D-meson decay constants with overlap fermions on domain wall fermion configurations

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Outline

- **Motivation**
- **Lattice setup**
- **Renormalization**
- **Preliminary results**
- **Summary**

arXiv:1710.08678(PRD97.094501, 2018)
arXiv:xxxx.xxxxx

Yujiang Bi, Hao Cai, Ying Chen, Wei-Feng Chiu,
Ming Gong, Keh-Fei Liu, Yi-Bo Yang

What we calculate

- $f_{D_{(s)}^{(*)}}$ and their ratios

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 c|P(p)\rangle = f_P p_\mu, \quad q = d, s$$

$$(m_q + m_c)\langle 0|\bar{q}\gamma_5 c|P(p)\rangle = f_P m_{PS}^2$$

$$\langle 0|\bar{q}(0)\gamma^\mu q'(0)|V(p,\lambda)\rangle = \boxed{f_V} m_V e^\mu_\lambda$$

- f_V^T / f_V

$$\langle 0|\Big(\bar{q}(0)\sigma^{\mu\nu}q'(0)\Big)(\mu)|V(p,\lambda)\rangle = i\boxed{f_V^T(\mu)}\left(e^\mu_\lambda p^\nu - e^\nu_\lambda p^\mu\right)$$

Motivation

- CKM elements

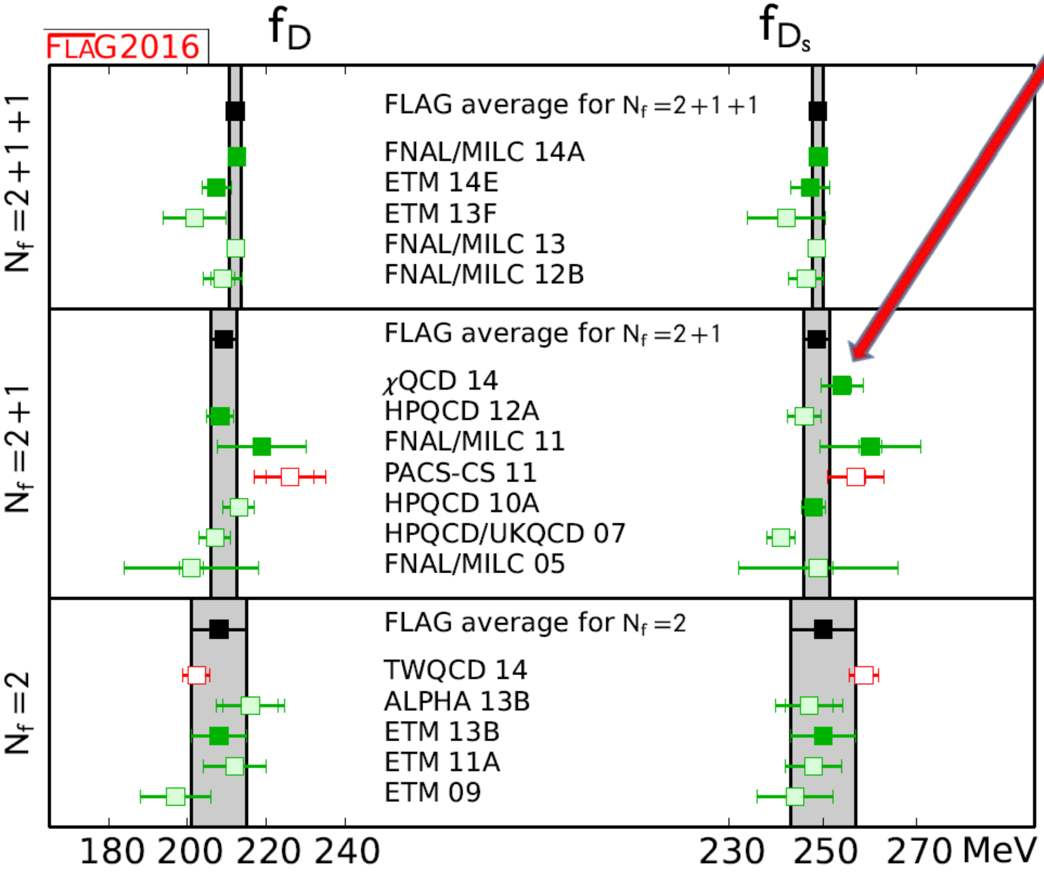
$$\Gamma(P \to \ell\nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

- f_V is not easy to measure
  - Leptonic decay BRs are much smaller than those of strong decays
- If there are exp. measurements, can also determine CKM matrix elements
- Test the accuracy of HQET,

$$f\_V/f\_PS = 1 + O(1/m_Q)$$

- Test the accuracy of factorization in studies of nonleptonic *B* decays, e.g., $B \to D^{(*)} M$

LQCD results



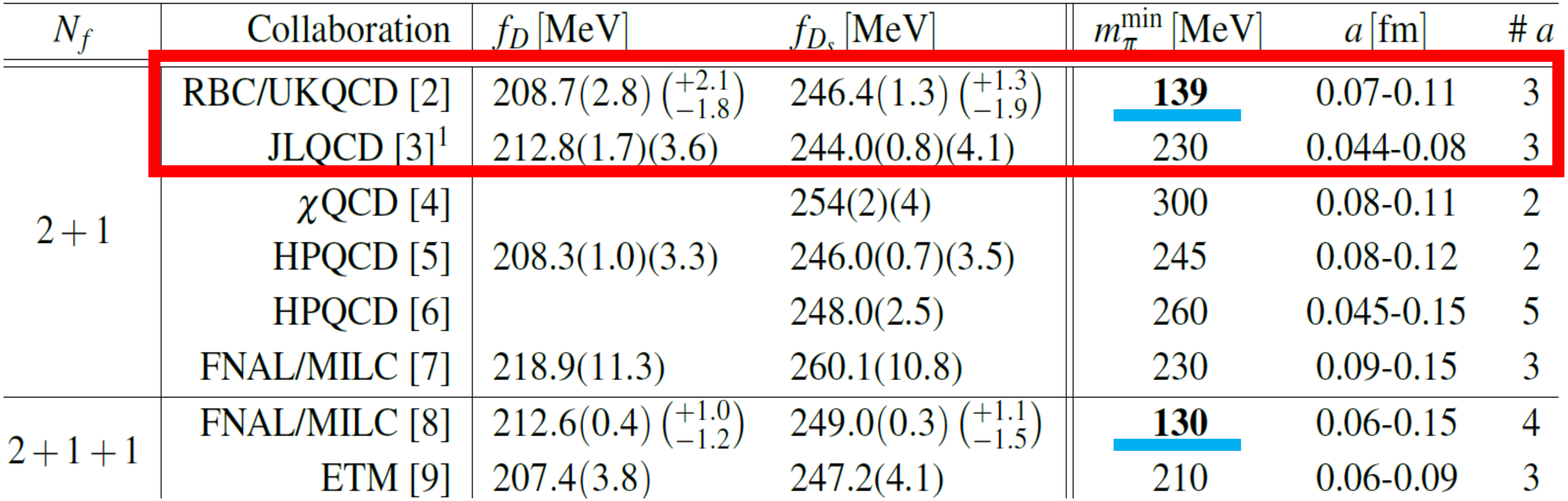
Y.-B. Yang et al., PRD92, 2015

- $N_f = 2+1$:
  $f_{D_s} = 249.8(2.3)$ MeV
- $N_f = 2+1+1$:
  $f_{D_s} = 248.83(1.27)$ MeV
- PDG2016 (CPC40,100001):
  $f_{D_s^+}^{exp} = 257.8(4.1)$ MeV
- Agreement in 2σ
- Exp. expected to catch up in 2020 (BESIII, BelleII)

