Lattice computation of $|V_{cd}/V_{cs}|$ and $|V_{td}/V_{ts}|$

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for the RBC-UKQCD Collaborations

Based on arXiv:1812.08791

Warwick

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THE UNIVERSITY *of* EDINBURGH

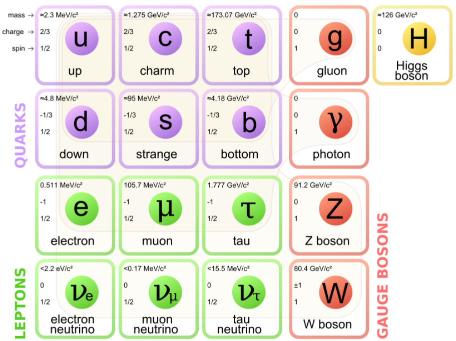
Outline

Motivation

The SM is very successful, but...

- Matter/Antimatter asymmetry?
- Why hierarchy of masses?
- Why three generations?
- What is dark matter?
- What is dark energy?
- ...

The Standard Model



Source: Wikipedia

Motivation

The SM is very successful, but...

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... **not the end of the story!**

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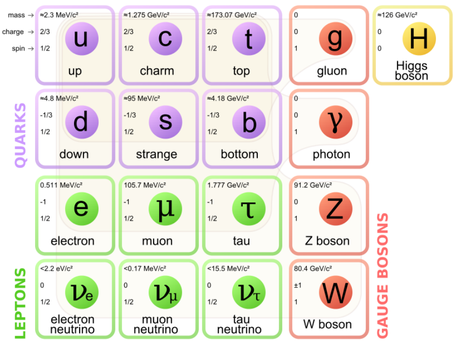

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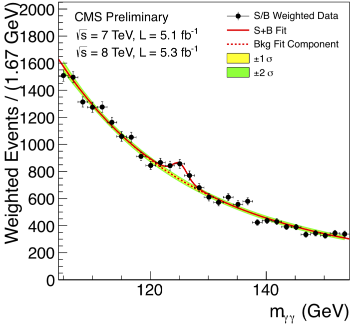
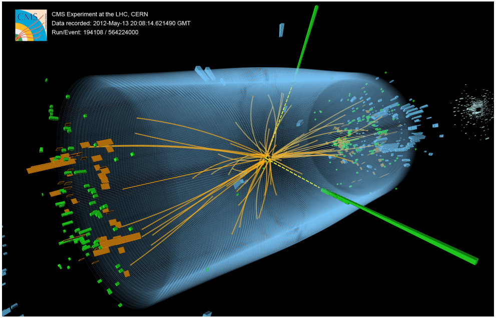
⇒ Search for New Physics!

Where to find New Physics?

1. Direct searches:
   ⇒ *Bump in the spectrum*



source:CERN

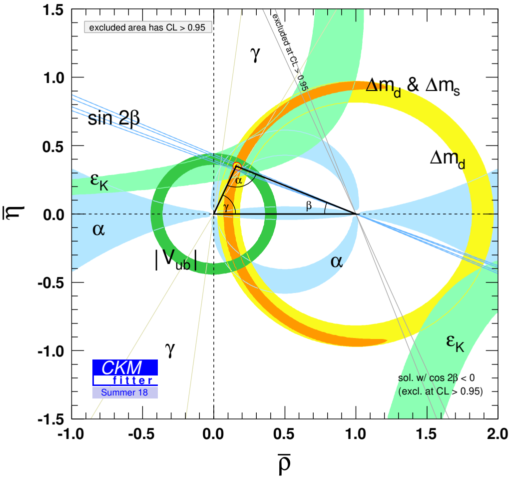


e.g. Higgs discovery in 2012

Where to find New Physics?

1. Direct searches:
   $\Rightarrow$ *Bump in the spectrum*
2. Indirect searches:
   Precision tests of SM:
   $\rightarrow$ Quantum corrections due to new particles modify SM predictions
   $\rightarrow$ NP shows as discrepancy between experiment and theory

$\Rightarrow$ **Over-constrain SM**



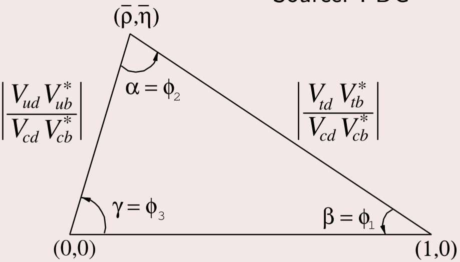
CKM

CKM Matrix

- 3 generations
- relates flavour eigenstates (d', s', b') to mass eigenstates (d, s, b)
- complex
  $\Rightarrow$ allows for $\not{CP}$
  via a single phase
- unitary
  e.g. 2nd row:
  $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 \stackrel{?}{=} 1$

Unitarity Triangle

Source: PDG



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = -1.$$

$\Rightarrow$ Test SM by determining CKM matrix elements

Why charm and bottom sector?

- Huge experimental efforts:

Belle II at SuperKEKB, Tsukuba

LHC at CERN, Geneva

First collision on 26/04/2018

and CLEO-c, BaBar, BESIII, ...

Why charm and bottom sector?

