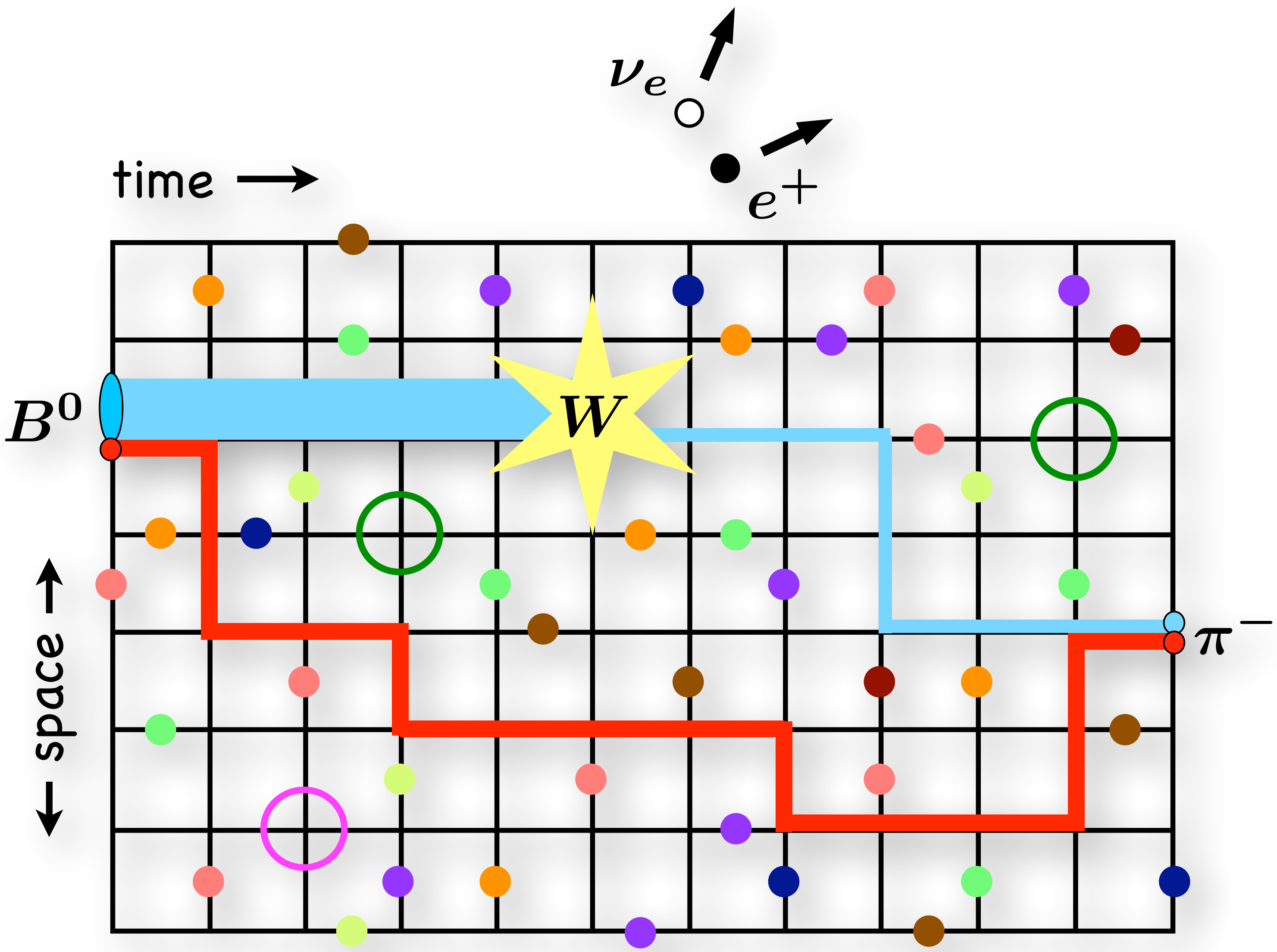
Lattice developments in *B* decays

M Wingate, DAMTP, University of Cambridge

LHCb-UK Annual Meeting, University of Warwick
3-4 Jan 2019

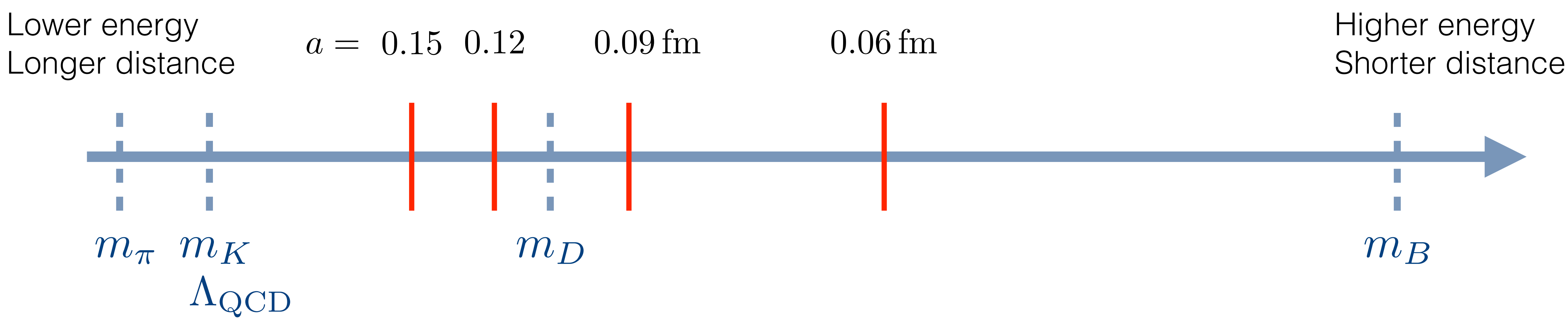
Lattice QCD



- Quantum FT → Statistical FT
- MC importance sampling
- Correlation functions
- Corr. length → hadron mass
- Amplitudes → Matrix elem.

Scales

Inversion of matrix developing zero eigenvalues as $m_u a \to 0$



- Low energy hadronic physics can be made free of lattice artifacts
- Option 1: use an EFT which separates m_b physics from $\Lambda_{\rm QCD}$ physics
- Option 2: with improved actions + a lot of lattice data, extrapolate in spacing and heavy quark mass simultaneously

Outline

- $b \to c$
- $b \to u$
- $c \to d, s$ (b spectator)
- [If time permits:] $b \to s$

Talk by T Tsang later today.

Outline

What can lattice QCD do to resolve/confirm discrepancies?

- $b \to c$
- $b \to u$
- $c \to d, s$ (b spectator)
- [If time permits:] $b \to s$

Talk by T Tsang later today.

Outline

What can lattice QCD do to resolve/confirm discrepancies?

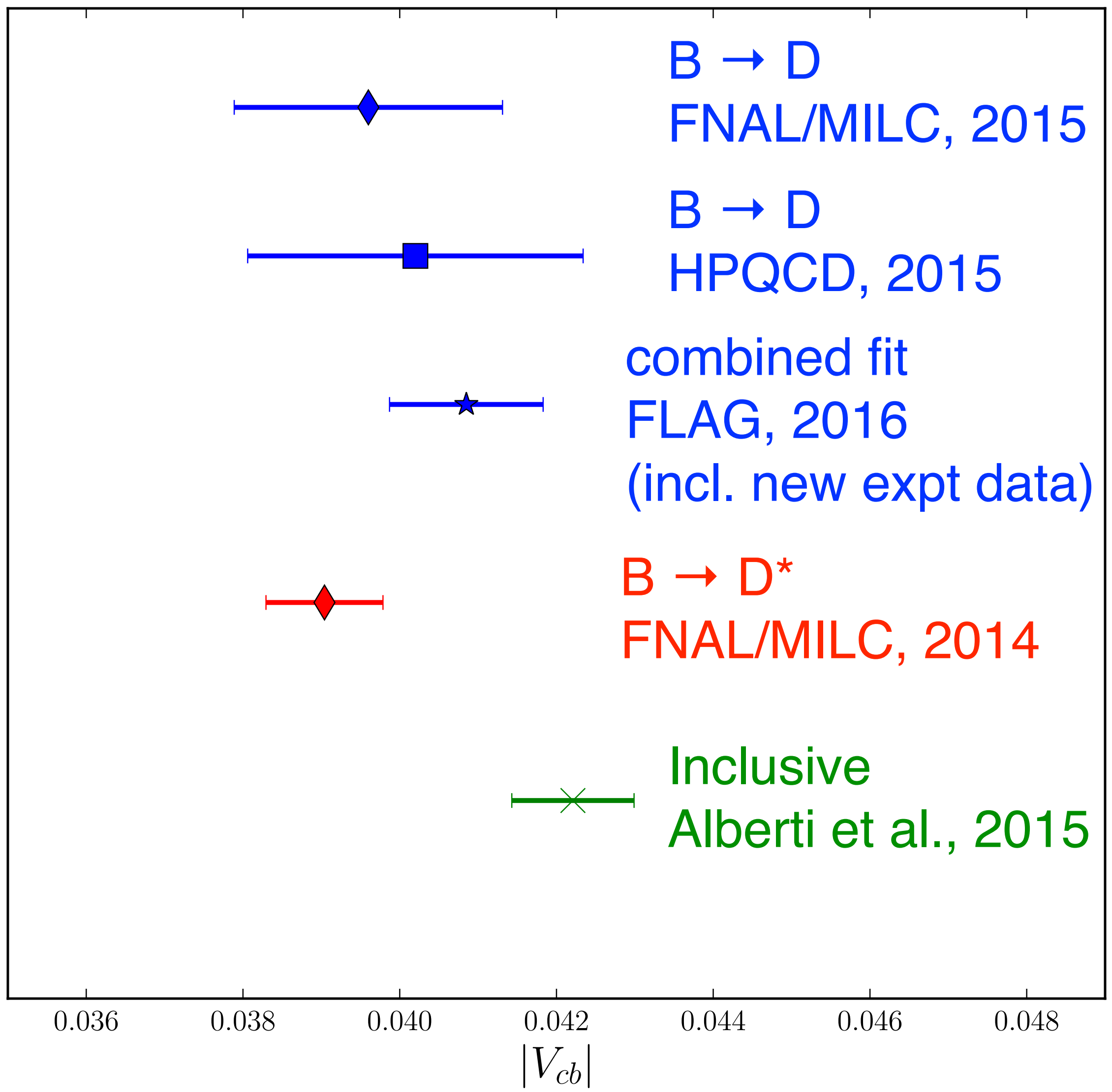
- $b \to c$ puzzle? anomaly?
- $b \to u$ puzzle?
- $c \to d, s$ (b spectator)
- [If time permits:] $b \to s$ anomalies?

Talk by T Tsang later today.

$$b \to c$$

Historic inclusive/exclusive $|V_{cb}|$

(before 2/2017)