FLAG2016, [arXiv:1607.00299]
results before Nov. 30, 2015

f_{D_s} and f_D

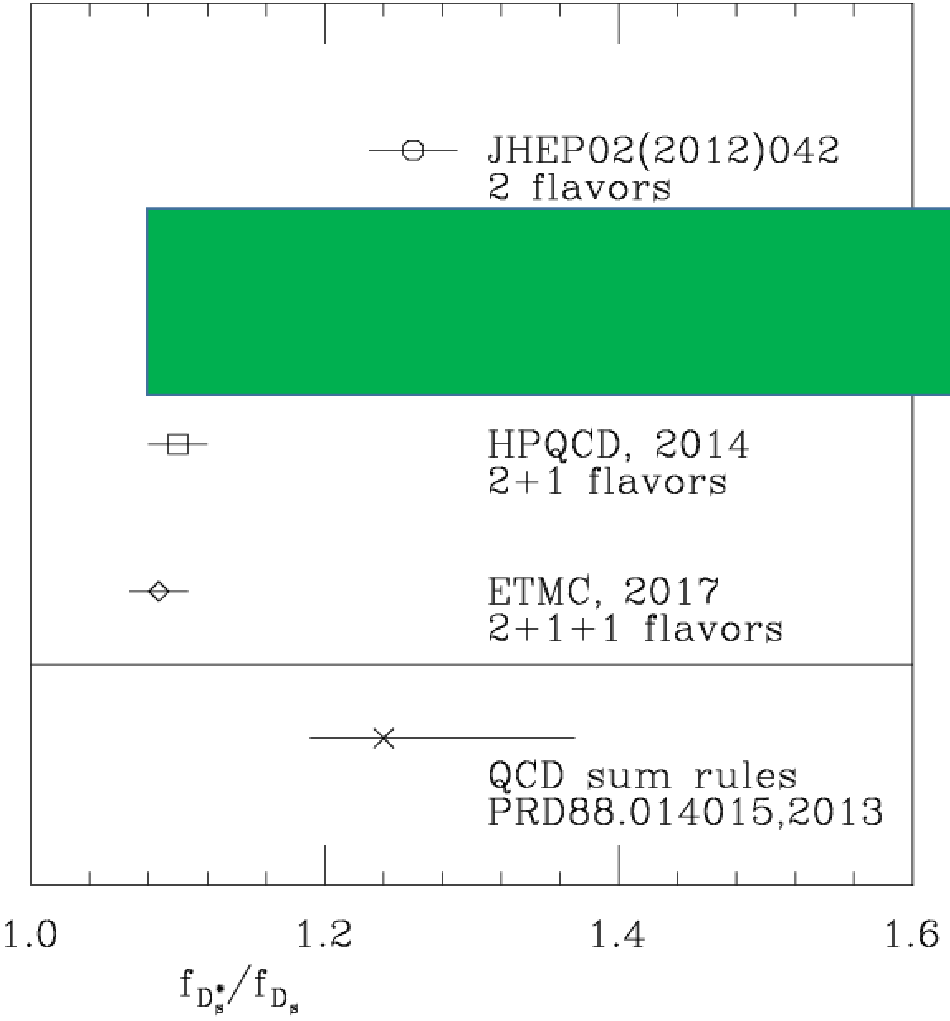
[1702.05360]



N_f	Collaboration	f_D [MeV]	f_{D_s} [MeV]	$m_\pi^{\min}$ [MeV]	a [fm]	# a
2+1	RBC/UKQCD [2]	$208.7(2.8)\left(^{+2.1}_{-1.8}\right)$	$246.4(1.3)\left(^{+1.3}_{-1.9}\right)$	**139**	0.07-0.11	3
	JLQCD [3][1]	212.8(1.7)(3.6)	244.0(0.8)(4.1)	230	0.044-0.08	3
	χQCD [4]		254(2)(4)	300	0.08-0.11	2
	HPQCD [5]	208.3(1.0)(3.3)	246.0(0.7)(3.5)	245	0.08-0.12	2
	HPQCD [6]		248.0(2.5)	260	0.045-0.15	5
	FNAL/MILC [7]	218.9(11.3)	260.1(10.8)	230	0.09-0.15	3
2+1+1	FNAL/MILC [8]	$212.6(0.4)\left(^{+1.0}_{-1.2}\right)$	$249.0(0.3)\left(^{+1.1}_{-1.5}\right)$	**130**	0.06-0.15	4
	ETM [9]	207.4(3.8)	247.2(4.1)	210	0.06-0.09	3

- **Results after 2015: RBC/UKQCD (1701.02644). JLQCD. 1803.03065**
- **JLQCD (lat2016, 1702.02303): Sys. error from scale setting only**
- **Blossier, Heitger, Post (1803.03065): Sys. error from scale setting**
  - $f_{D_s} = 238(5)(2)$ **MeV**
  - **2*a*, 2 flavors, Clover fermions**

$f_{D_s^*}/f_{D_s}$



- **Becirevic et al., JHEP02 (2012) 042**
  - **4*a*, 2-flavor, tmQCD**
- **HPQCD, PRL112.212002 (2014)**
  - **2*a*, 2+1-flavor, HISQ+asqtad**
- **ETMC, PRD96.034524 (2017)**
  - **3*a*, 2+1+1-flavor, tmQCD**
- **Sea quark effects?**