- Huge experimental efforts:
  $\Rightarrow$ LHC, Belle II, BESIII, ...
- Less explored than light quark sector

Absolute values (PDG 2018)

$$\begin{pmatrix} 0.97420(21) & 0.2243(5) & 0.00394(36) \\ 0.218(4) & 0.997(17) & 0.0422(8) \\ 0.0081(5) & 0.0394(23) & 1.019(25) \end{pmatrix}$$

Current uncertainties (PDG 2018)

$$\frac{|\delta V_{CKM}|}{|V_{CKM}|} = \begin{pmatrix} 0.02 & 0.22 & 9.1 \\ 1.8 & 1.7 & 1.9 \\ 6.2 & 5.8 & 2.5 \end{pmatrix} \%$$

Why charm and bottom sector?

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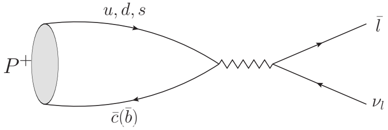
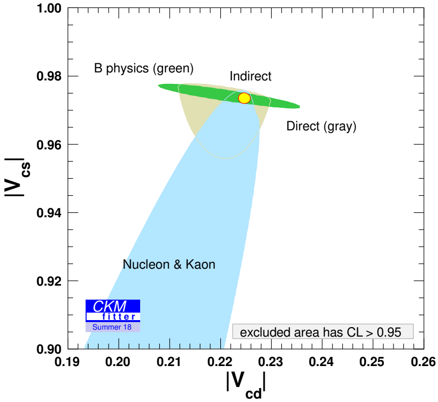
- Leptonic decays (tree):
  $\Rightarrow |V_{cd}|, |V_{cs}|, |V_{ub}|, \ldots$
- Semi-Leptonic decays (tree):
  $\Rightarrow |V_{cd}|, |V_{cs}|, |V_{cb}|, \ldots$





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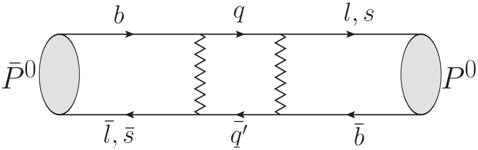

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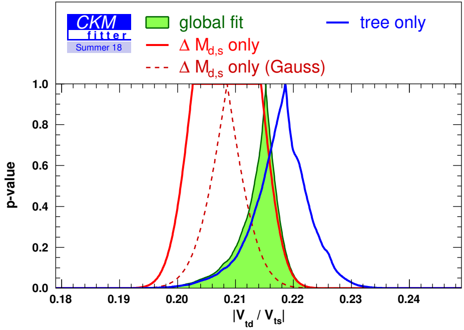
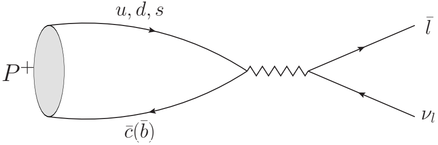
- Mixing (loop):
  $\Rightarrow |V_{td}/V_{ts}|$



Flavour Physics and CKM - leptonic decays

Experiment $\approx$ *CKM* $\times$ non-perturbative $\times$ (PT+kinematics)



Leptonic decays: $\Gamma(P \to l\nu_l) \approx |V_{q_2 q_1}|^2 \times f_P^2 \times \mathcal{K}_1$

where $\mathcal{Z}_A \langle 0 | \overline{c}\gamma_4\gamma_5 q | D_q(0)\rangle = f_{D_q} m_{D_q}, \qquad q = d, s$

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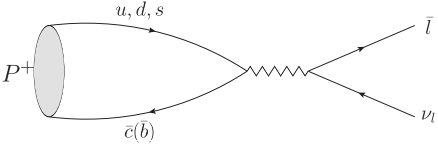

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[HFLAV] $f_D\,|V_{cd}| = (45.9 \pm 1.1)\,\mathrm{MeV}, \qquad f_{D_s}\,|V_{cs}| = (250.9 \pm 4.0)\,\mathrm{MeV}$

Computing f_{D_s}/f_D gives access to $|V_{cs}/V_{cd}|$

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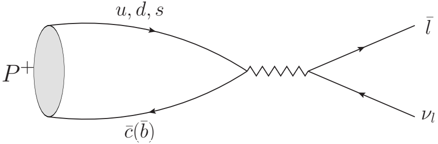

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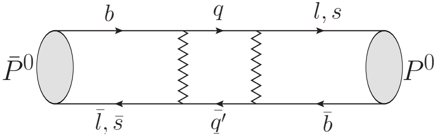
Neutral meson mixing:



$$\Delta m_P = \left| V^*_{tq_2} V_{tq_1} \right| \times f_P^2 m_P \hat{B}_P \times \frac{G_F^2 m_W^2}{6\pi^2} \mathcal{K}_2$$

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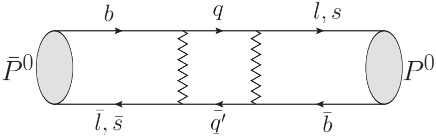

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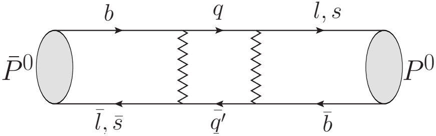
[HFLAV]

$$\Delta m_d = 0.5064 \pm 0.0019\,\mathrm{ps}^{-1}$$
$$\Delta m_s = 17.757 \pm 0.021\,\mathrm{ps}^{-1}$$

Flavour Physics and CKM - neutral meson mixing

Experiment $\approx$ *CKM* $\times$ non-perturbative $\times$ (PT+kinematics)

Neutral meson mixing:



