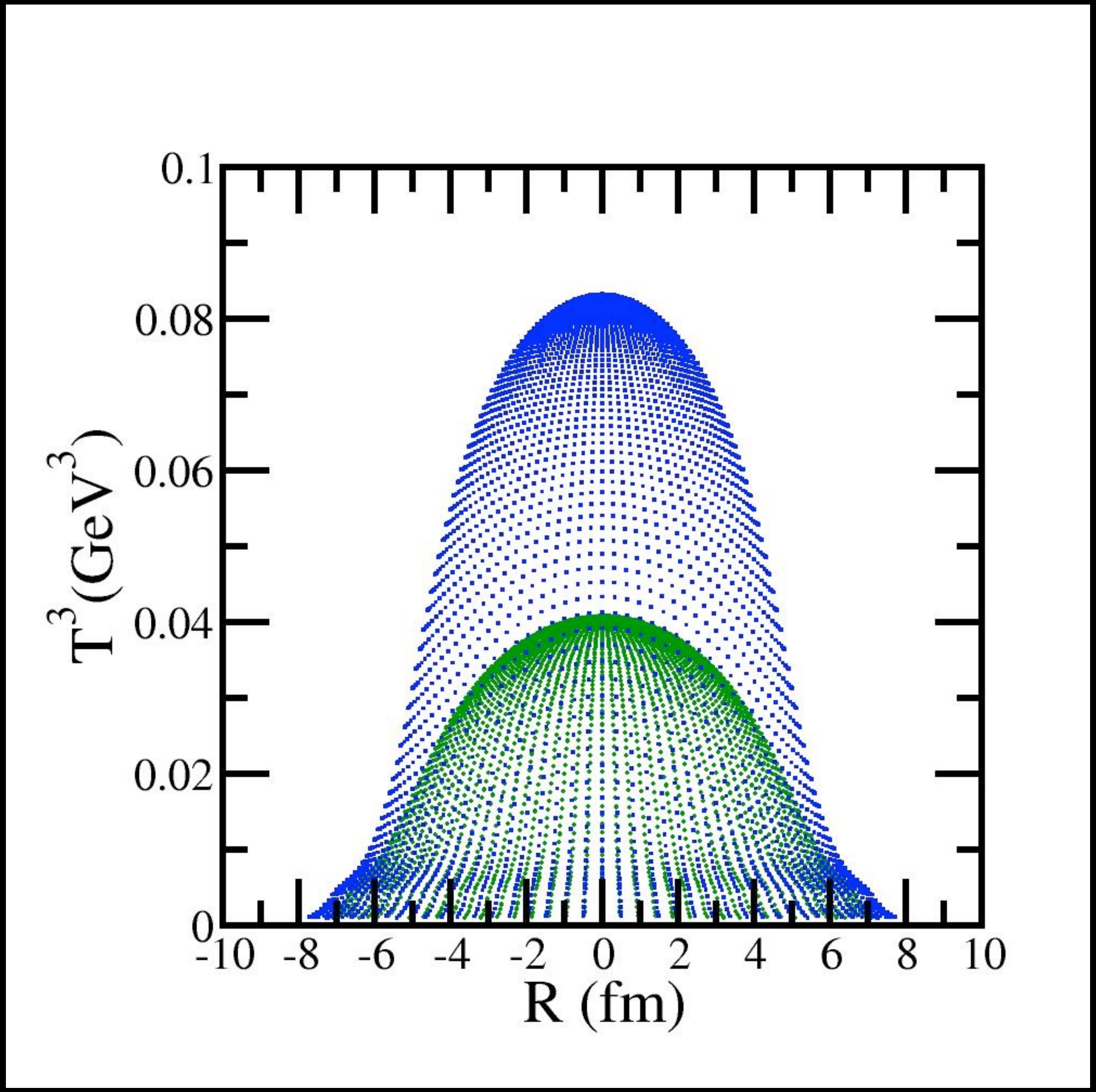
$$s_0 = s(T_0)$$

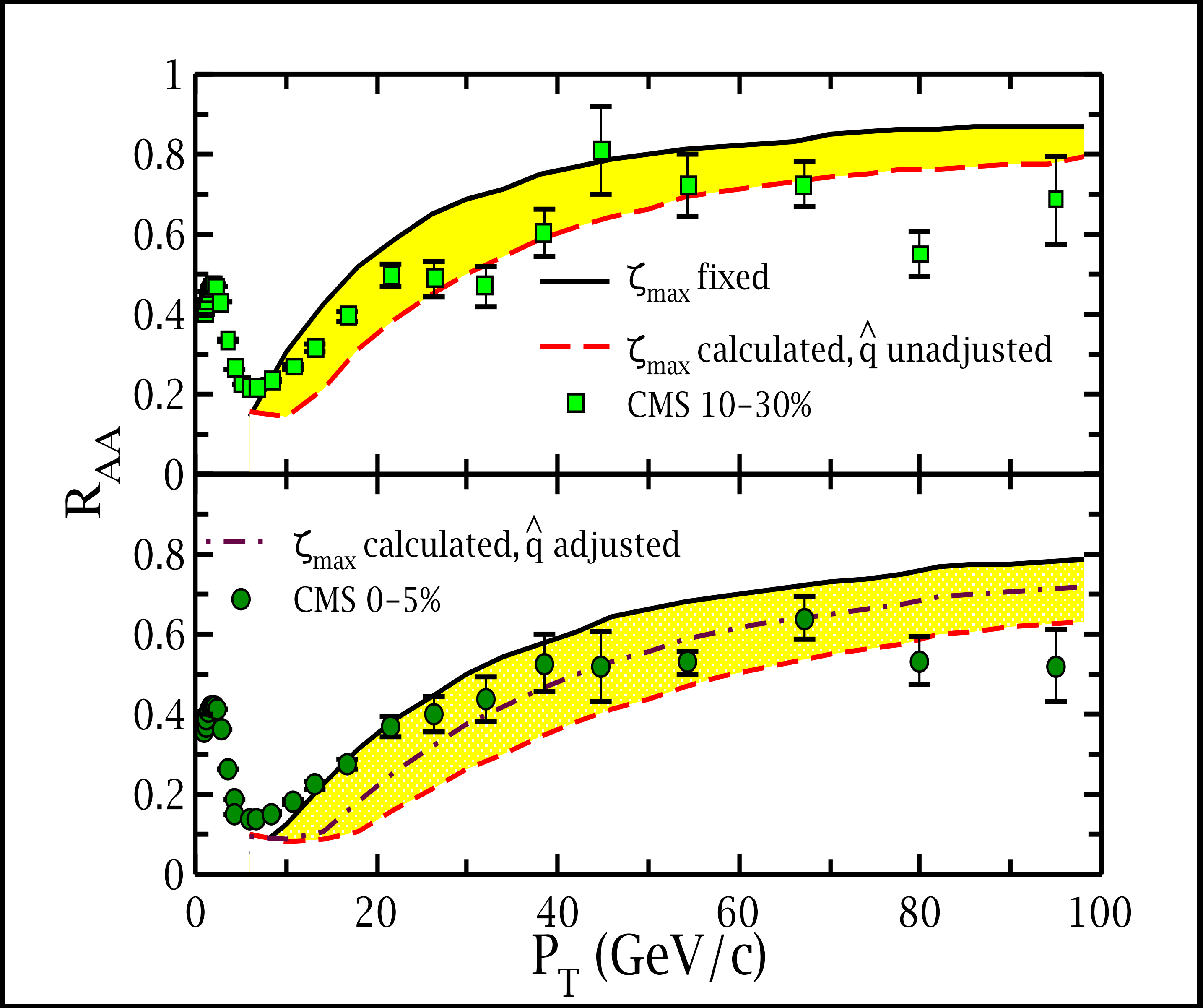


$$R_{AA} \sim \frac{\frac{dN_{AA}}{dp_T dy}}{N_{bin} \frac{dN_{pp}}{dp_T dy}}$$



From RHIC to LHC, refit hydro



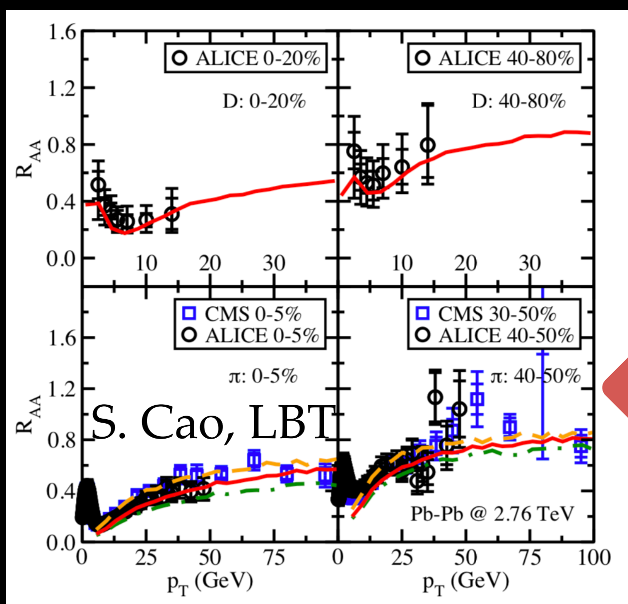


$\hat{q}$ should scale with an intrinsic quantity in the hydro

Necessity of Multi-scale models

Its the right thing to do.

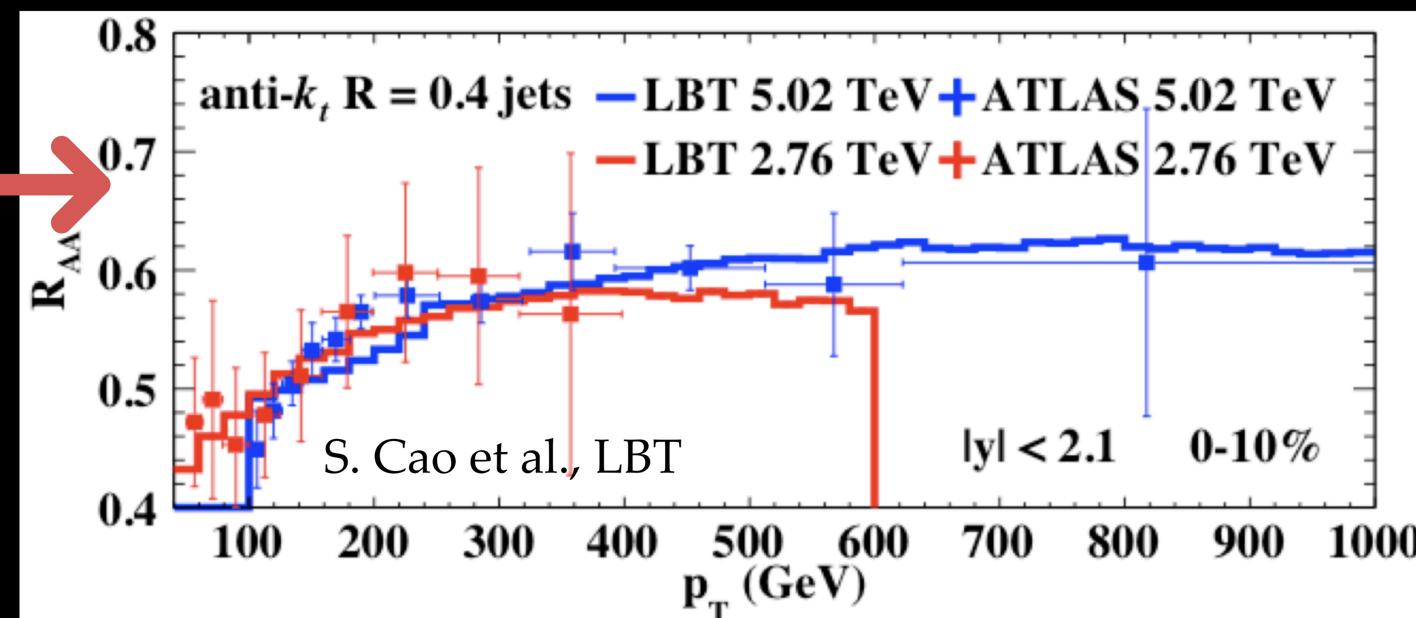
Pushing limited approaches past limits creates tension!