New lattice results

Judd Harrison, Christine Davies, MBW (HPQCD), arXiv:1711.11013

$$\mathcal{F}^{B\to D^*}(1) = h_{A_1}(1) = 0.895(10)_{\rm stat}(24)_{\rm sys}$$

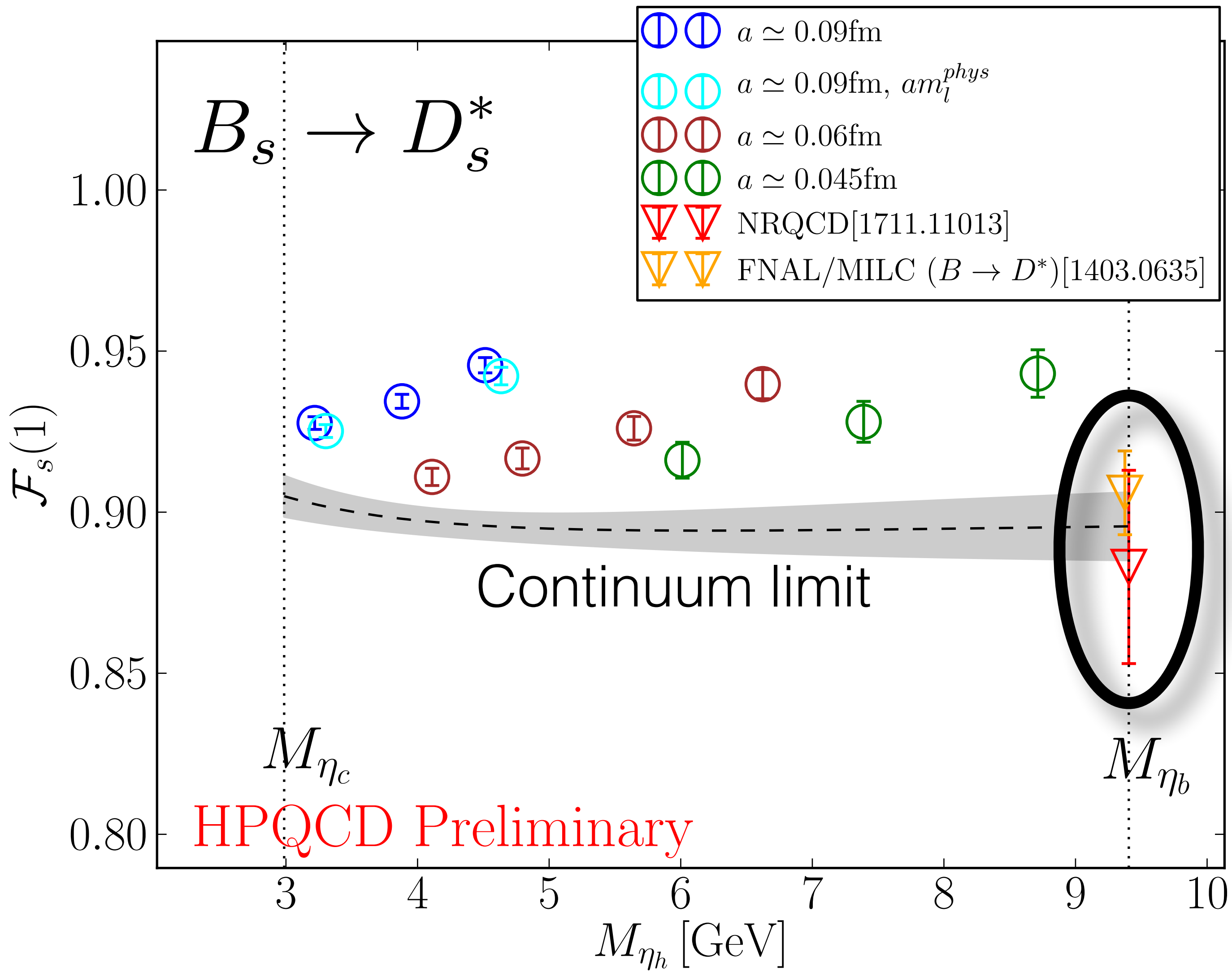
$$\mathcal{F}^{B_s\to D_s^*}(1) = h^s_{A_1}(1) = 0.883(12)_{\rm stat}(28)_{\rm sys}$$

$$\frac{\mathcal{F}^{B\to D^*}(1)}{\mathcal{F}^{B_s\to D_s^*}(1)} = \frac{h_{A_1}(1)}{h^s_{A_1}(1)} = 1.013(14)_{\rm stat}(17)_{\rm sys}$$

Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s\Lambda_{\rm QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{\rm QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^*D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2

- ***Good agreement*** with Fermilab/MILC result $h_{A1}(1) = 0.906(4)(12)$
- Independent lattices
- Different heavy quark formulations

Test of normalization



Extrapolate to b quark mass ⟶

- HISQ quarks for all quarks
- Conserved current, removes normalization uncertainty
- Good agreement between formulations

work by E McLean (HPQCD)

$B \rightarrow D^* l \nu$ shape ansätze

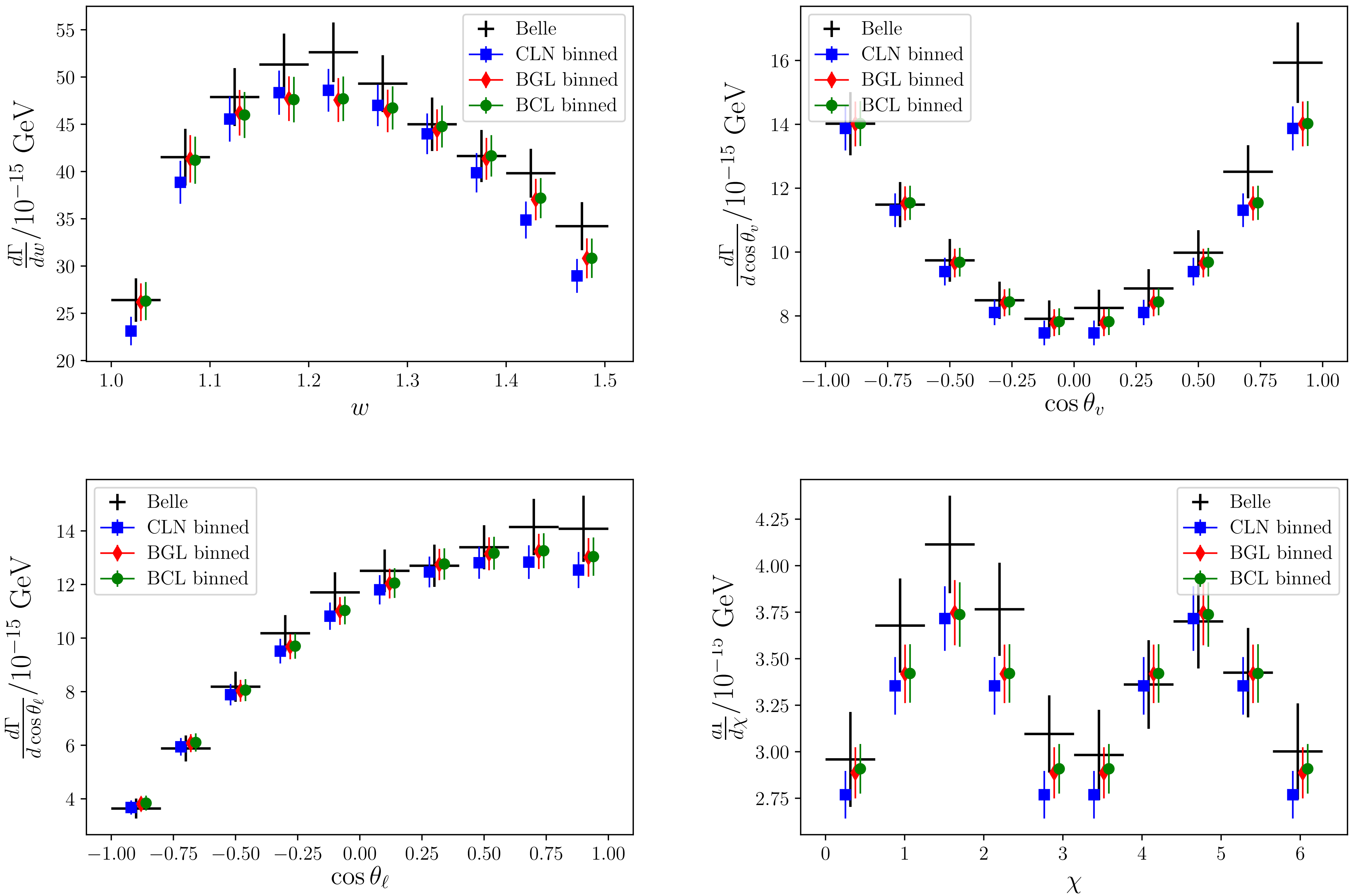
- Observables depend on 4 hadronic form factors. After removing poles, expand in ***power series*** about zero recoil point

- "Standard" procedure: Caprini-Lellouch-Neubert (CLN) parametrization using information from HQET and sum rules, ***without theory uncertainties on numerical coefficients***

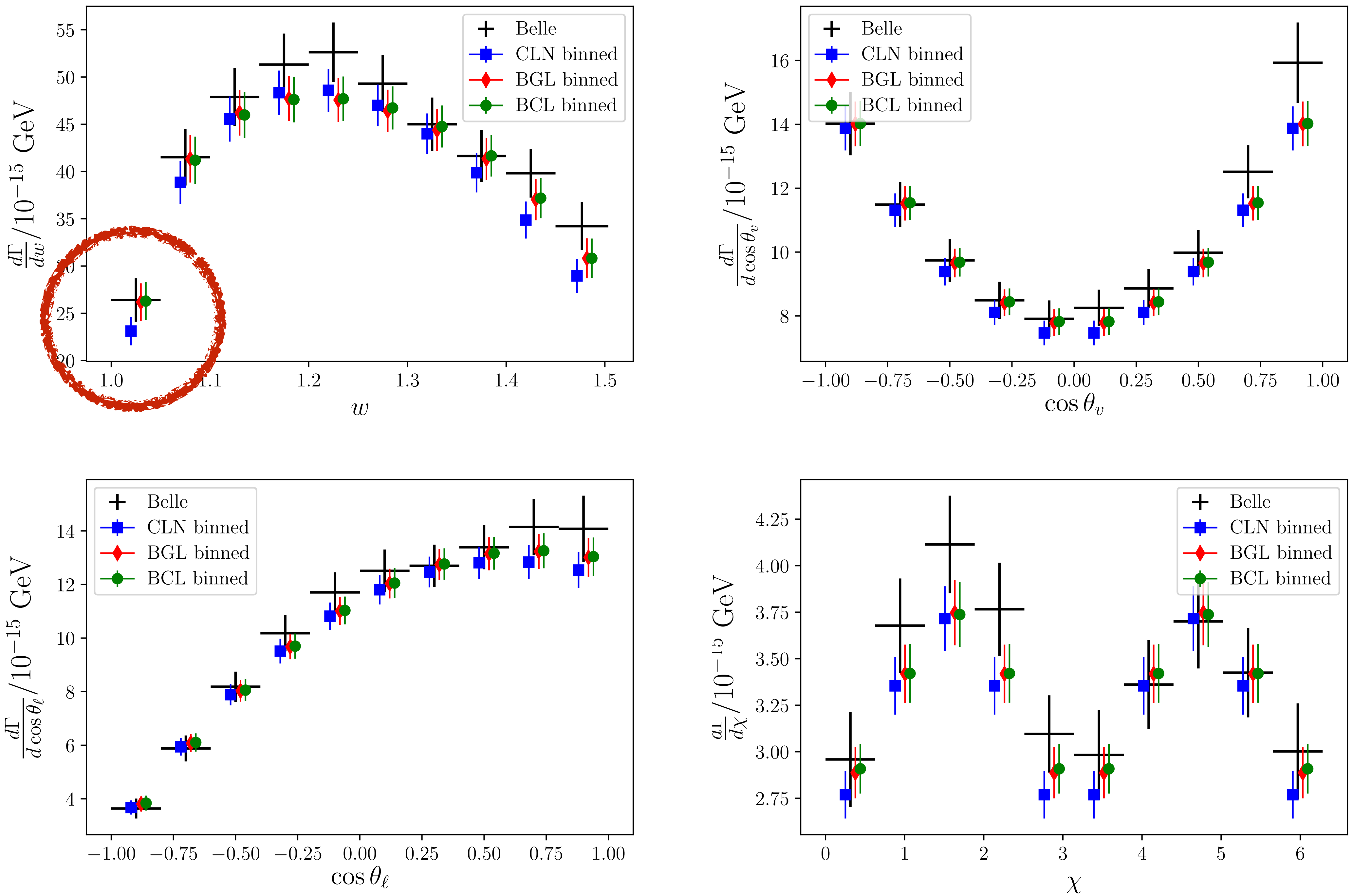
- Recently, Belle data has been unfolded [arXiv:1702.01521] and re-fit to more agnostic "*z*-parametrizations" Boyd-Grinstein-Lebed (BGL), Bourrely-Caprini-Lellouch (BCL)

Bigi, Gambino, Schacht, arXiv:1703.06124,
Grinstein & Kobach, arXiv:1703.08170,
Jaiswal, Nandi, Patra, arXiv:1707.09977,
Bernlochner, Ligeti, Papucci, Robinson , arXiv:1708.07134,