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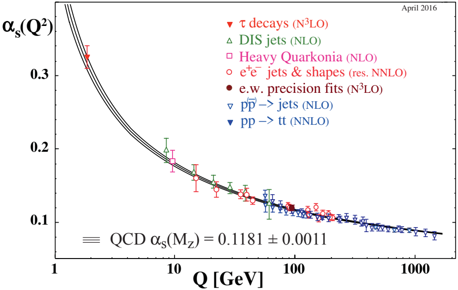
Computing ξ gives access to

$$\xi^2 = \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_B^2 \hat{B}_B} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

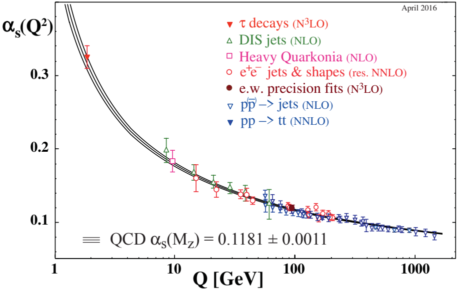
Non-Perturbative Physics



Source: PDG

- At *low energy scales* perturbative methods fail

Non-Perturbative Physics



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BG/Q in Edinburgh

$\Rightarrow$ Large scale computing facilities

- At *low energy scales* perturbative methods fail
- Lattice QCD simulations provides **first principle precision predictions** for phenomenology
- Calculations need to be improved for observables where the error is dominated by **non-perturbative physics**...

Lattice QCD methodology - the Path Integral

The path integral in Minkowski space:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}] \, \mathcal{D}[U] \, \mathcal{O}[\psi, \overline{\psi}, U] \, e^{iS[\psi, \overline{\psi}, U]}$$

Lattice QCD methodology - the Path Integral

Wick rotate ($t \to i\tau$) to Euclidean space:

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}] \, \mathcal{D}[U] \, \mathcal{O}[\psi, \overline{\psi}, U] \, e^{-S_E[\psi, \overline{\psi}, U]}$$

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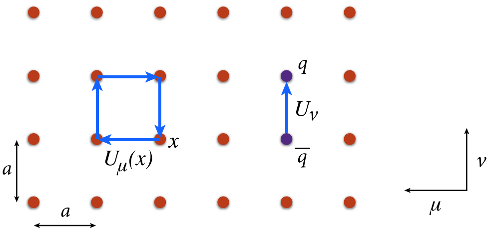
Calculate this **explicitly** by introducing *lattice*:



PDG

- Finite lattice spacing a
  $\Rightarrow$ UV regulator
- Finite Box of length L
  $\Rightarrow$ IR regulator

$\Rightarrow$ PI large **but finite** dimensional.

Parameters of QCD

$$S_{\rm QCD}[\psi,\overline{\psi},U] = S_G[U] + S_F[\psi,\overline{\psi},U]$$
$$= \int \mathrm{d}^4x \frac{1}{2g^2}\mathrm{Tr}\left[F_{\mu\nu}F_{\mu\nu}\right] + \sum_{N_f} \overline{\psi}_f\left(\gamma_\mu D_\mu + m_f\right)\psi_f$$

Coupling constant g + **quark masses** m_f $\Rightarrow$ defines QCD.

$$Z = \int \mathcal{D}[\psi,\overline{\psi}]\mathcal{D}[U] e^{-S[\psi,\overline{\psi},U]} = \int \mathcal{D}[U] \prod_{N_f} \det\left(D + m_f\right) e^{-S_G[U]}$$

Typical current simulations: $N_f = 2+1$. $\Rightarrow$ 3 parameters $(1+2)$

Multiple scale problem: back of the envelope

Control IR (Finite Size Effects) and UV (discretisation) effects

$$m_\pi L \gtrsim 4$$
$$a^{-1} \gg \text{Mass scale of interest}$$

For $m_\pi = m_\pi^{\rm phys} \sim 140\,\mathrm{MeV}$ and $\overline{m_c}(m_c) = 1.275(25)\mathrm{GeV}$ [PDG]:

$$L \gtrsim 5.6\,\mathrm{fm}$$
$$a^{-1} \sim 2.5\,\mathrm{GeV}$$

Requires $\Rightarrow L/a \gtrsim 70$
EXPENSIVE to satisfy both constraints simultaneously.

$\Rightarrow$ Need to carefully check discretisation effects

A Lattice Computation

Lattice vs Continuum

We simulate:

- at finite lattice spacing a
- in finite volume L^3
- lattice regularised
- Some bare input quark masses
  am_l, am_s, am_h
  In general: $m_\pi \neq m_\pi^{\mathrm{phys}}$

We want:

- $a = 0$
- $L = \infty$
- some continuum scheme
- $m_l = m_l^{\mathrm{phys}}$
- $m_s = m_s^{\mathrm{phys}}$
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$\Rightarrow$ Need to control all limits!

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$\Rightarrow$ Need to control all limits!