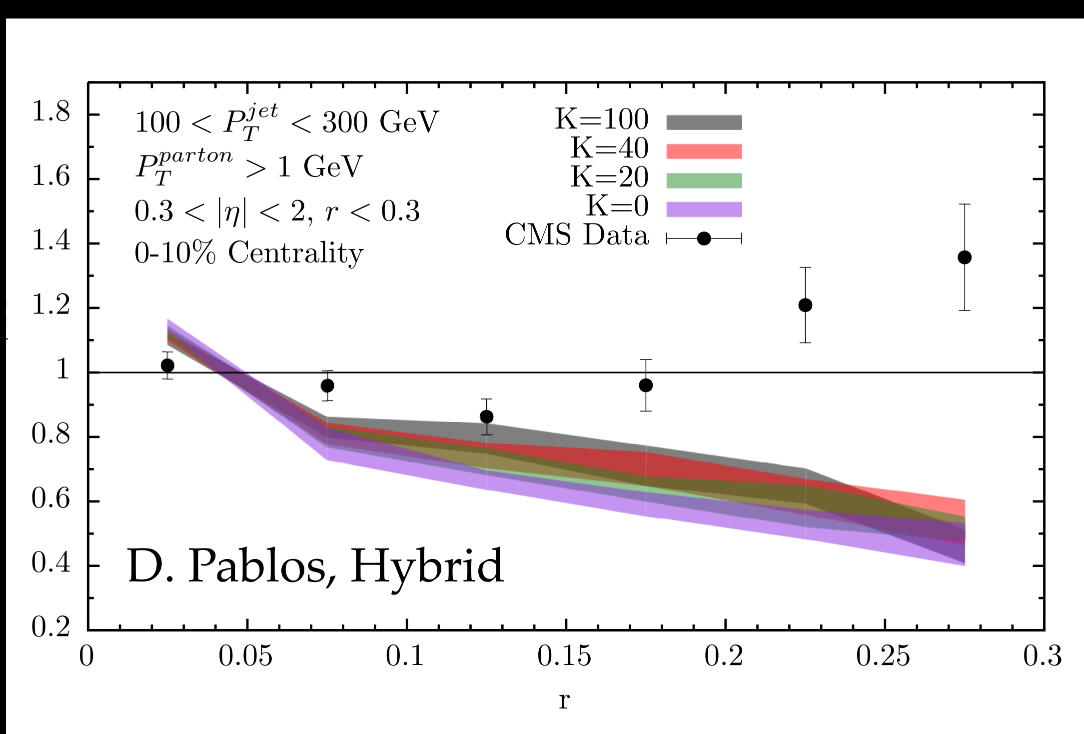


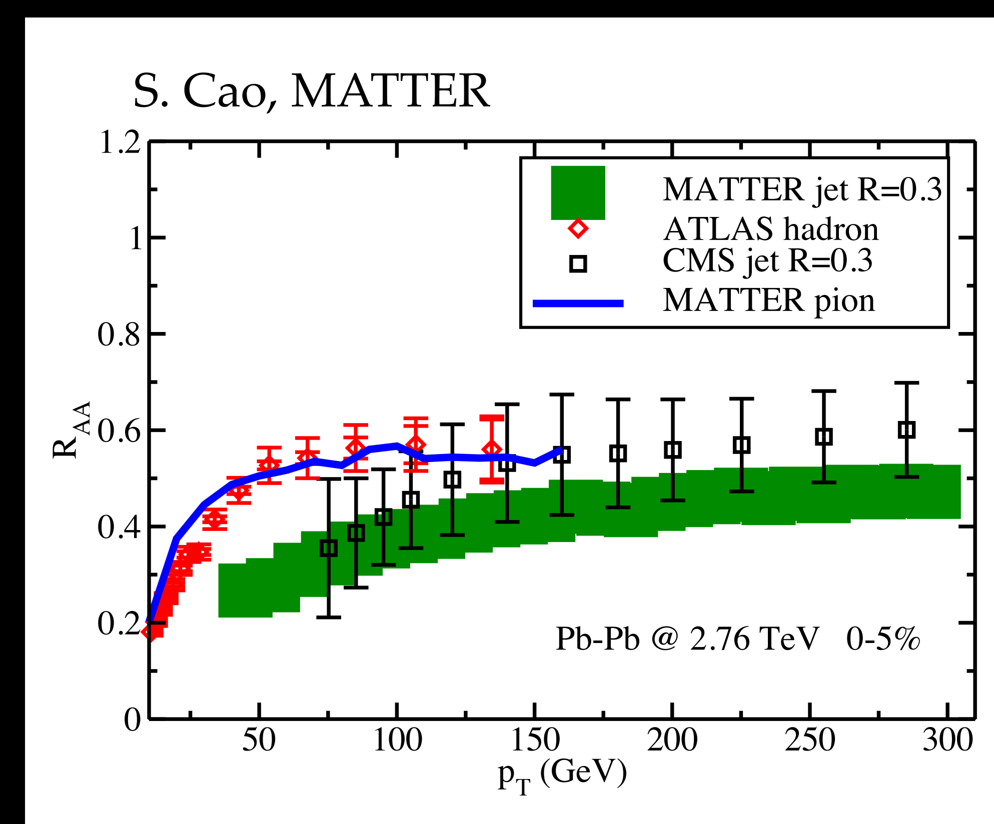
LBT

fixed α_S=0.15

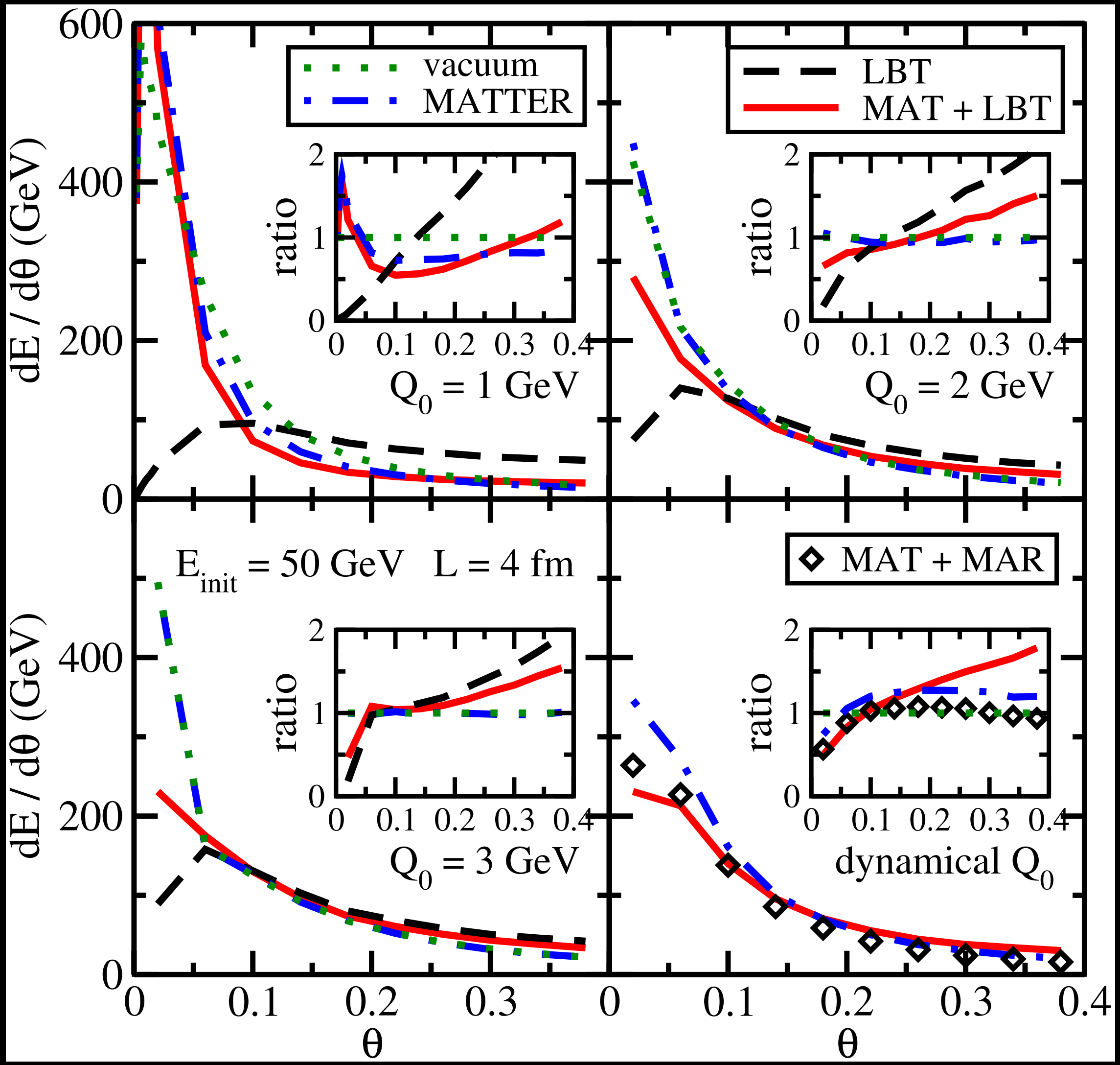
mean α_S=0.2

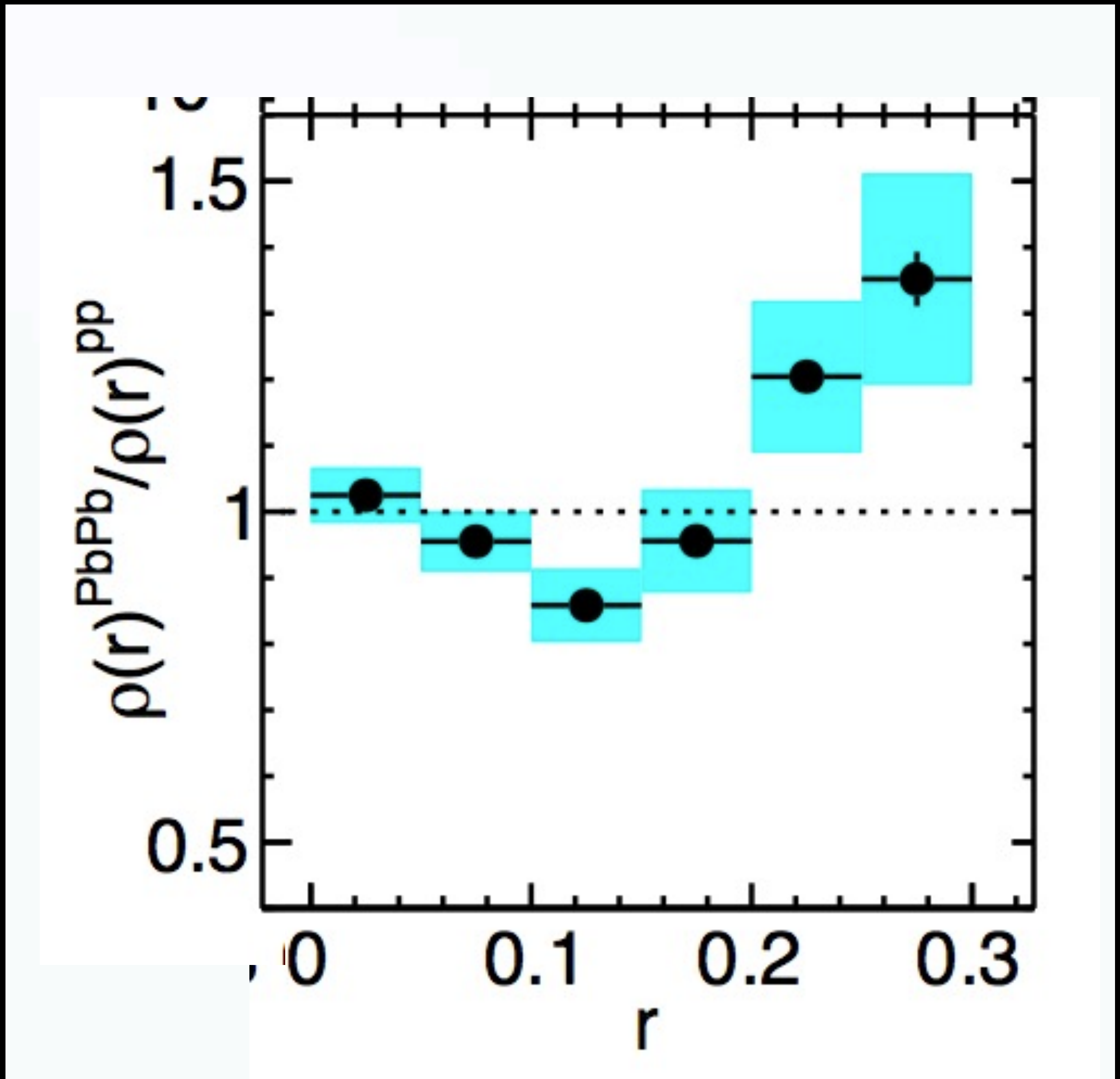






Evidence of multiple scales from multiple-stage Monte Carlos

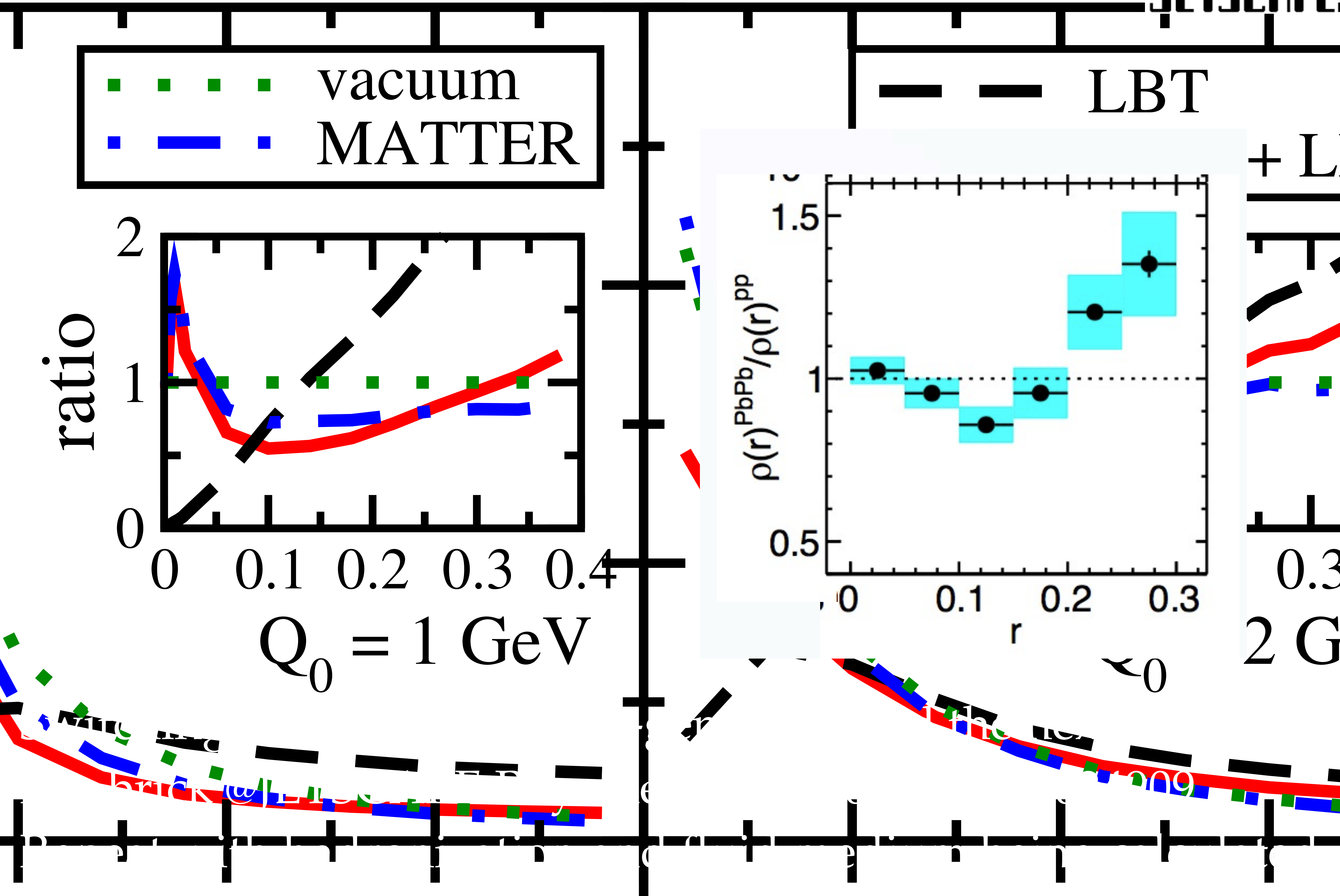




Switching between one event-generator and the next in a brick @JETSCAPE Phys.Rev. C96 (2017) no.2, 024909