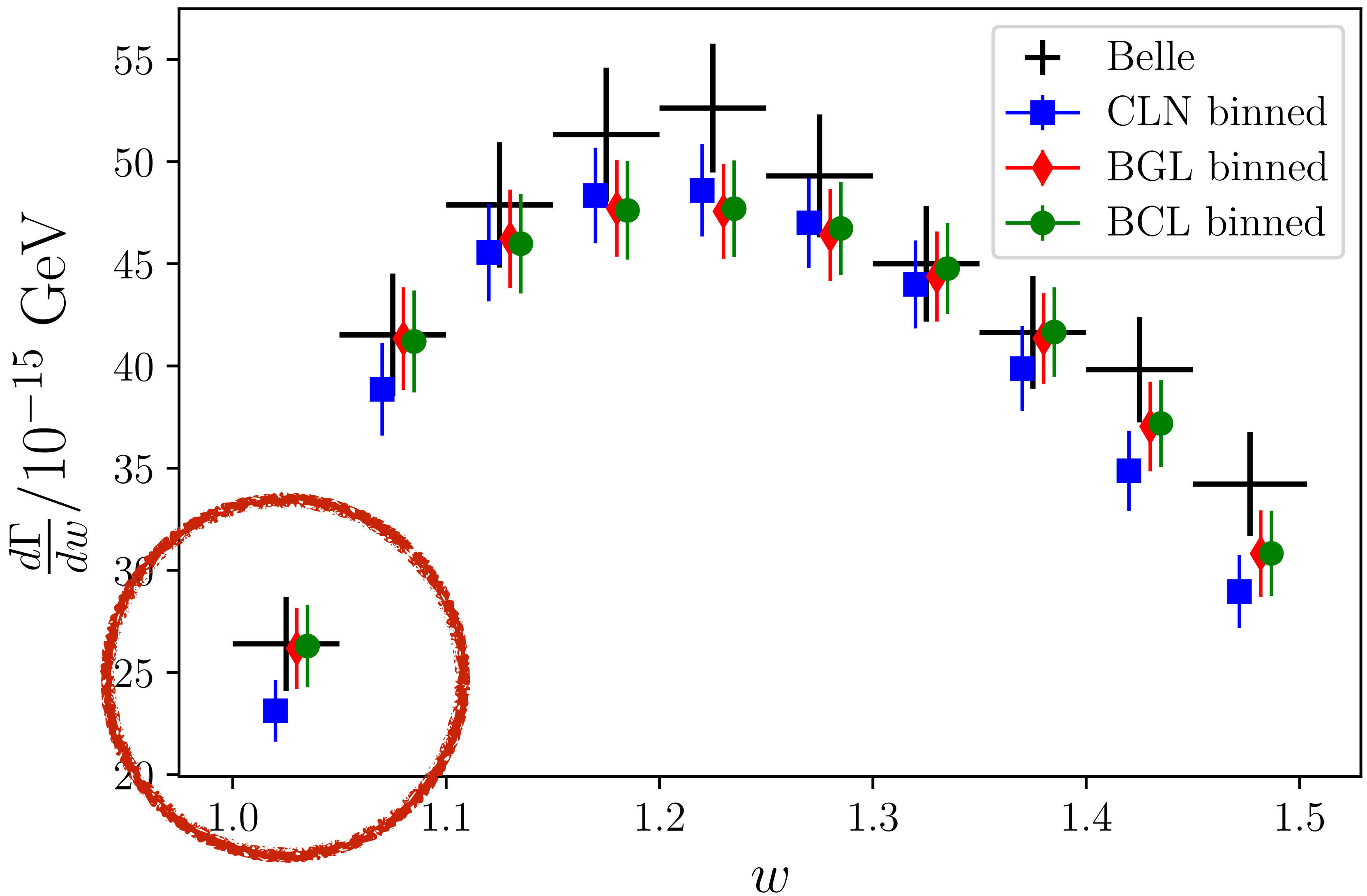
Fits to Belle data



Fits to Belle data

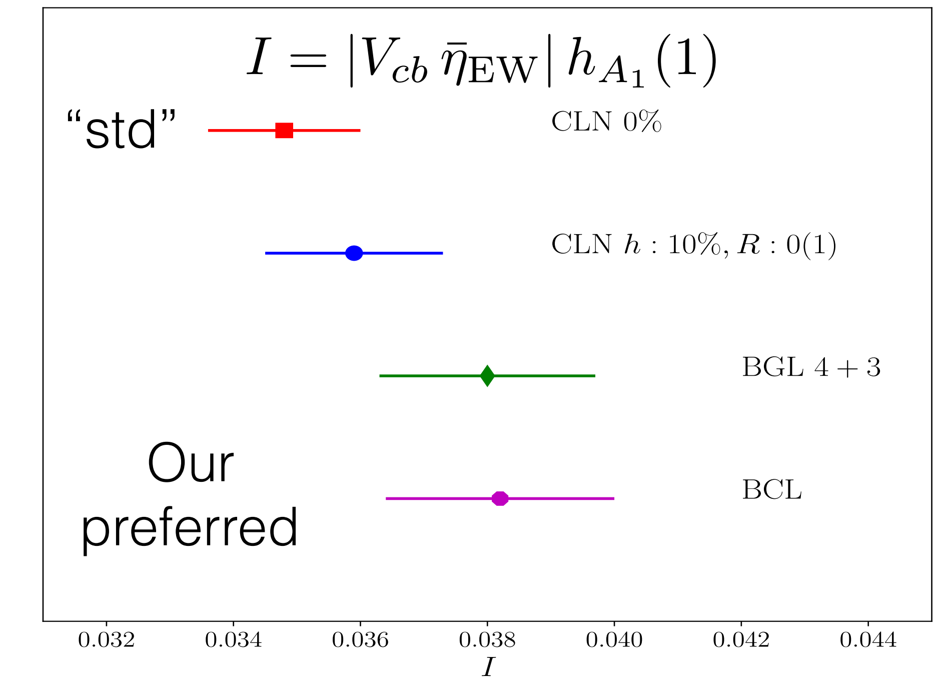


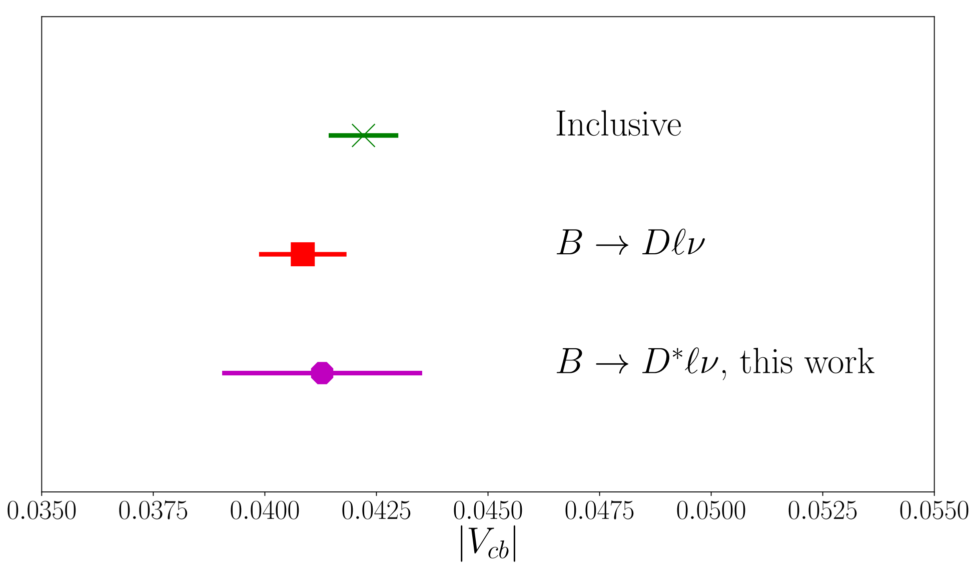
Fits to Belle data



Implications for V_{cb}

Different fit Ansätze





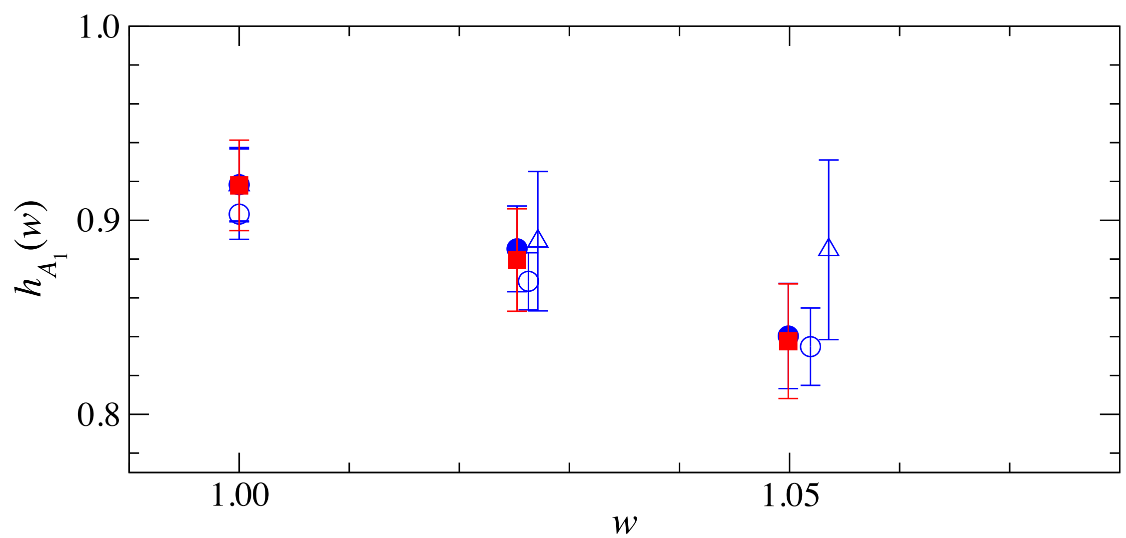
- Removal of theory assumptions resolves inclusive/exclusive tension, at least in Belle data
- Look forward to BaBar analysis
- Look forward to LQCD results at non-zero recoil

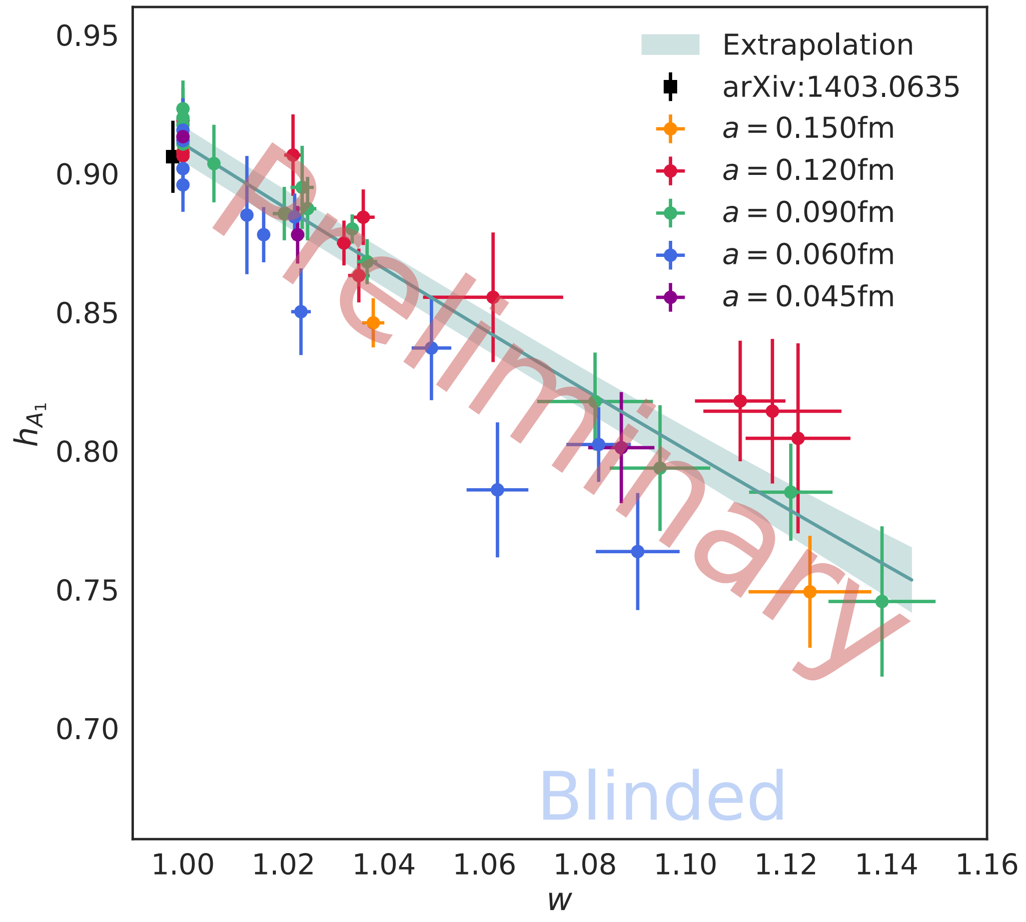
Harrison, et al., (HPQCD), arXiv:1711.11013

Nonzero recoil

All 4 form factors desirable:

- Complement experimental shape information
- Significant for massive τ lepton, R(D*)





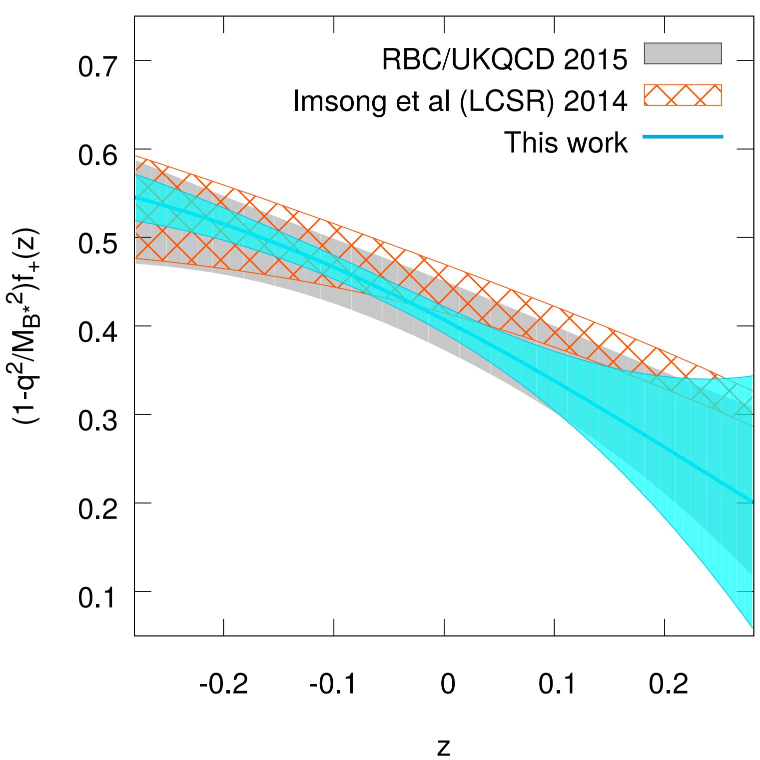
Kaneko et al. (JLQCD), [arXiv:1811.00794]