$\Rightarrow$ Decide on a fermion action:
Wilson, Staggered, Twisted Mass, **Domain Wall fermions**, $\cdots$

RBC/UKQCD $N_f = 2+1$ ensembles

	$L^3 \times T/a^4$	a^{-1}/GeV	m_π/MeV
C0	$48^3 \times 96$	**1.73**	**139**
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M0	$64^3 \times 128$	**2.36**	**139**
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1	$48^3 \times 96$	**2.77**	234

- Iwasaki gauge action
- Domain Wall Fermion action
  $\Rightarrow N_f = 2+1$ flavours in the sea
  $\Rightarrow M_5 = 1.8$ for light and strange
- **2 ensembles with physical pion masses** [PRD **93** (2016) 074505]
- 3 Lattice spacings [JHEP **12** (2017) 008]
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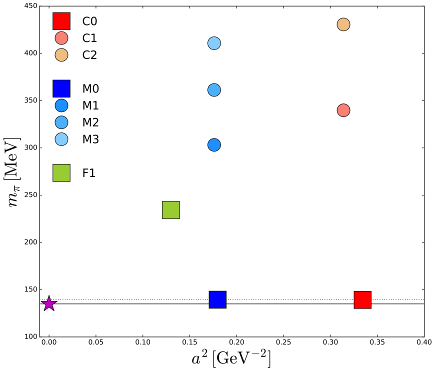

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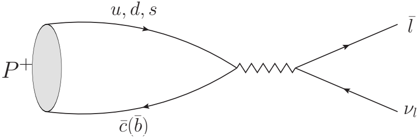
Charm: Stout smeared Möbius DW fermions with $M_5 = 1.0$, $L_s = 12$, $am_h \lesssim 0.7 \Rightarrow$ mixed action

chiral fermions $\Rightarrow$ **multiplicative renormalisation**

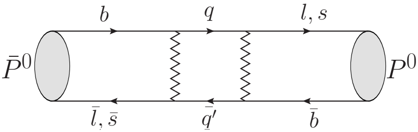
Measurement strategy

Leptonic decays



$$\mathcal{Z}_A \langle 0 | \overline{c} \gamma_4 \gamma_5 q \, | D_q(0) \rangle = f_{D_q} m_{D_q}$$

B-$\bar{B}$ mixing



$$B_P = \frac{\langle \bar{P}^0 | O_{VV+AA} | P^0 \rangle}{8/3 f_P^2 m_P^2}$$

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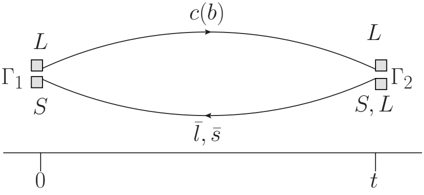

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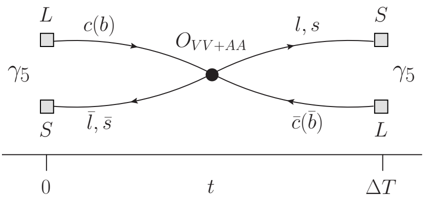
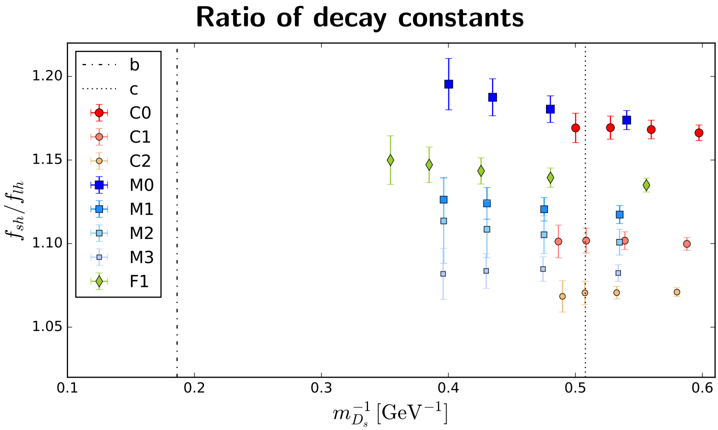


- Z_2-Wall sources on every 2nd time-slice
- Light and strange propagators Gaussian smeared sources (L and S sinks)
- Unitary light and physical strange quark masses
- Range of charm (and heavier) quark masses
- Many source-sink separations ΔT for 4-quark operator

Results of correlator fits



⇒ Renormalisation constants cancel

⇒ Mild linear behaviour with $1/m_H$ and a^2

⇒ Stat precision: 0.4 - 1.0 %

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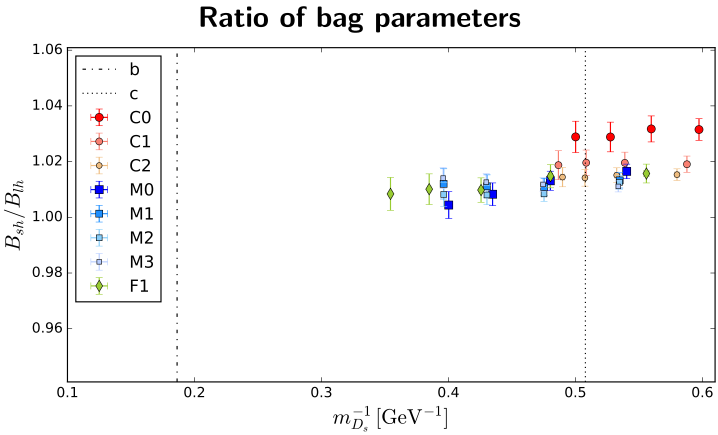

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Global fit form

Base fit

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\mathrm{phys}}, m_H^{\mathrm{phys}}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$