Repeat with hadronization and fluid medium being calculated

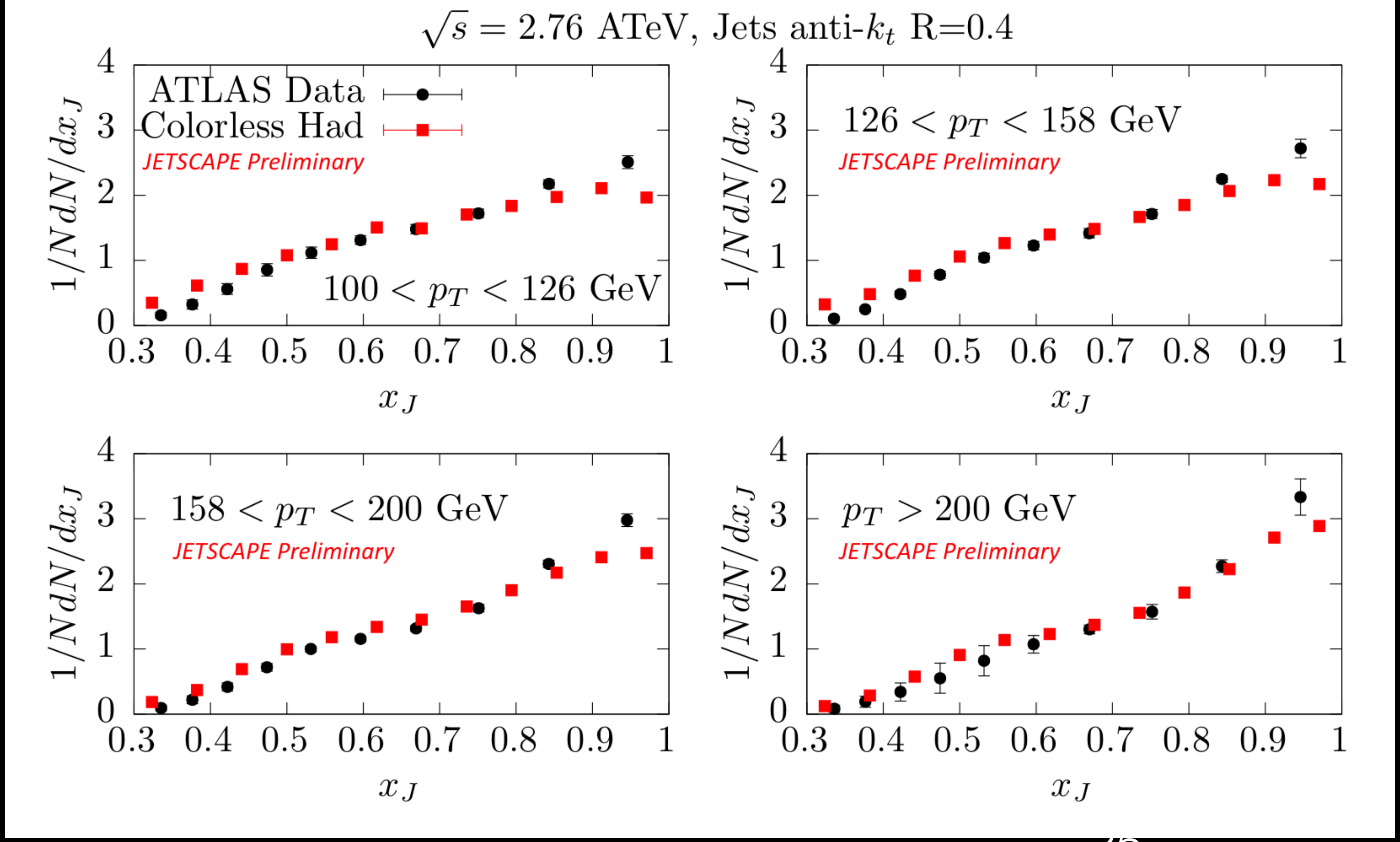
Evidence of multiple scales from

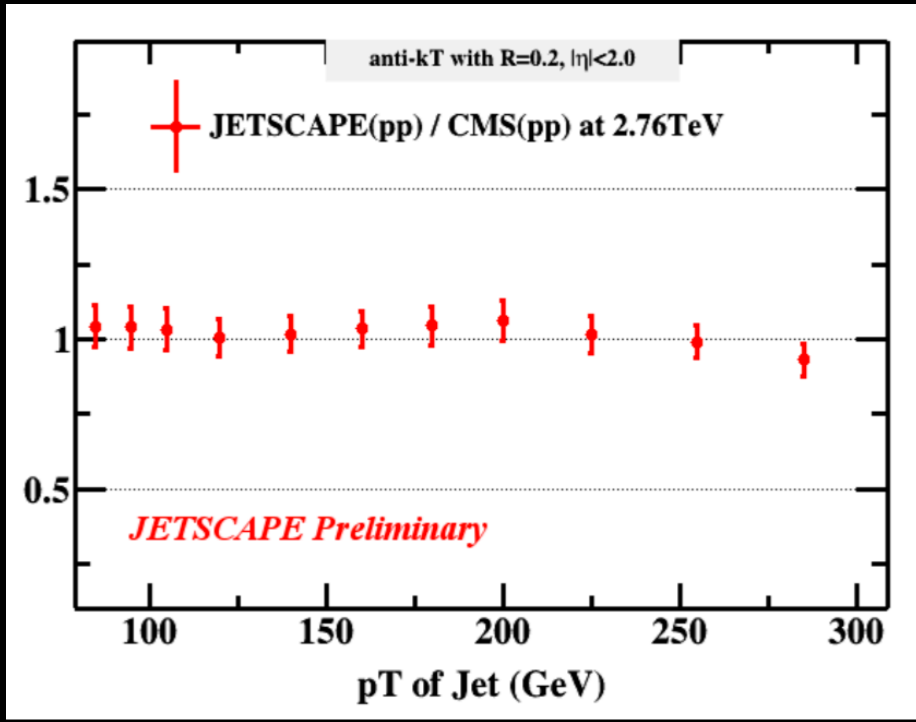


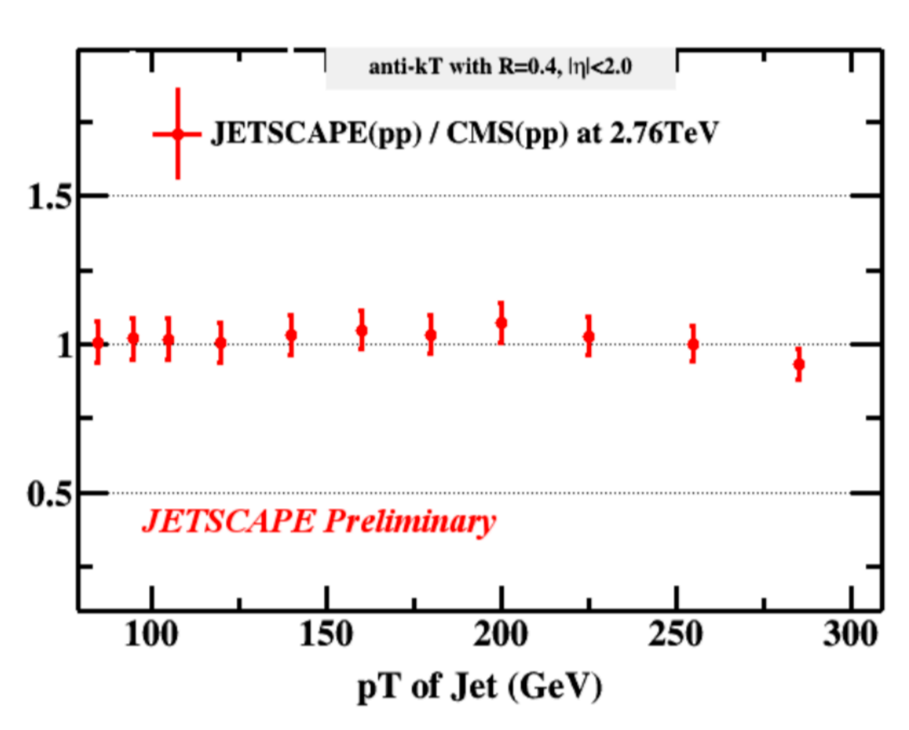
Using the full event generator

- Any good event generator needs a good p-p baseline

PYTHIA for initial state
MATTER for all final state partons > 1GeV
PYTHIA based hadronization of final partons







Preliminary results from JETSCAPE

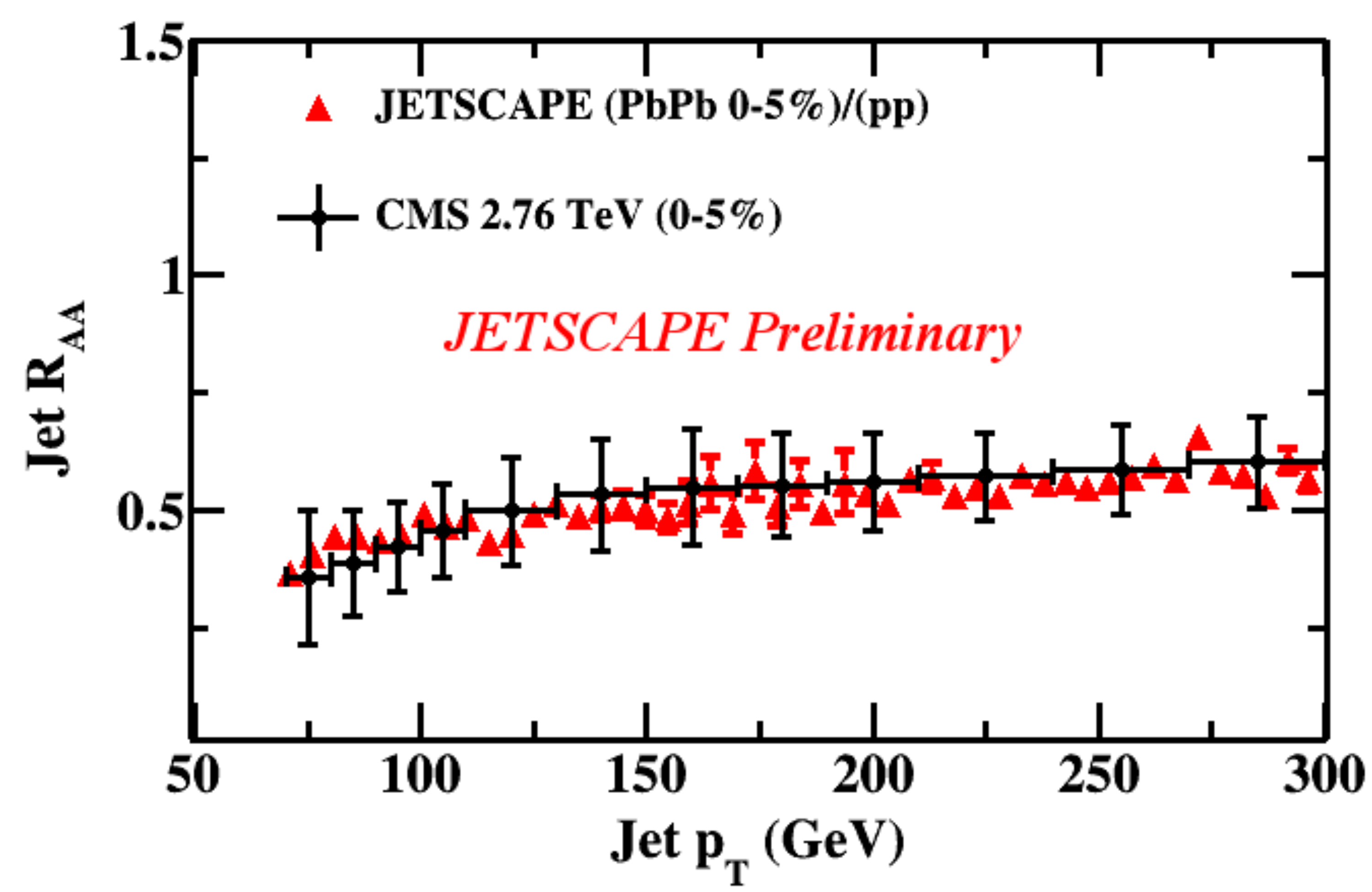
Initial state with TRENTO for both hydro and jets

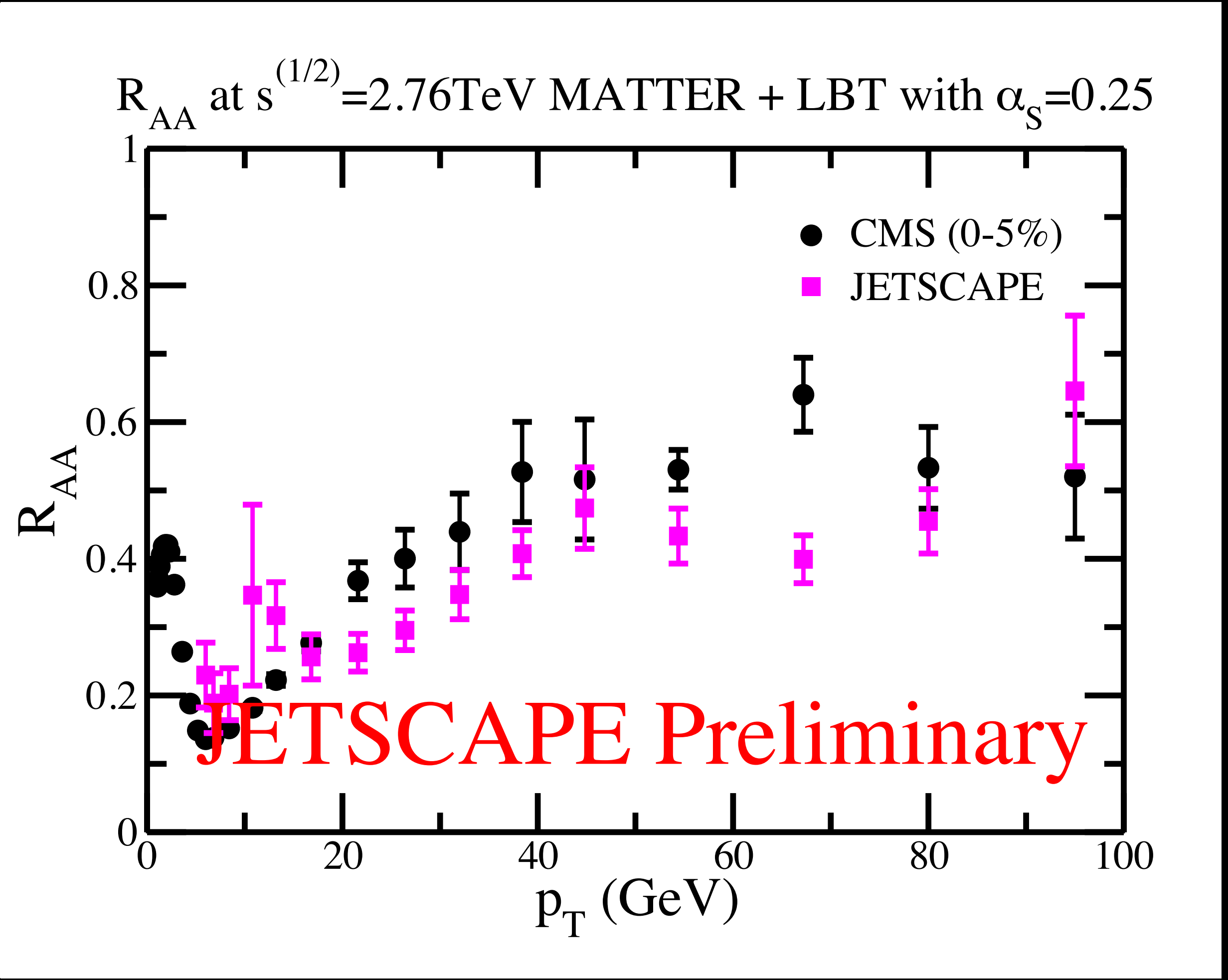
TRENTO —> PreEquib—> MUSIC —> Soft Hadronization

TRENTO —> PYTHIA init

—> (MATTER/LBT/MARTINI/AdS) + MUSIC profile

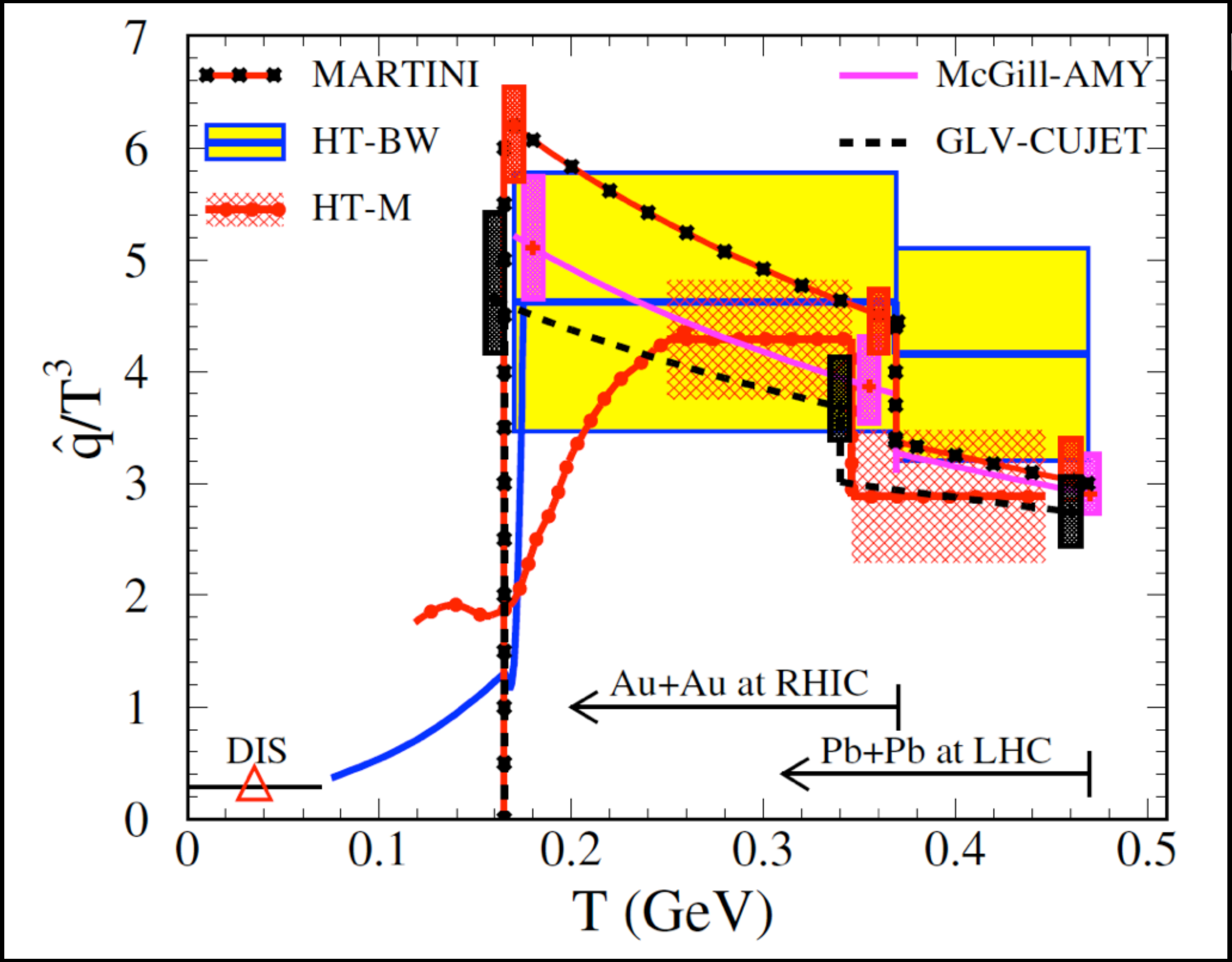
—> PYTHIA based hadronization





Consistent with Results from the JET collaboration

K. Burke et al.



Did separate fits to the RHIC and LHC data for maximal $\hat{q}$ without assuming any kink in the $\hat{q}$ vs T^3 curve

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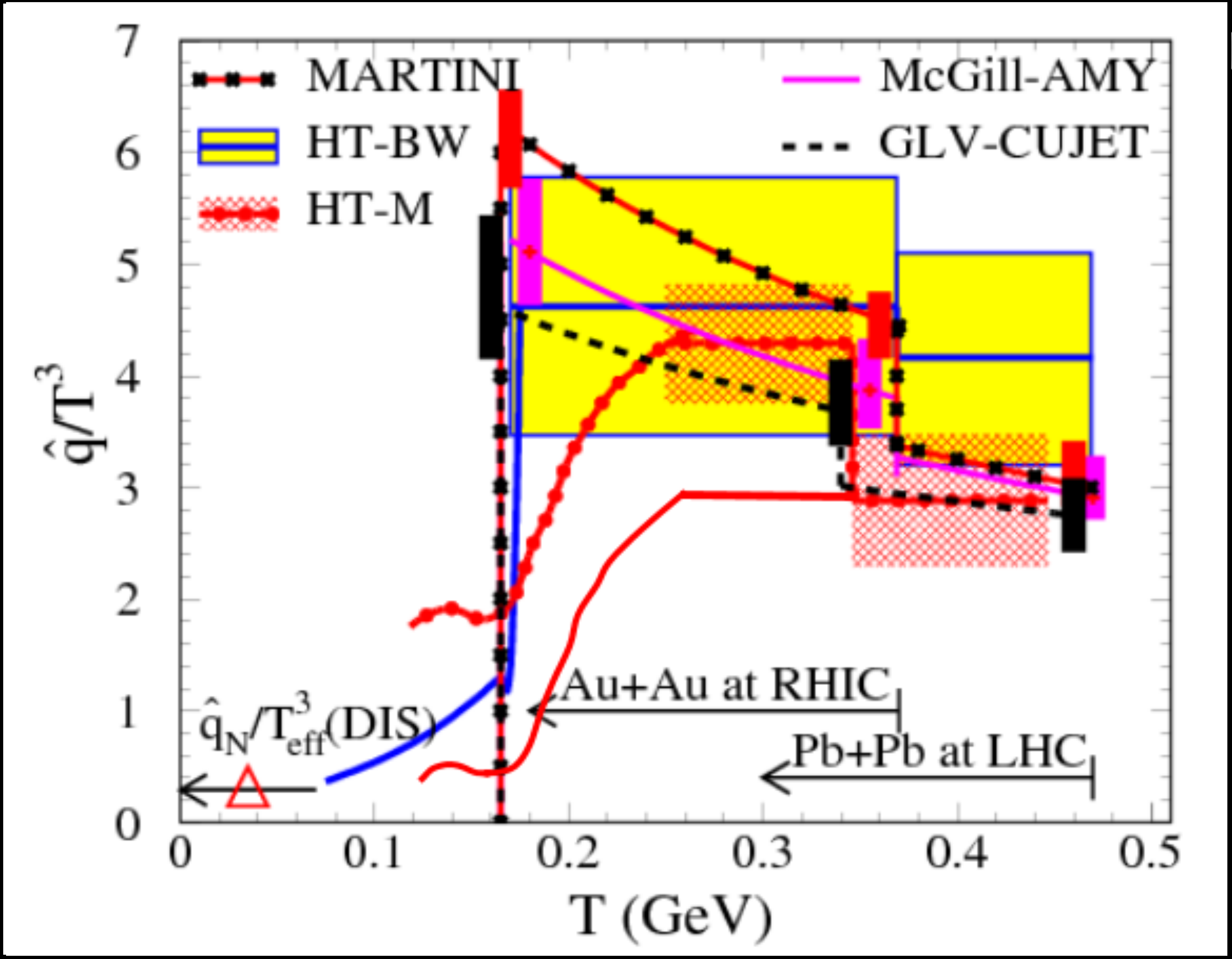