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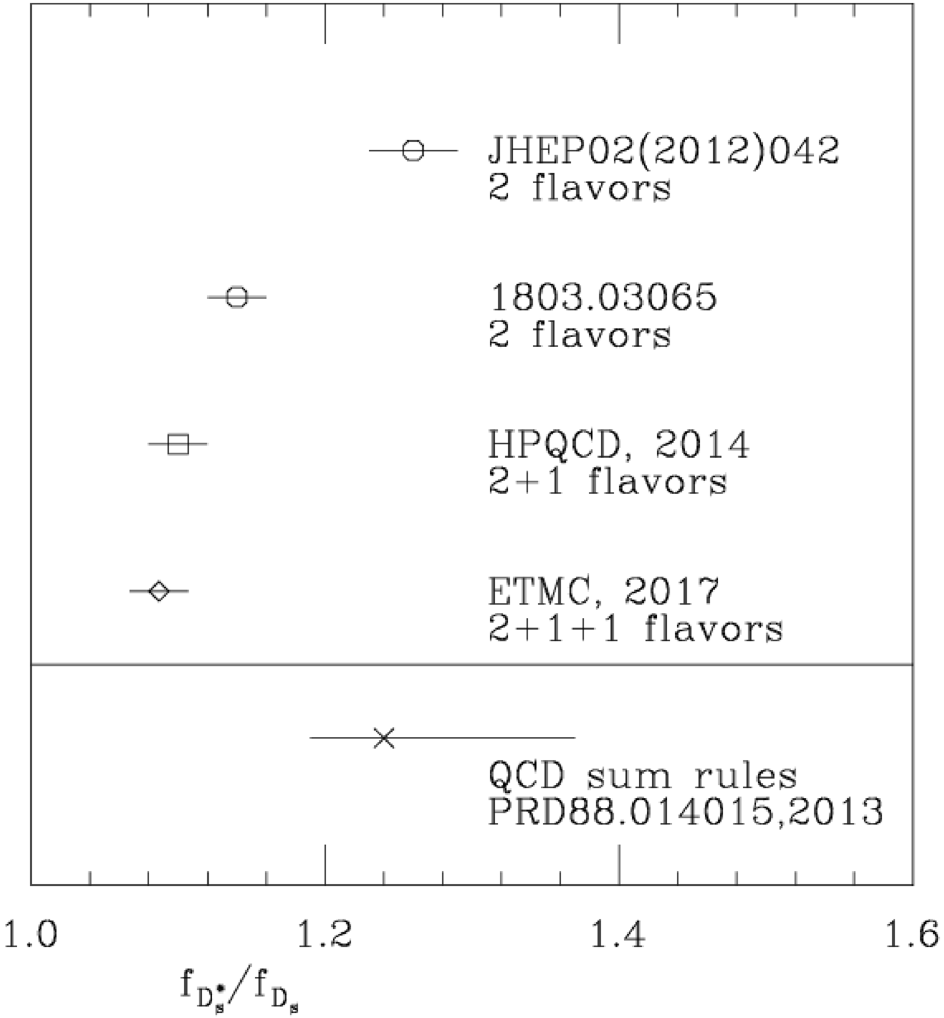

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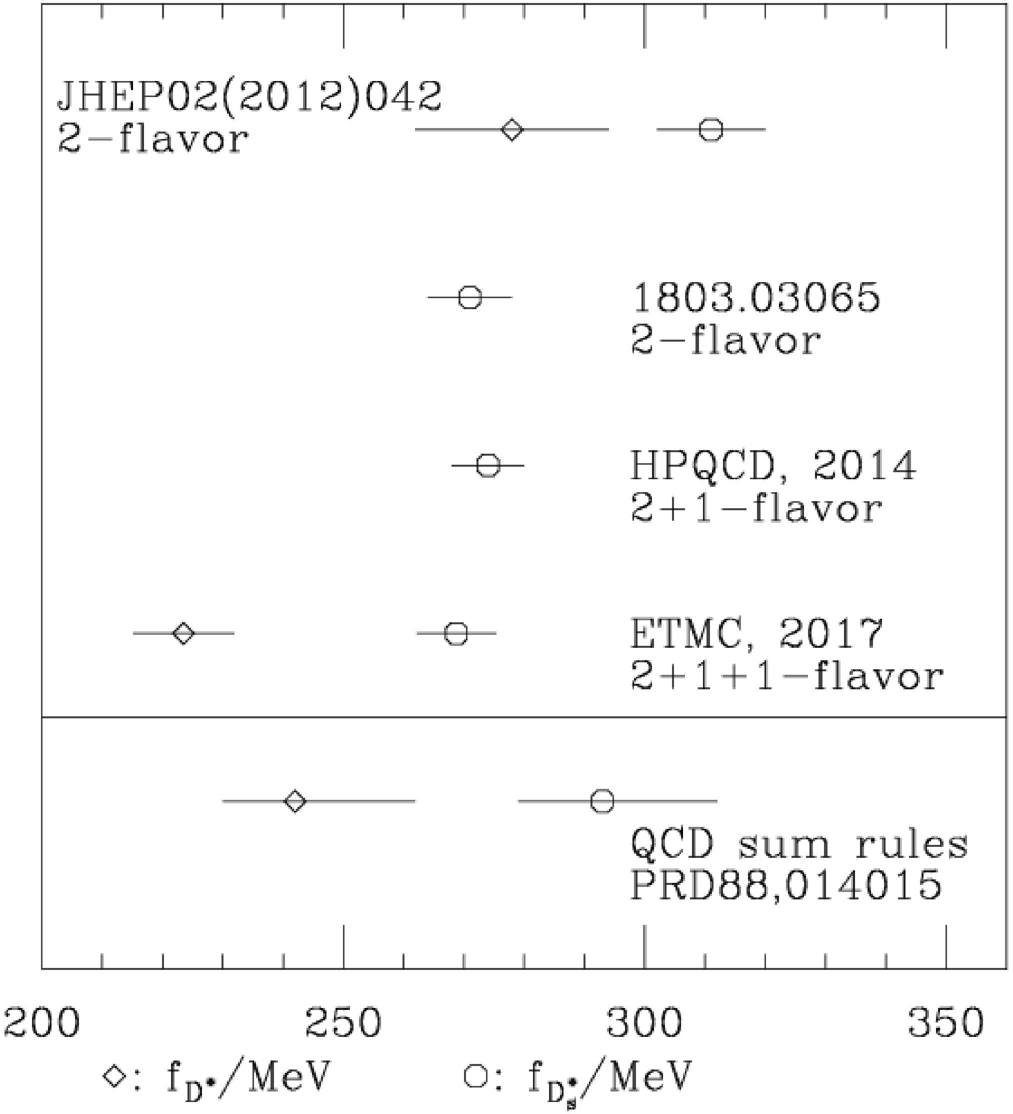
f_{D^*} and $f_{D_s^*}$



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- **Sea quark effects?**

Lattice setup

- **Overlap fermions on 45 domain wall fermion configurations**

$1/a(\mathrm{GeV})$	label	am_{sea}	volume
1.730(4)	48I	0.00078/0.0362	$48^3 \times 96$

- **2+1-flavor ensemble (RBC/UKQCD Collab.)**
- **Physical sea quark mass: $m_\pi = 139.2(4)$ MeV**
- **4 light val. quark masses: $m_\pi \sim 114 - 208$ MeV**
- **2 strange val. quark masses, slightly $\lesssim m_s^{phy.}$**

$am_l^{(val)}$	$am_s^{(val)}$	$am_c^{(val)}$
0.0017,0.0024, 0.0030,0.0060	0.0580, 0.0650	0.6800,0.7000, 0.7200,0.7400

m_π, f_π, m_K

- m_π and m_K are extracted from pseudoscalar density 2-point functions
- Will be used to fix the physical light and strange valence quark masses

am_q^{val}	0.0017	0.0024	0.0030	0.0060
m_π/MeV	114(2)	135(2)	149(2)	208(2)
f_π/MeV	130.3(9)	131.0(9)	131.6(8)	---