Vaquero et al. (Fermilab/MILC), Lattice 2018

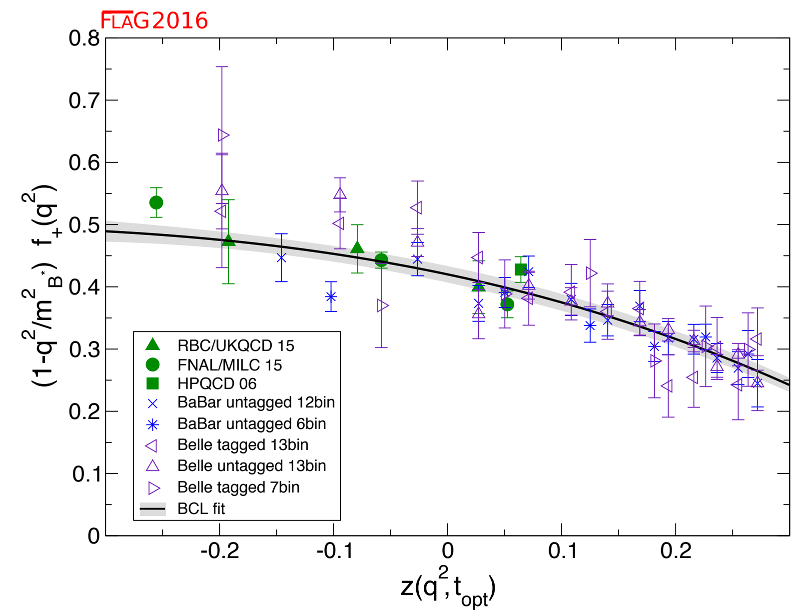
$$b \to u$$

Semileptonic decays

$B \to \pi \ell \nu$

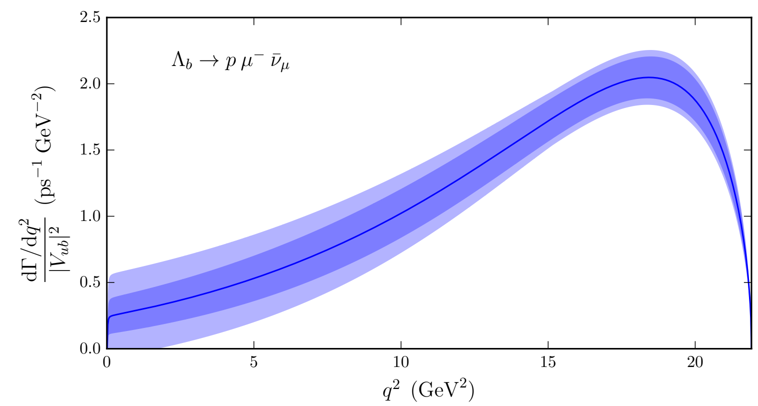


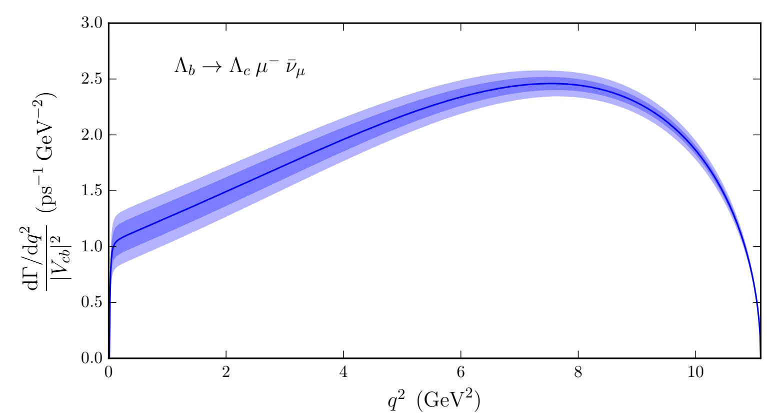
Bailey et al. arXiv:1503.07839
2+1 flavours



FLAG, arXiv:1607.00299
2+1 flavours

Λ_b decay





Detmold, Lehner, Meinel. arXiv:1503.01421
2+1 flavours

$$B \to \pi \ell \nu \implies |V_{ub}| = 3.62(14) \times 10^{-3}$$

FLAG, arXiv:1607.00299

$$\frac{\mathcal{B}(\Lambda_b \to p\mu\nu)}{\mathcal{B}(\Lambda_b \to \Lambda_c \mu\nu)} \implies$$

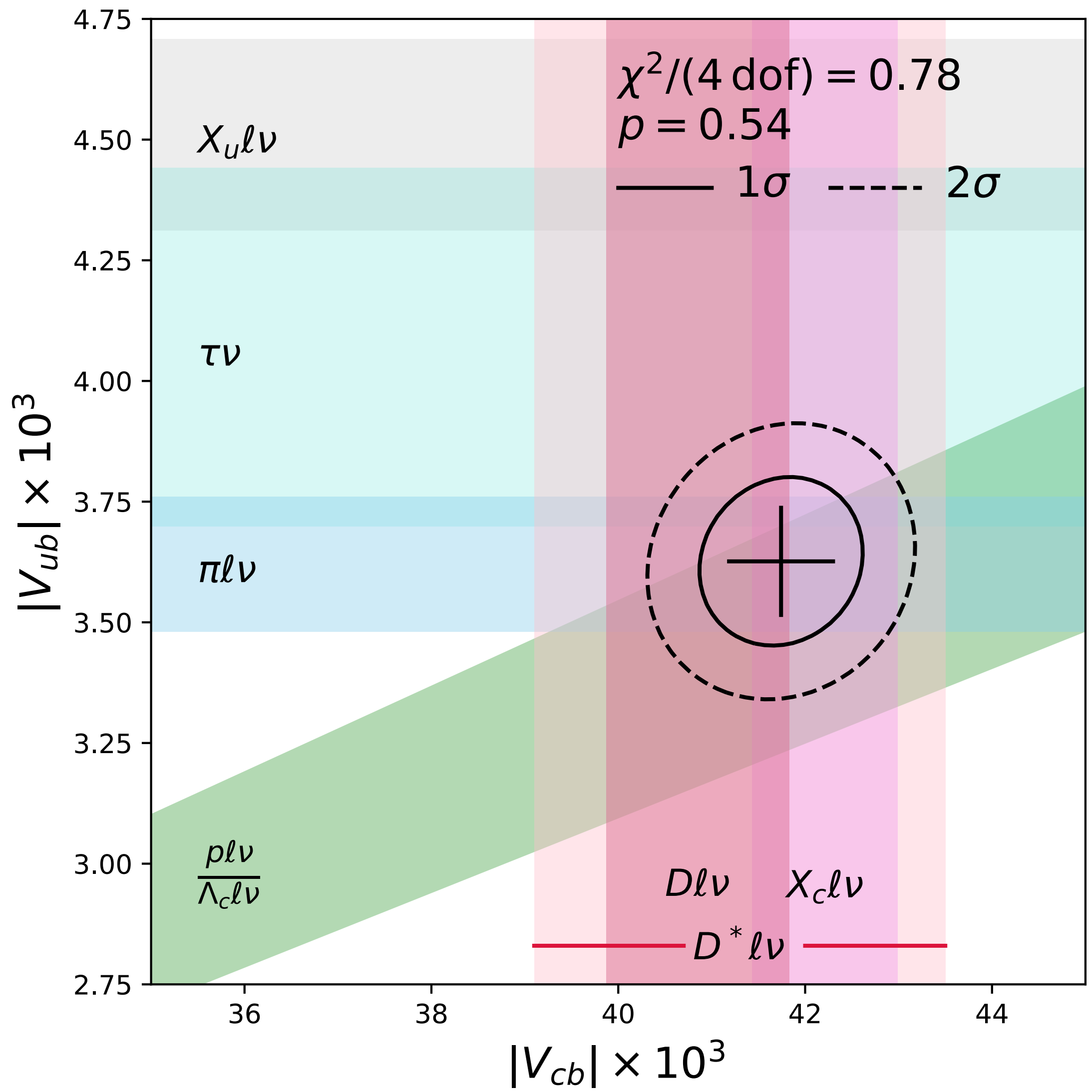
$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083(4)(4)$$

LHCb, arXiv:1504.01568

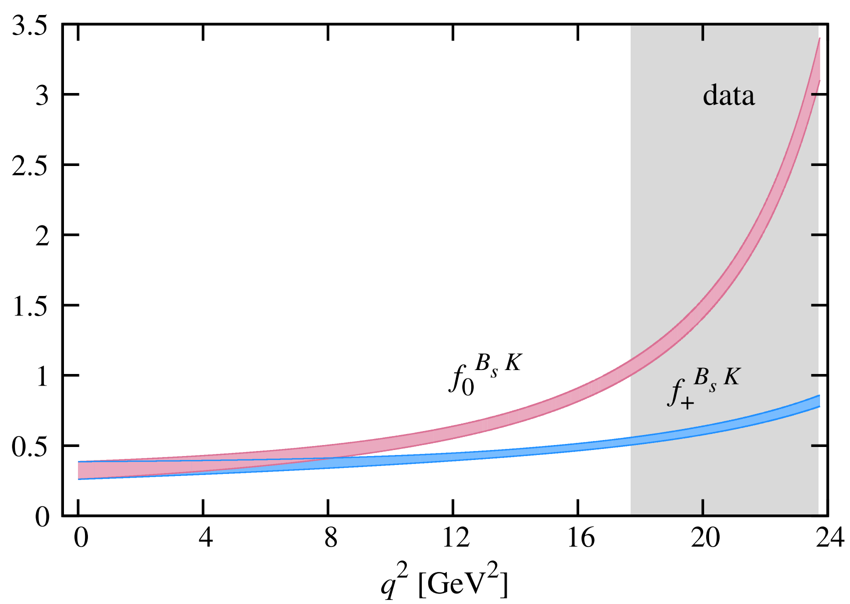
HPQCD and Fermilab MILC working on updates using MILC 2+1+1 lattices

$|V_{ub}|$, $|V_{cb}|$ fit

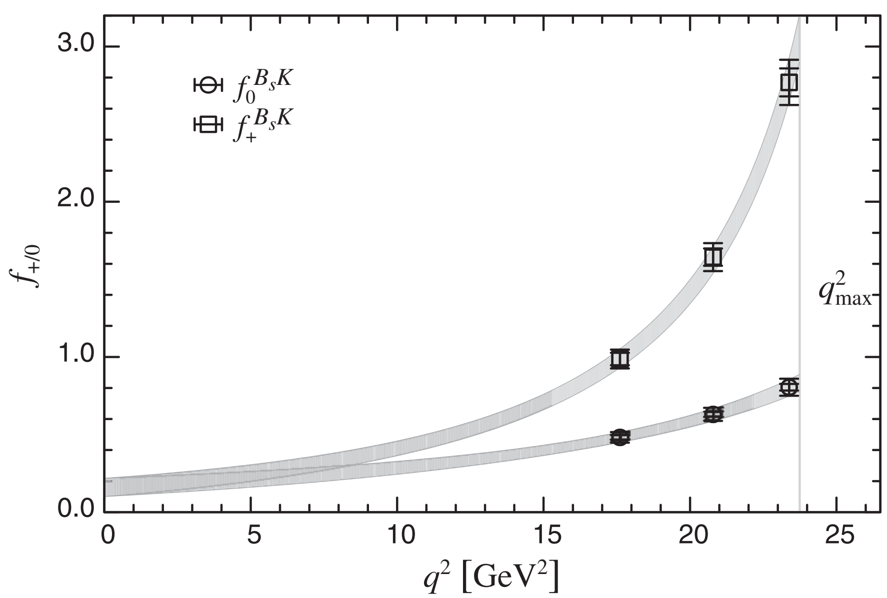
Omitting inclusive $|V_{ub}|$ and earlier $B \to D^*l\nu$ $|V_{cb}|$ one finds a good fit.