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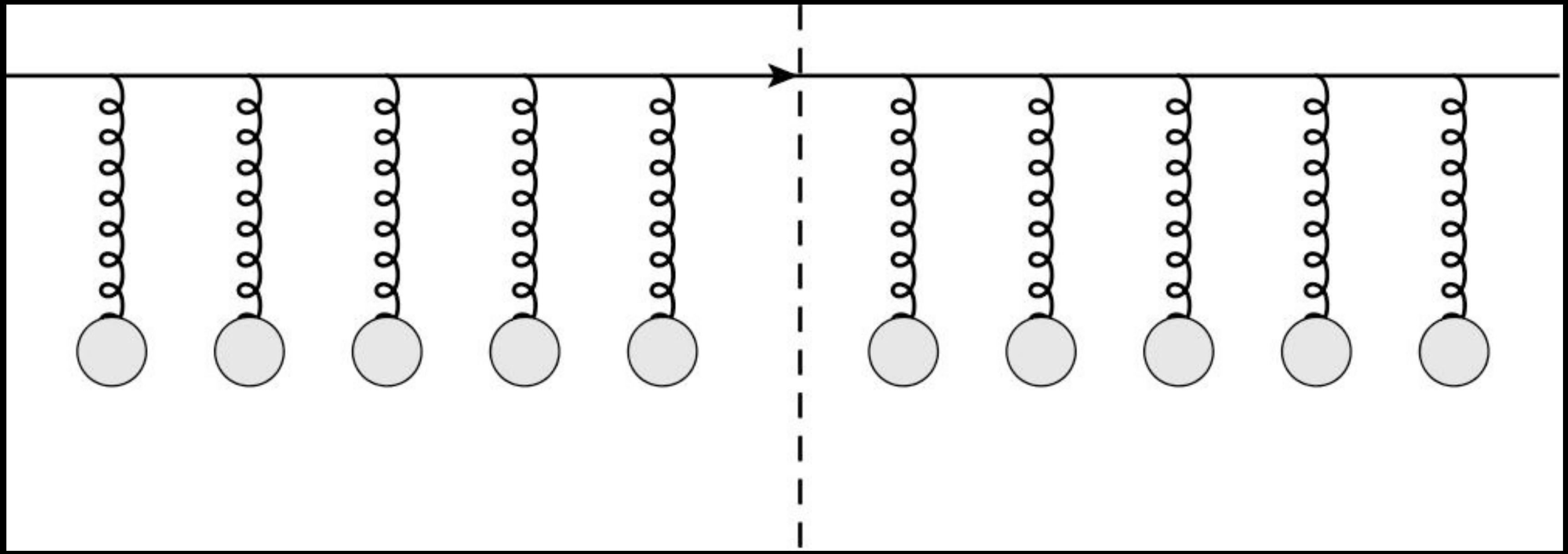
Back to the question of how the medium effects the parton.

A parton in a jet shower, has momentum components

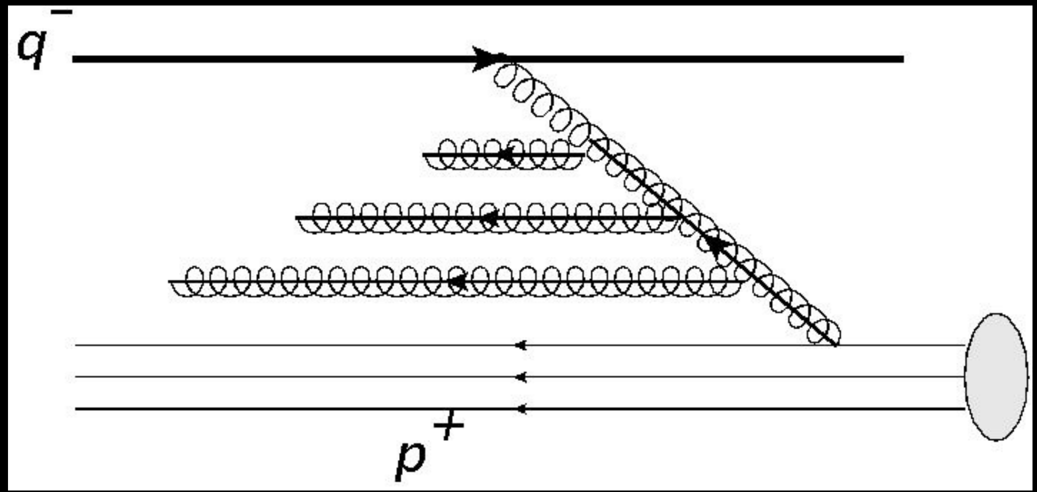
$q = (q^-, q^+, q_T) = (1, \lambda^2, \lambda)Q$, Q: Hard scale, $\lambda \ll 1$, $\lambda Q \gg \Lambda_{QCD}$



hence, gluons have

$$k_\perp \sim \lambda Q, \qquad k^+ \sim \lambda^2 Q$$

could also have $k^- \sim \lambda Q$



Assuming the medium has a large length.

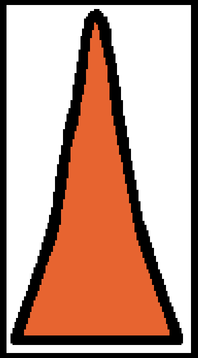
Or, the parton has a long life time, $1/(\lambda^2 Q)$

Multiple independent scattering dominates over multiple correlated scattering

Resumming gives a diffusion equation for the p_T distribution

$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$

$$\langle p_\perp^2 \rangle = 4Dt$$



$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_s C_R}{N_c^2 - 1} \int d\tilde{t} \langle F^{\mu\alpha}(\tilde{t}) v_\alpha F_\mu^\beta(0) v_\beta \rangle$$

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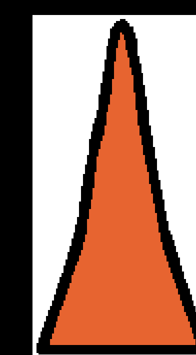
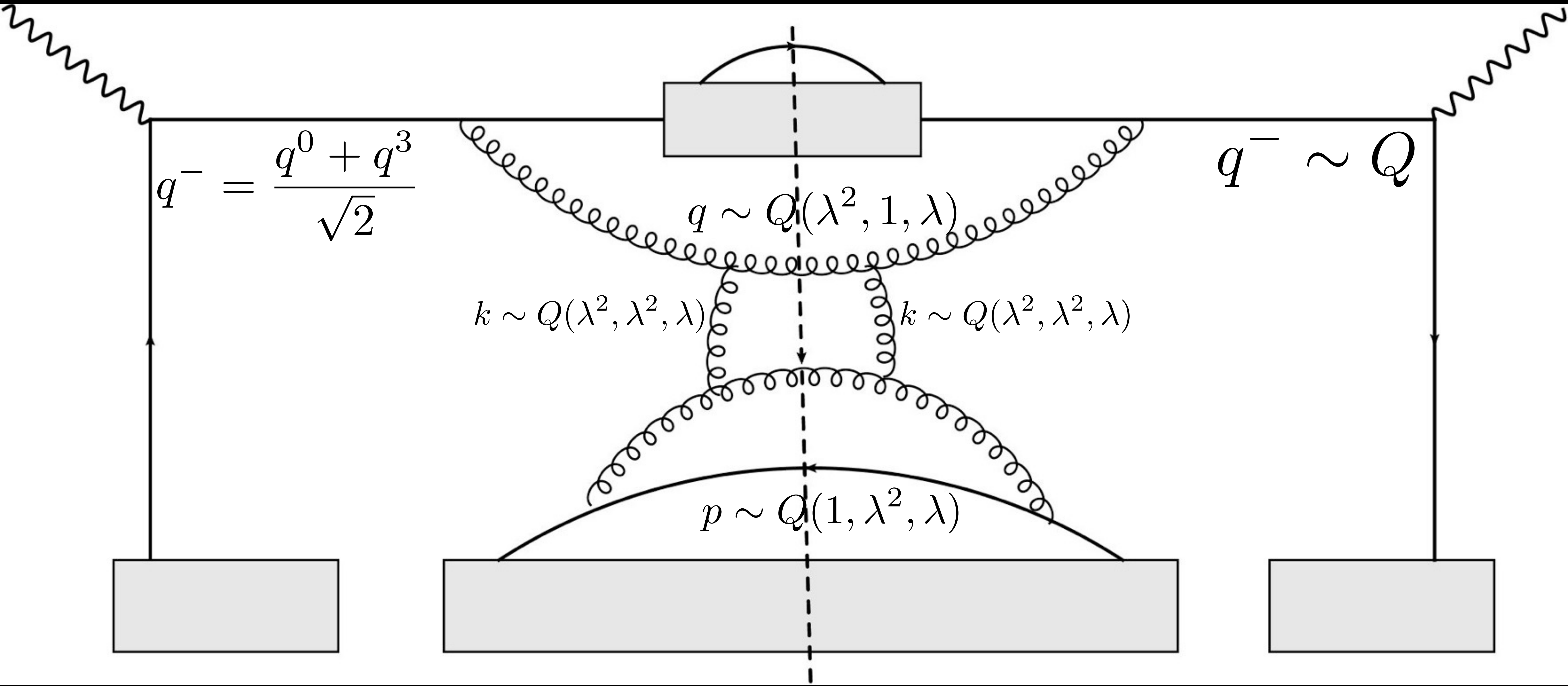


$$\langle p_\perp^2 \rangle = 4Dt$$

$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_S C_R}{N_c^2 - 1} \int dt \left\langle X \left| \mathrm{Tr}\left[\mathrm{U}^\dagger(\mathrm{t}, \mathrm{vt}; 0) \mathrm{t}^{\mathrm{a}} \mathrm{F}^{\mathrm{a}\mu\rho} \mathrm{v}_\rho \mathrm{U}(\mathrm{t}, \mathrm{vt}; 0) \mathrm{t}^{\mathrm{b}} \mathrm{F}^{\mathrm{b}\sigma}_{\ \ \mu}(0) \mathrm{v}_\sigma \right] \right| X \right\rangle$$

A factorized picture



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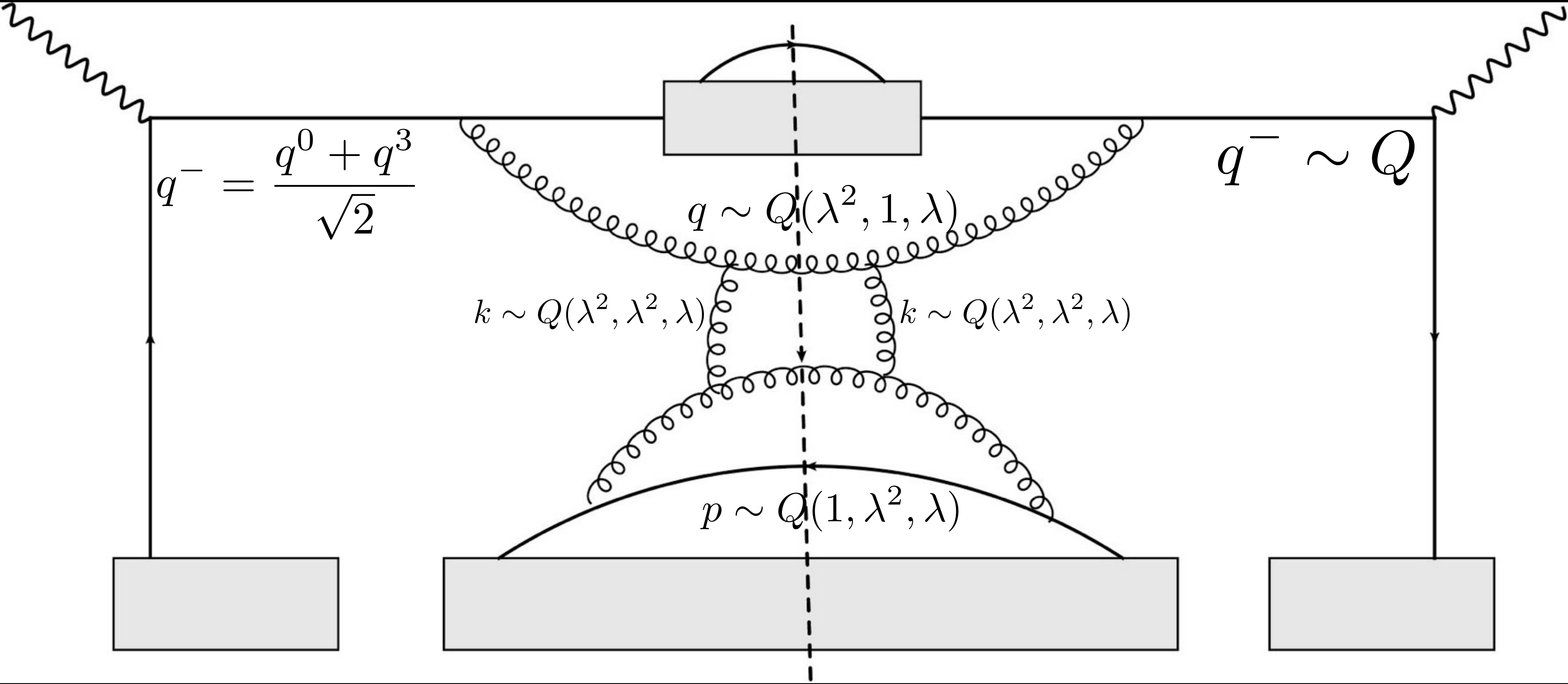
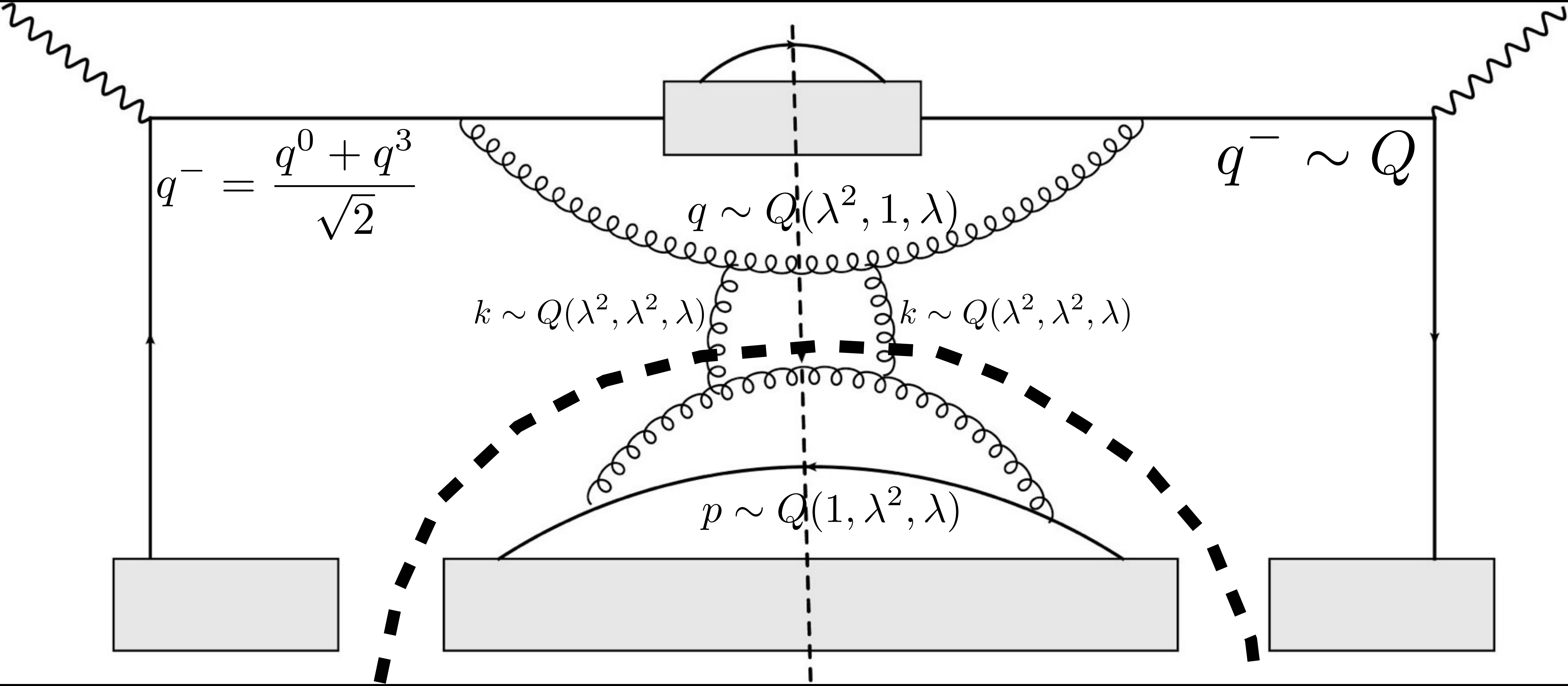