- A linear interp. in m_π^2 gives $f_\pi = 131.3(6)$ MeV
- Consistent with the RBC/UKQCD result on the same ensemble [arXiv:1411.7017(hep-lat)]

D-meson 2-point functions

- **Coulomb gauge wall source propagators are used to improve overlapping with the ground state**
- **Sink operators are with spacial displacement**

$$\boldsymbol{O_\Gamma(\vec{x}, t;\ \vec{r}) = \bar{\psi}_1(\vec{x}, t)\Gamma\psi_2(\vec{x} + \vec{r}, t)}$$

$\boldsymbol{\Gamma = \gamma_5}$ **or** $\boldsymbol{\gamma_i}$

$\boldsymbol{\vec{r} = 0}$**: local operator**

- **Same** $\boldsymbol{r = |\vec{r}|}$ **averaged to get the correct** $\boldsymbol{J^P}$

$$\begin{aligned} C_P(r,t) &= \frac{1}{N_r}\sum_{\vec{x},|\vec{r}|=r}\langle 0|O_P(\vec{x},t;\vec{r})O_P^{(W)\dagger}(0)|0\rangle, \\ C_V(r,t) &= \frac{1}{3N_r}\sum_{\vec{x},i,|\vec{r}|=r}\langle 0|O_V(\vec{x},t;\vec{r})O_V^{(W)\dagger}(0)|0\rangle \end{aligned}$$

$$C^W(t) = \langle 0|O^{(W)}(t)O^{(W)\dagger}(0)|0\rangle$$

Data analysis

1. **Simultaneous correlated fitting to all correlators**

   Common parameter: $\boldsymbol{m_H}$

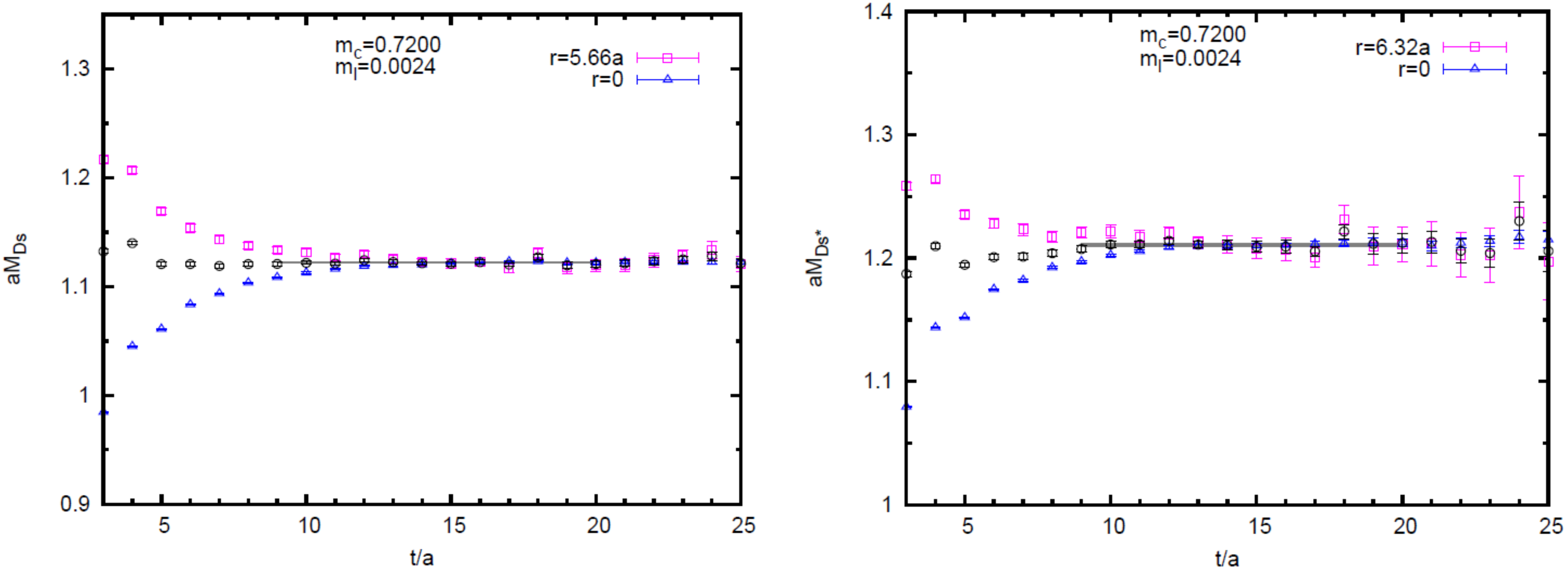
2. **Fit a simply combined correlator**

$$\boldsymbol{C(\omega, t) = C(r = 1, t) + \omega C(r, t)}$$

Adjust $\boldsymbol{r}$ and $\boldsymbol{\omega}$ to get the best mass plateau

- **The two methods give consistent $\boldsymbol{m_H}$**
- **The result of $\boldsymbol{m_H}$ is insensitive to $\boldsymbol{\omega}$**
- **Combine the spectral weights from $\boldsymbol{C(r = 0, t)}$ and $\boldsymbol{C^W(t)}$ to get the decay constants**