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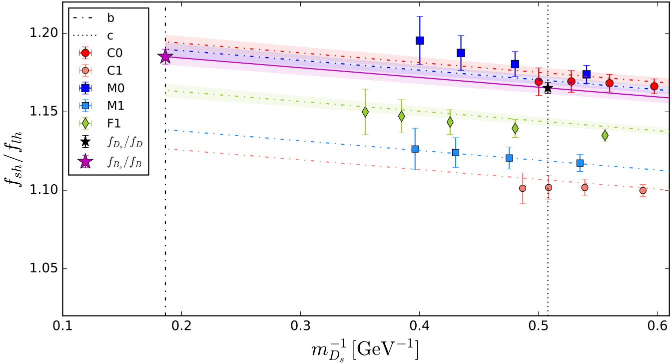
Assess systematic errors by

- varying cuts on pion mass
- using $m_H = m_D$, m_{D_s} and m_{η_c}
- varying inclusion/exclusion of heaviest data points
- varying inclusion/exclusion of fit parameters
- including/estimating higher order terms (a^4, $\left(\Delta m_\pi^2\right)^2$, $\left(\Delta m_H^{-1}\right)^2$)

$\Rightarrow$ All fits are fully correlated.

Global fit results - ratio of decay constants

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\rm phys}, m_H^{\rm phys}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

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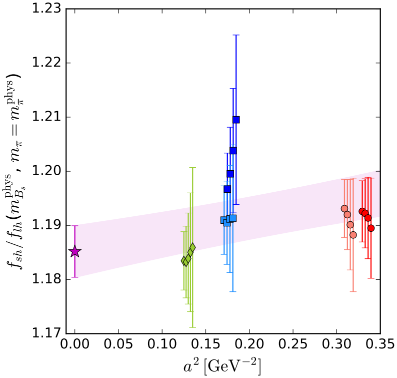

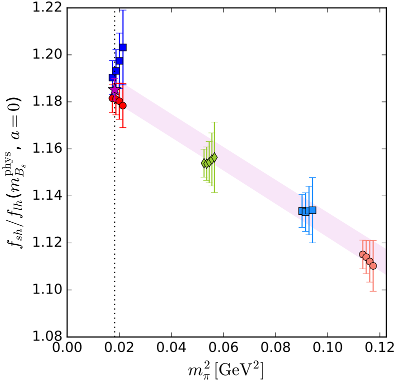

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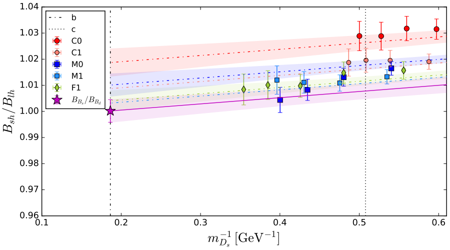
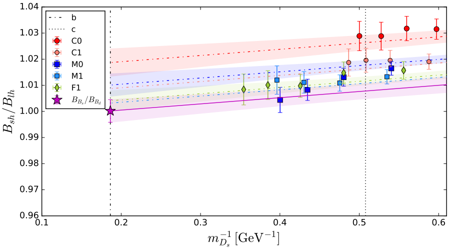
$B_{B_s}/B_B(a, m_\pi, m_H)$



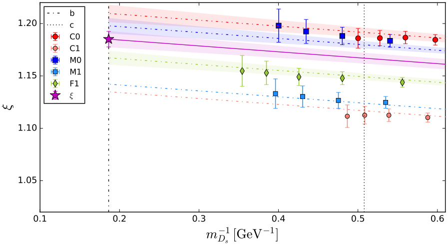
Recall: $$\xi \equiv f_{B_s}/f_B \times \sqrt{B_{B_s}/B_B}$$

1. chiral-CL of product of ratios
2. product of chiral-CL of ratios.

$B_{B_s}/B_B(a, m_\pi, m_H)$



$\xi(a, m_\pi, m_H)$

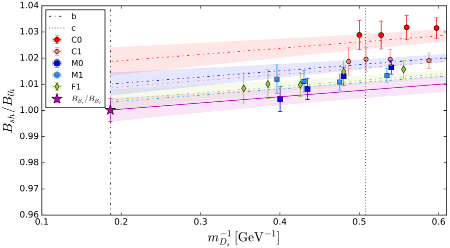


Recall: $\xi \equiv f_{B_s}/f_B \times \sqrt{B_{B_s}/B_B}$

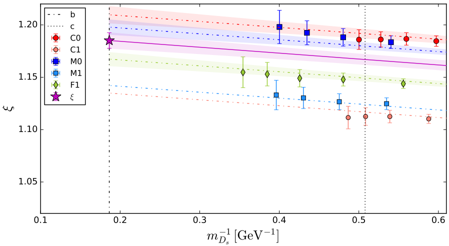
1. chiral-CL of product of ratios
2. product of chiral-CL of ratios.