Q is the hard scale of the jet ~ E

$Q\lambda$ is a semi-hard scale ~ $(ET)^{1/2}, \lambda \longrightarrow 0$

$\hat{q}$ contains all dynamics below $Q\lambda$

A factorized picture

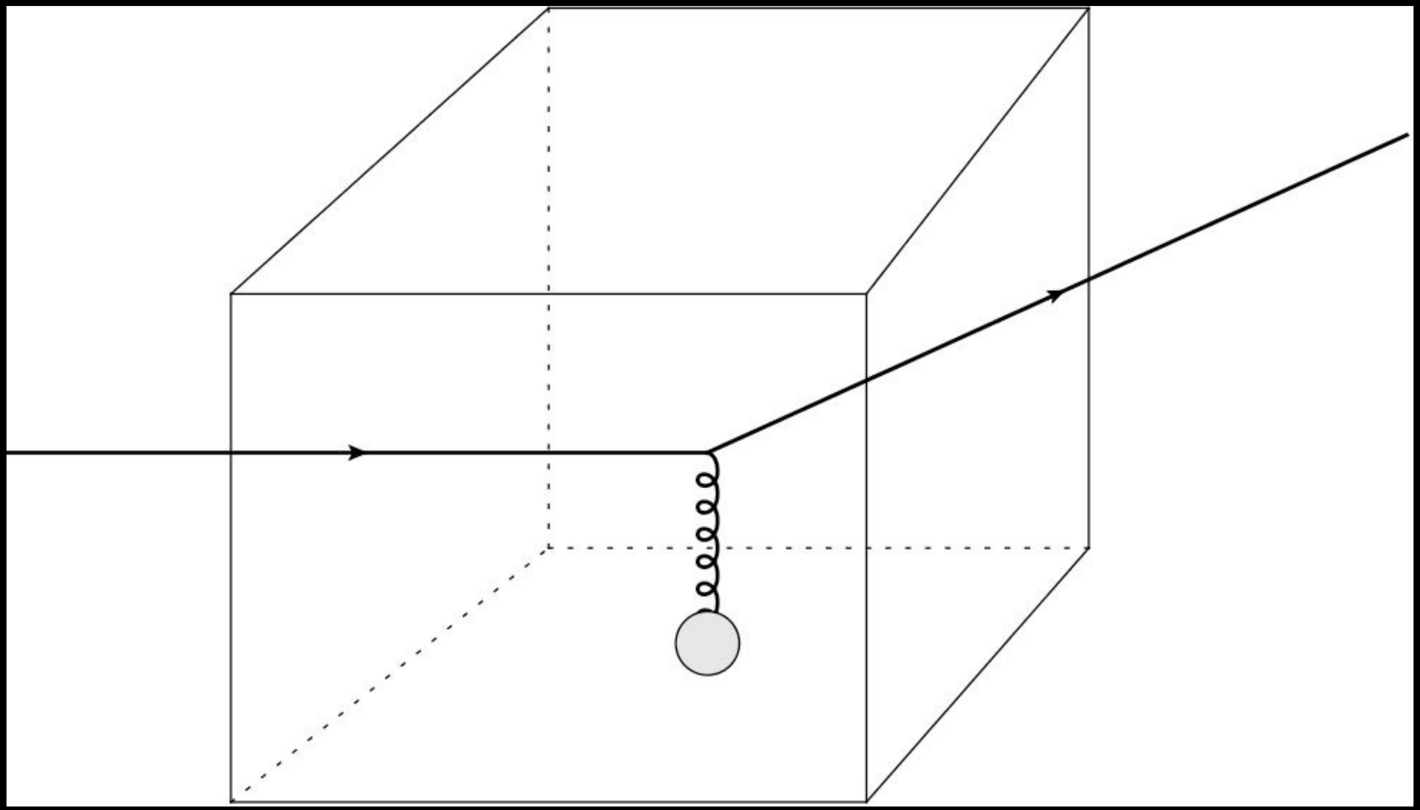


Q is the hard scale of the jet ~ E

$Q\lambda$ is a semi-hard scale ~ $(ET)^{1/2}, \lambda \longrightarrow 0$

$\hat{q}$ contains all dynamics below $Q\lambda$

A first principles method to calculate $\hat{q}$



$$W(k) = \frac{g^2}{2N_c}\langle q^-;M| \int d^4x d^4y \bar{\psi}(y)\ \not{A}(y)\psi(y)$$
$$\times\ |q^- + k_\perp;X\rangle\langle q^- + k_\perp;X|$$
$$\times\ \bar{\psi}(x)\ \not{A}(x)\psi(x)|q^-;M\rangle$$

in terms of W, we get $$\hat{q} = \sum_k k_\perp^2 \frac{W(k)}{t},$$

Final state is ``on-shell''

$$\delta[(q+k)^2] \simeq \frac{1}{2q^-}\delta\left(k^+ - \frac{k_\perp^2}{2q^-}\right).$$

Also we are calculating in a finite temperature heat bath

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3} d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}\cdot y^- + i\vec{k}_\perp\cdot\vec{y_\perp}}$$

$$\langle n|\frac{e^{-\beta E_n}}{Z}F^{+,}{}_\perp(y^-)F_\perp^+(0)|n\rangle$$

physical $\hat{q}(q^-, q^+)$ where $q^+ \sim \lambda^2 Q$

Consider a more general object

$$\hat{Q} \;=\; \frac{4\pi^2\alpha_s}{N_c}\int \frac{d^4y d^4k}{(2\pi)^4} e^{ik\cdot y} \frac{2(q^-)^2}{\sqrt{2}q^-} \frac{\langle M|F^{+\perp}(0)F^{+}_{\perp,}(y)|M\rangle}{(q+k)^2 + i\epsilon}.$$

Consider q^- large (~Q) and fixed

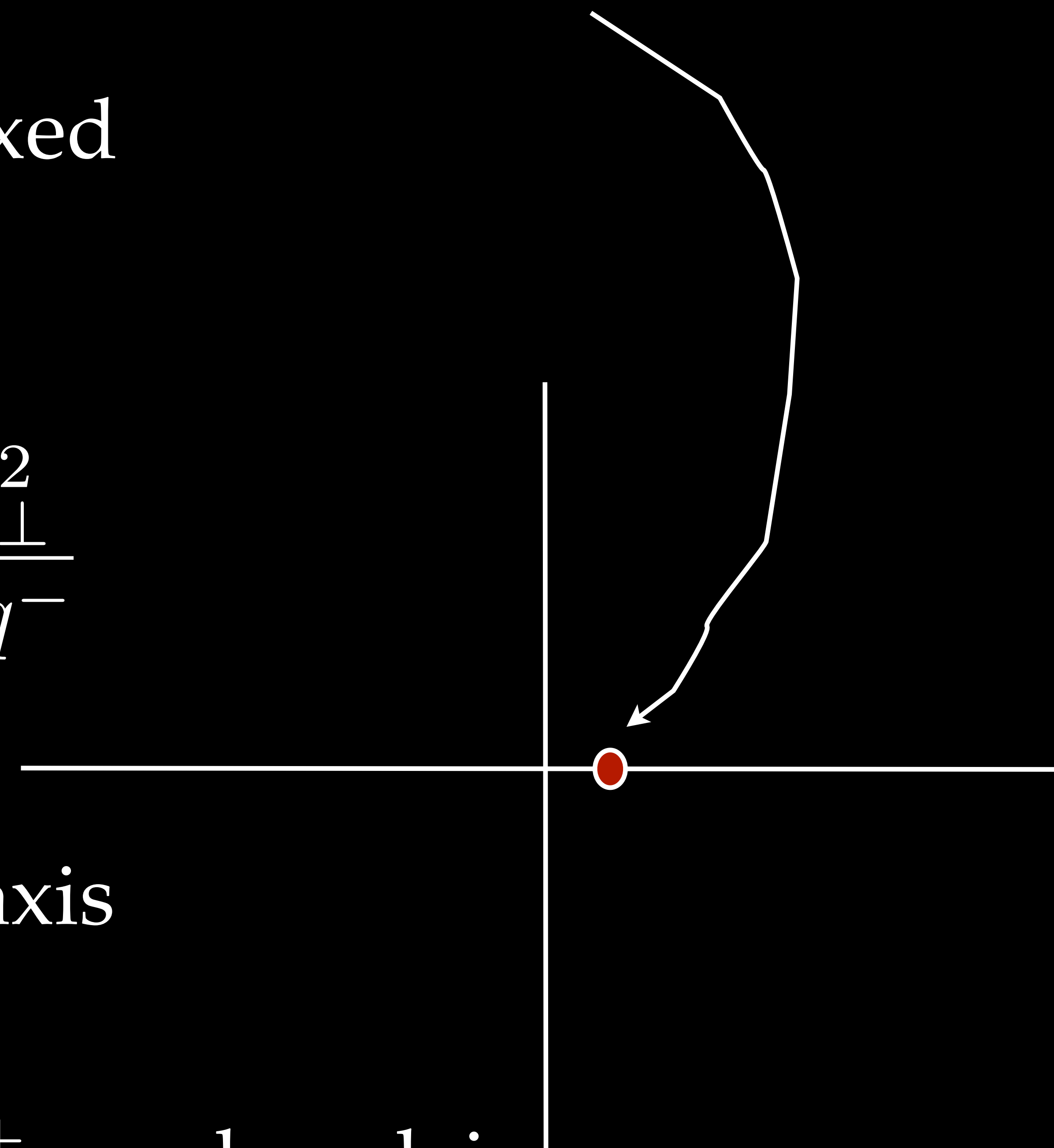
Consider q^+ to be a variable

$\frac{d^2\hat{Q}}{dk_\perp^2}$ has a pole at $q^+ = \frac{k_\perp^2}{2q^-}$

$\hat{Q}$ has a branch cut on the real axis

at $q^+ \sim \lambda^2$ Q

$$\hat{q} = Im(\hat{Q})$$



q^+ complex plain

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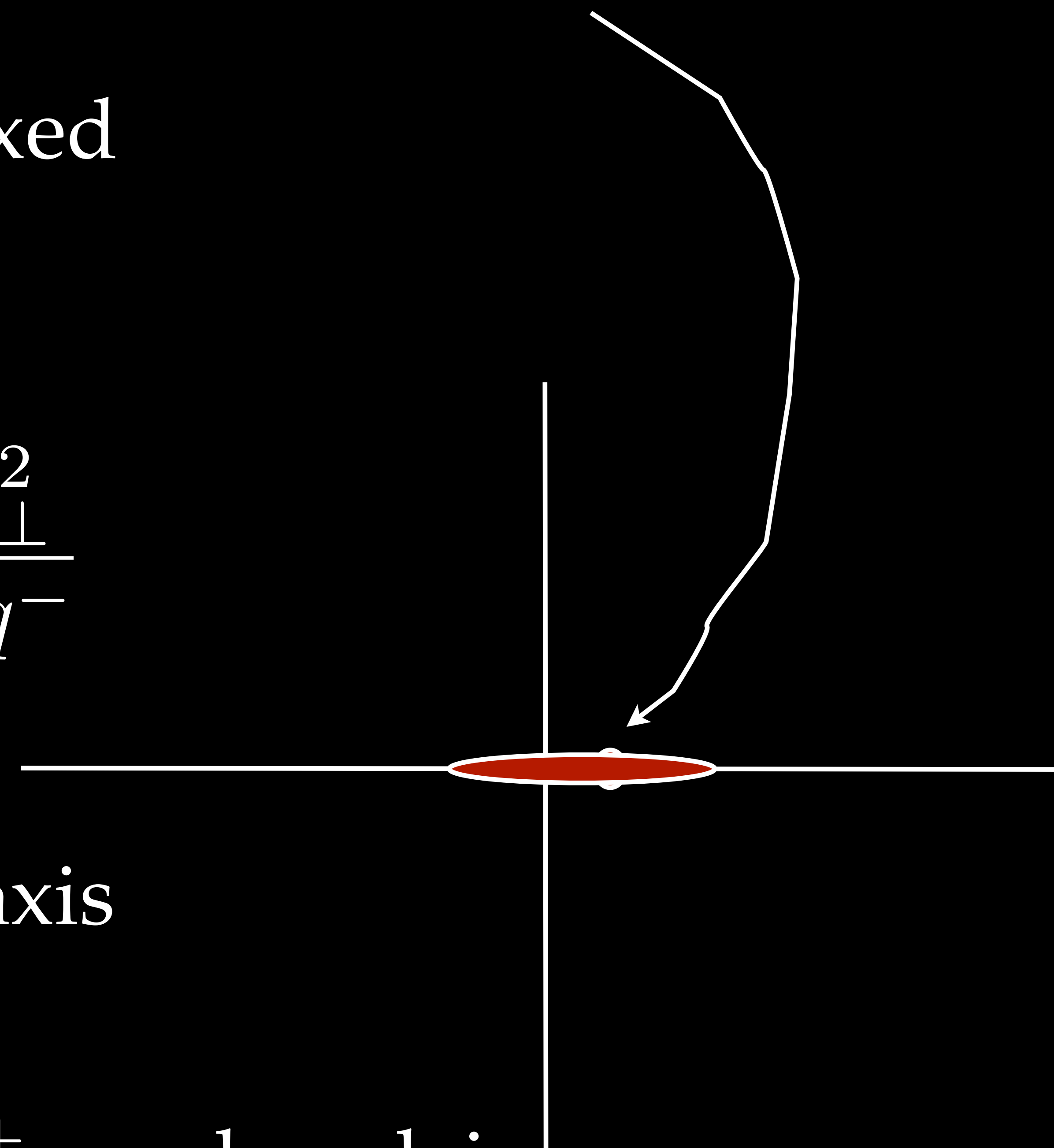

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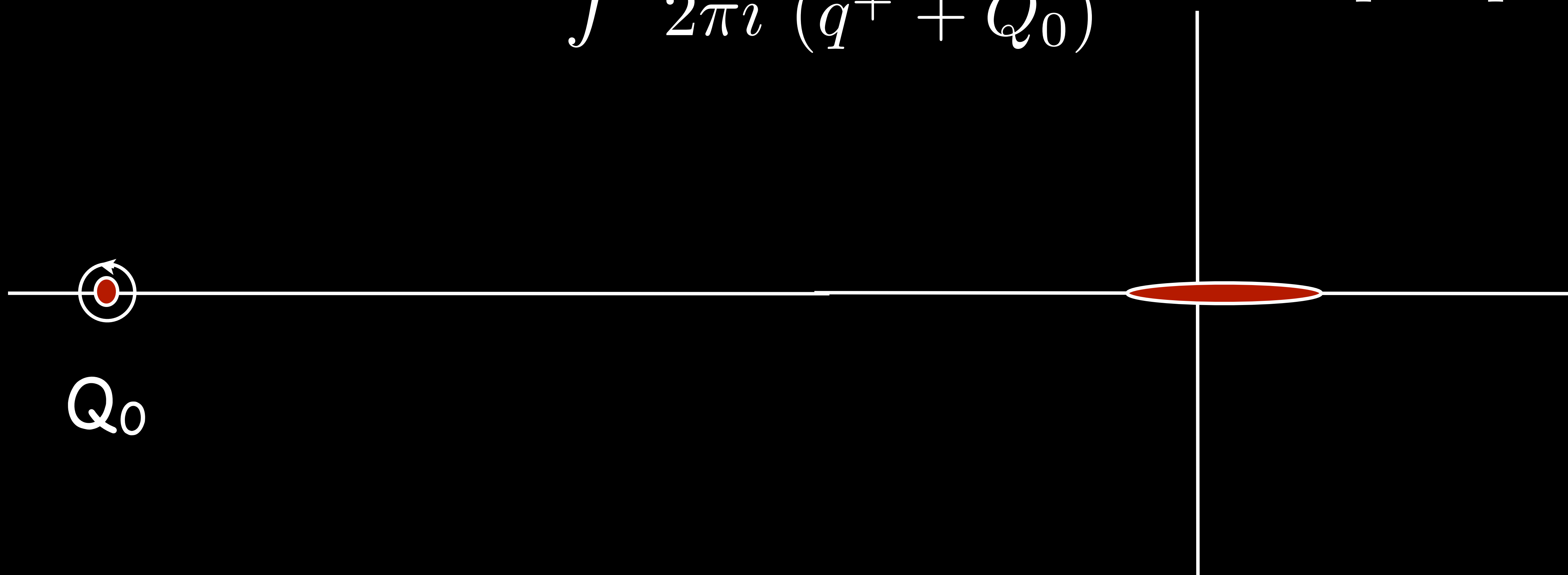
q^+ complex plain

Consider the following integral

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + Q_0)}$$

q^+complex plain



For Q_0 ~ -Q, can Taylor expand $\hat{Q}$ in terms of local operators

$$I_1 = \frac{4\sqrt{2}\pi^2\alpha_s \langle M| F_\perp^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-q\cdot i\mathcal{D} - \mathcal{D}_\perp^2}{2q^- Q_0}\right)^n F^+_{\perp,\mu} |M}{N_c 2Q_0}$$