Mass plateau and fitting results

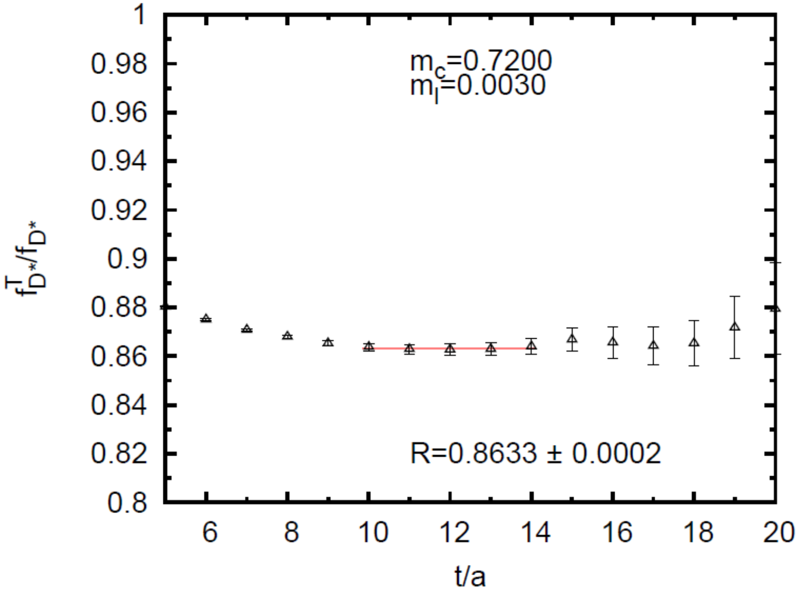


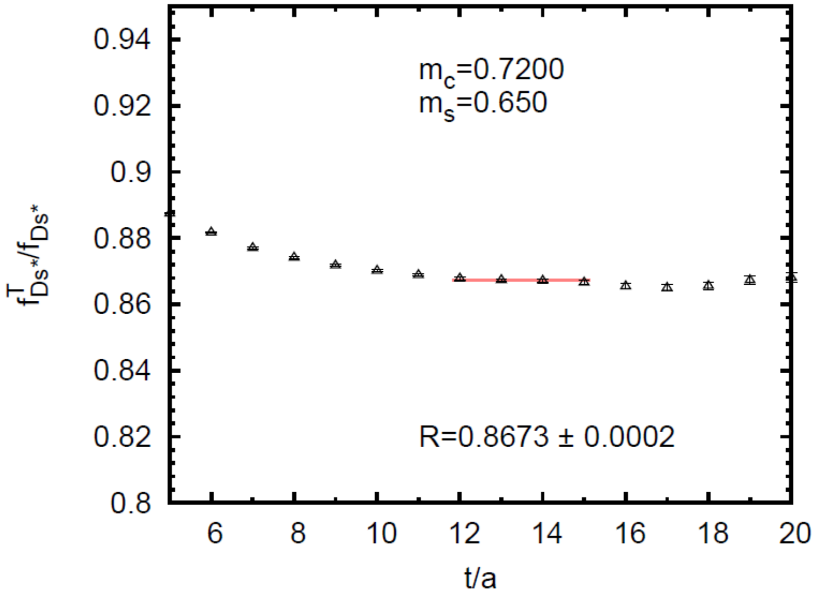
- Black circles: $C(\omega, t) = C(r = 1, t) + \omega C(r, t)$
- t_{max}: $\delta C(t)/C(t)$<10%(5%) for V(PS) mesons
- t_{min}: varied to get stable results, χ^2/dof<1.0

f_V^T / f_V

$$R(t) = C_T(t)/C_V(r=0,t) = \frac{\sum_i \langle T_{0i} O_V^{(W)\dagger} \rangle}{\sum_i \langle V_i O_V^{(W)\dagger} \rangle} \xrightarrow{t\to\infty} \frac{f_T}{f_V}$$

$$C_T(t) \quad = \quad \sum_{\vec{x},i} \langle 0 | O_T(\vec{x},t) O_V^{(W)\dagger}(0) | 0 \rangle$$





D*

Bare values

D_s*

Interp./extrap. to physical point

- **$m_\pi^2, m_{ss}^2 \equiv 2m_K^2 - m_\pi^2$ and m_{D_s} are used to set the physical quark masses**
- **Our quark masses are close to their physical values**
- **Linear Interp./extrap. in m_π^2, m_{ss}^2 and m_{D_s}**
- **For a meson mass or decay constant:**

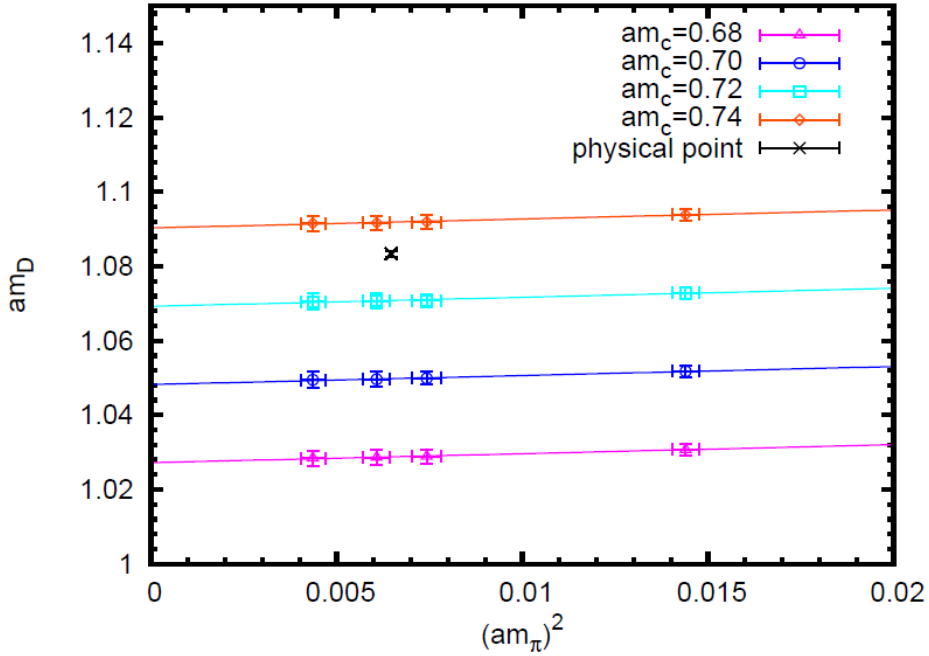
$$A(m_{u/d}, m_s, m_c) = A^{(phy)} + b_1 \Delta m_\pi^2(m_{u/d}) + b_2 \Delta m_{ss}^2(m_s) + b_3 \Delta m_{D_s}(m_c)$$

$$\Delta m^2 = m^2 - m_{phy}^2 \,, \quad \Delta m_{D_s} = m_{D_s} - m_{D_s}^{phy}$$

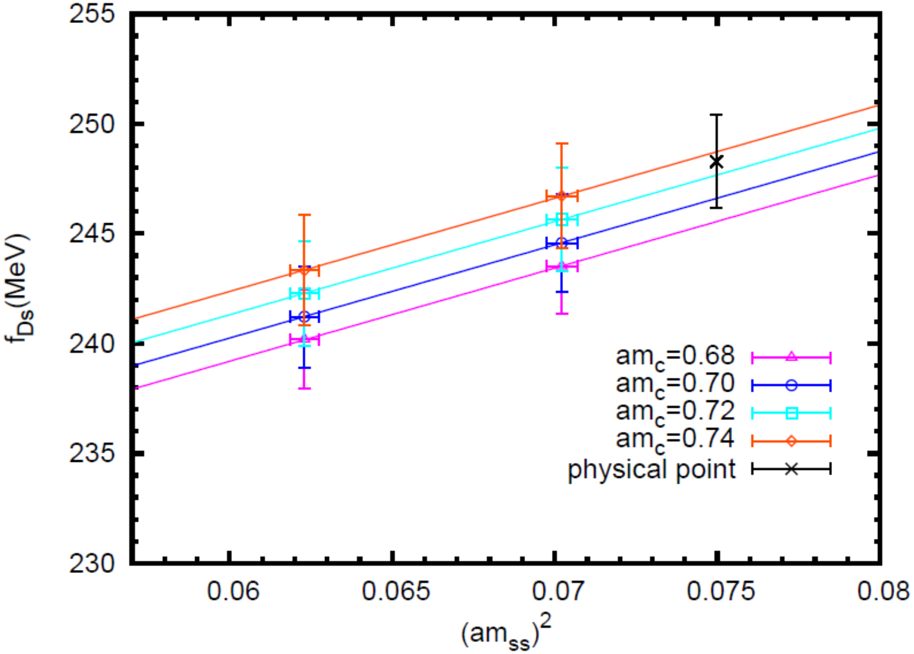
- **Supported by the data with good χ^2/dof**