Global fit results - ratio of bag parameters and ξ

$B_{B_s}/B_B(a, m_\pi, m_H)$



$\xi(a, m_\pi, m_H)$



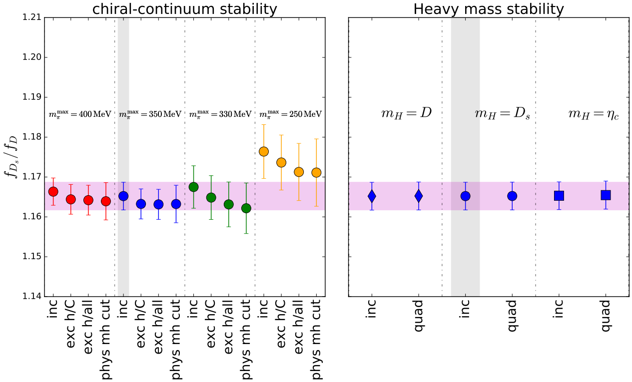
$$\lim_{a\to 0; m_q \to \mathrm{phys}} \left[f_{hs}/f_{hl} \sqrt{B_{hs}/B_{hl}} \right] (a, m_\pi, m_H) = 1.1851(74)_{\mathrm{stat}}$$

$$[f_{B_s}/f_B]_{\mathrm{phys}} \times \sqrt{[B_{B_s}/B_B]_{\mathrm{phys}}} = 1.1853(54)_{\mathrm{stat}}$$

chiral continuum limit of individual ratios gives better signal

Systematic Errors - variations of cuts to data for f_{D_s}/f_D

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{D_s}/f_D = 1.1652(35)_{\mathrm{stat}} \left({}^{+120}_{-\ 52}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for f_{B_s}/f_B

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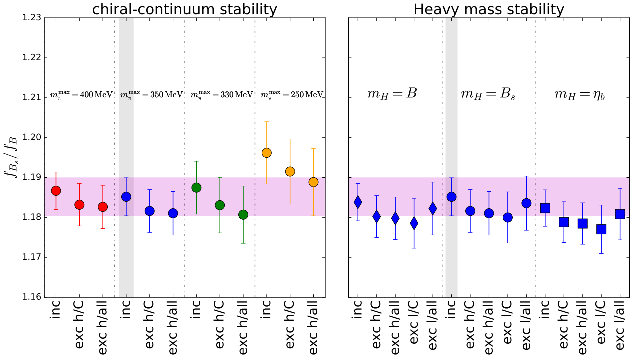

$$f_{B_s}/f_B = 1.1852(48)_{\mathrm{stat}} \left({}^{+134}_{-145}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for B_{B_s}/B_B

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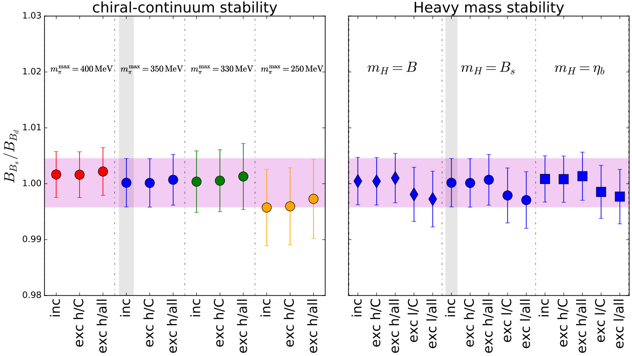
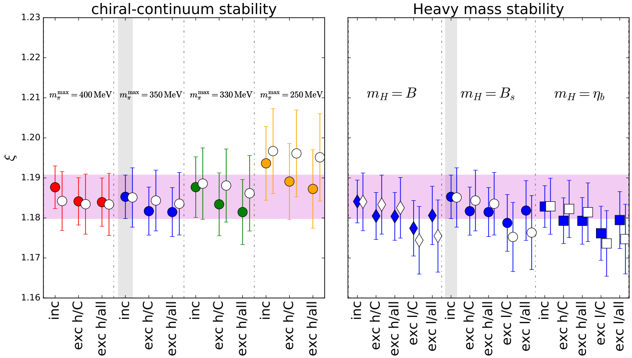


$$B_{B_s}/B_B = 1.0002(43)_{\text{stat}} \binom{+60}{-82}_{\text{sys}}$$

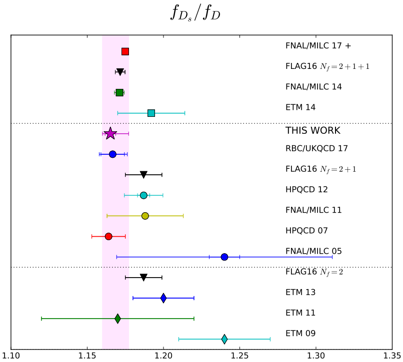
Systematic Errors - variations of cuts to data for ξ

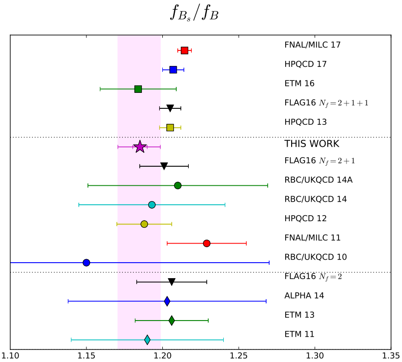
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$\xi = 1.1853(54)_{\rm stat}\, \binom{+116}{-156}_{\rm sys}$$

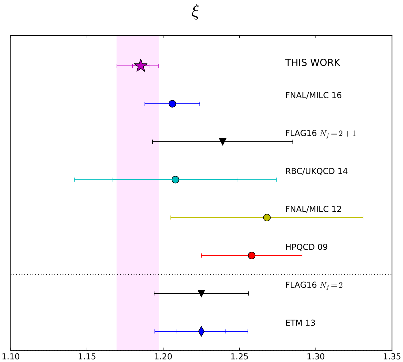
Comparison to literature - ratio of decay constants