$B_s \to K\, l\, \nu$

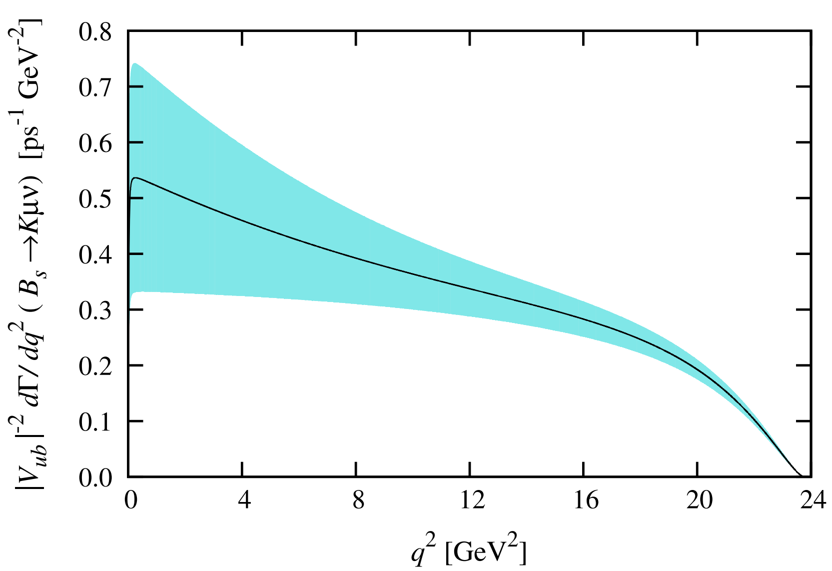


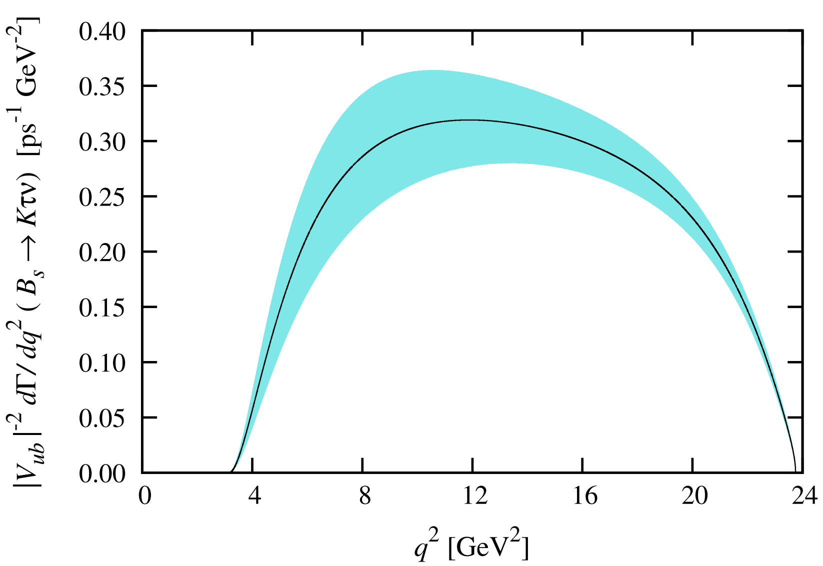
Bouchard et al (HPQCD), arXiv:1406.2279



Flynn et al (RBC-UKQCD), arXiv:1501.05373

Predicted differential decay rates





$B_c \to D\ell\nu$ form factors in future plans

Ongoing work

- NRQCD semileptonic B form factors being computed on 2+1+1 flavour MILC lattices. Independent, improved calculations compared to 2+1 flavour MILC lattices.

- RBC-UKQCD carrying forward semileptonic B decay programme using domain wall fermions and relativistic heavy *b*.

- JLQCD preliminary results for B to π, $D^{(*)}$ form factors, using Möbius domain wall for all quarks [Colquhoun et al., arXiv:1811.00227]

- Fermilab/MILC beginning all-staggered semileptonic programme on 2+1+1. They expect errors of 1-2% in form factors.

$$c \to d, s$$

with spectator *b*

$B_c \rightarrow B_{(d,s)}\, l\, \nu$

- With $|V_{cd}|$ & $|V_{cs}|$ as input: SM predictions for decay rate.
- With experimental data: Novel method to determine $|V_{cd}|$ & $|V_{cs}|$.

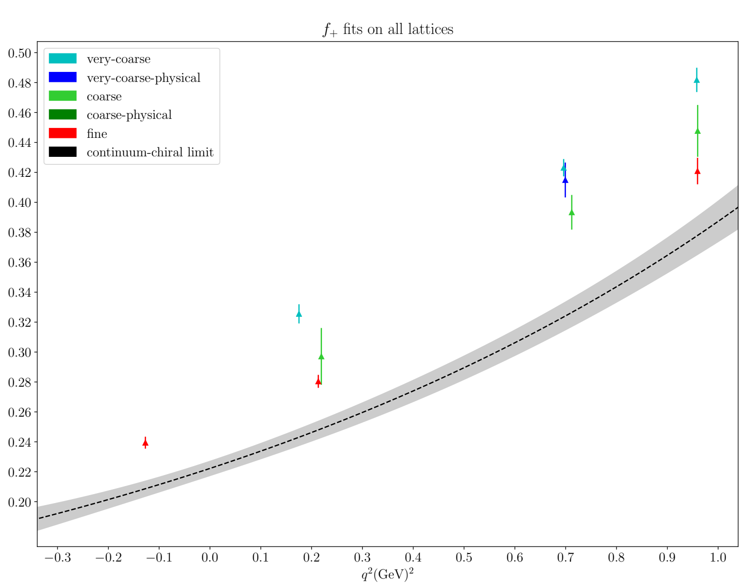
Using NRQCD on 3 MILC lattices:

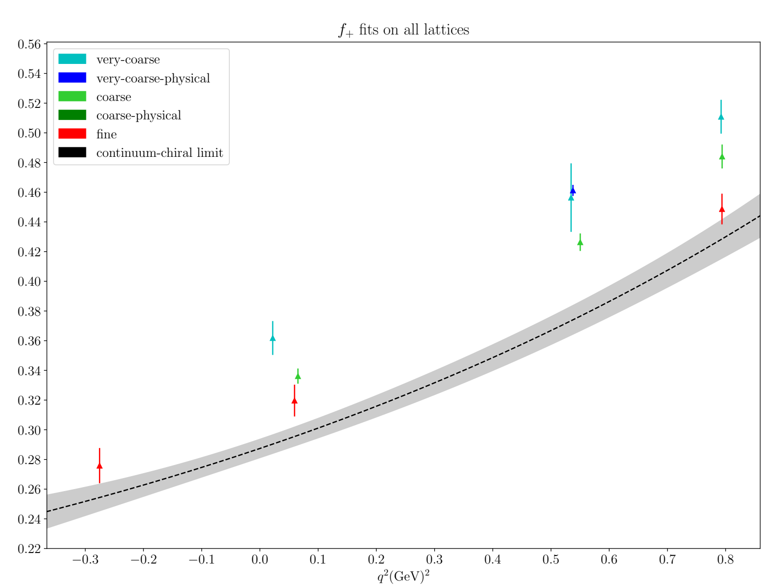
Heavy HISQ for b

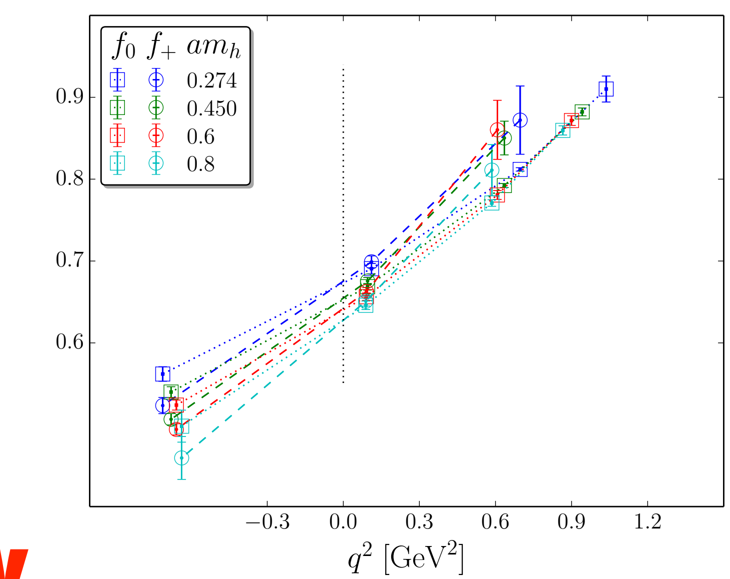
$$f_+^{B_c \rightarrow B^0}(q^2)$$

$$f_+^{B_c \rightarrow B_s^0}(q^2)$$

$$f_+^{B_c \rightarrow B_s^0}(q^2)$$







ALL VERY PRELIMINARY

work by L Cooper (HPQCD)

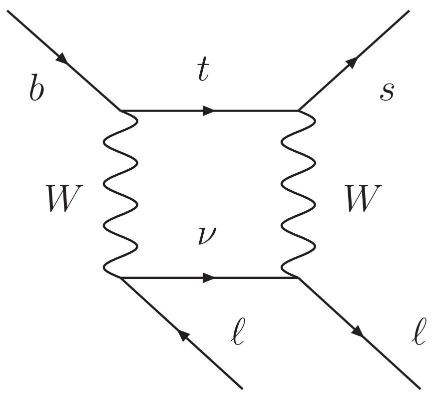
work by J Komijani (HPQCD)

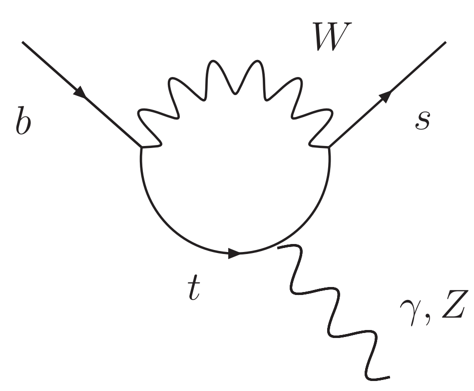
$$b \to \boldsymbol{s}$$

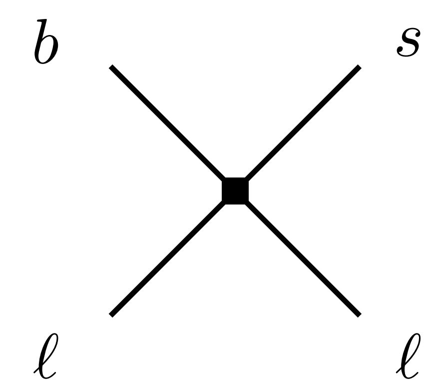
Rare decays

Short-distance = straightforward:

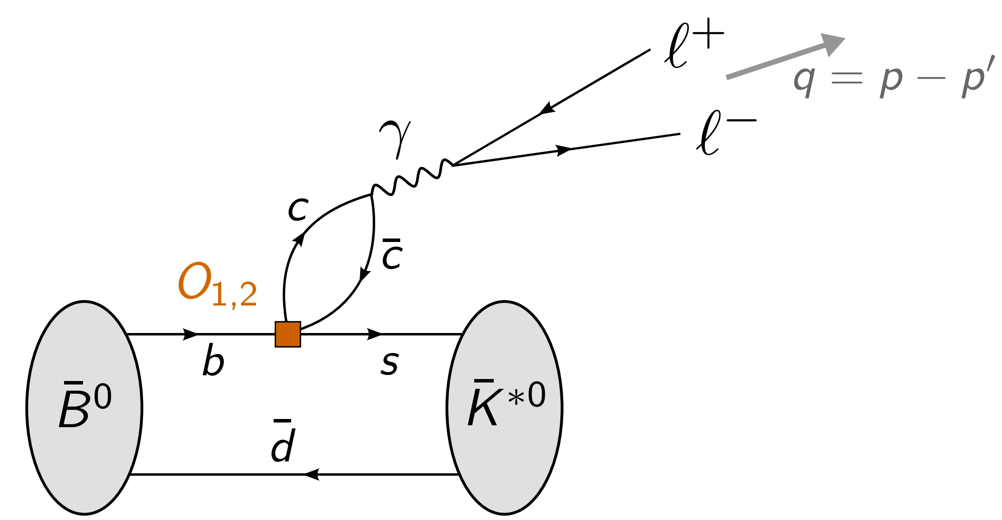
(2 quark-2 lepton operators, i.e. form factors):