Interp./extrap. to physical point

m_D



f_{D_s}



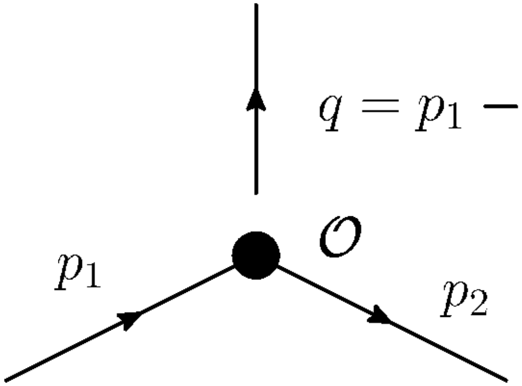
RI/(S)MOM renormalization

- Renormalization constants are needed to get
  - $\boldsymbol{f_P}$ using the local axial vector current
  - $\boldsymbol{f_V}$ using the local vector current
  - $\boldsymbol{f_V^T}$ from the tensor operator

- RI/MOM and RI/SMOM schemes are used and matched to the $\overline{\mathrm{MS}}$ scheme

- Two schemes are used to get more control on the systematics

RI/(S)MOM



- **RI-MOM with Ward Identity**
  - Z_A from WI: $\partial_\mu A^\mu = 2m_q P$
  - Then calculate Z_q and others
  - $p_1 = p_2$
- **RI-SMOM with WI**
  - Z_A from WI, then determine the other Z-factors
  - $p_1^2 = p_2^2 = (p_1 - p_2)^2 = q^2$
  - Nonperturbative effects are suppressed by $1/p^6$
  - Conversion ratio $Z^{\overline{\mathrm{MS}}}/Z^{\mathrm{SMOM}}$ may converge faster than $Z^{\overline{\mathrm{MS}}}/Z^{\mathrm{MOM}}$, true for Zs
  - Less momentum modes satisfy the symmetric condition

Renormalization of overlap quark bilinear operators

Bi et al., arXiv:1710.08678(PRD97.094501)

- **Matching hadronic matrix elements calculated on the lattice to the continuum**

$$\bar{\psi}\Gamma\psi, \qquad \Gamma = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$$

- **81 configurations are used**

$1/a$(GeV)	label	am_l^{sea}/am_s^{sea}	volume	$N_{\rm conf}$
1.730(4)	48I	0.00078/0.0362	$48^3 \times 96$	81

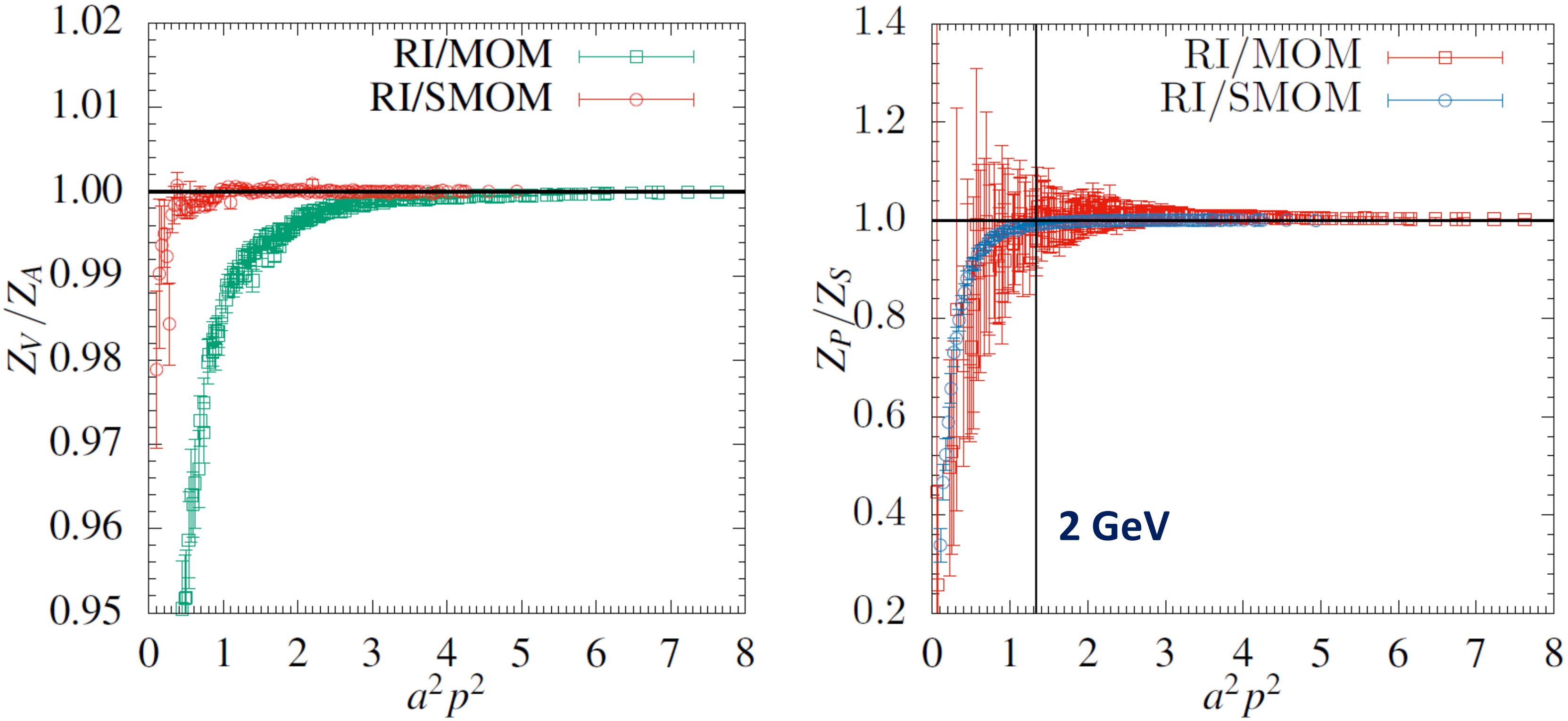
- **Point source quark propagators in Landau gauge**
- **"democratic" condition used in RI/MOM**

$$\frac{p^{[4]}}{(p^2)^2} < 0.29, \quad \text{where } p^{[4]} = \sum_\mu p_\mu^4, \quad p^2 = \sum_\mu p_\mu^2$$