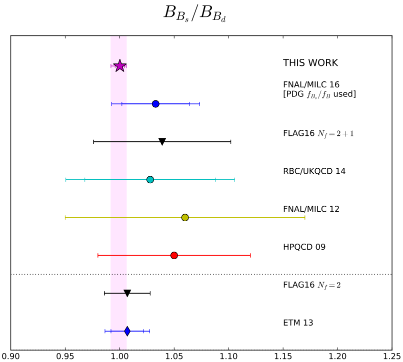




- Self consistent with RBC/UKQCD17: JHEP **12** (2017) 008
- Complimentary to (most) literature - no effective action for b.
- One of few results with physical pion masses.
- $|V_{cd}/V_{cs}| = 0.2148(56)_{\rm exp} \left({}^{+22}_{-10}\right)_{\rm lat}$

Comparison to literature





- Complimentary - no effective action needed for b
- Complimentary - **no operator mixing**!
- **First time with physical pion masses**
- $|V_{td}/V_{ts}| = 0.2018(\ 4)_{\rm exp} \left({}^{+20}_{-27}\right)_{\rm lat}$

Conclusions and Outlook

DONE

- **arXiv:1812.08791**
- f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_B and ξ
- $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$
- 3 lattice spacings, 2 m_π^{phys}
- First result for ξ and B_{B_s}/B_B with m_π^{phys}
- m_h from below m_c to $\sim m_b/2$
  $\Rightarrow$ extrapolation to b for ratios
  $\Rightarrow$ fully relativistic
- Good continuum scaling and self-consistent
- Competitive precision

Conclusions and Outlook

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Next steps

- Renormalisation of mixed action: $f_{D_{(s)}}$, $f_{B_{(s)}}$, bag parameters, BSM mixing
- 3rd lattice spacing at m_π^{phys}

Conclusions and Outlook

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Next steps

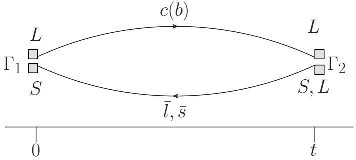
- Renormalisation of mixed action: $f_{D_{(s)}}$, $f_{B_{(s)}}$, bag parameters, BSM mixing
- 3rd lattice spacing at $m_\pi^{\rm phys}$

related projects

- $B_{(s)} \to D_{(s)}$, $B \to \pi$, $B_s \to K$
  Advanced draft
- BSM $K - \bar{K}$ mixing
  Advanced draft
- m_c and $a_\mu^{\rm LOHVP,c}$
- semi-leptonics (planned)
  $D \to \pi$, $D \to K$, $D_s \to K$

ADDITIONAL SLIDES

Correlator Fitting of two-point functions I

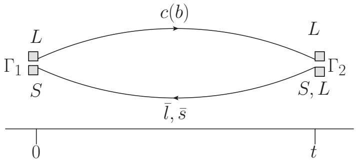


$$C_{ij}(t) = \sum_{n=0}^{\infty} (\psi_n)_i (\psi_n^*)_j e^{-E_n t}$$

with $E_n < E_{n+1}$ and $(\psi_n)_i = \frac{\langle 0|O_i|n\rangle}{\sqrt{2E_n}}$ for $O = \bar{c}_2^L \Gamma q_1^X$ where $X = S, L$.

Consider $\Gamma = \gamma_5$ (**P**seudo scalar) and $\Gamma = \gamma_4\gamma_5$ (**A**xial vector current).

Correlator Fitting of two-point functions I



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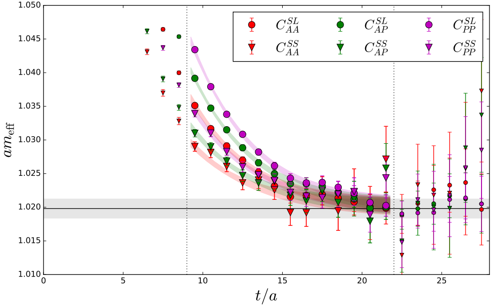
Consider $\Gamma = \gamma_5$ (**P**seudo scalar) and $\Gamma = \gamma_4\gamma_5$ (**A**xial vector current).

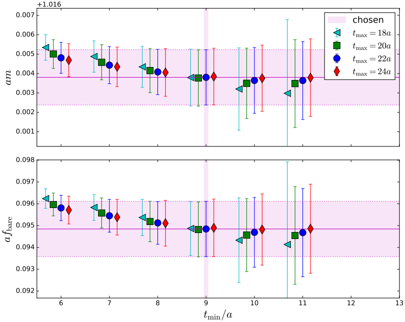
ISSUE: Exponential noise growth i.e. **signal-to-noise problem**

$\Rightarrow$ Simultaneous uncorrelated excited state fits to 6 channels:
$\langle AA\rangle^{SL}$, $\langle AP\rangle^{SL}$, $\langle PP\rangle^{SL}$, $\langle AA\rangle^{SS}$, $\langle AP\rangle^{SS}$ and $\langle PP\rangle^{SS}$

Correlator Fitting - two point functions II



Example fit (heavy-light meson with $am_h = 0.68$ on M0).