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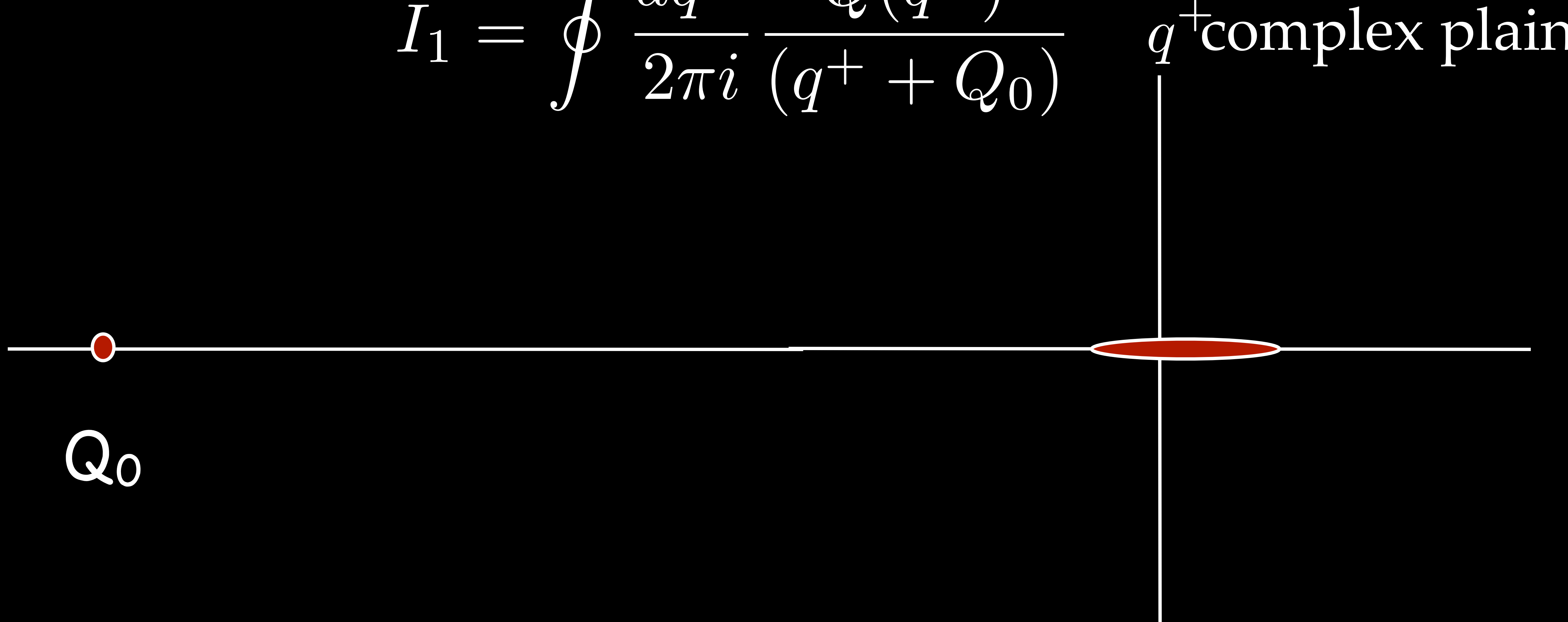

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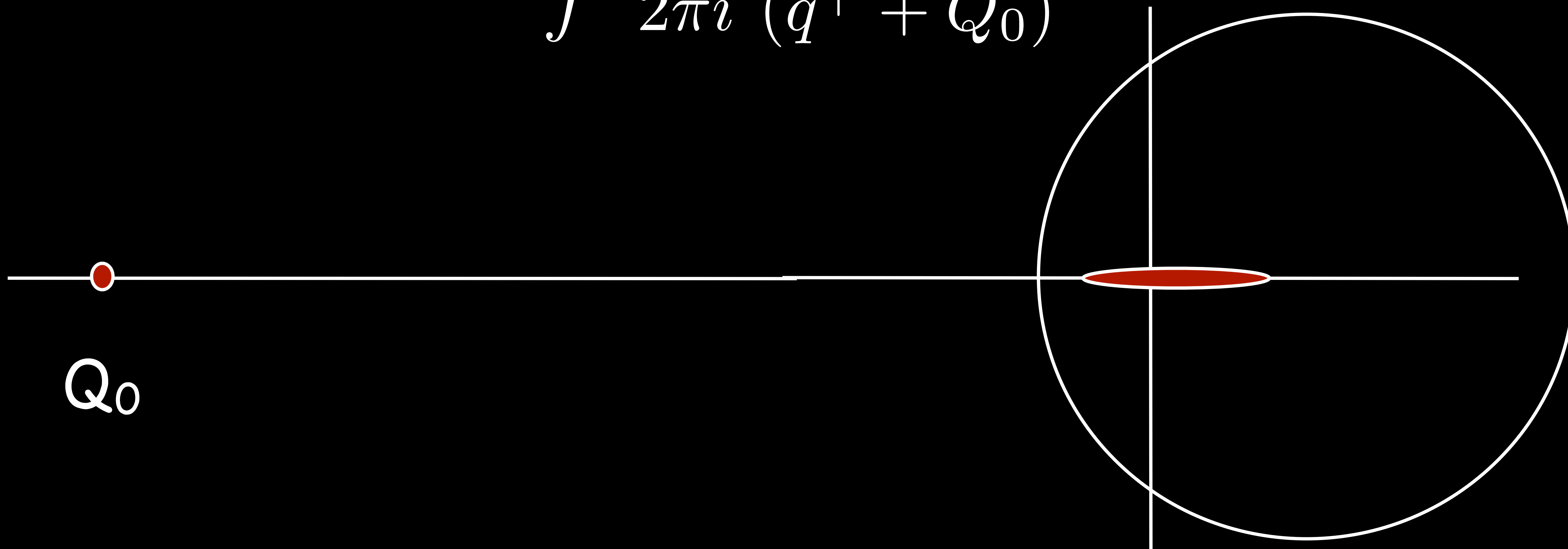
q^+complex plain



For Q_0 ~ $-Q$, can Taylor expand $\hat{Q}$ in terms of local operators

$$I_1 = \frac{4\sqrt{2}\pi^2 \alpha_s \langle M| F_\perp^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-q\cdot i\mathcal{D} - \mathcal{D}_\perp^2}{2q^- Q_0}\right)^n F_{\perp,\mu}^+ |M}{N_c 2 Q_0}$$

Consider the following integral

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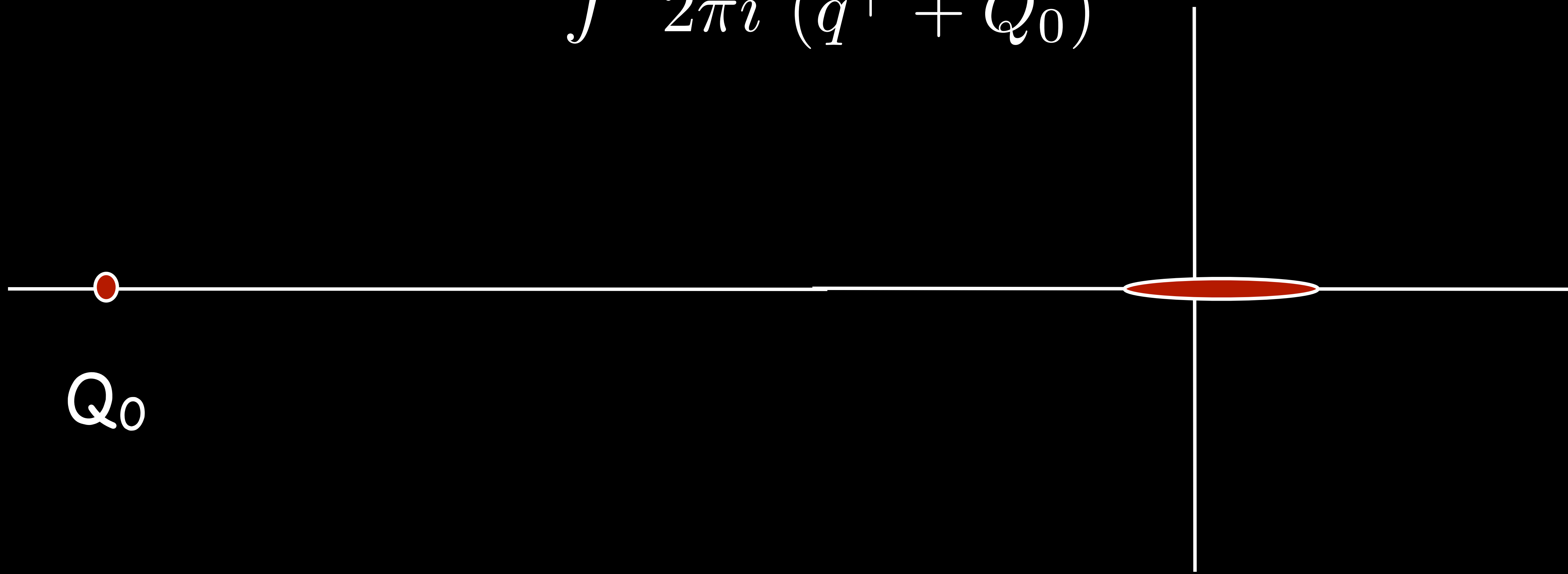

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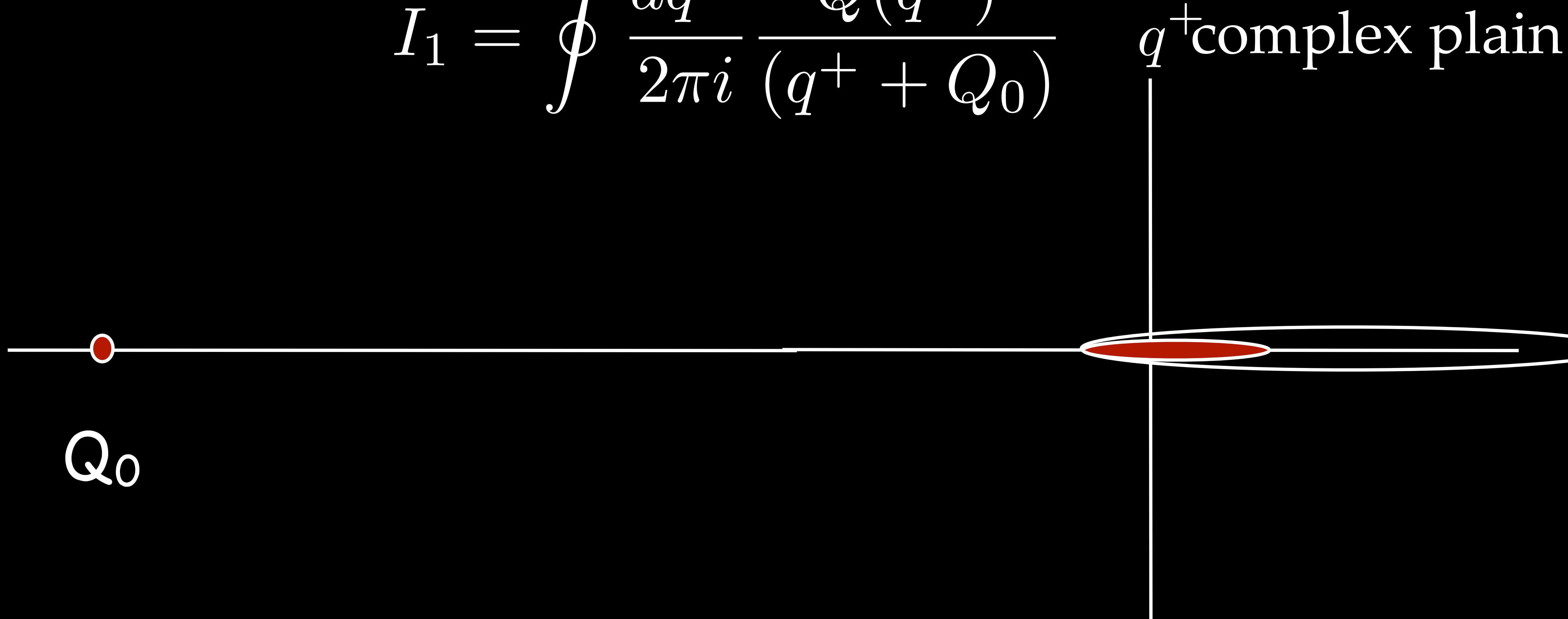


For $Q_0 \sim -Q$, can Taylor expand $\hat{Q}$ in terms of local operators

$$I_1 = \frac{4\sqrt{2}\pi^2\alpha_s \langle M| F_\perp^{+\mu} \sum_{n=0}^{\infty} \left(\frac{-q\cdot i\mathcal{D} - \mathcal{D}_\perp^2}{2q^- Q_0}\right)^n F^+_{\perp,\mu} |M}{N_c 2Q_0}$$

Deforming the contour

$$I_1 = \int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \frac{\hat{q}(q^+)}{q^+ + Q_0} + \int_0^\infty dq^+ V(q^+)$$

set $Q_0 = q^-$

Taylor expand I_1 on the real side and do the integral

$$\begin{aligned} \hat{\bar{q}}(Q^+) 2Q^+ &= \int_{-Q^+}^{Q^+} dq^+ \hat{q}(q^+) \\ &\simeq 2\hat{q}Q^+ + \frac{\hat{q}''(Q^+)^3}{3} \end{aligned}$$

Match powers of q^-

Easy to calculate local operators on the Lattice

Consider the unordered correlator

$$\mathcal{D}^{>}(t) = \sum_n \langle n| e^{-\beta H} \mathcal{O}_1(t) \mathcal{O}_2(0) |n\rangle$$

convert thermal weight to evolution in imaginary time

$$\mathcal{D}^{>}(-i\tau) = \Delta(\tau) = \mathrm{Tr}\left[e^{-\int_0^\beta d\tau H(\tau)} \mathcal{O}_i(\tau) \mathcal{O}_2(0) \right] .$$

with time derivatives

$$\mathcal{D}^{>}(-i\tau) = i^{N_t} \Delta(\tau)$$

But local operators are super simple

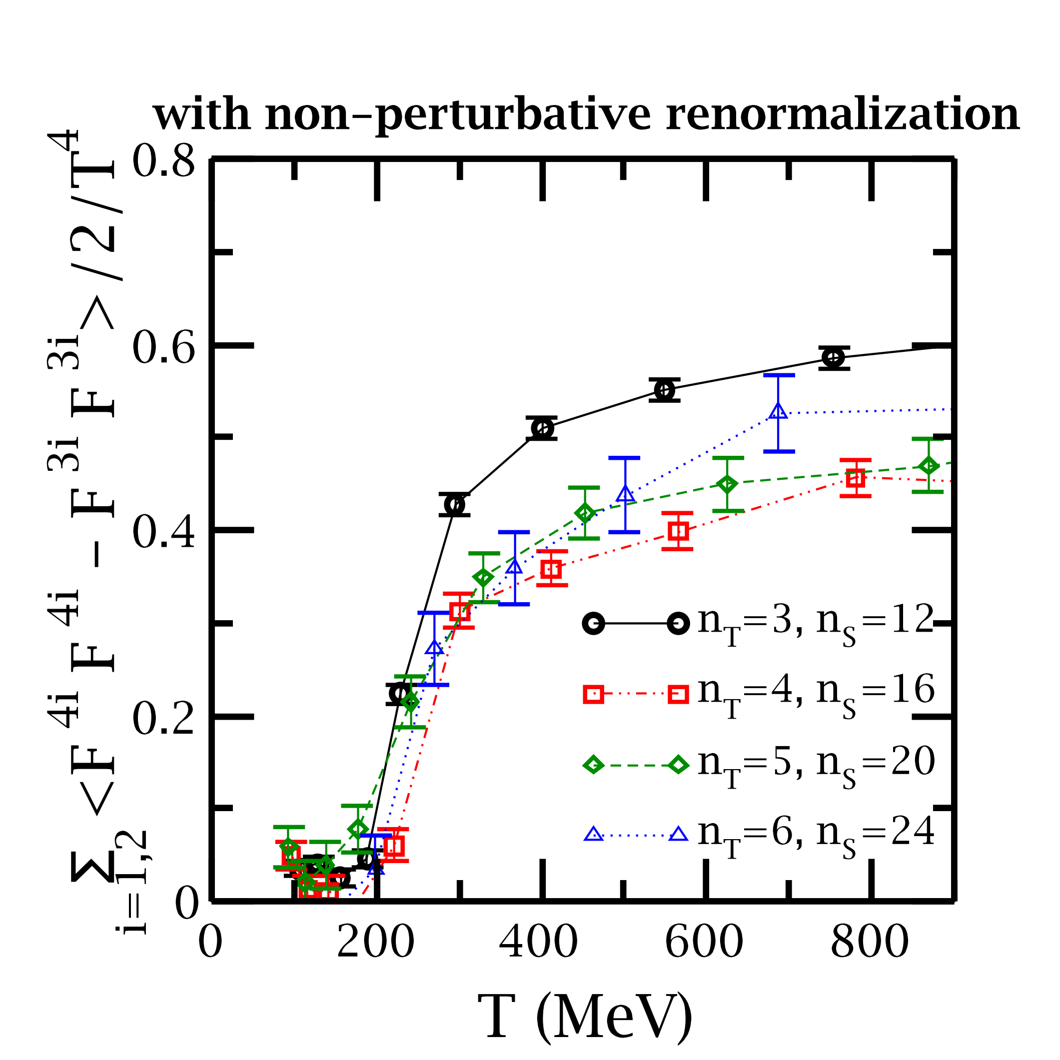
$$\mathcal{D}^{>}(t=0) = i^{N_t} \Delta(\tau = 0)$$