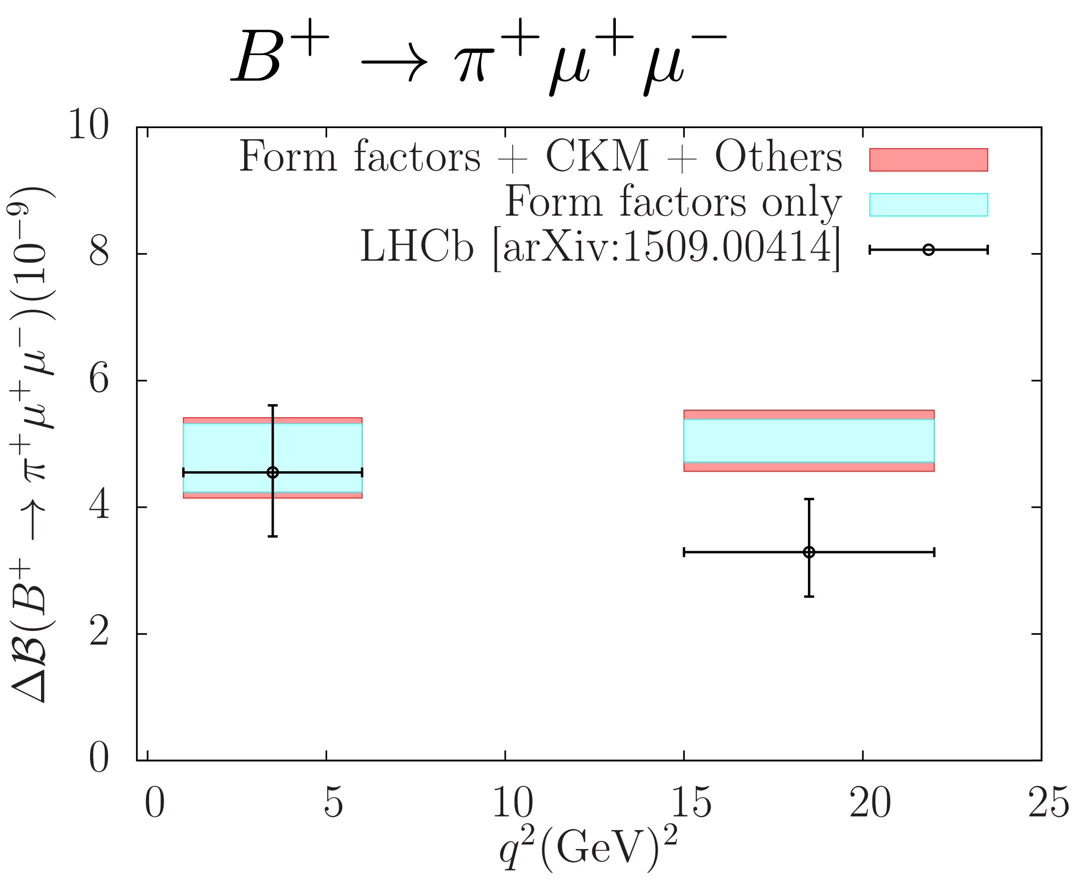


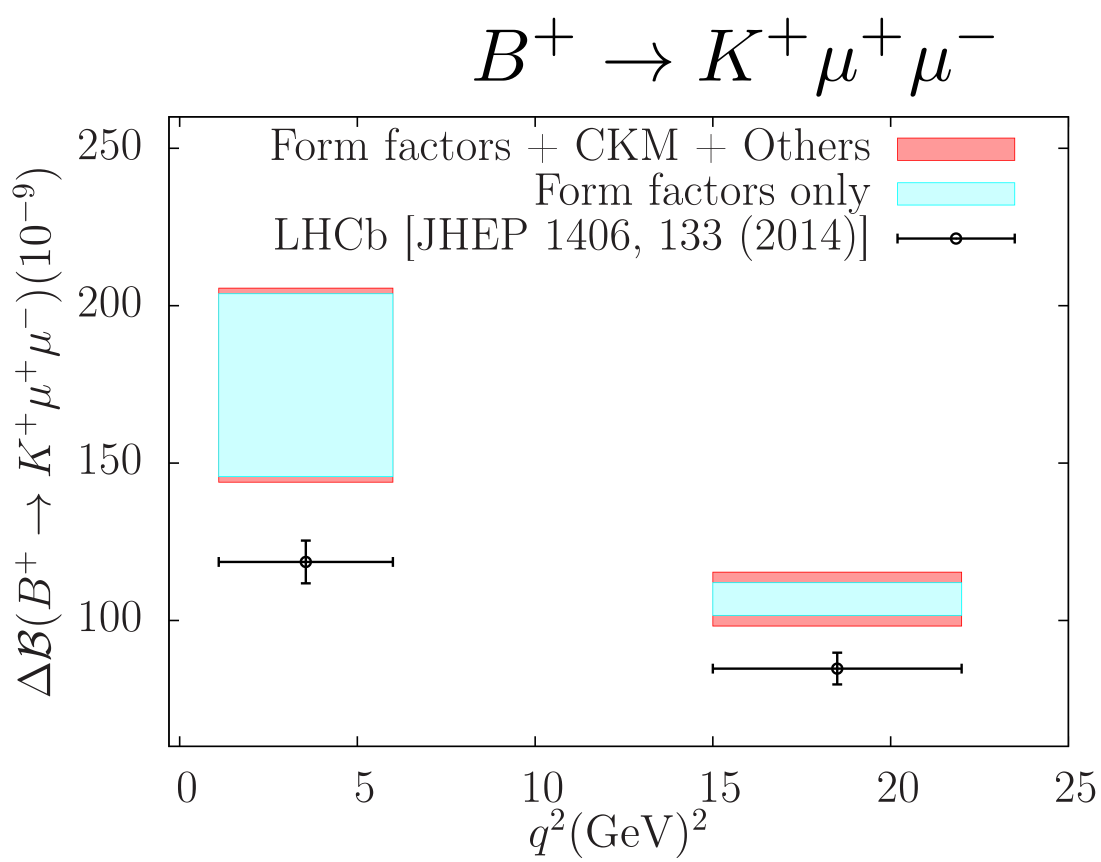


Long-distance = big challenge:



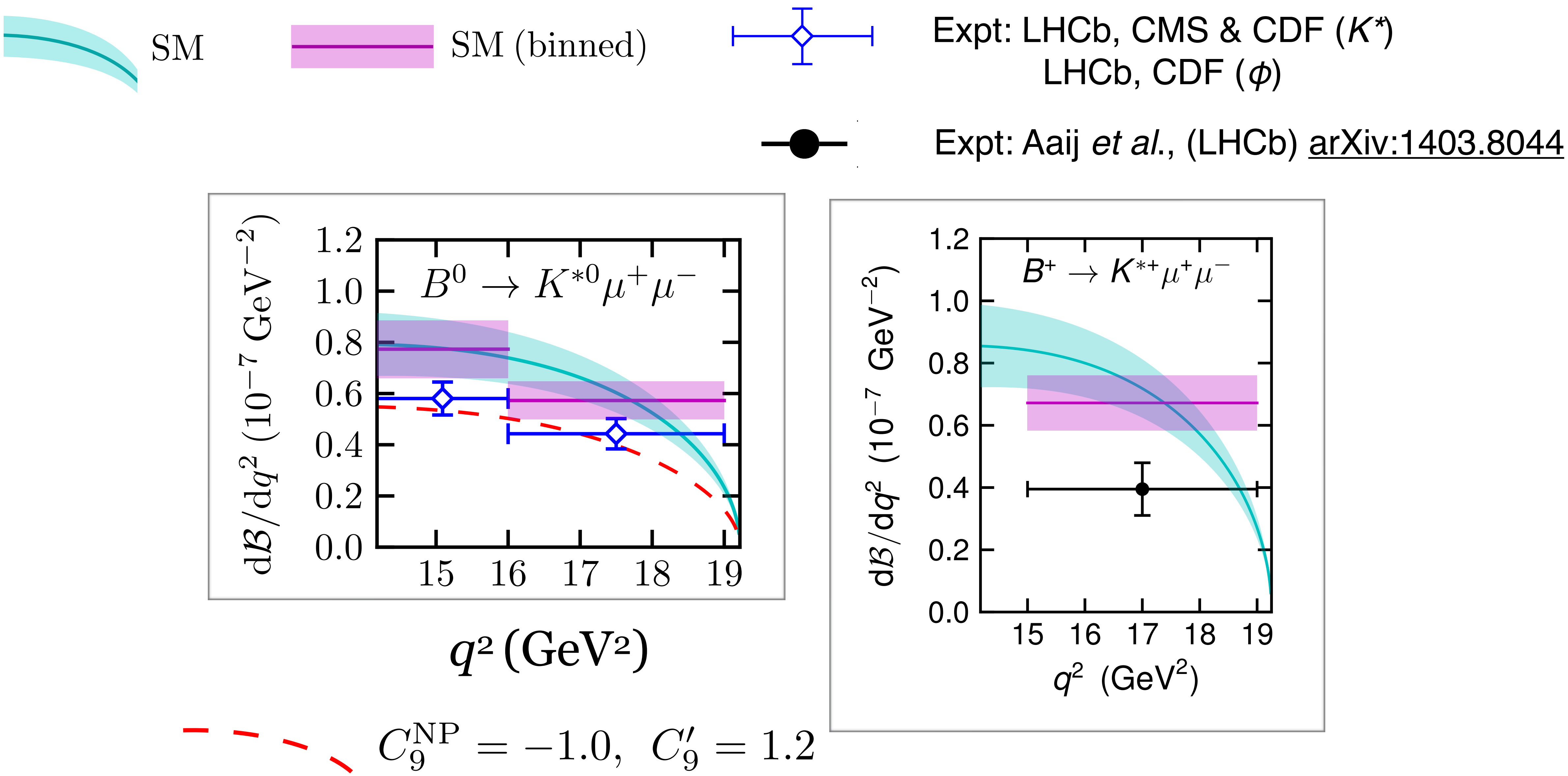
$B \to \pi\, \mu^+\mu^-$ & $B \to K\, \mu^+\mu^-$