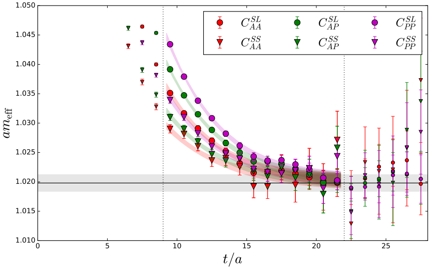


Stability

Correlator Fitting - two point functions III

$$C_{AP}^{LS}(t) \approx A_0^L P_0^S e^{-E_0 t} + A_1^L P_1^S e^{-E_1 t}$$

$$C_{AP}^{SS}(t) \approx A_0^S P_0^S e^{-E_0 t} + A_1^S P_1^S e^{-E_1 t}$$



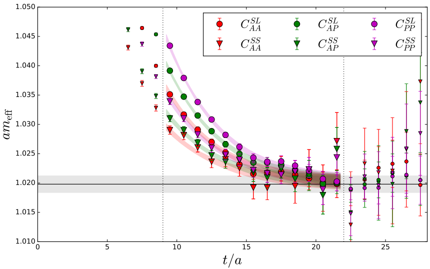
Construct Linear Combination

$$\begin{aligned} C_1^{AP}(t) &\equiv C_{AP}^{LS}(t) X^S - C_{AP}^{SS}(t) X^L \\ &\approx P_0^S \left(A_0^L X^S - A_0^S X^L \right) e^{-E_0 t} \\ &+ P_1^S \left(A_1^L X^S - A_1^S X^L \right) e^{-E_1 t} \end{aligned}$$

Correlator Fitting - two point functions III

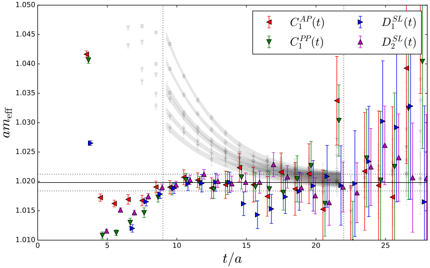
$$C_1^{AP}(t) \approx P_0^S \left(A_0^L X^S - A_0^S X^L\right) e^{-E_0 t} + P_1^S \underbrace{\left(A_1^L X^S - A_1^S X^L\right)}_{\text{small}} e^{-E_1 t}$$

Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.



Correlator Fitting - two point functions III

$$C_1^{AP}(t) \approx P_0^S \left(A_0^L X^S - A_0^S X^L\right) e^{-E_0 t} + P_1^S \underbrace{\left(A_1^L X^S - A_1^S X^L\right)}_{\text{small}} e^{-E_1 t}$$



Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.

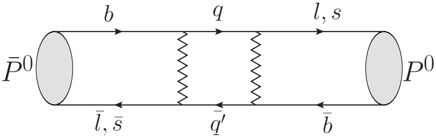
⇒ Removes (most of) excited state

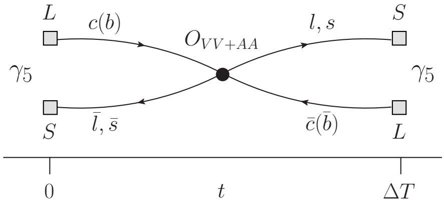
⇒ Strong *a posteriori* check of fit range

⇒ Possible to *refit* with smaller $t_{\min}$/fewer coefficients

⇒ Use this as optimised source for 3-point functions

Correlator Fitting of 4-quark operators I

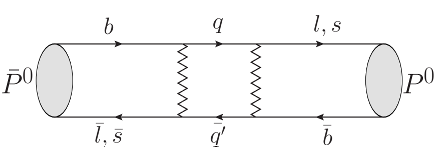


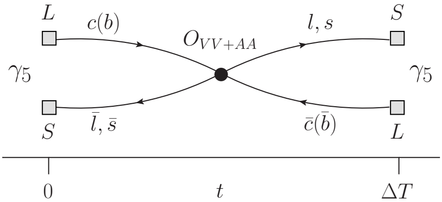


$$C_3(t, \Delta T) \equiv \left\langle P(\Delta T) O_{VV+AA}(t) \bar{P}^\dagger(0) \right\rangle$$

$$R(t, \Delta T) = \frac{C_3(t, \Delta T)}{8/3 C_{PA}(\Delta T - t) C_{AP}(t)} \to B_P \quad \text{for} \quad t, \Delta T \gg 0$$

Correlator Fitting of 4-quark operators I





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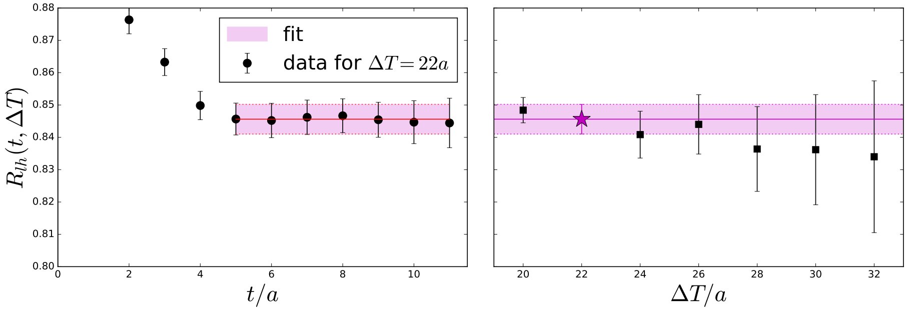
RBC/UKQCD's $K - \bar{K}$ BSM mixing calculation

Peter Boyle, Nicolas Garron, Jamie Hudspith, Andreas Jüttner, Julia Kettle, Ava Khamseh, Christoph Lehner, Amarjit Soni, JTT

[1710.09176, 1812.04981, in preparation]

Ex: $am_h = 0.68$ on M0



Correlator Fitting of 4-quark operators II

Ex: $am_h = 0.68$ on M0