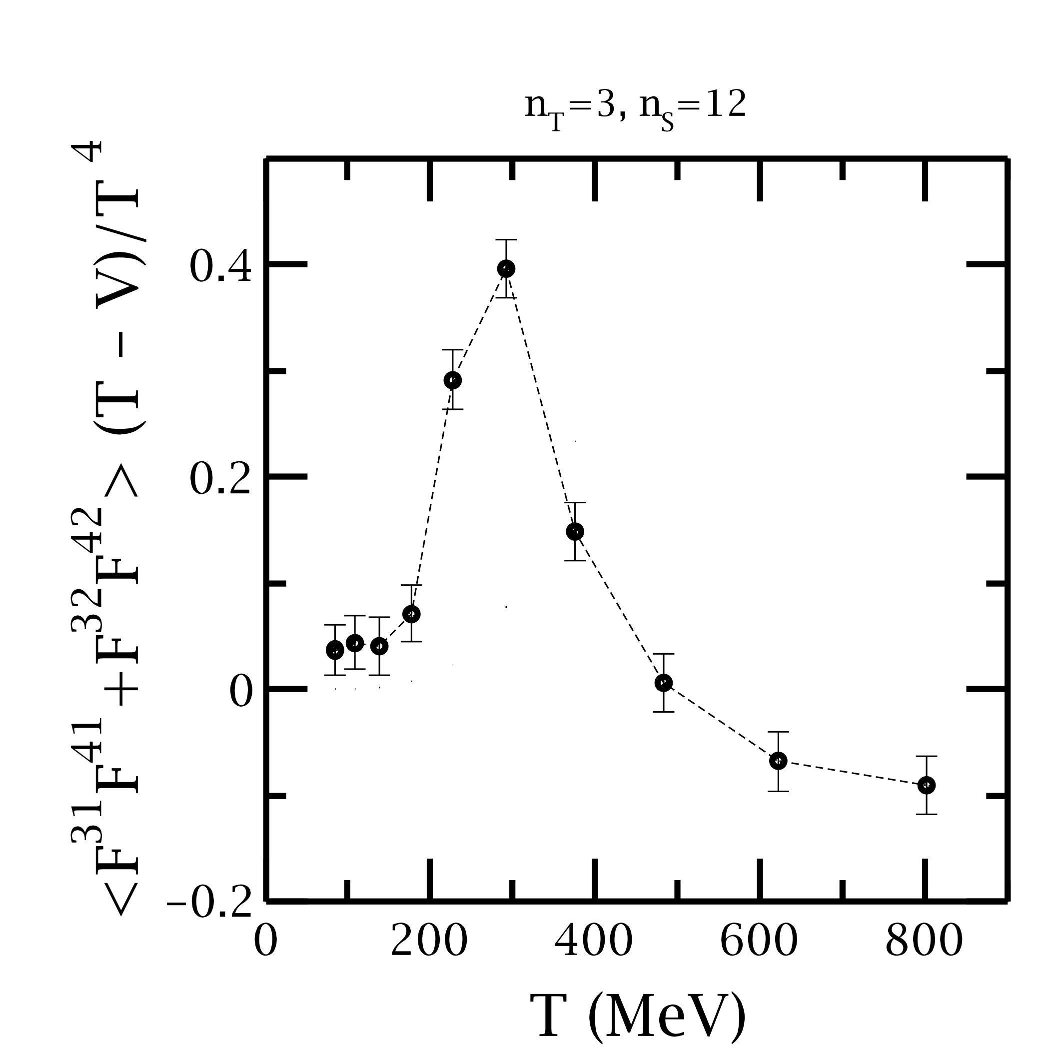
Rotating everything to Euclidean space and calculating

$$x^0 \to -ix^4 \text{ and } A^0 \to iA^4$$
$$\to F^{0i} \to iF^{4i}$$

$$\hat{q} \sim F^{+i}F^{+i} + F^{+i}\frac{\mathcal{D}_z}{q^-}F^{+i}$$

Calculate in quark less SU(2) gauge theory





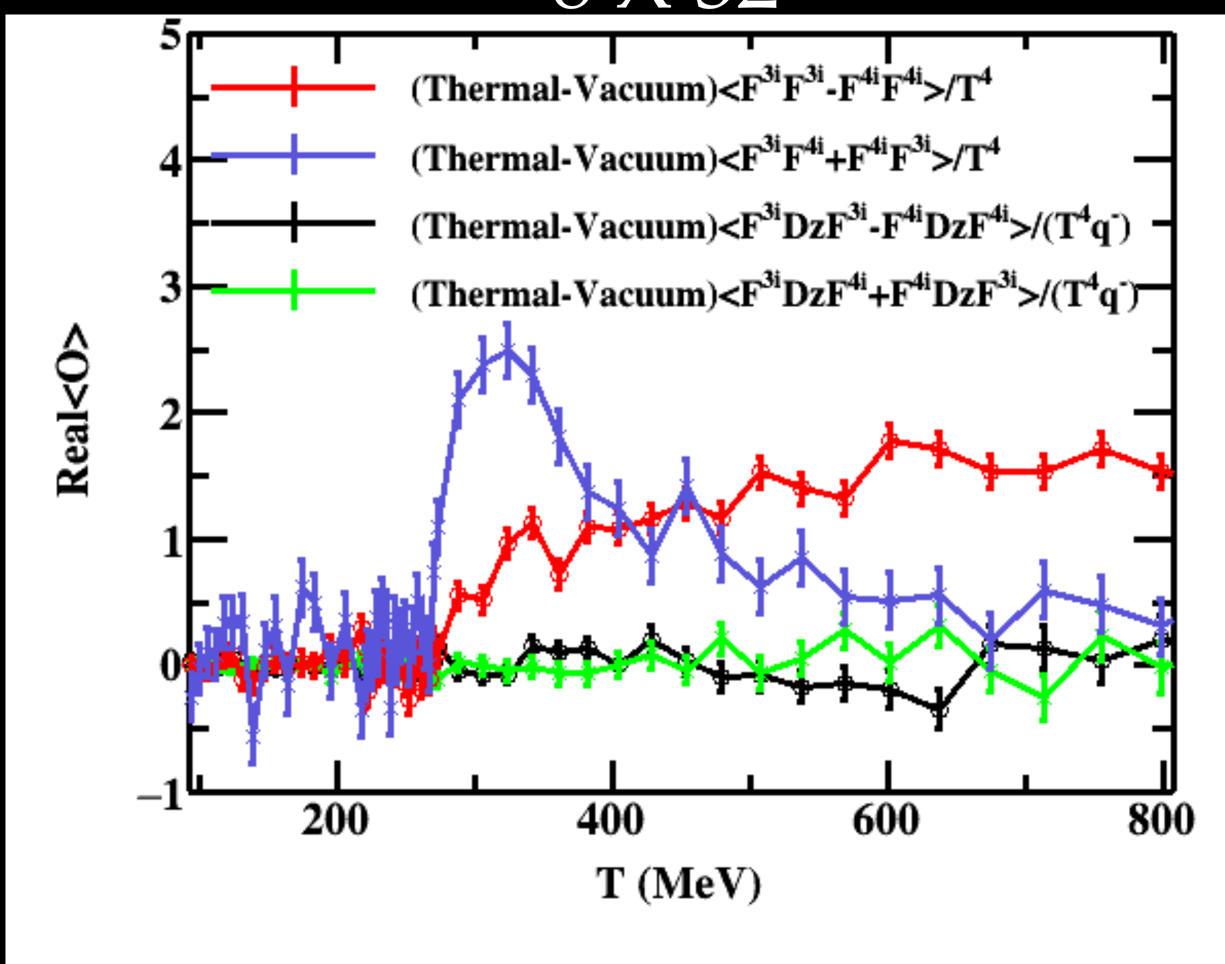
New Results in SU(3)

Full expansion of terms

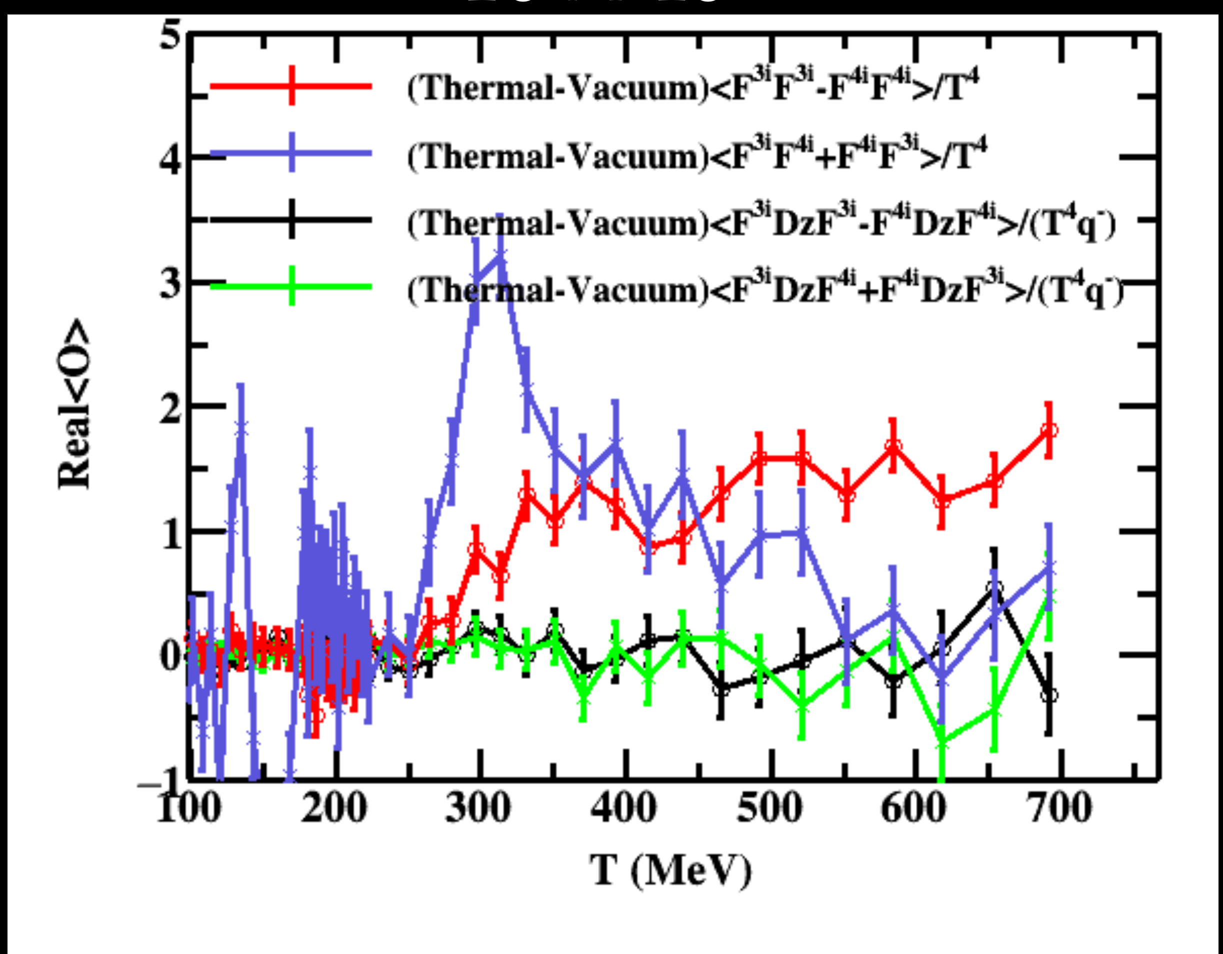
$$\hat{q} \sim \sum_{n=0} \langle m | F^{+i} \left(c_n \frac{\mathcal{D}_z}{q^-} \right)^n F^{+i} | m \rangle$$