Z_V/Z_A and Z_S/Z_P

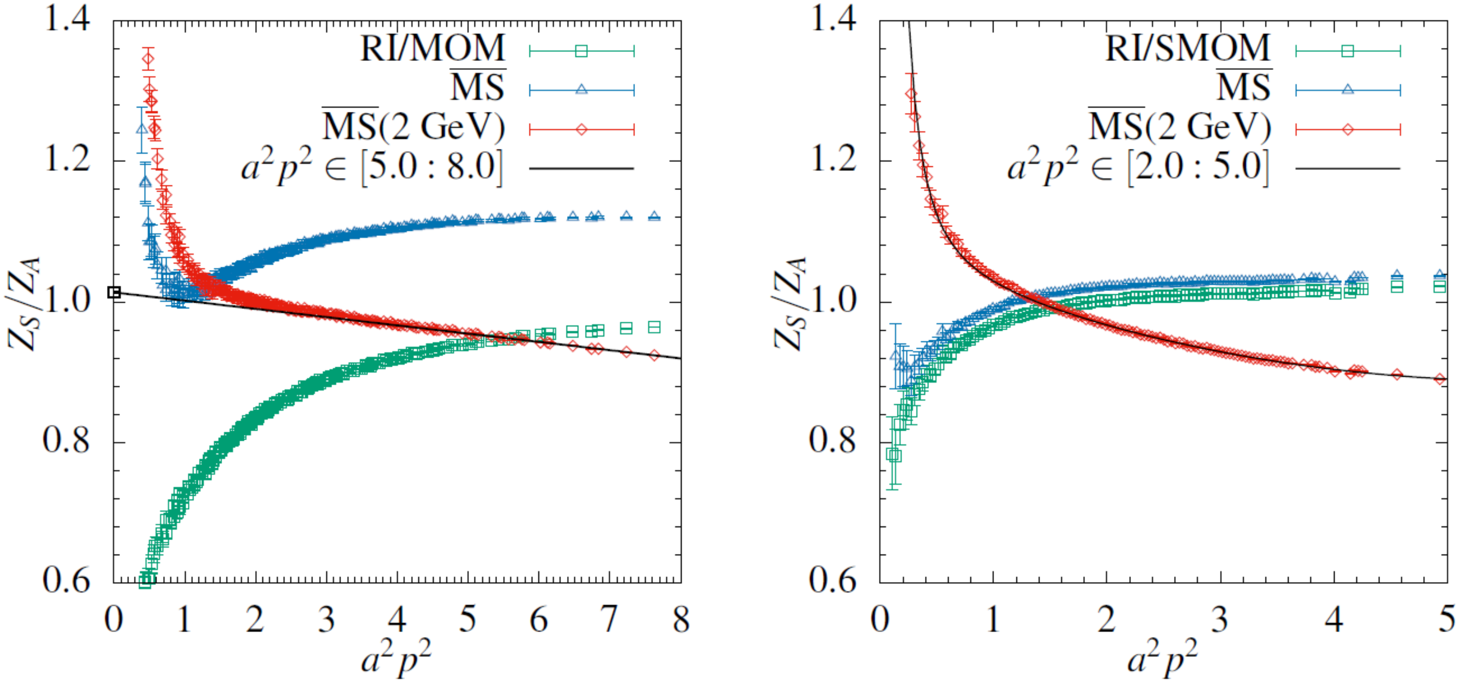
Bi et al., arXiv:1710.08678

- **$Z_V = Z_A$ and $Z_S = Z_P$ are expected and confirmed for overlap fermions**



Z_S

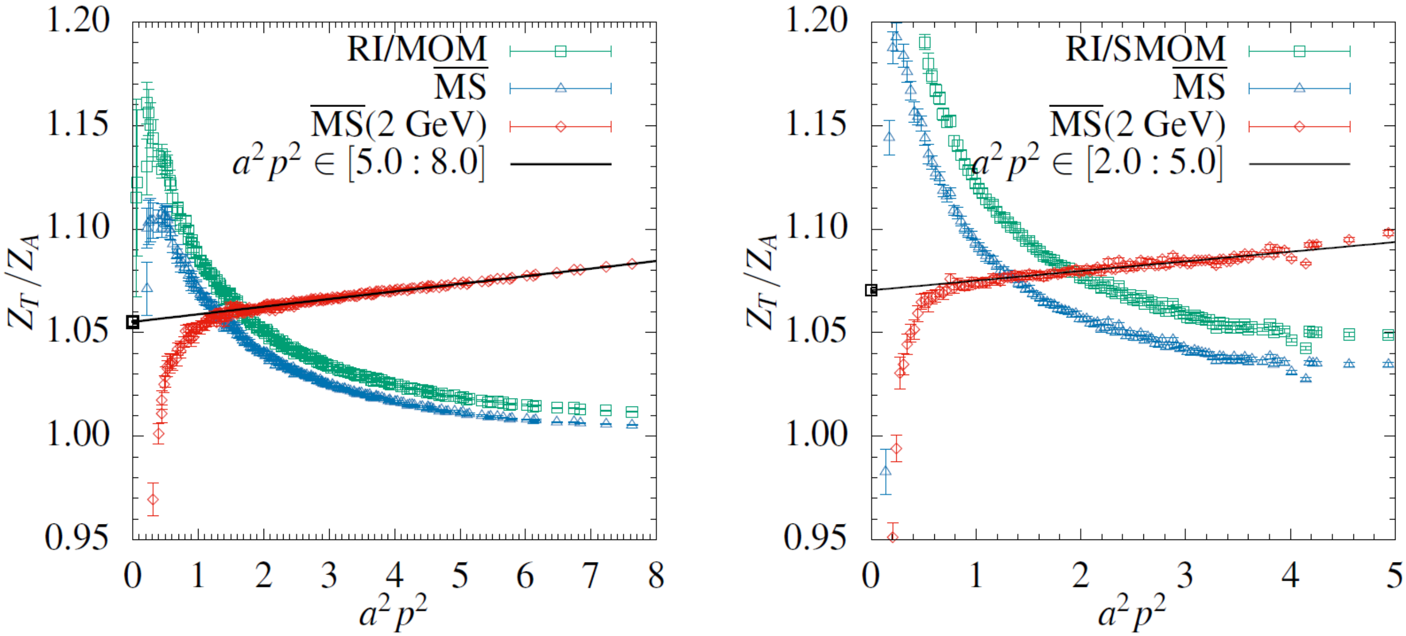
Bi et al., arXiv:1710.08678



- **Z_S in the RI/MOM, RI/SMOM and $\overline{\text{MS}}$ schemes**
- **$Z_S^{\overline{\text{MS}}}/Z_S^{\text{SMOM}}$ converges much faster than $Z_S^{\overline{\text{MS}}}/Z_S^{\text{MOM}}$**
  (corresponding sys. error: 1.5%→0.2%)
- **However $O((a^2p^2)^2)$ discretization effects seem large**

Z_T and the final results

Bi et al., arXiv:1710.08678



- **(Above) Z_T in the RI/MOM, RI/SMOM and $\overline{\mathrm{MS}}$ schemes**
- **(Below) Final results for all renormalization constants**

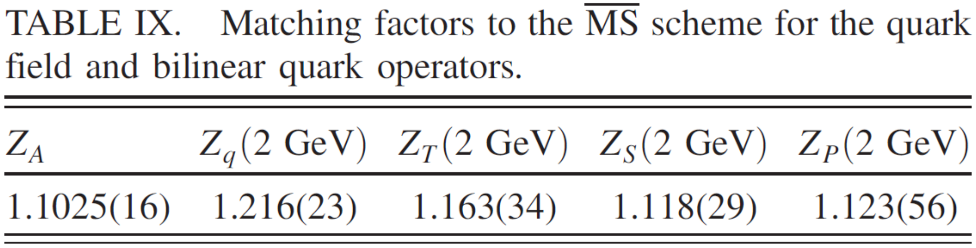


TABLE IX. Matching factors to the $\overline{\mathrm{MS}}$ scheme for the quark field and bilinear quark operators.