Du *et al.*, (FNAL/MILC) arXiv:1510.02349

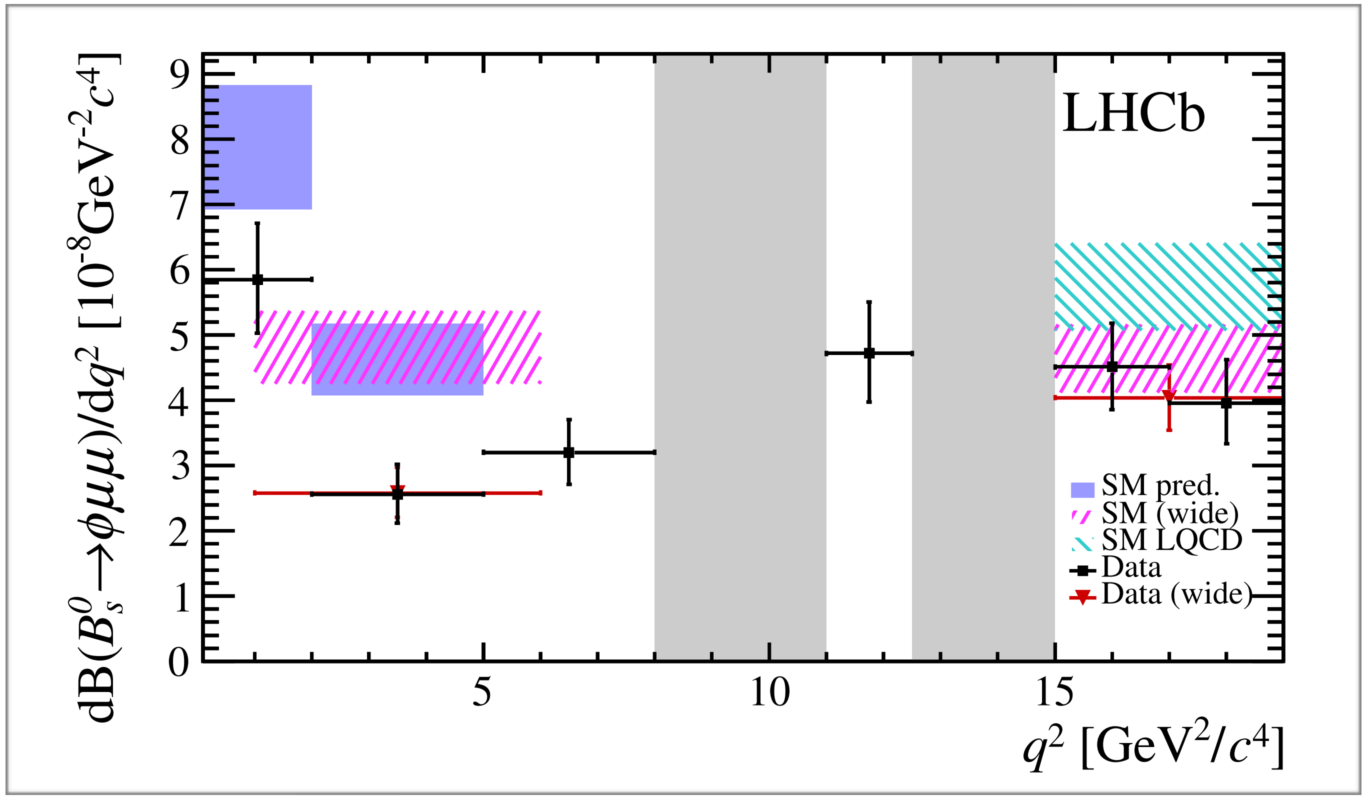
B → K* μ⁺ μ⁻



Horgan *et al.*, arXiv:1310.3887; S Meinel, Paris Workshop 2014

$B_s \to \phi\, \mu^+\mu^-$

Expt. measurement from Aaij *et al.*, (LHCb), arXiv:1506.08777

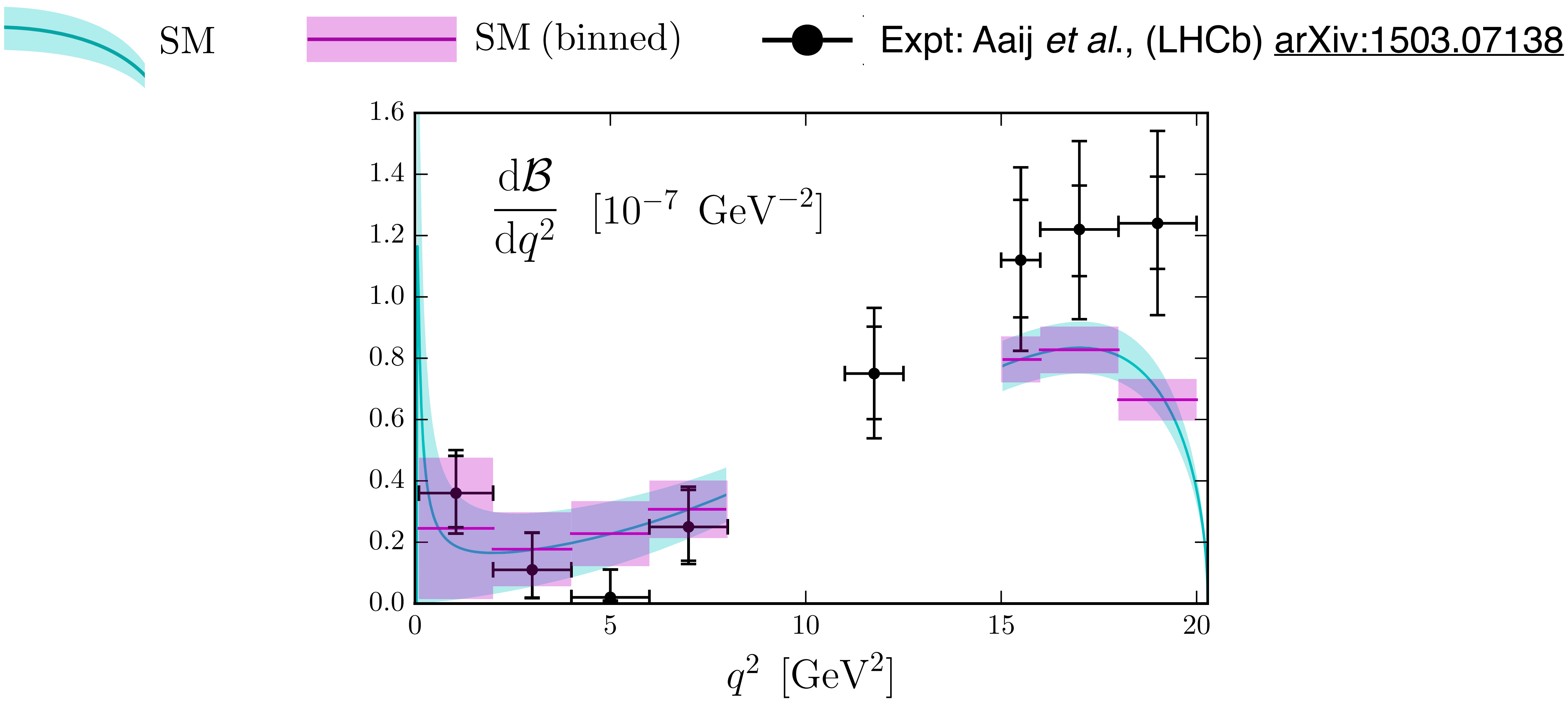


Bharucha, Straub, Zwicky, arXiv:1503.05534
Altmannshoher & Straub, arXiv:1411.3161

Update of Horgan *et al.*, arXiv:1310.3887

Difference in high q^2 SM prediction due in part to: inclusion of low q^2 LCSR form factors, formulation for virtual corrections from O_1, O_2; also inputs.

$\Lambda_b \to \Lambda \mu^+\mu^-$



- Contrary to rare B branching fractions, here the measured data at low recoil exceed the SM prediction. Detmold & Meinel, arXiv:1602.01399.
- $\Lambda_b \to \Lambda(1520)$ ll calculation in progress, Meinel & Renton, arXiv:1608.08110.

$b \to s\ l^+\ l^-$ decays

- Past 5 years: new unquenched form factors for $b \to s$ semileptonic decays of B, B_s, Λ_b . Intriguing difference between SM and expt.

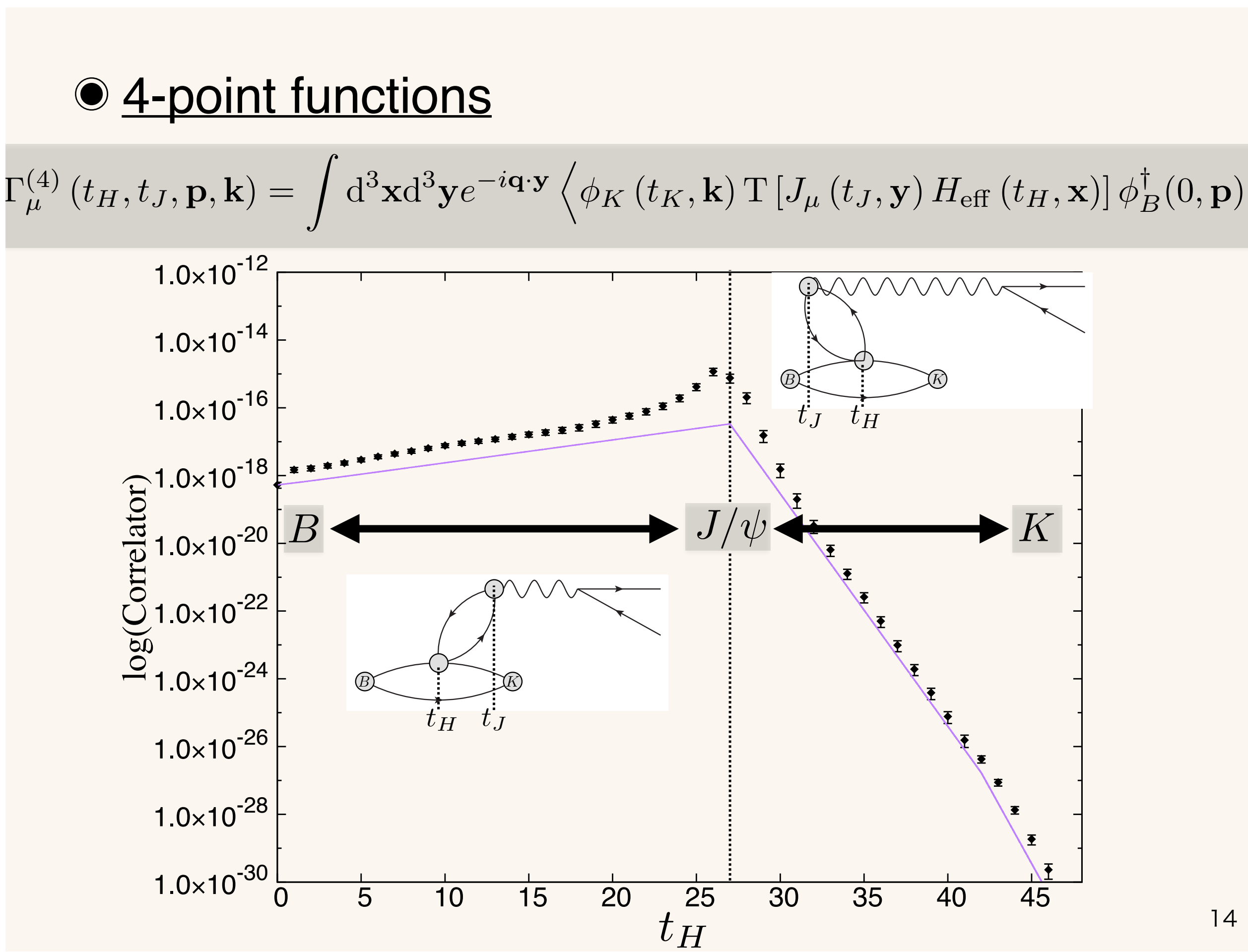
- "Gold-standard" if final state hadron is stable to strong decays. Likely to be improved as part of updating FCCC decays. *Smaller discretisation errors, data at physical pion mass, data at lower q^2.*

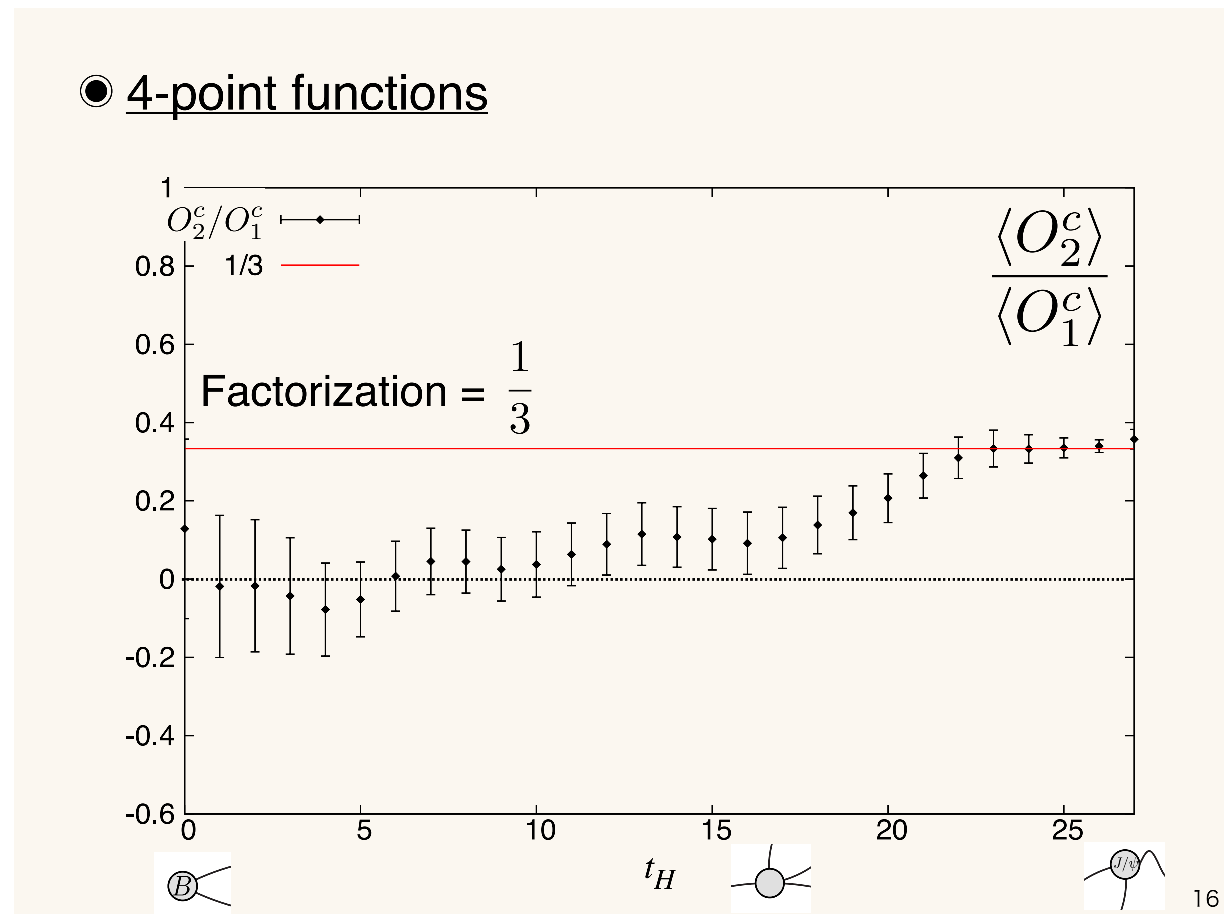
- Dealing with finite width of vector meson final states appears solvable [Briceño, Hansen, Walker-Loud], but there still is a lot of work to do.

- What benefit do smaller form factor errors have in the context of contributions from non-local operators?
  [One answer: $B \to K^{(*)}\ \bar{\nu}\ \nu$, to be measured by Belle II]

Long distance

Exploratory calculations presented by Nakayama & Hashimoto, Lattice 2018





Extending methods developed by RBC-UKQCD for rare K decays.