Similar to expansion in **Xiangdong Ji, PRL 110, 262002 (2013)**

8 X 32^3



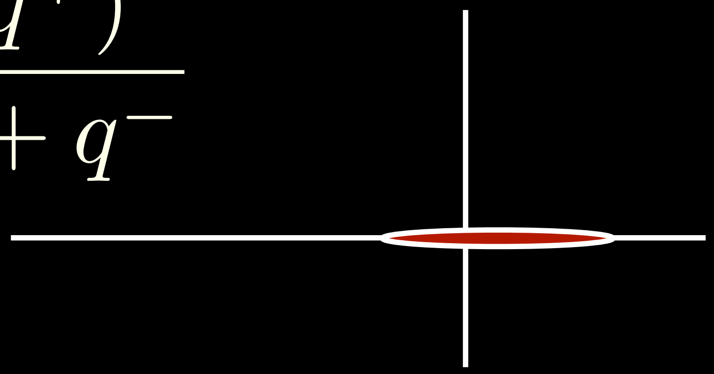
10 X 40^3



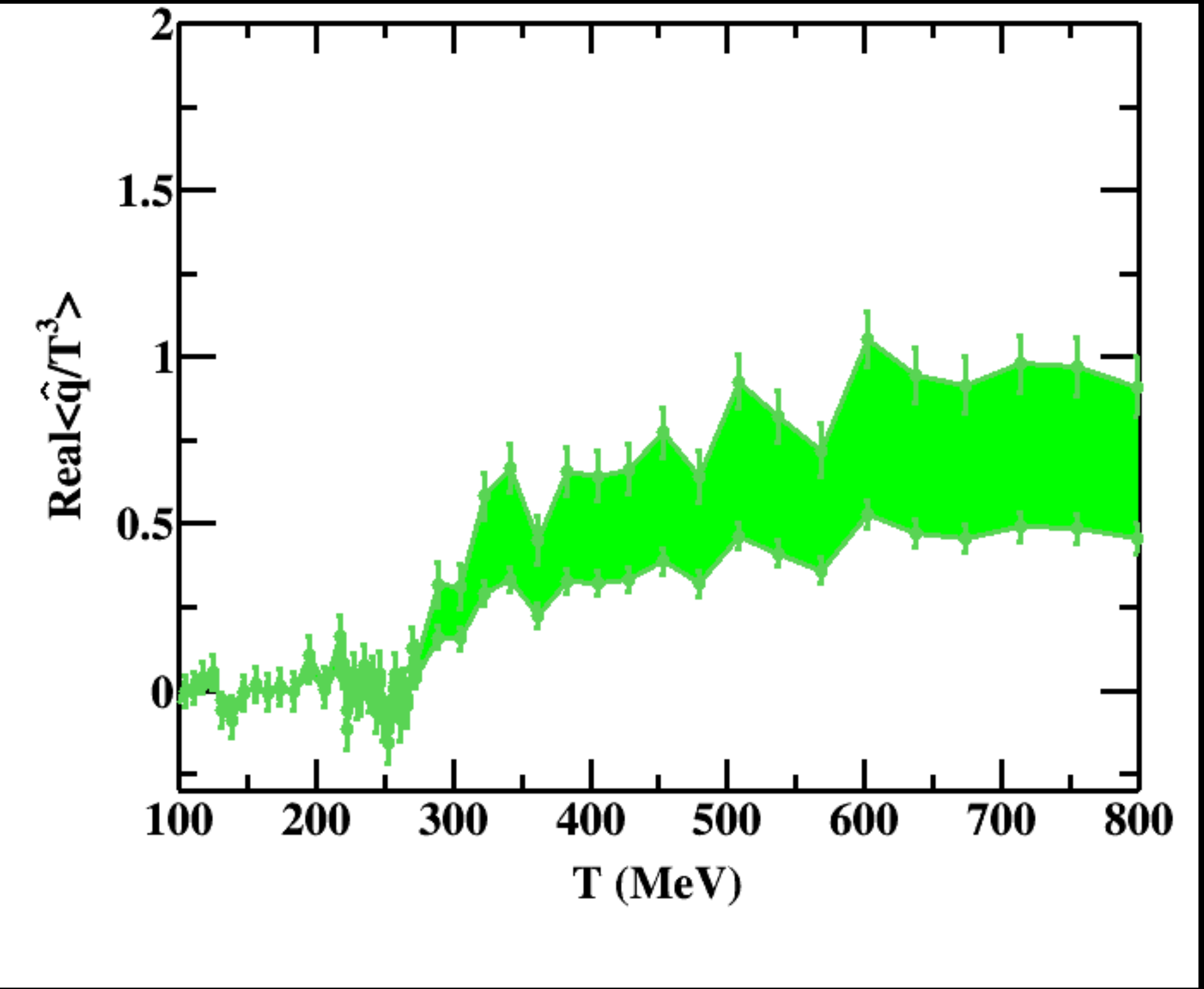
Extracting q̂

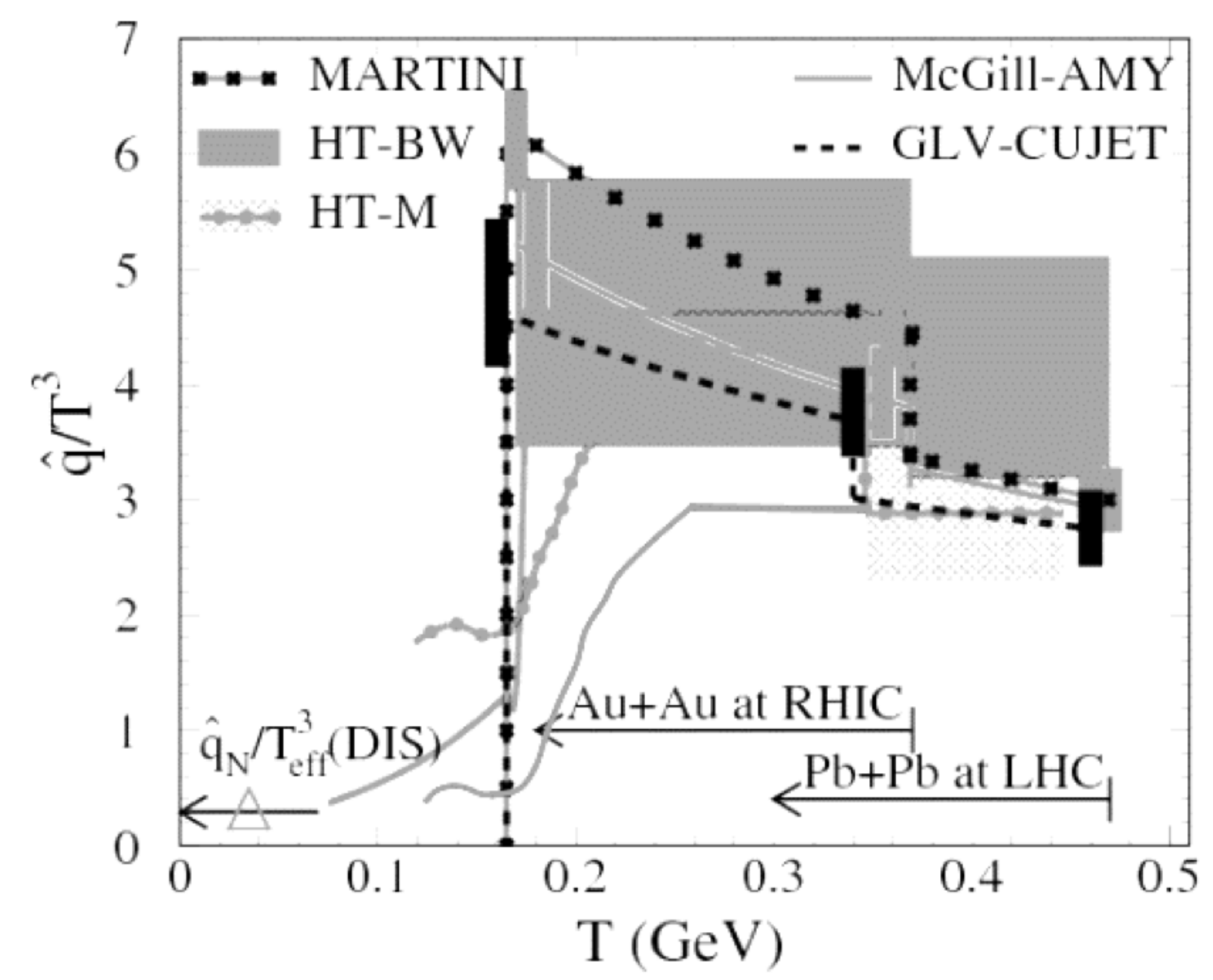
Systematic uncertainty from estimating the range of the thermal cut

$$\int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \frac{\hat{q}(q^+)}{q^+ + Q_0} \rightarrow \int_{Q_{min}}^{Q_{max}} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-}$$



$Q_{min/max}$ from HTL theory

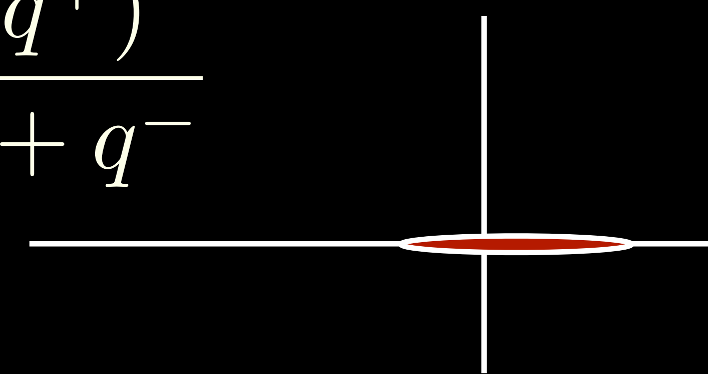




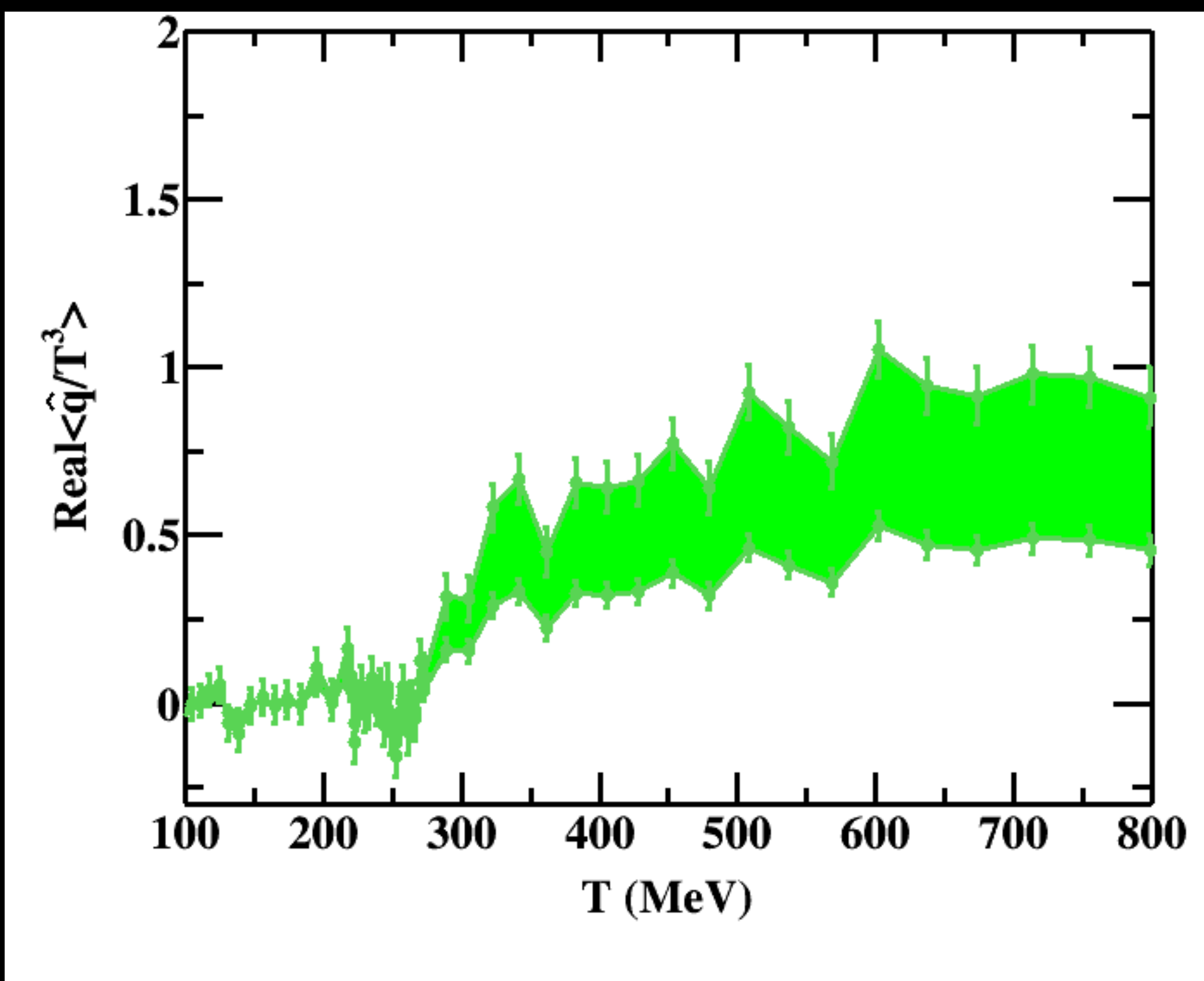
Extracting $\hat{q}$

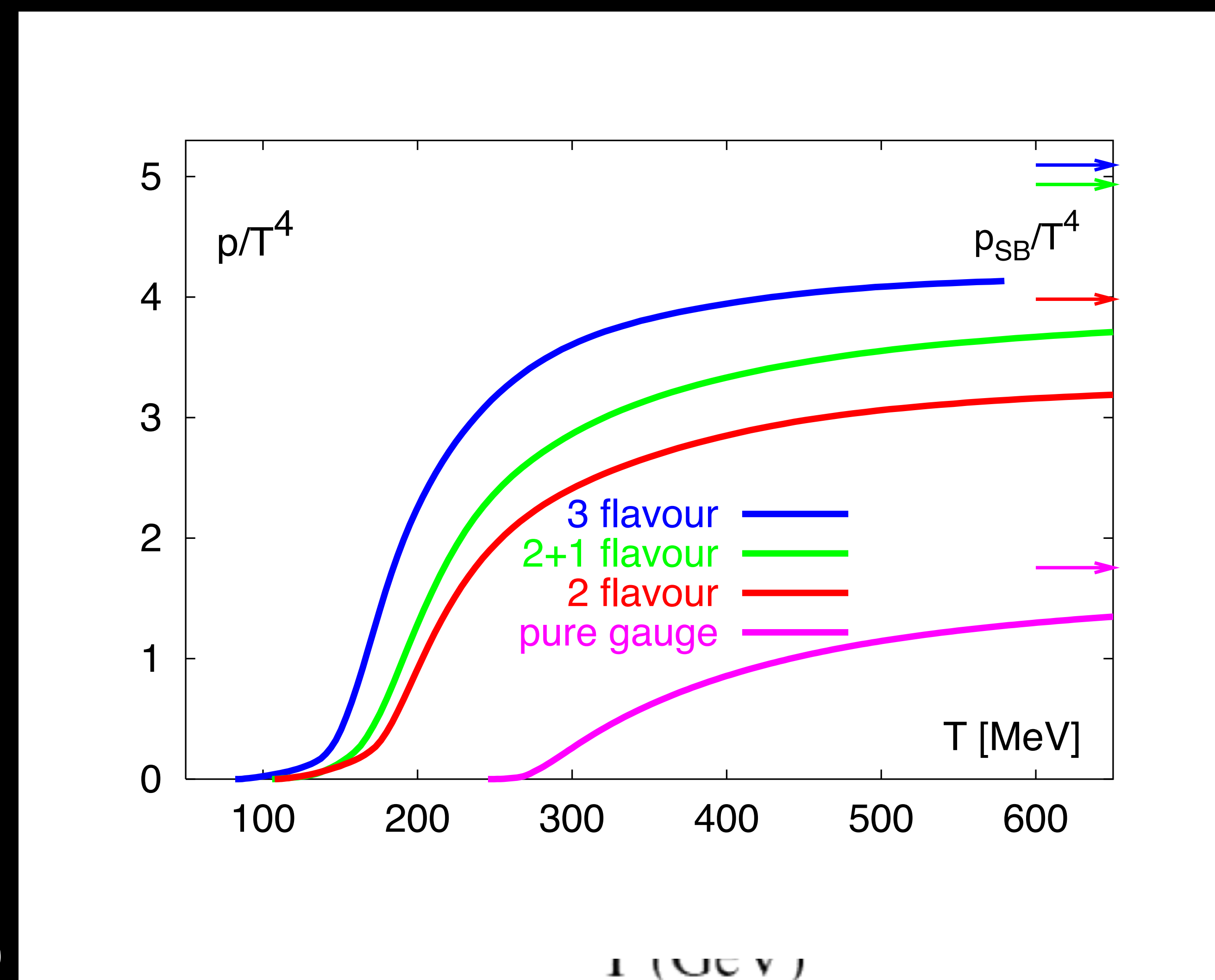
Systematic uncertainty from estimating the range of the thermal cut

$$\int_{-\lambda^2 Q}^{\lambda^2 Q} dq^+ \frac{\hat{q}(q^+)}{q^+ + Q_0} \rightarrow \int_{Q_{min}}^{Q_{max}} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-}$$



$Q_{min/max}$ from HTL theory





Outlook

- HPC is now almost an essential component of nuclear physics
- Large scale simulations are being set up to model the multi-scale phenomena in heavy-ion collisions
- Requires elaborate, compute intensive calibration procedure
- New methods being developed to look at jet transport coefficients from first principles.
- Preliminary results consistent with phenomenological extraction

Thank you for your attention!

Non perturbative re-normalization

- **Expectation value of Polyakov loop:**

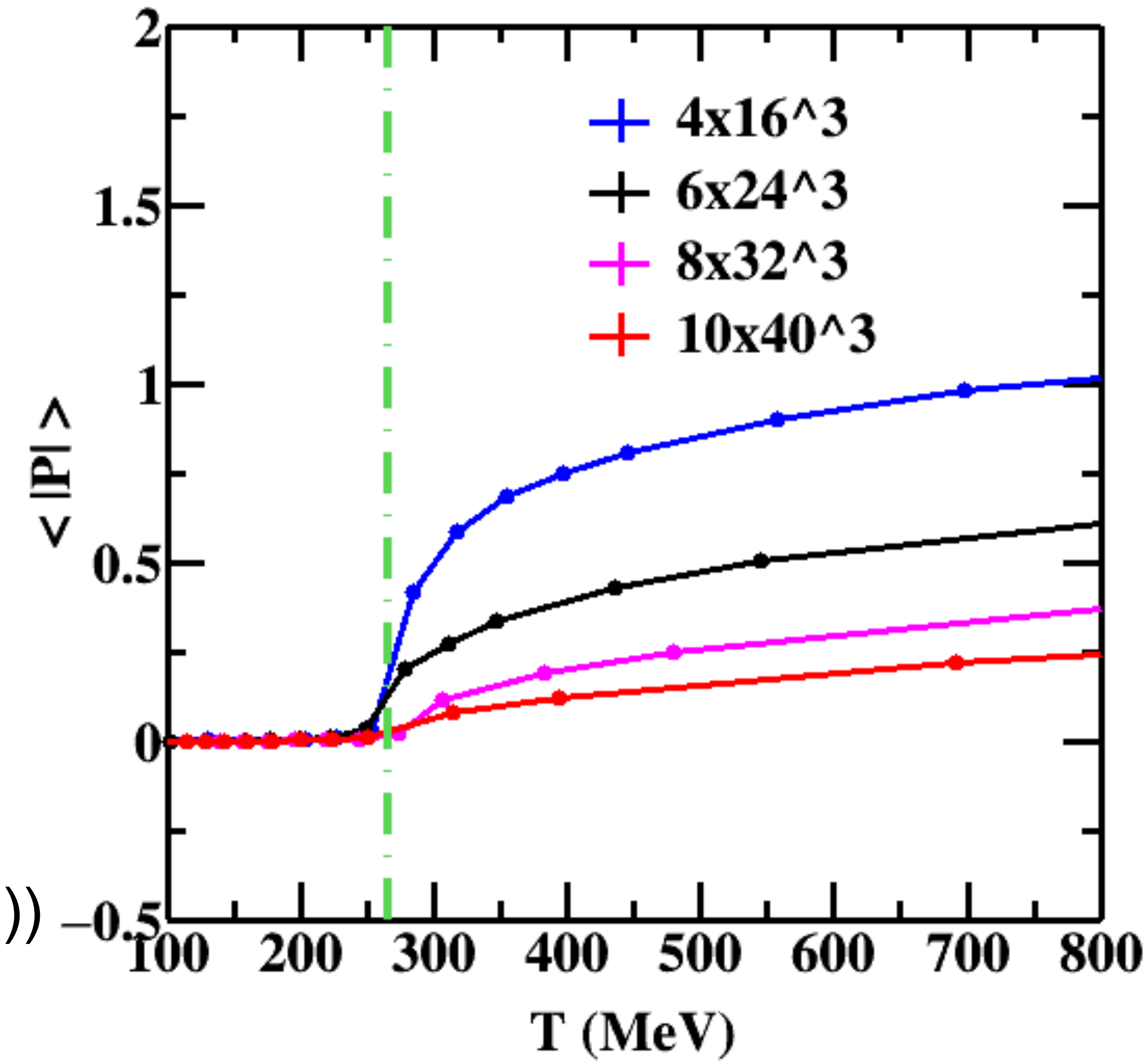
$$P = \frac{1}{n_x n_y n_z} tr\left[\sum_{\vec{r}} \prod_{n=0}^{n_t-1} U_4(na, \vec{r})\right]$$

- **Two loop beta function**

$$a_L = \frac{1}{\Lambda_L}\left(\frac{11}{16\pi^2 g^2}\right)^{-\frac{51}{121}} exp\left(-\frac{8\pi^2}{11g^2}\right)$$

$$Temperature, T = \frac{1}{n_t a_L}$$

(Pure SU(3))



- **Nonperturbative correction**

$$Tune\ \frac{T_c}{\Lambda_L}\ is\ independent\ of\ g$$