Z_A	Z_q(2 GeV)	Z_T(2 GeV)	Z_S(2 GeV)	Z_P(2 GeV)
1.1025(16)	1.216(23)	1.163(34)	1.118(29)	1.123(56)

Preliminary results (stat. + ren. errors)

	D	D^*	D_s	D_s^*
f_M/MeV	213(1)	235(3)	248(2)	275(5)
f_V^T/f_V		0.91(3)		0.92(3)
Mass/MeV	1874(1)	2028(2)	-	2121(2)
M^{exp}/MeV	1869.6	2010.3	1968	2112

- **The masses are (0.2-1%) higher than experiments**
- f_{D_s} **agrees with FLAG2016 (**$N_f = 2 + 1$**)**
- f_D **agrees with FLAG2016 (2+1): 209.2(3.3) MeV**

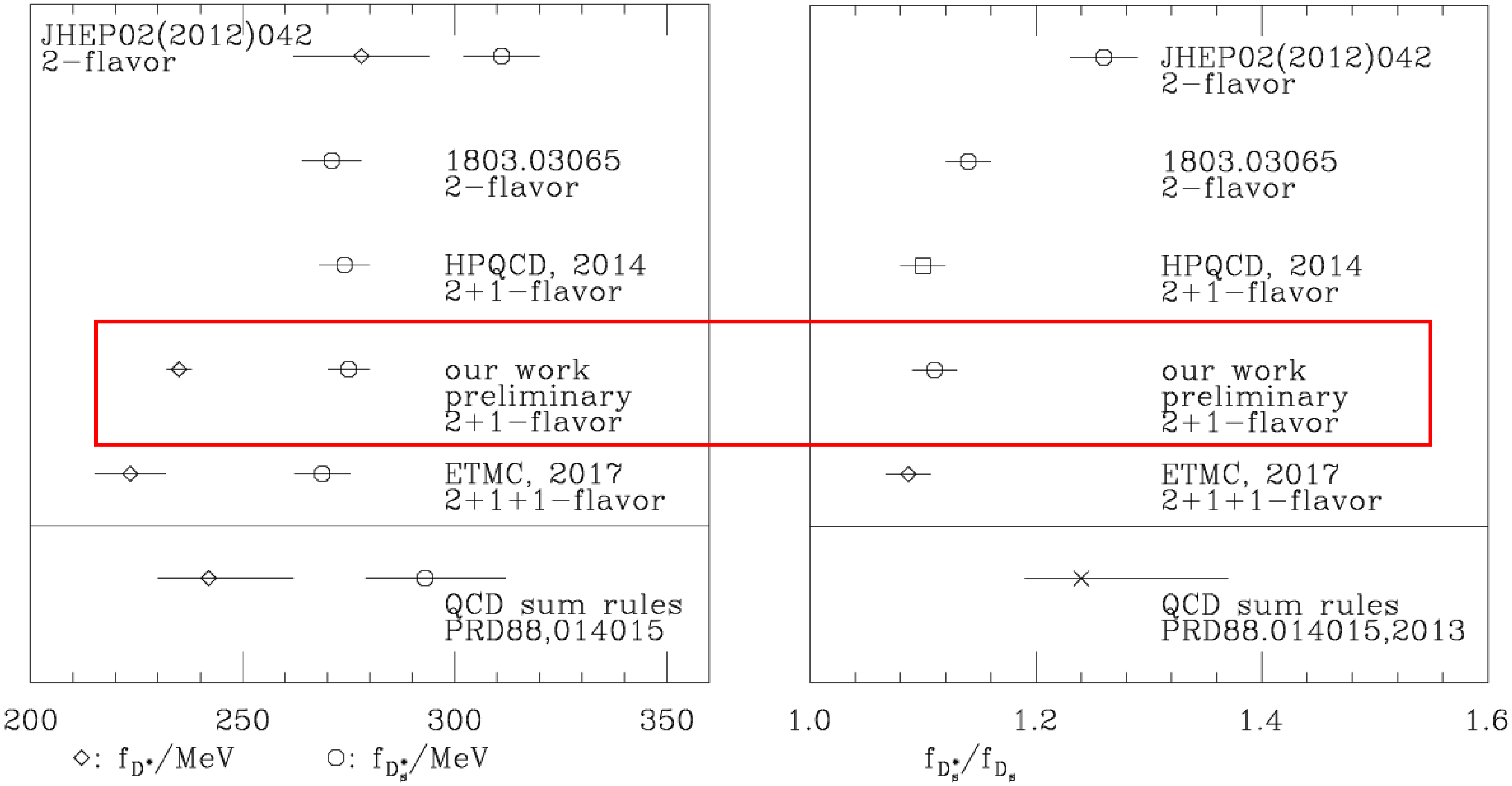
Preliminary results

- **Test of HQET: $f\_V/f\_PS = 1 + O(1/m_Q)$**
  - **~10%: f_{D^*}/f_D = 1.10(3), $f_{D_s^*}/f_{D_s}$ = 1.11(2)**

- **SU(3) flavor symmetry breaking**
  - **~17%: $f_{D_s}/f_D = 1.16(1)$, $f_{D_s^*}/f_{D^*} = 1.17(3)$**

- **f_V^T/f_V for D^* and D_s^* are inputs for LCSR in calculations of $B \to V$ form factors at low q^2**

f_{D^*} and $f_{D_s^*}$



- **HPQCD, PRL112, 212002 (2014)** **2 lattice spacings, 2+1-flavor**
- **ETMC, PRD96, 034524 (2017)** **3 a's, 2+1+1-flavor**
- **Becirevic et al., JHEP02 (2012) 042** **4 a's, 2-flavor**
- **Blossier, Heitger, Post (1803.03065)** **2 a's, 2-flavor, Clover fermions**

Summary

- **$f_{D_{(s)}^{(*)}}$ and $f_{D_{(s)}^{*}}^{T}/f_{D_{(s)}^{*}}$ are calculated with overlap on domain wall fermion configurations**
- **RI/(S)MOM are used for renormalization**
- **More lattice spacings are needed to better control discretization effects**
- **Finalize systematic uncertainties**

Summary

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Thanks for your attention!