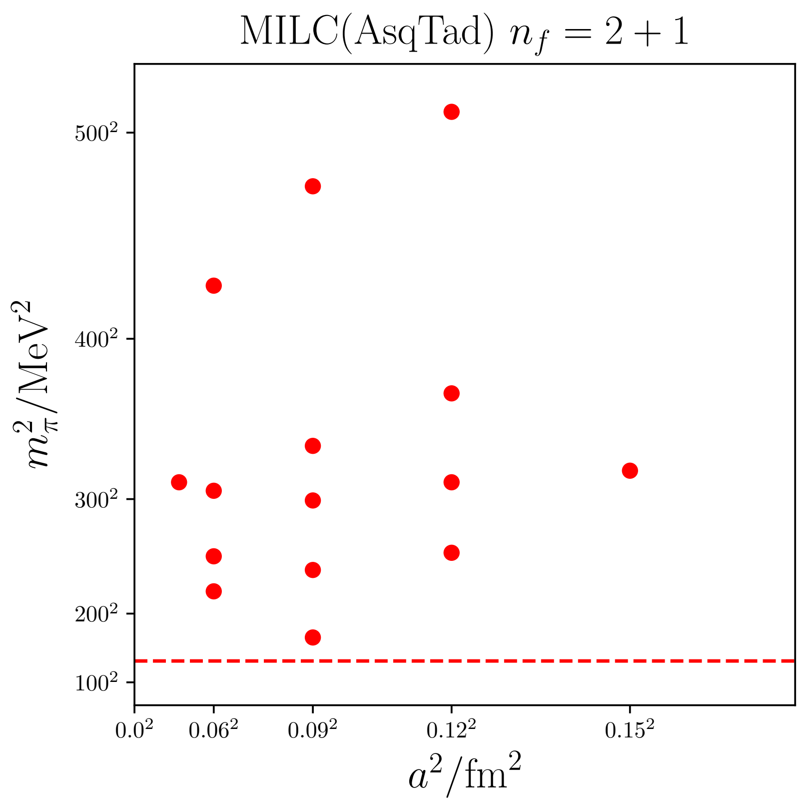
Conclusions

- Lots of activity among several groups using differing formulations, methods, configurations
- Many other quantities that could be shown here, e.g. $B_c \to J/\psi$, mixing, decay constants
- Hadronic matrix elements at increasing precision
- Interesting problems still to solve

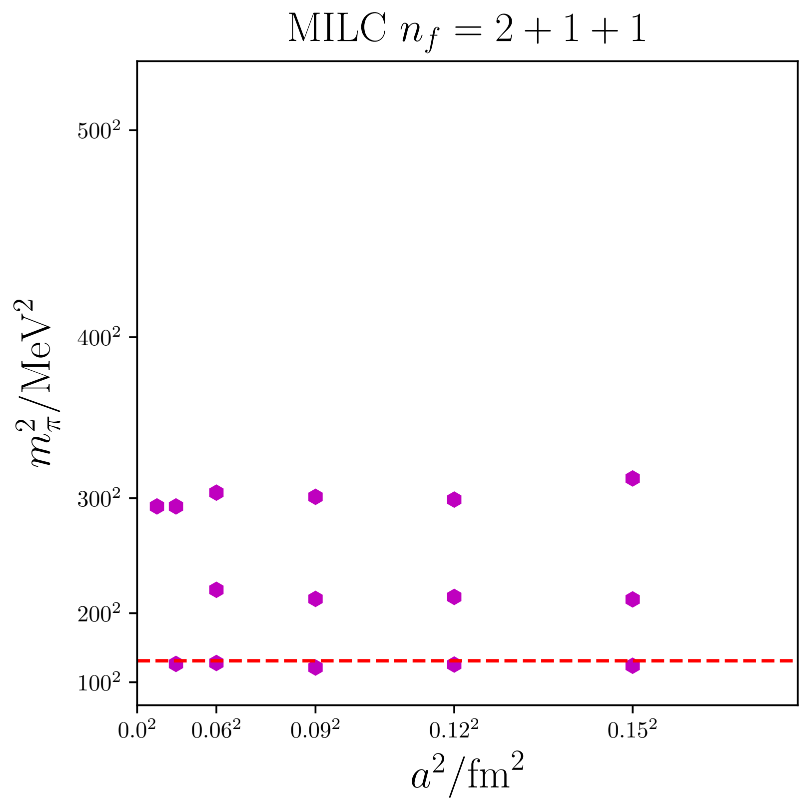
Back-up

Lattice ensembles

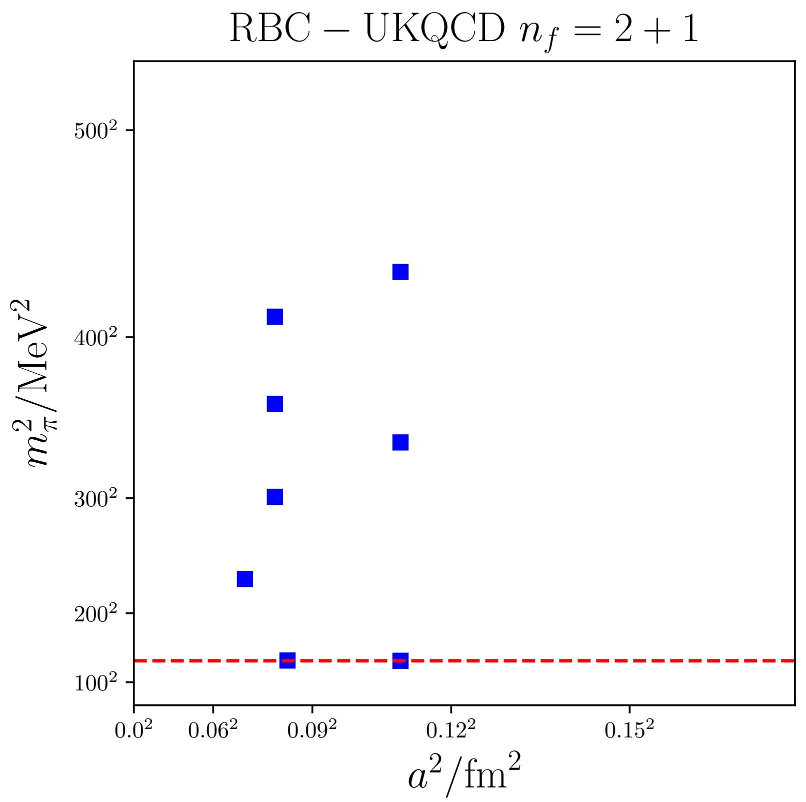
Results presented here use lattices from one of these ensembles:



improved staggered quarks



highly improved staggered quarks



domain wall quarks

Groups are also working on flavour physics with Wilson fermions, twisted-mass fermions, other types of staggered fermions, etc.

CLN parametrization

Form factors entering helicity amplitudes (massless leptons)

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
$$R_1(w) = R_1(1) + r_{11}(w-1) + r_{12}(w-1)^2$$
$$R_2(w) = R_2(1) + r_{21}(w-1) + r_{22}(w-1)^2$$

$$w = v \cdot v'$$

Fixed:

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$
$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$

Using this "tight" CLN parametrization

$$I = |\bar{\eta}_{EW} V_{cb}| h_{A_1}(1)$$

$$I_{\rm Belle} = 0.0348(12)$$ (unfolded)

$$I_{\rm HFLAV} = 0.03561(11)(44)$$

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CLN uncertainties

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2 z + (r_{h2r}\rho^2 + r_{h2})z^2 + (r_{h3r}\rho^2 + r_{h3})z^3]$$
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Coefficients calculated through *Λ/m* using HQET & sum rules

$$r_{h2r} = 53\,, r_{h2} = -15\,, r_{h3r} = -231\,, r_{h3} = 91$$ **BIG!**

$$r_{11} = -0.12\,, r_{12} = 0.05\,, r_{21} = 0.11\,, r_{22} = -0.06$$ small!

Ratios $V(q^2) = \frac{R_1(w)}{r'} h_{A_1}(w)$ $A_2(q^2) = \frac{R_2(w)}{r'} h_{A_1}(w)$

What are the uncertainties for the *r* ’s? 20%? 100%?

Bigi, Gambino, Schacht, arXiv:1703.06124,
Grinstein & Kobach, arXiv:1703.08170,
Jaiswal, Nandi, Patra, arXiv:1707.09977,
Bernlochner, Ligeti, Papucci, Robinson , arXiv:1708.07134,

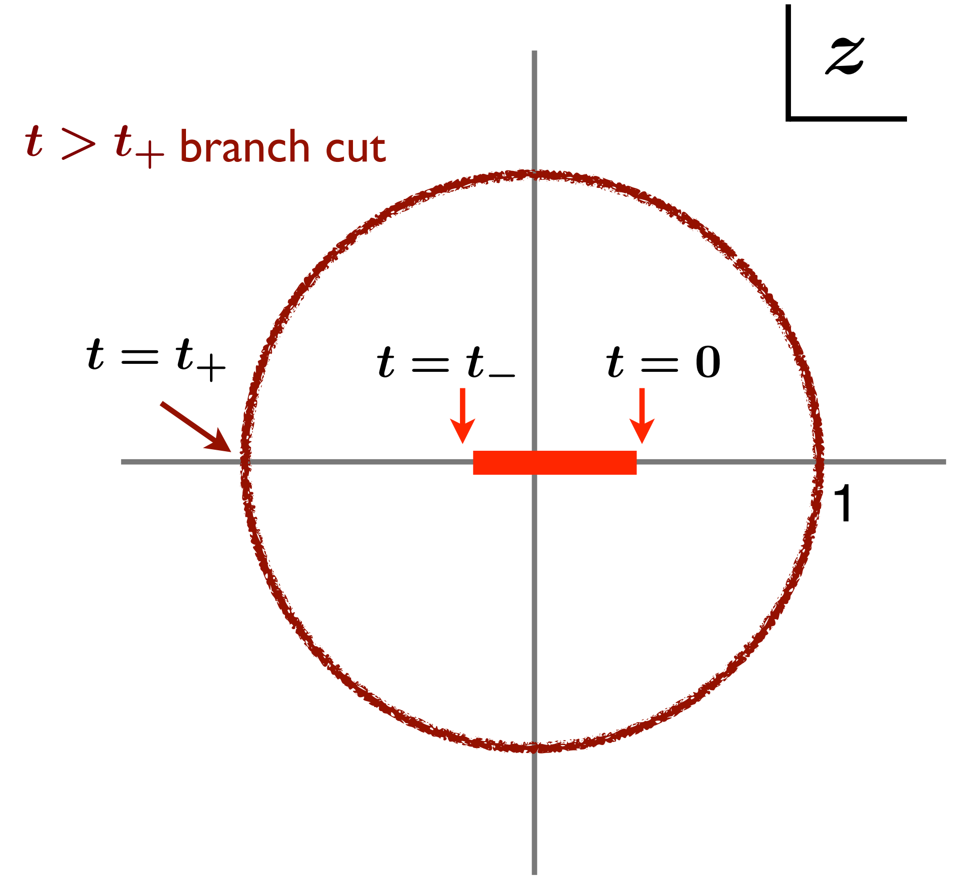
z-expansion

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = t_-$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\rm res}^2} \sum_n a_n z^n$$

BGL parametrization

$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t, t_0) \qquad Q_F(t) = \frac{1}{B_n(z)\phi_F(z)}$$

Blaschke factor

$$B_n(z) = \prod_{i=1}^{n} \frac{z - z_{P_i}}{1 - z z_{P_i}} \qquad z_{P_i} = z(M_{P_i}^2, t_-)$$

Unitarity bounds

$$S_{fF} = \sum_{k=0}^{K_f-1} [(a_k^{(f)})^2 + (a_k^{(F_1)})^2] \leq 1 \qquad S_g = \sum_{k=0}^{K_g-1} (a_k^{(g)})^2 \leq 1$$

Predictions for B_c vector & axial vector resonances

$$M_B + M_{D^*} = 7.290 \text{ GeV}$$

M_{1^-}/GeV	method	Ref.	M_{1^+}/GeV	method	Ref.
6.335(6)	lattice	[77]	6.745(14)	lattice	[77]
6.926(19)	lattice	[77]	6.75	model	[79, 80]
7.02	model	[79]	7.15	model	[79, 80]
7.28	model	[81]	7.15	model	[79, 80]

BCL parametrization

Simple form which uses less theoretical information.

$$F(t) = Q_F(t) \sum_{k=0}^{K_F-1} a_k^{(F)} z^k(t, t_0) \qquad Q_F(t) = \frac{N_F}{1 - \frac{t}{M_P^2}}$$

Using BGL as a guide, choose N_f = 300, N_{F1} = 7000, N_g = 5

Clean baseline, against which affects of theoretical input (HQET, unitarity bounds) can be measured

fit	n_B^+	n_B^-	K	I	$a_0^{(f)}$	$a_1^{(f)}$	$a_0^{(F_1)}$	$a_1^{(F_1)}$	$a_0^{(g)}$	$a_1^{(g)}$	S_{fF}	S_g
BCL	–	–	2	0.0367(15)	0.01496(19)	−0.047(27)	0.002935(37)	−0.0029(27)	0.027(13)	0.77(44)	0.0025(26)	0.60(69)
BCL	–	–	3	0.0378(17)	0.01496(19)	−0.065(40)	0.002935(37)	−0.0135(82)	0.026(13)	0.82(46)	0.08(38)	0.67(75)
BCL	–	–	4	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)
BCL	–	–	5	0.0382(18)	0.01497(19)	−0.310(42)	0.002936(37)	−0.0151(83)	0.109(16)	−0.29(37)	0.143(67)	0.10(22)

[link]